Measurements of Helical Magnetic Fields Using Flat Rotating Coils

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1 Introduction

Rotating coils are commonly used to measure the magnetic field coefficients (a_n, b_n) inside straight magnets [1]. They can also be employed to determine the multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Ref. [2]) in helical magnets.

We use a cylindrical coordinate system (r, θ, s) where s designates the coordinate along the longitudinal magnet axis. The area of a flat rotating coil ranges from r_1 to r_2 and from s_1 to s_2. The magnetic flux through the coils is

$$\Phi(\theta) = N \int_{s_1}^{s_2} \int_{r_1}^{r_2} B_\theta(r, \theta) \, dr \, ds \tag{1}$$

where N is the number of coils windings. For rotating coils one has $\theta = \omega t$ and the induced voltage

$$U = -\frac{d\Phi}{dt} \tag{2}$$

is proportional to the angular velocity ω.

We will present formulae for the magnetic flux through a rotating coil for straight and helical magnets. Assuming the induced voltage is parameterized in terms of ordinary multipole coefficients (a_n, b_n) conversion formulas will be given to obtain the helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$.

2 Straight Magnetic Fields

The azimuthal field in straight magnets can be expressed in multipole coefficients (a_n, b_n) as [2]

$$B_\theta = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[b_n \cos\left((n+1)\theta\right) - a_n \sin\left((n+1)\theta\right)\right]. \tag{3}$$

B_0 is the main magnetic field and r_0 a reference radius. The magnetic flux (1) becomes

$$\Phi(\theta) = NB_0(s_2 - s_1) \sum_{n=0}^{\infty} K_n \left[b_n \cos\left((n+1)\theta\right) - a_n \sin\left((n+1)\theta\right)\right] \tag{4}$$

where the coefficients K_n are defined by

$$K_n = \frac{r_0}{n+1}\left[\left(\frac{r_2}{r_0}\right)^{(n+1)} - \left(\frac{r_1}{r_0}\right)^{(n+1)}\right]$$
(5)

and result from the integration over r in (1).

3 Helical Magnetic Fields

The azimuthal helical field can be written in terms of helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Ref. [2]) as

$$B_\theta = \frac{B_0}{kr}\sum_{n=0}^{\infty} f_n I_{n+1}\left((n+1)kr\right)\left[\tilde{b}_n \cos\left((n+1)(\theta - ks)\right) - \tilde{a}_n \sin\left((n+1)(\theta - ks)\right)\right],$$
(6)

with

$$f_n = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}\frac{1}{r_0^n k^n}.$$
(7)

Here B_0 denotes the transverse component of the main field close to the magnet axis. This field is vertical at the location $s = 0$. The magnetic flux (1) can be expressed as

$$\Phi(\theta) = NB_0\sum_{n=0}^{\infty} R_n\left[\hat{b}_n \cos\left((n+1)\theta\right) - \hat{a}_n \sin\left((n+1)\theta\right)\right]$$
(8)

with new coefficients

$$R_n = \frac{f_n}{k}\int_{r_1}^{r_2}\frac{1}{r}I_{n+1}\left((n+1)kr\right)dr.$$
(9)

The integral in (9) can be computed numerically. In (8) new magnetic multipole coefficients

$$\begin{aligned}
\hat{a}_n &= +\tilde{a}_n T_n + \tilde{b}_n S_n,\\
\hat{b}_n &= -\tilde{a}_n S_n + \tilde{b}_n T_n.
\end{aligned}$$
(10)

are used for which

$$\begin{aligned}
S_n &= \frac{1}{(n+1)k}\left[\cos\left((n+1)ks_2\right) - \cos\left((n+1)ks_1\right)\right]\\
&= -\frac{2}{(n+1)k}\sin\frac{(n+1)k(s_2 - s_1)}{2}\sin\frac{(n+1)k(s_2 + s_1)}{2}
\end{aligned}$$
(11)

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and

$$T_n = \frac{1}{(n+1)k}\left[\sin\left((n+1)ks_2\right) - \sin\left((n+1)ks_1\right)\right]$$
$$= +\frac{2}{(n+1)k}\sin\frac{(n+1)k(s_2 - s_1)}{2}\cos\frac{(n+1)k(s_2 + s_1)}{2} \tag{12}$$

have been defined.

4   Conversion

We assume now a device that parameterizes the voltage (2) in terms of multipole coefficients (a_n, b_n) for straight magnets. If the measured magnetic field has helical symmetry, the coefficients $(\tilde{a}_n, \tilde{b}_n)$ in Eq. (8) can be derived as

$$\tilde{a}_n = \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n T_n - b_n S_n}{S_n^2 + T_n^2},$$
$$\tilde{b}_n = \frac{K_n}{R_n}(s_2 - s_1) \cdot \frac{a_n S_n + b_n T_n}{S_n^2 + T_n^2}. \tag{13}$$

We consider three special cases.
(a) Measuring coil of one helical wavelength with $s_1 = s$, $s_2 = s + \lambda$.
From equations (11) and (12) we obtain

$$S_n = T_n = 0 \tag{14}$$

and with (10)

$$\hat{a} = \hat{b} = 0. \tag{15}$$

The magnetic flux (8) is therefore zero and the coefficients can not be obtained.

(b) Measuring coil of half helical wave length with $s_1 = 0$, $s_2 = \lambda/2$.
In this case one has

$$S_n = \begin{cases} -\frac{2}{(n+1)k} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases} \quad \text{and} \quad T_n = 0.$$

Only coefficients with n even (i.e. helical dipole, sextupole etc. coefficients) can be measured. For those we have

$$\tilde{a}_n = +\frac{K_n}{R_n}\frac{(n+1)\pi}{2}b_n,$$
$$\tilde{b}_n = -\frac{K_n}{R_n}\frac{(n+1)\pi}{2}a_n. \tag{16}$$

(c) Infinitely short measuring coil with $s_1 = s$, $s_2 = s + ds$.
Expanding (11) and (12) to first order in ds we obtain

$$S_n = -\sin\left((n+1)ks\right)ds,$$
$$T_n = +\cos\left((n+1)ks\right)ds \tag{17}$$

and

$$\tilde{a} = \frac{K_n}{R_n}\left[+a_n\cos\left((n+1)ks\right)+b_n\sin\left((n+1)ks\right)\right],$$
$$\tilde{b} = \frac{K_n}{R_n}\left[-a_n\sin\left((n+1)ks\right)+b_n\cos\left((n+1)ks\right)\right]. \tag{18}$$

If in addition $s = 0$, the $(\tilde{a}_n, \tilde{b}_n)$ can by obtained from the (a_n, b_n) by multiplication with K_n/R_n.

References

[1] P. Schmüser, "Magnetic measurements of the superconducting HERA magnets and analysis of systematic errors", DESY HERA-p 92-1 (1992).

[2] W. Fischer, "Magnetic field error coefficients for helical dipoles", RHIC/AP/83 and AGS/RHIC/SN/17 (1996).