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SOME CALCULATIONS FOR A PLUTONIUM-FUELED ZPR-III

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SOME CALCULATIONS FOR A PLUTONIUM-FUELED ZPR-III

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Abstract

Calculations of critical size, reactivity coefficients and delayed neutron fractions are described for mockups of a plutonium-fueled EBR-I (Mark IV) and five possible metal-fueled power reactors. The calculations for critical size were performed using the SNG transport theory code. The reactivity coefficients were estimated by SNG calculations or by one-group perturbation theory desk calculations. The delayed neutron fractions were estimated by a new approximate method, which starts with a calculation of β for a bare core and then makes a correction for the small effect of the blanket. The reactivity effects studied were the effect of fuel plate and whole core expansion and control rod worths.

1. Introduction

The calculations described in this report were performed to provide background information for work with plutonium with ZPR-III.⁽¹⁾ The reactivity coefficients of the plutonium-fueled ZPR-III will differ from those of the uranium-fueled assembly because the plutonium fuel will be canned. The external dimensions of the pieces will be the same as standard ZPR-III constituents, $\frac{1}{8}$ in. thick, 2 in. wide, and 1, 2 and 3 in. long. The plutonium piece within each can will be retained at one end of the can by a spring which will allow expansion. The cans will usually be loaded with the springs on the ends away from the plane of division of ZPR-III, so that fuel expansion will give a negative reactivity change, but this change will be smaller than the effect that would occur if the fuel column could expand as a whole.

The reactivity changes described in this report are all expressed in terms of the reactivity, $\Delta k/k$, for a given fractional change in length, $\Delta L/L$. These may be converted to temperature coefficients of reactivity by assuming a coefficient of linear expansion for the plutonium alloy (containing 1% aluminum by mass) of 11×10^{-6} per $^{\circ}\text{C}$. Since the total expansion gap in each can is 4% of the length, the gap would not be closed at melting temperature.

The effect of fuel expansion is studied because this is the only prompt effect that might shut down the reactor in an accident (independently of the control system). Fuel expansion for a mockup of EBR-I is considered in Section 2.1, by means of spherical SNG calculations, and in Section 2.2 by means of one-group perturbation theory. Larger plutonium assemblies are considered in succeeding sections: critical size in Section 3.1, control rod worths in Sections 3.2 and 3.3, and the effects of fuel expansion in Sections 3.4-3.7. The delayed neutron fractions for all the assemblies are evaluated in Section 4 by a new approximate method. This method starts with a calculation of β for a bare core and then makes a correction for the small effect of the blanket.

Some of the methods and results should be noted because they are of general interest and application. In Sections 2.2 and 3.7, it is shown that one-group perturbation theory is adequate to calculate the effect of fuel expansions. In Section 2.3, there is a comparison of fundamental mode calculations for the same core using one, four and eleven groups. Provided a good spectrum is used when weighting the few-group constants, the same material buckling is obtained in each case. In Section 3.4 it is shown that, when estimating axial effects for finite cylindrical cores using slab machine codes, it is necessary to adjust the assumed radial buckling to allow for the variation of reflector savings with core density. An original way of evaluating the buckling and reflector savings from a SNG calculation is explained in Appendix 1.

2. EBR-I Mockup

2.1 SNG Calculations

The proposed mockup of the plutonium-fueled EBR-I (Mark IV) has the same dimensions and composition as the Mark III mockup⁽¹⁾ except that the core length is 20.3 cm and the core and end-blanket compositions correspond to the following plate arrangements: core, Pu, 11; Zr, 2; 100% Al, 1; 63% Al, 2; end-blanket, depleted U, $6\frac{1}{2}$; Zr, $4\frac{1}{2}$; 63% Al, 2; 45% Al, 3. The effect of axial expansion of the fuel was studied by transport theory calculations (SNG code) for the spherical version of this mockup described in Table 1. The eleven-group cross sections given by Loewenstein and Okrent⁽²⁾ were used in all the calculations (one volume of zirconium was replaced by one volume of iron). The results are given in Table 2. The basic case was that of a homogeneous core without gaps. The various effects were studied by subsequent calculations in which the plutonium only was contracted by 4% to make gaps at the radii corresponding to the expansion gaps of the cans. These radii enclosed the same proportions of the volume of the spherical core as the corresponding planes of the can ends will of the volume of the actual cylindrical cores.

TABLE 1

Specification of the Spherical Model of the ZPR-III Mockup
of EBR-I, Mark IV

Outer radius of region (cm)	11.24*	21.90	31.10	44.00
Volume fractions**				
Pu ²³⁹	0.2428	0	0	0
Pu ²⁴⁰	0.0116	0	0	0
U ²³⁵	0	0.00094	0	0.0018
U ²³⁸	0	0.44100	0	0.8240
Fe	0.3751	0.27800	0.543	0.0930
Al	0.1329	0.10200	0.161	0

*Inner boundary of core region is $r = 0$.

**The numbers of atoms per unit volume assumed were those given by Loewenstein and Okrent.(2)

TABLE 2

Results of Calculations for the Spherical Model of the
ZPR-III Mockup of EBR-I, Mark IV

	SNG Calculation	Perturbation Calculation	
		Simple Diffusion Theory	Asymptotic Diffusion Theory
Criticality parameter, k	1.0254		
Assumed loading of total plutonium, kg	28.8		
Reactivity loss per fractional fuel expansion axially for corresponding cylindrical core of length, 8 in., viz., ($\Delta k/k$) ($1/\Delta l$)			
(i) 2 + 2 in. plates in each half*	0.140	0.115	0.128
(ii) 1 + 3 in. plates in each half*	0.148	0.111	0.130
(iii) 3 + 1 in. plates in each half*	0.177	0.157	0.170
(iv) 4 in. plate in each half**	0.265	0.187	0.215

*The first plate mentioned in each case is the one nearer the plane of division of ZPR-III.

**There is no plan to use 4-in. plates, but this case is included for comparison.

2.2 Perturbation Calculations

It was not possible to calculate by the SNG code all the effects for all the reactors studied in Section 3. Estimates for other reactor sizes were obtained by perturbation theory from the results for Reactor C of Table 4. The accuracy of one-group perturbation theory was therefore tested by repeating the calculations of Section 2.1.

From the basic equation of one-group perturbation theory given by Codd, Shepherd and Tait,⁽³⁾ it can be shown that the change in reactivity, $\Delta k/k$, on contracting the plutonium to leave a volume, ΔV , void of plutonium is given by

$$\Delta k/k = A \left\{ (\beta - \alpha)(\overline{n^2} - \overline{n_G^2}) + 3D_0^2 \alpha \left[\overline{(\text{grad } n)^2} - \overline{(\text{grad } n)_G^2} \right] \right\} \Delta V$$

where

$$1/A = \sum_i \int N_i \nu_i \sigma_{fi} n^2 dV \quad (\text{sum over fissionable isotopes, integral over reactor}),$$

N_i = number of atoms per unit volume of fissionable isotope i ,

ν_i = number of neutrons per fission for isotope i ,

σ_{fi} = fission cross section of isotope i ,

n = flux normalised to unity at core center,

β = contribution of plutonium alloy to $(\nu - 1)\Sigma_f - \Sigma_c$ for core, viz., β_0 .

α = contribution of plutonium alloy to $\Sigma_f + \Sigma_c + \Sigma_{tr}$ for core, viz., α_0 .

Σ_f = macroscopic fission cross section,

Σ_c = macroscopic capture cross section,

Σ_{tr} = macroscopic scattering cross section modified to allow for nonisotropic angular distribution,

$\overline{n^2}$ and $\overline{(\text{grad } n)^2}$ = averages for core,

$\overline{n_G^2}$ and $\overline{(\text{grad } n)_G^2}$ = averages for gap, ΔV ,

D_0 = diffusion constant for core material.

The diffusion constant according to simple diffusion theory is given by

$$D_0 = 1/(3\alpha_0) \quad .$$

According to asymptotic diffusion theory, it is given by:

$$D_0 = (\beta_0 - \alpha_0)/K^2$$

$$= 1/[3\alpha_0 (1 + 4f/5 + 4f^2/175 + \dots)] \quad (4)$$

where

$$f = (\beta_0 - \alpha_0)/\alpha_0$$

It is adequate to take only two terms in the series in the calculations of this section.

The values of n and $\text{grad } n$ as functions of radius were estimated from the results of the SNG calculation for the homogeneous core, that is, for the basic case. The one-group constants were obtained from the eleven-group set⁽²⁾ using the spectrum at the core center to weight the groups. The effects described in Section 2.1 were recalculated by one-group perturbation theory, and the results are also given in Table 2. It can be seen that asymptotic diffusion theory gives somewhat better results than simple diffusion theory. Asymptotic diffusion theory calculates these reactivity effects to an accuracy of 20% or better. The accuracy is best when the changes that are made mainly affect the inner part of the core. This was expected, since the one-group picture breaks down near the core-blanket interface, where there is rapid degradation of the neutron spectrum.

2.3 Fundamental Mode Calculations

The work described in this section refers to the same model of EBR-I as the work in Sections 2.1 and 2.2. The remainder of this report can be understood without reading this section. Estimates of material buckling, K^2 , will be compared with those that have been obtained by one-, four- and eleven-group fundamental mode calculations.⁽⁵⁾ The three recipes for calculating the diffusion constant, D , described in ANL-5800,⁽⁶⁾ will all be compared and the influence of the recipe on the resulting K^2 will be demonstrated.

The value of k for a bare sphere of geometrical buckling $K_0^2 = 0.021 \text{ cm}^{-2}$ (radius = 21.68 cm) was calculated by a fundamental mode calculation⁽⁵⁾ for the core composition given in Table 1, using the eleven-group constants given by Loewenstein and Okrent.⁽²⁾ The result is given in Table 3. Using as weights the spectrum obtained from this calculation, four-group constants were derived. The three recipes for the diffusion constant, D , noted in ANL-5800,⁽⁶⁾ were all used. These are the group-sum equivalents of:

$$D = 1/3 \left\{ \sum_z N^z \frac{\int \phi(E) dE}{\int \phi(E)/\sigma_{tr}^z dE} \right\} \quad (1)$$

$$D = \frac{\int \phi(E) dE}{3 \sum_Z N^Z \int \phi(E) \sigma_{tr}^Z dE} \quad (2)$$

$$D = \left[\int \left\{ \phi(E) dE / \sum_Z N^Z \sigma_{tr}^Z \right\} \right] / \left[3 \int \phi(E) dE \right] \quad (3)$$

These differ in the order of the processes, averaging over groups, mixing of elemental constants to produce core constants, and inverting a transport cross section to produce a diffusion constant. The order in the three cases above is

- (1) Invert, average, mix.
- (2) Average, mix, invert.
- (3) Mix, invert, average.

It is pointed out in ANL-5800 that the ideal recipe (when producing a multigroup library or, as here, when reducing a multigroup library to a few-group library) is the third recipe. In practice, it is necessary to accept the first recipe. It can be seen from Table 3 that, for four-group fundamental mode calculations, the choice of recipe does not significantly alter the value of k .

TABLE 3

Values of k , for Bare Sphere, Radius 21.68 cm, of Same Composition as Core of Mockup of EBR-I, Mark IV

		k		
		1st Recipe for D	2nd Recipe for D	3rd Recipe for D
Fundamental Mode Calculations	1-group (asymptotic)	1.117	1.192	1.121
	1-group (simple)	1.002	1.075	1.006
	4-group	0.990	0.999	0.991
	11-group	0.998		
SNG Calculation	11-group	1.069 ± 0.006		

One-group values of the material buckling K^2 for the EBR-I composition were obtained from the values of $(\beta_0 - \alpha_0)$ and α_0 of Section 2.2 from the formula

$$K^2 = (\beta_0 - \alpha_0) / D_0$$

From each value of K^2 , the value of the criticality parameter k (Table 3) for a sphere of geometrical buckling K_0^2 was calculated using the following formula:

$$\frac{1}{k} = \frac{K_0^2}{K^2} \left[\frac{\nu - 1 - \alpha^*}{\nu} \right] + \left[1 - \frac{\nu - 1 - \alpha^*}{\nu} \right] ,$$

which in this case reduces to

$$1/k = 0.63 (0.021/K^2) + 0.37 .$$

(In this formula, $\alpha^* = \Sigma_c/\Sigma_f$ for the core.)

Finally a value of K^2 was estimated from an SNG calculation (Section 2.1) by the method described in Appendix 1. This was reduced to a value of k (Table 3) using the formula above.

The following conclusions may be drawn from the data in Table 3:

- (i) The first recipe for estimating an average D is superior to the second one for diffusion calculations.
- (ii) One-group simple diffusion theory calculations agree with multigroup diffusion theory calculations, provided an accurate spectrum is used for weighting.
- (iii) Asymptotic (one-group) diffusion theory calculations do not agree better with the SNG calculation than simple diffusion theory. The discrepancy is of the same magnitude, but of opposite sign.

It should also be noted that the difference between the SNG result and the result by eleven-group diffusion theory was 7% in $\Delta k/k$. A comparison of SNG and eleven-group diffusion theory calculations for the reactor model described in Table 1 gave 9% in $\Delta k/k$.

3. Power Reactor Mockups

3.1 Critical Sizes

Calculations of critical size, using the SNG programme, were performed for five spherical reactors typical of those that may be built in ZPR-III (Table 4). The core composition in each case was close to typical values for a future metal-fueled fast reactor, namely, fuel, 35%; iron, 20% and sodium 45%. It was assumed that 0.56 volume of aluminum is approximately equivalent to 1 volume of sodium. The assumed blanket composition was that of the medium blanket of the mockup of the Enrico Fermi reactor.

TABLE 4

Properties of Some Spherical Reactors Studied

Reactor	A	B	C	D	E
Number of plutonium plates per drawer*	1	$1\frac{1}{2}$	2	3	4
Core Volume Fractions					
Pu ²³⁹	0.0221	0.0331	0.0441	0.0662	0.0883
Pu ²⁴⁰	0.0010	0.0016	0.0021	0.0032	0.0042
U ²³⁵	0.0007	0.0006	0.0006	0.0005	0.0004
U ²³⁸	0.3493	0.3244	0.2994	0.2495	0.1996
Fe	0.1572	0.1655	0.1737	0.1902	0.2068
Al	0.2450	0.2459	0.2467	0.2483	0.2500
Blanket Thickness (cm)**	97.8	63.1	56.4	56.0	59.1
Critical Conditions:					
Core radius (cm)	211.91	56.82	39.49	27.08	21.66
Core Volume (L)	39863	768.2	257.9	83.21	42.58
Mass of Total Pu (kg)	16783	484.4	227.0	109.97	71.63

*The plate arrangement in core drawers was in all cases: Pu + depleted U, 8 (total); Fe, 1; 100% Al, 1; 63% Al, 6.

**The blanket volume fractions were in all cases: 0, 0.0011, 0.4870, 0.2100, and 0.1347 respectively.

In Table 4 it should be noted that the blanket thickness is different for each case because the SNG programme, when adjusting the core radius to achieve criticality, also alters the blanket thickness. In every case, the blanket is effectively infinite.

3.2 SNG Calculations of Control Rod Worths for Reactor C

The worths of control rod drawers in ZPR-III for the mockup of Reactor C were calculated by the method used by Molino and Davidson at KAPL.⁽⁷⁾ The basic calculation for this work was an SNG calculation for an infinite cylindrical version of Reactor C. Axial leakage was allowed for by equivalent group capture cross section.⁽¹⁾ Further calculations were then performed with the core density reduced to 50% within a central cylinder of 2.5-cm radius to obtain the worth of a central drawer, and with the core density reduced to 95% within an annular cylinder of 2.5-cm thickness at the core edge, to obtain the worth of an edge drawer. The results, given in Table 5, are approximate because this method does not take account of the influence of the axial blankets on control rod worth. Note that the figures refer to one drawer in one half only.

TABLE 5

Reactivity Effects for Reactors with Cylindrical Cores (with $L/D = 1$)
of the Same Composition as the Spherical Cores of Table 4

Reactor	A	B	C	D	E
Reactivity loss per fractional fuel expansion axially, $(\Delta k/k)(1/\Delta L)$	0.008	0.031	0.045*	0.066	0.082
Reactivity loss per fractional expansion axially of whole core (assumed homogeneous), $(\Delta k/k)(L/\Delta L)$					
(i) expanding into void gap	0.19	0.19	0.19*	0.19	0.19
(ii) expelling blanket	0.34	0.34	0.34*	0.34	0.34
Worth of one ZPR-III drawer, $\Delta k/k\%$					
(i) at core center	0.021	0.24	0.45*	0.86	1.26
(ii) at core edge	0.0004	0.043	0.13*	0.40	0.73

*These are the only results obtained by SNG calculations. The remaining results were estimated approximately from the results for Reactor C by semi-empirical methods described in the text.

3.3 Calculations of Control Rod Worths for Reactors A, B, D and E

In order to economize in machine time, the worths of control rods for reactors other than Reactor C were estimated from the SNG results for Reactor C by a semi-empirical method based on one-group perturbation theory. An approximate formula was derived for the worth of a control rod from the complete one-group formula given in Section 2.2 by making two assumptions:

- (i) that the transport contribution to the numerator, dependent on $\text{grad } n$, can be neglected, and
- (ii) that the blanket contribution to the denominator, $1/A$, can be neglected.

The approximate formula is:

$$\frac{\Delta k}{k} = \left(\frac{\nu - 1 - \alpha^*}{\nu} \right) \left(V_{cr}/V_c \right) \left[\overline{n_{cr}^2} / \overline{n_c^2} \right] ,$$

where $\alpha^* = \Sigma_c / \Sigma_f$ for core material; V_{cr} and V_c are the volumes of the fuel region of one drawer and of the whole core, respectively; and $\overline{n_{cr}^2}$ and $\overline{n_c^2}$ are the mean square normalized fluxes for drawer and core, respectively.

The first factor, $(\nu - 1 - \alpha^*)/\nu$, was assumed to have the same value in all reactors as for Reactor C, namely, 0.39. The second factor may be written $\frac{1}{2}A/\pi R^2$, where $A = 30.64 \text{ cm}^2$ is the frontal area of unit cell of the ZPR-III matrix and R is the radius of the cylindrical core (assumed to have $L/D = 1$). If a is the radius of the spherical core of the same volume, then

$$R^2 = 0.76 a^2 \quad .$$

On compounding all these numerical factors, the formula becomes:

$$\frac{\Delta k}{k} = \frac{2.5}{a^2} \left[\overline{n_{cr}^2} / \overline{n_c^2} \right]$$

In order to make it simple to evaluate the bracket, it was assumed that the flux in the cylindrical core was separable and also of parabolic shape in both the r and z directions, as follows:

$$n = \left[1 - (1 - n_1)(z/H)^2 \right] \left[1 - (1 - n_2)(r/R)^2 \right] \quad ,$$

where n_1 and n_2 are the values of flux at the core edge axially and radially. It can then be shown that for a central control drawer

$$\overline{n_{cr}^2} / \overline{n_c^2} = \frac{3}{1 + n_2 + n_2^2}$$

and for an edge control drawer

$$\overline{n_{cr}^2} / \overline{n_c^2} = \frac{3n_2^2}{1 + n_2 + n_2^2} \quad .$$

From the above formulas it is possible to calculate the worth of control drawers knowing a and n_2 . (It was assumed that n_2 was approximately equal to the normalized flux at the edge of the spherical model.)

On making the calculation for central and edge drawers of Reactor C, the results obtained were 0.32% and 0.042%, in poor agreement with the values in Table 5, obtained by SNG calculations. The discrepancies are mainly due to the neglect of the grad n term in the numerator. The approximate formula was forced to give the SNG values by increasing the numerical factor from 2.5 to 3.5 for central drawers and to 7.9 for edge drawers. The other results in Table 5 were then calculated by the approximate formula. The error in the central worths should be no more than about 30%, and in the edge worths than about a factor of 2.

3.4 SNG Calculations of Axial Expansion for Reactor C

The effect of axial expansion of the core material as a whole is included in Table 5 for comparison with the smaller effect of expansion of fuel only. Again, as in Section 3.3, the effects for Reactor C were calculated by a set of SNG calculations and the effects for other reactors were estimated from these results by empirical methods. The basic case in this case was an infinite slab version of Reactor C. The other SNG calculations were performed for this system with the core contracted in length by 4%, one with blanket filling the gap left by the contracted core and the other with half-density blanket filling the gap.

The correction for radial leakage had to be made with care because the core density was not the same in all three cases. It can be seen from Glasstone and Edlund⁽⁸⁾ that the reflector savings are proportional, approximately, to the diffusion constant of the core and so inversely proportional to the core density. The procedure adopted was therefore as follows:

A round number, 35 cm, was chosen for the core radius for the basic case such that $L/D = 1$, approximately, for the finite cylindrical core. The radial reflector savings were then assumed to be 14.2 cm on the evidence of the spherical calculation for Reactor C (see Appendix 1). This determined the extrapolated radius, $R_e = 49.2$ cm, and the radial buckling, given by

$$B_r^2 = (2.4048/R_e)^2$$

The effect of radial leakage was then allowed for by including additional effective group capture cross sections⁽¹⁾ given by

$$(\Sigma_c)_i = D_i B_r^2$$

where D_i is the group diffusion constant. For the two cases with the core length reduced by 4% (that is, core density increased by a factor 100/96), the whole procedure was repeated with the reflector savings multiplied by a factor 96/100. In this way it was shown that for a core length change, with blanket keeping contact with the core, the reactivity was given by

$$\frac{\Delta k}{k} = -0.34 \left(\frac{\Delta L}{L} \right)$$

To show that this procedure was reliable, the effect of uniform expansion of a finite cylindrical core was compared with the effect for a spherical core. A calculation similar to the above showed that the effect of radial expansion for a cylinder was given by

$$\frac{\Delta k}{k} = -0.62 \left(\frac{\Delta R}{R} \right)$$

The two results together imply that for the uniform expansion of a finite cylinder

$$\frac{\Delta k}{k} = - 0.32 \left(\frac{\Delta V}{V} \right) .$$

This result is in satisfactory agreement with the calculated result for a spherical core:

$$\frac{\Delta k}{k} = - 0.36 \left(\frac{\Delta V}{V} \right) .$$

It can be seen that the effect of increasing the core length for a cylinder by 1% is approximately one-third of the effect of increasing the radius of a spherical core by 1%, as is usually assumed. The method described here is more generally applicable. It could be used for example when the axial and radial reflectors were different.

3.5 Calculations of Axial Expansion for Reactors A, B, D and E

It can be shown by an analysis similar to that in Section 3.3 that the effect of linear expansion of the core (expansion into a small gap-fixed blanket boundary) is given by

$$\frac{\Delta k}{k} = - p \left(\frac{\Delta L}{L} \right) ,$$

where

$$p = 4 \left(\frac{\nu - 1 - \alpha^*}{\nu} \right) \left(\frac{2 + n_1 - 3n_1^2}{8 + 4n_1 + 3n_1^2} \right) .$$

On evaluating p for Reactor C, a value of 0.31 was obtained, in fair agreement with the value 0.19 given in Table 5 that was obtained by SNG calculations.

As the reactor size decreases, the second bracket in the above formula decreases, but the total change from Reactor A to Reactor E is only 40%. Conversely, when the reactor size decreases, the first factor probably increases slightly. Therefore, it seemed reasonable to assume the same values of p (for fixed and moving blankets separately) for all reactors in the study (Table 5).

3.6 SNG Calculations of Fuel Plate Expansion for Reactor C

The plutonium assemblies for ZPR-III will usually be constructed with the springs in the plutonium cans on the edge away from the plane of division of the reactor. In this section, the reactivity associated with the expansion of fuel alone will be studied; this is the effect associated with the prompt power coefficient. In a typical assembly, Reactor C, there will be 7 two-inch-long plutonium cans in each half and the springs in the cans will lie in 7 thin sections, in each half, parallel to the plane of division of ZPR-III. When the plutonium expands by a fraction, $\Delta l/l$, there will be a change in reactivity given by

$$\frac{\Delta k}{k} = -p \frac{\Delta l}{l}$$

Various calculations of p for typical reactors are presented in this section.

The same problem for an EBR-I mockup was described in Section 2. In that case, calculations were performed in spherical geometry with the spring-gap sections replaced by equivalent spherical shells. Since this is a poor approximation for large assemblies for the sections close to the plane of division, this problem for Reactors A through E was studied in cylindrical geometry.

The basic case (Reactor C) was a cylindrical model with the plutonium spread uniformly over the whole core, that is, there were no spring gaps. An SNG calculation was performed in slab geometry with corrections for radial leakage, as described in Section 3.4. The slab half-thickness was 35 cm, which corresponds to a reactor with $L/D = 1$. Another SNG calculation was then performed with the plutonium contracted by 4% to make plutonium-free gaps 5 cm apart. There was, of course, no central gap. The radial buckling assumed was the same for both calculations. From the change in k of the SNG calculations, it was shown that $p = 0.045$ (Table 5).

The calculations above refer to two-inch plates only. The effects for other sizes will be so similar (compare Section 2) as not to warrant separate calculation.

3.7 Perturbation Theory Calculations of Fuel Plate Expansion

Because of the uncertainty in the effect of fuel plate expansion on the radial buckling correction, the SNG calculations for Reactor C are not necessarily more accurate than the one-group perturbation calculations that will now be described. The same basic assumptions were made as in Sections 3.3 and 3.5. It was then possible to calculate n^2 and $(\text{grad } n)^2$ for

the whole core and for the spring gaps in terms of n_1 , the normalized flux at the core boundary. In this case, the $\text{grad } n$ term was included, but the contribution of the blanket to the denominator was neglected, as before. The result for Reactor C was

$$p = 0.0485 \quad ,$$

which confirms the result by SNG calculations.

When the fuel contracts, there is a change in average flux squared and in average gradient squared over the plutonium. Both can be expressed as a polynomial in $1/H$, where l is the fuel plate length (5 cm) and H is the core half-height. If all powers higher than the first are neglected, there is only a slight loss of accuracy. If, further, the $\text{grad } n$ term in the numerator and the contribution of uranium fissions to the denominator are both neglected, the formula for p can be simplified to

$$p = \left(\frac{\nu - 1 - \alpha^*}{\nu} \right)_{\text{Pu}} \left[\frac{15}{2} \frac{(1 - n_1^2)}{(8 + 4n_1 + 3n_1^2)} \right] \left(\frac{1}{H} \right) .$$

On evaluating this expression for Reactor C, a value

$$p = 0.057$$

was obtained, in good agreement with more accurate methods.

The values of p in Table 5 were obtained assuming that p is inversely proportional to the radius for the spherical model, and normalizing to the SNG result for Reactor C. In fact, the second bracket in the expression for p above decreases by a factor of two as reactor size decreases from Reactor A to E, but the other approximations made all produce small errors in the compensating sense.

4. Delayed Neutron Fractions

The delayed neutron fractions for the EBR-I mockup and the five mockups of larger power reactors were estimated by means of the approximate formula derived in Appendix 2, namely;

$$\beta_{\text{eff}} = \left\{ \frac{\int \phi_0^* \gamma_D dE}{\int \phi_0^* \gamma dE} \right\} \left[\sum_x (\beta_x \alpha_{xc} \lambda_c + \beta_x \alpha_{xb} \lambda_b) \right] .$$

The first bracket was evaluated in each case by a fundamental mode diffusion theory calculation using the HOBO code.⁽²⁾ The α 's and λ 's in the second bracket were evaluated from the SNG calculations used for estimating the critical masses. The values of β for the constituents of the reactors were taken from ANL-5800.⁽⁶⁾ The various steps in the calculation are summarized in Table 6.

TABLE 6

Summary of Calculations of Effective Delayed Neutron Fraction*

Reactor	λ_c	λ_b	β'_c	β'_b	β'	$\frac{\sum \gamma D_i \phi_i^*}{\sum \gamma_i \phi_i^*}$	β_{eff}
A	0.9995	0.0005	0.4851	1.29	0.4855	0.8088	0.393
B	0.990	0.010	0.4516	1.33	0.4604	0.8691	0.400
C	0.981	0.019	0.4247	1.35	0.4423	0.9095	0.402
D	0.967	0.033	0.3790	1.38	0.4122	0.9627	0.397
E	0.956	0.044	0.3371	1.40	0.3838	0.9996	0.384
EBR-I	0.953	0.047	0.2109	1.52	0.2723	1.0869	0.296

*Notation: β'_c and β'_b are delayed neutron fractions for core and blanket separately unmodified by worth of delayed neutrons, β' is delayed neutron fraction for whole reactor, unmodified.

It can be seen from the above description that the first step in the approximate calculation of β_{eff} is the estimation of β for a bare core of the same composition as the actual core. Because each λ_b is small, the blanket contribution is small and therefore may be estimated with sufficient accuracy by a one-group method. The absolute accuracy of the estimates of β is about 10% because of experimental errors in the values of β for the constituents.⁽⁶⁾ The relative accuracy will be about 5%. The approximate method will slightly overestimate β for small reactors; the rise in β with dilution, therefore, seems significant.

The approximate method of calculation β was checked against exact diffusion theory calculations for Reactor C. The multigroup fluxes and adjoint fluxes required for the integrations were obtained using the diffusion theory codes RE-7 and RE-27, respectively.⁽²⁾ The integrations were performed by the code RE-185. This method yielded 0.386% for β_{eff} in good agreement with 0.402% by the approximate method.

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Appendix 1

Estimation of Buckling and Reflector Savings

When performing the one-dimensional SNG calculations, described in Sections 3.2, 3.4 and 3.6, for cylindrical reactors, it was necessary to make a correction for the transverse leakage. This required a knowledge of the reflector savings in the transverse direction. The reflector savings were calculated from another SNG calculation for a spherical system of the same core and blanket compositions. An estimate of the buckling, K^2 , was first made from the detailed flux shape near the core center. Then the extrapolated radius, R_e , was calculated from the formula

$$R_e = \pi/K \quad .$$

Finally the reflector savings were obtained as the difference between the actual and extrapolated core radii.

For simplicity the numerical details will be explained in slab geometry. Near the center of a reflected slab reactor, the flux satisfies the following equation:

$$\nabla^2 \phi + K^2 \phi = 0 \quad .$$

Hence, for a zone near the center of the reactor bounded by $z = 0$ and z_0 ,

$$\begin{aligned} K^2 &= - \frac{\int_0^{z_0} (d^2\phi/dz^2) dz}{\int_0^{z_0} \phi dz} \\ &= - \frac{(d\phi/dz)_{z=z_0}}{\int_0^{z_0} \phi dz} \quad . \end{aligned}$$

Now the SNG programme prints out fluxes at the center of the mesh intervals. Suppose the mesh intervals are all of the same size, h ; then the above formula reduces to

$$K^2 = \frac{\phi(n) - \phi(n+1)}{h^2 \sum_{m=1}^n \phi(m)} \quad ,$$

where $\phi(m)$ is the flux in mesh interval m . For spherical geometry, the corresponding formula is

$$K^2 = \frac{n^2 [\phi(n) - \phi(n+1)]}{h^2 \sum_{m=1}^n (m^2 - m + 1/3) \phi(m)}$$

In practice, the fission rate was used in place of ϕ in the above formulas. A series of values of K^2 were obtained for various values of mesh number, n , and then an extrapolation was made to zero n .

Appendix 2

Approximate Method for Calculating the Effective Delayed Neutron Fraction

For any reactor, the effective delayed neutron fraction is given by

$$\beta_{\text{eff}} = \frac{\int dE \int dV \left\{ \phi^*(\underline{r}, E) \gamma_D(E) \int \sum_x [\beta_x \nu_x(E') \Sigma_{fx}(\underline{r}, E') \phi(\underline{r}, E') dE'] \right\}}{\int dE \int dV \left\{ \phi^*(\underline{r}, E) \gamma(E) \int \sum_x [\nu_x(E') \Sigma_{fx}(\underline{r}, E') \phi(\underline{r}, E') dE'] \right\}},$$

where

dE = energy increment,

dV = volume increment,

$\phi(\underline{r}, E)$ = flux of energy, E , at point, $\underline{r}$,

$\phi^*(\underline{r}, E)$ = adjoint flux,

$\gamma(E)$ = fission spectrum,

$\gamma_D(E)$ = delayed neutron spectrum,

β_x = delayed neutron fraction for isotope x , assumed independent of energy of neutron causing fission,

$\nu_x(E)$ = number of neutrons per fission for isotope x , when fission is caused by neutron of energy, E ,

$\Sigma_{fx}(\underline{r}, E)$ = macroscopic fission cross section for isotope x at $\underline{r}$ for neutrons of energy E .

We now proceed to make approximations. First, we assume that the worth, $\phi^*(\underline{r}, E)$, can be separated into a function of $\underline{r}$ and a function of E . Guided by the form for one-group theory, namely, $\phi^*(\underline{r}) = \phi(\underline{r})$, we assume

$$\phi^*(\underline{r}, E) = \phi^*(0, E) n(\underline{r})$$

where

$$\phi^*(0, E) = \phi_0^* = \text{worth at core center,}$$

$$n(\underline{r}) = \sum_i N_i(\underline{r}) = \text{total flux at } \underline{r}.$$

Then we are able to perform the E integration as follows:

$$\beta_{\text{eff}} = \left\{ \frac{\int \phi_0^* \gamma_D dE}{\int \phi_0^* \gamma dE} \right\} B$$

where

$$B = \frac{\int dV \left\{ n(\underline{r}) \int_{\Sigma_{\underline{x}}} \left[\beta_{\underline{x}} \nu_{\underline{x}}(E') \Sigma_{f\underline{x}}(\underline{r}, E') \right] \phi(\underline{r}, E') dE' \right\}}{\int dV \left\{ n(\underline{r}) \int_{\Sigma_{\underline{x}}} \left[\nu_{\underline{x}}(E') \Sigma_{f\underline{x}}(\underline{r}, E') \right] \phi(\underline{r}, E') dE' \right\}}$$

We now write $F_{\underline{x}}(\underline{r})$ for the total fission rate in isotope $\underline{x}$ at $\underline{r}$, and let

$$F(\underline{r}) = \sum_{\underline{x}} F_{\underline{x}}(\underline{r}) \quad .$$

We separate the volume integrals in the numerator into core and blanket contributions. Thus

$$B = \frac{\int_c dV \left\{ n(\underline{r}) \sum_{\underline{x}} \left[\beta_{\underline{x}} F_{\underline{x}}(\underline{r}) \right] \right\} + \int_b dV \left\{ n(\underline{r}) \sum_{\underline{x}} \left[\beta_{\underline{x}} F_{\underline{x}}(\underline{r}) \right] \right\}}{\int_{c+b} dV \left\{ n(\underline{r}) F(\underline{r}) \right\}} \quad .$$

So far we have made only one approximation. We now make our second and last approximation: that in the core

$$\frac{F_{\underline{x}}(\underline{r})}{F(\underline{r})} = \alpha_{\underline{x}c}, \text{ a constant independent of } \underline{r},$$

and in the blanket

$$\frac{F_{\underline{x}}(\underline{r})}{F(\underline{r})} = \alpha_{\underline{x}b}, \text{ a constant independent of } \underline{r} \quad .$$

Hence our complete formula is

$$\beta_{\text{eff}} = \left\{ \frac{\int \phi_0^* \gamma_D dE}{\int \phi_0^* \gamma dE} \right\} \left[\sum_{\underline{x}} \left(\beta_{\underline{x}} \alpha_{\underline{x}c} \lambda_c + \beta_{\underline{x}} \alpha_{\underline{x}b} \lambda_b \right) \right]$$

where

$$\lambda_c = \frac{\int_c dV \left\{ n(\underline{r}) F(\underline{r}) \right\}}{\int_{c+b} dV \left\{ n(\underline{r}) F(\underline{r}) \right\}}$$

$$\lambda_b = 1 - \lambda_c \quad .$$

We choose to set

$$\alpha_{xb} = \frac{\int_b dV \{F_x(\underline{r})\}}{\int_b dV \{F(\underline{r})\}} .$$

We might have chosen

$$\alpha_{xc} = \frac{\int_c dV \{F_x(\underline{r})\}}{\int_c dV \{F(\underline{r})\}} ,$$

but chose instead

$$\alpha_{xc} = \frac{F_x(0)}{F(0)} .$$

The difference between the two values of α_{xc} defined above is a few per cent for the reactors in our study. The true value lies between; we prefer the latter form because the neutrons produced near the core center are the most important ones.