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ELECTRICAL ENGINEERING

QUARTERLY PROGRESS REPORT NO 16

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

REPORT NO. COO-1695-33

March 15, 1971

Madison, Wisconsin 53706

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Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

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U.S. Atomic Energy Commission

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INTRODUCTION

J.L. Shohet

This report covers the period between December 15, 1970 and September 14, 1971.

Basically the work performed covers three areas. They are

1. Polarization measurements of the bremsstrahlung x-rays
2. Plasma Simulation
3. Plasma Diagnostics

Both theory and experimental work have been done.

The bremsstrahlung measurements have been made to determine the polarization distribution of the free-free bremsstrahlung exciting radially along the mid-plane. The scattering method has been used with beryllium as the scattering material. Use of this material, together with improved pulse counting techniques has resulted in a 5-fold increase in the scattered x-ray flux, necessitating counting times of only 200-400 minutes. Both wall and free-free bremsstrahlung polarization measurements were made using this method.

The theoretical work in this area is concentrating on a direct evaluation of the scattering matrix elements to obtain a definitive measure of the anisotropy of the free-free bremsstrahlung.

Plasma simulation has continued in conjunction with S. J. Gitomer, J. C. Sprott and A. J. Hatch. There are three approaches made to this problem. The first is consideration of simulation of electron cyclotron resonance in a 2-dimensional region considering interparticle coulomb fields. The second approach considers heating

of a collection of particles without interparticle reactions.

Finally, r.f. confinement of a much larger number of particles than previously is now possible after modification of an already existing program.

In the area of plasma diagnostics, a method of determining the amounts of cold and hot plasma present in the device is being developed. The method is based on a different response to microwaves in a cavity when the plasma is considered warm than when it is cold. Further diagnostic work in the optical spectrum will commence shortly when a monochromator is installed on the system. Hopefully, optical bremsstrahlung will be measured. It is anticipated that a Lisitano type r.f. source will be installed in the low frequency system to improve coupling characteristics. Finally, further calibration of the scintillator probe used to measure the distribution of electrons in the plasma has been made.

I. BREMSSTRAHLUNG MEASUREMENTS

A. Wall Polarization

D.G.S. Greene

- a) Experimental Set-up.
- b) Polarization of wall bremsstrahlung.
- c) Comparison to theory.

The intensities of radial x-rays scattered from a graphite block were compared for scattering in the r-z plane to scattering in the r- θ plane. The experimental set-up is shown in Figure 1.

Considering conservation of energy and momentum, the frequency change for Compton scattering of x-rays is given in equation (1):

$$\frac{\nu_0}{\nu} = 1 + \frac{h\nu_0}{m_0 c^2} (1 - \cos A) \quad (1)$$

Since where A is the angle between the incident photon and the scattered photon,

Since ν_0 is the frequency of the incident photon, and ν is the frequency of the scattered photon.

Since $h\nu_0 \ll m_0 c^2$, the energy of the scattered photon is approximately equal to the incident photon energy. For plane polarized photons the Compton scattering cross section is given in equation (2):

$$d\sigma = \frac{r_0^2}{4} d\Omega \left(\frac{\nu}{\nu_0}\right)^2 \left(\frac{\nu_0}{\nu} + \frac{\nu}{\nu_0} - 2 + 4 \cos^2 B\right) \quad (2)$$

where B is the angle between the polarization plane of the incident photon and the polarization plane of the scattered photon,

Experimental Apparatus for
Measuring Polarization of
Wall Bremsstrahlung

Figure 1

Graphite block and photomultiplier tube assembly are shown for scattering in the r - z plane. For scattering in the r - θ plane block and assembly are rotated 90° about the axis of pinholes so that assembly is perpendicular to this drawing.

$d\Omega$ is the element of solid angle into which the photon is scattered, and

r_0 is the classical electron radius.

Considering the results of equation (1), that $v = v_0$, ds is directly proportional to $\cos^2 B$. The angles involved in the photon scattering are illustrated in Figure 2. For this figure the direction of incidence is chosen as the x direction, and the plane of polarization is chosen as the y direction. As defined above, A is the angle which the scattered photon makes with the x axis. Since in this experiment the photomultiplier tube is only oriented at right angles to the radial x-rays, A equals 90° . For photon scattering into the y direction (angle C = 0°) the scattering cross section is essentially zero, since angle B would be 90° . For photon scattering perpendicular to the x-y plane (angle C = 90°) the cross section is not zero, but at a maximum, being enhanced over the y directed scattering by a factor of about $(2m_0 c^2 / h\nu_0)^2$. Thus the x-ray polarization direction may be determined by finding the directions of maximum and minimum intensities of scattering. Because of the cylindrical symmetry of the plasma producing the radial x-rays, the extremum directions were assumed to be θ and z . The percent polarization is given by equation (3):

$$P = \frac{-I(\text{hor}) + I(\text{vert})}{I(\text{hor}) + I(\text{vert})}$$

where P is the percent of polarization,
 $I(\text{hor})$ is the intensity of x-rays scattered into the photomultiplier tube oriented in the r-z plane (horizontal), and

Compton Scattering Geometry

- ν_0 incident photon (in x direction)
- ν scattered photon
- ϵ_0 polarization direction of ν_0 (in y direction)
- ϵ polarization direction of ν
- B angle between ϵ_0 & ϵ
- A angle between ν_0 & ν
- C angle between y & ν

Figure 2

I(vert) is for the phototube in the $r-\theta$ plane (vertical).

The movable brass plug shown in Figure 1 was inserted into the plasma until a maximum flux of radial x-rays was observed (this experiment has been previously described¹). The spectra of the x-rays scattered during 20 minutes by the graphite block were recorded as counts by the photomultiplier tube - pulse height analyzer combination for the two orientations of the graphite block - photomultiplier tube assembly. A coincidence circuit allowed counts only during plasma production. The log of the number of counts was shown in the display. A photograph of the overlapping of these two spectra was taken, and a drawing of this photo is shown in Figure 3a. When either the graphite block was removed, or a lead block was used to cover the pinholes, virtually no counts were observed, verifying that the counts were from photons scattered from the graphite. The photomultiplier tube - pulse height analyzer combination was calibrated in both orientations using a Cs^{137} source. During calibration the pinhole was covered by a block of lead, but otherwise the plasma remained on and also the magnetic field remained unchanged. A photograph of the overlapping of these two calibration spectra was taken and a drawing of this photo is shown in Figure 3b. Notice that the calibration spectra are not the same for the horizontal and vertical orientations of the photomultiplier tube. The 32 keV calibration peak occurs at channel #36 for the horizontal position and at channel #38 for the vertical. Evidently the magnetic field is sufficiently different in these two different orientations to affect the sensitive dynodes in the tube. Correc-

tions must be made to remove this effect. The heights of each channel display and the heights of the decade markings were measured.

A computer program was written to calculate the percent polarization as a function of x-ray energy. First the counts per channel were calculated from the measured heights. An interpolation was done between the counts per channel values to establish counts per energy intervals of 1 keV for both the spectra from the scattered radial x-rays and for the spectra from the cesium calibrations. Since the calibration peak locations were different for the horizontal and vertical spectra, different interpolation schemes were used for the two orientations. The percent difference between the horizontal and vertical calibration spectra at a given energy was attributed to the magnetic field variation, and so this percent difference was used to correct the values of the spectra from the scattered radial x-rays. Finally the percent polarization was calculated using equation (3). The percent polarization as a function of x-ray energy is shown in Figure 4. There is considerable spread in these experimental points. The percent polarization expected for radial x-rays from a loss cone distribution² with characteristic temperature of 31 keV is shown in Figure 4 as a solid line.

Percent Polarization vs Photon Energy

Correction

In Progress Report No. 13 (#C00-1695-27) from this lab on page 15, equation (13) defines the percent polarization as:

$$P(k) = \frac{\int dI(P_z) - \int dI(P_\theta)}{\int dI}$$

This is consistent with the usual definitions of percent polarization, but it is inconsistent with the remainder of Progress Report No. 13. To achieve consistency, all polarization scales in Figures 5 and 6 (pages 16-21 inclusive) must have their signs changed.

B. Free-Free Bremsstrahlung Polarization Measurements ---

P. A. Raimbault

The experimental work designed to measure the polarization of the free-free bremsstrahlung x-rays in the radial direction was continued.

Some important improvements were made in the experiment to lower the background. The basic principle of the device remained the same.

- 1) For low energy x-rays, the lighter the element, the better the scattering. Thus, in the place of graphite, beryllium was used and the scattering of 30 keV x-rays coming out from a source, gave a larger intensity by a factor of about 2.

- 2) The flux of x-rays was maximized by optimizing the pressure and the magnetic field.

The best magnetic field corresponded to a current in mV of 55.70 which placed the resonant zone in the midplane of the machine. The pressure was set at 7.5×10^{-4} torr.

- 3) A longer vacuum extension was used to reduce the effect of the magnetic field on the phototube. To further reduce the magnetic effects, the phototube was covered with two cylinders of netic-conetic foil and a piece of soft iron which is demagnetized regularly. The result has been such that now the magnetic field does not introduce any appreciable change in the measurements.

- 4) The exit windows were collimated to give an image only of the other side window and not of the walls of the chamber so that free-free bremsstrahlung may be obtained.
- 5) The incident microwave averaged power used for heating is now continuously measured and is held constant during the entire experiment.
- 6) A multichannel analyzer is used in preference to a single channel analyzer because of the difficulty in obtaining a stable window of energy for the latter.
- 7) An improved coincidence circuit was developed to count the x-rays after each pulse, which results in a lower background level. The magnetron firing is now delayed some 50 microseconds after the initialization of the system. The system is shown in Figure 5. The reason for this change is as follows.

It was found that the pulse generator feeding the coincidence terminal of the analyzer, actually disabled the analyzer for 20-30 microseconds. As a result, no counts were made during the very time when the plasma was the hottest, i.e., immediately after the heating pulse. The delay has resulted in a marked improvement. Figure 6 shows the present delayed pulse configuration.

- 8) The multichannel analyzer was calibrated with Cs 137. In Figure 7B, the peak is at 32 keV.

Results of the Experiments.

In Figure 7A we observed the plasma with no scattering from the Beryllium. The spectrum showed that most of the x-rays had energies

Figure 5

Figure 6

Figure A

Figure B $32 \uparrow$ KeV

Figure C

Figure D

Figure E

between 5 and 30 keV. (Fig. 7A)

The background was counted for 300 minutes and seemed to be uniform. It averaged about 3 counts per channel. (Fig. 7C)

The Be scattered x-rays measured in the horizontal position of the phototube are shown in Figure 7D. When the phototube is in this position it only counts photons whose electric vector is vertical (θ -direction).

The Be scattered x-rays when the tube was vertical are shown in Figure 7E. This position counts photons whose electric vectors are in the axial direction (z -direction). It can be seen that in both of the two positions the scattered intensity is clearly above the background between 10 and 30 keV. In addition, the shape of the spectra appeared to be similar to the unscattered spectrum. By comparing Figure 7D and Figure 7E we see a difference. There seem to be more x-rays whose electric field is in the θ -direction than in the z -direction.

The same measurements will be made from another set of points displaced along the axis from the mid-plane of the machine.

C. Theory of Free-Free Bremsstrahlung

P. A. Raimbault

1. Polarization.

We shall define the process of bremsstrahlung as follows: A photon of radiation is emitted whenever an electron which passes a positive ion is decelerated.

The problem we consider is to find the polarization of the emitted radiation. For this purpose a quantum mechanical theory³ is used. The results are compared with the experimental ones.

Let us define some variables for the problem.

$\vec{p}_0, E_0$	momentum and energy of the incident electron
$\vec{p}, E$	momentum and energy of the recoil electron
$\vec{k}$	momentum of the emitted photon
$\vec{j}$	direction of polarization perpendicular to $\vec{k}$. $ \vec{k} = h\nu/c$

Conservation of energy permits us to write $h\nu = E_0 - E$.

Momentum is not conserved because of the finite mass of the ion. A diagram of these vectors is shown in Figure 8. We call u_0 the incident wave function in the potential $V(r)$ of the ion. Its asymptotic form is a plane wave of momentum p_0 plus an outgoing spherical wave. u is a wave function whose asymptotic form is a plane wave plus an incoming spherical wave. Thus we can write the cross section of the problem as

$$\sigma(E, \Omega_p, \Omega_k, \vec{j}) = \frac{e^2 h\nu}{2 m^2 c^3} |D|^2 \quad (4)$$

with

$$D = \frac{1}{h} \int u^*(r) p_j e^{-i\vec{k} \cdot \vec{r}/h} u_0(r) d^3r$$

Figure 8

Figure 9

In equation (4) $\sigma(E, \Omega_p, \Omega_k, \vec{j}) dE d\Omega_p d\Omega_k$ represents the probability for an incident electron of momentum $\vec{p}_0$ to be scattered in such a way that its energy will be in the range E to $E + dE$. The direction of its momentum $\vec{p}$ will be in the cone of solid angle $d\Omega_p$ and the photon will be emitted with a polarization direction $\vec{j}$ and with a momentum that is in a cone of solid angle $d\Omega_k$.

2. Approximations and Comparison with the Experimental Case.

1) We assume that the ion is the center of the fixed Coulomb potential $V(r)$.

$$V(r) = - \frac{Ze^2}{r} \quad (5)$$

We need its Fourier transform which is, in atomic units:

$$\hat{V}(q) = - \frac{Z}{2\pi^2 q^2} \quad (6)$$

2) The problem is nonrelativistic which means $E_0 \ll mc^2$. In the experimental case, $E_0 \sim 30$ keV. This implies that $m_0 c^2 \approx 20 E_0$ so that we are then in the limit of application of the method.

We can then write

$$E_0 = \frac{p_0^2}{2m}, \quad E = \frac{p^2}{2m}, \quad \text{and} \quad k = \frac{p_0^2 - p^2}{2mc}$$

Now $p_0 \ll mc$ implies $k \ll |p_0 - p|$

3) The Born approximation can be used if $Z\alpha \ll 1$ and E_0 , and $E \gg \frac{mc^2}{2} (Z\alpha)^2 = I_1$. $Z\alpha = \frac{4}{137}$ and E_0 and $E \approx 100 I_1$ since we are using Helium.

3. Computation of the Cross Section.

The Fourier transforms of u_0 and u are:

$$\hat{u}_0 = \frac{(2\pi)^{3/2}}{\sqrt{p_0}} \psi_0(p') \quad (7)$$

$$\hat{u} = \sqrt{p} \psi(p') \quad (8)$$

Using the fact that $\vec{k}$ must be perpendicular to $\vec{j}$, we can find

$$D = 1 \left(\frac{p(2\pi)}{p_0} \right)^{3/2} \int d^3p' \psi^*(\vec{p}') p_j' \psi_0(\vec{p}'+\vec{k}) \quad (9)$$

p_j' component of $\vec{p}'$ in the $\vec{j}$ direction

Using the Born approximation we can find $\psi_0(\vec{p}')$ and $\psi(\vec{p}')$, namely,

$$\psi_0(\vec{p}') = \delta^{(3)}(\vec{p}' - \vec{p}_0) + \frac{2}{p_0^2 - p'^2} V'(\vec{p}' - \vec{p}_0) \quad (10)$$

$$\psi(\vec{p}') = \delta^{(3)}(\vec{p}' - \vec{p}) + \frac{2}{p^2 - p'^2} V'(\vec{p}' - \vec{p}) \quad (11)$$

If we substitute (10) and (11) in (9) the term involving δ will cancel. δ represents the free electron part of the wave function. The term in V^2 will be neglected. As a result only the cross terms will remain so that D is:

$$D = 2iV'(q) \left(\frac{p(2\pi)^3}{p_0} \right)^{1/2} \left(\frac{p_{0j}}{p^2 - |\vec{p}_0 - \vec{k}|^2} + \frac{p_j}{p_0^2 - |\vec{p} + \vec{k}|^2} \right) \quad (12)$$

where $\vec{q} = \vec{p}_0 - \vec{p} - \vec{k}$

If we recall that $k \ll |\vec{p}_0 - \vec{p}|$ and call the angle between $\vec{p}$ and $\vec{j}$: θ and the angle between $\vec{p}_0$ and $\vec{j}$: θ_0 we then find

$$D = -2iV'(|\vec{p}_0 - \vec{p}|) \sqrt{\frac{p}{p_0}} (2\pi)^3 \left(\frac{p_0 \cos\theta_0 - p \cos\theta}{p_0^2 - p^2} \right) \quad (13)$$

This results in a value for σ of

$$(E, \Omega_p, \Omega_k, \vec{j}) = z^2 \alpha^3 \frac{\hbar^2}{\pi^2} \frac{p}{p_0} \frac{1}{E_0 - E} \frac{(p_0 \cos\theta_0 - p \cos\theta)^2}{|\vec{p}_0 - \vec{p}|^4} \quad (14)$$

4. Polarization and Application to the Experimental Case.

1) We can immediately integrate over the solid angle Ω_p to eliminate the angular dependence of σ on $\vec{p}$.

2) We shall now study the case $\vec{k}$ perpendicular to $\vec{p}_0$ which is our experimental situation. That is, the velocity of the electrons is a minimum in the direction perpendicular to the field lines. We can observe the polarization in a direction perpendicular to the field lines.

We define the polarization as:

$$P(p_0, p) = \frac{J_{||} - J_{\perp}}{J_{||} + J_{\perp}} \quad (15)$$

where $J_{||}$ and $J_{\perp}$ are the relative probabilities for polarization $\vec{j}$ in the x-direction and z-direction, respectively.

then

$$P = \frac{(1-3R^2) \log\left(\frac{1+R}{1-R}\right) + 6R}{(3-R^2) \log\left(\frac{1+R}{1-R}\right) + 2R}$$

where

$$R = \frac{p}{p_0}$$

If $R \rightarrow 0$ v is a maximum and $P \rightarrow 1$.

If $R \rightarrow 1$ v is a minimum and $P \rightarrow -1$.

If we plot $v/v_{\max}$ versus P we get the graph shown in Fig. 9.

In the present experimental case the photons have energies around 30 keV. Since the electron energies are of the same magnitude, this implies that $v \rightarrow v_{\max}$ and therefore $P = 1$. The polarization must therefore be in the parallel direction (with respect to the field lines). This result agrees with the latest experimental data which have been shown previously.

The theoretical results were obtained assuming a monoenergetic beam of particles in the x direction. However, we must use an anisotropic distribution function of the velocities to have a more accurate result, but the basic conclusions should not be changed.

II. PROBE MEASUREMENTS OUTSIDE THE MIRRORS

D.G.S. Greene

- a) Agreement with a loss cone hypothesis.
- b) An estimate of electron temperature.

Experiment

The potential of a Langmuir probe across 10K to ground was measured at various axial positions. The experimental setup is shown in Figure 10. A graph of probe voltage versus distance from machine center is shown in Figure 11. Analysis of the shape of this graph follows in the next two sections. The magnetic field was measured with a Hall Effect gauss meter from the machine center to the end plate. This graph is shown in Figure 12.

Region A

The region beyond the mirror throats out to a weak magnetic field of about 200 gauss is called region A. The probe voltage variation with probe position in this region may be explained as follows: The flux of particles, ϕ , traveling along a tube of magnetic flux, ψ , is constant and is given in equation (17).

$$\phi = nv_{\parallel} A \quad (17)$$

where n is the density of electrons,

$v_{\parallel}$ is their parallel velocity (in the direction of the magnetic field), and

A is the area of the flux tube.

$$\psi = BA \quad (18)$$

where B is the magnetic field.

Axial Probe Position Relative to Plasma and Magnetic Mirrors for Measurement of V versus D

Figure 10

Probe Potential versus Probe Position

Magnetic Field on Axis as a Function
of Distance from Plasma Center
(mirror ratio is 1:1.19)

Figure 12

Eliminating A between equations (17) and (18) and solving for n yields equation (19).

$$n = \frac{\phi}{\psi} \frac{B}{v_{\parallel}} \quad (19)$$

The magnetic moment, μ , is given in equation (20).

$$\mu = v_{\perp}^2 / B \quad (20)$$

where $v_{\perp}$ is the perpendicular velocity of the electrons.

The total velocity of the electrons is given by equation (21).

$$v = \sqrt{v_{\parallel}^2 + v_{\perp}^2} \quad (21)$$

Eliminating $v_{\perp}$ between equations (20) and (21) and solving for $v_{\parallel}$ yields equation (22).

$$v_{\parallel} = \sqrt{v^2 - \mu B} \quad (22)$$

Introducing the loss cone hypothesis, it is assumed that particles leaving the plasma have about 0 parallel velocity at the mirror throats where the magnetic field is at a maximum, B_m . In equation (22) setting $v_{\parallel}$ equal to 0 where $B = B_m$ yields equation (23).

$$v^2 = \mu B_m \quad (23)$$

Eliminating v^2 between equations (22) and (23) and substituting the resulting expression for $v_{\parallel}$ into equation (19) yields equation (24).

$$n = \frac{\phi}{\psi} \frac{B}{\sqrt{B_m - B}} \quad (24)$$

The current to the spherical probe is proportional to the particle density and the particle velocity. Thus the variation of the probe potential with the magnetic field may be developed as follows:

$$\text{Probe potential} = V \sim nv \sim n \frac{B}{\sqrt{B_m - B}} \quad (25)$$

where the following constants have been included in the proportionality constant: electrical resistance, probe area, particle velocity, flux of particles per tube of magnetic flux, and the magnetic moment.

The expression $B/\sqrt{B_m - B}$ is calculated and is the solid line on Figure 11. The experimental data fits this solid line quite well in region A, which supports the loss cone hypothesis. The departure of the experimental data from the solid line nearer the end plates is discussed in the next section.

Region B

The weaker magnetic field region, farther from the confined plasma than region A, is called region B. Because the magnetic field is weak, in this region charged particles with high radial velocities at the machine axis may strike the vacuum cylinder walls. In such a case the current to the probe is decreased by this loss of the higher energy particles. The radius of gyration for a charged particle in a magnetic field is given by equation (26).

$$r = \frac{v_{\perp} m}{e B} \quad (26)$$

Since the vacuum chamber has a radius of 7.4 cm, a charged particle passing through the axis of the machine (where it might contribute to the probe current) must turn with a radius of 3.7 cm or less.

Setting this limit of 3.7 cm for r , particles with velocities greater than $v_{3.7}$ are lost, where $v_{3.7}$ is the velocity in equation (26) with a subscript 3.7 to describe the turning radius. Solving for $v_{3.7}$ yields

$$v_{3.7} = 6.5 \times 10^7 B \quad (27)$$

where $v_{3.7}$ is measured in cm/sec, and

B is measured in gauss.

Figure 13a is a graph of a Maxwellian distribution of velocities. v_p is the most probable velocity in the distribution; it is also the velocity of a particle with energy kT , the characteristic temperature of the distribution. Figure 13b shows the percent of particles in the Maxwellian distribution with velocities greater than a given velocity. The probe potential of equation (25) is reduced by this same percentage (the percent of particles lost). Velocity may be related to the magnetic field through equation (27), and the magnetic field in turn may be related to the probe position through Figure 12. In particular for v_p equal to 3×10^9 cm/sec (which corresponds to a Maxwellian distribution with a characteristic temperature of 2.6 keV) the probe voltage is given by the dashed line in Figure 11. Probe voltages corresponding to distributions with characteristic temperatures of 10.2 keV and 1.1 keV are also shown in Figure 11.

Maxwellian Velocity Distribution

Figure 13a

V_p is the most probable velocity

$$V_p = \left(\frac{2KT}{m} \right)^{1/2}$$

See Below
for Continuation ->

Particles With Velocities
Less Than V
versus V
(for above
Maxwellian)

Figure 13b

Further experimental verification

A smaller cylinder (inside diameter equal to 7.2 cm) was placed inside the vacuum chamber. For the probe centered in this smaller cylinder, the graph of probe voltage versus distance from machine center, D , is shown in Figure 14. Again the solid line is from $B/\sqrt{B_m - B}$ and again the dashed curve is for v_p equal to 3×10^9 . Since a radius of gyration greater than 1.8 cm will allow a charged particle to strike the wall of the smaller cylinder, equation (27) must be modified using this smaller radius to equation (28).

$$v_{1.8} = 3.2 \times 10^7 B \quad (28)$$

Interpretation

Based on the above experiment, what is the energy distribution of the particles at the center of the hot plasma? It has been assumed that a loss cone distribution with characteristic temperature kT exists at the plasma center. No energy is added or removed by the magnetic field as the particles in the loss cone pass the mirror throats and with increasing parallel velocities spiral down the cylinder toward the probe. Conserving the magnetic moment, the decrease in the perpendicular component of velocity can be calculated in terms of the magnetic field, as the velocity changes from totally in the perpendicular direction at the mirror throats to mostly in the parallel direction in the weak magnetic field of region B. The particles which are lost by striking the cylinder walls are those with sufficiently large perpendicular components

Probe Potential

versus

Probe Position

(inside 7.2 cm diameter cylinder)

Probe Potential (log scale) $\rightarrow$

4

2

1

.4

.2

.01

Distance from Center of Plasma, D (cm) $\rightarrow$

80

90

100

110

120

130

140

Figure 14

2.6 keV

of velocity. The ratio squared of the perpendicular component of velocity for a particle which is lost at the point where it is lost to the perpendicular component of velocity at the mirror throat equals the ratio of the magnetic fields at these points.

$$\frac{v_{\perp}^2}{v_{\perp T}^2} = \frac{B}{B_m} \quad (29)$$

where $v_{\perp T}$ is the perpendicular component of velocity of a particle at the mirror throat.

This ratio of magnetic fields is about 1/100, and since the perpendicular components of velocity seem to be fitted by a characteristic temperature of 2.6 keV at the point of loss, the characteristic temperature of the electrons at the mirror throat would be about 260 keV. This temperature is higher by a whole order of magnitude than previous estimates of the electron temperature in the plasma.

The effects of collisions

If collisions are included, such a high temperature of 260 keV may not be needed in the confined plasma to produce the measured 2.6 keV temperature in region B. The mean free path is the average distance a particle travels between collisions. Various kinds of collisions, some of which might even be multiple encounters with many other particles, are possible, which leads to various definitions of a mean free path. Define a mean free path, L , to be the average distance between collisions which randomly deflect the direction of the particle's velocity by an average angle of ω .

During such a collision the magnitude of the electron's velocity changes by $x\%$. However the expectation of probe potential versus probe position in the magnetic field of region A as previously discussed, must not be greatly altered by these collisions, since experiments closely match these expectations. After each collision a particle begins a new spiraling trajectory. The new spiral increases in diameter toward regions of smaller values of B , following the same dependence on B as did the old spiral. Hence the electron density, n , is given by the same expression as before, equation (8). Thus between collisions the current to the probe tip decreases at the same rate as previously described without collisions. During the collision the density of charged particles doesn't change, but the average velocity does change by $x\%$. This will cause a modification in equation (9). This modified equation is written as equation (30).

$$V \sim nv_c \sim nv(1-x\%)^{d/L} (1-x\%)^{d/L} \frac{B}{\sqrt{B_m - B}} \quad (30)$$

where v is the velocity without collisions,

v_c is the velocity with collisions,

L is the mean free path (in the length L , v is decreased by $x\%$ to the value v_c), and

d is the distance traveled by the electron.

The new factor $(1-x\%)^{d/L}$ introduced by collisions tends to cause the probe current to fall below the solid line of Figure 11. Since the actual experimental value of the probe current does not fall

below this solid line, the influence of this new factor must be negligible. Either $x\%$ must be very small, which means that the electron velocities far from the confined plasma are not much different from electron velocities near the plasma, or else L must be very large, which means that collisions very seldom occur and therefore the collisionless analysis leading to the rather high value of 260 keV for the plasma temperature would be valid. Therefore assume that $x\%$ is negligibly small, but that L is of some influential value. This means that several collisions may occur outside the plasma producing changes in the directions of the electron velocities, but not in the magnitude of these velocities. In further support of this conclusion: if the electrons were cooling appreciably as they passed through region B (that is, if $x\%$ were large), the scatter of experimental points about a single dashed curve in either Figure 11 or Figure 14 would be qualitatively different. Nearest the plasma (at say $D = 80$ cm) the points would be below the dashed curve, since below the curve corresponds to higher temperatures. Farther from the plasma (at say $D = 130$ cm) the points would be above the dashed curve corresponding to a lower temperature distribution. If any tendency is present in the experimental scatter, it appears to be just the opposite, which would correspond to higher temperatures occurring farther from the plasma.

Considering the angular relaxation per path length L , an approximate relation between the electron temperature or the confined plasma and the electron temperature of the dashed curve in Figure 11 which best fits the experimental data may be derived (as

follows in the remainder of this section). From 80 to 140 cm the log of magnetic field versus D is very nearly a straight line (see Figure 12). This linear relation is expressed in equation (31):

$$B = B_0 \text{ EXP}(-D/C) \quad (31)$$

where $B_0 = 3000$ gauss, and

$$C = 28.6 \text{ cm.}$$

A charged particle moving away from the confined plasma along a typical path in this magnetic field describes a helix with expanding radius and increasing pitch. As it moves a distance L , equal to a mean free path, along the helical path, it moves a distance p in the parallel direction. p is related to L by equation (32):

$$\text{EXP}(-p/C) = 4A/(A(1+R) + (1-R))^2 \quad (32)$$

where $A = \text{EXP}(-L/C)$,

L = mean free path along the helical path,

$C = 28.6$ cm (from equation (31)),

$R = V_{\parallel}/V$,

V = total velocity of particle, and

$V_{\parallel}$ = parallel component of V at the start of D .

A graph of $\text{EXP}(-p/C)$ versus R for various values of L is given in Figure 15.

Between collisions the $V_{\perp}^2$ of an average particle changes from $V_{\perp a}^2$ to $V_{\perp b}^2$ where a is the initial value of the position D and b is the final value of D . An average particle travels a parallel dis-

tance p between collisions so that b minus a equals p . With no collisions the magnetic moment is conserved as follows:

$$\mu = v_{\perp a}^2 / B_a = v_{\perp b}^2 / B_b \quad (33)$$

where the subscript a means evaluate B or $v_{\perp}^2$ at the position where D equals a and the subscript b , where D equals b .

Substituting equation (31) for B into equation (33) yields equation (34):

$$v_{\perp b}^2 = v_{\perp a}^2 \text{ EXP}(-p/C) \quad (34)$$

where the exponential factor may be evaluated from equation (32) or Figure 15.

During a collision an average particle changes its perpendicular component of velocity. If this change during collision is equal to the change expressed in equation (34), and, two, this change is from $v_{\perp A}$ for an average particle in the actual distribution to an average $v_{\perp I}^2$ for an isotropic distribution, an approximation for the average $v_{\perp A}^2$ may be made in terms of the average energy of the particles. Thus the perpendicular temperature may be related to the actual temperature. The average perpendicular velocity component is a certain fraction, γ , of the average total velocity.

$$v_{\perp}^2 = \gamma v^2 \quad (35)$$

For an isotropic distribution: since

$$v_{\perp I}^2 = v_x^2 + v_y^2 = \frac{2}{3} v^2, \quad \gamma = \frac{2}{3} \quad (36)$$

where the I subscript means isotropic.

For the actual distribution:

$$v_{\perp A}^2 = \alpha v^2 \quad (37)$$

where α is a fraction to be determined, and the subscript A means actual.

The change in $v_{\perp}^2$ during collision is:

$$v_{\perp I}^2 - v_{\perp A}^2 = \text{change} \quad (38)$$

This change is assumed equal to the loss in v^2 as the charged particle spirals toward lower magnetic fields:

$$v_{\perp a}^2 - v_{\perp b}^2 = \text{change} = v_{\perp I}^2 - v_{\perp A}^2 \quad (39)$$

Substituting for the various $v_{\perp}^2$ the expressions in equations (34), (36), and (37), into equation (39) yields:

$$\alpha v^2 - \text{EXP}(-p/C) v^2 = \frac{2}{3} v^2 - \alpha v^2 \quad (40)$$

Thus

$$\alpha = \frac{2}{3} / (2 - \text{EXP}(-p/C)) \approx \frac{1}{3} \quad (41)$$

The average of the squares of the perpendicular components of velocities are related to the average of the squares of the total velocities just as the measured perpendicular temperature is related to the temperature of the particles.

$$\alpha = \frac{V_{\perp}^2}{V^2} = \frac{kT_{\perp}}{kT} \quad (42)$$

for $kT_{\perp}$ equaling about 2.6 keV, it is found for a mean free path of about 25 cm that kT equals about 7 keV. This value for kT is actually a lower limit, because the fraction γ in equation (36) is too large. γ results from assuming that collisions give a particle a random parallel velocity in the forward direction. Actual collisions don't change the particle's direction very much; on the average the velocity is probably confined to an angle of less than 30° to the axis. This means that the $\frac{2}{3}$ factor in equation (41) would be replaced by something less than $\frac{1}{4}$. For a factor of $\frac{1}{6}$, kT would be about 31 keV. Note that a particle has an average $V_{\perp}$ sometime during the collision, not at the beginning, and likewise average $V_{\perp}$ occurs sometime during the change from $V_{\perp a}$ to $V_{\perp b}$, not at the position a. Therefore the α in equation (37) is too large, and the α to be introduced into equation (39) is too small. These two corrections tend to cancel each other.

Computer Simulation

The average perpendicular component of velocity of 100 escaping particles is calculated at various distances from the confined plasma. For initial conditions the particles are placed in the mirror throat and are assigned velocities in the perpendicular direction which approximate a Maxwellian velocity distribution:

$$f(V_{\perp}) = V_{\perp}^2 \text{EXP}(-V_{\perp}^2/\alpha) \quad (43)$$

where α characterizes the temperature of the distributions.

A mean free path of 25 cm is chosen (which corresponds to helium gas at 6×10^{-4} torr). A distance $\frac{p}{5}$ (traveled in the parallel direction while the particles travel 5 cm along their spiral paths) is calculated from equation (32). The new perpendicular velocities at this location in the smaller magnetic field (the field is found from figure 13) are calculated using equation (34). Since the particles have traveled $\frac{1}{5}$ of a mean free path, 13% or 13 particles are randomly selected to undergo collisions. Each collision consists of changing the direction of the particle's velocity by y° , where a new y is randomly selected for each collision from a group of angles which match the distribution of angles expected from Rutherford scatterings of 8° or more. For this distribution of angles the average angle μ equals 14° . And finally the average of the squares of the perpendicular components of velocities is calculated. This average is proportional to the perpendicular temperature as described in equation (42). Then the calculation cycle is repeated: a new $\frac{p}{5}$ is calculated considering the new ratio of parallel velocity to total velocity, etc. A graph of the perpendicular temperature versus the distance from the confined plasma's center is given in Figure 16. The expected perpendicular temperature variations with distance for two other cases are also shown in Figure 16 for twice the mean free path (50 cm) and half the average scattering angle, and also for no collisions. The experimental values from Figure 14 (of about 2 keV at 110 cm, about 2.6 keV at 120 cm, and about 3 keV at 125 cm) are shown as diamonds in Figure 16.

Perpendicular Temperature Versus Distance from Center of Confined Plasma (D)

Figure 16

There are two comments to be made at this point regarding the fortuitous "fit" of the curve produced by the computer simulation of escaping particles and the experimental data which shows an increasing perpendicular temperature at greater distances from the plasma. First, from the rough approximation of a plasma by only 100 particles in the simulation, rather large variations may be expected to result: apparently the increase of temperature with distance is such a variation, since at greater distances, at say 200 cm, the temperature has fallen to 1.9 keV. Second, for different values of mean free path and average scattering angle a variety of temperatures at 120 cm may result from a 31 keV temperature confined plasma. However, neither great accuracy nor uniqueness are claimed for the above plausible interaction mechanisms, but merely a consistent relationship between believable values of plasma parameters is demonstrated.

If the magnetic field is removed or somehow held constant, the perpendicular temperature, as calculated in the computer simulation, approaches 20.3 keV after many collisions. This is the fraction of 31 keV expected by equation (36) for an isotropic electron energy distribution. Thus the magnetic field decrease with distance is responsible for the reduction of the perpendicular temperature from the 20.3 keV value. A more nearly constant magnetic field in the 110 cm to 130 cm region than is illustrated in Figure 12 could account for the apparent temperature increase with distance which the spread of experimental points in Figure 14 seems to indicate. The sort of rise to be expected for a constant magnetic field in the 100 cm region is indicated by circles in Figure 16.

III. PLASMA SIMULATION

J. L. Shohet

A. 2-Dimensional Electron Cyclotron Resonance - Multi-Particle

With the help of S.J. Gitomer and the Los Alamos Scientific Laboratory, 36,864 simulation particles were utilized in an electron cyclotron resonance geometry. Figure 17 shows the results for the total energy and the field energy of the system for an applied uniform field of 10^5 volts/meter.

The determination of heating will be made by examining the change in entropy of the system. If entropy is increasing, the heating will be said to have taken place. In addition, the distribution function of the simulation particles will be observed, to ascertain whether coulomb collisions alone are sufficient to produce heating.

Figure 18 shows similar results for various values of electric field. It appears that the heating rate is independent of the magnitude of the electric field.

The program will be modified to consider a 2 dimensional region in which the magnetic field will be essentially parallel to one axis, rather than perpendicular to both of them. In addition, axial variations will now be considered.

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Fig 18

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PARAMETERS: 32×32 MESH, 36864 SIMULATION PARTICLES

$B_0 = 318$ GAUSS, $n_0 = 1.0 \times 10^{16}$ ELECTRONS/ m^3

$w_p/w_c = 1.0$, CELL SIZE = PARTICLE SIZE

$\Delta x = 1.0 \lambda_D$, $\Delta y = 1.0 \lambda_D$, $\Delta t = 0.04 \tau_{ce} = 0.04 \tau_{pe}$

POISSON SOLVER: HOCKNEY METHOD

PARTICLE EQ. OF MOTION: LEAP-FROG TIME CENTERED

DIMENSIONS: x, y PLANE $\vec{B} = B_0 \hat{z}$

Fig 17

B. 2-Dimensional Electron Resonance

1. Guiding Center Model -- J.L. Shohet & P.A. Raimbault

With the help of J.C. Sprott a multiple particle r.f. heating simulation program has been studied. Coulomb reactions have been neglected, but a large number of particles can be followed in a magnetic mirror field. The original problem solved, by finite difference methods, the following equation

$$\ddot{y} + \omega_c^2 y = \frac{eE_{\perp}}{m} \sin \omega t \sin (kx - \phi) \quad (44)$$

$$\ddot{x} + \frac{\mu_m}{m} \frac{\partial B}{\partial x} = \frac{eE_{\parallel}}{m} \sin \omega t \sin (kx - \psi) \quad (45)$$

where μ_m is the magnetic moment. $E_{\perp}$ and $E_{\parallel}$ are the magnitudes of the externally applied r.f. heating fields. The magnetic field is assumed to point and to vary in the x direction. ω may be set to cyclotron resonance if desired.

Equation (44) is the exact solution for the motion in the y-direction, if it is assumed that there is no external field in the z-direction. Equation (45) is the guiding center solution to the motion parallel to the magnetic field.

Sprott's results show that, for a collection of particles, assuming a distribution of particle velocities, all launched at one point, that heating corresponding to a single particle theory¹ does occur. However, a question as to the validity of the guiding center approximation arises, since the external electric field does vary significantly over the distance the particle moves in the x-direction.

As a result, the exact solution will be obtained. This means that equation (45) must be modified to appear as

$$\ddot{x} = \frac{eE_{\parallel}}{m} \sin \omega t \sin(kx - \psi) + \frac{e}{m} (\dot{y}B_z - \dot{z}B_y) \quad (46)$$

Since $\dot{z}$ appears, we must solve the z equation of motion as well

$$\ddot{z} = \frac{e}{m} [\dot{x}B_y - \dot{y}B_x] \quad (47)$$

Finally, the y equation of motion must also be modified to be

$$\dot{y} = \frac{eE_{\perp}}{m} \sin \omega t \sin(kx - \phi) + \frac{e}{m} (\dot{z}B_x - \dot{x}B_z) \quad (48)$$

2. Exact Solution -- S.K. Nijhawan

Since a charged particle is reflected in a magnetic mirror machine along a gradient in the magnetic field it is considered essential to solve the exact equation of motion of the particle.

The z component of the magnetic field was taken as $B_0(1 + \alpha|z|)$. $B_0(1 + \alpha z^2)$ is sometimes a better approximation just to make sure that the particle is reflected somewhere under all circumstances because its gradient is continuous. The x , y , z components of the magnetic field that we considered are as follows:

$$B(z) = B_0(1 + \alpha|z|) \quad (49)$$

$$B(x) = -\frac{B_0 \cdot \alpha \cdot x}{2} \cdot \frac{|z|}{z} \quad (50)$$

$$B(y) = -\frac{B_0 \cdot \alpha \cdot y}{2} \cdot \frac{|z|}{z} \quad (51)$$

The desire to examine the validity of the program and to observe difference in the motion of the particle and, from one orbit to another required us to consider a very low magnetic field and a high α (B_0 as low as 3 gauss and $\alpha=3$).

A rough estimate for calculating B_0 is as follows. We would like that the particle is reflected in 10 cyclotron orbits within a distance of 20 cm. If the particle travels 2 cm. in the z-direction in one orbit then the average velocity in z-direction $\approx 10^6$ m/sec $\Rightarrow$ the particle should therefore complete one orbit in $2 \times 10^{-2} / 10^6$ sec. Thus ω_c should be of the order of $\frac{10^8}{2}$, and the magnetic field should be of the order of 3 gauss.

The nonvalidity of the guiding center theory due to rapidly changing magnetic fields does not affect our results since we are solving the exact equations.

The equations of motion now become

$$\vec{F} = \frac{q}{m} (\vec{E} + \vec{V} \times \vec{B}) \quad (52)$$

If we assume $\frac{q}{m} = 1.76 \times 10^{11}$ and resolve the equation into component form we have

$$\frac{dV_x}{dt} = -1.76 \times 10^{11} [E_x + V_y \cdot B_z - V_z \cdot B_y] \quad (53)$$

$$\frac{dV_y}{dt} = -1.76 \times 10^{11} [E_y + V_z \cdot B_x - V_x \cdot B_z] \quad (54)$$

$$\frac{dV_z}{dt} = -1.76 \times 10^{11} [E_z + V_x \cdot B_y - V_y \cdot B_x] \quad (55)$$

The problem was scaled to simulate the large velocities, high frequency, and small radius of gyration that exhibit in reality. The values used for scaling are the following.

$$\begin{aligned}
 x &= 2x_c \\
 y &= 2y_c \\
 z &= .2z_c \\
 V_x &= 10^8 V_{cx} \\
 V_y &= 10^8 V_{cy} \\
 V_z &= 10^7 V_{cz} \\
 t &= .2 \times 10^{-7} \tau
 \end{aligned}$$

where the symbol c represents the variable in the computer and V_x, V_y, V_z, x, y, z represent the velocities and position in x, y, z, direction. τ and t are computer and real time, respectively.

The values of the magnetic field after scaling and assuming $B_0 = 3 \text{ gauss} = 3 \times 10^{-4} \text{ webers}$ becomes the following.

$$B_x = -\frac{3 \times 10^{-4}}{2} 2 x_c \cdot \alpha \frac{|z_c|}{z_c} \quad (56)$$

$$B_y = -3 \times 10^{-4} y_c \cdot \alpha \frac{|z_c|}{z_c} \quad (57)$$

$$B_z = 3 \times 10^{-4} \left(1 + \alpha \cdot \frac{|z_c|}{5}\right) \quad (58)$$

The scaling also changed the differential equations of motion as shown, assuming $E_y = E_z = 0$.

$$\frac{dx_c}{d\tau} = V_{cx} \quad (59)$$

$$\frac{dy_c}{d\tau} = V_{cy} \quad (60)$$

$$\frac{dz_c}{d\tau} = V_{cz} \quad (61)$$

$$\frac{dV_x}{d\tau} = -3.5 \left[\frac{E_x}{10^5} + .30 V_{cy} \left(1 + \frac{|z_c|}{5} \right) + .03 V_{cz} \times y_c \frac{|z_c|}{2} \right] \quad (62)$$

$$\frac{dV_y}{d\tau} = 1.05 \left[\frac{V_{cz} \times x_c}{10} + V_c \times \left(1 + \frac{|z_c|}{5} \right) \right] \quad (63)$$

$$\frac{dV_z}{d\tau} = -10.50 [V_{cy} \cdot x_c - V_{cx} \cdot y_c] \frac{|z_c|}{z_c} \quad (64)$$

The new scaled flow diagram is shown on the next page.

Before we made an attempt to run the program on the analog computer, the same equations were solved by numerical integration methods on the digital computer. The following results were obtained.

Case A No electric field.

- 1) The total energy remained constant.
- 2) As the particle goes to the higher magnetic fields its radius of gyration keeps on decreasing.
- 3) The particle velocity in the z direction is a sine function of time.

Fig. 20 shows the particle motion projected on the x,y plane and a graph of V_z vs. time.

Case B Electric field $E_x = 100$ V/m.

Fig. 21 shows the projection on xy planes of the motion.

Fig 19

PROJECTION OF PARTICLE MOTION in
x-y plane.

$$E_0 = 0.$$

Fig 20

PROJECTION OF PARTICLE MOTION
in x-y plane.

$E_0 = 100$

Fig 21

C. R.F. Confinement.

Successful demonstration of r.f. confinement for both single particle and coulomb collision-multiple particle models has been demonstrated in 2 dimensions. To lessen the stray effects of collisions, the number of simulation particles is being increased. It is anticipated that this method will result in the same effects, but with a more relativistic representation. This work is being conducted with the help of A.J. Hatch.

IV. WARM PLASMA IN A MICROWAVE CAVITY

J. L. Shohet

The eigenvalue equation of a warm plasma in a microwave cavity has been derived. Currently, a study of the eigenfrequencies as a function of temperature and plasma density is underway.

The fundamental equations used to obtain the eigenvalue equation are

$$\frac{mV_T^2}{n_0} \nabla n = - \frac{m}{n_0} \frac{\partial \vec{\Gamma}}{\partial t} + q\vec{E} \quad (65)$$

$$\nabla \cdot \vec{\Gamma} = - \frac{\partial n}{\partial t} \quad (66)$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (67)$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + q\vec{\Gamma} \quad (68)$$

$$\vec{\Gamma} = n_0 \vec{V} \quad (69)$$

Here V_T the thermal speed, n_0 the equilibrium density, $\vec{\Gamma}$ the particle current, and $\vec{V}$ is the perturbed velocity. The equations may be combined to give an equation in E_z , the z-component of the electric field of the wave.

This equation is, in cylindrical coordinates

$$-V_T^2 \nabla^4 E_z + \omega^2 \left[1 + \frac{V_T^2}{c^2} \right] \nabla^2 E_z - \frac{\omega^4}{c^2} \left[1 - \frac{\omega_p^2}{c^2} \right] E_z = 0 \quad (70)$$

where c is the velocity of light and ω_p is the plasma frequency. This biquadratic equation may be factored. If the radial variation goes as $\exp(jkr)$, we then get

$$[\nabla_{\perp}^2 + \lambda_1^2][\nabla_{\perp}^2 + \lambda_2^2] E_z = 0 \quad (71)$$

where $\nabla_{\perp}^2$ is the gradient perpendicular to z . This may further be separated if it is noted that

$$\lambda_1^2 = k_1^2 - \left(\frac{N\pi}{L}\right)^2 \quad (72)$$

$$\lambda_2^2 = k_2^2 - \left(\frac{N\pi}{L}\right)^2 \quad (73)$$

whence k_1^2 and k_2^2 are the two roots of the equation

$$-V_T^2 k^4 - \omega^2 \left(1 + \frac{V_T^2}{c^2}\right) k^2 - \frac{\omega^4}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right) = 0 \quad (74)$$

The quantity $N\pi/L$ is based on the axial variation fixed by the height L of a cylindrical cavity.

Now the electric field in the z -direction inside the plasma will vary as

$$E_z = [a_1^1 J_0(\lambda_1 r) + a_2^1 J_0(\lambda_2 r)] \exp(i\ell\theta + i \frac{N\pi}{L} z - \omega t) \quad (75)$$

and similarly, the H_z field will go as

$$H_z = [b_1^1 J_0(\lambda_1 r) + b_2^1 J_0(\lambda_2 r)] \exp(i\ell\theta + i \frac{N\pi}{L} z - \omega t) \quad (76)$$

The a 's and b 's are arbitrary coefficients. Equations (75) and (76) assume that the plasma is a cylindrical column concentric with the cavity axis. If the plasma does not fill the cavity completely, then we must also know the fields external to the plasma column. They are

$$E_z^0 = [a^0 J_\ell(\lambda_0 r) + c^0 Y_\ell(\lambda_0 r)] \exp(i\ell\theta + i\frac{N\pi}{L} z - \omega t) \quad (77)$$

and

$$H_z^0 = [b^0 J_\ell(\lambda_0 r) + d^0 Y_\ell(\lambda_0 r)] \exp(i\ell\theta + i\frac{N\pi}{L} z - \omega t) \quad (78)$$

Again, the fields are expressed in terms of arbitrary coefficients.

λ_0 is defined as

$$\lambda_0^2 = k_0^2 - \left(\frac{N\pi}{L}\right)^2 = \frac{\omega^2}{c^2} - \left(\frac{N\pi}{L}\right)^2 \quad (79)$$

Knowing the forms of E and H in the z-direction permits us to find all of the components in terms of them. The eigenvalue equations are then determined from the boundary conditions on the system. These are then solved simultaneously to determine the eigenfrequency, given values of cavity radius, cavity height, plasma frequency, and thermal speed.

The boundary conditions are:

1. Across the plasma vacuum interface, the tangential components of $\vec{E}$ and $\vec{H}$ are continuous.
2. At the cavity outer radius, the tangential component of $\vec{E}$ and the normal gradient of $\vec{H}$ must be zero, assuming that the cavity walls are perfect conductors.

It should be noted that equations (72) and (73) show that the plasma has 2 modes. These modes are not coupled, thus the eigenvalue equations for each mode must be solved separately.

Preliminary results have been obtained for the TM_{010} mode. The data are displayed as a function of plasma frequency with temperature as a parameter. They are shown in Figure 22. Note that two modes are clearly visible. The upper frequency corresponds to that related to the empty cavity mode, while the lower one is purely "acoustic" in origin. Note that the lower mode is affected by temperature, while the upper one is essentially not.

Further studies of this and other modes are currently under way. The aim of this work is to provide a means for differentiating between warm and cold plasma when they are both present.

V. PYREX SYSTEM

R. T. Pedley

Since the last progress report, the pyrex system has been utilized in the calibration of a scintillator probe. This probe was used by Dave Greene to obtain velocity distributions which are outlined elsewhere in this report.

At one end of the system, a small, heated cathode was attached to the end plate. The probe is entered into the vacuum chamber through a port on the other end plate. A high accelerating voltage is then applied between the end plates. The filament voltage, the accelerating voltage, and the distance between the probe tip and the filament, can be varied. The results are not yet complete due to failures at various times of the amplifier, the high voltage power supply, and the analyzer.

A scanning monochrometer with resolution of less than an angstrom has been purchased. It will be used in conjunction with ion cyclotron resonance work to measure Doppler broadening and thus calculate ion temperatures. It will also be used for continuum measurements.

In the Annual Progress Report (December 16, 1970), a V.I. characteristic of a Langmuir probe for a r.f. plasma was shown. The current is zero for probe voltages between 0 and -10 volts. Similar results have been obtained at the University of Michigan⁴ for a plasma generated with a Lisitano coil. Therefore, a Lisitano coil is being designed for the pyrex system in order to duplicate the results at Michigan. Also, double Langmuir probes will be tried in order to further investigate this phenomena.

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