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SNAP 2
STRUCTURAL AND DYNAMIC ANALYSIS

AEC Research and Development Report

ATOMICS INTERNATIONAL

A DIVISION OF NORTH AMERICAN AVIATION, INC.

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NAA-SR-9054
GENERAL, MISCELLANEOUS,
AND PROGRESS REPORTS
TID-4500 (29th Ed.)
SNAP REACTORS,
SNAP PROGRAM
M-3679 (34th Ed.)
125 PAGES

SNAP 2
STRUCTURAL AND DYNAMIC ANALYSIS

By
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ATOMICS INTERNATIONAL

A DIVISION OF NORTH AMERICAN AVIATION, INC.
P.O. BOX 309 CANOGA PARK, CALIFORNIA

CONTRACT: AT(11-1)-GEN-8
ISSUED: MAY 1, 1964

DISTRIBUTION

This report has been distributed according to the category "General, Miscellaneous, and Progress Reports," as given in "Standard Distribution Lists for Unclassified Scientific and Technical Reports" TID-4500 (29th Ed.), and "SNAP Reactors, SNAP Program, M-3679 (34th Ed.), March 15, 1964. A total of 780 copies were printed.

ABSTRACT

The structural design criteria and the system basic loads for the SNAP 2 compact power unit are presented. Included in these structural criteria are the following environments: (1) ground, (2) prelaunch, (3) launch, (4) preorbital-startup and, (5) orbital startup and continuous operation. These data were then used to determine the structural integrity of the system and to satisfy the reliability requirements. A summary is presented of the more significant system structural and dynamic analyses generated on the SNAP 2 compact power unit.

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I. INTRODUCTION

This report summarizes the structural analysis work on the SNAP-2 system (Figure 1). The structural environment and the basic structural loads were determined; a stress and vibration analysis was completed. All aspects of the SNAP-2 program were considered.

The SNAP 2 system is a self-contained auxiliary power unit capable of supplying 3 to 5 kwe (Figure 2). It consists of three basic subsystems; the reactor heat source, the power conversion unit and the radiator-condenser heat rejection system. NaK is circulated through the reactor core and through the NaK to mercury boiler-superheater. In the boiler-superheater, the reactor heat is transferred to the mercury working fluid of the Rankine power conversion cycle. The mercury is converted into a superheated vapor which is expanded through a turbine. The resulting mechanical power output of the turbine is converted to electrical power by the alternator. The mercury vapor exhaust from the turbine is then condensed in the radiator-condenser to complete the cycle. All of the power conversion system rotating components, i. e., mercury pump, alternator, and turbine, are mounted on a single rotating shaft unit called the Combined Rotating Unit (CRU).

The SNAP-2 basic mechanical design philosophy is not founded solely upon a minimum weight objective. The design is sometimes strongly influenced by performance and/or manufacturing parameters. After these parameters are factored into the design, structural analysis is performed to the design criteria to ensure that the reliability objectives (design margins) have been met for structural integrity. Then a minimum weight analysis is performed to obtain a structurally optimized article. The structural basic loads were developed for the prelaunch and launch conditions; consideration was given to ground surface winds, shock and vibration on lift-off coupled with acoustical noise, aerodynamic gust loading creating an angle of attack at the maximum dynamic pressure, and inertia loading. Data was also generated to determine the amount of aerodynamic heating to which the vehicle may be exposed during the inertia condition. This data pertained to the RC-11 (honeycomb radiator-condenser); the RC-13 and 14 (aluminum-steel radiator-condensers) and the instrumentation compartment. Other environments, although generally less severe, were also included, where necessary, for structural analysis.

SNAP 2 UNIT

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Figure 1. Structural Illustration of Snap 2 Unit

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Figure 2. SNAP 2 Schematic

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The previous data were used to determine the structural integrity of the system and to satisfy the reliability requirements. Numerous components, subsystems, and assemblies were subjected to structural analysis.

The 2PSM2A developmental structural test system was analyzed to determine structural integrity (Figure 3). This was done to determine whether or not the system will withstand the scheduled structural qualification testing, and to determine the structural margin of safety and therefore the critical conditions. This analysis was then used to generate the test requirements document required for structural testing.

Also included in the work scope of the analyses were a great many vibration and dynamic analyses. These were concerned mainly with 2PSM2A developmental structural test and are presently being expanded to the flight design as test correlation becomes available.

Also, component design curves were developed so that design loads could be generated in the absence of a complicated dynamic analysis.

Although the specific SNAP 2 Flight Test Program has been deferred, continuing analysis and testing will be carried on under the SNAP Mercury-Rankine System Development Program.

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Figure 3. 2PSM2A Configuration

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II. STRUCTURAL DESIGN CRITERIA

The basic SNAP-2 Unit has many design requirements which affect the structural adequacy of the vehicle. Since the SNAP-2 Unit has many performance parameters, (i. e. thermal isolation requirements between the radiator condenser and the instrument compartment, meteoroid survival in orbit) sometimes more structural material is available than necessary and where this is true, advantage has been taken to increase the structural integrity with a minimum weight penalty.

A. STRUCTURAL ENVIRONMENT

The SNAP-2 Unit is designed to survive the total environment through one year of service in orbit.¹ Included in this environment are the following phases: (1) ground environment, (2) prelaunch, (3) launch environment, (4) pre orbital-startup, and (5) orbital startup and continuous one year operation.

1. Ground Environment

This includes the performance and structural acceptance testing, the ground transportation, and various minor environments. Thermal shock and stress during normal thermal testing must be checked to insure that fatigue failures will not occur during the life of the unit because of ground operation or that material properties do not degrade so that launch survival is not certain. Ground handling is accomplished so the mechanical shock and vibration loads do not become critical when compared to the launch environment.

2. Prelaunch

This criteria involves the protection of the system against an equivalent 40-mph ground wind velocity which consists of a 25-mph steady wind and a 15-mph gust while on the launch pad in the vertical position. For this condition, instrument modules, at the base of the SNAP-2 Unit, must be removable for access and the resulting discontinuous structure was then analyzed.

3. Launch Environment

This trajectory imposes the most severe environment so far as the structure is concerned (Figure 4). The vibration and acoustic environment at lift-off and through MACH 1 is enveloped by qualification shock and vibration testing.

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Figure 4. Typical Trajectory Loading

The unit is vibrated and shocked from the base along each axis. The basic loads for these conditions are dependent on the estimated system damping properties and results in one of the major structural design conditions. After the vehicle passes through MACH 1, the product of dynamic pressure and angle of attack becomes a maximum. It has been estimated that this occurs with a dynamic pressure of 860 psf and at an angle of attack of 8° which results in critical hoop stresses in the shell of the unit. Another critical launch load is near the end of the booster burn where the aerodynamic heating on the vehicle induces loads and simultaneously the inertia loads are maximum. Loads and temperatures have been computed for the various times of this trajectory under these conditions.

The SNAP unit, after booster separation, must eject its nose cone which results in shock or impulse loads to the structure.

4. Pre Orbital-startup

Structural considerations of this period are keyed to the effect of sun-shade transients and to the thermally induced loads.

5. Orbital Startup and Continuous One Year Operation

Another important factor of the design criteria is where thermal shock loads and thermal deflections are introduced into the system which primarily affect the shield brackets and the piping system. In addition, vibratory input from the CRU imbalance must be checked. Creep rupture is important during continuous operation at system pressure.

B. FAILURE CRITERIA AND RELIABILITY DESIGN MARGIN OBJECTIVES

The criteria for determining structural failure is that the structure must be capable of reacting the maximum expected flight load (defined as limit load) without causing a mission failure. The margin of safety (MS) is defined for every component in the structure and is

$$MS = \frac{F}{kf} - 1$$

where

F is allowable stress or load

f is maximum applied (limit) stress or load

k is load factor

The value of the load factor is determined by reliability criteria² which states that component strength shall have a margin of four standard deviations (4σ) of the margin distribution and that structural components shall be designed to a nominal strength of at least 136% of maximum design load. This criteria means that the probability of failure due to understrength is 0.0000317.

The material values used in the stress analysis are at least 99% probable. The coefficient of variation of strength³ (C_{V_s}) or one standard deviation, measured in percent is 4 or 5% for structural materials. The mean value is then the minimum guaranteed (99% probable) divided by one minus the number of standard deviations between the mean and the 99% probability value (2.33) times the coefficient of variation of strength or;

$$\text{Mean Strength} = \frac{\text{Strength (99\% probable)}}{1 - 2.33 C_{v_s}}$$

$$= 1.09 \text{ Strength (99\% probable)}$$

In the structural analyses presented the 99% probable strength allowables are used and the load factor is $1.36/1.09 = 1.25$. The maximum applied (limit) load times the load factor (1.25) is referred to as the ultimate load in this report. The relationship of these factors is shown in Figure 5.

Where ground personnel safety is involved the load factor has arbitrarily been increased to 1.50.

In some cases a zero margin of safety for one load condition, as defined above, would allow the structure to deform (yield), which would cause redistribution of load and permanent deformation upon release of the load. This in turn may affect the load distribution for another case and cause failure; whereas, the structural margins prior to this deformation were adequate. Because of this,

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Figure 5. Strength Reliability Factors

the above yielding is not allowed without full consideration of the above. The definition of the margin of safety is then revised to account for yielding, and becomes

$$MS = \frac{\text{Material Yield Strength}}{\text{Maximum Applied Stress}} - 1$$

or

$$MS = \frac{\text{Material Ultimate Strength}}{k \times \text{Maximum Applied Stress}} - 1$$

The minimum acceptable margin of safety is zero except in some special cases with respect to yielding.

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III. BASIC STRUCTURAL LOADS

A. GROUND HANDLING

Ground handling was accomplished in such a manner so as to eliminate it as a critical factor in the design of the compact power system; thus, no basic loads were derived.

B. PRELAUNCH

The basic structural loads represent the calculation of the loads necessary to determine the stresses within the unit and thus determine the structural margin of safety. Loads are derived for the conditions described in the design criteria.

The prelaunch 40 mph ground wind loads are determined by taking the cross-sectional drag data about a body of revolution. The bending moment and shear were computed by integrating this data over the surface of the SNAP-2 Unit.

$$V = \int C_D q ds$$

$$M = \int V dx$$

where

C_D is coefficient of drag

q is dynamic pressure

s is surface area

V is shear

M is moment

X is length

These loads were then superimposed with an assumed inertia load one inch off the axis. The resulting moment at the base is 15,075 in-lbs.

C. LAUNCH

1. Shock and Vibration

The launch basic loads include the shock and vibration of the SNAP Unit experienced on lift-off and through MACH 1. The vibration levels were established by the vehicle contractor to envelop the vibratory loading that would be experienced in flight. The following levels represent the qualification test which in turn are used for design, since actual environmental levels as a function of time are not available.

TABLE 1
QUALIFICATION TEST USED FOR DESIGN

		Cps
Longitudinal Axis (X axis)	1/2 Inch DA	5 to 9
	2.0g	9 to 250
	3.5g	250 to 400
	7.5g	400 to 3000
Lateral Axis (y and z axis)	1/2 Inch DA	5 to 7.5
	1.5g	7.5 to 250
	2.0g	250 to 400
	7.5g	400 to 3000

Each test consists of a 45 min sweep from low to high frequency at a constant octave sweep rate. The lateral response at the reactor is limited to 5.0g, except that vibration input at resonance shall not be reduced below 0.2g.

a. Body Bending Loads

One of the critical structural design loads is derived when the lateral vibration on the qualification test, produces a response of 5g's on the reactor. The accelerations were estimated and the bending moment diagrams drawn as shown in Figure 6. This was considered acceptable and conservative until the analysis of page 112 is incorporated in the design criteria and data from 2PSM2A testing becomes available.

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Figure 6. Equivalent Static Loads for Lateral Vibration Conditions

b. Shield Bracket Loads

The estimated vibration loading of 5.0g laterally and 1g longitudinally at the shield, as determined in the design criteria, defines one of the critical loads on the unit near the shield brackets. The equivalent moment and shear are 52,700 in-lbs and 3,065 lbs at station 124 where the shield-radiator condenser interface is located. The loads are distributed to the brackets as shown in Figure 7.

The radiator-condenser was assumed to function as two structural rings reacting the loads imparted by the shield brackets. These rings were assumed to be station 124 at the top of the bracket and station 117.5 at the bottom of the bracket. The radiator-condenser internal loading as a result of this action is shown in Figure 8. These same loads were then induced into the shield (Figure 9).

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a. Analysis

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b. Cutaway

Figure 7. Shield Bracket Loads (1b) — Vibration Condition

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Figure 8. Radiator-Condenser Internal Loading-Vibration Condition

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Figure 9. Shield Support Structure

The typical shield bracket shown below distributed these loads into the shield structural rings. The load action on these rings is shown in Figure 10.

RING A
STATION 124

RING B
STATION 128

7636-0120

Figure 10. Induced Shield Loads (lb) – Vibration Condition

c. Piping Loads

For system design, the simulated vibratory environment was considered using a 30g side acceleration uniformly along the length of the system. A summary of the 2PSM2A system and loads are shown in Figures 11 and 12.

d. Component Bracket Loading

To determine component bracket dynamic loading, a simplified design analysis was needed; therefore, the tanks were assumed to obey the weight-dynamic force curve which were supplied by the vehicle contractor (Figure 13) for a one degree of freedom system. However, in order to obtain a more realistic system, and yet keep the problem simple, both translational and rotational degrees of freedom were considered. Using the conservation of energy, the one degree of freedom information was used to determine a force and moment as shown below. The bracket frequencies were to be designed above the system fundamental frequency so that acoustic energy input and vibratory coupling would be minimized. The minimum design bracket frequency considered was 125 cps.

coupled natural frequency

$$\omega^2 = \frac{-(k_{\alpha} m + m \rho^2 k_x) \pm \sqrt{(k_{\alpha} m + m \rho^2 k_x)^2 + 4(m^2 \epsilon^2 - m^2 \rho^2)(k_x k_{\alpha})}}{2(m^2 \epsilon^2 - m^2 \rho^2)}$$

FIGURE 11 2PSM2A PIPING ASSEMBLY
NaK AND Hg LINES

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NaK SYSTEM

UPPER LINE

LOWER LINE

MERCURY SYSTEM

UPPER SECTION

LOWER SECTION

ALL LOADS ARE LIMIT
30g LOADING

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Figure 12. 2PSM2A Piping Basic Loads

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Figure 13. Mounting Bracket Design Forces

ratio of moment to force

$$\frac{M}{F} = \frac{k_{\alpha} \epsilon}{-k_{\alpha} + m \rho^2 \omega^2 - \epsilon^2 m \omega^2} + \epsilon$$

energy equation for force and moment

$$F^2 \left(\frac{k_x}{m \omega^2} \right) = F^2 + \rho^2 \left(\frac{k_x}{k_{\alpha}} \right)^2 M^2 + 2\epsilon \left(\frac{k_x}{k_{\alpha}} \right) M F$$

These equations are to be solved for F and M

ω = coupled natural frequency

k_x = linear spring constant lb/inch

k_{α} = rotational spring constant inch/radian about the elastic axis

m = mass acting at center of gravity

ρ = radius of gyration about elastic axis

ϵ = distance between center of gravity and elastic axis (assume 1 inch unless known)

M = moment inch-lb

F = force-lb

*Figure 13

2. Aerodynamic Loading

The critical point on the trajectory so far as the aerodynamic pressure loading is concerned is where the product of dynamic pressure and angle of attack is a maximum. This point is described below.

Alt	= 30,000 feet
Pressure	= 4.36 psi
Dynamic Pressure	= 860 psf
MACH Number	= 1.4
Angle of Attack	= 8°

Several points on the trajectory were evaluated and the analysis shows the peak moment to be rather sharp with respect to time, and at the end of 90 seconds of flight time the resulting moment has dropped to approximately 1% of the maximum (Figure 14).

Pressures and basic loads were computed for the nose cap and cone, radiator-condenser, and base load ring of the SNAP Unit.

The configuration is examined in sections and the upstream disturbances' effect is neglected.

a. Nose Cone

The inviscid flow field over the hemispherical region of the nose cap was analyzed by the method described in reference 4 where the properties of the field are derived from the solution of the transformed field equations. The equations become soluble by numerical techniques when written in terms of a coordinate system described by an assumed shock. This transformation leads to a scheme of point to point numerical definition of the flow field which eventually yields the body that is associated with the assumed shock.

The flow properties beyond the sonic line on the hemisphere was described by using Prandl-Meyer Expansion on a finitely truncated approximation of the body.

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FLIGHT TIME

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Figure 14. Relative Flight Loads

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b. Conical Surface

On the conical surfaces the slender body theory⁵ was used as a first approximation to determine the loads. The equations for the flow field are linearized by neglecting product partials of potential and mixed partials of potentials which yield

$$\beta^2 \phi_{xx} - \frac{1}{r} \phi_r - \phi_{rr} - \frac{1}{r^2} 2 \phi_{\Phi\Phi} = 0$$

where

$$\beta = \sqrt{M^2 - 1}$$

ϕ = First order cross flow potential

r, Φ = coordinates

This is true, if the ratio's of perturbation velocities to free stream velocity are small compared to unity, or if the body is slender (i. e., if the rate of change of radius along the axis is small).

The pressure on the system is then made up of two parts, (1) predicts pressure due to axial flow; (2) the effects of the cross component;

$$C_p = \left(\frac{\Delta p}{q} \right)_{\alpha=0} + (C_{p1})$$

where $(\Delta p/q)_{\alpha=0}$ is evaluated directly from charts of the exact solution for cones, $(C_p)_1 = \alpha \left[-4 \frac{dr}{dx} \cos \phi + \alpha (2 \cos 2\phi - 1) \right]$. The resulting pressure distribution and basic loads are shown in Figures 15, 16, 17 and 18.

A more rigorous method proposed by Van Dyke, Hybrid Theory,⁶ was used to get a better approximation to the actual pressure distribution. This theory combines the first order cross-flow and the second order axial-flow solutions. The first order solutions are gained by making the assumptions made in the slender body potential; thus,

$$\beta^2 \phi_{xx} - \frac{1}{r} \phi_r - \phi_{rr} - \frac{1}{r^2} 2 \phi_{\Phi\Phi} = 0$$

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Figure 15. Aerodynamic Loading at Maximum -
Dynamic Pressure - Slender Body Theory

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Figure 16. Pressure Distribution - Nose Cone - Slender Body Theory

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Figure 17. Pressure Distribution - Radiator-Condenser - Slender Body Theory

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Figure 18. Pressure Distribution - Instrumentation Compartment - Slender Body Theory

The first order problem is satisfied by a linear combination of an axial potential (ϕ_0) and a cross-flow potential (ϕ_1).

This breaks the original first order equation into two parts. The equations governing cross-flow:

$$\phi_{1rr} + \frac{1}{r} (\phi_{1r}) - \frac{1}{r^2} (\phi_1) - \beta^2 \phi_{1xx} = 0$$

The first order equations for axial flow:

$$\phi_{0rr} + \frac{1}{r} (\phi_{0r}) - \beta^2 \phi_{0xx} = 0$$

Solution of these equations for ϕ_1 with the conditions of undisturbed flow upstream and tangency at the body produces, in conical coordinates ($t = \frac{\beta r}{x}$), for a cone:

$$\phi_1(t) = Bx \left(\frac{\sqrt{1-t^2}}{t} - t \operatorname{SECH}^{-1} t \right)$$

where constant (B) derived from tangency condition.

The required derivatives for velocity components are:

$$\begin{aligned} \phi_{1_x} &= \frac{B \sqrt{1-t^2}}{t} \\ \phi_{1_r} &= -\beta \frac{B}{2} \left[\frac{\sqrt{1-t^2}}{t^2} + \operatorname{SECH}^{-1} t \right] . \end{aligned}$$

Solution for ϕ_o with boundary conditions of undisturbed flow upstream and tangency is $\phi_o = Ax (\operatorname{SECH}^{-1} t - \sqrt{1-t^2})$ where constant (A) is from tangency condition.

The second order velocity potential was gained by substituting the first order potential into the terms of the total potential equations which were neglected for the solution of the first order potential. The solution to the resulting nonhomogeneous wave equation is the second order potential. The second order potential becomes the sum of the first order potential and the particular solution from the nonhomogeneous terms; thus,

$$\begin{aligned} \Phi_o &= -Cx \left(\operatorname{SECH}^{-1} t - \sqrt{1-t^2} \right) + A^2 M^2 \left[\left(\operatorname{SECH}^{-1} t \right)^2 \right. \\ &\quad \left. - (N+1) \sqrt{1-t^2} \operatorname{SECH}^{-1} t - \frac{1}{4} \beta^2 A \left(\frac{1-t^2}{t^2} \right)^{3/2} \right] , \end{aligned}$$

where $N = \left(\frac{\gamma+1}{2} \right) \left(\frac{M}{\beta} \right)^2$, M is MACH number, and C is solved from the tangency conditions.

The required derivatives for velocity components are:

$$\begin{aligned}\Phi_{O_x} &= -C \operatorname{SECH}^{-1} t + A^2 M^2 \left[\left(\operatorname{SECH}^{-1} \right)^2 - (N-1) \frac{\operatorname{SECH}^{-1}}{\sqrt{1-t^2}} t \right. \\ &\quad \left. - (N-1) - \frac{3}{4} \beta^2 A \frac{\sqrt{1-t^2}}{t^2} \right] \\ \Phi_{O_r} &= C \beta \frac{\sqrt{1-t^2}}{t} + A^2 M^2 \beta \left[-\frac{2}{t} \sqrt{1-t^2} \operatorname{SECH}^{-1} t + \frac{(N+1)}{t} \right. \\ &\quad \left. + (N-1) t \frac{\operatorname{SECH}^{-1}}{\sqrt{1-t^2}} t + \frac{1}{4} \beta A (2+t^2) \frac{\sqrt{1-t^2}}{t^2} \right] .\end{aligned}$$

The exact isentropic expression for coefficient of pressure is:

$$C_p = \frac{2}{\gamma M^2} \left\{ \left[1 + \frac{\gamma-1}{2} M^2 \left(1 - \frac{Q^2}{U^2} \right) \right]^{\gamma/\gamma-1} - 1 \right\} ,$$

where

$$\frac{Q^2}{U^2} = \left(\frac{u}{U} \right)^2 + \left(\frac{v}{U} \right)^2 + \left(\frac{w}{U} \right)^2 .$$

The normalized axial, radial, azimuthal velocity components with reference to body coordinates is given by

$$\frac{u}{U} = \cos \alpha (1 + \Phi_{O_x}) + \phi_{1_x} \sin \alpha \cos \phi$$

$$\frac{v}{U} = \sin \alpha \cos \phi (1 + \phi_{1_r}) + \Phi_{O_r} \cos \alpha$$

$$\frac{w}{U} = -\sin \phi \sin \alpha \left(1 + \frac{B}{\epsilon} \frac{\sqrt{1-t^2}}{t} - t \operatorname{SECH}^{-1} t \right) .$$

Pressure distributions for the conical surfaces were calculated by evaluating the derivatives of the two potentials on the surface.⁵ With this to evaluate $(Q/U)^2$, C_p was computed for several points on the body. The resulting pressures and basic loads are shown in Figures 19, 20, 21 and 22.

These pressures and loads were less severe, over the nose cone and radiator condenser, than the slender body predictions. The Hybrid Theory was used as reference in later analyses because of experimental verification⁶ and because of the apparent limitations of the slender body theory pressure distribution for a cylinder.

c. Inertia Loading

The inertia loads accompanying these conditions were computed using the 2PSM2A weight distribution and assuming values of acceleration of 2.1g aft and 0.4g side (as supplied by the vehicle contractor). This data is used in combination with the Hybrid theory pressure distribution data to complete the basic loads for the aerodynamic pressure condition. These inertia loads are shown in Figure 23.

3. End Boost Loading

a. Inertia

The end boost condition occurs after the maximum dynamic pressure condition as shown in Figure 4. The basic loads for this condition were computed assuming that the maximum inertia loads (7.6g axial acceleration + 1.0g side acceleration) were coupled with the most severe aerodynamic heating conditions. The bending moment, shear and axial loads due to inertia are shown in Figure 24. The aerodynamic heating temperature data is given in paragraph b.

b. Aerodynamic Heating

The thermal history due to aerodynamic heating was solved by subjecting an idealized model of the structure to a heat environment dependent on the model's temperature response. The procedure starts by replacing the body with a network of fictitious rods or abstract linear heat paths. Heat sink values are assigned to nodes which simulate the heat capacity of the material. With this lattice, heat balances about the nodes can be made. The temperature of a node is found by assuming, then correcting, the temperature at a node then checking the constraint of heat balance at all nodes. This iteration proceeds on until all the conditions within a limit are met.

Figure 19. Aerodynamic Loading at Maximum Dynamic Pressure Hybrid Theory

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Figure 20. Pressure Distribution - Nose Cone Hybrid Theory

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Figure 21. Pressure Distribution - Radiator-Condenser-Hybrid Theory

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Figure 22. Pressure Distribution - Instrumentation Compartment-Hybrid Theory

11-3-63

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Figure 23. Limit Inertia Loads for Maximum Dynamic Pressure Condition

11-3-63

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Figure 24. Limit Inertia Loading - End Boost Condition

Boundary nodes that describe the heat flow of radiation and conduction through the boundary layer are added constraints. Heat flow due to radiation is dependent on the temperature attained by the node. Iteration of assuming the node temperature and checking heat balance yields the temperature at the radiating node.

The condition on the boundary layer nodes are governed by a temperature, time, heat flow surface. For a particular point in time, a temperature is assumed. The surface yields the dependence of heat flow on temperature. Then performing the iteration from point in time on the trajectory dependent surface, the structure's temperature history is found.

The temperature, time, heat flow surface is obtained by solving the boundary layer conduction equation for a fixed wall temperature condition through the trajectory. By solving for several wall temperatures and extrapolating for intermediate wall temperatures the heat surface can be described.

The rate of heat flow for Laminar conditions:

$$q_w = h (T_r - T_w)$$

T_r = Recovery Boundary Layer Temperature

T_w = Wall Temperature

h = Heat Transfer Coefficient

$$T_r = T_\delta \left\{ 1 + r \left(\frac{\gamma - 1}{2} \right) M_\delta^2 \right\}$$

T_δ = Temperature Freestream

M_δ = Freestream MACH Number

r = Recovery Factor

γ = Ratio of Specific Heats

$$h = C_{H_\delta} \rho_\delta \mu_\delta C_p$$

$$C_{h_{\delta}} = \text{Stanton Number} = 1/s \frac{C_{f_s}}{2}$$

ρ_{δ} = Density

μ_{δ} = Coefficient of fluid viscosity

s = Reynolds Analogy Factor

C_{f_s} = Skin Friction coefficient

Therefore

$$q_w = C_{h_{\delta}} \rho_{\delta} \mu_{\delta} C_p \left[T_s \left(1 + r \left(\frac{\gamma - 1}{2} \right) M_{\delta}^2 \right) - T_w \right]$$

where

$$r = \sqrt{P_{r_{\delta}}} \text{ (recovery factor)}$$

$$P_r = \text{Prandtl No.} = \frac{C_p \mu}{K}$$

Turbulent flow is described by the same equation for (q_w) with $C_{H_{\delta}}$ recovery factor redefined.

$$\begin{aligned} \text{Recovery factor } r = P_{r_T} & \left[1 + \frac{2}{K} \frac{\sqrt{C_f}}{2} \left(1 - P_{r_T} \right) \left[\frac{\pi^2}{6} + \frac{3}{2} \left(1 - P_{r_T} \right) \right] \right. \\ & + 25 \frac{C_f}{2} \left\{ \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) + 2 \ln \left[1 + \frac{5}{6} \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) \right] \right. \\ & \left. \left. + \ln 6 \cdot \ln \left[1 + \frac{7}{8} \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) \right] - \ln 6 \cdot \ln \left[1 + \frac{1}{4} \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) \right] \right\} \right] \end{aligned}$$

$$C_{H_\delta} = \frac{1}{s} \left(\frac{C_{f_s}}{2} \right) \text{ Stanton Number}$$

C_{f_s} solved from;

$$\frac{0.242}{\left[C_{f_s}^{1/2} \left(\frac{\gamma - 1}{2} M_\delta^2 \right) \right]^{1/2}} \left(\sin^{-1} \alpha + \sin^{-1} \beta \right) = 0.41$$

$$+ \log_{10} \left(\text{Re}_{x_\delta} C_{f_s} - f(n) \log_{10} \left(\frac{T_w}{T_\delta} \right) \right)$$

$$\alpha = (2A^2 - B) / (B^2 + 4A^2)^{1/2}$$

$$\beta = B / (B^2 + 4A^2)^{1/2}$$

$$A^2 = \left(\frac{\gamma - 1}{2} M_\delta^2 \right) / \left(\frac{T_w}{T_\delta} \right)$$

$$B = \left[\left(1 + \frac{\gamma - 1}{2} M_\delta^2 \right) / \left(\frac{T_w}{T_\delta} \right) \right] - 1$$

Reynolds factor

$$S = P_{r_T} \left[1 + 5 \sqrt{C_{f/2}} \left\{ \frac{1}{5K} \left(1 - P_{r_T} \right) \left[\frac{\pi^2}{6} + \frac{3}{2} \left(1 - P_{r_T} \right) \right] \right. \right. \\ \left. \left. + \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) + \ln \left[1 + \frac{5}{6} \left(\frac{P_{r_L}}{P_{r_T}} - 1 \right) \right] \right\} \right]$$

P_{rL} = Prandtl No. Laminar

P_{rT} = Prandtl No. Turbulent

K = Mixing Length Proportionality Factor

A Reynolds Number of 5×10^5 was used as a criteria for transition from turbulent to laminar flow. The method described has been set up in two North American IBM programs.

Temperatures were generated for RC-11 (honeycomb) and RC-13, RC-14 (aluminum-steel).^{7, 8} The data generated from these programs is presented in Figures 25 through 27.

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Figure 25a. Radiator-Condenser 11 - Ascent Temperature History - Station 11
(Sheet 1 of 4)

NAA-SR-9054

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Figure 25b. Radiator-Condenser 11 - Ascent Temperature History - Station 71
(Sheet 2 of 4)

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Figure 25c. Radiator-Condenser 11 - Ascent Temperature History - Station 107
(Sheet 3 of 4)

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Figure 25d. Radiator-Condenser 11 - Ascent Temperature History - Station 125
(Sheet 4 of 4)

a. Station 71

b. Station 107

7636-0129

Figure 26a & b. Radiator-Condenser 13 - Ascent Temperature History
 (Sheet 1 of 2)

c. Station 107

d. Station 125

7636-0129

Figure 26c & d. Radiator-Condenser 13 - Ascent Temperature History
(Sheet 2 of 2)

a. Station 11

b. Station 71

7636-0130B

Figure 27a & b. Estimated Radiator-Condenser 14 - Ascent Temperature History (Sheet 1 of 2)

c. Station 107

d. Station 125

7636-0130

Figure 27c & d. Estimated Radiator-Condenser 14 - Ascent Temperature History (Sheet 2 of 2)

c. Shield Bracket Loading

For the shield bracket region, the inertia induced loads are computed identically to those of the vibration condition considering two rings, one at Station 124 and one at Station 117.5 (Figure 7). The load reactions are shown in Figure 28. The thermally induced loads are assumed to be reacted only by the upper simulated frame at Station 124 for the honeycomb R/C. These thermally induced loads assume a very stiff shield with respect to the radiator-condenser in the radial direction. The internal load distribution for the higher loaded station 124 is shown in Figure 29.

For the thermal loads:

$$\Delta R = R\alpha\Delta T = 0.0465 \text{ inches}$$

$$W = \frac{2\Delta REI}{R^3 \left[\frac{1}{S^2} \left(\frac{1}{2} \theta + \frac{1}{2} SC \right) - \frac{1}{\theta} \right]} = 624 \text{ lb}$$

$$\theta = 22-1/2^\circ$$

$$S = \sin \theta$$

$$C = \cos \theta$$

The loads induced into the shield are computed similarly to those for the vibration conditions on page 25. The inertia loads are shown in Figure 30 and the thermal load input will be equal and opposite from that induced in the radiator-condenser at station 124.

7636-0131

Figure 28. Loading (lb) Induced Into Radiator-Condenser 11 - Inertia Condenser

7636-0132

Figure 29a & b. Radiator-Condenser 13 - Internal Loading Inertia Condition

7636-0133

Figure 30. Shield Induced Loading (lb)
- Inertia Condition

D. PRE ORBITAL STARTUP

The system prior to startup is orbiting and cooling, reflecting the temperature gradients due to the solar environment. The basic loads for this phase are purely thermal and the data is contained in Reference 9.

E. ORBITAL STARTUP AND CONTINUOUS OPERATION

The orbital startup causes many thermal transients in the system. These transients are the basic loads used for analysis and they appear in Reference 9.

The basic loads for continuous operation combine the steady state thermal environment (Reference 10) with the vibration environment of the CRU. This vibration environment is insignificant compared to the launch vibration.

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IV. STRUCTURAL ANALYSIS

A. INTRODUCTION

A myriad of structural analysis has been generated on the SNAP-2 Unit in support of the many facets of several designs. Many small and/or minor analyses have been performed but only the larger and more applicable are presented.

The method used in determining the degree of structural integrity is the margin of safety. The margin of safety (MS) is based on the ratio of allowable stress (F) to the applied stress (f) for a particular stress state

$$MS = \frac{F}{f} - 1 \quad .$$

The margin of safety is used to determine whether any yielding has occurred by comparing the actual limit stress to the yield allowable of the material or by using an ultimate stress (1.25 times limit stress) and comparing it to the ultimate allowable of the material or the basic stability allowable of the system. The ultimate stress allowables used represent at least 99% probable values and thus meet the reliability objectives of 136% with respect to the mean strength as described on Page 19. The basic loads are assumed to be absolute and therefore no reliability factors are associated with it.

B. RADIATOR-CONDENSER 10 ANALYSIS^{11, 12}

1. Introduction

This is the first fabricated aluminum-steel radiator-condenser consisting of a cone-cylinder configuration (Figure 31). The tubes were originally 405 SS and then later changed to Haynes 25 alloy during the manufacturing sequence. The loading conditions for this radiator are different than those currently used, but the analysis is valid for the loads used. The unit design was analyzed and then re-analyzed for the "as-built" condition.

TYPICAL CROSS-SECTION AT STA. 134.300

TYPICAL CROSS-SECTION AT STA. 8.500

- 24 LINEARLY TAPERED TUBES-.063 1100-0 AL.
- SKIN 1 IS CONSTANT
- 134 FT² AREA
- TUBES & ARMOR L-605
- AL. FRAMES 6061-T6

7562-0156

Figure 31. Aluminum-Steel Radiator-Condenser 10
(as built)

2. Minimum Margins of Safety

TABLE 2
MINIMUM MARGINS OF SAFETY
RADIATOR-CONDENSER 10

Component	Stress	Condition	Margin of Safety
Load entry bracket	tension	End boost	0.05 Yield
Tube columns	buckling	Maximum $q\alpha$	0.01 Ultimate
Station 44 frame	compression	Maximum $q\alpha$	0.01 Yield
Station 33.125 frame	crippling	Maximum $q\alpha$	0.01 Ultimate
Station 94.250 frame	crippling	Maximum $q\alpha$	-0.15 Ultimate

The negative margin of safety for the load condition on the frame at Station 94.25 is due to circumferential aerodynamic pressure. The frame is assumed to be completely annealed, whereas, realistically this will not be the case. To determine the amount of thermal material property degradation, a test is planned before structural qualification.

3. Tube Analysis

The tubes, including effective skin, were checked as columns to determine the frame spacing necessary.

The critical l/ρ (length divided by radius of gyration) was determined using either the Johnson equation for short columns

$$F_c = F_{co} \left[1 - \frac{F_{co} (l/\rho)^2}{4\pi^2 E} \right],$$

or the long column Euler equation

$$F_c = \frac{\pi^2 E_T}{(l/\rho)^2}.$$

The transition l/ρ is defined by $\sqrt{2\pi} \sqrt{E/F_{co}}$.

where

F_{co} is the column yield stress.

l is the effective column length.

ρ is the radius of gyration.

E is the modulus of elasticity.

E_T is the tangent modulus of elasticity.

The results of these calculations are shown in Figure 32.

7636-0143

Figure 32. Radiator-Condenser 10-Frame Spacing

4. Frame Analyses

After the frame spacing was established through the column analysis, the frames were then checked for stiffness to ensure support of the tube columns.

The analysis utilized the method developed by Timoshenko¹³ where the frame spring constant is established by

$$\alpha = \frac{m P_{cr}}{\beta l} \quad .$$

where

α = spring constant of frame

m = number of spans

l = frame spacing

β = buckling coefficient

$$P_{cr} = \frac{\pi^2 E_T}{(l/\rho)^2} \quad .$$

This analysis verified the assumption of relatively rigid frames established earlier in the column buckling analyses for frame spacing.

The inertial loads from the reactor-shield were computed and the stress distribution found. Also, several component loadings were introduced into the shell and analyzed.

C. RADIATOR-CONDENSER 13 ANALYSIS¹⁴

1. Introduction

The structural integrity of RC-13 is shown assuming that the structure has accumulated 100 hours of thermal operation prior to launch, partially annealing the structure, and then subjected to the launch and orbital environments.

The RC-13 design is basically a more advanced radiator-condenser than RC-10, and it is a cone of semimonocoque construction. The skin is 1100-0 aluminum, a low strength material selected for its high thermal conductivity. The stringer tubes (32 linearly tapered, equally spaced) are made of Haynes 25 alloy which possesses high strength and compatibility with mercury (Figure 33).

32 Linearly Tapered Tubes
(equally spaced)

0.030 1100-0 Aluminum Skin
0.010 1100-0 Aluminum Doubler at
Forward End is Provided to Reduce
Aerodynamic Heating

The Armor Width is Increased in the
Aft Section to Provide Adequate
Structural Material

Frame Material is 2014-T6 Aluminum
Except in the R-C - Shield Bracket
Area Where it is A-286 Steel

Component Loads are Introduced to the
R-C Shell via Brackets, Intercostals,
and Frames

Radiating Area is Approximately
 120 Ft^2

Approximate Dry Weight is 210 Lb

7636-0142

Figure 33. Radiator-Condenser 13 - Basic Description

The circumferential frames are made of aluminum and steel, depending on the station. The frame at Station 8.00 is made of two materials, namely, A-286 steel and Haynes 25 alloy. The frames at Stations from 26.50 to 107.00 are made of 2014-T6 aluminum. The frames at Stations 20.00, 118.00, and 124.00 are made of A-286 steel. The 2014-T6 was chosen because of its lesser degradation of properties after a 100-hr soak at 600°F when compared to other lightweight materials.

At the time when the radiator-condenser was designed, the reactor and shield configurations were not decided. Therefore, three configurations were studied and the more critical used. None of these three correspond to the final configuration, but the present design is satisfactory by comparison.

TABLE 3

SHIELD CONFIGURATIONS RADIATOR-CONDENSER 13

Case	Shield Wt	C. G.	Reactor Wt	C. G.
1	300	133.5	256	154.8
2	434	138.4	256	162.4
3	504	130.4	256	154.4
Final RC-13	355	132.6	276	154.5

The results of the analysis established that the radiator-condenser structurally yields in the areas listed below.

<u>Area of Yield</u>	<u>Reason</u>	<u>Condition</u>
Station 94 Frame	Thermal Compression	End Boost
Station 118.0 Frame	Thermal Compression	End Boost

Even though the radiator-condenser yields structurally, it is still capable of withstanding the ultimate loads imposed upon it. Frame material substitutions are currently being investigated for RC-13 to remedy this situation.

The minimum margin of safety for RC-13 are given in Table 4

2. Minimum Margins of Safety

TABLE 4
MINIMUM MARGINS OF SAFETY

Component	Load Case	Critical Stress	MS *Ultimate †Yield
Vapor Manifold	Orbital Operation	Tension	1.28* 0.06†
Vapor Riser	Vacuum	Compression	0.045†
Station 8.00	Lateral Vibration	Rivet Shear	0.035*
Station 20.00	Lateral Vibration	Rivet Shear	0.05*
Airload Stations 27 to 107	Aero Pressure	Compression	0.25†
Station 94.00 Thermal	Aerodynamic Heating	Compression	0.178* -0.346†
Station 118.00 Thermal	Aero Heating and Inertia	Tension	0.02* -0.135†

3. Tube Analysis

The RC-13 frame spacing was determined by the column equation and the tangent modulus theory and assuming the ends pinned.

$$l = \sqrt{\frac{\rho^2 \pi^2 E_T}{F_c}}$$

where

ρ = radius of gyration

E_t = tangent modulus

F_c = allowable stress

l = column length

The loading condition considered to arrive at the spacing as shown in Figure 33 are the aerodynamic pressure, lateral vibration, and inertia or end boost loading.

4. Vapor Manifold and Riser

The stress in the vapor manifold was determined statically and a factor of 2.0 was then applied to account for thermal shock.

$$f = E\alpha(T - T_{ave})$$

$$\alpha = 7.19 \times 10^{-6} \text{ in/}^\circ\text{F}$$

$$E = 27.5 \times 10^6 \text{ psi}$$

$$\Delta T = 450^\circ\text{F}$$

$$f = 27.5 \times 10^6 \times 7.19 \times 10^{-6} \times \frac{450}{2} = 44,500 \text{ psi}$$

A 0.06 margin of safety with respect to yielding but a 1.28 margin with respect to ultimate failure is represented.

The vapor riser is critical for bending when there is a vacuum internally with atmospheric pressure on the outside of the riser. This occurs on the ground prior to pulling a vacuum in the ground test tank and also prior to launch.

The vapor riser is made of Haynes 25 steel and has a thickness of 0.040 in.

Using an elastic energy analysis, a moment due to the above loading is obtained.

$$M = 11.7 \text{ in-lb}$$

Thermal stresses in the tube were considered in conjunction with the end boost loading and found to be non-critical.

The vapor manifold was checked for the working pressure at 10 psi in a vacuum at 600°F, also the condition of a vacuum in the system with atmospheric pressure externally was checked which gives a minimum margin of safety of 0.045 with respect to yielding.

5. Frame Analysis

The two main structural frames at Station 8.5 and 20 were checked for the PCS inertia loading and simulated vibratory loading. The basic frame was not critical for any of the load conditions (minimum margin of safety is 2.08); however, the rivets joining the two frame sections at Station 8.5 have a minimum margin of safety of 0.035. The rivets which connect the frame to the meteoroid armor of the tubes to form a continuous load path at Station 20 have a margin of safety of 0.05.

The frames at Stations 27.07 through 107 are known as the airload stations. The pressure distribution used for analysis is shown below.

A typical frame cross-section is shown on the following page.

The material properties and environmental conditions are given below.

Material - 2014-T6 aluminum

Prior exposure - 100 hours at 600°F

Test temperature - room temperature

Tension yield stress (F_{ty}) = 11,800 psi allowable

Tension ultimate stress (F_{tu}) = 30,800 psi allowable

The critical part is element No. 1 shown above which is critical for local yielding and the minimum margin of safety is 0.25 (Stress = 9,422).

The airload frame at Station 94 is considered the most critical for aerodynamic heating as the highest thermal gradient occurs in this frame. The boundary conditions were the curves of Figures 26a through 26d on the outer surface and the transient thermal conduction computed to determine the temperature gradient across the frame. The most critical time was found to be at 105 sec after launch. The temperature distribution analyzed is shown below. The frame is then allowed to grow radially with a net axial load of zero and induced stress due to the moment caused by the thermal gradient, with the resulting stress are shown in Figure 34.

The inner flange is critical for a tension yield stress of 13,500 psi and has a negative margin of safety of -0.12, but has a positive margin of safety of 0.57 when compared to ultimate stresses and therefore meets the reliability objectives. The outer flange is critical for compression yield and has a negative margin of safety of -0.346, with respect to yielding, but, again, it is positive with regard to ultimate stresses. The aerodynamic heating transients indicated in Figure 34 occur after the period of high frame loading (maximum dynamic pressure) portion of the launch trajectory, thus, yielding was allowed. This situation was corrected in the more advanced radiator systems.

The frame at Station 118 was also considered for aerodynamic heating and will have different temperature gradients, because it is constructed of A-286 steel; whereas, the other frames are of aluminum. This particular frame is critical later in the flight trajectory and occurs at 145 sec after launch.

7636-0143

Figure 34. Frame Temperature and Induced Stress - Station 94

The frame at Station 118 is analyzed identically to Station 94 and the resulting stress are much higher because of the added resistance. The temperature and stress levels are shown in Figure 35.

7636-0144

Figure 35. Frame Temperature and Induced Stress-Station 118

Superimposed on this condition are the inertial effects, on the frame, of the reactor-shield structure which has 7.6g acting down and 1.0g acting in a lateral direction. The resulting stress on the inner flange is 109,830 psi indicating that the material will yield but the ultimate margin is 0.02.

Several reactor-shield weight combinations were considered and then the load envelope used for design. The critical total load introduced into the radiator-condenser at Station 124 at the shield brackets is 69,000 in-lbs of moment and 4,750 lbs of shear. This is due to an equivalent vibration condition

of 5g load acting laterally and 1.0g acting down at the reactor-shield center of gravity. Station 124 is not a continuous ring and therefore represents a somewhat complicated load picture.

The shear load in the + Z direction is reacted by shear in the skin. This is proportioned by the effective skin in the Z direction. The frame loads for Station 124 are shown below:

The load taken by each load point is also proportional to its effective shear length. The frame loads were then balanced because of their discontinuities. The kick moment produces an unbalanced axial load in the shield which is added to the shield shear load and balanced.

The loads being determined, the moments were defined by isolating a segment as shown below and taking the moments at various sections to resolve the maximum moment. The maximum bending moment occurs in segment No. 2, with a value of 521 in.-lb with a resulting high margin of safety.

D. RADIATOR-CONDENSER 14 ANALYSIS¹⁵

1. Introduction

The Radiator-Condenser 14 (RC-14) (Figure 36) is an aluminum-steel concept and represents a structurally improved RC-13. The outer skin is 1100-0 aluminum similar to RC-13 except that it is backed with a titanium corrugation to restrain it from buckling. The stringer tubes (40 linearly tapered, equally spaced) are made of Haynes 25 alloy which possesses high strength and compatibility with mercury. Weight-strength analyses were made on the frames to determine the best material considering 2014-T6 aluminum, A-286 steel and 6AL4V titanium. The results indicated a minimum weight design with adequate strength using the titanium.

The dry weight of RC-14 was calculated to be 168 pounds which represents an improvement of approximately 20 lb over RC-13 and a non-yielding structure. The basic loads used in the analysis are similar to RC-13. The critical areas and their margins of safety are shown below. Only the areas which differed significantly from RC-13 analysis are presented here although the summary of minimum margins of safety is complete

STA 129.6

TUBE GEOMETRY

STA 8.3

40 LINEARLY TAPERED TUBES

SKIN — 0.020 STA 9.0 TO 107.0
 0.040 STA 107.0 TO 128.6
 1100-0 ALUMINUM
 TUBES — L-605 CRES
 FRAMES — 6AL-4V TITANIUM

7636-0145

Figure 36. Radiator-Condenser 14 - Basic Geometry

2. Minimum Margin of Safety - Radiator-Condenser 14

TABLE 5
CRITICAL MARGINS OF SAFETY
RADIATOR-CONDENSER 14

Component	Load Case*	Temperature (°F)	Critical Stress	Minimum Margin of Safety *Ultimate † Yield
Tube-Skin Column	3	R.T.	Buckling	0.005*
Tube	1	450	Tension	0.15†
Airload Frame	2	R.T.	Crippling	1.23*
Station 90 Frame, Thermal	1	500	Tension	2.00†
Station 71.0 Frame, Thermal	1	500	Tension	4.35†
Station 118.0 Frame, Thermal	1	500	Tension	4.84†
Titanium Back-up	5	600	Tension	4.00*
Aft End Structure	3	R.T.	Buckling	0.03*
PCS Attach Frames	1	500	Crippling	0.37*
Vapor Manifold	4	R.T.	Compr.	0.58†
Vapor Riser	4	R.T.	Compr.	0.87†
Shield Bracket, Segment Beam	3	R.T.	Tension	0.57†
Shield Bracket, Shear Panel	3	R.T.	Shear	0.33†
Shield Bracket, Bracket	3	R.T.	Crippling	1.75*
Shield Bracket, Station 118 Frame	3	R.T.	Crippling	1.10*
Skin Shear Panel	3	R.T.	Shear	0.25†
Typical Frame Splice	1	500	Bearing	0.94†

* Load Cases

1. End Boost
2. Maximum Dynamic Pressure
3. Maximum Vibration (Static Equivalent)
4. Helium Leak Test of Pressure System
5. Ground Performance Testing

3. Tube Analysis

The frame spacing was accomplished in the same manner as RC-13 (Reference Page 72) the results are shown in Figure 37.

4. Frame Analysis

A weight-strength study was made of the frame at Station 71 which was considered the most critical and also typical of the airload reaction frames. The first step was to apply the pressure distribution of Figure 21 on the frame, to satisfy the maximum dynamic pressure condition, and to size the frame using various materials. A summary of materials considered, thickness required and frame weight is tabalized below.

<u>Material</u>	<u>Thickness</u>	<u>Weight</u>	<u>Margin of Safety</u>
2014-0 Aluminum	0.063	2.07	0.02
A-286 Steel	0.020	1.86	2.02
6AL 4V Titanium	0.020	1.04	1.23

The steel and titanium frames show high margins of safety since 0.020 was considered minimum gage. These frames were then checked for the aerodynamic heating condition in the same manner as RC-13. The resulting temperatures and stresses are given in Figures 38 and 39. It was obvious in reviewing the above data that the titanium frames were best suited for use in the structure. The load condition for the maximum dynamic pressure condition was more critical for this frame and the minimum margin of safety is 1.23

7636-0146

Figure 37. Radiator-Condenser 14 - Frame Spacing

which is more than adequate. Stiffness requirements of the frames to insure column stability of the tubes was checked and found adequate.

The upper portion of RC-14 in the region of the shield brackets at Station 125.75 is space limited due to passage of NaK plumbing and electrical wiring. It was impossible to have a continuous load ring at this station and therefore the loads at this station were reacted by the comparatively rigid shield. The shear loads and shield ring balance loads were reacted in the four ring segments of Figure 40. The loads for the assumed 5.0g response of the reactor shield combination are the most severe. The internal load distribution is found for the critical segment as shown in Figure 41. These loads were used to

7636-0147

Figure 39.
Radiator-Condenser 14 -
Frame Stress Station 71

7636-0148

7636-0149

Figure 40. Applied Loads (lb)
and Shear Reactions
Station 125.75

7636-0150

Figure 41. Internal Load Distribution —
Ring Segment Station 125.75

analyze the ring segments. The critical part of the segment is that where there is a cutout for the tube the equivalent section is

$$\bar{y} = 0.213 \quad I = 0.010 \quad A = 0.1903 \quad (\text{Titanium as Reference})$$

$$\frac{E_H}{E_T} = 2.0$$

The critical stress is in the 0.040 Haynes 25 element

$$f = \frac{E_H}{E_T} \left[\frac{MC}{I} + \frac{P}{A} \right] = 2 \left[\frac{830 \times 0.213}{0.010} + \frac{650}{0.1903} \right] = 42600 \text{ psi}$$

The minimum margin of safety is 0.57 with respect to yield.

The frame at Station 118 is used to react the kick loads from the shield brackets. These inplane loads are shown below:

Maximum Vibration Loads (lb)

Maximum Inertia Loads (lb)

Internal load distribution was determined for these two cases.

The maximum internal load distribution for the lateral vibration are Moment = 392 in. -lb, Axial load = 369 lb and for the inertia condition Moment = 165 in. -lb and Axial load = 28. Again a weight-strength analysis of this force was generated considering A-286 steel and 6AL-4V titanium. The aerodynamic heating thermal gradient stresses were computed for the materials (Figures 42 and 43) and it was obvious that the titanium frame was lighter and structurally adequate. The minimum margin of safety is 1.10.

7636-0151

Figure 42. Radiator-Condenser —
Aerodynamic Heating Temperature —
Station 118 (A) Case II (B) Case V

NAA-SR-9054

Figure 43. Frame Thermal Stress Distribution - Station 118

5. Fin Substructure Analysis

Unique to RC-14 is the titanium back-up structure for the aluminum skin. This is required to prevent local skin buckles in the center of the aluminum areas defined by the tubes and frames. These local distortions are caused when heat is applied to the shell and the skin tends to grow at a faster rate than the Haynes-25 tubes allow. The following sketch shows a cross-sectional view of the titanium back-up structure selected to prevent the local distortions.

Properties of Materials

	<u>1100-0 Al</u>	<u>6AL-4V Ti</u>	<u>Haynes-25</u>
Typical Area (In. ²)	0.070	0.021	0.022
FTY (10 ³ psi) R. T.	3.5	145.0	67.2
600°F	1.2	97.5	40.4
E (10 ⁶ psi) R. T.	10.0	15.2	30.0
600°F	7.0	12.9	27.0
α (10 ⁻⁶ in./in./°F) 70-600°F	15.1	5.0	7.6

The mathematical model is

$$\delta_1 = \alpha L \Delta T + \frac{P_1 L}{AE} - \frac{M_1 C_1 L}{EI}$$

$$\delta_3 = \alpha L \Delta T + \frac{P_3 L}{AE} - \frac{M_3 C_3 L}{EI}$$

$$M_1 = P_1 C_1$$

$$M_3 = P_3 C_3$$

$$P_1 + P_2 + P_3 = 0$$

where

δ is displacement at interface

α is coefficient of thermal expansion

L is length

ΔT is temperature differential (600°F-70°F)

P is axial load

A is cross sectional area

E is modulus of elasticity

M is moment

C is distance from neutral axis to interface.

At 600°F the yield strength of the 1100 aluminum is 1200 psi. The limiting load P_2 in the aluminum panel is then (1200 psi) (0.07 sq in.) = 84 lb

Solving the above equations for a temperature difference of 530°F the maximum stress in the titanium backup structure is 16,800 psi indicating a margin of safety of 3.650.

E. THE 2PSM2A SYSTEM STRUCTURAL MOCKUP ANALYSIS

1. Introduction

The PSM2A stress analysis¹⁶ represents an accumulation of all the analysis of most major flight type components as of approximately October 1962.

2. Minimum Margins of Safety

The summary of the minimum margins of safety are shown below for the system. All of the conditions pertaining to the flight design were checked although 2PSM2A is a structural mockup and will not see the total environment.

TABLE 6
MARGINS OF SAFETY

Part	Condition	Critical Mode	Margin of Safety *Ultimate †Yield
RADIATOR-CONDENSER			
Forward End	Maximum ΔT on Face Sheets plus Inertia Loading	Intercell Buckling	0.18*
Aft End	Maximum Dynamic Pressure	Intercell Buckling	0.70*
Splice Joint (Forward End)	5.0g Side Vibration	Compression	0.11*
Station 124	5.0g Side Vibration	Hoop Bending	0.11*
Vapor Manifold	Internal Vacuum External 1 atm.	Compression Buckling	1.22†
Shield Bracket	Inertia + Aero Heat	Weld Shear	0.73*
Pipe Support Spot Braze	Piping Vibration	Tension	0.08*
BASE LOAD RING			
Panel	Maximum Dynamic Pressure	Primary Buckling	1.65*
Fiber Glass Ring	Maximum Dynamic Pressure	Bearing	0.26†

TABLE 6 (CONTINUED)

Part	Condition	Critical Mode	Margin of Safety *Ultimate †Yield
BASE LOAD RING (CONTINUED)			
Instrument Package	Vibration	Bending of Clip	0.15 †
ADAPTER RING			
Flange	Maximum Dynamic Pressure	Tension Yield	0.00 †
Bolt Pad	Maximum Dynamic Pressure	Bending	0.01 †
Tie Down Bolt	Maximum Dynamic Pressure	Tension	0.40 *
THERMAL BARRIER			
Barrier Material	Inertia	Bending	0.31 *
NOSE CONE			
Basic Cone	All	-	>2.00
Ejection Support Fitting	Ejection Load	Crippling	0.27 *
PIPING			
NaK Pipe in CRU	Vibration	Bending	0.15 †
Boiler Bracket Station 54.92	Vibration	Bolt Bearing	0.27 †
NaK Pipe Bracket Station 44.00	Vibration	Rivet Bearing	0.24 †
NaK Pipe Station 44.00	Vibration	Hoop Bending	0.20 *

TABLE 6 (CONTINUED)

Part	Condition	Critical Mode	Margin of Safety *Ultimate †Yield
PIPING (CONTINUED)			
Bracket Clamp Station 44.00	Vibration	Bending	0.14 †
Spring Support	Vibration	Crushing (Bending)	0.25*
SHIELD CAN			
Ring - Station 126.00	Vibration	Bending	0.27 †
Can - Lower Part	Operating Pressure	Tension	0.82*
Fitting - Pad	Vibration	Bending	0.01 †
REACTOR SUB ASSEMBLY			
Vessel Support	Spring Preload	Bending	0.12 †
Fuseable Link Assembly	Spring Preload	Tension	0.29 †
COMPONENTS			
Expansion Compensator Mockup	Vibration	Pipe Bending	0.78 †
Orbital Startup Pump	Operating Pressure	Bending	0.05 †

3. Radiator-Condenser - RC-11 Analysis¹⁷

The radiator-condenser for 2PSM2A (RC-11) design is basically a steel honeycomb cone (Figure 44). The tubes are provided with meteoroid armor and are an integral part of the honeycomb structure. The honeycomb is capable of carrying the design loads on its own accord.

All material is PH 15-7 Mo SS except for the tapered copper fins which are neglected in the analysis due to the low strength properties of the copper.

40 Linearly Tapered Tubes
 Tube Top Shown Above
 Tube Bottom Shown Below
 Copper Tapered Fins
 All Other Material
 PH 15-7 Mo TH1050 SS
 Total Area = 120 Ft²
 Useful Area = 117 Ft²
 Meteoroid Armor
 90% Probability
 0.089 in. of Steel

7636-0153

Figure 44. Radiator-Condenser 11 - Basic Geometry

Steel doublers are provided on the inner face sheets to provide an adequate structural buildup of material for attaching components to the R-C shell. The doublers are brazed into position in the original braze cycle.

The inertia loads of the reactor shield are introduced to the R-C structure via eight uniformly spaced load brackets near the forward end. Doublers are provided in this area to enable the eight concentrated loads to be distributed about the R-C circumference as they move longitudinally through the shell. The load leaves the structure at the aft end to the base load ring uniformly.

The end boost case considered the axial load plus the thermal induced hoop loads. These loads were relatively low except near the shield brackets and the resulting margins of safety for this condition are adequate.

The maximum face sheet temperature differences were computed (Figures 25-27) and combined with the inertia loads with the following results:

TABLE 7
MAXIMUM FACE SHEET TEMPERATURE DIFFERENCES
COMBINED WITH THE INERTIA LOADS

Section	f_x Psi	f_y Psi	γ	T °F	F_{ci_x} Psi	Margin of Safety
Aft	-26,600	$\pm 10,520$	± 0.396	450	55,800	0.67
Mid	-26,600	$\pm 10,300$	± 0.387	465	55,200	0.66
Forward	-28,400	$\pm 7,810$	± 0.275	535	55,100	0.55

The loads in the upper portion of the radiator-condenser were developed by assuming the shield brackets entered the structure at two fictitious rings each with a depth of 4 in. (Figure 7).

The condition considered included end boost where the inertia and temperature loads were additive. The temperature loads are induced by the cold-rigid shield compared to the hot flexible radiator-condenser with a temperature differential across the face sheets. Station 124 loading is shown on the following page:

The margin of safety for this condition is 0.18 and is critical for intra-cell buckling on the face sheets. The total stress for this condition is 90,600 psi compression on the inner face sheet. This consists of 9400 psi due to inertia reaction, 62,400 psi due to basic restraint of the brackets with respect to the radiator condenser and 18,800 psi due to the temperature difference across the face sheet. The lateral vibration condition considering ultimate loads develops stress in this same simulated ring of 136,400 psi at point A below. The resulting margin of safety is 0.11.

The aerodynamic pressure causes hoop bending around the periphery of the cone and also an axial component due to the body bending caused by the pressure. The hoop bending internal stress distribution is solved by use of elastic energy methods assuming a ring of constant cross sectional properties with the external pressure loading applied. This system is three times redundant. The associated stresses are maximum near the aft end. The limit stress is 29,100 psi due to hoop bending combined with 13,900 psi due to body bending. The resulting margin of safety is 0.70.

All other load conditions showed higher margins of safety. The complete stress analysis of RC-11 proved valuable during assembly when analysis of damaged areas was needed. The analysis was used to determine whether re-work was needed.

The lower portion of RC-11 (Figure 45) was analyzed for the loads induced by the power conversion system tankage (Figure 3). The induced bending in the two structural rings at Stations 8.50 and 20 had adequate margins of safety. The attachment of the fiberglass ring to the base of the structure was also analyzed and found to be adequate.

7636-0154

Figure 45. Base Structure 2PSM2A

4. Piping Bracket Attachment Test

The brackets which hold the piping and other miscellaneous items are spot brazed to the honeycomb. Because of the lack of structural knowledge of spot brazing and other attachments to thin face sheet honeycomb test samples were called for and the results used for analysis of the RC-11 bracketry (Figure 46). The test specimens included area brazed, riveted, spot brazed and mechanical fastener type of attachments. These attachments were used to fasten a tee to the honeycomb test panels with and without doublers brazed to the honeycomb. It was found that the area braze, spot braze and riveted attachments were tension critical, failing at approximately 800 psi, 100 lb and 380 lb respectively. The mechanical fasteners were good for above 600 lb. Spot brazing was used for 2PSM2A and supplemented by rivets where necessary.

5. Base Load Ring

The PSM2A base load ring differed slightly from the flight article in that the panels were of a different material and the instrument packages were mockup parts. The analysis, however, conservatively assumed a flight type configuration where 6061-T6 material was used and that an emittance coating was applied at 600°F. The material allowables were reduced to relative low values because of the application of the coatings. The margins of safety of the panel were still adequate. The fiberglass ring at the lower part of the unit has a low margin (0.26) in compression bearing. However, tests have indicated this value to be quite conservative.

A preliminary print of the adapter ring was received from the vehicle contractor and stress checked for the 2PSM2A loads and found to be inadequate and was redesigned per analysis. The analysis considered a curved beam with an upward load and restrained at each tiedown point. For an arc of 45°, the resulting equations then became

$$\begin{aligned} & \frac{1}{EI} \left[0.285 T_o + 0.500 M_o - 0.025 wR^2 \right] \\ & + \frac{1}{KG} \left[1.284 T_o - 0.500 M_o + 0.028 wR^2 \right] \\ & 0.707 M_o - 1.707 T_o - 0.039 wR^2 = 0 \end{aligned}$$

4-10-64

7636-0199

Figure 46. Typical Bracket Installation 2PSM2A

It was analyzed for a load of 200 lb/in tension (limit) and found to develop 11,400 psi due to differential bending from the torque and direct bending of the moment. The yield allowable is 12,000 psi so a 0.05 margin of safety exists. It was checked for fatigue and found to be adequate for the number of cycles involved.

6. Thermal Barrier

The thermal barrier consists of Thompson glass and is supported by the radiator-condenser at Station 7.16 and then dished down in the center of the vehicle to Station -4.5 where it is attached to a simulated Agena. The material was found to have extremely low stress allowables through testing (62 psi tension). A rigorous analysis was completed to demonstrate the structural integrity of this unsupported barrier. Energy considerations were used to equate the deflections at A and B which considering loading of the three separate sections independently. The resultant loading is shown in Figure 47. The minimum margin of safety is 0.31.

7. Nose Cone

The nose cone for 2PSM2A represents a mass and structural mockup of the flight system which was investigated for the aerodynamic and vibratory loading and all margins of safety were found to be adequate.

7636-0156

Figure 47. Applied Loading - Thermal Barrier 2PSM2A

8. Piping

The piping system for 2PSM2A was conservatively analyzed assuming a 30g acceleration force in any lateral direction and treating the pipe as a beam on multiple supports. Although expansion joints were in the piping system, they served no useful purpose other than to simulate the flight design. The thermal condition was not analyzed for the 2PSM2A piping since temperature will not be applied and the thermal analysis effort diverted to the flight design. The system was then designed and analyzed for the assumed 30g response. The loading summaries were presented in Figure 12 for both the NaK and mercury systems.

The critical section for the piping is at Station 17.5 and the NaK pipe CRU interface with a moment of 2076 in. -lb. The stress is 28,700 psi with a margin of safety of 0.15 with respect to yield.

9. Shield Container

The 2PSM2A shield container (Figure 2) is made of 316 corrosion resistant steel and the shield material is a plastic substance with the same density and similar material properties of the flight shield material. The reactor mock-up is fastened to a plate structure supported by a cone which in turn is supported

by a cone of a lesser angle. The base of the latter cone has structural rings which react the radial kick loads due to load exit into the radiator-condenser. These rings are critical for the 5g vibration condition with a minimum margin of safety of 0.27.

The can is analyzed as two conical sheets with a flat plate closure at the upper end. The upper cone consists of the sloped section, making a 60° angle with the longitudinal axis of the can, and the lower cone consists of the main can body which makes a 7.45° angle with the longitudinal axis. The smaller end of the upper cone is closed by a flat plate which forms the mounting base for the reactor and its associated controls.

Discontinuity expressions were developed for the redundant forces at the junctures between the plate and upper cone and between the two cones. Another set of expressions for the stresses were developed as functions of the various accelerations representing the loading conditions. For each condition of loading, stresses were determined assuming (1) no bond between the shield material and can, and (2) full bond between the shield and can. In either case the structural capacity of the shield was not considered, and the buckling of the can is judged to be non-critical. Therefore, even without a bond between the shield and can, the shield material will resist and limit, by direct bearing on the can prevents excessive distortion. The margins of safety were found to be adequate. The fitting pads were analyzed very conservatively and have a margin of safety of 0.01.

10. Reactor Mockup

The 2PSM2A reactor mockup weighs 265 lb and represents, as nearly as possible, the flight reactor reflector system. The loading is primarily vibratory and inertia induced. The most critical sections were found to have loads induced by the reflector ejection spring forces. The base plate is critical in bending with a margin of safety of 0.12. The fusible link assembly was preloaded and the margin of safety was 0.29. All other margins were considered adequate.

Other miscellaneous components for 2PSM2A were analyzed and found to be satisfactory. In the PCS subsystem the tooling induced loads were investigated to insure that nothing would be damaged during assembly.

F. FLIGHT DESIGN SUBSYSTEMS

1. Thermal Barrier

It was evident that the thermal barrier used on 2PSM2A was heavier than necessary because of the deep dished effect to support it on the Agena. Also it was desired to have thermal interchange between electronic components in the instrumentation section. The strength criteria that was set up was to keep the lowest resonant frequency above the longitudinal frequency of the SNAP Unit. The simplest structure available was a cone. Because of space requirement, a portion of the cone under the CRU had to be flat and thus continuity of the cone was lost. The cone vertex angle was 150° — a very shallow cone making standard analysis questionable. A thorough stability analysis was made to insure structural integrity with a 7.6g load acting on the insulation blanket, structure and approximately 30 lb of cabling. Under the CRU a flat plate was used and beaded in order to keep the resonant frequency above 100 cps. The load

reactions were computed on the structure and the plate loads were beamed through a straight beam to cantelevered supports in the instrument compartment. The analysis of the shallow cone was indeed complex and basically followed the analysis presented in a January 1963 AIAA Journal, "Elastic Stability of Orthotropic Conical and Cylindrical Shells Subjected to Axisymmetric Loading Conditions." The critical pressure was 0.6 psi compared to the applied pressure of 0.21 psi. The material considered was 0.020 aluminum headed for additional stiffness. The minimum stability margin of safety was 2.00.

2. Vapor Manifold¹⁸

The mercury vapor manifold was analyzed for the thermal stresses which developed as a result of the transient temperatures in RC-11. The thermal stresses were developed in the long leg of the manifold assuming clamped edges along the top and bottom edges. The temperature gradient (Figure 48) produces a curvature as shown below in the legs similar to pure bending of a plate to a spherical surface.

$$f_{\max} = \frac{\alpha E \Delta T}{2(1 - \mu)}$$

The manifold material is PH 15-7 Mo and has a yield allowable for this condition of 143,000 psi. The moment generated was compared to the burn down weld and the thickness of weld needed. The thickness needed was 0.025 compared to the 0.030/0.060 available when welded to specification.

The stresses were computed with the thermal gradient at 0.005 sec. The stress level including a dynamic shock factor was 83,100 psi. The margin of safety for this part is 0.72.

The differential expansion of the manifold with respect to the radiator condenser was considered and the loads computed assuming flexibility of the radiator condenser and lower part of the manifold as well as the tubing (Figure 49). The moment in the manifold joint is 49 in. -lb while that induced into the radiator condenser is 42 in. -lb. The margins of safety associated with this condition are adequate.

7636-0157

Figure 48. Vapor Manifold Thermal Gradients

7636-0158

Figure 49. Radiator-Condenser 11 - Vapor Manifold Geometry

3. NaK Piping System¹⁹

Extensive analyses were performed on the flight NaK piping system. The NaK system is made up of two piping loops, a hot line and a cold line (see Figure 1). The hot line is the piping carrying the hot NaK from the reactor to the boiler. The cold line carries the NaK from the boiler back to the reactor. The terms "hot" and "cold" are only relative as the NaK is always in the 1000°F range. The piping is of 316 SS throughout the system. The structural requirements for the piping system are based on various types of loading such as static acceleration, vibratory loads and thermally induced loads during operation. The primary stress problem is related to the thermally induced loading. The natural growth of the system due to the increase in temperature (room temperature to above 1000°F) meets resistance due to the geometry of the system. Expansion joints are used to make the longitudinal resistance to growth a minimum. The system analyzed is shown below.

HOT LINE

XY PLANE

represents an anchor point - support in all directions.

represents radial and lateral support - support in all directions except longitudinal, along centerline of system.

represents an expansion joint - relatively free to expand longitudinally only the spring constant of the bellows offers resistance. This expansion joint is not to be capable of moment transfer.

Thermoelectric pump

HOT LINE

YZ PLANE

COLD LINE CONFIGURATION

XY PLANE

YZ PLANE

The analysis of the piping system for the 30 g's loading in any direction is not critical. Various calculations made on hot line and cold line show no stresses of major significance. There are torsional stresses present due to the geometry of the system. These stresses are also of a small magnitude. The stresses on the system due to thermal conditions are of a critical nature. The stress levels are about those that are given as allowables for creep rupture at 10,000 hr.

The critical portion of the hot NaK line is that portion from station 95 to station 140.5 in the Y Z plane.

The piping from station 125.5 to 140.5 grows due to temperature. The resistance to this growth is only in the direction perpendicular to the longitudinal direction. There is no resistance (neglecting spring constant of expansion joint) in the longitudinal direction. The piping at point Z, Station 125.5 wants to deflect δ_H outboard.

$$\delta_H = \alpha \Delta T l \sin 5^\circ$$

$$= (10.5 \times 10^{-6}) (1250 - 70) 15 \sin 5^\circ = 0.0162 \text{ in.}$$

The piping system is analyzed to determine the induced stresses due to this deflection.

The preceding beam is solved for a 100 lb load at point Z, Station 125.5. The resulting solution shows a deflection of 0.0139 in. at point Z. Thus a beam with an applied load of $(0.0162/0.0139) 100$ or 116 lb gives the desired deflection at point Z.

The maximum moment on the minimum cross-section is

$$M_Z = 1.16 (72 \times 8 - 11 \times 22) = 388 \text{ in. -lb}$$

$$f_B = \frac{388 (0.6875 - 0.01)}{0.020} = 13,100 \text{ psi}$$

This stress is above the creep rupture allowable for 10,000 hr (11,650 psi). A creep relaxation analysis was begun to determine the actual life of the system for this condition.

V. VIBRATION ANALYSIS

A. INTRODUCTION

These analyses are performed on the SNAP-2 Unit to insure that structural integrity will be maintained during the various life-phases of the unit. The launch condition through MACH 1 is the most critical. It is necessary to decouple major component frequencies from system frequencies and to design the unit to withstand the vibratory loads.

As mentioned in the design criteria, the vibration-flight environment is covered by the qualification testing when the unit is positioned on the shake table and excited through the frequency range of approximately 10 cps to 3000 cps on all three axes. Since this is the known vibration environment, it forms the most severe design condition for the analytical model, that is the base restrained against rotation.

The SNAP-2 Unit consists of several subsystems of significance to a vibration environment. Of major importance is the response of the reactor and shield to this environment through which loads are transmitted through the shell (radiator-condenser) to the base load ring from the lower part of the unit. This produces the maximum lateral loading in the shell and in the reactor-shield subsystem. The fundamental frequency in the longitudinal direction is 98.85 cps compared to the estimated Atlas-Agena-SNAP combination of between 10 and 50 cps for this mode so there should be little coupling. The lateral frequency is 19.22 cps, while that of the Atlas-Agena-SNAP between lift-off and MACH 1 is between 2 to 20 cps; thus, possibly causing some vehicle coupling for the higher modes. The next most significant subsystem is the power conversion system tank arrangement. The majority of this subsystem lies between Stations 8.5 and 20. The components are quite heavy and being cantilevered from the shell, causing a problem in this area. Two structural rings are present in the radiator-condenser to react the loads. The analysis of this area indicates the fundamental frequency to be 61 cps for the S2PSM-2A system and as yet, the frequencies for flight systems are undetermined. Because of the complexity of the above analysis and the need for design information, a simple design equation was derived for use in determining loads for a particular component supported from a plane and subjected to a

vibration environment. The results of these equations are shown on page 30. The piping system was of concern and in order to not have cross-coupling between systems and components, the brackets were spaced to produce a minimum piping frequency of 125 cps.

B. 2PSM2A BODY BENDING ANALYSIS

Lateral vibration analysis²⁰ of S2PSM-2A was generated using a modified Myklestad analysis. This analysis is modified to contain the effects of rotary inertia and shear stiffness as well as pure bending stiffness. The method is essentially a step-by-step solution of the differential equation of motion of a lumped mass distribution spring system as shown below.

Equations expressing the moment, shear, slopes, and deflections are written for each station. The following equations, in matrix notation, results

$$\begin{bmatrix} S_{i+1} \\ M_{i+1} \\ \theta_{B_{i+1}} \\ \theta_{T_{i+1}} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & m_i \omega^2 \\ L_i & 1 & I_{zz_i} \omega^2 & 0 & m_i \omega^2 L_i \\ -\frac{L_i^2}{2EI_i} & -\frac{L_i}{EI_i} & 1 - I_{zz_i} \frac{\omega^2 L_i}{EI_i} & 0 & -\frac{m_i \omega^2 L_i^2}{2EI_i} \\ -\frac{L_i^2}{2EI_i} + \frac{1}{kAG_i} & -\frac{L_i}{EI_i} & 1 - I_{zz_i} \frac{\omega^2 L_i}{EI_i} & 0 & -\frac{m_i \omega^2 L_i^2}{2EI_i} + \frac{m_i \omega^2}{kAG_i} \\ \frac{L_i^3}{6EI_i} - \frac{L_i}{kAG_i} & \frac{L_i^2}{2EI_i} & -L_i + I_{zz_i} \frac{\omega^2 L_i}{2EI_i} & 0 & 1 + \frac{m_i \omega^2 L_i^3}{6EI_i} - \frac{L_i m_i \omega^2}{kAG_i} \end{bmatrix} \begin{bmatrix} S_i \\ M_i \\ \theta_{B_i} \\ \theta_{T_i} \\ Y_i \end{bmatrix}$$

ω = frequency = radians per second

m_i = lumped mass = $\frac{\text{pounds second}^2}{\text{inches}}$

θ_B = slope due to bending = radians

X = longitudinal deflection

Y = lateral deflection = inches

M = moment = inch-pounds

S = shear = pounds

θ_S = slope due to shear = radians

θ_T = total slope = radians

EI = bending stiffness = pounds-inch²

kAG = shear stiffness = pounds

I_{zz} = mass moment of inertia = pounds-inches second²

L = length = inches

F_i = axial force = pounds

AE_i = longitudinal stiffness = pounds

The large matrix is denoted by D_i . Therefore,

$$\begin{bmatrix} S_{i+1} \\ M_{i+1} \\ \theta_{B_{i+1}} \\ \theta_{T_{i+1}} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} D_1 \end{bmatrix} \begin{bmatrix} S_i \\ M_i \\ \theta_{B_i} \\ \theta_{T_i} \\ Y_i \end{bmatrix}$$

As the iteration takes place from lumped mass to lumped mass, a product of matrices results

$$\begin{bmatrix} D_T \end{bmatrix} = \begin{bmatrix} D_N \end{bmatrix} \begin{bmatrix} D_{N-1} \end{bmatrix} \cdot \cdot \cdot \begin{bmatrix} D_i \end{bmatrix} \cdot \cdot \cdot \begin{bmatrix} D_1 \end{bmatrix}$$

The matrix D_T , when expanded, will appear as follows:

$$\begin{vmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} \\ N_{21} & N_{22} & N_{23} & N_{24} & N_{25} \\ N_{31} & N_{32} & N_{33} & N_{34} & N_{35} \\ N_{41} & N_{42} & N_{43} & N_{44} & N_{45} \\ N_{51} & N_{52} & N_{53} & N_{54} & N_{55} \end{vmatrix}$$

For a cantilever beam, the initial conditons are

$$\begin{aligned} S_1 &= 0 \\ M_1 &= 0 \\ \theta_{B_1} &= \theta_{T_1} \\ Y_1 &= 1 \\ \theta_{B_n} &= 0 \\ Y_n &= 0 \end{aligned}$$

A solution for the natural frequencies is found by setting the determinant equal to zero and utilizing iteration techniques. When the frequencies are determined those values of moments, shears and relative displacements are calculated.

A total of 35 lumped masses were assumed and thus 35 springs evaluated for bending and shear stiffness. The resulting mode shapes, moments and shears were computed for the first three modes. (Figure 50)

C. 2PSM2A LONGITUDINAL MODEL ANALYSIS

Longitudinal vibration analysis of S2-PSM-2A was generated similarly to the lateral case (Figures 48a, b, and c)

$$\text{where } K_i = \frac{E_i A_i}{L_i} = \text{pounds per inch}$$

Therefore,

$$\begin{bmatrix} F_{i+1} \\ X_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & m_i \omega^2 \\ -\frac{1}{K_i} & 1 - \frac{m_i \omega^2}{K_i} \end{bmatrix} \begin{bmatrix} F_i \\ X_i \end{bmatrix}.$$

Again,

$$\begin{bmatrix} D_T \end{bmatrix} = \begin{bmatrix} D_N \end{bmatrix} \begin{bmatrix} D_{N-1} \end{bmatrix} \cdot \cdot \cdot \begin{bmatrix} D_i \end{bmatrix} \cdot \cdot \cdot \begin{bmatrix} D_1 \end{bmatrix}$$

$$\begin{bmatrix} D_T \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

where the symbols are as defined previously.

For the axial rod, lumped mass idealization of SNAP-2, the boundary conditions are $F_1 = 0$, $X_n = 0$, and $X_1 = 1.0$. The resulting output for the first three modes is shown in Figures 51a, b, c, d, e, and f.

*-NORMALIZED TO A 1-IN TIP DEFLECTION

a. First Mode - 19.226 cps

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Figure 50. 2PSM2A Vibration Analysis Lateral Frequencies and Mode Shape (Sheet 1 of 3)

*-NORMALIZED FOR A 1in TIP DEFLECTION

b. Second Mode - 109.142 cps

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Figure 50. 2PSM2A Vibration Analysis Lateral Frequencies and Mode Shape (Sheet 2 of 3)

* -NORMALIZED FOR A 1-IN TIP DEFLECTION

c. Third Mode - 190,896 cps

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Figure 50. 2PSM2A Vibration Analysis Lateral Frequencies and Mode Shape (Sheet 3 of 3)

a.b. 1st Mode - 98.854 cps

7637-0160

Figure 51. 2PSM2A Longitudinal Vibration Analysis

c. d. 2nd Mode - 395.361 cps

7636-0160

Figure 51. 2PSM2A Longitudinal Vibration Analysis

e. f. 3rd Mode - 658.259 cps

7636-0160

Figure 51. 2PSM2A Longitudinal Vibration Analysis

D. 2PSM2A POWER CONVERSION SUBSYSTEM ANALYSIS

The power conversion subsystem analysis is completed for the S2PSM-2A system.²¹ This system considers only the translation degrees of freedom, while the system for flight is expanded to consider all of the degrees of freedom of the subsystem. The analysis is computed using the basic matrix iteration method derivable from conservation of energies of the system. The end equation in matrix form is

$$\begin{bmatrix} K \end{bmatrix}^{-1} \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} Y \end{bmatrix} - \frac{1}{\omega^2} \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} Y \end{bmatrix} = 0$$

where

- K^{-1} = Influence coefficient matrix where each element k_{ij} is the deflection of the i^{th} element due to the j^{th} unit load
- M = Mass matrix where only diagonal terms are considered and each element m_{ii} is the mass of that element
- Y = A column matrix of the normalized deflections of each element
- ω = A scalar representing the circular frequency of the system

The equations are solved for the first natural mode and the second, third, . . . , j^{th} mode are determined through orthogorality relationships.

After the frequencies and mode shapes have been obtained, the system internal loads are then computed assuming a maximum deflection response at a certain element. For S2PSM-2A, 30g was assumed acting on the nitrogen tank.

$$\begin{bmatrix} P \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} Y \end{bmatrix} = - \omega^2 \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} Y \end{bmatrix}$$

where K , Y , M , ω are as previously defined and P is the element load.

This data is then used to determine the equivalent static stresses in the system at resonance, verification by test is needed to better define the assumed 30g load.

The normalized deflection of the tanks is given in Table 8 for the first three modes. The S2PSM-2A power conversion geometry is shown below. (Figure 52).

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Figure 52. Node Points and Degrees of Freedom
2PSM2A Power Conversion System

TABLE 8
S2PSM2A PCS MODE SHAPES AT RESONANCES

Natural Frequency		1st Mode 61.00 cps	2nd Mode 116.42 cps	3rd Mode 144.08 cps
(REF: Figure 52)				
Normalized	1	0.040	-0.060	0.028
	2	0.216	0.129	-0.446
Mode	3	-0.082	0.138	-0.336
	4	0.171	0.004	-0.252
Shape	5	0.253	0.095	-0.129
	6	0.240	0.064	-0.291
Node Points	7	1.000	-1.000	-0.034
	8	0.929	0.725	1.000
	9	0.575	-0.110	0.049
	10	-0.003	0.054	0.271
	11	-0.029	0.053	-0.213
	12	-0.292	-0.058	-0.133

These loads were then transmitted from the tanks to the structural rings in the radiator-condenser. The maximum stress in the ring at Station 20 is 37,100 psi if the assumption of 30g on the nitrogen tank is valid. The margin of safety of the structural rings is greater than 2.0 for this condition.

E. SYSTEM BODY BENDING ANALYSIS USING RC-13

A lateral vibration analysis of a system utilizing RC-13 was computed using an eigen value technique described earlier and neglecting shear deflection and rotary inertia. The analysis results are shown in Figure 53.

7636-0162

Figure 53. Normalized Mode Shapes — Radiator-Condenser 13

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