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ANNUAL REPORT
COMPUTER SYSTEMS RESEARCH

SEPTEMBER 1, 1969 to AUGUST 31, 1970

PART A: PROFESSOR GEAR; PART B: PROFESSOR POPPELBAUM

MASTER

In Part A this year's accomplishments include:

1. The preparation of support programs for generalized modeling including programs for the PDP-8.
2. Further work was done on stiff differential equations and led to a number of highly useful subroutines.
3. Further work on test matrices was performed with special attention to singular matrices.
4. The design of a display processing unit for a multi-terminal network was successfully completed.
5. The responsibility for the time-sharing system was handed over to the Service Computation Group.

In Part B the highlights of the year's achievements are:

1. Completely new system design for the On-Line Fourier Transform (OLFT).
2. The near completion of the linear image transformer, TRANSFORMATRIX.
3. The successful demonstration of stereo television without polarizing filters using our BLAST tube.
4. The complete circuit design for STEREOMATRIX, the 3-D display system for computer output.
5. The complete circuit and systems design for EIDOLYZER, the pattern recognition machine for colored landscapes.

Details of the work on Part A and Part B will be found in the subsequent pages.

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ANNUAL REPORT PART A
SOFTWARE SYSTEMS RESEARCH
SEPTEMBER 1, 1969 to AUGUST 31, 1970

Summary

The program for the last contract period covered the following items:

- A-1. Generalized Modeling
- A-2. Numerical Algorithm Development
- A-3. Test Matrices
- A-4. Hardware
- A-5. Timesharing

The work in each of these sections is described below.

1. Generalized Modeling

A major part of our effort was devoted to preparing support programs for the system, planning details of the system and investigating suitable numerical techniques. As a result of the latter work, a number of simplifications have been introduced. In particular, the symbolic differentiation and analysis to partition the equations into different types (algebraic and differential) has been shown to be unnecessary. It has also been discovered how to avoid multiple inversions of sparse matrices (which might have led to loss of sparseness). This work is mentioned in more detail under section 2.

A preliminary version of the proposed system has been designed (see Gear, C. W., et.al. THE SIMULATION AND MODELING SYSTEM--A Snapshot View, DCS File No. 824). It has been partially implemented. A summary flowchart for the system as it currently stands is shown in Figure 1. The various components are discussed briefly below and the status of their implementation is given. Detailed descriptions are available in referenced reports and quarterly reports.

Figure 1
Flowchart for Preliminary Generalized Simulation System

Graphic Network Input and PDP-8 Support Programs. Software for the PDP-8 to support the graphical input of network definitions includes:

- a. Multi-terminal and communications support monitor (GLASP) (see Michel, M. GRAPHICAL REMOTE-ACCESS SIMULATION SYSTEM (GRASS) The Communication and Monitor Components (GASP/GLASP), DCS Report No. 346, DCS File No. 824, and quarterly reports).

This package, to multiplex the interactive graphics processing of several consoles and to provide high-speed communication between the PDP-8 and 360/75 CPU's, has been completed.

- b. Generalized drawing facility (see DCS File No. 824, and quarterly reports). This graphics input package, running under GLASP, is the principal software that will run in the PDP-8. With this group of programs, a user interactively creates a network definition using previously defined elements, plus equations, parameters, line drawings, and comment text. It is 80% coded, and most of that code is debugged.

To aid in the preparation of the above, an Editor-Assembler-Lister-Utilities package was created for the PDP-8. It is in constant use as the principal system for PDP-8 program generation, allowing large programs to be written and debugged on the PDP-8 in a stand-alone configuration with its disk. This avoids relying on the 360 for assemblies or using the very slow DEC tapes and DECTAPE operating system. (see DCS File No. 824 and quarterly reports)

Storage and Retrieval. The local library system has been designed. It will store and retrieve data both locally (on the PDP-8 disk) and remotely (in the 360 via the 2701 channel). The data structure used on the PDP-8 disk is designed for rapid access, in that disk latency delays have been reduced considerably. The 360 will store both the original data structure as transmitted from the PDP-8 and the transformed output from item analysis.

Item Analysis. This program will convert the data structure representation of the network elements as graphic elements into a form suitable for rapid analysis when an instance of an element is used as a part of a larger network. It is currently being coded.

Global Analysis. This program analyzes the network to be simulated in order to generate all of the equations. The global analysis has been divided into two parts: information retrieval and equation generation. The item analyses will construct a filing system in which all references will be symbolic. The information retrieval portion of global analysis will extract information and resolve symbolic references pertinent only to the network on which analysis is requested. This modified information is stored in memory for use by the second part of global analysis, the equation generation. A detailed discussion of the equation generation can be found in Chapter 5 and the Appendix of DCS File 824. This second part written as two recursive subroutines, one to analyze elements and one to analyze networks, is nearly debugged.

Elimination. This processes the equations in order to remove trivial identities and to classify the variables according to their usage. It is designed.

Compiler 1. This generates three machine language subroutines which will evaluate the residuals of the equations, compute those variables dependent on input parameters only, and compute those variables dependent explicitly on time. It is coded.

Sparse Structure Detector. This generates a data structure representing the nonzero elements of the Jacobians of the equations. It has been coded and is being debugged.

Sparse Pivot Selector. This determines a pivot strategy for inverting the Jacobian and manipulates the data structure into a factored LU form of the inverse. It is complete.

Compiler 2. This is provided information by the Pivot Selector from which it can generate code to form the numerical elements of the factored LU form of the inverse and premultiply a vector by that inverse. It is being coded.

Transient Analysis. This performs a simultaneous solution of ordinary differential and algebraic equations. It has been completed and tested by other groups. (see section 2)

2. Numerical Algorithm Development and Testing

Some of this work was performed in conjunction with the simulation system. A compact subroutine was prepared from the earlier work on stiff equations. This has been tested and submitted to the algorithms section of the CACM (see Gear, C. W. Algorithm for the Solution of Ordinary Differential Equations by Adams or Stiff Methods with Automatic Step and Order Control). A paper describing its operation has been submitted simultaneously (see Gear, C. W. Automatic Integration of Ordinary Differential Equations). Further work has resulted in the discovery of stiffly stable methods of order up to eleven (see Jain, M. K. and Srivastava, V. K. High Order Stiffly Stable Methods for Ordinary Differential Equations). The previous highest order known was eight, found by Dill in his Master's thesis last year.

An effort was made to find numerical methods that were suitable for simultaneously solving equations of the form

$$\begin{aligned} \underline{y}' &= \underline{f}(\underline{y}, \underline{u}, t) & \underline{y}(0) &= \underline{y}_0 \\ 0 &= \underline{g}(\underline{y}, \underline{u}, t) \end{aligned}$$

at the same time that we were investigating symbolic techniques for manipulating the equations that arose in the simulation system into this

form. It was found that the stiff methods could be extended to handle equations of the form

$$F(y, y', t) = 0$$

directly, regardless of whether the derivatives were present or not. A new subroutine has been written which handles these equations and will be the principal numerical package for the transient analysis package. (It may also provide the DC analysis--see next year's proposal.) This subroutine has been incorporated by TRW into the SCEPTRE network analysis program. It is showing a fifty-fold improvement in speed on the basis of some tests (see internal TRW Report #5324.30-67, The TESS Transient Study, by Virgil Stokes (May 1970)). A paper describing the technique has been prepared and submitted to the IEEE publications (see Gear, C. W. Simultaneous Solution of Differential Algebraic Equations).

3. Test Matrices

The following material represents the results of our work this year in collecting information on test matrices. It is self-explanatory except for the definition of condition numbers. The notation used here is defined as follows:

$$C(j) = ||A||_j ||A^{-1}||_j$$

where

$$||A||_1 = \max_j \{ \sum_i |a_{ij}| \}$$

$$||A||_2 = \{ \text{maximum eigenvalue of } A^*A \}^{1/2}$$

$$||A||_E = \{ \sum_i \sum_j |a_{ij}|^2 \}^{1/2}$$

For singular matrices we have used

$$C(P) = |\lambda_{\max}| / |\lambda_{\min}| \quad (|\lambda_{\min}| \neq 0)$$

NOTE: $|\lambda_{\min}|$ is the modulus of the smallest nonzero eigenvalue.

The identification number code is to be interpreted
as follows:

x_1 x_2 x_3 x_4 x_5 x_6

$x_1 = \begin{cases} 0 & \text{real matrix} \\ 1 & \text{complex matrix} \\ 2 & \text{either} \end{cases}$

$x_2 = \begin{cases} 0 & \text{Hermetian} \\ 1 & \text{non-Hermetian} \\ 2 & \text{either} \end{cases}$

$x_3 = \begin{cases} 0 & \text{singular} \\ 1 & \text{non-singular} \\ 2 & \text{either} \end{cases}$

x_4 x_5 x_6 = sequence number

x_1 x_2 x_3 = 999 designates the definition of a wide class
of test matrices, or statements of useful
properties for generating test matrices.

IDENTIFICATION: 001001

MATRIX: $A(I,J)=2$ IF $I=J$,
 $A(I,J)=-1$ IF $ABS(I-J)=1$,
 $A(I,J)=0$ OTHERWISE.

EXAMPLE (N=5)

```

      2 -1  0  0  0
     -1  2 -1  0  0
A =   0 -1  2 -1  0
      0  0 -1  2 -1
      0  0  0 -1  2
  
```

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR,
 POSITIVE DEFINITE, TRIDIAGONAL.

INVERSE: $AINV(I,J)= I*(N+1-J)/(N+1)$ IF $I \leq J$,
 $AINV(I,J)= AINV(J,I)$.

EXAMPLE (N=5)

```

      5  4  3  2  1
      4  8  6  4  2
AINV=(1/6)* 3  6  9  6  3
      2  4  6  8  4
      1  2  3  4  5
  
```

EIGENVALUES: $EIVAL(I)= 2*(1-COS(I*PI/(N+1)))$,
 FOR $I=1,2,...,N$.

EXAMPLE (N=5)

```

EIVAL(1)= 0.26794 91924 31122 70650
EIVAL(2)= 1.00000 00000 00000 00000
EIVAL(3)= 2.00000 00000 00000 00000
EIVAL(4)= 3.00000 00000 00000 00000
EIVAL(5)= 3.73205 08075 68877 29353
  
```

EIGENVECTORS: $EIVEC(I,J)= SIN(I*J*PI/(N+1))$

EXAMPLE (N=5)

```

EIVEC(1,1)= 0.50000 00000 00000 00000
EIVEC(2,1)= 0.86602 54037 84438 64675
EIVEC(3,1)= 1.00000 00000 00000 00000
EIVEC(4,1)= 0.86602 54037 84438 64675
EIVEC(5,1)= 0.50000 00000 00000 00000
EIVEC(1,2)= 0.86602 54037 84438 64675
EIVEC(2,2)= 0.86602 54037 84438 64675
EIVEC(3,2)= 0.00000 00000 00000 00000
EIVEC(4,2)=-0.86602 54037 84438 64675
EIVEC(5,2)=-0.86602 54037 84438 64675
EIVEC(1,3)= 1.00000 00000 00000 00000
EIVEC(2,3)= 0.00000 00000 00000 00000
EIVEC(3,3)=-1.00000 00000 00000 00000
EIVEC(4,3)= 0.00000 00000 00000 00000
EIVEC(5,3)= 1.00000 00000 00000 00000
EIVEC(1,4)= 0.86602 54037 84438 64675
EIVEC(2,4)=-0.86602 54037 84438 64675
EIVEC(3,4)= 0.00000 00000 00000 00000
EIVEC(4,4)= 0.86602 54037 84438 64675
EIVEC(5,4)=-0.86602 54037 84438 64675
EIVEC(1,5)= 0.50000 00000 00000 00000
EIVEC(2,5)=-0.86602 54037 84438 64675
  
```

```
EIVEC(3,5)= 1.00000 00000 00000 00000
EIVEC(4,5)=-0.86602 54037 84438 64675
EIVEC(5,5)= 0.50000 00000 00000 00000
```

DETERMINANT: DET(A) = N+1

CONDITION NUMBERS:

```
C(1)= ((N+1)**2-1)/2 FOR N EVEN,
      = ((N+1)**2)/2 FOR N ODD,
C(2)= (1-COS( N*PI/(N+1)))/(1-COS(PI/(N+1))),
C(E)= SQRT((6*N-2)*(2*N**6+12*N**5+35*N**4+60*N**3
      +53*N**2+18*N)/180)/(N+1).
```

EXAMPLES

N	C(1)	C(2)	C(E)
5	1.80(1)	1.39(1)	2.07(1)
20	2.20(2)	1.78(2)	5.06(2)
40	8.40(2)	6.81(2)	2.74(3)
60	1.86(3)	1.51(3)	7.42(3)
80	3.28(3)	2.66(3)	1.51(4)
100	5.10(3)	4.13(3)	2.63(4)

REMARKS: MATRICES OF THIS TYPE ARISE IN THE DISCRETIZATION OF THE HEAT EQUATION (REF. 1). THIS MATRIX IS A SPECIAL CASE OF A WIDE CLASS OF MATRICES DESCRIBED BY GEAR (REF. 4). WHEN N+1 IS A POWER OF 2 THE INVERSE CAN BE EXACTLY REPRESENTED IN A BINARY MACHINE PROVIDED N.LT.2**L WHERE L IS THE NUMBER OF BITS IN THE MANTISSA (EXCLUDING SIGN). NOTICE THAT MAXIMUM ELEMENT OF THE INVERSE IS LESS THAN N. ALL OF THE EIGENVALUES ARE CONTAINED IN THE INTERVAL (0, 4). AS N INCREASES THE SMALLEST EIGENVALUE APPROACHES 0 AND THE LARGEST EIGENVALUE APPROACHES 4. THIS MATRIX HAS BEEN USED FREQUENTLY IN TESTING (REF. 2,3). WHEN LU DECOMPOSITION OR GAUSS ELIMINATION IS PERFORMED ON THIS MATRIX NO ROW EXCHANGES ARE MADE, SINCE THE COMPUTED PIVOTS ARE ALWAYS MAXIMUM. THEREFORE, USE OF A AS A TEST MATRIX IN A LINEAR EQUATION ROUTINE WHICH USES PARTIAL PIVOTING FOR SIZE WILL NOT TEST THE ROW EXCHANGE PORTION OF THE ROUTINE. WHEN LU DECOMPOSITION OR GAUSS ELIMINATION IS PERFORMED ON THE MATRIX AINV IT IS ALSO TRUE THAT NO ROW EXCHANGES ARE MADE. NOTICE THAT THE MINOR OF THE (1,1) ELEMENT OF THE MATRIX OBTAINED BY PERFORMING THE FIRST STEP OF GAUSS ELIMINATION ON THE MATRIX (N+1)AINV(ORDER N) IS (N+1)AINV(ORDER N-1) AND THAT AINV(1,1).GE.AINV(I,1) FOR ANY I AND N.

REFERENCES:

1. E.U. CONDON AND HUGH ODISHAW, EDS., HANDBOOK OF PHYSICS, PART 1, P123, MCGRAW-HILL, YR1967.
2. M.H. LIETZKE, R.W. STOUGHTON AND MARJORIE P. LIETZKE, A COMPARISON OF SEVERAL METHODS FOR INVERTING LARGE SYMMETRIC POSITIVE DEFINITE

- MATRICES, MATH COMP, V18, PP449-456, YR1964.
3. THOMAS C. DOYLE, INVERSION OF SYMMETRIC COEFFICIENT MATRIX OF POSITIVE-DEFINITE QUADRATIC FORM, MTAC, V11, PP55-58, YR1957.
 4. C. W. GEAR, A SIMPLE SET OF TEST MATRICES FOR EIGENVALUE PROGRAMS, MATH COMP, V23, PP119-125, YR1969.

IDENTIFICATION: 001002

MATRIX: $A(I,J)=1$ IF $ABS(I-J)=1$,
 $A(N,N)=-1$,
 $A(I,J)=0$ OTHERWISE.

EXAMPLE (N=5)

A=

0	1	0	0	0
1	0	1	0	0
0	1	0	1	0
0	0	1	0	1
0	0	0	1	-1

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR,
 TRIDIAGONAL.

INVERSE: N=EVEN; $AINV(I,J)=(-1)**(INT(I/2)+INT((J-1)/2))$ IF I IS ODD
 & I.LE.J,
 $AINV(I,J)=0$ IF I IS EVEN & I.LE.J,
 $AINV(I,J)=AINV(J,I)$.
 N=ODD; $AINV(I,J)=(-1)**(INT(I/2)+INT(J/2)+1)$
 IF I IS ODD & I.LE.J,
 $AINV(I,J)=0$ IF I IS EVEN & I.LE.J,
 $AINV(I,J)=AINV(J,I)$.

EXAMPLE (N=5)

AINV=

-1	1	1	-1	-1
1	0	0	0	0
1	0	-1	1	1
-1	0	1	0	0
-1	0	1	0	-1

EIGENVALUES: $EIVAL(I)=2*\cos(2*I*PI/(2*N+1))$
 FOR I=1,2,...,N.

EXAMPLE (N=5)

EIVAL(1)= 1.68250 70656 62362 33772
 EIVAL(2)= 0.83083 00260 03772 85106
 EIVAL(3)= -0.28462 96765 46570 28089
 EIVAL(4)= -1.30972 14678 90570 12811
 EIVAL(5)= -1.91898 59472 28994 77978

EIGENVECTORS: $EIVC(I,J)=\sin(2*I*J*PI/(2*N+1))$
 FOR I=1,2,...,N.

EXAMPLE (N=5)

EIVC(1,1)= 0.54064 08174 55597 58211
 EIVC(2,1)= 0.90963 19953 54518 37141
 EIVC(3,1)= 0.98982 14418 80932 73238

```

EIVEC(4,1)= 0.75574 95743 54258 28377
EIVEC(5,1)= 0.28173 25568 41429 69771
EIVEC(1,2)= 0.90963 19953 54518 37141
EIVEC(2,2)= 0.75574 95743 54258 28377
EIVEC(3,2)=-0.28173 25568 41429 69771
EIVEC(4,2)=-0.98982 14418 80932 73238
EIVEC(5,2)=-0.54064 08174 55597 58211
EIVEC(1,3)= 0.98982 14418 80932 73238
EIVEC(2,3)=-0.28173 25568 41429 69771
EIVEC(3,3)=-0.90963 19953 54518 37141
EIVEC(4,3)= 0.54064 08174 55597 58211
EIVEC(5,3)= 0.75574 95743 54258 28377
EIVEC(1,4)= 0.75574 95743 54258 28377
EIVEC(2,4)=-0.98982 14418 80932 73238
EIVEC(3,4)= 0.54064 08174 55597 58211
EIVEC(4,4)= 0.28173 25568 41429 69771
EIVEC(5,4)=-0.90963 19953 54518 37141
EIVEC(1,5)= 0.28173 25568 41429 69771
EIVEC(2,5)=-0.54064 08174 55597 58211
EIVEC(3,5)= 0.75574 95743 54258 28377
EIVEC(4,5)=-0.90963 19953 54518 37141
EIVEC(5,5)= 0.98982 14418 80932 73238

```

DETERMINANT: DET(A)=(-1)**(1+INT((N-1)/2))

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
5	1.00(1)	6.74	1.16(1)
20	4.00(1)	2.60(1)	9.05(1)
40	8.00(1)	5.15(1)	2.55(2)
60	1.20(2)	7.70(1)	4.67(2)
80	1.60(2)	1.02(2)	7.18(2)
100	2.00(3)	1.28(2)	1.00(3)

REMARKS: THE INVERSE OF THIS MATRIX HAS BEEN USED BY FRANK (REF. 1) FOR TESTING EIGENVALUE ALGORITHMS.

ALL OF THE ELEMENTS OF THIS MATRIX AND ITS INVERSE ARE 1, -1 OR 0. LU DECOMPOSITION AND GAUSS ELIMINATION SHOULD PRODUCE EXACT RESULTS. BACK SUBSTITUTION WILL BE EXACT FOR RIGHT SIDES WHICH ARE INTEGERS.

ALL OF THE ELEMENTS IN THE LU DECOMPOSITION ARE 1, -1 OR 0 AND AT EVERY INTERMEDIATE STAGE OF THE DECOMPOSITION THE ELEMENTS REMAIN EQUAL TO 1, -1 OR 0. THIS IS TRUE FOR THE DECOMPOSITION OF A AND FOR THE DECOMPOSITION OF AINV.

WHEN LU DECOMPOSITION OR GAUSS ELIMINATION IS PERFORMED ON THE MATRIX AINV, THE MINOR OF THE (1,1) ELEMENT OF THE MATRIX OBTAINED BY PERFORMING THE FIRST STEP OF GAUSS ELIMINATION ON THE MATRIX AINV (ORDER N) IS AINV(ORDER N-1).

THE FOLLOWING PAIRS OF ROWS MUST BE EXCHANGED IN THE COURSE OF GAUSS ELIMINATION OF A: (1,2), (3,4), ... AND IN GENERAL (2*I-1,2*I).

NO ROW EXCHANGES ARE REQUIRED IN THE COURSE OF GAUSS ELIMINATION OF AINV WHEN PARTIAL PIVOTING FOR SIZE

IS USED.

ALL OF THE EIGENVALUES ARE LOCATED IN THE INTERVAL $(-2, 2)$. AS N INCREASES, THE MINIMUM EIGENVALUE APPROACHES -2 , AND THE MAXIMUM EIGENVALUE APPROACHES 2 .

REFERENCES:

1. WERNER L. FRANK, COMPUTING EIGENVALUES OF COMPLEX MATRICES BY DETERMINANT EVALUATION AND BY METHODS OF DANILEWSKI AND WIELANDT, JSIAM, V6, PP378-392, YR1958.
2. C. W. GEAR, A SIMPLE SET OF TEST MATRICES FOR EIGENVALUE PROGRAMS, MATH COMP, V23, PP119-125, YR1969.

IDENTIFICATION: 001003

MATRIX: $A(I,1)=A(1,I)=1$

$A(I,J)=A(I-1,J)+A(I,J-1)$ OTHERWISE.

THE ARRAY IS A SQUARE SEGMENT OF A PASCAL TRIANGLE WITH THE APEX IN THE UPPER LEFT CORNER.

EXAMPLE (N=5)

```

      1  1  1  1  1
      1  2  3  4  5
A=    1  3  6 10 15
      1  4 10 20 35
      1  5 15 35 70

```

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR, POSITIVE DEFINITE.

INVERSE: THE FOLLOWING FORTRAN STATEMENTS WILL PRODUCE THE ELEMENTS $AINV(I,J)$ OF THE INVERSE OF THE MATRIX A.

```

      DO 40 II=1,N
      I=N-II+1
      AINV(I,I)=1.0
      IF(I.EQ.N) GO TO 20
      DO 10 J=2,II
10    AINV(I,I)=AINV(I,I)-A(I,J)*AINV(I+J-1,I)
      IF(I.EQ.1) GO TO 50
20    DO 40 K=2,I
      AINV(I-K+1,I)=0
      NIK=N-I+K
      DO 30 J=2,NIK
30    AINV(I-K+1,I)=AINV(I-K+1,I)
      1      -A(I-1,J)*AINV(I-K+J,I)
40    AINV(I,I-K+1)=AINV(I-K+1,I)
50    CONTINUE

```

EXAMPLE (N=5)

```

      5 -10  10  -5   1
     -10  30 -35  19  -4

```

AINV= 10 -35 46 -27 6
 -5 19 -27 17 -4
 1 -4 6 -4 1

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: DET(A)= 1

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
5	1.56(4)	8.52(3)	8.55(3)
6	2.05(5)	1.11(5)	1.11(5)
7	2.87(6)	1.49(6)	1.50(6)
8	3.96(7)	2.06(7)	2.07(7)
9	5.72(8)	2.91(8)	2.91(8)
10	8.13(9)	4.16(9)	4.16(9)

REMARKS: THE ELEMENTS OF A AND AINV ARE INTEGERS,
 HENCE BOTH CAN BE EXACTLY REPRESENTED.
 THE CONDITION NUMBERS GIVEN ABOVE INDICATE THAT THIS
 MATRIX IS ILL-CONDITIONED.
 THE ELEMENTS OF AINV MAY ALSO BE COMPUTED BY
 FORMULAS GIVEN BY STOCKMAL (REF. 2).
 THOUGH WE HAVE NOT PROVED IT, IT SEEMS CERTAIN THAT
 THE EIGENVALUES OF THIS MATRIX APPEAR IN RECIPROCAL
 PAIRS. THUS IF EIVAL IS AN EIGENVALUE, THEN SO IS
 1/EIVAL AND IF N IS ODD, THEN 1 IS AN EIGENVALUE.

REFERENCES:

1. JOHN CAFFREY, ANOTHER TEST MATRIX FOR DETERMINANTS AND INVERSES, CACM, V6, P310, YR1963.
2. FRANK J. STOCKMAL, THE INVERSE OF A TEST MATRIX, CACM, V6, P615, YR1963.

IDENTIFICATION: 001004

MATRIX: FOR $N=P-1$ WHERE P IS AN ODD PRIME

$QR(K,P)=0$ IF P DIVIDES K,
 $QR(K,P)=1$ IF K IS CONGRUENT TO A SQUARE
 INTEGER MODULO P,
 $QR(K,P)=-1$ OTHERWISE.

(THIS IS THE LEGENDRE-JACOBI QUADRATIC RECIPROCITY
 SYMBOL) THEN

$A(I,J)= QR(I+J,P).$

EXAMPLE (N=6)

	1	-1	1	-1	-1	0
	-1	1	-1	-1	0	1
A=	1	-1	-1	0	1	1
	-1	-1	0	1	1	-1
	-1	0	1	1	-1	1
	0	1	1	-1	1	-1

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR.

INVERSE: $A_{INV}(I,J) = (QR(I+J,P) - QR(I,P) - QR(J,P))/P$
 EXAMPLE (N=6)

		-1	-3	1	-3	-1	0
		-3	-1	-1	-3	0	1
A _{INV} =	1/7*	1	-1	1	0	3	3
		-3	-3	0	-1	1	-1
		-1	0	3	1	1	3
		0	1	3	-1	3	1

EIGENVALUES: $EIVAL(I) = 1, -1, \sqrt{P}, -\sqrt{P}$,
 THE LAST TWO EACH HAVING MULTIPLICITY $(N-2)/2$.

EXAMPLE (N=6)

$EIVAL(1) = 1$
 $EIVAL(2) = -1$
 $EIVAL(3) = \sqrt{7}$
 $EIVAL(4) = \sqrt{7}$
 $EIVAL(5) = -\sqrt{7}$
 $EIVAL(6) = -\sqrt{7}$

EIGENVECTORS: NOT KNOWN.

DETERMINANT: $DET(A) = P^{((N-2)/2)}$ FOR $(N-2)/2$ ODD
 $DET(A) = -P^{((N-2)/2)}$ FOR $(N-2)/2$ EVEN

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
6	6.43	2.65	8.78
16	2.21(1)	4.12	2.60(1)
36	5.20(1)	6.08	6.06(1)
58	8.41(1)	7.68	9.87(1)
78	1.14(2)	8.89	1.33(2)
96	1.42(2)	9.85	1.65(2)

REMARKS: THE ELEMENTS OF THE INVERSE WILL ALWAYS HAVE
 THE FORM $1/P, -1/P, 3/P, -3/P$, OR 0. THUS, EXCEPT
 FOR 0, OR $P=3$, THEY CAN NOT BE REPRESENTED EXACTLY
 IN A BINARY MACHINE.
 THE CONDITION NUMBER C(2) IS CONSIDERABLY SMALLER
 THAN C(1) AND C(E).

THOUGH WE HAVE NOT PROVED IT, IT SEEMS CERTAIN THAT
 THE NON-ZERO COMPONENTS OF THE EIGENVECTOR WITH
 THE EIGENVALUE 1 OR -1 ARE EQUAL, AND THAT IF THE
 I-TH COMPONENT OF THE EIGENVECTOR WITH EIGENVALUE 1
 IS NOT ZERO, THEN THE I-TH COMPONENT OF THE EIGEN-
 VECTOR WITH EIGENVALUE -1 IS 0 AND CONVERSELY.
 THEREFORE WE CAN GET THE TWO EIGENVECTORS WITH
 EIGENVALUES 1 AND -1 FROM THE COLUMN VECTOR WHICH IS
 THE PRODUCT OF A AND THE COLUMN VECTOR WHOSE
 COMPONENTS ARE ALL 1. FOR EXAMPLE IN THE CASE OF
 $N = 6$ THE TRANSPOSE OF THE PRODUCT OF A AND THE
 TRANSPOSE OF $(1, 1, 1, 1, 1, 1)$ IS $(-1, -1, 1, -1, 1,$
 $1)$, AND THEREFORE THE EIGENVECTORS WITH EIGENVALUES

1 AND -1 ARE (0, 0, 1, 0, 1, 1) AND (1, 1, 0, 1, 0, 0) RESPECTIVELY.

REFERENCES:

1. MORRIS NEWMAN AND JOHN TODD, EVALUATION OF MATRIX INVERSION PROGRAMS, JSIAM, V6, PP466-476, YR1958.

IDENTIFICATION: 001005

MATRIX: $A(I,J) = 1/J$ FOR $I \leq J$,
 $A(I,J) = J/I$ FOR $I > J$.

EXAMPLE (N=5)

	1	1/2	1/3	1/4	1/5
	1/2	1	2/3	2/4	2/5
A=	1/3	2/3	1	3/4	3/5
	1/4	2/4	3/4	1	4/5
	1/5	2/5	3/5	4/5	1

CHARACTERISTICS: REAL, SYMMETRIC, POSITIVE DEFINITE, OSCILLATORY.

INVERSE: $AINV(I,I) = (4*I**3)/(4*I**2 - 1)$ FOR $I \leq N$,
 $AINV(N,N) = (N**2)/(2*N - 1)$,
 $AINV(I,J) = -I*(I+1)/(2*I + 1)$ FOR $J \leq I+1$,
 $AINV(I,J) = 0$ FOR $J > I+1$,
 $AINV(I,J) = AINV(J,I)$.

EXAMPLE (N=5)

	4/3	-2/3	0	0	0
	-2/3	32/15	-6/5	0	0
AINV=	0	-6/5	108/35	-12/7	0
	0	0	-12/7	256/63	-20/9
	0	0	0	-20/9	25/9

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: NOT KNOWN.

CONDITION NUMBERS: NO SIMPLE EXPRESSIONS.

EXAMPLES

N	C(1)	C(2)	C(E)
5	2.68(1)	1.97(1)	2.53(1)
20	4.73(2)	3.72(2)	7.43(2)
40	1.92(3)	1.57(3)	4.16(3)
60	4.33(3)	3.62(3)	1.14(4)
80	7.71(3)	6.51(3)	2.35(4)
100	1.21(4)	1.03(4)	4.09(4)

REMARKS: NO SIMPLE EXPRESSIONS FOR EIGENVALUES AND EIGENVECTORS, OR THE DETERMINANT ARE KNOWN TO US.

REFERENCES:

1. MORRIS NEWMAN AND JOHN TODD, EVALUATION OF MATRIX

INVERSION PROGRAMS, JSIAM, V6, PP466-476, YR1958.

IDENTIFICATION: 001006

MATRIX: $A(I,I) = 2$ FOR $I.NE.1$ AND $I.NE.N$,
 $A(1,1) = 3$
 $A(N,N) = 1$
 $A(I,J) = -1$ FOR $ABS(I-J).EQ.1$,
 $A(I,J) = 0$ OTHERWISE.

EXAMPLE (N=5)

$A = \begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$

CHARACTERISTICS: REAL, SYMMETRIC, TRIDIAGONAL,
 POSITIVE DEFINITE.

INVERSE: $AINV(I,J) = (2*I-1)/2$ FOR $I.LE.J$,
 $AINV(J,I) = AINV(I,J)$.

EXAMPLE (N=5)

$AINV = 1/2 * \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 & 3 \\ 1 & 3 & 5 & 5 & 5 \\ 1 & 3 & 5 & 7 & 7 \\ 1 & 3 & 5 & 7 & 9 \end{bmatrix}$

EIGENVALUES: $EIVAL(I) = 2*(1-COS((2*I-1)*PI/(2*N)))$

EXAMPLE (N=5)

$EIVAL(1) = 9.78869$ 67409 69285 57671 (-2)
 $EIVAL(2) = 8.24429$ 49541 50537 41663 (-1)
 $EIVAL(3) = 2.00000$ 00000 00000 00000
 $EIVAL(4) = 3.17557$ 05045 84946 25834
 $EIVAL(5) = 3.90211$ 30325 90307 14423

EIGENVECTORS: $EIVC(I,J) = SIN((2*I-1)*(2*J-1)*PI/(4*N))$

EXAMPLE (N=5)

$EIVC(1,1) = 0.15643$ 44650 40230 86901
 $EIVC(2,1) = 0.45399$ 04997 39546 79156
 $EIVC(3,1) = 0.70710$ 67811 86547 52440
 $EIVC(4,1) = 0.89100$ 65241 88367 86236
 $EIVC(5,1) = 0.98768$ 83405 95137 72619
 $EIVC(1,2) = 0.45399$ 04997 39546 79156
 $EIVC(2,2) = 0.98768$ 83405 95137 72619
 $EIVC(3,2) = 0.70710$ 67811 86547 52440
 $EIVC(4,2) = -0.15643$ 44650 40230 86901
 $EIVC(5,2) = -0.89100$ 65241 88367 86236
 $EIVC(1,3) = 0.70710$ 67811 86547 52440
 $EIVC(2,3) = 0.70710$ 67811 86547 52440
 $EIVC(3,3) = -0.70710$ 67811 86547 52440
 $EIVC(4,3) = -0.70710$ 67811 86547 52440
 $EIVC(5,3) = 0.70710$ 67811 86547 52440
 $EIVC(1,4) = 0.89100$ 65241 88367 86236

EIVEC(2,4)=-0.15643 44650 40230 86901
EIVEC(3,4)=-0.70710 67811 86547 52440
EIVEC(4,4)= 0.98768 83405 95137 72619
EIVEC(5,4)=-0.45399 04997 39546 79156
EIVEC(1,5)= 0.98768 83405 95137 72619
EIVEC(2,5)=-0.89100 65241 88367 86236
EIVEC(3,5)= 0.70710 67811 86547 52440
EIVEC(4,5)=-0.45399 04997 39546 79156
EIVEC(5,5)= 0.15643 44650 40230 86901

DETERMINANT: DET(A)= 2

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
5	5.00(1)	3.99(1)	5.65(1)
20	8.00(2)	6.48(2)	1.79(3)
40	3.20(3)	2.59(3)	1.01(4)
60	7.20(3)	5.84(3)	2.79(4)
80	1.28(4)	1.04(4)	5.72(4)
100	2.00(4)	1.62(4)	1.00(5)

REMARKS: THIS MATRIX IS A SPECIAL CASE OF A WIDE CLASS OF MATRICES DESCRIBED BY GEAR (REF. 2).

THE ELEMENTS OF THIS INVERSE CAN BE EXACTLY REPRESENTED IN A BINARY MACHINE PROVIDED THE LARGEST ELEMENT $(2N - 1)/2$ DOES NOT EXCEED THE CAPACITY OF THE REPRESENTATION.

ALL OF THE EIGENVALUES OF THIS MATRIX ARE CONTAINED IN THE INTERVAL $(0, 4)$. AS N INCREASES THE SMALLEST EIGENVALUE APPROACHES ZERO AND THE LARGEST EIGENVALUE APPROACHES FOUR.

WHEN LU DECOMPOSITION OR GAUSS ELIMINATION IS PERFORMED ON THIS MATRIX NO ROW EXCHANGES ARE MADE, SINCE THE COMPUTED PIVOTS ARE ALWAYS MAXIMUM. THEREFORE, USE OF A AS A TEST MATRIX IN A LINEAR EQUATION ROUTINE WHICH USES PARTIAL PIVOTING FOR SIZE WILL NOT TEST THE ROW EXCHANGE PORTION OF THE ROUTINE.

WHEN LU DECOMPOSITION OR GAUSS ELIMINATION IS PERFORMED ON AINV THE ELEMENTS IN THE PIVOT COLUMN ON THE PRINCIPAL DIAGONAL AND BELOW WILL ALL BE EQUAL TO ONE (EXCEPTING THE FIRST COLUMN WHERE ALL OF THE ELEMENTS ARE EQUAL TO $1/2$).

REFERENCES:

1. MORRIS NEWMAN AND JOHN TODD, EVALUATION OF MATRIX INVERSION PROGRAMS, JSIAM, V6, PP466-476, YR1958.
2. C. W. GEAR, A SIMPLE SET OF TEST MATRICES FOR EIGENVALUE PROGRAMS, MATH COMP, V23, PP119-125, YR1969.

IDENTIFICATION: 001007

MATRIX: $A(I,J) = 1/(I+J-1)$

EXAMPLE (N=5)

	1	1/2	1/3	1/4	1/5
	1/2	1/3	1/4	1/5	1/6
A=	1/3	1/4	1/5	1/6	1/7
	1/4	1/5	1/6	1/7	1/8
	1/5	1/6	1/7	1/8	1/9

CHARACTERISTICS: REAL, SYMMETRIC.

INVERSE: $A_{INV}(I,J) = (-1)^{I+J} / ((I+J-1) * H(I,N) * H(J,N))$

WHERE $H(K,N) = \text{FAC}(N+K-1) / (\text{FAC}(N-K) * (\text{FAC}(K-1) ** 2))$

$\text{FAC}(P) = P * (P-1) * \dots * 1$

EXAMPLE (N=5)

	25	-300	1050	-1400	630
	-300	4800	-18900	26880	-12600
A_{INV}=	1050	-18900	79380	-117600	56700
	-1400	26880	-117600	179200	-88200
	630	-12600	56700	-88200	44100

EIGENVALUES: NO EXPRESSION IN CLOSED FORM. TABULATIONS HAVE BEEN MADE (REF. 4).

EXAMPLE (N=5, FROM REF. 4)

EIVAL(1)= 1.56705 06910 98231
 EIVAL(2)= 2.08534 21861 10133 (-1)
 EIVAL(3)= 1.14074 91623 41981 (-2)
 EIVAL(4)= 3.05898 04015 11917 (-4)
 EIVAL(5)= 3.28792 87721 71863 (-6)

EIGENVECTORS: NO EXPRESSION IN CLOSED FORM. TABULATIONS HAVE BEEN MADE (REF. 4).

EXAMPLE (N=5, FROM REF. 4)

EIVEC(1,1)= 1.00000 00000 00000
 EIVEC(2,1)= 5.80566 92249 80478 (-1)
 EIVEC(3,1)= 4.18800 95256 90560 (-1)
 EIVEC(4,1)= 3.30061 05409 17674 (-1)
 EIVEC(5,1)= 2.73258 24401 62320 (-1)
 EIVEC(1,2)= 1.00000 00000 00000
 EIVEC(2,2)= -4.58425 80576 61740 (-1)
 EIVEC(3,2)= -7.05925 82907 15063 (-1)
 EIVEC(4,2)= -7.37537 92074 31147 (-1)
 EIVEC(5,2)= -7.12798 94314 80946 (-1)
 EIVEC(1,3)= -2.95833 43954 91379 (-1)
 EIVEC(2,3)= 1.00000 00000 00000
 EIVEC(3,3)= 1.66348 46563 67509 (-1)
 EIVEC(4,3)= -4.27528 04665 91248 (-1)
 EIVEC(5,3)= -7.80543 77407 62442 (-1)
 EIVEC(1,4)= 7.06702 26210 87525 (-2)
 EIVEC(2,4)= -6.48336 02593 66261 (-1)
 EIVEC(3,4)= 1.00000 00000 00000
 EIVEC(4,4)= 3.49178 63233 06241 (-1)
 EIVEC(5,4)= -8.35542 93387 42830 (-1)
 EIVEC(1,5)= -8.04735 96573 69526 (-3)
 EIVEC(2,5)= 1.52103 86654 52718 (-1)
 EIVEC(3,5)= -6.59762 08136 21921 (-1)
 EIVEC(4,5)= 1.00000 00000 00000
 EIVEC(5,5)= -4.90419 53143 50719 (-1)

DETERMINANT: $\text{DET}(A) = E/F$

$E = (\text{FAC}(1) * \text{FAC}(2) * \dots * \text{FAC}(N-1)) ** 2$

$F = N ** N * ((N ** 2 - 1 ** 2) ** (N-1))$

$* (N ** 2 - 2 ** 2) ** (N-2)$

$* \dots * (1) ** 1$

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
2	2.70(1)	1.93(1)	1.93(1)
3	7.48(2)	5.24(2)	5.26(2)
4	2.84(4)	1.55(4)	1.56(4)
5	9.44(5)	4.77(5)	4.81(5)
6	2.91(7)	1.50(7)	1.51(7)
8	3.39(10)	1.53(10)	1.55(10)
10	3.54(13)	1.60(13)	1.63(13)
12	4.12(16)	1.71(16)	1.75(16)

REMARKS: THIS MATRIX IS A FINITE SEGMENT OF A HILBERT MATRIX. IT IS PROBABLY THE MOST OFTEN CITED EXAMPLE OF AN ILL-CONDITIONED MATRIX. THIS MATRIX ARISES IN LEAST-SQUARE APPROXIMATION (REF. 5). THE INVERSE IS TABULATED FOR $N=2(1)10$ IN REF. 1. FOR GENERALIZATION SEE TM222001, TM0010101.

REFERENCES:

1. RICHARD SAVAGE AND EUGENE LUKACS, TABLES OF INVERSES OF FINITE SEGMENTS OF THE HILBERT MATRIX, NBSAMS, V39, PP105-108, YR1954.
2. JOHN TODD, THE CONDITION OF THE FINITE SEGMENTS OF THE HILBERT MATRIX, NBSAMS, V39, PP109-116, YR1954.
3. THOMAS C. DOYLE, INVERSION OF SYMMETRIC COEFFICIENT MATRIX OF POSITIVE-DEFINITE QUADRATIC FORM, MTAC, V11, PP55-58, YR1957.
4. HENRY E. FETTIS AND JAMES C. CASLIN, EIGENVALUES AND EIGENVECTORS OF HILBERT MATRICES OF ORDER 3 THROUGH 10, MATH. COMP, V21, PP431-441, YR1967.
5. EUGENE ISAACSON AND HERBERT BISHOP KELLER, ANALYSIS OF NUMERICAL METHODS, PP194-199, JOHN WILEY & SONS, YR1966.

IDENTIFICATION: 001008

MATRIX: $A(I, I) = 1 - K * I ** 2$ FOR $I=1, 2, \dots, N-1,$

$A(I, N) = K * I$ FOR $I=1, 2, \dots, N-1,$

$A(N, I) = A(I, N)$

$A(N, N) = -K$

$A(I, J) = -K * I * J$ OTHERWISE,

WHERE $K = 6 / (N * (N+1) * (2 * N - 5)).$

EXAMPLE (N=5)

24	-2	-3	-4	1
-2	21	-6	-8	2

$$A = (1/25) \begin{pmatrix} -3 & -6 & 16 & -12 & 3 \\ -4 & -8 & -12 & 9 & 4 \\ 1 & 2 & 3 & 4 & -1 \end{pmatrix}$$

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR,
MULTIPLE EIGENVALUES.

INVERSE: $AINV(I,I) = 1$ FOR $I=1,2,\dots,N-1$,
 $AINV(I,N) = I$
 $AINV(N,I) = AINV(I,N)$
 $AINV(I,J) = 0$ OTHERWISE.

EXAMPLE (N=5)

$$AINV = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

EIGENVALUES: $EIVAL(I) = 1$ FOR $I=1,2,\dots,N-2$,
 $EIVAL(N-1) = (X + \sqrt{Y})^{**}(-1)$
 $EIVAL(N) = (X - \sqrt{Y})^{**}(-1)$

WHERE $X = (N+1)/2$
 $Y = 1/K + X^{**}2$

EXAMPLE (N=5)

$EIVAL(1) = 1$
 $EIVAL(2) = 1$
 $EIVAL(3) = 1$
 $EIVAL(4) = 1/(3 + \sqrt{34})$
 $EIVAL(5) = 1/(3 - \sqrt{34})$

EIGENVECTORS: NOT KNOWN.

DETERMINANT: $DET(A) = -6/(N*(N+1)(2*N-5))$

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
5	2.40(1)	8.83	1.67(1)
20	4.59(2)	6.11(1)	3.11(2)
40	1.92(3)	1.65(2)	1.27(3)
60	4.37(3)	2.97(2)	2.89(3)
80	7.83(3)	4.52(2)	5.16(3)
100	1.23(4)	6.26(2)	8.08(3)

REMARKS: THIS MATRIX IS FAIRLY WELL CONDITIONED. THE
 INVERSE IS COMPOSED OF INTEGERS AND THUS IS EXACTLY
 REPRESENTABLE IN A BINARY MACHINE.

THE EIGENVALUE 1 HAS MULTIPLICITY N-2, SO IT CAN BE
 EXPECTED THAT THIS MATRIX IS USEFUL IN TESTING
 EIGENVALUE ALGORITHMS IN THE PRESENCE OF MULTIPLE
 EIGENVALUES.

REFERENCES:

1. M. J. AEGERTER, CONSTRUCTION OF A SET OF TEST
 MATRICES, CACM, V2, PP10-12, YR1959, MOAUG.

IDENTIFICATION: 001009

MATRIX: $A(I,J) = \text{SQRT}(2/(N+1)) * \sin(I * J * \pi / (N+1))$
FOR $I, J = 1, 2, \dots, N$.

EXAMPLE (N=5) . LET $T = \text{SQRT}(3)$

	$1/2$	$T/2$	1	$T/2$	$1/2$
	$T/2$	$T/2$	0	$-T/2$	$-T/2$
$A=1/T *$	1	0	-1	0	1
	$T/2$	$-T/2$	0	$T/2$	$-T/2$
	$1/2$	$-T/2$	1	$-T/2$	$1/2$

CHARACTERISTICS: REAL, SYMMETRIC, NON-SINGULAR,
ORTHOGONAL.

INVERSE: $A_{INV}(I,J) = A(I,J)$

EIGENVALUES: $EIVAL(I) = 1$ FOR $I = 1, 2, \dots, N/2$
AND N EVEN,
 $EIVAL(I) = -1$ FOR $I = N/2 + 1, \dots, N$
AND N EVEN,
 $EIVAL(I) = 1$ FOR $I = 1, 2, \dots, (N+1)/2$
AND N ODD,
 $EIVAL(I) = -1$ FOR $I = (N+3)/2, \dots, N$
AND N ODD.

EXAMPLE (N=5)

$EIVAL(1) = 1$
 $EIVAL(2) = 1$
 $EIVAL(3) = 1$
 $EIVAL(4) = -1$
 $EIVAL(5) = -1$

EIGENVECTORS: EXPRESSION IN TERMS OF ELEMENTARY
FUNCTIONS IS UNKNOWN.

DETERMINANT: $+1$ OR -1 .
 $+1$ IF N EVEN OR IF $N \equiv 1 \pmod{4}$,
 -1 OTHERWISE.

CONDITION NUMBERS:

$C(1) = ((\sin(\pi/(N+1))) / (1 - \cos(\pi/(N+1)))) ** 2$

$C(2) = 1$

$C(E) = N$

EXAMPLES

N	C(1)	C(2)	C(E)
5	4.64	1.00	5.00
20	1.70(1)	1.00	2.00(1)
40	3.32(1)	1.00	4.00(1)
60	4.94(1)	1.00	6.00(1)
80	6.56(1)	1.00	8.00(1)
100	8.19(1)	1.00	1.00(2)

REMARKS: THIS MATRIX WAS SUGGESTED BY NEWMAN AND TODD
(REF. 1). IT HAS THE SMALLEST POSSIBLE CONDITION
NUMBER $C(2)$ HENCE MATRIX INVERSION ROUTINES CAN BE
EXPECTED TO PRODUCE THE INVERSE OF THIS MATRIX VERY

ACCURATELY.

REFERENCES:

1. MORRIS NEWMAN AND JOHN TODD, THE EVALUATION OF MATRIX INVERSION PROGRAMS, JSIAM, V6, PP466-476, YR1958.

IDENTIFICATION: 001010

MATRIX: $A(I,J) = 1.0/(P+I+J-1)$ FOR $I,J = 1,2,\dots,N$
AND P A NONNEGATIVE INTEGER
EXAMPLE (P=2, N=5)

	1/3	1/4	1/5	1/6	1/7
	1/4	1/5	1/6	1/7	1/8
A=	1/5	1/6	1/7	1/8	1/9
	1/6	1/7	1/8	1/9	1/10
	1/7	1/8	1/9	1/10	1/11

CHARACTERISTICS: REAL, SYMMETRIC.

INVERSE: $AINV(I,J) = (-1)**(I+J)*F(I)*F(J)/(P+I+J-1)$
 $F(K) = FAC(P+K+N-1)/(FAC(K-1)*FAC(N-K)*FAC(P+K-1))$
EXAMPLE (P=2, N=5)

	3675	-29400	79380	-88200	34650
	-29400	250880	-705600	806400	-323400
AINV=	79380	-705600	2041200	-2381400	970200
	-88200	806400	-2381400	2822400	-1164240
	34650	-323400	970200	-1164240	485100

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: NOT KNOWN.

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
2	8.17(1)	6.63(1)	6.63(1)
3	4.61(3)	3.09(3)	3.09(3)
4	2.11(5)	1.24(5)	1.24(5)
5	7.97(6)	4.64(6)	4.64(6)
6	2.95(8)	1.67(8)	1.67(8)
8	4.01(11)	2.05(11)	2.06(11)
10	4.99(14)	2.43(14)	2.44(14)
12	5.85(17)	2.82(17)	2.83(17)

REMARKS: THIS IS A GENERALIZED FORM OF A FINITE SEGMENT OF A HILBERT MATRIX (TM001007). FOR ANOTHER GENERALIZATION SEE TM222001.

AS FOR THE HILBERT MATRIX, THIS MATRIX IS VERY ILL CONDITIONED.

REFERENCES:

1. A. R. COLLAR, ON THE RECIPROCATION OF CERTAIN MATRICES, PROC. ROY. SOC. EDINBURGH, V59, PP195-206, YR1939.
2. A. R. COLLAR, ON THE RECIPROCAL OF A SEGMENT OF A GENERALIZED HILBERT MATRIX, PROC. CAMB. PHIL. SOC., V47, PP11-17, YR1951.

IDENTIFICATION: 001011

MATRIX: $A(I,J) = N - \text{ABS}(I-J)$
 EXAMPLE (N=5)

A=

5	4	3	2	1
4	5	4	3	2
3	4	5	4	3
2	3	4	5	4
1	2	3	4	5

CHARACTERISTICS: REAL, SYMMETRIC, POSITIVE DEFINITE.

INVERSE: $\text{AINV}(I,I) = (N+2)/(2*N+2)$ FOR $I=1$, OR N ,
 $\text{AINV}(I,I) = 1$ OTHERWISE,
 $\text{AINV}(I,J) = -1/3$ FOR $\text{ABS}(I-J)=1$ AND $N=2$,
 $\text{AINV}(I,J) = -1/2$ FOR $\text{ABS}(I-J)=1$ AND $N \neq 2$,
 $\text{AINV}(I,J) = 1/(2*N+2)$ FOR $\text{ABS}(I-J)=N-1$,
 $\text{AINV}(I,J) = 0$ OTHERWISE.

EXAMPLE (N=5)

AINV=

7/12	-1/2	0	0	1/12
-1/2	1	-1/2	0	0
0	-1/2	1	-1/2	0
0	0	-1/2	1	-1/2
1/12	0	0	-1/2	7/12

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: NOT KNOWN.

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
5	3.80(1)	3.13(1)	4.30(1)
20	6.00(2)	5.38(2)	1.50(3)
40	2.40(3)	2.16(3)	8.62(3)
60	5.40(3)	4.86(3)	2.39(4)
80	9.60(3)	8.64(3)	4.92(4)
100	1.50(4)	1.35(4)	8.60(4)

REMARKS: THIS MATRIX WAS USED AS A TEST MATRIX IN REFERENCE 1.

IF THE EIGENVALUES ARE DISTINCT THEN THE FOLLOWING OBSERVATIONS HOLD. BY THE GIVEN FORM OF THE N BY N MATRIX, IT IS CLEAR THAT IF $(V(1), V(2), \dots, V(N))$ IS AN EIGENVECTOR THEN SO IS $(V(N), \dots, V(2), V(1))$

FOR THE SAME EIGENVALUE. THEREFORE

$$V(I) = V(N-I+1) \text{ OR } -V(N-I+1), I = 1, 2, \dots, N$$

BY THE FOLLOWING ARGUMENTS:

$$V(I) = C \cdot V(N-I+1) \quad \text{AND}$$

$$V(N-I+1) = C \cdot V(I), \quad I = 1, 2, \dots, N$$

WHICH IMPLIES $C^2 = 1$. HENCE $C = 1$ OR -1 .

IF N IS ODD AND $V(I)$ ARE THE COMPONENTS OF AN EIGEN-VECTOR SUCH THAT $V(I) = -V(N-I+1)$, THEN $V((N+1)/2) = 0$.

USING THE JACOBI EXPANSION OF THE DETERMINANT IT CAN BE SHOWN THAT THE NUMBER OF EIGENVECTORS SUCH THAT $V(I) = V(N-I+1)$ IS $N/2$ OR $(N+1)/2$ WHEN N IS EVEN OR ODD, RESPECTIVELY.

REFERENCES:

1. M. H. LIETZKE, R. W. STOUGHTON AND MARJORIE D. LIETZKE, A COMPARISON OF SEVERAL METHODS FOR INVERTING LARGE SYMMETRIC POSITIVE DEFINITE MATRICES, MATH COMP, V18, PP449-456, YR1964.

IDENTIFICATION: 000001

MATRIX:

611	196	-192	407	-8	-52	-49	29
196	899	113	-192	-71	-43	-8	-44
-192	113	899	196	61	49	8	52
407	-192	196	611	8	44	59	-23
-8	-71	61	8	411	-599	208	208
-52	-43	49	44	-599	411	208	208
-49	-8	8	59	208	208	99	-911
29	-44	52	-23	208	208	-911	99

CHARACTERISTICS: REAL, SYMMETRIC, SINGULAR,
SOME CLOSELY SPACED EIGENVALUES, A PAIR OF
EQUAL EIGENVALUES.

INVERSE: NONE.

EIGENVALUES:

EIVAL(1)= 10*SQRT(10405)
= 1.02004 90184 29996 82385 (3)
EIVAL(2)= 1020
EIVAL(3)= 510+100*SQRT(26)
= 1.01990 19513 59278 48300 (3)
EIVAL(4)= 1000
EIVAL(5)= 1000
EIVAL(6)= 510-100*SQRT(26)
= 9.80486 40721 51699 71776 (-2)
EIVAL(7)= 0
EIVAL(8)=-10*SQRT(10405)
=-1.02004 90184 29996 82385(3)

EIGENVECTORS:

EIVEC(1,1)= 2

```
EIVVEC(2,1)= 1
EIVVEC(3,1)= 1
EIVVEC(4,1)= 2
EIVVEC(5,1)= 102-SQRT(10405)
              =-4.90184 29996 82384 63137 (-3)
EIVVEC(6,1)= 102-SQRT(10405)
              =-4.90184 29996 82384 63137 (-3)
EIVVEC(7,1)=-204+2*SQRT(10405)
              = 9.80368 59993 64769 26275 (-3)
EIVVEC(8,1)=-204+2*SQRT(10405)
              = 9.80368 59993 64769 26275 (-3)
EIVVEC(1,2)= 1
EIVVEC(2,2)=-2
EIVVEC(3,2)=-2
EIVVEC(4,2)= 1
EIVVEC(5,2)= 2
EIVVEC(6,2)=-2
EIVVEC(7,2)= 1
EIVVEC(8,2)=-1
EIVVEC(1,3)= 2
EIVVEC(2,3)=-1
EIVVEC(3,3)= 1
EIVVEC(4,3)=-2
EIVVEC(5,3)= SQRT(26)
              = 5.09901 95135 92784 83002
EIVVEC(6,3)=-5-SQRT(26)
              =-1.00990 19513 59278 48300
EIVVEC(7,3)=-10-2*SQRT(26)
              =-2.01980 39027 18556 96601 (1)
EIVVEC(8,3)= 10+2*SQRT(26)
              = 2.01980 39027 18556 96601 (1)
EIVVEC(1,4)= 1
EIVVEC(2,4)=-2
EIVVEC(3,4)=-2
EIVVEC(4,4)= 1
EIVVEC(5,4)=-2
EIVVEC(6,4)= 2
EIVVEC(7,4)=-1
EIVVEC(8,4)= 1
EIVVEC(1,5)= 7
EIVVEC(2,5)= 14
EIVVEC(3,5)=-14
EIVVEC(4,5)=-7
EIVVEC(5,5)=-2
EIVVEC(6,5)=-2
EIVVEC(7,5)=-1
EIVVEC(8,5)=-1
EIVVEC(1,6)= 2
EIVVEC(2,6)=-1
EIVVEC(3,6)= 1
EIVVEC(4,6)=-2
EIVVEC(5,6)= 5-SQRT(26)
              =-9.90195 13592 78483 00282 (-2)
EIVVEC(6,6)=-5+SQRT(26)
              = 9.90195 13592 78483 00282 (-2)
EIVVEC(7,6)=-10+2*SQRT(26)
              = 1.98039 02718 55696 60056 (-1)
```

```

EIVEC(8,6)= 10-2*SQRT(26)
          =-1.98039 02718 55696 60056 (-1)
EIVEC(1,7)= 1
EIVEC(2,7)= 2
EIVEC(3,7)=-2
EIVEC(4,7)=-1
EIVEC(5,7)= 14
EIVEC(6,7)= 14
EIVEC(7,7)= 7
EIVEC(8,7)= 7
EIVEC(1,8)= 2
EIVEC(2,8)= 1
EIVEC(3,8)= 1
EIVEC(4,8)= 2
EIVEC(5,8)= 102+SQRT(10405)
          = 2.04004 90184 29996 82385 (2)
EIVEC(6,8)= 102+SQRT(10405)
          = 2.04004 90184 29996 82385 (2)
EIVEC(7,8)=-204-2*SQRT(10405)
          =-4.08009 80368 59993 64769 (2)
EIVEC(8,8)=-204-2*SQRT(10405)
          =-4.08009 80368 59993 64769 (2)

```

DETERMINANT: DET(A)= 0

CONDITION NUMBERS: C(R)= 1.040(4)

REMARKS: THIS MATRIX WAS CONSTRUCTED BY ROSSER, LANZOS, HESTENES, AND KARUSH (REF. 1) FOR TESTING CERTAIN EIGENVALUE EIGENVECTOR ALGORITHMS. IT HAS SEVERAL CLOSE TOGETHER EIGENVALUES AND THEREFORE IT IS USEFUL FOR TESTING THE ABILITY OF AN ALGORITHM TO SEPERATE CLOSE EIGENVALUES.

REFERENCES:

1. ROSSER, J.B., LANZOS, C., HESTENES, M.R., AND KARUSH, W., SEPARATION OF CLOSE EIGENVALUES OF A REAL SYMMETRIC MATRIX, J. RES. NBS, V47, PP291-297, YR1951.

IDENTIFICATION: 002001

MATRIX: A(I,J)=1 FOR I.NE.J,

A(I,I)=1 + X

EXAMPLE (N=5, X=3)

```

      4 1 1 1 1
      1 4 1 1 1
A=    1 1 4 1 1
      1 1 1 4 1
      1 1 1 1 4

```

CHARACTERISTICS: REAL, SYMMETRIC, SINGULAR IF X=0 OR -N.

INVERSE: AINV(I,J)= -1/(X**2 + N * X) FOR I.NE.J,

$$\text{AINV}(I,I) = 1/X - 1/(X**2 + N * X)$$

EXAMPLE (N=5, X=3)

$$\text{AINV} = 1/24 * \begin{pmatrix} 7 & -1 & -1 & -1 & -1 \\ -1 & 7 & -1 & -1 & -1 \\ -1 & -1 & 7 & -1 & -1 \\ -1 & -1 & -1 & 7 & -1 \\ -1 & -1 & -1 & -1 & 7 \end{pmatrix}$$

EIGENVALUES: X (N-1 TIMES),
X + N

EXAMPLE (N=5, X=3)

EIVAL(1)= 3
EIVAL(2)= 3
EIVAL(3)= 3
EIVAL(4)= 3
EIVAL(5)= 8

EIGENVECTORS: FOR THE ISOLATED EIGENVALUE (X + N),
THE EIGENVECTOR IS

EIVEC(I,N)= 1 FOR I=1,2,...,N,
AND ANY VECTOR ORTHOGONAL TO THIS ONE IS AN
EIGENVECTOR WITH EIGENVALUE X.

DETERMINANT: DET(A)= (X+N)*X**(N-1)

CONDITION NUMBERS: ANY CONDITION NUMBER GREATER THAN
ONE CAN BE OBTAINED BY APPROPRIATE CHOICE OF A
POSITIVE VALUE FOR X.

C(1)= 1 + (2*N-2)/X FOR X.GT.0,

C(2)= 1 + N/X FOR X.GT.0,

C(E)= SQRT(U*V) FOR X.GT.0

U= N*(X**2+2*X+N)

V= N*((X+N)**2-2*X-N)/(X*(X+N))**2.

EXAMPLES (X=3)

N	C(1)	C(2)	C(E)
5	3.67	2.67	6.78
20	1.37(1)	7.67	3.85(1)
40	2.70(1)	1.43(1)	9.76(1)
60	4.03(1)	2.10(1)	1.72(2)
80	5.37(1)	2.77(1)	2.58(2)
100	6.70(1)	3.43(1)	3.56(2)

EXAMPLES (X=1)

N	C(1)	C(2)	C(E)
5	9.00	6.00	1.27(1)
20	3.90(1)	2.10(1)	9.35(1)
40	7.90(1)	4.10(1)	2.59(2)
60	1.19(2)	6.10(1)	4.72(2)
80	1.59(2)	8.10(1)	7.24(2)
100	1.99(2)	1.01(2)	1.01(3)

EXAMPLES (X=0.01)

N	C(1)	C(2)	C(E)
5	8.01(2)	5.01(2)	1.00(3)
20	3.80(3)	2.00(3)	8.72(3)
40	7.80(3)	4.00(3)	2.50(4)
60	1.18(4)	6.00(3)	4.61(4)
80	1.58(4)	8.00(3)	7.11(4)
100	1.98(4)	1.00(4)	9.95(4)

REMARKS: ANY CONDITION NUMBER MAY BE ASSIGNED TO THIS MATRIX BY APPROPRIATE CHOICE OF X . HENCE THIS MATRIX IS USEFUL FOR TESTING THE SENSITIVITY OF ALGORITHMS TO ILL-CONDITIONED SYSTEMS. THIS ADVANTAGE WAS POINTED OUT BY NEWMAN AND TODD (REF. 1). THIS MATRIX IS AN EXAMPLE OF A CIRCULANT OR CYCLIC MATRIX. RULES FOR THE DETERMINATION OF THE INVERSE, EIGENVALUES, AND EIGENVECTORS ARE KNOWN FOR THIS CLASS OF MATRICES (REF. 1,2). A BRIEF FLURRY OF PAPERS (REF. 3,4,5,6) APPEARED IN THE CACM BY PEOPLE WHO SEEMED UNAWARE OF THE KNOWN FACTS ABOUT THIS CLASS OF MATRICES. ANOTHER GENERALIZATION OF THIS FORM WAS POINTED OUT BY CLINE (REF. 7).

REFERENCES:

1. MORRIS NEWMAN AND JOHN TODD, THE EVALUATION OF MATRIX INVERSION PROGRAMS, SIAMJ, V6, PP466-476, YR1958.
2. R. ZURMUHL, MATRIZEN, 4TH ED., P231, SPRINGER-VERLAG, YR1964.
3. M. L. PEI, A TEST MATRIX FOR INVERSION PROCEDURES, CACM, V5, P508, YR1962.
4. WILLIAM S. LASOR, TEST MATRIX FOR INVERSION, CACM, V6, P102, YR1963.
5. ROBERT D. RODMAN, A NOTE ON A SET OF TEST MATRICES FOR INVERSION, CACM, V6, P515, YR1963.
6. A. C. R. NEWBERRY, PEI MATRIX EIGENVECTORS, CACM, V6, P515, YR1963.
7. RANDALL E. CLINE, A CLASS OF MATRICES TO TEST INVERSION PROCEDURES, CACM, V7, PP724-725, YR1964.

IDENTIFICATION: 011001

MATRIX: $A(I,J) = 1$ FOR $(J-I)=1$,
 $A(N,1) = E$
 $A(I,J) = 0$ OTHERWISE.

EXAMPLE (N=5)

$A =$	0	1	0	0	0
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	1
	E	0	0	0	0

CHARACTERISTICS: REAL, NON-SYMMETRIC, NON-SINGULAR.

INVERSE: $AINV(I,J) = 1$ FOR $(I-J)=1$,
 $AINV(1,N) = 1/E$
 $AINV(I,J) = 0$ OTHERWISE.

EXAMPLE (N=5)

$AINV =$	0	0	0	0	$1/E$
	1	0	0	0	0
	0	1	0	0	0

```

0 0 1 0 0
0 0 0 1 0

```

EIGENVALUES: EIVAL(I) = (EXP(2*PI*M*I)*ABS(E))**(1/N)
 FOR I=0,1,2,...,N-1 AND M=SQRT(-1).

EXAMPLE (N=5, E= 0.00001)

```

EIVAL(1)= 0.1
EIVAL(2)= 0.1*(COS(2*PI/5)+M*SIN(2*PI/5))
EIVAL(3)= 0.1*(COS(4*PI/5)+M*SIN(4*PI/5))
EIVAL(4)= 0.1*(COS(6*PI/5)+M*SIN(6*PI/5))
EIVAL(5)= 0.1*(COS(8*PI/5)+M*SIN(8*PI/5))

```

EIGENVECTORS: EIVEC(I,J) = (EIVAL(J))**(I-1)

EXAMPLE (N=5, E=0.00001)

```

EIVEC(1,1)= 1.0
EIVEC(2,1)= 0.1
EIVEC(3,1)= 0.01
EIVEC(4,1)= 0.001
EIVEC(5,1)= 0.0001
EIVEC(1,2)= 1.0
EIVEC(2,2)= 0.1*(COS(2*PI/5)+M*SIN(2*PI/5))
EIVEC(3,2)= 0.01*(COS(4*PI/5)+M*SIN(4*PI/5))
EIVEC(4,2)= 0.001*(COS(6*PI/5)+M*SIN(6*PI/5))
EIVEC(5,2)= 0.0001*(COS(8*PI/5)+M*SIN(8*PI/5))
EIVEC(1,3)= 1.0
EIVEC(2,3)= 0.1*(COS(4*PI/5)+M*SIN(4*PI/5))
EIVEC(3,3)= 0.01*(COS(8*PI/5)+M*SIN(8*PI/5))
EIVEC(4,3)= 0.001*(COS(2*PI/5)+M*SIN(2*PI/5))
EIVEC(5,3)= 0.0001*(COS(6*PI/5)+M*SIN(6*PI/5))
EIVEC(1,4)= 1.0
EIVEC(2,4)= 0.1*(COS(6*PI/5)+M*SIN(6*PI/5))
EIVEC(3,4)= 0.01*(COS(2*PI/5)+M*SIN(2*PI/5))
EIVEC(4,4)= 0.001*(COS(8*PI/5)+M*SIN(8*PI/5))
EIVEC(5,4)= 0.0001*(COS(4*PI/5)+M*SIN(4*PI/5))
EIVEC(1,5)= 1.0
EIVEC(2,5)= 0.1*(COS(8*PI/5)+M*SIN(8*PI/5))
EIVEC(3,5)= 0.01*(COS(6*PI/5)+M*SIN(6*PI/5))
EIVEC(4,5)= 0.001*(COS(4*PI/5)+M*SIN(4*PI/5))
EIVEC(5,5)= 0.0001*(COS(2*PI/5)+M*SIN(2*PI/5))

```

DETERMINANT: DET(A) = (-1)**(N-1)*E

REMARKS: THIS IS A GENERALIZATION OF A CYCLIC OR
 CIRCULANT MATRIX (REF. 2). ACCORDING TO REFERENCE 1
 IT WAS SUGGESTED AS A TEST MATRIX BY FORSYTHE. THE
 CONDITION NUMBER OF THIS MATRIX CAN BE MADE ARBITR-
 ARILY LARGE BY CHOICE OF E. IT WAS USED AS THE BASIS
 OF A TEST MATRIX BY PARLETT (REF. 3).
 WHEN A PARTIAL PIVOTING STRATEGY IS USED IN GAUSS
 ELEMINATION OR LU DECOMPOSITION ROW EXCHANGE WILL BE
 REQUIRED IN THE SELECTION OF EVERY PIVOT.

CONDITION NUMBERS:

EXAMPLES (E=0.00001)

N	C(1)	C(2)	C(E)
5	1.00(5)	1.00(5)	2.00(5)
20	1.00(5)	1.00(5)	4.36(5)

40	1.00(5)	1.00(5)	6.24(5)
60	1.00(5)	1.00(5)	7.68(5)
80	1.00(5)	1.00(5)	8.89(5)
100	1.00(5)	1.00(5)	9.95(5)

REFERENCES:

1. PAUL A. WHITE, THE COMPUTATION OF EIGENVALUES AND EIGENVECTORS OF A MATRIX, JSIAM, V6, PP393-437, YR1958.
2. R. ZURMUHL, MATRIZEN, 4TH ED., P231, SPRINGER-VERLAG, YR1964.
3. BERESFORD PARLETT, LAGUERRE'S METHOD APPLIED TO THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18, PP464-485, YR1964.

IDENTIFICATION: 010001

MATRIX: $A(I,J) = 1$ FOR $I=1$ AND $J=1,2,\dots,N$,
 $A(I,J) = 1/(I+J-1)$ FOR $I=2,3,\dots,N$,
AND FOR $J=1,2,\dots,N$.

EXAMPLE (N=5)

	1	1	1	1	1
	1/2	1/3	1/4	1/5	1/6
A=	1/3	1/4	1/5	1/6	1/7
	1/4	1/5	1/6	1/7	1/8
	1/5	1/6	1/7	1/8	1/9

CHARACTERISTICS: NON-SYMMETRIC, EIGENVALUE OF MAXIMUM MODULUS IS POSITIVE, EIGENVALUE OF MINIMUM MODULUS IS NEGATIVE, ELEMENTS OF THE INVERSE ARE INTEGERS.

INVERSE: $AINV(I,1) = (-1)**(N-I) * C(N+I-1,I-1)*C(N,I)$
FOR $I=1,2,\dots,N$,
 $AINV(I,J) = (-1)**(I-J-1)*I*C(I+J-1,J-1)$
 $*C(I+J-2,J-2)*C(N+I-1,I+J-1)$
 $*C(N+J-1,I+J-1)$
FOR $I=1,2,\dots,N$
AND FOR $J=2,3,\dots,N$.

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: $DET(A) = (-1)**(N-1)/DEL(N)$,
WHERE $DEL(N)$ IS GIVEN BY THE RECURRENCE RELATION
 $DEL(1) = 1$
 $DEL(N+1) = C(2*N,N-1)*C(2*N,N)*(2*N+1)*DEL(N)$
AND $C(I,J)$ IS THE COMBINATORIAL SYMBOL.

EXAMPLE (N=5)

$DEL(5) = 5.33433 \ 60000$
 $DET(A) = 1/DEL(5)$

CONDITION NUMBERS:

EXAMPLES

N	C(1)	C(2)	C(E)
2	1.80(1)	1.41(1)	1.42(1)
3	8.64(2)	4.83(2)	4.85(2)
4	3.79(4)	1.70(4)	1.71(4)
5	1.44(6)	5.92(5)	5.96(5)
6	5.63(7)	2.04(7)	2.06(7)
8	8.16(10)	2.38(10)	2.41(10)
10	1.09(14)	2.77(13)	2.80(13)
12	1.37(17)	3.20(16)	3.24(16)

REMARKS: THIS MATRIX IS CLOSELY RELATED TO A FINITE SEGMENT OF THE HILBERT MATRIX (TM 001007). IT IS ILL-CONDITIONED, LIKE THE HILBERT MATRIX, AND IT IS NON-SYMMETRIC, UNLIKE THE HILBERT MATRIX.

REFERENCES:

1. MARK LOTKIN, A SET OF TEST MATRICES, MTAC, V9, PP153-161, YR1955.
2. P. J. EBERLEIN, A JACOBI-LIKE METHOD FOR THE AUTOMATIC COMPUTATION OF EIGENVALUES AND EIGENVECTORS OF AN ARBITRARY MATRIX, JSIAM, V10, PP74-88, YR1962.

IDENTIFICATION: 010002

MATRIX: A IS THE BLOCK MATRIX

$$A = \begin{pmatrix} B & 2*B \\ 4*B & 3*B \end{pmatrix}$$

AND B IS THE MATRIX

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 0 \end{pmatrix}$$

CHARACTERISTICS: REAL, NONSYMMETRIC.

INVERSE: AINV IS THE BLOCK MATRIX

$$\begin{pmatrix} -3*BINV & 2*BINV \\ 4*BINV & -1*BINV \end{pmatrix}$$

$$AINV = 1/5$$

AND BINV IS THE MATRIX

$$BINV = \begin{pmatrix} 0 & 0 & 0 & 0 & 1/E \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

EIGENVALUES: EIVAL(K) = 5*((EXP(2*PI*W*K)*ABS(E)))**0.2
 FOR K=1,2,3,4,5, AND W=SQR(-1),
 EIVAL(K) = -((EXP(2*PI*W*K)*ABS(E)))**0.2
 FOR K=6,7,8,9,10, AND W=SQR(-1).

EXAMPLE (E=0.00001)

$$EIVAL(K) = 0.5*\cos(2*PI*K/5) + 0.5*W*\sin(2*PI*K/5)$$

```
FOR K=1,2,3,4,5,
EIVAL(K)=-0.1*COS(2*-I*K/5)-0.1*W*SIN(2*-I*K/5)
FOR K=6,7,8,9,10.
```

```
EIGENVECTORS: EIVEC(J,K)= (EIVAL(K))**(J-1)
FOR J=1,2,3,4,5; K=1,2,3,4,5,
EIVEC(J,K)=2*(EIVAL(K))**(J-1)
FOR J=6,7,8,9,10; K=1,2,3,4,5,
EIVEC(J,K)= (EIVAL(K))**(J-1)
FOR J=1,2,3,4,5; K=6,7,8,9,10,
EIVEC(J,K)= -(EIVAL(K))**(J-1)
FOR J=6,7,8,9,10; K=6,7,8,9,10.
```

DETERMINANT: DET(A)=-3125*E**2

CONDITION NUMBERS:

EXAMPLE (E=0.00001)

N	C(1)	C(2)	C(E)
10	7.00(5)	5.83(5)	1.20(6)

REMARKS: A SPECIAL CASE OF THE MATRIX A WITH E=0.00001 WAS USED BY PARLETT AND EBERLEIN AS A TEST MATRIX (REF. 1,2).

THE MATRIX B IS THE EXAMPLE FOR THE MATRIX 011001, AND, REPLACING B WITH SOME MATRIX, WE CAN GET ANOTHER TEST MATRIX (SEE MATRICES 010004, 010005). THE FORMULAS FOR THE EIGENVALUES AND EIGENVECTORS ARE DERIVED BY THE FOLLOWING PROPOSITION:
LET A BE AN M*N BY M*N BLOCK MATRIX WHOSE (I,J)-TH M BY M MATRIX BLOCK IS C(I,J)*B, WHERE B IS AN M BY M MATRIX. IF E AND E' ARE EIGENVALUES OF B AND THE COEFFICIENT MATRIX C, RESPECTIVELY, AND V=(V(1), V(2), ..., V(M)) AND V'=(V'(1), V'(2), ..., V'(N)) ARE THE CORRESPONDING EIGENVECTORS OF THE TWO MATRICES, THEN E'*E IS AN EIGENVALUE OF A AND (V'(1)*V, V'(2)*V, ..., V'(N)*V) IS THE CORRESPONDING EIGENVECTOR OF A.

REFERENCES:

1. BERESFORD PARLETT, LAGUERRE'S METHOD APPLIED TO THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18, PP464-485, YR1964.
2. P. J. EBERLEIN, A JACOBI-LIKE METHOD FOR THE AUTOMATIC COMPUTATION OF EIGENVALUES AND EIGENVECTORS OF AN ARBITRARY MATRIX, JSIAM, V10, PP74-88, YR1962.

IDENTIFICATION: 010003

MATRIX: A(I,J)= 0 FOR J-I.GT.1,
A(I,J)= N-I FOR J-I.EQ.1,
A(I,J)= N-I+1 FOR I.GE.J.

EXAMPLE (N=5)

5 4 0 0 0

A=

4	4	3	0	0
3	3	3	2	0
2	2	2	2	1
1	1	1	1	1

CHARACTERISTICS: HESSENBERG FORM.

INVERSE: AINV(I,J)= -1 FOR I-J.EQ.1,
 AINV(I,J)= 0 FOR I-J.GT.1,
 AINV(1,1)= 1
 AINV(1,J)= (-1)**(J+1)*FAC(N-1)/FAC(N-J)
 AINV(I,J)= (-1)**(I+J)*(N-I+2)*FAC(N-I)
 /FAC(N-J)

EXAMPLE (N=5)

1	-4	12	-24	24
-1	5	-15	30	-30
0	-1	4	-8	8
0	0	-1	3	-3
0	0	0	-1	2

EIGENVALUES: NOT KNOWN.

EIGENVECTORS: NOT KNOWN.

DETERMINANT: DET(A)= 1

CONDITION NUMBERS:

N	C(1)	C(2)	C(E)
2	9.00	6.85	7.00
3	3.50(1)	2.76(1)	2.87(1)
4	1.68(2)	1.20(2)	1.27(2)
5	8.91(2)	6.47(2)	6.91(2)
6	5.87(3)	4.21(3)	4.55(3)
8	3.76(5)	2.76(5)	3.03(5)
10	3.84(7)	2.85(7)	3.18(7)
12	5.68(9)	4.27(9)	4.79(9)

REMARKS: THIS MATRIX WAS USED BY FRANK (REF. 1) AND
 BY PARLETT (REF. 2) AS A TEST MATRIX. IT HAS BEEN
 DISCUSSED BY WILKINSON.

REFERENCES:

1. WERNER L. FRANK, COMPUTING EIGENVALUES OF COMPLEX MATRICES BY DETERMINANT EVALUATION AND BY METHODS OF DANILEWSKI AND WIELANDT, JSIAM, V6, PP378-392, YR1958.
2. BERESFORD PARLETT, LAGUERRES METHOD APPLIED TO THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18, PP464-485, YR1964.
3. J. H. WILKINSON, ERROR ANALYSIS OF FLOATING-POINT COMPUTATION, NUM MATH, V2, PP319-340, YR1960.

IDENTIFICATION: 010004

MATRIX: A IS THE BLOCK MATRIX

$$A = \begin{pmatrix} B & 2*B \\ 4*B & 3*B \end{pmatrix}$$

AND B IS THE BLOCK MATRIX

$$B = \begin{pmatrix} 5*C & -C \\ 5*C & C \end{pmatrix}$$

AND C IS THE MATRIX

$$C = \begin{pmatrix} -2 & 2 & 2 & 2 \\ -3 & 3 & 2 & 2 \\ -2 & 0 & 4 & 2 \\ -1 & 0 & 0 & 5 \end{pmatrix}$$

CHARACTERISTICS: REAL, NONSYMMETRIC, ALL EIGENVALUES ARE COMPLEX.

INVERSE: AINV IS THE BLOCK MATRIX

$$AINV = \frac{1}{5} \begin{pmatrix} -3*BINV & 2*BINV \\ 4*BINV & -BINV \end{pmatrix}$$

AND BINV IS THE BLOCK MATRIX

$$BINV = \frac{1}{10} \begin{pmatrix} CINV & CINV \\ -5*CINV & 5*CINV \end{pmatrix}$$

AND CINV IS THE MATRIX

$$CINV = \frac{1}{12} \begin{pmatrix} 30 & -20 & -5 & -2 \\ 18 & -8 & -5 & -2 \\ 12 & -8 & 1 & -2 \\ 6 & -4 & -1 & 2 \end{pmatrix}$$

EIGENVALUES: BELOW W=SQRT(-1)

$$\begin{aligned} EIVAL(1) &= 60+W*20 \\ EIVAL(2) &= 60-W*20 \\ EIVAL(3) &= 45+W*15 \\ EIVAL(4) &= 45-W*15 \\ EIVAL(5) &= 30+W*10 \\ EIVAL(6) &= 30-W*10 \\ EIVAL(7) &= 15+W*5 \\ EIVAL(8) &= 15-W*5 \\ EIVAL(9) &= -12+W*4 \\ EIVAL(10) &= -12-W*4 \\ EIVAL(11) &= -9+W*3 \\ EIVAL(12) &= -9-W*3 \\ EIVAL(13) &= -6+W*2 \\ EIVAL(14) &= -6-W*2 \\ EIVAL(15) &= -3+W \\ EIVAL(16) &= -3-W \end{aligned}$$

THE ABOVE EIGENVALUES CAN BE OBTAINED BY THE PROPOSITION GIVEN IN THE REMARKS FOR THE MATRIX 010002 COMBINING THE EIGENVALUES GIVEN BELOW.

$$\begin{aligned} EIVAL(1) &= -1 \quad \text{FOR THE COEFFICIENT MATRIX OF A} \\ EIVAL(2) &= 5 \end{aligned}$$

$$\begin{aligned} EIVAL(1) &= 3+W \quad \text{FOR THE COEFFICIENT MATRIX OF B} \\ EIVAL(2) &= 3-W \end{aligned}$$

$$\begin{aligned} EIVAL(1) &= 1 \quad \text{FOR THE MATRIX C} \\ EIVAL(2) &= 2 \\ EIVAL(3) &= 3 \\ EIVAL(4) &= 4 \end{aligned}$$

EIGENVECTORS: THE CORRESPONDING EIGENVECTORS OF EACH OF THE ABOVE MATRICES ARE GIVEN BELOW, AND THE EIGENVECTORS OF THE MATRIX A CAN BE OBTAINED BY THE PROPOSITION REFERRED TO ABOVE.

EIVEC(1,1)= 1 FOR THE COEFFICIENT MATRIX OF A
EIVEC(2,1)=-1
EIVEC(1,2)= 1
EIVEC(2,2)= 2

EIVEC(1,1)= 1 FOR THE COEFFICIENT MATRIX OF B
EIVEC(2,1)= 2-W
EIVEC(1,2)= 1
EIVEC(2,2)= 2+W

EIVEC(1,1)= 4 FOR THE MATRIX C
EIVEC(2,1)= 3
EIVEC(3,1)= 2
EIVEC(4,1)= 1
EIVEC(1,2)= 3
EIVEC(2,2)= 3
EIVEC(3,2)= 2
EIVEC(4,2)= 1
EIVEC(1,3)= 2
EIVEC(2,3)= 2
EIVEC(3,3)= 2
EIVEC(4,3)= 1
EIVEC(1,4)= 1
EIVEC(2,4)= 1
EIVEC(3,4)= 1
EIVEC(4,4)= 1

DETERMINANT: DET(A)=1296(16)

CONDITION NUMBERS:

N	C(1)	C(2)	C(E)
16	1.99(3)	8.99(2)	1.12(3)

REMARKS: THIS MATRIX WAS USED AS A TEST MATRIX BY EBERLEIN (REF. 1) AND BY PARLETT (REF. 2).

REFERENCES:

1. P. J. EBERLEIN, A JACOBI-LIKE METHOD FOR THE AUTOMATIC COMPUTATION OF EIGENVALUES AND EIGENVECTORS OF AN ARBITRARY MATRIX, JSIAM, V10, PP74-88, YR1962.
2. BERESFORD PARLETT, LAGUERRE'S METHOD APPLIED TO THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18, PP464-485, YR1964.

IDENTIFICATION: 010005

MATRIX: A IS THE BLOCK MATRIX
A= B 2*B

4*B 3*B
 AND B IS THE BLOCK MATRIX
 B= 3*C 3*C
 5*C C
 AND C IS THE BLOCK MATRIX
 6*D -D D 0
 C= 8*D 0 D 2*D
 -2*D 0 D 2*D
 5*D -D -D D
 AND D IS THE MATRIX
 -2 2 2 2
 D= -3 3 2 2
 -2 0 4 2
 -1 0 0 5

CHARACTERISTICS: REAL, NONSYMMETRIC, ALL EIGENVALUES
 ARE COMPLEX.

INVERSE: AINV IS THE BLOCK MATRIX

AINV= 1/5 -3*BINV 2*BINV
 4*BINV -BINV

AND BINV IS THE BLOCK MATRIX

BINV= 1/12 -CINV 3*CINV
 5*CINV -3*CINV

AND CINV IS THE BLOCK MATRIX

0 DINV -DINV 0
 CINV= 1/10 -6*DINV 6*DINV -4*DINV -4*DINV
 4*DINV 0 2*DINV -4*DINV
 -2*DINV DINV 3*DINV 2*DINV

AND DINV IS THE MATRIX

30 -20 -5 -2
 DINV= 1/12 18 -8 -5 -2
 12 -8 1 -2
 6 -4 -1 2

EIGENVALUES: BELOW W=SQRT(-1)

EIVAL(1)= 360+W*120
 EIVAL(2)= 360-W*120
 EIVAL(3)= 270+W*90
 EIVAL(4)= 270-W*90
 EIVAL(9)= 180+W*60
 EIVAL(10)= 180-W*60
 EIVAL(5)= 120+W*240
 EIVAL(6)= 120-W*240
 EIVAL(7)= 90+W*180
 EIVAL(8)= 90-W*180
 EIVAL(13)= -120+W*40
 EIVAL(14)= -120-W*40
 EIVAL(11)= 60+W*120
 EIVAL(12)= 60-W*120
 EIVAL(15)= 90+W*30
 EIVAL(16)= -90+W*30
 EIVAL(17)= -90-W*30
 EIVAL(18)= 90-W*30
 EIVAL(21)= -72+W*24
 EIVAL(22)= -72-W*24
 EIVAL(27)= -60+W*20

EIVAL(28)= -60-W*20
 EIVAL(19)= -40+W*80
 EIVAL(20)= -40-W*80
 EIVAL(29)= -54+W*18
 EIVAL(30)= -54-W*18
 EIVAL(23)= 30+W*60
 EIVAL(24)= -30+W*60
 EIVAL(25)= -30-W*60
 EIVAL(26)= -30+W*60
 EIVAL(37)= -36+W*12
 EIVAL(38)= -36-W*12
 EIVAL(31)= -24+W*48
 EIVAL(32)= -24-W*48
 EIVAL(39)= -30+W*10
 EIVAL(40)= -30-W*10
 EIVAL(33)= -20+W*40
 EIVAL(34)= -20-W*40
 EIVAL(35)= -18+W*36
 EIVAL(36)= -18-W*36
 EIVAL(41)= -12+W*24
 EIVAL(42)= -12-W*24
 EIVAL(43)= 24+W*8
 EIVAL(44)= 24-W*8
 EIVAL(45)= 10+W*20
 EIVAL(46)= 10-W*20
 EIVAL(47)= 18+W*6
 EIVAL(48)= -18+W*6
 EIVAL(49)= -18-W*6
 EIVAL(50)= 18-W*6
 EIVAL(51)= 8+W*16
 EIVAL(52)= 8-W*16
 EIVAL(53)= 6+W*12
 EIVAL(54)= -6+W*12
 EIVAL(55)= -6-W*12
 EIVAL(56)= 6-W*12
 EIVAL(57)= 12+W*4
 EIVAL(58)= 12-W*4
 EIVAL(59)= 4+W*8
 EIVAL(60)= 4-W*8
 EIVAL(61)= 6+W*2
 EIVAL(62)= 6-W*2
 EIVAL(63)= 2+W*4
 EIVAL(64)= 2-W*4

THE ABOVE EIGENVALUES CAN BE OBTAINED BY THE
 PROPOSITION GIVEN IN THE REMARKS FOR THE MATRIX
 010002 COMBINING THE EIGENVALUES GIVEN BELOW.

EIVAL(1)=-1 FOR THE COEFFICIENT MATRIX OF A
 EIVAL(2)= 5

EIVAL(1)=-2 FOR THE COEFFICIENT MATRIX OF B
 EIVAL(2)= 6

EIVAL(1)= 1+W*2 FOR THE COEFFICIENT MATRIX OF C
 EIVAL(2)= 1-W*2
 EIVAL(3)= 3+W
 EIVAL(4)= 3-W

EIVAL(1)= 1 FOR THE MATRIX D
 EIVAL(2)= 2
 EIVAL(3)= 3
 EIVAL(4)= 4

EIGENVECTORS: THE CORRESPONDING EIGENVECTORS OF EACH
 OF THE ABOVE MATRICES ARE GIVEN BELOW, AND THE
 EIGENVECTORS OF THE MATRIX A CAN BE OBTAINED BY THE
 PROPOSITION REFERRED TO ABOVE.

EIVEC(1,1)= 1 FOR THE COEFFICIENT MATRIX OF A
 EIVEC(2,1)=-1
 EIVEC(1,2)= 1
 EIVEC(2,2)= 2

EIVEC(1,1)= 3 FOR THE COEFFICIENT MATRIX OF B
 EIVEC(2,1)=-5
 EIVEC(1,2)= 1
 EIVEC(2,2)= 1

EIVEC(1,1)= 0 FOR THE COEFFICIENT MATRIX OF C
 EIVEC(2,1)= W
 EIVEC(3,1)= W
 EIVEC(4,1)=-1
 EIVEC(1,2)= 0
 EIVEC(2,2)= W
 EIVEC(3,2)= W
 EIVEC(4,2)= 1
 EIVEC(1,3)= 1
 EIVEC(2,3)= 3-W
 EIVEC(3,3)= 0
 EIVEC(4,3)= 1
 EIVEC(1,4)= 1
 EIVEC(2,4)= 3+W
 EIVEC(3,4)= 0
 EIVEC(4,4)= 1

EIVEC(1,1)= 4 FOR THE MATRIX D
 EIVEC(2,1)= 3
 EIVEC(3,1)= 2
 EIVEC(4,1)= 1
 EIVEC(1,2)= 3
 EIVEC(2,2)= 3
 EIVEC(3,2)= 2
 EIVEC(4,2)= 1
 EIVEC(1,3)= 2
 EIVEC(2,3)= 2
 EIVEC(3,3)= 2
 EIVEC(4,3)= 1
 EIVEC(1,4)= 1
 EIVEC(2,4)= 1
 EIVEC(3,4)= 1
 EIVEC(4,4)= 1

DETERMINANT: DET(A)= 12**32*3**16(64)

CONDITION NUMBERS:

N C(1) C(2) C(E)

64 2.93(4) 7.27(3) 1.20(4)

REMARKS: THIS MATRIX HAS BEEN USED BY PARLETT AS A
TEST MATRIX (REF. 1).

REFERENCES:

1. BERESFORD PARLETT, LAGUERRE'S METHOD APPLIED TO
THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18,
PP464-485, YR1964.

IDENTIFICATION: 010006

MATRIX: A IS THE BLOCK MATRIX

$$A = \begin{pmatrix} 8*B & 4*B \\ -5*B & -B \end{pmatrix}$$

AND B IS THE BLOCK MATRIX

$$B = \begin{pmatrix} C & 2*C \\ 4*C & 3*C \end{pmatrix}$$

AND C IS THE BLOCK MATRIX

$$C = \begin{pmatrix} 4*D & 3*D \\ -2*D & -D \end{pmatrix}$$

AND D IS THE MATRIX

$$D = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

CHARACTERISTICS: REAL, NON-SYMMETRIC, REAL AND COMPLEX

INVERSE: AINV IS THE BLOCK MATRIX

$$AINV = \begin{pmatrix} 1/12 & -BINV & -4*BINV \\ 5*BINV & 8*BINV \end{pmatrix}$$

AND BINV IS THE BLOCK MATRIX

$$BINV = \begin{pmatrix} 1/5 & -3*CINV & 2*CINV \\ 4*CINV & -CINV \end{pmatrix}$$

AND CINV IS THE BLOCK MATRIX

$$CINV = \begin{pmatrix} 1/2 & -DINV & -3*DINV \\ 2*DINV & 4*DINV \end{pmatrix}$$

AND DINV IS THE MATRIX

$$DINV = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

EIGENVALUES: BELOW W= SQRT(-1)

$$\begin{aligned} EIVAL(1) &= 3 \\ EIVAL(2) &= 4 \\ EIVAL(3) &= 6 \\ EIVAL(4) &= 1.5+W*1.5*SQRT(3) \\ EIVAL(5) &= 1.5-W*1.5*SQRT(3) \\ EIVAL(6) &= -1.5-W*1.5*SQRT(3) \\ EIVAL(7) &= -1.5+W*1.5*SQRT(3) \end{aligned}$$

EIVAL(8) = 8
EIVAL(9) = $2+W*2*\text{SQRT}(3)$
EIVAL(10) = $2-W*2*\text{SQRT}(3)$
EIVAL(11) = $-2-W*2*\text{SQRT}(3)$
EIVAL(12) = $-2+W*2*\text{SQRT}(3)$
EIVAL(13) = $3+W*3*\text{SQRT}(3)$
EIVAL(14) = $3-W*3*\text{SQRT}(3)$
EIVAL(15) = $-3-W*3*\text{SQRT}(3)$
EIVAL(16) = $-3+W*3*\text{SQRT}(3)$
EIVAL(17) = -15
EIVAL(22) = -20
EIVAL(23) = -30
EIVAL(24) = $7.5+W*7.5*\text{SQRT}(3)$
EIVAL(25) = $7.5-W*7.5*\text{SQRT}(3)$
EIVAL(26) = $-7.5-W*7.5*\text{SQRT}(3)$
EIVAL(27) = $-7.5+W*7.5*\text{SQRT}(3)$
EIVAL(28) = -40
EIVAL(29) = $10+W*10*\text{SQRT}(3)$
EIVAL(30) = $10-W*10*\text{SQRT}(3)$
EIVAL(31) = $-10-W*10*\text{SQRT}(3)$
EIVAL(32) = $-10+W*10*\text{SQRT}(3)$
EIVAL(33) = $15+W*15*\text{SQRT}(3)$
EIVAL(34) = $15-W*15*\text{SQRT}(3)$
EIVAL(35) = $-15-W*15*\text{SQRT}(3)$
EIVAL(36) = $-15+W*15*\text{SQRT}(3)$
EIVAL(37) = $20+W*20*\text{SQRT}(3)$
EIVAL(38) = $20-W*20*\text{SQRT}(3)$
EIVAL(39) = $-20-W*20*\text{SQRT}(3)$
EIVAL(40) = $-20+W*20*\text{SQRT}(3)$
EIVAL(18) = $4+W*4*\text{SQRT}(3)$
EIVAL(19) = $4-W*4*\text{SQRT}(3)$
EIVAL(20) = $-4-W*4*\text{SQRT}(3)$
EIVAL(21) = $-4+W*4*\text{SQRT}(3)$

THE ABOVE EIGENVALUES CAN BE OBTAINED BY THE
PROPOSITION GIVEN IN THE REMARKS FOR THE MATRIX
010002 COMBINING THE EIGENVALUES GIVEN BELOW.

EIVAL(1) = 3 FOR THE COEFFICIENT MATRIX OF A
EIVAL(2) = 4

EIVAL(1) = -1 FOR THE COEFFICIENT MATRIX OF B
EIVAL(2) = 5

EIVAL(1) = 1 FOR THE COEFFICIENT MATRIX OF C
EIVAL(2) = 2

EIVAL(1) = -1 FOR THE MATRIX D
EIVAL(2) = $0.5+W*0.5*\text{SQRT}(3)$
EIVAL(3) = $0.5-W*0.5*\text{SQRT}(3)$
EIVAL(4) = $-0.5+W*0.5*\text{SQRT}(3)$
EIVAL(5) = $-0.5-W*0.5*\text{SQRT}(3)$

EIGENVECTORS: THE CORRESPONDING EIGENVECTORS OF EACH
OF THE ABOVE MATRICES ARE GIVEN BELOW, AND THE
EIGENVECTORS OF THE MATRIX A CAN BE OBTAINED BY THE
PROPOSITION REFERRED TO ABOVE.

EIVEC(1,1) = 4 FOR THE COEFFICIENT MATRIX OF A
EIVEC(2,1) = -5

EIVC(1,2)= 1
EIVC(2,2)=-1

EIVC(1,1)= 1 FOR THE COEFFICIENT MATRIX OF B
EIVC(2,1)=-1
EIVC(1,2)= 1
EIVC(2,2)= 2

EIVC(1,1)= 1 FOR THE COEFFICIENT MATRIX OF C
EIVC(2,1)=-1
EIVC(1,2)= 3
EIVC(2,2)=-2

EIVC(1,1)= 1 FOR THE MATRIX D
EIVC(2,1)= 0
EIVC(3,1)= 1
EIVC(4,1)= 0
EIVC(5,1)= 1
EIVC(1,2)=-1-W*SQR(3)
EIVC(2,2)= -W*2*SQR(3)
EIVC(3,2)= 2-W*2*SQR(3)
EIVC(4,2)= 3-W*SQR(3)
EIVC(5,2)= 2
EIVC(1,3)=-1+W*SQR(3)
EIVC(2,3)= W*2*SQR(3)
EIVC(3,3)= 2+W*2*SQR(3)
EIVC(4,3)= 3+W*SQR(3)
EIVC(5,3)= 2
EIVC(1,4)= 1-W*SQR(3)
EIVC(2,4)= 2
EIVC(3,4)= 0
EIVC(4,4)= 1-W*SQR(3)
EIVC(5,4)= 2
EIVC(1,5)= 1+W*SQR(3)
EIVC(2,5)= 2
EIVC(3,5)= 0
EIVC(4,5)= 1+W*SQR(3)
EIVC(5,5)= 2

DETERMINANT: DET(A)= 12**20(20)

CONDITION NUMBERS:

N	C(1)	C(2)	C(E)
40	7.64(3)	4.42(3)	7.15(3)

REMARKS: THIS MATRIX HAS BEEN USED AS A TEST MATRIX BY EBERLEIN (REF. 1) AND BY PARLETT (REF. 2). IN EBERLEIN'S PAPER THE EIGENVALUES LISTED SHOULD BE MULTIPLIED BY 2. IN PARLETT'S PAPER FOUR EIGENVALUES ARE OMITTED, THEY ARE IDENTIFIED HERE AS EIVAL(18), EIVAL(19), EIVAL(20), AND EIVAL(21).

REFERENCES:

1. P. J. EBERLEIN, A JACOBI-LIKE METHOD FOR THE AUTOMATIC COMPUTATION OF EIGENVALUES AND EIGENVECTORS OF AN ARBITRARY MATRIX, JSIAM, V10, PP74-88, YR1962.

2. BERESFORD PARLETT, LAGUERRE'S METHOD APPLIED TO
THE MATRIX EIGENVALUE PROBLEM, MATH COMP, V18,
PP464-485, YR1964.

4. Hardware

The work this year has included designing the multiple terminal display system, upgrading the PDP-8 to a state in which it can adequately perform system tasks such as symbolic editing and assembly in a high speed stand-alone state, linking the PDP-8 directly to the IBM 360/75 and performing the required hardware maintenance on the PDP-8 and PDP-7. Reports on the latter are found in the quarterly progress reports, so will not be elaborated further here.

A display processing unit (DPU) has been designed (see Hostovsky, R. Design of a Display Processing Unit in a Multi-Terminal Environment, DCS Report No. 343). It will "execute" display files from the PDP-8 memory and generate an incremental display code to drive a scope. This will be buffered into a small core and then put on the disk. Since the DPU can (sequentially) execute many different display files and put the results onto different tracks of the disk, multiple displays can be achieved by adding low cost display controllers and scopes as described in Hostovsky, R., et.al. Display Controller, DCS File No. 829. One such display controller is now working from the disk, although the incremental code must at present be generated by the PDP-8. An alternative to the disk storage of the incremental code is to drive a storage tube display directly from the output of the display processing unit. This is under study. In addition, a PDP-8/I--connected to the PDP-8 via a high speed CPU-CPU link--is available to aid in this evaluation. It can be programmed to simulate the design of the DPU, and generate incremental code for the disk or storage tube displays as desired.

During the year the disk was interfaced to the PDP-8 and made available for program and data storage as well as storage of incremental display code. However, the disk is still giving problems. It appears to be very temperature and power supply sensitive. The present software reads back any information written onto it and compares it with the original before continuing. This makes it possible to operate, but it slows down the execution by a factor of five or more (depending on the current temperament of the disk). The disk connection is described in

Lopeman, H. and Miller, R. Data Disk Control, DCS File No. 818. A second disk interface has been designed, built, and is in the final debugging stage. It is hoped that this interface will have a sharply reduced error rate, as well as being more suited to digital program storage. The addition of an Inktronic printer to the PDP-8 allows the programmer to work with the system in a stand-alone mode for editing, assembly and debugging of the large PDP-8 programs. The disk and the Inktronic printer have helped to overcome the problem that we have been unable to get a partition allocated to us in the 360 on any permanent basis. We can debug most of the 360 system via batch and only need full online operation for debugging the communications packages and monitors.

To provide for greater speed and flexibility, and to remove the graphics from the timesharing partition, the PDP-8 has been interfaced directly to an IBM 2701 communications adapter in a similar manner to the PDP-7 to 360 interface. Previously the PDP-8 was connected to the PDP-7 and could communicate as another timesharing terminal with a low data rate of 100 characters per second. The new interface is described in Carter, C. E., et.al. DEC PDP-8 Interface to the IBM 2701 PDA, DCS Report No. 372.

5. Timesharing

The responsibility for the timesharing system was handed over to the service computation group during this year as it was felt that the project had passed beyond the research stage and into the routine service and maintenance stage. Before relinquishing it we rewrote the file handling system. The status of the PL/I interpreter is described in the quarterly reports for the 4th Quarter 1969 and 1st Quarter 1970.

ANNUAL REPORT PART B
UNCOMMON FORMS OF INFORMATION PROCESSING
SEPTEMBER 1, 1969 TO AUGUST 31, 1970

1. SUMMARY OF ACHIEVEMENTS (PART B)

The last contract period was concerned with the following projects:

B-2: OLFT. This is the On-Line-Fourier Transform system using a KDP crystal (charged by an electron beam) as an "instant transparency". Coherent light is used to generate the Fourier Transform by purely optical means. In phase two it was desired to optimize the present system and to determine the specifications of a possible improved version. This has been done successfully and the specifications for a new system are now well determined (see proposal).

B-6: Creep Vidicon. This is the project involving a special vidicon in which the target is a photo-conductive plate receiving "boundaries" in the form of dark lines. Each illuminated island is (uniformly) charged and its potential -- and therefore its "interior points" -- detected in a subsequent scan. Unfortunately the only bid (ITT, Fort Wayne) was unacceptably high, and consequently the project was definitively shelved.

B-7: TRANSFORMATRIX. This was one of the two main projects for the contract period. Here the goal is to produce an "image transformer", in which 1024 output points show (programmed) linear combinations of the light values incident on 1024 input points. In particular Fourier Transforms can be produced on-line since the machine -- using stochastic methods -- contains 1024 parallel channels. The input and output sections as well as the 1024-fold arithmetic unit are completed and tested, and only the "coefficient generator" portion must be installed. It turned out that pattern recognition facilities and means for picture enhancement could be incorporated without an undue increase in hardware. If anything, TRANSFORMATRIX is ahead of schedule.

B-10: BLAST. (Binocular Lens Alternate Scan Television) This project exploits the idea of writing on each TV line "left eye" and "right eye" information and positioning in front of the phosphor vertical cylindrical lenses, so as to

visually separate the two information sets. The special tube, built by Electro-Vision, unfortunately showed several defects. Nevertheless it is clear at this point that the idea works. Some rework on the tube will, however, be necessary.

B-9: LASCOT. (Large Screen Color Television) This project, carried as B-9 in the contract period 1968/1969 -- and supposedly cancelled for 1969/1970 -- turned out to have implications which did not fall under project B-11 (STEREOMATRIX, see below). Consequently the project was revived but using color. The crux of the matter is to use a standard color TV-set as a buffer, scanning the screen in synchronism with vibrating mirrors used simultaneously for the projection of three colored light beams. Preliminary results look very encouraging.

B-11: STEREOMATRIX. This is the on-line 3-D display for computer graphics using two laser beams, polarized at right angles to each other. Variable windowing, read-in with a pointer and perspective modification with the observer's position are features of the system. During the contract period the hardware design (including some printed circuit layout) was completed and all prototype circuits tested. About 90% of all IC's have been ordered. The project is probably 3-4 months behind schedule, due to insufficient staffing. Since it is the second major undertaking of the group, efforts will be made to increase the staff for the (final) phase next year.

B-15: EIDOLYZER. (Eidon = picture). This project (based on preliminary work under an ONR contract) aims to show that recognition of simple colored landscapes does not necessitate mask-matching. The fundamental idea is to compare a sequence of horizontal colored bands to a "world model", in order to hypothesize such sequences as "sky-grass-pond-field". The design of all circuits in bread-board form is complete.

B-16: SEMANTRIX. In this work colored blocks are manipulated according to a "world model", so that the assembly represents a "possible picture". The "world model" is kept inside a general purpose computer, which is also in charge of the mechanical devices. The design of the "locator" for blocks of given attributes (color) is complete and a portion of the mechanical part is ready.

2. DETAILS OF ACHIEVEMENTS (PART B)

For a quarter-by-quarter report on all projects, giving full details, drawings, and photographs, the Quarterly Progress Reports for the contract period should be consulted. For convenience, however, we shall give below an overall view of the progress on each project.

B-2: OLFT. Experimental work on the OLFT system has concentrated on maximizing the performance of the present form of the electro-optical light valve (Ardenne tube). Specifically, the erase gun circuits and the secondary electron collection electrode have been thoroughly tested, as well as "high-resolution" 5-mil crystals. Erase gun operation improves resolution and reduces "lag", but is difficult to control because of variations in critical parameters such as crystal time-constant, and secondary-electron crossover-voltage, both of which are a function of temperature. The 5-mil crystals -- the best available -- produced a resolution of about 200 line-pairs. Since the goal is to obtain 500 line-pairs, it appears that a larger surface will become necessary. In addition, these very thin crystals are subject to stress birefringence, interference patterns, and severe surface "etching" caused by the high energy (20kV) write beam. In short, we have determined that the present light-valve system is severely limited by low resolution and lack of control of critical parameters. A theoretical investigation was therefore started to design a reflex-mode light-valve with its KDP crystal cooled to its Curie temperature (about -75°C), which should have ten times the resolution and require only three per cent of the modulation voltage of an identical crystal used at room-temperature. Depending on the available funds such an improved Pockels-Effect chamber (or "Ardenne Tube"), will be subcontracted for in the near future.

The attempts to obtain a CaBr adaptive filter (bleached by a "preceding" Fourier Transform) have shown that a new laser (argon) must be used. It also appears that it will be easier to pick up the Fourier Transform by a vidicon (necessitating a reference beam, i.e. using a Fourier Hologram) and to filter electronically, reconvert the hologram into a (filtered) image by a second pass through the system. The proposal gives further details.

B-6: Creep Vidicon. As mentioned in the summary, this project had to be abandoned due to outrageously high bids for an experimental version, namely \$45,000. It should be stated that this was actually the salary for a two-man team in Fort Wayne, which was to interact with us over a period of one year.

Since the solution to the problems to be solved by the device (determination of "inside points" of connected regions), can be found elsewhere (see "TRICOLOR CARTOGRAPH" for the preceding contract period), it seems wise at this point to abandon the project once and for all.

B-7: TRANSFORMATRIX. In addition to the Fourier Transform and the Coordinate Transform, a third type of operation has been incorporated into the Transformatrix system. This transformation is a form of pattern recognition by which the system can detect any 5 x 5 subsection of the input pattern selected by the operator, indicating its spatial location in the input pattern by a light dot in the output pattern. Also, the pattern recognition coefficient controls can be used to enhance the contrast of the input picture. The inclusion of this third operation resulted in only minor hardware modifications.

The majority of the logic and circuit design of the Transformatrix Coefficient Generator was completed during the past year. The scheme for producing the Fourier Transform coefficients requires 32 identical fast five bit adders and five bit encoders, operating in parallel to achieve the necessary speed. The adders and encoders have been designed and fabricated.

Once the coefficients needed for a particular output value are calculated (in digital form), they are stored (digitally) and converted into analog voltages. Since 1,024 coefficients are required per output point, an array of 1,024 5 bit registers and D/A converters is required. Because of the large number of identical circuits needed, a considerable amount of time was spent in the design and layout of the storage cell and D/A converter. Thirty-two such circuits are contained on a single large printed circuit board. In this way, thirty-two such boards contain all 1,024 coefficient storage registers and D/A converters. The design of the board has been finalized and the boards are entering production.

Approximately 80% of the digital control logic has been designed and fabricated. As yet not designed are the inverse coordinate transform analog circuitry for the Coordinate Transform and the integrating circuitry for the product Synchronous Random Pulse Sequences.

B-9: LASCOT. Figure 1 shows the schematic of LASCOT. The idea is to scan the picture received on a standard color television set and project a reproduction of it on a large screen.

* VERTICAL AND HORIZONTAL MIRRORS ARE SYNCHRONIZED

Figure 1. Schematic Block Diagram of LASCOT.

Scanning System -- It consists of a vertical deflection mirror and a horizontal deflection mirror with associated optics. The deflection mirrors are of the vibrating galvanometer type and are driven by a triangular waveform.

With half of these mirrors and lenses a spot on a TV screen is focused on the photomultipliers, where it is split into its different color components by the two dichroic mirrors. The output of the photomultipliers is fed to amplifiers and drivers.

The scanning system is working and has been tested. The diagrams of the triangular wave generator and photomultiplier detector are given in Figures 2 and 3.

Projection System -- A narrow collimated light beam is split into three principal colors (red, green and blue) by narrow band interference filters. The optical half power bandwidth of the filters is 10nm. These three colored beams are passed through three electro-optical light modulators (EOLMs) which receive their modulating signals from the driver outputs of the photomultipliers. The colored beams are combined with the help of dichroic mirrors and are sent to a set of vertical and horizontal deflection mirrors.

The EOLM used is the Crystalab EOM-3050A, which has a half wave retardation voltage of 450 volts at $6328\text{\AA}$. In order to avoid a 0 to 450 volt swing for the driving voltage, the EOLM is driven with a -225 to +225 voltage and a quarterwave plate is placed behind the polarizer.

B-10: BLAST. The pair of closed circuit television cameras has been received and is mounted on a special tripod on which the cameras can be adjusted for different optical axis angles and separation. Experiments were undertaken to investigate the stereo effect which could be achieved using a tri-color monitor and colored glasses. The results were quite striking.

The special cathode ray tube for BLAST was designed, quoted on, built and received by January 1, 1970. When received, the tube leaked and had to be returned to Electro-Vision Industries for repairs. After again receiving the tube and working with it, it was determined that the conducting stripes, which were to be used for synchronization, had become electrically disconnected in various regions of the tube face and could not be used. As yet, the tube has not been replaced or repaired. It appears that it may be possible to use the tube in its present state: By adjusting both the scan and the chopper frequency,

Figure 2. LASCOT Detector.

Figure 3. LASCOT Triangular Waveform Generator.

the chopped picture has been aligned under the lenses and optical separation of the images has been achieved. Due to non-linearities in the scan, this alignment has been achieved only in a small central region of the picture. It is felt that total alignment can be achieved by properly varying the chopper frequency across the picture and by linearizing the scan.

The video chopper and mixer have been built and are operating. The circuitry for recovering the signal from the conducting stripes has been built and tested using the tube and the signal from the intact sections of the screen has been recovered. If the tube is rebuilt, this circuitry can be used.

B-11: STEREOMATRIX.

Transformer -- The Stereomatrix "transformer" requires fast bipolar D/A converters, fast analog gates, fast analog multipliers and fast analog dividers. This year has been devoted completely to designing these circuits and testing them for satisfactory operation up to 10MHz. All power supplies and components have been procured.

Stereomatrix also requires about 55 analog gates (which can switch $\pm 10V$ at a 100MHz rate) for the windowing unit and a D/A converter. Commercially available units are both too slow and too costly. Thus a fast bipolar analog gate, which works up to 10MHz and can switch ± 15 volts, has been designed and tested. The total cost of all the components is much less and the electrical characteristics of the circuit much better than available units. A D/A converter utilizing these gates has been designed and a circuit board laid out.

Stereomatrix also contains twelve multipliers and one divider. These must operate up to 5MHz. In the beginning, commercially available multipliers were tested: The Intronic multiplier works up to 200KHz and costs \$300.00, as does the multiplier manufactured by Hybrid Systems, which works up to 4MHz. Thus, the total cost of the multipliers and divider would be about \$4,000.00. In order to reduce this expense, it was decided to design a multiplier which is both faster and less costly.

A linear transconductance multiplier has been designed and tested up to 20MHz, complete with its level shifting network. The total cost of the components in this multiplier is less than ten dollars. The advantage of this circuit is that its output can be directly summed with the output of the other multipliers, thus saving many fast-inverting op-amps. The multiplier can be used directly as a divider using only one fast inverting amplifier, rather than the two needed in other circuits.

In the size control, conventional methods for changing the gain of op-amp summers could not be used. The various methods available were studied, such as utilizing an FET transistor as a variable resistor. These methods are not suitable at high frequency and thus a "Raysistor" (a light sensitive resistor) is used to control the gain of the op-amp summer. A circuit which linearizes the Raysistor resistance was designed and tested.

Coefficient Generator and Cursor -- After surveying the market and analyzing the cost and feasibility of the digital and analog method of generating coefficients, the digital method was chosen: The digital system is much less expensive and more accurate.

The overall system of the Stereomatrix Coefficient Generator was determined and designed during the fall of 1969. More than ten types of D/A converters were studied and sampled before ordering the most suitable one.

The cursor generator and the inverse matrix transformer were designed in the winter of 1969. The inverse transformer is an analog device requiring fifty-eight analog multipliers and fifty operational amplifiers.

Other components and subsystems were studied and selected. This includes: (1) digital integrated circuits (about 1000 packages), (2) cabinets and bench for the entire machine, (3) power supplies for the entire system, and (4) digital displays (30 each) for rotation angle and translation indication. The above items were ordered by the end of January 1970.

One portion of the coefficient generator is now in the electronics shop for fabrication. Since there are 14 each of the same card required and each has about 30 ICs, a tester has been designed and constructed to test the cards. The work to be done at the present is the timing and control logic design. The operator control box is also under consideration.

Display -- Receipt of the Argon ion laser light source in July 69 seemed to be the start of a running battle to get it operating and keep it going. Two separate short circuits, apparently caused by shipping vibrations, necessitated the replacement of first the power supply in October and then the laser head in November. The power supply has subsequently been modified twice to improve its reliability. There is still some work to be done on it, however, to eliminate mistriggering of the SCR voltage controls at low power levels.

After much research concerning the state-of-the-art, availability, cost and delivery delays, acousto-optic light beam deflectors were selected and specifications written. Bids were received and the contract awarded to Orlando Research Corporation. Delivery of the deflectors is scheduled for fall, 1970. The question of how fast the beam can be moved from spot to spot caused some study of frequency modulation. Apparently the bandwidth of the wideband power amplifier which drives the ultrasonic transducer will allow a one megahertz rate of movement as desired. However, the varactor oscillator may not be able to change frequencies that rapidly, and most importantly, the acousto-optic crystal, which does the actual deflection, may be much slower.

The search for a projection screen has narrowed the choice to three possible rear projection types of which two samples are yet to be received. If these are not satisfactory, a front projection metallized lenticular screen will be used.

Preliminary designs of the optical mountings and some reflectivity measurements have been made, and the driver for the electro-optic cell intensity modulator is being designed.

Position Detector -- Preliminary experiments have shown the possibility of detecting an observer's position (transmitter!) by using 3m waves, emitted by a non-resonating antenna. The phase difference between two receiving antennas gives a reasonably good indication of the transmitter's position.

B-15: EIDOLYZER. A picture analysis system was designed based upon the idea that each level of context, proceeding from more general to specific, is used to constrain the inferences made concerning the identity of objects and conditions. In the present case, two levels of context will be considered. The gross context, level 1, is assumed, arbitrarily, to be landscape scenes. (See Figure 4 for block diagram.)

A slide of the desired scene is projected onto a 16 x 16 array of groups of photocells, the Scan Array. At each of the 256 points of the Scan Array, three photocells, reacting to only red, green or blue light respectively, will decode the color of the slide at that point. A total of five colors are used to make the slides: red, green, blue, brown and white.

The next step in the process is to determine the "color" of each of the 16 rows. It has been decided that if any row has a definite majority (e.g., 9 out of 16) of one color then that row shall be assigned that color (Color Detector).

Figure 4. EIDOLYZER: Block Diagram.

Simultaneous with the determination of the colors of the rows, the colors of the details in the rows are determined. The Region Analyzer then divides the rows into contiguous blocks of the same predominant color, while the Detail Analyzer assigns the corresponding details to the proper regions.

The color-plus-region information provides sufficient data to hypothesize the nature of the regions (level 2). This decision is subject to change if subsequent data -- details (level 3) or imperceived conditions like temperature and humidity (level 4) -- are strongly contradictory.

Finally, hypothesized conditions will be presented as labels of regions and details on a display panel.

In the past few months the circuitry involved in determining the colors of the rows and details, the location and sizes of the regions, and the nature of the scene has been designed. Printed circuit boards have been laid out, parts have been ordered, and wiring lists are being prepared.

The only problem remaining is the determination of the colors of the slide at each of the 256 groups of photocells. The input slide is projected through a vertical beam divider, with one image projected on a rear projection screen for display purposes, and the other image impinging on the photocell array in a light-tight compartment inside the cabinet. Since tungsten projector lamps emit considerably more energy in the "red" end of the visible spectrum than in the "blue" end, and, since most photocells react more strongly to red light, it is not difficult to determine if red or brown (a combination of red and green) light is impinging on a particular photocell. To determine if blue or green light is projected, however, a combination of light conversion and narrow bandpass filters are needed. These filters are obtained in a thin gelatin form, so that they can be punched out and inserted into the photocell by the manufacturer before the photocells are hermetically sealed. Presently, tests are being run on various photocells and filters to determine the combination which finds the color of the slide with the highest degree of accuracy.

B-16: SEMANTRIX.

Project Outline -- SEMANTRIX acts as a semantic jig-saw puzzle, where the adjective "semantic" means that the puzzle pieces are manipulated according to a "world model", which is meant to be in some sense representative of the real world.

Design Procedure -- The project falls naturally into two parts: the world model, which will be programmed in a general purpose computer, and the hardware for communicating with the blocks. The work so far has been concerned with the latter.

The requirements of the hardware are: (1) that the position and attributes of each block be communicated to the computer, and (2) repositioning of the blocks must be accomplished upon reception of a suitable signal from the computer. Several schemes were investigated to satisfy these requirements.

Operation -- The system selected associates a number with each block; the attributes are implicit in this, e.g.:

<u>Attribute</u>	<u>Block #</u>
Red	{1,2,3,4
Green	{3,5,7

An infra-red signal is transmitted with the block numbers encoded sequentially in the signal. The block, on the reception of its number, retransmits an infra-red signal, which is received by a scan device under the plane of blocks. The position of each block is then obtained from the position of the scan when it receives the retransmission.

A scene is constructed by the machine by first allowing it to choose a block at random and then complete the scene by referring to a big world model for subsequent block positions. The blocks are moved by means of an overhead electro-mechanical block selector.

The feasibility of the infra-red system was tested on a simulated board, and a low power (the blocks have to carry their own power supply) high dc noise immunity (ambient light has a high IR component) circuit was devised (see Figure 5), which operates with Motorola MRTL logic, used in a slightly modified mode.

The selector has been designed, and a positional control system has been built (see Figure 6 for block diagram, Figure 7 for circuit diagram) using proportional control based on pulse area signals. The techniques used here are those currently employed in model airplanes.

Figure 5. SEMANTRIX Infrared Sensor Circuit.

Figure 6. SEMANTRIX: Block Diagram of Positional Control System.

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PROPOSAL FOR THE PERIOD SEPTEMBER 1, 1970 to AUGUST 31, 1971

PART A: PROFESSOR GEAR; PART B: PROFESSOR POPPELBAUM

This proposal covers both Part A, i.e. the work of Professor Gear's group, and Part B, namely the work of Professor Poppelbaum's group. It should be noted that Part A uses new code numbers, namely A-1 for software, A-2 for theory, A-3 for hardware and A-4 for applications. Part B continues to use the code numbers introduced at the beginning of the contract so as to conserve continuity for all projects.

Part A proposes four areas of investigation:

1. Software. Work on the generalized simulation system will continue.
2. Theory. Work will be done on pivot selection for maximum accuracy, symbolic detection of singularities, and steady state and small signal analysis.
3. Hardware. No new projects will be started, but the display processing unit will be built and interfaced to the disc presently driving only one display unit.
4. Applications. Potential users will be contacted in order to select applications of the network analysis work.

Further details will be found in the following pages.

Part B proposes the following list of projects:

- B-2. Phase 3 of OLFT
- B-9. Completion of the Large Screen Color Television project
- B-10. Correction of certain design deficiencies in the BLAST tube
- B-11. Completion of STEREOMATRIX
- B-15. Assembly of EIDOLYZER
- B-16. Actual construction of SEMANTRIX
- B-17. Preliminary investigation of an image recoding scheme using magnetic bubbles in ortho-ferrite
- B-18. Random to serial converter for display units
- B-19. SCANTRIX, a line at a time image amplifier
- B-20. CERAVISION, a color TV system with synchronized ceramic filters.

It will be noted that B-17 through B-20 are new and correspond to our entry into the area of image recoding. Details of all these projects will be found in the subsequent pages.

PROPOSED PROGRAM PART A
SOFTWARE SYSTEMS RESEARCH
SEPTEMBER 1, 1970 to AUGUST 31, 1971

Summary

The proposed activity in the coming twelve months can be classified into the four areas:

- A-1. Software
- A-2. Theory
- A-3. Hardware
- A-4. Applications

The work in these areas will all be directed toward the central problem of generalized modeling and simulation. The interaction between the efforts in the different areas is a fundamental part of the research program as it is our belief that it is past time to study the solution of problems by the use of techniques from more than one area. However, in the next twelve months it is proposed to concentrate more of the efforts on the earlier items in the above list. Software will receive the most attention since we are now at a state in which the theory of a preliminary system is fully understood. Hence, its implementation should be completed in order to verify the concepts and experiment with the embodiment.

The ultimate goal of the work is to provide a flexible extendable system which can be used on a large class of problems, and hence to promote the study of the application of simulation in areas where it is not now productive. Additional theoretical problems must be tackled in order to allow the system to be extended and to execute more efficiently. The hardware and systems to support more than one console will continue to be studied, but at a reduced level because of decreasing budgets and attendant staffing problems.

1. Software

An overall flowchart of the system was given in Figure 1 of the Annual Report. The principal effort in software will be to complete the sections of the first version and to check out the complete system. The report indicates approximately the amount of work remaining in each. New sections that should be written this year include:

Diagnostic I/O

Tabular Data I/O

Graphic Data I/O

Procedural Control by Code

The diagnostic I/O section should allow the user to determine the value of any variable in the system and possibly to set it to a desired value. Tabular data output is the usual output facility for printing results. As input it provides a facility for specifying arbitrary functions. Internally an approximation must be fitted through the specified points. Graphic I/O is an extension of tabular I/O to a more responsive interactive medium. The user should be allowed to define functions by drawing their graphs on the tablet. Points from the graph would be sampled in order to fit polynomials to it. (It is not feasible to store all of the approximately 500 points that represent a graph on the face of a CRT.) Procedural control refers to the ability to state a procedure for the repeated analysis of a network with varying parameters. The user should be able to use iteration loops to request multiple analyses to study the behavior for different initial conditions or different values of certain elements.

2. Theory

Areas of theoretical interest to the development include:

1. Pivot selection for numerical accuracy as well as sparseness preservation.
2. Provision of diagnostics in the user's source language.

3. Symbolic detection of singularities, both those that do not permit a solution and those that allow multiple solutions. (One of these should be found.)
4. Provision of capabilities for the definition of primitive elements by procedural code.
5. Handling of discrete valued variables.
6. Steady state (DC) analysis of the system.
7. Analysis of small perturbations about a steady state (small signal AC analysis).
8. Isolation of inactive areas of networks to gain efficiency.

Initially it is planned to work on items 1, 2, and 6 since they are essential for the reasonable functioning of a system. Item 4 will receive some attention as soon as the preliminary system is working because it will extend the flexibility and ease of application. The other items need to be investigated when man power is available from the earlier tasks although it is unlikely that they will be started this year. A discussion of the problems to be solved for each item follows.

Pivot Selection. Currently we have implemented a method proposed by Dantzig for the selection of pivot elements to retain sparseness. This technique only considers the number of nonzero elements in each row and column. (We have modified it to also consider the complexity of the element, preferring to pivot on a simple element like the integer 1 or other small integers rather than complex expressions--see the 4th Quarterly Report 1969 p. 38.) Unfortunately, this may mean that the pivots could be small relative to other elements in the matrix being inverted, causing large numerical errors. We have observed that the numerical accuracy of the matrix inversion is not as important as other parts of the integration process since it only affects the rate of convergence of the corrector iteration, so we can tolerate fairly poor pivot selection. It will only increase the execution time of the integration slightly. However, when a very bad pivot selection is made,

convergence may be prevented. The pivot selection is made before the integration starts in order to generate code for the multiplication of a vector by the inverse. Since the numerical values of many of the elements of the matrix are not known at that time, it is impossible to guarantee a good numerical choice. The most promising solution to the problem is to find some method of detecting that the choice is no good during the integration (this may change with real time as the values of variables change), and then to redo the inversion using both the sparseness information and the now available numerical information on the sizes of the elements. It is not obvious how to detect that the pivot choice is now too bad. Two possibilities to be examined are to use the nonconvergence of the corrector iteration as an indicator (although this can also occur because the step size is too large or the problem very ill-conditioned) or to compile code which tests some of the critical elements along with the matrix inversion code.

Diagnostics. During the course of an integration it is possible that one of the variables will get out of range or that the integrator will be unable to converge because of singularities in the system. In the former case it is certainly possible to report which variable has exceeded bounds by retaining all of the tables constructed during the various stages of translation. Unfortunately, there are many of these as there are four levels of translation (Item Analysis, Global Analysis, Elimination, and Compilation). It is important that a compact way be found to store the translation information in such a form that the address of an internal variable can be related to the information on the display and stored in the display computer (the PDP-8). This is nontrivial because a given variable internally may correspond to several variables on the display, due to the fact that any simple relations like $x = y$ have been eliminated and that a given node may have its "instance" in several different elements at several different macro levels.

The second problem, that of reporting a singularity, seems less tractable. By the time that the numerical stage has been reached, it is difficult to see how a given variable or set of variables can be "blamed"

for causing a singularity, although it is desirable to be able to tell the user that the singularity is due to a particular part of his network, such as two voltage generators connected in parallel (in network analysis). It is more probable that a solution to this problem will have to be sought symbolically as suggested below.

Symbolic Detection of Singularities. Since the system allows arbitrary elements to be defined, it does not seem possible to specify interconnection rules which will prevent singular networks from being constructed. Also it does not seem possible to find straightforward analysis rules which will eliminate unimportant singularities. As an example of the former type, suppose the user has defined the element

$$E(1) = E(2) - 1.5$$

$$I(1) = -I(2)$$

and draws a network that includes

The equations that the global analysis will generate for this will include

$$F_1(E_1, E_2, \dots) = E_1 - E_2 + 1.5 = 0$$

$$F_2(E_1, E_2, \dots) = E_1 - E_2 - 1.5 = 0$$

It is evident that these do not have a solution! This will become numerically apparent when the Jacobian of the system must be inverted.

An example of the second type of problem occurs when the simple network

is considered. Initially the set of four equations

$$F_1 = E_1 - E_2 + 1.5 = 0$$

$$F_2 = E_1 - E_2 - I_1 * R = 0$$

$$F_3 = I_1 + I_2 = 0$$

$$F_4 = -I_1 - I_2 = 0$$

will be generated by global analysis.

The Jacobian is

$$J = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & -R & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

and is singular. However, $F_3 = -F_4$ so the last two equations can be simultaneously satisfied leaving one degree of freedom, in this case the choice of either E_1 or E_2 . An electrical network analysis program handles this problem by considering only potential differences across elements ($E_1 - E_2$) and ignoring one of the Kirchoff current laws ($I_1 + I_2 = 0$). Unfortunately, the generality of the proposed system does not allow us to assume that I-type variables necessarily sum to zero on elements (in fluid flow problems an element could be a reservoir) or that only potential differences are important (in analog computer networks we

usually look at absolute voltage and do not explicitly display the reference voltage connection). The only way out of these problems seems to be to generate the Jacobian symbolically and to try and invert it symbolically to see if it is singular.

Procedural Definitions. The currently being developed system requires that all elements be definable by means of algebraic and differential equations. It is desirable that it be possible to use available procedural languages (FORTRAN, Algol, etc.) to define more general elements. Now that we have combined differential and algebraic equations into a single equations into a single equation of the form $F(\underline{y}, \underline{y}', t) = 0$ it is certainly feasible to ask the user only to provide code to compute F when $\underline{y}$, $\underline{y}'$, and t are given. However, this will raise some problems in analysis and numerical solution. It is impossible to form the symbolic "Jacobian" of a general program, although the numerical partial derivatives can be computed locally. This will further complicate the problem of singularity detection. Since a program can include branches, the functions can be discontinuous. Since the numerical techniques rely heavily on polynomial extrapolation, methods will have to be found to detect the discontinuities in order to avoid extrapolating through them.

Discrete Variables. This is in a sense an extension of the previous problem to variables which are piecewise constant. However, that does not appear to be a profitable way to view it. The user may wish to define integer valued variables in order to combine discrete and continuous simulation techniques. The problem is to find techniques to effectively find the time at which the next discrete variable changes so as to make as large a step as possible without passing through such points.

Steady State Analysis. This problem must be solved for the transient analysis problem in order to start the integration. In this case, certain of the $\underline{y}$ variables are assigned values at $t = 0$, the remaining $\underline{y}$ values and values of the $\underline{y}'$ must be found from the algebraic

equations $\underline{F}(\underline{y}, \underline{y}', 0) = 0$ in order to begin the transient analysis integration. In the DC problem, we must solve $\underline{F}(\underline{y}, \underline{y}', t_0) = 0$ for $\underline{y}' = 0$. If a solution $\underline{y}_0, \underline{y}'_0$ is known to $\underline{F}(\underline{y}_0, \underline{y}'_0, t_0) = 0$, it is possible to integrate $\underline{F}(\underline{y}, \underline{y}', t_0) = 0$ to a steady state to solve the latter, so the transient analysis program can be used. However, there is still the problem of finding the starting values $\underline{y}_0, \underline{y}'_0$. A method being explored by one of Dennis's students (Computer Science Department, Cornell) is to consider

$$\underline{F}(\underline{x}) - e^{-t} \underline{F}(\underline{x}_0) = 0$$

At $t = 0$ this has the solution $\underline{x} = \underline{x}_0$, at $t = \infty$ its solution satisfies $\underline{F}(\underline{x}) = 0$. $\underline{x}$ satisfies the differential equation

$$\underline{x}' = -J^{-1}(\underline{x}) \underline{F}(\underline{x})$$

where $J(\underline{x}) = \partial \underline{F} / \partial \underline{x}$. By integrating this to convergence we find a solution of $\underline{F}(\underline{x}) = 0$. By integrating the differential equation

$$\underline{x}' = -J_n^{-1} \underline{F}(\underline{x})$$

instead, where J_n is $\frac{\partial \underline{F}}{\partial \underline{x}}$ at some point $\underline{x}_n$ we will converge to a solution of $\underline{F}(\underline{x}) = 0$ if an equilibrium is achieved. In either case, accuracy of solution is not important since any equilibrium point is a solution of $\underline{F}(\underline{x}) = 0$, so it is an ideal problem in which to use stiffly stable methods with large time steps. The advantage of the second equation is that its Jacobian is

$$\partial \underline{x}' / \partial \underline{x} = -J_n^{-1} J(\underline{x}) = -I + (\underline{x} - \underline{x}_n)$$

Consequently, for values of $\underline{x}$ near $\underline{x}_n$, the Jacobian of the differential equation need not be evaluated and the approximation $-I$ can be used. When corrector convergence fails, we can update $\underline{x}_n$ to the current value of $\underline{x}$ and recompute J_n . Thus, it appears that the current integration package may also handle the DC analysis with only minor modifications.

Small Signal Analysis. This is a study of small changes about the stable state

$\underline{F}(\underline{y}_0, 0, t_0) = 0$. The problem is linearized by writing

$$\underline{F}(\underline{y}, \underline{y}', t_0) = \frac{\partial \underline{F}}{\partial \underline{y}} (\underline{y} - \underline{y}_0) + \frac{\partial \underline{F}}{\partial \underline{y}'} \underline{y}' = 0$$

where the partials are evaluated at $(\underline{y}_0, 0, t_0)$ and the eigenvalue-vector problem for

$$\left(\frac{\partial \underline{F}}{\partial \underline{y}} + \lambda \frac{\partial \underline{F}}{\partial \underline{y}'} \right) \underline{y} = 0 \text{ is solved.}$$

Since the matrices involved are large and sparse it will be necessary to locate or develop some programs for the sparse general eigenvalue problem. To our knowledge none exist at this time.

Isolation of Inactive Areas. When the currently envisaged system is capable of handling large networks, it will find itself in situations in which part of the network being analyzed is inactive for long periods of time, e.g. switching type circuits undergo a brief rapid change of state and then are quiescent for relatively long periods. In order to improve execution speed it is desirable to find techniques to eliminate the inactive sections from the network and replace them with equivalent simpler passive elements and "detection devices" which will indicate when their activity must be reconsidered. The location of these subnetworks may not be possible automatically, but in an interactive system we could ask the user to indicate their boundaries.

3. Hardware

Support for the next twelve months will be used to maintain existing equipment used in the remainder of the project and to complete already started projects. No new projects will be started. The display processing unit described in Report No. 343 will be built and interfaced to the disk which is already driving one display unit. If time permits, an additional display will be connected to the system. Much of this is

critically dependent on the reliability of the disk. It may yet be necessary to modify the disk controller and slow down the data rate from 3 MHz to 2 MHz.

4. Applications

The design of the preliminary system was based on existing network analysis and simulation systems. When it has been implemented we propose to approach a number of potential users, both in the University and in other AEC laboratories in order to select one or more applications which will be studied in detail so that the system can properly evolve toward the next level. Problems in network analysis and continuous simulation will only allow us to compare the system with existing specialized systems for speed and flexibility. We should, therefore, attempt to locate problems, such as might occur in continuous process control, which are relatively difficult to analyze by conventional means.

PROPOSED PROGRAM PART B
UNCOMMON FORMS OF INFORMATION PROCESSING
SEPTEMBER 1, 1970 TO AUGUST 31, 1971

1. SUMMARY OF PROPOSAL (PART B)

TRANSFORMATRIX will hopefully be completed before the next contract period. The machine will then undergo acceptance tests and some attempts at heuristic programming will be made. This leaves considerable work on OLFT (Phase three), LASCOT, SEMANTRIX, and, above all, STEREOMATRIX. The completion of BLAST and EIDOLYZER reasonably early in the year is anticipated, so that we shall again turn towards a new area of research, namely Image Recoding. This corresponds to projects B-17 through B-20. We therefore aim at the following list:

B-2: OLFT (On-Line Fourier Transform). Completely new reflecting, low temperature Ardenne tube. New ion laser. Adaptive filtering by picking up the Fourier Hologram and processing it electronically.

B-9: LASCOT (Large Screen Color Television). Alignment of the scan looking at a commercial tricolor tube with the motion of the three colored beams used for projection. Elimination of "venetian blind" effect when the local system is out of synchronization.

B-10: BLAST (Binocular Lens Alternate Scan Television). Restoration of the burnt-off (synchronizing) metal "comb" behind the cylindrical lenses. Possible attempts to replace it by an ultraviolet phosphor, using UV detection for synchronization.

B-11: STEREOMATRIX. Layout of the remaining circuit boards for the "transformer", the "coefficient generator" and the "cursor". Acceptance tests for the ultrasonic deflectors to be delivered in the fall. Refinements of the "position detector". Overall alignment. It does not appear certain that all this can be concluded during the coming contract period.

B-15: EIDOLYZER (Eidon = picture). Actual assembly and testing of the system is anticipated in late autumn. The completion by January is virtually certain.

B-16: SEMANTRIX. Most of the construction on this project is yet to be done. Some difficulties with the mechanical "block mover" are anticipated, but the completion before summer 1971 is likely.

B-17: DOC (Digital to Optical Converter). This is the first one of the new Image Recoding Projects. The idea is to use "magnetic bubbles" in orthoferrite to form a binary matrix, and to employ polarized laser light to produce interference patterns (like Fourier Transforms) by reflection. Note that a DOC could replace an Ardenne tube as long as no grey-scale is desired and the picture elements are evenly spaced! In an alternate approach a ferroelectric ceramic is locally polarized by electric fields: then grey-scale operation would be possible.

B-18: RASER (Random to Serial Converter). This is a converter specifically designed for line drawings. It uses a fast core memory with a 512 bit word length: A random (x,y) input is stored in word #y, while x is represented by an appropriate "1" in the sequence of 512 bits. Raster scan display then simply amounts to scanning one word after the other.

B-19: SCANTRIX. Here use will be made of our "on the fly" encoding capability of light values (as in the ORBIT system of the period 1967/68) to store a whole TV line (512 points, 8 light values per point) in a buffer. The buffer is connected to 512 control circuits, driving 512 fast response lights in a linear array. This (very bright) array is projected onto a screen by a rotating mirror or its electro-optical equivalent.

B-20: CERAVISION. An ultra-cheap color display is obtained by associating with a black-and-white camera-monitor system voltage controlled color filters, made of ferroelectric ceramics. This is, of course, in essence a color wheel system, except that no synchronization problem arises. The on-line recoding of a commercial color signal is a natural for the Image Recoding Project.

2. DETAILS OF PROPOSAL (PART B)

B-2: OLFT

The OLFT system will be modified and further developed as follows: first, the existing light valve will be replaced with a cooled-crystal device for improved resolution; second, the system will be redesigned for hologram-pickup of the Fourier Transform, followed by electronic processing and light-valve reconstruction.

Low Temperature Operation. The disadvantages of the present light valve (low resolution, high steering potentials) are such that major modifications (larger area, thinner crystals) would be necessary to retain room-temperature operation. Operating with the crystal cooled to the Curie temperature T_c (about -75°C) gives definite advantages: resolution is improved by a factor of 10 over the same crystal operated at room temperature, and the modulation voltage required is reduced by a factor of 30. Further, other critical parameters, such as crystal time constant, are greatly improved and stabilized by controlling the temperature. It is proposed that the light valve be redesigned according to Figure 8. Included in the modifications would be a pulsed argon laser system to replace the present low-contrast, inefficient beam-shutter/cw helium-neon laser combination.

Fourier Transform Processor. The proposed modifications of the light valve should yield a resolution of about 500 line pairs. This is excellent for video reproduction and should be adequate for direct reconstruction of the Fourier Transform. That is, we propose an OLFT processing system (as shown in Figure 9) which uses optical processing to obtain the Fourier Transform of an image, direct electronic processing of the Fourier Transform, followed by reconstruction of the modified image. The Fourier Transform $F(u,v)$ would be obtained by hologram techniques, so that the phase content of F is preserved. That is, a reference beam Ae^{jkx} is added to F and "mixed" by the high-resolution vidicon target:

$$\left| Ae^{jkx} + F(u,v) \right|^2 = A^2 + F^2 + A(Fe^{-jkx} + F^*e^{jkx})$$

When the vidicon scans the target, the output consists of low-frequency material ($A^2 + F^2$) and an amplitude-modulated carrier (the time-scanned version of $A(Fe^{-jkx} + F^*e^{jkx})$). A good band-pass filter followed by a detector would demodulate the desired Fourier Transform into a standard video signal. The Fourier Transform is then processed (modified) by an appropriate computer and the new image can be reconstructed by a second pass through the light-

Figure 8. OLFT Low-Temperature Light Valve.

Figure 9. OLFT Processing System.

valve and vidicon. The advantage of this system is that the Fourier Transform is available in the form of an electronic signal, allowing very sophisticated processing schemes.

B-9: LASCOT

This coming year we shall incorporate for the first time the modulation of the three colored beams (see Figure 10) by picking off the steering signal, from a commercial color TV receiver. Since the initial idea of mechanically linking the "sampling" mirrors (both horizontal and vertical) to those used for light beam deflection was dropped, some difficulties are anticipated concerning phase-lags.

Although the very light oscillating mirrors can possibly be made to follow phase-jumps in the incoming TV-signal, it is not contemplated to phase-lock the local mirror system. Instead it is proposed to compensate for the degradation of luminosity on the phosphor (e.g., when the scanning photocells look at line i of the field rather than the line k being traced at that moment -- see Figure 11) by sensing the delay τ and controlling a non-linear amplifier of variable gain. If this gain is made proportional to $e^{(k-i)}$, the amplifier will correct the loss of signal strength, which is clearly of the form $e^{(i-k)}$.

B-10: BLAST

The main goal will be to improve the rather fortuitous synchronization of the left and right camera with the position of the beam behind the cylindrical lenses of the BLAST-tube. Means for doing this (namely a metallized grid which -- when struck by the beam -- gives off a signal) were incorporated in the original tube. Unfortunately the grid was burnt off in preliminary experiments. We shall therefore open the tube and deposit a double comb according to Figure 12, using a thick chromium film instead of the (transparent) thin film. The double-comb structure, suggested by Turner of the University of Essex, allows each half of the grid to go back to equilibrium potential, while the other half produces the synchronization signal.

A possible alternative, to be realized in a second BLAST tube, is to use ultraviolet emitting phosphor: this can be a thin transparent layer. The "beam crossing" will then correspond to a shower of UV photons, detected by a suitably filtered photocell in the neck of the tube. One big advantage of this design is that the synchronization signal is available at the potential

* VERTICAL AND HORIZONTAL MIRRORS ARE SYNCHRONIZED

Figure 10. LASCOT Block Diagram.

Figure 11. Venetian Blind Compensator for LASCOT.

Figure 12. New Design for BLAST Tube.

of the cathode, i.e., no "step-down" games have to be played with 25 KV potentials (masking the small metal-grid signals).

B-11: STEREOMATRIX

Figure 13 reiterates the overall design of STEREOMATRIX. The principal difference compared to the original design is the substitution of a single laser (with a beam splitter) for the original two laser configuration. By arranging for the polarization of this laser to be at 45° to the horizontal, it is possible to extract components at 90° to each other by the simple interposition of a horizontal and a vertical polarizer. The cost advantage of this arrangement is obvious. Another money-saving trick will be used in the deflection: the horizontal deflection will be common to both beams.

Our principal problem will be the realization of the 256 point resolution in the ultrasonic deflectors and the associated driving circuitry. The preliminary experiments at Orlando Research Corporation indicate that we are at the limit of -- but not beyond -- the state of the art.

The actual stuffing of the printed circuit boards and the overall assembly are the main time-consuming items for next year and will absorb about 80% of the manpower on the project.

Refinements in the "position detector" should lead to a resolution of about 1" instead of the present 1'.

B-15: EIDOLYZER

Figure 14 shows the block diagram of EIDOLYZER, Figure 15 the Scan Array with color detectors and row analyzer and Figure 16 the Region Analyzer. These circuits -- tested in bread-board form -- must now be transferred to printed circuit boards and the machine must be assembled. Since a small (2 x 2 point) model has worked at the required speeds, it is thought that no essential difficulties will be encountered in the final (16 x 16 point) version.

It is planned to extend some of the ideas of EIDOLYZER (i.e., pattern recognition without mask-matching and the use of a "world model") to the analysis of line drawings. In particular it is felt that a rather simple signal analysis will be capable of identifying polygons. Some preliminary experimentation, involving a circle of photocells looking at a polygon as it "expands", has already been done.

Figure 13. Principle of STEREOMATRIX.

Figure 14. EIDOLYZER: Block Diagram.

EIDOLYZER: REGION ANALYZER

Figure 16. EIDOLYZER: Region Analyzer.

Some of the work on this project, namely the more theoretical aspect, is supported by an ONR contract.

B-16: SEMANTRIX

Figure 17 shows the anticipated layout of the SEMANTRIX system. The "block mover" is a glorified x-y plotter with crane facilities. The circuits for the location of a block with given attributes (the color is encoded in a signal sequence!) are designed, but some mechanical problems with the mover remain.

B-17: DOC (New)

This project consists of two parts: a Digital-to-Optical Converter (DOC); and an Optical Fast Fourier Transform (FFT) system utilizing the DOC, a Fourier-Transform lens, and a vidicon pickup.

The DOC. This device consists of a pulsed laser light source and a planar array of "dots" which spatially amplitude-modulate the wavefront of the collimated laser beam (much like pinholes in an opaque screen). Each dot is an independent modulator as well as a storage location, controlled by digital address and write signals similar to those of a standard core memory. There are two devices which could serve as a DOC array: "magnetic bubbles" contained in an orthoferrite crystal substrate, and switchable domains contained in a ferroelectric ceramic substrate (f.e.c.e.s.). Both devices utilize local saturable domains both for electrical storage and optical modulation. Briefly, some properties of interest are:

	<u>mag. bubbles</u>	<u>ferro. ceramic</u>
1. substrate	orthoferrite (single crystal)	hot-pressed ceramic
2. thickness	~75 nm	~10 nm
3. domain size	~25 nm	~ 5 nm
4. modulation	Faraday effect	Electro-optic effect
5. % modulation, max.	~3	~20
6. control signal	~100 mA	~40V
7. stability	fair	good
8. gray-scale	binary	~4 levels min.

Figure 17. Block Diagram of SEMANTRIX.

In both devices the switching times are less than 10 μ s but typically more than 1 μ s. Thus, an array of 500 x 500 dots, if arranged in the form of 500 parallel shift registers, could easily store a complete video image in less than a frame period. However, the digital nature of the DOC storage feature is better adapted, perhaps, to more general computer image processing.

The DOC/FFT System. Suppose that the DOC-array consists of a rectangular grid of $(2M + 1)$ by $(2N + 1)$ points with x and y separations of A and B respectively (see Figure 18). Then, ignoring the shape of each spot, the optical wavefront leaving the DOC would be of amplitude

$$f(x,y) = \sum_{m=-M}^M \sum_{n=-N}^N f_{mn} \delta(x-mA) \delta(y-nB),$$

where f_{mn} is the amplitude of the dot at $(x,y) = (mA,nB)$, as in Figure 18.

Using a lens in the standard configuration (see Figure 19) we obtain the Optical Fourier Transform $F(u,v)$ of $f(x,y)$:

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy e^{-j(ux+vy)} f(x,y) \\ &= \sum_{m=-M}^M \sum_{n=-N}^N f_{mn} e^{-j(umA+vnB)} \end{aligned}$$

To obtain the phase content of $F(u,v)$, one uses a reference beam, together with a high resolution vidicon pickup, followed by a band pass-detector, as in the OLFT hologram system; alternately, a low-resolution vidicon followed by a high-speed square-root device might be possible, if suitable restrictions are acceptable. The result is a very high speed Fast Fourier Transform system with high resolution. Not only is the two-dimensional transform available, but by the following relationship:

$$f(t) = \sum_{l=-L}^L f_l \delta(t-lT)$$

$$\begin{aligned} f_{mn} &= f_l, & l &= m+n (2M + 1) \\ & & L &= M+N (2M + 1) \end{aligned}$$

(this is a simple sequential relation: $f_{10} = f_1$, $f_{20} = f_2$, etc.)

it can be shown that $F(w)$, the Fourier Transform of $f(t)$, is simply:

Figure 18. D.O.C. Array.

Figure 19. D.O.C.F.F.T. (High Resolution Pickup).

$$F(w) = F \left[u = w \frac{T}{A}, v = w \frac{T}{B} (\ell M + 1) \right]$$

Thus the DOC/FFT can provide fast Fourier Transforms of very long strings of one-dimensional images!

B-18: RASER (New)

The fundamental idea of RASER is depicted in Figure 20. Coordinate pairs (x,y) -- with nine bits for each coordinate -- come into the converter at random times. The y-part determines the address in a 512 word (of 512 bits each) plated wire memory with roughly 1 μ s cycle time. The x-part is converted into a "one out of 512" code, i.e. a "one" in a string of zeros giving the position in (quantized) analog fashion. By sequentially displaying one word after the other, we obviously obtain a raster scan or serial version of the stored information. Note that no grey scale is provided: This would considerably increase the cost. In practice the memory is actually of the 8K word-32 bit type; reasonably simple address recoding makes the use perfectly acceptable.

B-19: SCANTRIX (New)

The difficulty (and cost) in building large screen TV displays -- such as the one in LASCOT or STEREOMATRIX -- is due to the relatively complicated mechanical or electro-optical mechanisms necessary for the horizontal deflection of light beams. Especially when high precision, linearity and good resolution are required (STEREOMATRIX!), the cost of this part of the system can be tens of thousands of dollars Since integrated circuits have now become fast and cheap enough, it is entirely possible to "freeze" a given TV line, by storing in a buffer a sampled version thereof. In Figure 21 we have assumed that the luminosity is sampled at 512 points, each point being assigned 8 levels (3 bits). Our background with ORBIT (On-Line Reduced Bandwidth Information Transmission) allows us the design of this fast buffer. In practice one must actually switch between two buffers: one is filled as the other one is read-out in parallel.

The read-out (line by line) is obtained by connecting to the buffer a linear array of 512 fast response lamps of the light emitting diode or of the quartz halogen type: These have driving requirements certainly within the scope of solid state circuitry. Assuming 10 W per source, we have 5KW in one line: this obviously calls for considerable cooling. The remaining

Figure 20. Fundamental Idea of RASER.

Figure 21. SCANTRIX Block Diagram.

problem is now simply to project the "frozen" lines in 512 positions above one another (this neglects interlacing). This objective can be attained by a device as simple as a slowly rotating mirror, or as sophisticated as a solid-state deflector. In a first model the cheaper solution will be implemented. Note that projects such as this one are only possible because of the fantastic drop in price of integrated circuits!

B-20: CERAVISION (New)

The considerable increase in cost and complication of color TV displays over black-and-white versions is due to the fact that a dot-sequential system is used. In the original CBS field-sequential system a color-wheel was employed, and receiver synchronization was difficult and inelegant. This system was recently revived for the moon color camera built by Westinghouse: it necessitated a reasonably complicated recoding (storage tubes!) in order to be interconnectable with network TV. It is clear that the one great objection to the CBS system is the cumbersome motor plus filterwheel design and the impossibility of this high-inertia system to follow phase-jumps in synchronization.

Recent progress in electrically controlled color-filters of the ferro-electric ceramic type make the replacement of mechanical color wheels by purely electronically synchronized color filters attractive. Our background (OLFT!) in controlling fast rise and fall time high voltage devices makes work in this area very promising. It is therefore proposed to build a simple camera-monitor setup with color filters of the ceramic type, steered in parallel from a common synchronization generator. At the beginning it is anticipated that the monitor must have a relatively small screen, i.e., about 2" x 2" -- such miniature monitors are now available. Figure 22 shows the general layout.

Figure 22. Principle of CERAVISION Color System.