Phenomenological Bounds in Inclusive Neutrino Interactions*

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Abstract

Using expressions for the ν and $\bar{\nu}$ charged and neutral current cross sections and the electroproduction structure function integral, and positivity requirements of the sea contribution, we obtain bounds on $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$, and $\sigma_{nc}^{\bar{\nu}N}/\sigma_{nc}^{\nu N}$ in the standard model. A bound on $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ obtained with a $V+A$ term $\bar{p}'\gamma_{\mu}(1-\gamma_5)n$ is used to rule out such a term in the current. We introduce a plot of $\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu}N}$ versus $\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu}N}$ to analyze the neutral current data. We find a new relation connecting moments of y and y distributions at a particular point y_n for ν and $\bar{\nu}$ interactions. Our results do not depend on the neutral current data.

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The interest in inclusive neutrino interactions $\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \mu^\pm + X$ and $\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \nu_\mu(\bar{\nu}_\mu) + X$ has sparked increasing experimental^{1,2,3} and theoretical⁴ activity. In view of the uncertainties in the size of the neutrino cross sections and the results of the x and y distributions⁵ ($x = Q^2/2M\nu$, $y = \nu/E$), we concentrate on the integrated data. It is our aim here to parametrize the various structure functions in a phenomenological quark-parton approach;⁶ and to construct bounds on interesting physical quantities. The bounds are obtained both in the conventional V-A theory and also in the model including a V + A term proposed by de Rujula, Georgi, and Glashow.⁷

Among our conclusions: (a) The bound we obtain appears to rule out the proposal of Ref.7. (b) We find it useful to introduce a plot of $\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N}$ versus $\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}$. (c) We obtain a new relation connecting moments of y and y distributions at a particular point y_n for neutrino and antineutrino interactions.

We begin by writing the conventional weak charged current in the 4-quark scheme:

$$J_\mu^+ = \bar{p}\gamma_\mu(1+\gamma_5)n(\theta_c) + \bar{p}'\gamma_\mu(1+\gamma_5)\lambda(\theta_c), \quad (1)$$

where $n(\theta_c) = n \cos\theta_c + \lambda \sin\theta_c$ and $\lambda(\theta_c) = \lambda \cos\theta_c - n \sin\theta_c$, with θ_c the Cabibbo angle. The V+A charged current of Ref.7 leads to the modification

$$J_\mu^{'+} = J_\mu^+ + \bar{p}'\gamma_\mu(1-\gamma_5)n. \quad (2)$$

In both cases the neutral current is of conventional form $J_\mu^Z = J_\mu^3 - 2\sin^2\theta_w J_\mu^{\text{em}}$ where the J_μ^3 corresponds to the respective charged current expressions.

Our approach is to deal with the integrated valence and sea densities we obtain in our quark-parton picture. Let us define a factor $\theta = \theta(W - W_{\text{th}})$ (W is the invariant mass of the final hadron system) which is 0 below threshold and 1 above the threshold. We take the sea contribution of the quark and antiquark densities to be the same, and we use the

conventions $V = \int_0^1 dx x (u+d-\bar{u}-\bar{d})$, $2\zeta_N = \int_0^1 dx x (\bar{u}+\bar{d})$, $\zeta_S = \int_0^1 dx x s$,
and $\zeta_c = \int_0^1 dx x c$. Here the quark densities are denoted $u(x)$, $d(x)$,
 $s(x)$ and $c(x)$. In the conventional fashion (σ in units of G^2_{ME}/π),

$$\begin{aligned}\sigma^{\nu N} &= \frac{2}{3} \int_0^1 dx F_2^{\nu N}(x) - \frac{1}{3} \int_0^1 dx x F_3^{\nu N}(x) \\ &= (V \cos^2 \theta_c + 2\zeta_N \cos^2 \theta_c + 2\zeta_S \sin^2 \theta_c + \frac{2}{3}\zeta_N) \\ &\quad + \theta (V \sin^2 \theta_c + 2\zeta_N \sin^2 \theta_c + 2\zeta_S \cos^2 \theta_c + \frac{2}{3}\zeta_c), \quad (3)\end{aligned}$$

$$\begin{aligned}\sigma^{\bar{\nu} N} &= \frac{2}{3} \int_0^1 dx F_2^{\bar{\nu} N}(x) + \frac{1}{3} \int_0^1 dx x F_3^{\bar{\nu} N}(x) \\ &= (\frac{1}{3}V + 2\zeta_N \cos^2 \theta_c + 2\zeta_S \sin^2 \theta_c + \frac{2}{3}\zeta_N) \\ &\quad + \theta (2\zeta_N \sin^2 \theta_c + 2\zeta_S \cos^2 \theta_c + \frac{2}{3}\zeta_c), \quad (4)\end{aligned}$$

$$\frac{18}{5} \int_0^1 dx F_2^{eN}(x) = V + 4\zeta_N + \frac{4}{5}\zeta_S + \frac{16}{5}\zeta_c, \quad (5)$$

$$\begin{aligned}\frac{3}{2} (\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}) &= - \int_0^1 dx x G_3^{\nu N}(x) \\ &= \int_0^1 dx x \left\{ (\frac{1}{2} - \sin^2 \theta_W)(u+d-\bar{u}-\bar{d}) + (\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W)(c-\bar{c}) \right. \\ &\quad \left. + (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W)(s-\bar{s}) \right\} \\ &= (\frac{1}{2} - \sin^2 \theta_W) V, \quad (6)\end{aligned}$$

$$\begin{aligned}
\frac{3}{4} (\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N}) &= \int_0^1 dx G_2^{\nu N}(x) \\
&= \int_0^1 dx x \left\{ \frac{1}{2} (1 - 2 \sin^2 \theta_W + \frac{20}{9} \sin^4 \theta_W) (u + d + \bar{u} + \bar{d}) \right. \\
&\quad \left. + (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W + \frac{4}{9} \sin^4 \theta_W) (s + \bar{s}) + (\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W + \frac{16}{9} \sin^4 \theta_W) (c + \bar{c}) \right\} \\
&= (1 - 2 \sin^2 \theta_W + \frac{20}{9} \sin^4 \theta_W) (\frac{1}{2} V + 2 \zeta_N) \\
&\quad + (1 - \frac{4}{3} \sin^2 \theta_W + \frac{8}{9} \sin^4 \theta_W) \zeta_s + (1 - \frac{8}{3} \sin^2 \theta_W + \frac{32}{9} \sin^4 \theta_W) \zeta_c. \quad (7)
\end{aligned}$$

Equations (3) through (7) are averaged over proton and neutron targets.

Consider the data below the threshold for production of charmed particles. The Weinberg angle is then determined to be

$\sin^2 \theta_W = \frac{1}{2} - \frac{1}{2} (3 \cos^2 \theta_c - 1) \left(\frac{\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}}{\sigma_{\nu N} - \sigma_{\bar{\nu} N}} \right)$; the valence contribution is $V = (\sigma_{\nu N} - \sigma_{\bar{\nu} N}) / (\cos^2 \theta_c - 1/3)$. Since the charged current data is better determined than the neutral current data, we may use the required positivity of the sea contributions to derive allowed domains on the plot of $\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N}$ versus $\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}$, which are presented as the shaded areas in Fig. 1a. The electroproduction data used in the Figure are given by Miller et al. and A. Bodek et al.⁸

If we consider the expressions (3) to (7) above charm threshold, solutions are $V = \frac{3}{2} (\sigma_{\nu N} - \sigma_{\bar{\nu} N})$ and $\sin^2 \theta_W = \frac{1}{2} - \frac{\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}}{\sigma_{\nu N} - \sigma_{\bar{\nu} N}} \sim 0.4 - 0.5$, the standard result¹¹ . Only the sea combinations $\zeta_N + \zeta_s$ and $\zeta_N + \zeta_c$ can be determined. In addition, the equations become redundant, which

yields a necessary consistency condition (8) satisfied by the data ¹²,

$$\begin{aligned} \sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N} = \frac{40}{27} E \left(\frac{\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}}{\sigma^{\nu N} - \sigma^{\bar{\nu} N}} \right)^2 + \frac{8}{33} (\sigma^{\nu N} + \sigma^{\bar{\nu} N} - \frac{35}{18} E) \left(\frac{\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}}{\sigma^{\nu N} - \sigma^{\bar{\nu} N}} \right) \\ + \frac{5}{33} (\sigma^{\nu N} + \sigma^{\bar{\nu} N} + \frac{10}{9} E). \end{aligned} \quad (8)$$

Here $E \equiv \frac{18}{5} \int_0^1 dx F_2^{en}(x)$. Equation (8) is plotted in Fig. 1b. We

further require the positivity condition for $\zeta_N + \zeta_s$ and $\zeta_N + \zeta_c$; this implies

$$\frac{\sigma^{\bar{\nu} N}}{\sigma^{\nu N}} \geq \frac{1}{3} + \frac{10}{9} \left(1 - \frac{E}{\sigma^{\nu N}} \right), \quad (9.a)$$

and

$$\frac{\sigma^{\bar{\nu} N}}{\sigma^{\nu N}} \geq \frac{1}{3} + \frac{10}{57} \left(\frac{E}{\sigma^{\nu N}} - 1 \right). \quad (9.b)$$

We may also find a bound for $\sigma_{nc}^{\bar{\nu} N} / \sigma_{nc}^{\nu N}$ by obtaining this ratio with the aid of (6) and (7) and extremizing it with respect to $\sin^2 \theta_w$:

$$\frac{1+a}{3+a} \leq \frac{\sigma_{nc}^{\bar{\nu} N}}{\sigma_{nc}^{\nu N}} \leq \frac{31+b}{13+b}, \quad (10)$$

where $a = \frac{2}{11} (5E + 21\sigma^{\bar{\nu} N} - 12\sigma^{\nu N}) / (\sigma^{\nu N} - \sigma^{\bar{\nu} N})$ and $b = \frac{2}{11} (115E + 186\sigma^{\bar{\nu} N} - 177\sigma^{\nu N}) / (\sigma^{\nu N} - \sigma^{\bar{\nu} N})$.

The terms a and b in (10) are small (as expected) because they are due to sea contributions. The important feature we emphasize is that the sea terms serve to improve the bounds.

We may go through a similar analysis for the (V+A) term added as in (2). The electroproduction terms and the charged current terms below threshold are unaffected. Above threshold, we obtain

$$\sigma^{\nu N} = \frac{4}{3} V + \frac{10}{3} \zeta_N + 2 \zeta_S + \frac{8}{3} \zeta_C, \quad (11)$$

$$\sigma^{\bar{\nu} N} = \frac{1}{3} V + \frac{10}{3} \zeta_N + 2 \zeta_S + \frac{8}{3} \zeta_C, \quad (12)$$

$$\begin{aligned} \frac{3}{4} (\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N}) &= \left(\frac{3}{4} - \frac{4}{3} \sin^2 \theta_W + \frac{10}{9} \sin^4 \theta_W \right) (V + 4 \zeta_N) \\ &+ \left(1 - \frac{4}{3} \sin^2 \theta_W + \frac{8}{9} \sin^4 \theta_W \right) \zeta_S + \left(2 - \frac{16}{3} \sin^2 \theta_W + \frac{32}{9} \sin^4 \theta_W \right) \zeta_C, \end{aligned} \quad (13)$$

$$\frac{3}{2} (\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}) = V \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right). \quad (14)$$

From Eqs. (11), (12) and (14) we obtain $\sin^2 \theta_W = \frac{3}{8} - \frac{9}{4} \left(\frac{\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N}}{\sigma^{\nu N} - \sigma^{\bar{\nu} N}} \right)$

which yield $\sin^2 \theta_W = 0.15 - 0.36$ for the present data. We then

solve the equations just as before. There is no redundancy, so the seas

are individually determined. In particular, the requirement

$$\zeta_S = \frac{1}{24} (\sigma^{\bar{\nu} N} + \sigma^{\nu N} - \frac{5}{3} E) \geq 0 \quad \text{implies}$$

$$\frac{\sigma^{\bar{\nu} N}}{\sigma^{\nu N}} \geq \frac{5}{3} \frac{E}{\sigma^{\nu N}} - 1 \gtrsim \frac{2}{3} \quad \text{if } E > \sigma^{\nu N}. \quad (15)$$

This appears to rule out a (V+A) term¹⁵ of type (2). Thus we do not investigate the consequences of (2) further.

In a theory in which the neutral structure functions $G_2^{\nu N}(x)$, $G_3^{\nu N}(x)$ describe the data, we find the y -distribution using (6) and (7)

$$\begin{aligned} \frac{d\sigma_{nc}^{(\nu, \bar{\nu})N}}{dy} &= (1-y+\frac{1}{2}y^2) \int_0^1 dx G_2^{\nu N}(x) \mp y(1-\frac{1}{2}y) \int_0^1 dx x G_3^{\nu N}(x) \\ &= \frac{3}{4}(\sigma_{nc}^{\nu N} + \sigma_{nc}^{\bar{\nu} N})(1-y+\frac{1}{2}y^2) \pm \frac{3}{2}(\sigma_{nc}^{\nu N} - \sigma_{nc}^{\bar{\nu} N})y(1-\frac{1}{2}y). \quad (16) \end{aligned}$$

We obtain from (16)

$$\begin{aligned} \langle y^n \rangle_{nc}^{\nu, \bar{\nu}} &\equiv \left(\frac{1}{\sigma_{nc}} \int_0^1 dy y^n \frac{d\sigma_{nc}}{dy} \right)^{\nu, \bar{\nu}} \\ &= \frac{1}{n+1} \left(\frac{1}{\sigma_{nc}} \frac{d\sigma_{nc}}{dy} \right)^{\nu, \bar{\nu}} \Big|_{y=1-\sqrt{\frac{2}{(n+2)(n+3)}}} \quad (17) \end{aligned}$$

Note that (17) implies that $\left\{ (d\sigma_{nc}/dy)/\sigma_{nc} \langle y^n \rangle_{nc} \right\} \Big|_{y=1-\sqrt{\frac{2}{(n+2)(n+3)}}}$

is equal to $n+1$ for both ν and $\bar{\nu}$. The special case for $n=0$ was obtained previously by Ecker and Pietschmann¹⁶. The special points considered here are large enough to be unaffected by thresholds. Since all quantities in (17) are measurable in principle, we may use some of these quantities to determine other, more poorly determined, quantities. The relation (17) is independent of the valence-sea parametrization and the amounts of V and A currents.

While we have carried out a numerical analysis to determine the seas the neutral current errors are so large that we refrain from giving any results, except to mention that, for V-A the seas are generally less than 10% of the valence terms (and are consistent with zero). For the V+A case, the magnitude of the sea term can get large with respect to the valence term.

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the above threshold case), the seas' denominators vanish. For consistency, the numerators must vanish; all lead to the consistency condition (8).

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FIGURE CAPTIONS

Figure 1a. Plot of $\sigma_{\eta c}^{\nu N} + \sigma_{\eta c}^{\bar{\nu} N}$ versus $\sigma_{\eta c}^{\nu N} - \sigma_{\eta c}^{\bar{\nu} N}$ with the unit of $G^2 M E / \pi$ showing allowed domains,⁹ for the case that the present data is below charm threshold. For illustrative purposes, the data of Ref. 1 are used to determine the curves. The experimental point is from Ref. 10.

Figure 1b. Plot of $\sigma_{\eta c}^{\nu N} + \sigma_{\eta c}^{\bar{\nu} N}$ versus $\sigma_{\eta c}^{\nu N} - \sigma_{\eta c}^{\bar{\nu} N}$ with the unit of $G^2 M E / \pi$ showing Eq. (8) above charm threshold. The data points are taken from Refs. 9, 13 and 14, assuming these data to be above charm threshold.

Fig. 1a

Fig. 1b