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MASTER

**DEFICIENT COOLING
FIFTH QUARTERLY PROGRESS REPORT
JULY 1-SEPTEMBER 30, 1970**

U.S. ATOMIC ENERGY COMMISSION
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DEFICIENT COOLING
FIFTH QUARTERLY PROGRESS REPORT
July 1—September 30, 1970

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SUMMARY

A simplified transient analysis, for flow and/or pressure decay in a boiling system (reported in the fourth quarterly report, GEAP-10221), has been coded for use with FORTRAN IV. The program was used to predict time of CHF in two actual runs. The results are very close, on the conservative side, to the actual data and other predictions obtained from a more complex code (SCAT) which calculates thermal conditions in a reactor core.

Installation of the three-subchannel sampling system for the nine-rod test section was completed. Final inspection and hydro tests were completed to meet ASME code requirements.

The TAPER code, for acquisition and reduction of transient data, has been expanded from seven- to fourteen-channel capability. Analysis of the transient flow as obtained by an orifice shows that the inertia term is practically negligible for cases of interest in this program. Thus, transient flow data can be reduced, with little error (about 1 to 2%), by use of standard steady-state orifice techniques.

1. INTRODUCTION

1.1 GENERAL

The work on the Deficient Cooling Program is directed toward extension of out-of-pile heat transfer data in support of current nuclear-powered, water-cooled plants. A brief review of current technology reveals that the large bulk of data is of the steady- or quasi-steady-state class. Thus, a need exists for data taken under transient conditions of power, flow and pressure in combination with different geometries. Analysis and study of such data are expected to contribute to the following overall objectives:

- a. A firmer understanding and establishment of heat transfer safety limitations, and
- b. Extension of safe operating limits to higher power densities.

Deficient cooling conditions are considered to be those which result in inadequate cooling of the fuel. They may result in potentially excessive cladding temperature which, in combination with the internal gas pressure of the fuel, may cause some type of fuel disruption. Resultant temperatures depend on the mode or modes of heat transfer that arise during deficient cooling conditions. A brief description of these modes of heat transfer follows.

Three different modes of heat transfer can generally occur when heat is removed, during high quality conditions, from a surface such as a fuel rod: nucleate boiling, with a liquid film on the heated surface; transition boiling; and the liquid deficient region. Nucleate boiling occurs at very high surface heat transfer coefficients, usually in excess of 10,000 Btu/h-ft², with steady surface temperatures. Increase of heat flux eventually results in "film dryout" and a change of mode to transition boiling, which is usually characterized by temperature oscillations of varying amplitude on the heated surface. The transition mode of heat transfer exhibits relatively low heat transfer coefficients, on the order of 1000 Btu/h-ft² or less. Continued increase of heat flux eventually results in the liquid deficient regions, with some liquid droplets impinging on the heated surface which is mostly steam cooled.

The heat flux at which transition is made from the nucleate to transition mode of boiling, as evidenced by onset of surface temperature oscillations, is usually termed the critical heat flux (CHF). For low qualities, it is also known as the

departure from nucleate boiling (DNB), or burnout point. Considerable effort has been made on parametric definition of the CHF with pressure, coolant flow, vapor quality, and the geometry of the system. These parametric studies serve as design data and indicate the operating conditions beyond which fuel surface temperatures increase substantially, with greater risk to fuel cladding integrity.

Most of the work defining limits of CHF has been done with small incremental heat flux increases which would allow it to be classified as quasi-steady-state data. However, the actual modes of heat transfer become more complex in some cases involving transients, when the dynamic characteristics of the fuel-coolant system come into play. Therefore practical cases of transient conditions which are considered worthwhile for study and test under this Deficient Cooling Program are those involving:

- a. Transient reduction of flow,
- b. Transient increase of power,
- c. Oscillation of power into and out of the transition boiling region to determine effects of prolonged operation at CHF, and
- d. Transient decrease of pressure which causes flashing and potentially deficient cooling conditions due to increased void content.

Furthermore, it is proposed to study, under transient conditions, typical geometry changes such as those due to fuel spacer components and those caused by rod blowing and swelling. Such studies are expected to define more clearly heat transfer safety limitations, and will eventually result in extension of safe operation to higher limits.

1.2 PROGRAM OBJECTIVES

The objectives expected to be accomplished under this program are as follows:

- a. Determine experimentally the CHF and temperature regimes which may occur in water reactor fuel assemblies due to nonstandard cooling conditions that result from power, flow, or pressure transients, and determine these values relative to steady-state data.
- b. Evaluate temperature regimes likely to be experienced by water reactor fuel assemblies in loss-of-coolant accidents, and determine areas of similarity or relationship between these regimes and those found in transient critical conditions.
- c. Evaluate consequences of fuel rod geometry changes on heat transfer performance from simulated tests on single, electrically-heated rods.
- d. Provide a plan for further in- and out-of-pile work necessary for more complete performance and definition of the performance of water reactor fuel under deficient cooling conditions.

1.3 ORGANIZATION OF PROGRAM

The outline of tasks, as originally proposed for the entire program, is shown in Table 1 with brief reference to the equipment and type of tests. It is intended to preserve these task reference designations in this and subsequent reports so that documenting of work and progress can be made in a systematic manner.

Tasks 3, 4, 5, and 8 involve test work. Chronological sequence of testing will probably proceed in the numerical task order.

Table 1-1
TASK DESIGNATION AND CONTENT

No.	Title	Content
1	Planning and Analysis	<ul style="list-style-type: none"> a. Review current technology and plan appropriate tests to integrate with required safety programs b. Analysis as necessary to determine test parameters
2a	Design	<ul style="list-style-type: none"> a. Design new equipment or modifications of existing equipment to carry out test plans
2b	Manufacture	<ul style="list-style-type: none"> a. Manufacture or purchase new equipment required
3	One-Rod Exploratory Transient CHF Tests	<ul style="list-style-type: none"> Exploratory tests involving a. Steady-state and transient flow, power, and pressure b. Cycling of power and flow into the region of CHF transition boiling c. Prolonged exposure to CHF conditions d. Modifications of heaters for nonuniform axial heat flux
4	Multirod Transient Tests	<ul style="list-style-type: none"> Tests involving: a. Steady state and transient conditions b. Power and flow cycling c. Combinations of intra-bundle clearance, power distribution, and spacer components with a. and b.
5	One-Rod Tests with Geometry Changes	<ul style="list-style-type: none"> a. Effects of temperature and internal pressure on heater swelling b. Steady-state simulated swelling at constant coolant pressure drop
6	Data Reduction and Analysis	<ul style="list-style-type: none"> a. As required for test Tasks 1, 3, 4, 5, and 8
7	Project Administration, Reporting, and Recommendations	<ul style="list-style-type: none"> a. Administration of entire program b. Regular and topical reporting c. Future recommendations for continued efforts
8	Multirod Geometry Changes	<ul style="list-style-type: none"> a. Steady-state CHF data with spacer components and simulated fuel rod bowing and swelling

2. ANALYSIS (TASK 1)

2.1 TRANSIENT ANALYSIS

2.1.1 Transient Computer Code Development

A simplified transient analysis for flow and/or system pressure decay in a boiling system was performed, and reported in the fourth quarterly report, GEAP-10221, page 4. The solution, using the "method of characteristics," adopts the Lagrangian point of view and yields the transient local conditions of quality, and flow or pressure along the heated length. The transient equations have been programmed in FORTRAN IV, yielding a computer code, MAYU, which can handle system depressurization and/or flow decay transients. As discussed previously in GEAP-10221, the exponential flow decay case can be solved exactly, while the depressurization and combined pressure and flow transient must be solved numerically, using the constraint that the total mass in the convected control volumes remains constant.

A depressurization test case for MAYU was run in which a linear depressurization, at -200 psi/sec, was made for system pressure from 1000 to 800 psi. The results of this run for the first 0.35 second of depressurization are shown in Figure 2-1. The plot shows several characteristic and constant quality contours at each point in space and time. The main characteristic is the time-position trajectory of a control volume which was at the theoretical boiling boundary at $t = 0$. It can be seen that at the end of the heated length, $z = 6$ feet, the quality contour experiences a maximum when it crosses the main characteristic. Physically, this means that for times less than one two-phase transit time, from boiling boundary to end of heated length, the fluid "flashes off" quite strongly during depressurization, causing the exit mass flux to increase and the exit quality to decrease (see Figure 2-2). Once the fluid exiting the test section is fluid which was subcooled at $t = 0$, the expulsion process is less violent, so that the exit mass flux decreases and the exit quality then increases with time in a quasi-steady-state manner. It is thus apparent that the dynamics of the transient are considerably different for times less than one two-phase transit time.

Figure 2-2 gives the transient quality at the end of the heated length ($z = 6$ ft) for the entire 1-second depressurization from 1000 to 800 psi. The minimum in quality at $t = 0.265$ second is associated with one two-phase transit time, and thereafter the quality increases due to the decrease of mass flux as "flashing" subsides. The depressurization ends at 1 second, after which the quality increases quite rapidly until steady-state conditions at 800 psi are achieved. This rapid increase in quality, once "flashing" stops, has important implications on the CHF phenomena subsequent to depressurization.

2.1.2 Analysis of CHF Data

Transient CHF data have been taken in a single-rod annular test section with an inside diameter of 1.25 inches, containing a 0.540-inch outside diameter by 9-foot electrically heated stainless steel tube. Inlet flow to the test section was monitored with a Pace cell across a sharp-edged orifice. System pressure, at the end of the heated length, was monitored with a fast response Baldwin-Lima-Hamilton transducer. The data was corrected for inertia terms and lags, as described previously in the first quarterly report, GEAP-13048. In almost all cases, the observed CHF occurred after the blowdown valve closed. Thus CHF was not directly associated with the depressurization phenomena but with the rapid rise in quality (see discussion on quality increase in subsection 2.1.1).

Two runs in which CHF did occur during blowdowns in the range of 1000 to 800 psi were chosen for comparison of experimental time to CHF to that predicted by the transient computer program MAYU described in subsection 2.1.1. The system pressure and inlet flow variations for these runs are plotted in Figures 2-3 and 2-4. It can be seen that the depressurization transient can be approximated by two distinct linear slopes. These slopes are associated with single-phase steam being discharged initially, and then a discharge of two-phase mixture which greatly slows down the transient pressure decay. The time history of system pressure and inlet flow was input to MAYU, along with the initial conditions given in Table 2-1.

Figure 2-1. Particle Paths and Constant Quality Lines for Depressurization

Figure 2-2. Quality versus Time During and After Depressurization ($z = 6$ ft)

Figure 2-3. Depressurization Showing Experimental Conditions and Predicted CHF (Run 36)

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Figure 2-4. Depressurization Showing Experimental Conditions and Predicted CHF (Run 62)

Table 2-1

Run No.	Mass Flux (lb/h-ft ²)	Heat Flux (Btu/h-ft ²)	Exit Quality (%)	Inlet Temperature (°F)	System Pressure (psia)
36	0.506 X 10 ⁶	0.514 X 10 ⁶	22.8	514	996
62	1.06 X 10 ⁶	0.635 X 10 ⁶	13.6	527	994

The steady-state "best linear fit" CHF correlations, obtained previously in the same test section, were also input to MAYU, so that

$$q''_{CHF} [G(t), P(t)] = A [G(t), P(t)] [x(t)] + B [G(t), P(t)], \quad (2-1)$$

where it is noted that the slope A and intercept B are a function of time. MAYU calculated the exit quality, and q''_{CHF} by Equation (2-1), and compared it to the actual q'' to see whether CHF has occurred (i.e., when $MCHFR^\dagger = q''_{CHF}/q'' = 1.0$). The results of this calculation are summarized in Table 2-2.

Table 2-2

Run No.	MAYU Time ^(a) to MCHFR = 1 (sec)	Measured Time ^(a) of CHF (sec)	Duration of Depressurization Transient (sec)
36	2.03	2.30	3.0
62	1.29	1.40	2.7

(a) Times referenced to the start of the transient.

It may be noted in Table 2-2 that predictions by MAYU of transient pressure, flow, and quality at the exit, coupled with steady-state CHF correlations, yield conservative predictions for onset of CHF. These trends are in agreement with other investigators.† The conservatism in predicting time to CHF could well be caused by the assumptions implicit in MAYU. Future work in this area should clarify this point.

3. MANUFACTURE (TASK 2)

3.1 NINE-ROD, THREE-SUBCHANNEL SAMPLING SYSTEM INSTALLATION

Installation of the three-subchannel sampling system for the nine-rod test section was completed during this quarter. This installation is expected to result in more rapid and economical acquisition of data compared with the one-subchannel sampling system used in the past.

The sampling system was described briefly in the third quarterly report, GEAP-10196. The line diagram for the system is shown in Figure 3-1. This figure has been brought up to date to show locations of all resistance thermal detectors and thermocouples in the sample condensing and metering systems.

† Minimum Critical Heat Flux Ratio

‡ Moxon, D., and Edwards, P. A., "Dryout During Flow and Power Transients," AEEW-R-553, 1967.

Figure 3-2 shows the three sample lines emerging vertically from the top flange of the test vessel, and passing horizontally to the four heat exchangers in the upper background. Figure 3-3 is a different view, looking obliquely downward, showing the heat exchangers and sample and cooling water piping. The four-vane sample flowmeters are out of view underneath the piping in both Figures 3-2 and 3-3. The insulation has been removed to show more detail.

The Heliflow heat exchangers were made by Graham Manufacturing Company. HE1 is rated at 1500 psi/300°F on the shell side and 1500 psi/600°F on the tube side. The other three exchangers—HE1', HE2, HE3—are rated 225 psi on the shell, cooling water side; and 1500 psi/600°F on the tube, sample side. The entire system is fabricated from stainless steel, and is designed and installed in accordance with ASME Code requirements.

A feature of the first sample system, using exchangers HE1 and HE1', is that the sample is condensed in HE1 and then sufficiently subcooled in HE1' so it can be used in HE1 to condense the incoming sample. Since the sample and cooling water flow in HE1 are equal, they cancel in the heat balance. Thus, two sources of error in the quality determination of the sample are eliminated: sample flow measurement, and cooling water flow measurement.

3.2 INDIRECT CALROD TYPE HEATERS FOR NINE-ROD TEST SECTION

Twelve uniform flux, stainless-steel-jacketed calrods ordered from General Electric Company, Shelbyville, Indiana, were received and pre-tested at low pressure up to about 90% of full power. The pre-test was successful; however, after grooves were milled in the jackets for flow splitters, surface cracks began to develop. These cracks developed in the braze-filled thermocouple grooves as well as in the splitter grooves and were attributed to residual stresses from swaging operations during manufacture. Two or three rods had cracks so severe they were obviously unusable. The balance had cracks, which were just noticeable, but showed signs of progression. It was decided not to risk expensive assembly and operation with these questionable rods as they were very likely to leak and cause electrical shorting. The rods were returned for replacement. Design and manufacturing changes will be made by the vendor to avoid this occurrence on new calrods.

Figure 3-1. Flow Diagram for Three-Subchannel Sampling System (Nine-Rod Test Section)

Figure 3-2. Nine-Rod, Three-Subchannel Sampling System

Figure 3-3. Nine-Rod, Three-Subchannel Sampling System (another view)

4. EXPERIMENTAL WORK (TASK 4)

4.1 NINE-ROD SUBCHANNEL SAMPLING INSTALLATION

The subchannel sampling system described in subsection 3.1 includes several vane-type flowmeters for sample flow measurement, and two orifices for measurement of cooling water flow. These were calibrated for several points in their range of use by means of a weigh tank and calibration curves drawn.

4.2 TRANSIENT FLOW CONTROL VALVE

Problems encountered with the transient flow control valve were discussed in the fourth quarterly report, GEAP-10221, page 14. The two main problems encountered in attempts to obtain the required speed were: (a) a time lag in the instrument-to-control-air circuit, which caused oscillations; and (b) the reset rate of the GEMAC flow controller was apparently too slow for the speed required. A few additional tests were run with the GEMAC controller at its maximum range, but improvement in the speed of operation could not be obtained. Two changes were initiated to correct these difficulties. First, a Moore No. 74 regulator (valve positioner) was ordered, to eliminate the 150 to 200 msec transport delay in the instrument-to-control-air signal. Second, a GEMAC flow controller (Style 2) with a faster reset rate of up to 2000 repeats per minute has been requested. Final tests will be made when this equipment is available.

5. DATA RECORDING AND REDUCTION

The TAPER code described previously in the fourth quarterly report, GEAP-10221, page 17, has been expanded from 7-channel to 14-channel capability. Preliminary runs on transient CHF data taken in a single-rod annulus have indicated that the code is working satisfactorily. The transient orifice equation has been derived previously (GEAP-10164), and was found to be a nonlinear Riccati equation of the form,

$$\frac{dw}{dt} + B_1 w^2 = B_2 \Delta P(t). \quad (5-1)$$

If Equation (5-1) is linearized about the initial flow rate $w(o)$, then

$$\frac{d(\delta w)}{dt} + 2B_1 w(o) \delta w = B_2 \Delta P_o [e^{-\alpha t} - 1], \quad (5-2)$$

the homogeneous solution of which is

$$\delta w = C_3 e^{-2B_1 w(o) t}$$

Thus, the "initial" time constant of the orifice is

$$\tau = \frac{1}{2 w(o) B_1}. \quad (5-3)$$

It can be shown that for cases of interest here, this is normally quite small, and is obviously inversely proportional to the initial flow rate. This indicates that the "inertia term," $\frac{dw}{dt}$, should be quite small during most transients of interest, and hence can be neglected. This clearly implies that the transient data can be reduced using standard steady-state orifice techniques.

To investigate this point in detail, the exact solution of Equation (5-1) was synthesized (see Appendix) for the case in which

$$\Delta P = A e^{-\alpha t} \quad (5-4)$$

This case is quite typical of the Sanborn traces observed during actual testing and, in any event, can approximate the response of most flow decay transients of interest. It is shown in the Appendix that the normalized solution is of the form,

$$N \triangleq \frac{w^*(\tau)}{\sqrt{\Delta P^*(\tau)}} = - \left[\frac{I_1(\tau) - \beta K_1(\tau)}{I_0(\tau) + \beta K_0(\tau)} \right],$$

where,

$$\tau = \frac{2}{\alpha} \sqrt{B_1 B_2 A} e^{-\frac{\alpha t}{2}},$$

$$\tau_0^\dagger = \frac{2 B_1 w(o)}{\alpha},$$

$$\beta = - \left[\frac{I_1(\tau_0) + I_0(\tau_0)}{K_0(\tau_0) - K_1(\tau_0)} \right],$$

$w(o)$ = Initial flow at $t = 0$,

$$w^*(\tau) \triangleq w(\tau)/w(o),$$

$$\Delta P^*(\tau) \triangleq \frac{\Delta P(\tau)}{\Delta P(o)} = \frac{\Delta P(\tau)}{A},$$

and the I_1 , I_0 , K_1 and K_0 are modified Bessel functions.

Figure 5-1 gives a nondimensional plot of this solution and indicates that, except for small values of τ_0 , the inertial term is negligible (i.e., $N \cong 1.0$). A typical single-rod annulus flow decay transient from 1 lb/sec, with a time constant of 1/3 second, gives, for a 1-inch flange top orifice in a 3-inch pipe,

$$\begin{aligned} B_1 &= 261.4 \\ B_2 &= 8.93 \\ \tau_0 &= 174.3 \end{aligned}$$

Figure 5-2 indicates that the exact solution and the solution obtained when the steady-state orifice equation is used, agree extremely well at each point in time. Hence, the inertial corrections are negligible for single-rod and nine-rod transients of interest to the Deficient Cooling Program. Thus, the Runge Kutta scheme which was originally included in TAPER as an option has been eliminated.

† Ratio of the ΔP time constant in Equation (5-4) to the "initial" time constant of the orifice in Equation (5-3).

Figure 5-1. Flow Corrections versus τ/τ_0

Figure 5-2. Comparison of Quasi-Static and Corrected Flow Rates for Test Case

APPENDIX

THE EXACT SOLUTION OF THE ORIFICE EQUATION WITH
AN EXPONENTIAL FORCING FUNCTION

As derived previously in the first quarterly report, GEAP-13048, a momentum balance across a flange tap orifice yields a Riccati equation of the form,

$$\frac{dw}{dt} + B_1 w^2 = B_2 \Delta P(t). \quad (A-1)$$

To solve this equation for an arbitrary forcing function $\Delta P(t)$, it can be transformed by using a transformation of variables suggested by Davis: *

Let

$$w \triangleq \frac{1}{B_1 u} \frac{du}{dt},$$

hence,

$$\frac{dw}{dt} = \frac{1}{B_1 u} \frac{d^2 u}{dt^2} - \frac{1}{B_1 u^2} \left(\frac{du}{dt} \right)^2,$$

$$w^2 = \frac{1}{B_1^2 u^2} \left(\frac{du}{dt} \right)^2.$$

Introducing these into Equation (A-1),

$$\frac{d^2 u}{dt^2} - B_1 B_2 \Delta P(t) u = 0. \quad (A-2)$$

Consider the flow decay case of an exponential ΔP across the orifice,

$$\Delta P(t) \triangleq A e^{-\alpha t},$$

and defining

$$B_3 \triangleq B_1 B_2 A,$$

(A-2) becomes

$$\frac{d^2 u}{dt^2} - B_3 e^{-\alpha t} u = 0. \quad (A-3)$$

To put the latter into a standard form, let

$$v(\tau) \triangleq u(t), \quad (A-4)$$

* Davis, H. T., "Introduction to Nonlinear Differential and Integral Equations," U.S. Atomic Energy Commission, September 1960.

where

$$\tau \triangleq \frac{2}{\alpha} \sqrt{B_3} e^{-\frac{\alpha}{2} t}$$

Then

$$\frac{du}{dt} = \frac{dv}{d\tau} \left(\frac{d\tau}{dt} \right) = \frac{dv}{d\tau} (-\sqrt{B_3}) e^{-\frac{\alpha}{2} t},$$

and substituting from Equation (A-4) into the latter,

$$\frac{du}{dt} = - \frac{dv}{d\tau} \left(\frac{\alpha\tau}{2} \right), \quad (A-5)$$

where

$$\frac{d\tau}{dt} = - \frac{\alpha\tau}{2}$$

Next, the second derivative is formed,

$$\frac{d^2 u}{dt^2} = \frac{\alpha^2 \tau^2}{4} \frac{d^2 v}{d\tau^2} + \frac{\alpha^2 \tau}{4} \frac{dv}{d\tau} \quad (A-6)$$

Introducing Equations (A-4) and (A-6) into Equation (A-3), and cancelling $\alpha^2/4$,

$$\tau^2 \frac{d^2 v}{d\tau^2} + \tau \frac{dv}{d\tau} - \tau^2 v = 0. \quad (A-7)$$

Equation (A-7) is a form of the modified Bessel equation of order "0", which has the solution,

$$v(\tau) = C_1 I_0(\tau) + C_2 K_0(\tau).$$

We are now in position to evaluate the initial conditions. From the original transformation,

$$\frac{1}{u(o)} \left(\frac{du}{dt} \right)_{t=0} = B_1 w(o) \quad (A-8)$$

and

$$\frac{du}{dt} = \frac{dv}{d\tau} \frac{d\tau}{dt} = - \frac{\alpha\tau}{2} \frac{dv}{d\tau} = - \frac{\alpha\tau}{2} [C_1 I_1(\tau) - C_2 K_1(\tau)]$$

where a relationship from the Handbook of Mathematical Functions[†] has been used.

Now define

$$\tau_o \triangleq \frac{2}{\alpha} \sqrt{B_3} = \frac{2 B_1 w(o)}{\alpha} = \frac{(\text{Time Constant of Transient})}{(\text{"Initial" Time Constant of Orifice})}$$

$$\therefore \left(\frac{du}{dt} \right)_{t=0} = - \frac{\alpha}{2} \tau_o [C_1 I_1(\tau_o) - C_2 K_1(\tau_o)].$$

[†] "Handbook of Mathematical Functions," AMS-55, U.S. Department of Commerce, Nat. Bur. Stds., June 1964, p. 376, eq. 9.6.27.

Since in Equation (A-4), $v(\tau_0) = u(o)$, Equation (A-8) becomes

$$B_1 w(o) = \frac{-\frac{\alpha}{2} \tau_0 [C_1 I_1(\tau_0) - C_2 K_1(\tau_0)]}{[C_1 I_0(\tau_0) + C_2 K_0(\tau_0)]}$$

Thus,

$$C_2 = C_1 \left[\frac{-\frac{\alpha}{2} \tau_0 I_1(\tau_0) - B_1 w(o) I_0(\tau_0)}{K_0(\tau_0) B_1 w(o) - \frac{\alpha}{2} \tau_0 K_1(\tau_0)} \right] \triangleq C_1 \beta [\alpha, B_1, B_2, w(o)]. \quad (A-9)$$

Recalling that $\frac{\alpha}{2} \tau_0 = \sqrt{B_3}$, and $w(o) \triangleq \frac{\sqrt{B_3}}{B_1}$,

from the steady-state orifice equation, then Equation (A-9) becomes

$$\beta = - \left[\frac{I_1(\tau_0) + I_0(\tau_0)}{K_0(\tau_0) - K_1(\tau_0)} \right] \quad (A-10)$$

Equation (A-9) indicates C_1 and C_2 are proportional; hence one can write

$$v(\tau) = C_1 [I_0(\tau) + \beta K_0(\tau)],$$

and since $u(t) = v(\tau)$, it is obvious from the transformation $w = (1/B_1 u) (du/dt)$ that C_1 cancels and hence is arbitrary.

Thus we arbitrarily chose $C_1 \triangleq 1$, to obtain

$$w(t) = \frac{1}{B_1 u} \frac{du}{dt} = \frac{-\frac{\alpha \tau}{2} [I_1(\tau) - \beta K_1(\tau)]}{B_1 [I_0(\tau) + \beta K_0(\tau)]} \quad (A-11)$$

where

$$\tau \triangleq \frac{2}{\alpha} \sqrt{B_3} e^{-\frac{\alpha}{2} t} \quad (A-12)$$

It is interesting to note that $\Delta P(t) \triangleq A e^{-\alpha t}$, thus Equation (A-12) becomes

$$\tau = \frac{2}{\alpha} \sqrt{B_1 B_2} \sqrt{\Delta P(t)} \quad (A-13)$$

Hence Equation (A-11) can be written as

$$\frac{w(t)}{\sqrt{\Delta P(t)}} = - \sqrt{\frac{B_2}{B_1}} \left[\frac{I_1(\tau) - \beta K_1(\tau)}{I_0(\tau) + \beta K_0(\tau)} \right] \quad (A-14)$$

The latter can be normalized by defining

$$w(\tau)^* \triangleq \frac{w(\tau)}{w(o)} = \frac{w(\tau)}{\sqrt{\frac{B_2}{B_1} \Delta P(o)}} = \frac{w(\tau)}{\sqrt{\frac{B_2}{B_1} A}}$$

and

$$\Delta P(\tau)^* \triangleq \frac{\Delta P(\tau)}{\Delta P(o)} = \frac{\Delta P(\tau)}{A}$$

Thus Equation (A-14) becomes

$$N \triangleq \frac{w(\tau)^*}{\Delta P(\tau)^*} = - \left[\frac{I_1(\tau) - \beta K_1(\tau)}{I_0(\tau) + \beta K_0(\tau)} \right] \quad (A-15)$$

Accordingly, once β has been evaluated from Equation (A-10), Equation (A-12) can be used with Equation (A-15) to yield the normalized flow history, or Equation (A-11) to yield the time history of $w(t)$.

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