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VARIABLE MODERATOR REACTOR
DEVELOPMENT PROGRAM

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FOREWORD

Development of the Variable Moderator Reactor (VMR) is conducted under Contract No. AT(04-3)-250, Project Agreement No. 3, by Advanced Technology Laboratories, a Division of American-Standard, for the U. S. Atomic Energy Commission. This third quarterly technical progress report covers the continuation of work under Phase I of the contract, "Physics and Kinetics Analysis and Initial Evaluation," and the continuation of work under Phase II, "Critical Experiment and Analysis of Results."

The information contained in this progress report is preliminary and subject to change as the program evolves.

PROGRAM OBJECTIVES

The objectives of the work currently being performed, and of the work to be performed during the next quarter (Feb. 16, 1960 through May 15, 1960), in accordance with the agreed upon scope,¹ is as follows:

1. To continue modification of the core physics model PUREE',² as dictated by future checking against experimental data.
2. To use the hydrodynamic code BOCH³ and the physics codes PUREE' and VAL PROD⁴ to continue analysis of the relationship between voids, flux, and burnup, and to continue to investigate the relationship between $k_{\text{effective}}$ and void fraction, including the effects of the spatial distribution of steam voids, and neutron flux.
3. To use the analog representation of the VMR core kinetics, together with results from "2" above, to continue study of the transient characteristics of various VMR core arrangements. To compare the results of the analog representation with existing reactor transient response data.
4. To commence mechanical and system design and cost analysis of specific VMR cores that have been examined for nuclear and hydrodynamic feasibility.
5. To complete assembly of the critical experiment equipment, and to commence the experiments.

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1. "Program for Evaluation of the Variable Moderator Reactor under the new Reactor Concepts Program for the Atomic Energy Commission", Atomic Energy Division of American-Standard, Proposal P-470A, December 17, 1958.
 2. PUREE' is a non-dimensional physics code which computes homogenized constants for use in neutron-balance equations.
 3. BOCH is a two-dimensional hydrodynamic code which computes the non-uniform void distribution in the core.
 4. R. H. Stark and C. H. White, "VAL PROD, a Revision of the PROD II 630 Multigroup," General Electric, Atomic Power Equipment Division, GEAP-0952, January 9, 1957 (a one-dimensional physics code for solving criticality and reactivity).

STATUS OF PROGRAM

The following sections describe the progress in each of the above areas and in the areas outlined under "Objectives of Current Work" from the previous Quarterly Progress Report.⁵

A. Core Designs

The hydrodynamics code BOCH and the physics code PUREE' have been used in the analysis of specific VMR cores. Eight core designs of different geometries have been selected for initial evaluation. Extensive analysis of the first of these cores has been completed, and more general analysis of the other cores is in progress.

B. Kinetics

The analog representation of the VMR kinetics has been completed and applied to the analysis of the VMR over a large range of parameters for curves of positive void versus reactivity. It is presently being applied to curves of negative void versus reactivity.

C. Physics

Further checks of the first three energy groups of PUREE' against experimental data have been completed. The P_3 -POP method of computing the thermal group constants (as described in the second quarterly progress report⁵) has been used in conjunction with the first three groups to analyze the reference core designs and the critical experiment. The more complex blackness method of computing the PUREE' thermal group constants (as described in the first quarterly progress report⁶) appears to be functioning properly, and is being checked on very tight lattices to determine the magnitude of the change in fuel-pin blackness due to incident neutron distortion by neighboring fuel pins.

D. Mechanical and System Design

Mechanical and system design of selected cores has commenced. Computation of the temperature-induced stresses in the core structural members is in progress. Corrosion rate of the moderator systems has been computed, and preliminary analysis of a clean-up system has commenced.

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5. "Variable Moderator Reactor Development Program," Advanced Technology Laboratories, a Division of American-Standard, Quarterly Progress Report No. 2, ATL-A-103, 30 November 1959.
 6. Quarterly Progress Report No. 1, ATL-A-100, 30 August 1959.

E. Critical Experiments

Mallinckrodt Nuclear Corporation encountered difficulties in the manufacture of the completed fuel pins and was unable to meet the scheduled delivery date. It is expected that the fuel will be delivered early in March. All other items of equipment required for the initial experiments have been completed and shipped to Battelle Memorial Institute.

The Hazards Analysis Report has been submitted to the Chicago Operations Office of the Atomic Energy Commission.

SUMMARY OF RESULTS

A. Core Designs and Parametric Studies

Various parameters have been investigated as functions of moderator volume/ UO_2 volume, coolant volume/ UO_2 volume, and moderator volume/coolant volume. Core Design No. 1, with a moderator volume/coolant volume of 0.554, has a highly skewed axial power density distribution, indicating a marked dependence upon steam void distribution. This skewing of the power distribution results in a peak/average power density ratio of 1.76. It is expected that moderator-to-coolant ratios can be selected which will temper the effect of the axial void distribution and will flatten the power density distribution curve.

Core Design No. 1, with a coolant volume/ UO_2 volume ratio of 1.61, has a negative void coefficient of reactivity.

For a given moderator/ UO_2 ratio, reactivity (k_{eff}) does not appear to be sensitive to variations in pin spacing/pin diameter ratios within the range of 1.3 to 1.4. Since hydrodynamic considerations favor larger spacings, these wider pin lattices will probably be used (subject to further verification).

B. Kinetics

A complete description of the kinetics analog model is presented in Attachment I. This analog model has been used to analyze the VMR for positive void-reactivity curves. The model was found to be stable over a wide range of parameters.

C. Hydrodynamics

A complete description of the BOCH hydrodynamic code is presented in Attachment II.

D. Physics

A complete description of the PUREE' first three energy groups and the P_3 -POP method of computing the thermal group constants is presented in Attachment III. Further checks of energy groups 1, 2, and 3 over a range of experimental data indicates a computed effective multiplication within 1% of the measured values.

I. CORE DESIGNS AND PARAMETRIC STUDIES

In addition to Core Design No. 1, which was described in the last quarterly,⁵ computations are proceeding on Core Designs No. 2 through 8. Table I summarizes some of their significant parameters.

Detailed descriptions of the BOCH hydrodynamic code and the PUREE' physics code, which are being used to analyze these designs, are included in this report as Attachments II and III. Preliminary results of some of the core design computations are shown in Figures 1 through 17.

Figures 1 through 13 are specifically concerned with Core Design No. 1, and Figures 14, 15, and 16 are plots of data obtained from all of the core designs. Figure 1 gives the flux distribution through the small cell, while Figures 2 through 5 show the variations of the thermal group cross sections for the small cell as functions of coolant density and enrichment. Figures 6 through 10 show the same quantities for the large cell calculations. ("Small cell" is defined as a single fuel pin and its associated coolant. "Large cell" is defined as a complete fuel assembly with its associated coolant, shroud, and moderator.)

The initial calculations on Core Design No. 1 were made to evaluate the effect of average coolant density variation on reactivity. The coolant density chosen was based on an axial coolant density distribution computed by the BOCH hydrodynamic code for an assumed sine power distribution. Coolant density distributions for the assumed sine power distribution and for the actual skewed power distribution are shown in Figure 17. The change in both k_{eff} and k_{∞} with average density variation is shown in Figure 11.

The radial and axial power density distributions for Core Design No. 1 are shown in Figures 12 and 13. Axial power distribution was computed by dividing the core into six axial regions and utilizing a different average coolant density for each region. These different average coolant densities were determined by reference to the axial coolant density distribution computed by BOCH (Figure 18). The radial power density distribution was computed by assuming a constant average coolant density throughout the core.

Figures 14 through 16 are the results of preliminary analysis of Core Designs 2 through 8, in which an enrichment of 2.0% U 235 and an average coolant density of 0.41 gm/cc were used. The variation in reactivity with the actual coolant density distribution is being computed for select cases so that plots may be made of reactivity changes associated with actual and "average" coolant density variations.

Figures 14 through 17 were prepared in such a way that it is possible to determine the effect of the change in a particular reactor design parameter on other dependent parameters, while parameters not pertinent to the specific analysis being considered are held constant.

Due to the preliminary nature of the curves presented in this section, few conclusions can be drawn. However, some observations concerning apparent trends are appropriate.

Reference to Table I will indicate that Core Design No. 1 has the lowest moderator/coolant ratio of all the cases considered. This low moderator/coolant ratio should result in emphasis being placed on the nuclear importance of the coolant region; Figure 13 illustrates this emphasis. The highly skewed axial power distribution curve results from the decrease in reactivity with void generation in the coolant channels. As the moderator/coolant ratio is changed, it is expected that combinations can be found where the moderator region will have a tempering effect on the axial power density curve, tending to reduce the amount of skew and the peak-to-average power ratio. The amount of reactivity associated with changes in moderator level will be plotted as a function of the moderator/coolant ratio to determine the absolute and relative control associated with the moderator region.

The radial and axial peak-to-average power density ratios are shown for Core Design No. 1 in Figures 12 and 13 (the normalized values of 1.0 represent the average power). The radial average power density is based upon a volume weighted average (e.g., $P_{avg} = \frac{\sum PV}{V}$). It is seen that the peak-to-average power density ratio is 1.76 for the radial distribution and 1.77 for the axial distribution. The radial power density distribution was computed assuming constant exit steam quality across the core. Additional computations will be made to ascertain the power flattening effect produced by orificing so that the center channels are more highly voided than the outer channels.

As indicated in Figure 11, Core Design No. 1 has a negative void coefficient of reactivity (e.g., reactivity decreases as steam voids increase). It is noted from the graph that for an assumed length average coolant density of 35 lb/cu. ft (0.56 gm/cc), approximately 1.6% reactivity is associated with the steam voids.

TABLE I
VMR CORE DESIGN DATA

	Core Design No.							
	1	2	3	4	5	6	7	8
Nominal thermal output, mw	70							
Operating pressure, psia	1000							
Fuel pin cladding thickness, in.	0.030							
Fuel pin diametral gap, in.	0.005							
Fuel cladding material	Zr							
Shroud material	SS							
Shroud thickness, in.	0.050							
*Coolant volume/ UO_2 volume	1.61	1.61	1.61	1.24	1.24	1.24	1.24	1.61
Moderator volume/ UO_2 volume	0.892	2.0	3.0	3.0	1.0	3.0	3.0	2.0
*Total water volume/ UO_2 volume	2.50	3.61	4.61	4.24	2.24	4.24	4.24	3.61
*Moderator volume/coolant volume	0.554	1.24	1.86	2.41	0.807	2.41	2.41	1.24
Fuel pins per element	61							37
Fuel elements per core	121							
UO_2 in core, kg	12,560					7,310	17,840	7,610
Density UO_2 , gm/cc	10.41							
Active height of core, in.	72							
Core diam. across hex. corners, in.	74.4	84.2	92.0	89.4	71.6	69.0	103.6	56.4
Core diam. across hex. flats, in.	64.4	72.8	79.7	77.4	62.0	59.7	91.5	48.8
Fuel pellet diameter, in.	0.420					0.320	0.500	0.420
Fuel pin diameter (clad OD), in.	0.485					0.385	0.565	0.485
Fuel pin centerline spacing, in.	0.676			0.631	0.631	0.494	0.745	0.672
Fuel pin spacing/diameter	1.393			1.30	1.30	1.28	1.32	1.386
Fuel element centerline spacing, in.	6.20	7.01	7.67	7.44	5.98	5.75	8.81	5.48
Moderator gap between shrouds, in.	0.75	1.56	2.12	2.34	0.88	1.74	2.80	1.21
Distance between flats of shrouds (internal), in.	5.35			5.00	5.00	3.91	5.91	4.17

* Coolant volume is computed with zero voids.

SMALL CELL RADIAL NEUTRON FLUX DISTRIBUTION FOR CORE DESIGN NO. 1

FIGURE 1

057 012

SMALL CELL THERMAL ABSORPTION CROSS SECTION VERSUS COOLANT DENSITY
FOR CORE DESIGN NO. 1

FIGURE 2

057 013

SMALL CELL FISSION CROSS SECTION AT THERMAL ENERGY
VERSUS COOLANT DENSITY FOR CORE DESIGN NO. 1

FIGURE 3

SMALL CELL DIFFUSION CONSTANT AT THERMAL ENERGY
VERSUS COOLANT DENSITY FOR CORE DESIGN NO. 1

FIGURE 4

SMALL CELL TRANSPORT CROSS SECTION VERSUS COOLANT DENSITY
FOR CORE DESIGN NO. 1

FIGURE 5

LARGE CELL RADIAL NEUTRON FLUX DISTRIBUTION FOR CORE DESIGN NO. 1

FIGURE 6

LARGE CELL THERMAL ABSORPTION CROSS SECTION VERSUS COOLANT DENSITY
FOR CORE DESIGN NO. 1

FIGURE 7

LARGE CELL FISSION CROSS SECTION AT THERMAL ENERGY
VERSUS COOLANT DENSITY FOR CORE DESIGN NO. 1

FIGURE 8

LARGE CELL DIFFUSION CONSTANT AT THERMAL ENERGY
VERSUS COOLANT DENSITY FOR CORE DESIGN NO. 1

FIGURE 9

LARGE CELL TRANSPORT CROSS SECTION VERSUS COOLANT DENSITY
FOR CORE DESIGN NO. 1

FIGURE 10

REACTIVITY VERSUS AVERAGE COOLANT DENSITY FOR CORE DESIGN NO. 1

FIGURE 11

RADIAL POWER DENSITY DISTRIBUTION FOR CORE DESIGN NO. 1
(Assumed Coolant Density = $46.8 \text{ lb/ft}^3 = 0.75 \text{ gm/cm}^3$)

FIGURE 12

057 023

AXIAL POWER DENSITY DISTRIBUTION FOR CORE DESIGN NO. 1

FIGURE 13

REACTIVITY VERSUS MODERATOR/ UO_2 VOLUME

FIGURE 14

REACTIVITY VERSUS PIN SPACING/PIN DIAMETER

FIGURE 15

REACTIVITY VERSUS NUMBER OF PINS

FIGURE 16

057 027

COOLANT DENSITY DISTRIBUTIONS FOR CORE DESIGN NO. 1

FIGURE 17

II. KINETICS

The results obtained by applying the kinetics analog model shown in Attachment I to specific VMR problems are presented below.

The general procedure in the analog work was to set all parameters at their reference values (Table V of Attachment I) and vary from run to run only one parameter (or two, if closely related). In the following description of the analog work, it is thus to be understood that all parameters have their reference value except for those specifically mentioned. It also should be noted that no attempt was made in this preliminary work to optimize the moderator level control system. The purpose of the work was to gain an insight to the relative importance and effect of various system parameters rather than to study reactor control.

Figure 18a shows the response of the reference system to a 20% increase in steam demand. It will be noted that the maximum amplitude of the pressure disturbance is 30 psi and that the reactivity decrease due to increased fuel temperature very nearly compensates for the reactivity increase due to the decrease in coolant density.

Figure 18b shows the response to a 10% increase in steam demand for the case in which the transport time delay (K) associated with flow of water from the top of the riser to the core inlet plenum is zero.

Figures 19a and 19b show system response to a 10% change in steam demand when the fuel-pin time constant is, respectively 3 and 1/4 times the reference value of 9.5 seconds (28.5 and 2.4 sec). The reactivity change due to fuel temperature changes for the longer time constant can be observed to be quite large: the moderator level is increased in order to bring the net reactivity back to zero. In contrast, for the short time constant fuel pin, a decrease in moderator level is required. The large temperature effects noted are, of course, due to the fact that the fuel temperature change is proportional to the fuel-element time constant.

Additional analog runs (not illustrated) indicated that the values of the time constants τ_1 and τ_2 are of small importance. Simultaneous variation of τ_1 and τ_2 from 1/2 to 4 times the reference values (i. e., from 0.2 to 1.6 sec) produced no detectable difference in model behavior.

Representation of the fuel-element time response by first and second order approximations produced a detectable but negligible effect on system response.

A brief summary of the system parameters that were varied and an indication of the ranges over which the reactor response was found to be stable is given in Table II. Analog work is being resumed, using a positive void coefficient of reactivity, and will be followed with an investigation of the system with a negative void coefficient.

TABLE II

	<u>Range</u>
a) * Time constant for density change in response to step change in temperature of water entering core	0.15 sec to 1.2 sec
b) * Time constant for density change in response to step change in heat input to water	
c) Fuel-element time constant	
d) Transport time delay associated with flow of water from top of riser (and feedwater inlet) to core entrance	0 to 8 sec
e) Void coefficient of reactivity	
1) Without moderator level control	$\frac{1}{2}$ to $1\frac{1}{2}$ times reference value
2) With reference moderator level control	$\frac{1}{4}$ to 2 times reference value
f) Moderator level rate of control	From 0 to 32 times reference value
[i.e., $C \text{ in } \frac{dL}{dt} = -C (P - P_{\text{set}})$, where moderator L is moderator level]	
g) Fuel temperature reactivity coefficient	$1/8$ to 2 times reference value

* Parameters "a" and "b" were varied simultaneously.

RESPONSE OF REFERENCE KINETICS ANALOG MODEL

FIGURE 18

- (a) Reference conditions but with 28.5-sec fuel-element time constant.
 (b) Reference conditions but with 2.4-sec time constant.
 Both sets are in response to 10% step increase in steam demand.

RESPONSE OF REFERENCE KINETICS ANALOG MODEL

FIGURE 19

III. PHYSICS

A. Groups 1, 2, and 3 of the Four-Group PUREE' Model

Testing of the method and cross section data of PUREE' against various experiments has continued. The experimental lattices were some of those described by Kouts, performed at BNL and WAPD.^{7, 8} The test cases are identified as 1, 2, 4, and 6 in Table III.

TABLE III

Test Cases for PUREE' Energy Groups 1, 2 and 3

Case No.	Fuel Type	Ratio of Water to Uranium or to Uranium Oxide	Enrichment (%)	Fuel Rod Diameter (in.)	Computed Effective Multiplication
1	Metal	2	1.3	0.6	0.999
2	Metal	3	1.3	0.6	0.995
4	Metal	3	1.3	0.387	0.989
6	UO ₂	1.78	1.3	0.383	1.005

Experimental values of thermal utilization were employed in the calculation. For the larger metal-rod diameters the "standard"⁹ values of A (9.2) and μ (2.7) were used in the formula for effective resonance integral:

$$\int \sigma_{a_{28}} \frac{dE}{E} = A \left(1 + \mu \frac{S}{M} \right)$$

On the smaller rod sizes, use of these standard values gave somewhat inaccurate results, which is not surprising, since the values are old and are based on measurements with rod sizes more of interest for graphite and D₂O reactors, i. e., diameters of 1 to 2 inches.

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7. H. Kouts, et al, "Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water," Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Vol. 5, P/600 (1955).
 8. H. Kouts, et al, "Physics of Slightly Enriched, Normal Water Lattices (Theory and Experiment)," Proceedings of the Second United Nations International Conference on Peaceful Uses of Atomic Energy, Vol. 12, Reactor Physics, p 1841 (1958).
 9. J. B. Sampson and J. Chernick, "Resonance Escape Probability in Thermal Reactors," KAPL-1754, August 1, 1957.

The values used for the smaller rod sizes were those given by Hellstrand,¹⁰ i. e., $A = 7.8$ and $\mu = 3.87$ for the metal and $A = 10.5$ and $\mu = 2.17$ for UO_2 . These are stated to hold in the range $0.07 \leq \frac{S}{M} \leq 0.25$ for metal and $0.2 \leq \frac{S}{M} \leq 0.7$ for the oxide. Case 4 (metal) in Table III has $\frac{S}{M} = 0.218$, and Case 6 (oxide) has $\frac{S}{M} = 0.395$.

The computed multiplication of all the cases considered is essentially within 1% of the measured effective multiplication of 1.0.

Although the over-all multiplication is computed quite accurately by PUREE' as it stands, some of the component aspects of criticality such as the ratio of U 238 fissions to U 235 fissions, etc., do not compare with Kouts measurements as closely as would be desirable. It is planned to analyze this further and make improvements in PUREE' if possible.

PUREE', together with the cross-section library for hydrogen and oxygen, was used to calculate the "age" for water. It is well known that diffusion theory (and Selengut-Goertzel moderation of neutrons) gives an age that is too large compared with experiment. This was found also to be true in PUREE'. A diffusion coefficient for Group 1 that was used by WAPD and is found from transport theory leads to a much better value of age for water. This value is being used in the water reflector of the critical experiment and VMR calculations.

B. Thermal Group of the Four-Group PUREE' Model

Cross sections for the thermal neutron group used in the VMR calculation performed thus far have been obtained from a combination of an infinite medium Wigner-Wilkins calculation for thermal neutron spectrum with a P_3 spherical harmonics calculation of the thermal neutron spatial distribution. In addition, a computer program has been under development for the calculation of two effects on thermal neutron behavior which are not taken into account in this analysis:

- 1) A change in fuel-pin blackness due to the distortion by neighboring pins of the angular distribution of incident neutrons.
- 2) The difference between the thermal neutron spectrum in the pin and that in the coolant-moderator.

(See Quarterly Progress Report No. 1 for a description of the computational method.)

10. "Reactor Physics Constants," ANL-5800, p 344.

This program appears to be functioning properly, and its first use will be to determine the importance of the first effect by comparing, for tight lattices, the blacknesses obtained by this method with those of Stuart and Woodruff. Should the effect be significant, the 'PUREE' thermal group code will be used to generate curves of thermal group cross sections as functions of lattice parameters for further VMR calculations. Should it be negligible, further use of the code to account for effect 1 above will not be necessary.

The corrections to present calculations due to effect 2 will be combined with those due to the spectrum shift across the fuel assembly to obtain a total thermal neutron spectral shift correction. This correction can be expected to be small in the clean reactor because the U 235 fission and capture cross sections have a nearly $1/V$ dependence in the thermal range. That it is actually small is borne out by the smallness of the portion of the component of the temperature coefficient of reactivity associated with the energy shift of the thermal neutrons (see Attachment III). The thermal shift correction might be expected, however, to have a significant effect on fuel-cycle properties because of changes in the relative microscopic cross sections of U 235 and Pu 239.

C. Burnup Calculations

Table IV presents the design cores upon which the initial burnup calculations will be made for fuel-cycle analysis.

TABLE IV
BURNUP CALCULATIONS

<u>Core Conditions for Burnup Calculations</u>							
Case No.	a	b	c	d	e	f	g
Core Design No.	1	1	4	4	4	5	7
Average mwd/t of removed fuel	15,000	15,000	10,000	15,000	15,000	15,000	15,000
Plant load factor	0.8
No. of fuel regions in core	1	2	1	2	1	1	1
Fraction of core removed/cycle	1	$\frac{1}{2}$	1	$\frac{1}{2}$	1	1	1
Cycle condition	Steady state
Radial power distribution	Based upon constant exit steam quality across core.						

Information to be Obtained from Burnup Calculations

Initial enrichment.

Average final enrichment of removed fuel.

Pu concentration of removed fuel.

Number of operating days per cycle.

IV. CRITICAL EXPERIMENT

A. Physics

A number of calculations pertaining to the critical experiments have been completed using PUREE'. The results are summarized below.

The calculations were performed on the following configuration.

Coolant-to- UO_2 ratio = 1.8,

Pin pitch = 0.703 in.,

Shroud diameter (across flats) = 5.490 in.,

Number of shrouds = 37,

Coolant plus moderator-to- UO_2 ratio = 2.75,

Shroud spacing = 6.34 in.

This is the configuration of greatest coolant plus moderator-to- UO_2 ratio. The core was assumed to be reflected radially and at the bottom by an H_2O reflector of essentially infinite thickness (about 1 foot of H_2O reflector is essentially infinite thickness).

With H_2O as the coolant, the critical height was computed to be 2 feet 2 inches; with furfuryl alcohol, the result was 2 feet 7 inches. The PUREE' method thus predicts that the H_2O coolant gives a more reactive system than the furfuryl alcohol. Since the furfuryl alcohol simulates void, this indicates the configuration will be a negative void coefficient, which is in disagreement with the results of the "simplified physics model."

It was decided previously (based upon rough calculations) to put the flux-measuring foils at a distance of 1 foot from the bottom of the fuel rod. The above results substantiate the prediction that foils so placed will be well below the water level in all configurations.

The void coefficient of reactivity was also computed more directly by reducing the density of the H_2O coolant. For 25% voids in the coolant, the reactivity showed a decrease of 3.0%, or an average void coefficient of -0.12% per percent void over the void range of 0 to 25%. For a void increase to 50%, the reactivity showed an additional drop of 5.2%, indicating a void coefficient of -0.21% per percent void over the void range of 25 to 50%. For a void increase to 100%, the reactivity showed an additional drop of 19.2%, indicating a void coefficient of -0.38% per percent void over the void range of 50 to 100%.

A plot of these data on reactivity versus void percent shows that the void coefficient for void percents close to zero will be very much smaller than the values quoted above and

of uncertain sign, although probably negative. A calculation for a 5% void is being made to verify this conclusion.

The above calculations of void coefficient of reactivity were made under the assumption that the coolant and moderator are at the same height and that there is no rise in coolant height when coolant voids are introduced. The case in which the coolant rises in the shroud above the level of the moderator upon the introduction of homogeneous voids was also computed. The magnitude of the void coefficient is quite sharply reduced under this assumption. In the void range 0 to 25%, the coefficient of reactivity is -0.04% per percent void, and in the void range 25 to 50%, it is -0.09% per percent void.

The next computation was to determine the component of the temperature of reactivity associated with the upward energy shift of the thermal neutrons when the moderator and coolant temperature rises. This effect causes a decrease in the effective thermal absorption and fission cross sections. The value of the coefficient was found to be -0.81×10^{-5} per $^{\circ}\text{C}$ (-0.45×10^{-5} per $^{\circ}\text{F}$).

The last calculations were to determine various components of the temperature coefficient due to a rise in fuel temperature. The effect on reactivity of the Doppler broadening of the U 238 resonances was studied first. The value of this coefficient was found to be -3.1×10^{-5} per $^{\circ}\text{C}$ (-1.7×10^{-5} per $^{\circ}\text{F}$) in the fuel temperature range 293 to 500 $^{\circ}\text{K}$ and -1.9×10^{-5} per $^{\circ}\text{C}$ (-1.0×10^{-5} per $^{\circ}\text{F}$) in the range 500 to 1000 $^{\circ}\text{K}$.

The second of these calculations was concerned with the Doppler coefficient due to broadening of U 235 fission and capture resonances. The data on these resonances are somewhat meager. However, taking into account all resonances for which there are data, the coefficient was found to be -0.03×10^{-5} per $^{\circ}\text{C}$ (-0.02×10^{-5} per $^{\circ}\text{F}$). It thus appears that the Doppler effect of U 235 will be small compared to that of U 238, even if all the resonances are taken into account.

The third and last of these calculations was to find the effect on reactivity of the vertical expansion of the fuel rods due to a fuel-temperature increase. It is assumed that since the water level is only part way up the fuel, the vertical expansion of the fuel leads to a fuel density decrease in the active portion computed from the linear expansion coefficient. The coefficient for this was found to be 0.52×10^{-5} per $^{\circ}\text{C}$ (0.29×10^{-5} per $^{\circ}\text{F}$).

B. Equipment Manufacture

Figures 20 through 24 show various items of equipment being manufactured for the VMR critical experiment.

SHROUD CAN SECTIONS AND BOTTOM PLUGS
AT TOLERANCE CHECKING STATION

SHROUD CANS BEING SET UP ON
WELDING JIG FOR BOTTOM PLUGS

ASSEMBLED SHROUD AND COMPONENT PARTS

STAINLESS STEEL FUEL-ELEMENT SHROUDS
(Manufactured by Airline Welding and Engineering, Gardena, Calif.)

CONTROL ROD IN
WELDING JIG

WELDING THREE-BLADED
CONTROL ROD

SINGLE-BLADED AND THREE-BLADED CONTROL RODS

BORAL CONTROL RODS
(Manufactured by Advanced Technology Laboratories)

FIGURE 21

057 040

DRUM OF UO_2 POWDER

SINTERED UO_2 FUEL PELLETS

INSPECTION OF UO_2 FUEL PELLETS

URANIUM OXIDE FUEL PELLETS
(Manufactured by Mallinckrodt Nuclear Corp., St. Louis, Mo.)

FIGURE 22

FUEL-PIN GRID PLATE PARTS
AND DRILLING JIG

MILLING OPERATION, FUEL-PIN GRID PLATES

INSPECTION, FUEL-PIN GRID PLATE

FUEL-PIN GRID PLATES
(Manufactured by Advanced Technology Laboratories)

FIGURE 23

057 042

COOLANT DISTRIBUTION HEADER

ALUMINUM BASE SUPPORT PLATE FOR SUPPORTING CORE
(ONE OF THREE PLATES, EACH WITH DIFFERENT HOLE
SPACINGS)

HEXAGONAL SUPPORT FRAME PRIOR TO
ASSEMBLY AND FINAL MACHINING

CORE SUPPORT STRUCTURE
(Manufactured by Todd Shipyards, Alameda, Calif.)

FIGURE 24

V. MECHANICAL AND SYSTEM DESIGN

A few general computations concerned with the VMR mechanical and system design are in progress in the broad categories indicated below.

A. Thermal Stress Analysis of Core Support Structure

The effects of gamma heating in the core for steady-state conditions have been investigated. The maximum gamma heating rate was found to be approximately 0.5 watt/gm of structural material. This heating rate, coupled with the transfer of heat from the coolant to the moderator region, was found to produce very small tensile stresses (less than 2000 psi) in the shrouds during steady-state operation.

The differential expansion between the shrouds and supporting barrel (thermal shield), which occurs as a result of rapid pressure (hence temperature) change in the reactor, is currently under investigation.

B. Moderator Clean-up System

The maximum corrosion rate in the moderator system was found to be approximately 20 pounds per year of metal. To maintain less than 1 ppm of impurity in the moderator system, a clean-up stream of 2300 lb/hr of moderator water must be processed.

ATTACHMENT I

KINETICS MODEL FOR
BOILING WATER REACTORS

ATL-226 Preliminary

29 February 1960

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057 046

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INTRODUCTION

This report presents the derivation of a kinetics model of a nuclear reactor cooled by boiling water and a few preliminary results obtained by applying the model to a specific reactor, the VMR. These preliminary results, restricted to the case of a VMR with a positive void coefficient of reactivity, show the model to be stable over a wide range of system parameters. Additional analog work presently planned will include study of a reactor with a negative void coefficient of reactivity.

The model is based on small-signal analysis in order to avoid the formidable difficulties involved in representing the hydrodynamic behavior of the boiling coolant when subjected to very large disturbances. The system equations are thus linear with constant coefficients, with two exceptions: the nonlinear term of the neutron kinetics equations and the transport time delay associated with the flow of water from the top of the convection riser to the core inlet plenum are retained in the system description.

One of the limits to stable nonoscillatory operation of a boiling water reactor is a phenomenon termed "chugging." The model does not include provision for representing this chugging phenomenon, inasmuch as chugging is dependent, at least in part, on a hydrodynamic description involving the compressibility of the two-phase mixture in the fuel-element flow channels. The model does, however, permit representation of oscillatory variations of either constant, decreasing, or increasing amplitude in coolant density (and thus in pressure, heat generation ratio, etc.), and these phenomena have been found in analog studies of the model. Because chugging has been observed in heat transfer loops without coupling between heat generation and the observed chugging (as can be the case in a reactor), it is at least possible that some of the chugging phenomena observed in nuclear reactors may be dependent on purely hydrodynamic conditions. In this case, it may be noted that avoidance of chugging is a hydrodynamic problem and can be treated independently of the representation of the kinetic behavior of a reactor as hereindescribed. In any event, the presently available understanding of chugging phenomena does not permit complete description in the present model.

The kinetics model and the analog representation of the model are both made with reference to use in kinetics analyses of the VMR. The derivations of the relations, however, are quite general and are not restricted to boiling water or to any specific type of reactivity control.

The following sections of this report present a complete derivation of the kinetics model, which is summarized in Figure 9. The preliminary analog results are briefly presented in Section II of VMR report ATL-A-107^{*}. Parameters in the analog work are based on VMR Core Design No. 1.^{*}

* "Variable Moderator Reactor Development Program," Advanced Technology Laboratories, a Division of American-Standard, Quarterly Progress Report No. 3, ATL-A-107, February 29, 1960.

I. NEUTRON KINETICS EQUATIONS

The neutron kinetics equations relate the time variation of neutron density, n , to excess multiplication, Δk . The equations are:

$$\begin{aligned}\frac{dn}{dt} &= \frac{\Delta k}{\ell} n - \frac{\beta n}{\ell} + \sum_{i=1}^6 \lambda_i C_i \\ \frac{dC_i}{dt} &= a_i \frac{\beta n}{\ell} - \lambda_i C_i\end{aligned}\quad (1)$$

Here, ℓ = the average time between successive neutron generations,

β = the fraction of neutrons that are delayed,

λ_i = the decay constant of the i^{th} group,

C_i = the concentration of neutrons held in the i^{th} group of delayed neutrons,

a_i = the fraction of delayed neutrons falling in the i^{th} group.

The numerical constants a_i , λ_i , and β are given in Table I for uranium-fueled thermal reactors. (For kinetics purposes, the data in Table I can be applied to uranium-fueled reactors without regard to the neutron energy spectrum.)

TABLE I
Six-Group Delayed Neutron Constants
for Reactors Fueled with U 235*

$$\beta = 0.0064$$

Group	a_i	λ_i
1	0.033	0.0124
2	0.219	0.0305
3	0.196	0.111
4	0.395	0.301
5	0.115	1.13
6	0.042	3.00

* "Reactor Physics Constants," ANL 5800, 1958.

Since the neutron density, n , is directly proportional to the rate of heat generation, Q_g , in the fuel, equation 1 can be rearranged, replacing the group $\frac{\beta n}{l}$ by Q_g and $\frac{\Delta k}{\beta}$ by R , the excess multiplication measured in dollars. This procedure gives

$$\frac{l}{\beta} \frac{d}{dt} Q_g = Q_g (R - 1) + \sum_{i=1}^6 \lambda_i C'_i,$$

and

$$\frac{dC'_i}{dt} = a_i Q_g - \lambda_i C'_i. \quad (2)$$

For water-moderated reactors such as the EBWR and VMR, the average neutron lifetime is of the order of 6×10^{-5} second; the ratio l/β therefore has a value of the order of 10^{-2} .

The influence of the term $(l/\beta) dQ_g/dt$ on kinetic behavior will thus be negligible for all but the most rapid changes in heat generation and is omitted in the following work.

Because this report considers the behavior of the reactor when disturbed from some initial steady-state condition of operation, it proves convenient to use, as the dependent variable, the fractional change in heat generation rate rather than Q_g . Denoting the initial steady-state heat generation rate by Q_0 , we put

$$\Delta'Q_g \equiv \frac{Q_g - Q_0}{Q_0}$$

and obtain from equations 2 the neutron kinetics equations used in subsequent work.

$$\Delta'Q_g (1 - R) - R = \sum_{i=1}^n \lambda_i b_i, \quad (3)$$

and

$$b_i = a_i \Delta'Q_g - \lambda_i C'_i.$$

Here b_i is a variable proportional to the change in delayed neutron concentration in the i^{th} group and need not be further defined. For steady-state operation prior to a disturbance the initial conditions are

$$b_i = \Delta'Q_g = R = 0.$$

Representation of the neutron kinetics using six groups of delayed neutrons is unnecessarily complex for the objectives of the VMR kinetics analysis and produces a potential

source of time-consuming set-up errors for analog computation. For the objectives of this work, it is believed that the number of groups can be reduced to two with a loss in accuracy of system representation that is small compared to the loss due to approximations involved in representing other parameters of the system. Skinner and Hetrick* suggest that the constants $(\lambda'_1, \lambda'_2, a'_1, a'_2)$ for a two-group model be computed using the following relations:

$$\begin{aligned} \sum_1^2 a'_i / \lambda'_i &= \sum_1^6 a_i / \lambda_i = 13.04 \\ \sum_1^2 a'_i / \lambda'^2_i &= \sum_1^6 a_i / \lambda_i^2 = 470.4 \\ \sum_1^2 a'_i \lambda'_i &= \sum_1^6 a_i \lambda_i = 0.4037 \\ \sum_1^2 a'_i &= \sum_1^6 a_i = 1 \end{aligned} \quad (4)$$

The constants evaluated by these equations are given in Table II.

TABLE II
Two-Group Constants

Group	λ_i (sec ⁻¹)	a_i
1	0.5623	0.705
2	0.02506	0.295

* "The Transfer Function of a Water Boiler Reactor," Nuclear Science and Engineering, Vol. 3, pp 573-594, (1958).

II. MULTIPLICATION

For analysis of reactor kinetics over short time intervals (relative to times associated with poison changes and other longer term effects), the excess multiplication is considered to be a function of three variables: coolant density, fuel temperature, and moderator level. For the small-signal-analysis method used in this report, suitable approximations to the dependency of excess multiplication, R , on these three variables are given by the relations

$$\frac{\partial R}{\partial \rho} = \alpha_3, \quad (5)$$

$$\frac{\partial R}{\partial T_f} = \alpha_1, \quad (6)$$

$$\frac{\partial R}{\partial L} = \alpha_2. \quad (7)$$

The constants, α , are determined by reactor physics. The relationships between coolant density, ρ , fuel temperature, T_f , moderator level, L , and the other system variables on which they are dependent are described in sections III, V and IX.

III. COOLANT DENSITY

Changes in coolant density exert an influence on reactivity in accordance with the location of the density change: a given density change has more weight in a high-flux region of the reactor core than the same change in a low-flux region. For this reason, reactivity changes are properly related to a weighted density function rather than to a simple mean or volume average density. The proper weighting function for VMR conditions is not known at the present time, but both flux and flux square weighting have been used in previous investigations. The weighted mean density for a channel is given by the relation

$$\bar{\rho} = \frac{\int_0^L \omega dz}{\int_0^L dz} \quad (8)$$

in which $\omega(z)$ is the selected weighting function and L the active length of the channel. The weighted density is calculated for the core, using the same equation but integrating over the volume of the core with $\omega = \omega(r, z)$. Changes in the three density functions (average, flux, flux square weighted) are compared for a single fuel element in Table III.

The steady-state distribution of density in a core channel is expressible as a function of four variables: a) pressure, P , b) heat input distribution, $Q_c(Z)$, c) water inlet temperature, T_{in} , and d) mass flow rate, m . The time variation of density is related to the time variations of these four variables in paragraphs A through D below. The method used involves the idealization that the steady-state density changes that obtain after changes in the four variables are related to the variable changes by the equation

$$\Delta\rho = \frac{\partial\rho}{\partial P} \Delta P + \frac{\partial\rho}{\partial\Delta'Q} \Delta'Q + \frac{\partial\rho}{\partial T_{in}} \Delta T_{in} + \frac{\partial\rho}{\partial m} \Delta m, \quad (9)$$

where $\Delta\rho$ represents, as the case may be, a local density, an average density, or a weighted density function. The time variation of $\Delta\rho$ in response to changes in the four variables is postulated to be of the first order form

$$\Delta\rho(s) = \frac{\partial\rho}{\partial P} \frac{\Delta P(s)}{1 + \tau_0 s} + \frac{\partial\rho}{\partial\Delta'Q} \frac{\Delta'Q(s)}{1 + \tau_1 s} + \frac{\partial\rho}{\partial T_{in}} \frac{\Delta T_{in}(s)}{1 + \tau_2 s} + \frac{\partial\rho}{\partial m} \frac{\Delta m(s)}{1 + \tau_3 s} \quad (10)$$

Only density changes in the two-phase region of the core are considered, since these changes are very much larger than those occurring in the single-phase regions.

A. Pressure Dependence of Density

An increase in pressure causes vapor to condense and, by condensation of the vapor, causes the liquid-phase temperature (and the vapor temperature) to increase to the new higher saturation temperature. The increased saturation temperature also increases the non-boiling length and consequently the density in the region of the increment of the boiling length. The converse effects accompany a decrease in pressure.

For flow in a channel, the local liquid-vapor mixture density, ρ , is given by the relation

$$\rho = \frac{\sigma + \frac{\chi}{1-\chi}}{\sigma V_\ell + \frac{\chi}{1-\chi} V_v}, \quad (11)$$

in which σ is the slip ratio (the ratio of vapor-phase velocity to liquid-phase velocity), V_ℓ and V_v are the specific volumes of the saturated liquid and vapor phases, respectively, and χ , the quality, is the ratio of the local vapor mass rate to the total mass rate of both phases. Postulating that the slip ratio is independent of small changes in pressure, differentiation of equation 11 with respect to pressure gives

$$\frac{d\rho}{dP} = \frac{\rho}{\left(\sigma + \frac{\chi}{1-\chi}\right)} \left[\frac{1}{(1-\chi)^2} (1 - V_v \rho) \frac{d\chi}{dP} - \rho \left(\sigma \frac{dV_\ell}{dP} + \frac{\chi}{1-\chi} \frac{dV_v}{dP} \right) \right] \quad (12)$$

The pressure derivative of quality in this equation can be obtained from a heat balance on the mixture flowing through the channel.

$$\int_0^z \frac{q}{L} dz = m [h_{\text{sat}} - h_{\text{in}} + \chi \Delta h] \quad (13)$$

Maintaining constant the mass rate, m , the channel inlet enthalpy h_{in} , and the heat input distribution q/L , and differentiating with respect to pressure gives

$$\frac{d\chi}{dP} = -\frac{1}{\Delta h} \frac{dh_{\text{sat}}}{dP} - \frac{\chi}{\Delta h} \frac{d\Delta h}{dP} \quad (14)$$

The property derivatives in equations 12 and 14 are readily obtained from steam tables for any desired pressure. By specifying a value of quality (for given pressure and slip ratio), the mixture density ρ is obtained from equation 11, the derivative $\frac{dx}{dP}$ from equation 14, and the pressure derivative of mixture density from equation 12. In this manner, the relation between $\frac{\partial \rho}{\partial P}$ and ρ is determined. This relation is shown in Figure 1 for a system pressure of 1000 psia and for four values of slip ratio. The effect of pressure level on the density derivative is shown in Figure 2 for a slip ratio of unity. The curves in Figure 2 are closely fitted by the empirical relation

$$\frac{d\rho}{dP} = \frac{6.72 \rho^{1.8}}{P^{1.5}} \quad (15)$$

Inspection of Figures 1 and 2 (and equation 15) shows that the change in density accompanying a given change in pressure will be smaller as 1) the slip ratio is increased, 2) the operating pressure of the reactor is increased, and 3) as the mixture density is decreased. Consequently, reactors operating at high pressure and large exit void fraction should exhibit less dependence on pressure fluctuations than reactors operating at low pressure and with low exit void fraction.

Pressure-induced changes in the axial distribution of density for a coolant channel is shown for a particular case in Figure 3. It may be observed that, for this case, the change in the non-boiling length represents a negligible contribution to the change in average density caused by a change in pressure.

For a step change in reactor pressure, the change in density can be expected to be quite abrupt, inasmuch as pressure disturbances are propagated at high velocity. Thus, the time constant τ_0 (see equation 10) is assumed zero. This simplification is further justified for natural-circulation reactors with a relatively large mass of liquid in the convection riser, because pressure changes in such reactors appear to be sufficiently slow to insure that the density will follow the pressure with no appreciable error.

B. Heat Input Dependence of Density

The local change in density following a change in heat input to the coolant is directly obtainable from the heat balance equation (equation 13) upon postulating that the axial distribution of the change in heat input is identical to the initial heat input distribution. With this condition, one obtains

PRESSURE DERIVATIVE OF DENSITY FOR
WATER LIQUID-VAPOR MIXTURES AT 1000 PSIA

FIGURE 1

PRESSURE DERIVATIVE OF DENSITY FOR
WATER LIQUID-VAPOR MIXTURES AT A SLIP RATIO OF UNITY

FIGURE 2

EFFECT OF PRESSURE CHANGES ON
 AXIAL DISTRIBUTION OF COOLANT DENSITY

FIGURE 3

$$\frac{dQ}{Q_0} = \frac{dh(z)}{h(z) - h_{in}} \quad (16)$$

where dQ is the change in heat input for the entire channel, and Q_0 is the initial heat input rate. Restricting attention to the two-phase region of the channel, this equation can be written as

$$\frac{dQ}{Q_0} = \frac{\Delta h d\chi}{\Delta h \chi + c_p \Delta T_{sub}} \quad (17)$$

in which ΔT_{sub} is the inlet subcooling of the coolant and $d\chi$ the change in local quality. The change in liquid-vapor mixture density corresponding to a change in quality is, from equation 10:

$$\frac{d\rho}{d\chi} = \frac{(1 - V_v \rho)}{(\sigma V_l + \frac{\chi}{1 - \chi} V_v) (1 - \chi)^2} \quad (18)$$

Thus, from equations 17 and 18, the local change in density due to a fractional change, $\frac{dQ}{Q_0}$, in channel heat input rate is

$$\frac{d\rho}{dQ/Q_0} = \left(\chi + \frac{c_p \Delta T_{sub}}{\Delta h} \right) \left(\frac{1 - V_v \rho}{\sigma V_l + \frac{\chi}{1 - \chi} V_v} \right) \frac{1}{(1 - \chi)^2} \quad (19)$$

The change in the axial distribution of density due to an increase and decrease of 10% of the heat output per pin is shown in Figure 4 for a specific case. It will be observed that, for this case, neglect of the change in non-boiling length has only a small effect on the length-average density.

The time constant, τ_1 (equation 10) can be expected to be of the order of the time required for a parcel of coolant to flow through the channel. Because the mass rate per channel will depend on the radial position of the channel in the core and also on the orificing, if any, of the fuel elements, it is evident there is no uniquely definable simple time constant. However, a seemingly reasonable procedure is to evaluate τ_1 as the time required for a parcel to flow through the central channel of the core, inasmuch as channels in the central high-flux region of the core are the most influential in determining changes in reactivity. With this definition, τ_1 can be approximated by the relation

EFFECT OF HEAT OUTPUT CHANGES ON
AXIAL DISTRIBUTION OF COOLANT DENSITY

FIGURE 4

$$\tau_1 = \int_0^L \frac{\rho(z)}{G} dz, \quad (20)$$

in which ρ is the local coolant density and G the mass flux in the central channel. For the conditions shown in Figure 4, the time constant is 0.8 second. (The ratio $\frac{G}{\rho}$ is a fictitious local average velocity.)

C. Dependence of Density on Water Inlet Temperature

The steady-state change in local density corresponding to a change in channel inlet temperature is derived from equations 13 and 18. Differentiating equation 13 and holding mass rate, heat input, and pressure constant gives

$$\frac{d\chi}{dT_{in}} = \frac{c_p}{\Delta h} \quad (21)$$

Combining this relation with equation 18 gives the local steady-state density change caused by a change in channel inlet temperature:

$$\frac{d\rho}{dT_{in}} = \frac{c_p}{\Delta h} \frac{(1 - V_v \rho)}{(\sigma V_\ell + \frac{\chi}{1 - \chi} V_v)(1 - \chi)^2} \quad (22)$$

The axial density distribution change due to a $\pm 2^\circ\text{F}$ change in inlet temperature is shown for a specific case in Figure 5.

The time constant τ_2 (equation 10) can be expected to be of the same order of magnitude as τ_1 but probably somewhat larger because the disturbance due to a change in inlet temperature is initiated at the entrance to the channel, while the disturbance due to a change in heat input is distributed along the length of the channel.

D. Dependence of Density on Mass Flow Rate

Differentiating equation 13 with inlet temperature, heat input, and pressure held constant gives

$$\frac{d\chi}{dm/m_o} = \frac{h_{sat} - h_{in} + \chi\Delta h}{\Delta h} \quad (23)$$

Combining this relation with equation 18 gives the local change in density due to a fractional change, dm/m_o , in coolant mass rate:

EFFECT OF COOLANT ENTRY TEMPERATURE CHANGES ON
 AXIAL DISTRIBUTION OF COOLANT DENSITY

FIGURE 5

$$\frac{d\rho}{dm/m_o} = \left(\frac{c_p}{\Delta h} \Delta T_{sub} + \chi \right) \frac{(1 - V_v \rho)}{(\sigma V_l + \frac{\chi}{1 - \chi} V_v) (1 - \chi)^2} \quad (24)$$

The density distribution change due to a $\pm 10\%$ change in coolant mass rate is shown in Figure 6 for a specific case.

The time constant τ_3 (equation 10) can be expected to be of the order of τ_2 .

E. Comparison of Density Derivatives

Derivatives of the three density functions (length-average, flux, and flux square weighted) are presented in Table III for the specific case upon which Figures 3 to 6 are based. A sine heat output distribution was used in the calculations. Column 3 of Table III presents the length-average density derivative evaluated directly from the length-average densities computed by the BOCH code* and listed on Figures 3 to 6. The remaining columns show density derivatives computed using the local derivatives derived in paragraphs A through D above. For this purpose, equation 8 was used, with the proper local density derivative replacing the local density, ρ .

TABLE III
WEIGHTED DENSITY DERIVATIVES

(1)	(2)	(3)	(4)	(5)	(6)
		Length-Average		Weighting	
	Units	BOCH*	Hand	Flux	Flux Square
$\frac{\partial \rho}{\partial P}$	lb/ft ³ psi	+0.055	0.0515	0.0623	0.0686
$\frac{\partial \rho}{\partial T}$	lb/ft ³ /°F	-0.45	-0.402	-0.487	-0.536
$Q \frac{\partial \rho}{\partial Q}$	lb/ft ³	-10	-9.72	-10.57	-11.43
$M \frac{\partial \rho}{\partial M}$	lb/ft ³	+8.5	7.33	7.70	8.04

* An IBM 650 program made for use in calculating the steady-state hydrodynamic performance of boiling water reactors.

EFFECT OF COOLANT MASS RATE CHANGES ON
 AXIAL DISTRIBUTION OF COOLANT DENSITY

FIGURE 6

IV. HEAT INPUT TO COOLANT

Following a step change in heat generation rate within the fuel, the heat transfer rate to the coolant will follow from its initial value to the final value with an approximately first order time response characterized (with UO_2 fuel elements) by one of the longest time constants in the kinetics system. The evaluation of the transfer function relating heat transfer to the coolant in response to a change in heat generation rate is thus an item of considerable interest in the kinetics analysis.

The thermal conductivity of UO_2 changes appreciably with temperature. However, uranium oxide fuel pellets are known to fracture under exposure to reactor operating conditions, and the ability to predict the resultant effective thermal conductivity of the fractured pellet is, of course, low. It has thus become customary to assign, for an effective conductivity, a fictitious value that experiment and experience indicate to be consistent with specific observed behavior. As a consequence, the thermal conductivity of UO_2 fuel pellets is considered to be uniform and constant. The value of the effective thermal conductivity is frequently taken to be in the range from 1 to 1.3 Btu/hr-ft² (°F/ft).

In boiling heat transfer, the temperature of the interface between the boiling liquid and the heat transfer surface is effectively independent of heat flux. Thus, heat is transferred by conduction from the surface of the pellet to the effectively constant outside surface temperature of the clad. The equivalent unit conductance of this conduction path is given, with sufficient accuracy, by the equation

$$\frac{1}{h} = \frac{\Delta_g}{k_g} + \frac{\Delta_c}{k_c} \quad (25)$$

in which Δ_g and Δ_c are the thicknesses of the gas gap and the clad and k_g and k_c are the thermal conductivities of the gas and clad, respectively.

For a cylindrical fuel element losing heat through a constant unit conductance to a constant ambient temperature and having a fuel radius, r_o , uniform heat generation per unit axial length, and uniform and constant thermal properties, the heat transferred per unit length from the surface in response to a step change in heat generation is

$$\frac{Q_c}{Q_g} = 1 - \sum_{n=1}^{\infty} \frac{4B^2}{\lambda_n^2 (\lambda_n^2 + B^2)} \exp \left(- \frac{\lambda_n^2}{r_o^2} \text{at} \right) \quad (26)$$

The eigenvalues, λ , are roots of the eigenfunction

$$\lambda J_0'(\lambda) + B J_0(\lambda) = 0 \quad (27)$$

In these two equations, a = thermal diffusivity of the cylinder, t = time, Q_c = heat flow from the cylinder in response to the step change in heat generation, Q_g , and B is defined as the modulus $\frac{hr_o}{k}$. The general form of the series is

$$\frac{Q_c}{Q_g} = 1 - \gamma_1 e^{-t/\zeta_1} - \gamma_2 e^{-t/\zeta_2} - \gamma_3 e^{-t/\zeta_3} - \dots \quad (28)$$

The first several coefficients and the corresponding squares of the eigenvalues are presented in Table IV with the modulus B as argument.

For the case in which the heat generation rate varies with fuel radius as $I_0(p \frac{r}{r_o})$, the corresponding equation is

$$\frac{Q_c}{Q_g} = 1 - 2 p^2 B \left[1 + \frac{B}{p} \frac{I_0(p)}{I_0'(p)} \right] \sum_{n=1}^{\infty} \frac{\exp\left(-\frac{\lambda_n^2}{r_o^2} at\right)}{(B^2 + \lambda_n^2)(p^2 + \lambda_n^2)} \quad (29)$$

in which the symbols and eigenfunctions are as defined previously.

The transfer function relating heat flow from the cylinder (i. e., to the coolant) to the rate of heat generation is simply derived from equation 28 and has the form

$$\frac{Q_c(s)}{Q_g(s)} = \frac{\gamma_1}{1 + s \zeta_1} + \frac{\gamma_2}{1 + s \zeta_2} + \dots \quad (30)$$

in which

$$\zeta_n = \frac{r_o^2}{a \lambda_n^2} ,$$

and s is the La Place transform variable.

TABLE IV
EIGENVALUES AND COEFFICIENTS FOR STEP CHANGE IN
UNIFORM HEAT GENERATION IN CYLINDER

B	γ_1		γ_2		γ_3	
$\frac{hr_o}{k}$	λ_1^2	$\frac{4B^2}{\lambda_1^2 (B^2 + \lambda_1^2)}$	λ_2^2	$\frac{4B^2}{\lambda_2^2 (B^2 + \lambda_2^2)}$	λ_3^2	$\frac{4B^2}{\lambda_3^2 (B^2 + \lambda_3^2)}$
0.1	0.195	1.0				
0.2	0.381	0.999				
0.3	0.557	0.998				
0.4	0.725	0.997				
0.5	0.885	0.995				
0.6	1.037	0.994				
0.8	1.320	0.989				
1.0	1.577	0.984	16.64	0.0136		
1.5	2.12	0.970	17.56	0.0259		
2.0	2.56	0.954	18.41	0.0388		
3.0	3.20	0.922	19.92	0.0624		
5.0	3.96	0.872	22.2	0.0952	58.0	0.0207
6.0	4.20	0.853	23.1	0.1058	59.3	0.0254
8.0	4.53	0.824	24.4	0.1188	61.6	0.0331
10.0	4.75	0.804	25.3	0.1260	63.3	0.0387
15.0	5.06	0.772	26.8	0.1333	66.3	0.0466
20.0	5.23	0.754	27.6	0.1354	68.1	0.0502
∞	5.78	0.692	30.5	0.1313	74.9	0.0534

V. VOLUME MEAN TEMPERATURE OF FUEL

Broadening of the U 238 resonances with increasing UO_2 temperature produces a negative fuel-temperature coefficient of reactivity (α_1). The temperature change of the UO_2 in response to a change in heat generation is thus of interest.

The volume mean fuel temperature, $\bar{T}$, is defined as

$$\bar{T} = \frac{1}{\pi r_o^2} \int_0^{r_o} 2 \pi r T dr \quad (31)$$

The rate of energy storage per unit length of element is

$$(\rho c) \pi r_o^2 \frac{d\bar{T}}{dt}, \quad (32)$$

in which (ρc) is the heat capacity per unit volume of fuel. An energy balance requires that the rate of storage equal the rate of heat generation less the rate of heat transfer from the fuel.

$$\pi r_o^2 (\rho c) \frac{d\bar{T}}{dt} = Q_g - Q_c \quad (33)$$

The transfer function relating a change in fuel temperature to a change in heat generation is thus

$$\frac{\bar{T}(s)}{Q_g(s)} = \frac{1}{\pi r_o^2 \rho c} \frac{1}{s} \left(1 - \frac{Q_c(s)}{Q_g(s)} \right) \quad (34)$$

It will be noted from equation 33 that, following a step change in heat generation rate, the fuel temperature will continue to change indefinitely unless Q_c becomes equal to Q_g . Thus, if the series given by equation 28 is terminated with the n^{th} term, it is necessary that the coefficient γ_n be assigned the value such that $\sum_{i=1}^n \gamma_i = 1$. Normally, only the first term of the series will be required for kinetics analysis. In this case,

$$\gamma_1 = 1$$

VI. REACTOR PRESSURE CHANGES

Pressure changes are propagated at high velocity within the reactor. As a consequence, a pressure change can be considered to be uniformly and instantaneously distributed throughout the reactor.

Pressure changes within the reactor are considered to be expressible as a function of two excitations; a change in steam demand, D (lb/hr), or a change in steam generation rate, G . In the following, steam demand D is considered to be an independent variable, one that is governed by processes independent of those occurring within the reactor.

With steady-state operation, the two steam rates are equal and the steam generation rate, G , is expressible, by a heat balance across the core, as

$$G = \frac{Q}{\Delta h} - \frac{c_p}{\Delta h} M(T_{\text{sat}} - T_{\text{in}}) \quad , \quad (35)$$

In this equation, Q = total heat transferred to the coolant in the core, M = total coolant mass rate through the core, and T_{in} = temperature of the coolant entering the core. During a disturbance in the four variables, T_{in} , T_{sat} , M and Q , it will be postulated that the steam rate is given by substitution of the instantaneous values of these four variables in the above equations. This simplification is justified, inasmuch as the time constants associated with a change in G in response to changes in the four variables are small relative to the time responses associated with changes in the four variables.

We now inquire about the change in reactor pressure that accompanies a change in either G or D . The basic postulate for this purpose is that the vapor and liquid phases in the riser and the steam plenum above the riser remain in thermodynamic equilibrium during a pressure disturbance. (The coolant mass in the core is small relative to the coolant mass in the riser and steam plenum and will be ignored.) With this idealization, an increase in pressure will require that a part of the vapor phase condense in order to bring the liquid phase to the new higher saturation (equilibrium) temperature corresponding to the increased pressure. Although the mass of the liquid phase will thereby be increased, the increase is very small relative to the initial mass of liquid and will be ignored.

An increase in pressure increases the equilibrium temperature, T_{sat} , by an amount

$$\Delta T_{\text{sat}} = \frac{dT_{\text{sat}}}{dP} \Delta P \quad . \quad (36)$$

If the mass of the liquid phase is W_o , then the heat that must be transferred to the liquid phase to increase its temperature this amount is

$$W_o c_p \frac{dT_{sat}}{dP} \Delta P \quad (37)$$

This energy is gained by condensation of the vapor phase. The mass of vapor which must condense to furnish this energy is

$$\frac{W_o c_p}{\Delta h} \frac{dT_{sat}}{dP} \Delta P \quad (38)$$

so that the rate of condensation of steam, dM_s/dt , may be written as

$$\frac{dM_s}{dt} = \frac{W_o c_p}{\Delta h} \frac{dT_{sat}}{dP} \frac{dP}{dt} \quad (39)$$

Applying a mass balance to the flow of steam requires that the rate, G , at which steam is delivered from the core must equal the rate at which steam condenses plus the rate, D , at which steam leaves the core.

$$G - D = \frac{W_o c_p}{\Delta h} \frac{dT_{sat}}{dP} \frac{dP}{dt} \quad (40)$$

For not-too-large departures from a nominal operating condition, the factor multiplying dP/dt can be considered a constant. The properties group $\frac{c_p}{\Delta h} \frac{dT_{sat}}{dP}$ is shown in Figure 7 as a function of pressure.

PROPERTIES GROUP AS A FUNCTION OF
SATURATION PRESSURE

FIGURE 7

VII. CHANGE IN CORE INLET TEMPERATURE

A heat balance taken at the point of feedwater entry to the reactor vessel gives the relation

$$M T_D = (M - G) T_{\text{sat}} + F T_{\text{feed}} \quad (41)$$

in which M , G , and F are the coolant mass rate through the core, the steam generation rate, and the feedwater rate, respectively, and T_D is the temperature of the water entering the downcomer after mixing with the feedwater. (In steady-state reactor operation, the feedwater rate, F_o , and steam generation rate, G_o , are equal and $T_D = T_{\text{in}_o}$ in which T_{in} is the coolant temperature at entrance to the core.) Differentiation of equation 41 gives the change in T_D as a function of changes in five variables M , G , T_{sat} , F , T_{feed} .

$$\begin{aligned} dT_D = \frac{dM}{M_o} (T_{\text{sat}_o} - T_{\text{in}_o}) - T_{\text{sat}_o} \frac{dG}{M_o} + \frac{G_o}{M_o} dT_{\text{feed}} + T_{\text{feed}_o} \frac{dF}{M_o} \\ + \left(1 - \frac{G_o}{M_o}\right) dT_{\text{sat}} \end{aligned} \quad (42)$$

The change in inlet temperature, dT_{in} , after establishment of a new steady state following a change in any or all of the changes (differentials) indicated above will be equal to dT_D . However, the change in T_{in} will lag the change in T_D by the time, K , required for a parcel of water to flow from the feedwater inlet location (which is assumed to be at the top of the downcomer) to the core inlet. Because the velocity distribution in the downcomer is not uniform and also because of heat exchange with the downcomer walls, a step change in T_D will not produce a subsequent step change in T_{in} . Rather, the change in T_{in} will exhibit, very roughly, an exponential-type response somewhat as shown in Figure 8, in which τ_4 is the time constant of the exponential response.

In LaPlace transform notation, the relation between dT_D and dT_{in} is

$$\Delta T_{\text{in}}(s) = \frac{1}{1 + \tau_4 s} e^{-Ks} \Delta T_D(s) \quad (43)$$

For electronic analog representation of this relation, the transport time delay (K) is conveniently represented by using a tape recorder to read in a signal which is reproduced after a delay, in analog time, determined by tape speed and the tape length between recording and reading heads.

FIGURE 8

VIII. COOLANT MASS RATE VARIATIONS

The rate at which hydraulic power is delivered to the circulating coolant can be considered to be the product of the coolant mass rate, $\dot{m}$, and the difference between hydrostatic heads in the downcomer and the core plus riser.

$$\text{Hydraulic Power} = \frac{\dot{m}}{\rho_\ell} [\rho_\ell (h_c + h_r) - (\bar{\rho}_c h_c + \bar{\rho}_r h_r)] \quad , \quad (44)$$

in which h_c and h_r are heights of the core and riser and $\bar{\rho}$ is a height-average density. Expressed in different terms, this is the pump power that would be required in the absence of a gravitational field (supposing the flow phenomena to be otherwise unchanged). In steady-state operation, this power input must equal the rate of frictional dissipation in the flow circuit. In non-steady-state operation the excess of the hydraulic power input over the rate of frictional dissipation is equal to the rate of increase of kinetic energy of the flowing fluid (supposing potential energy changes to be negligible). Thus,

$$\frac{d(\text{KE})}{dt} = \frac{-\dot{m}}{\rho_\ell} (\Delta P_s + \Delta P_f) \quad , \quad (45)$$

in which ΔP_s is the change in hydrostatic pressure head in the core plus riser, and ΔP_f is the change in friction pressure loss, the changes being departures from an initial steady-state condition of operation.

As a first approximation to the kinetic energy of the fluid in the flow circuit, the velocity will be assumed uniform at any cross section of the flow. The kinetic energy of an element of fluid is then

$$d(\text{KE}) = \frac{u^2}{2} \rho A dl \quad , \quad (46)$$

in which u is the velocity, A the cross-sectional area normal to flow, and dl an incremental length of flow path. Substituting the continuity relation $\dot{m} = uA\rho$ and integrating around the flow path gives

$$\text{KE} = \frac{\dot{m}^2}{2} \int \frac{dl}{A\rho} \quad . \quad (47)$$

Differentiation of this equation gives, supposing that the value of the integral is substantially independent of time,

$$\frac{d(KE)}{dt} = m \frac{dm}{dt} \int \frac{dl}{A\rho} \quad (48)$$

Substituting this relation into equation 45 gives

$$I \frac{dm}{dt} = \frac{-1}{\rho_l} (\Delta P_s + \Delta P_f) \quad (49)$$

in which I is the integral in equation 48.

Since flow friction losses occur almost entirely in flow from the core inlet plenum to the bottom of the riser, the friction losses in the remainder of the flow circuit can be ignored. The static pressure difference ΔP_c across the core is the sum of the friction drop and the hydrostatic head in the core. (The difference between the dynamic heads in the plenum and the bottom of the riser is relatively very small and can be ignored.) The static pressure drop, ΔP_c , can be considered to be a function of mass rate and length-average density, $\bar{\rho}$, in the element. Thus, in general,

$$d\Delta P_c = \frac{\partial \Delta P_c}{\partial \bar{\rho}} \Delta \bar{\rho} + \frac{\partial \Delta P_c}{\partial \frac{m}{m_o}} \frac{\Delta m}{m_o} \quad (50)$$

For the particular case summarized in Figures 3 through 6, this relation becomes

$$d\Delta P_c = -5.88 \Delta \bar{\rho} + 325 \frac{\Delta m}{m_o} \quad (51)$$

Thus, noting that the hydrostatic head is the product of the length-average density and the core height, one obtains

$$d\Delta P_f = d\Delta P_c - h_c d\bar{\rho}_c \quad (52)$$

or

$$d\Delta P_f = -11.9 \Delta \bar{\rho}_c + 325 \frac{\Delta m}{m_o} \quad (53)$$

in which ΔP_f is the static pressure drop due to friction, and the height of the core is 6 feet.

The change in hydrostatic pressure in the core and riser can be approximated by assuming that the density in the riser is equal to the density at exit to the fuel element. For the case presented in Figures 3 through 6, the exit density is 0.7 of the length-average density.

Thus,

$$\Delta P_s = \bar{\rho}_c (h_c + 0.7 h_r) \quad , \quad (54)$$

which gives, with $h_r = 10$ ft and $h_c = 6$ ft,

$$d\Delta P_s = 13 \Delta \bar{\rho}_c \quad . \quad (55)$$

Substituting equations 55 and 53 into 45 gives

$$\rho_l I \frac{dm}{dt} = -1.1 \Delta \bar{\rho}_c - 325 \frac{\Delta m}{m_o} \quad . \quad (56)$$

For a heavily orificed element, essentially all of the friction loss will occur across the orifice. Assuming that the axial density distribution and static pressure drop for the orificed element are the same as for the unorificed element one obtains

$$\rho_l I \frac{dm}{dt} = -13 \Delta \rho - 385 \frac{\Delta m}{m} \quad . \quad (57)$$

If the reactor changes from one operating condition to another steady-state operating condition at which the change in $\Delta \rho$ is 1 lb/ft^3 , then equation 56 indicates that $\frac{\Delta m}{m} = \frac{1.1}{325}$. The change in density due to this fractional change in mass rate is, using the information in Table III,

$$\frac{-8.5 \times 1.1}{325} = 0.029 \text{ lb/ft}^3 \quad , \quad (58)$$

or less than 3% of the postulated density change. Consequently, the change in mass rate has, for this case, a negligible effect on kinetic behavior of the reactor. For the extreme case of the heavily orificed element, the corresponding density change due to the change in mass rate is -0.29 lb/ft^3 . However, heavy orificing, if used, would be used only for elements at the periphery of the core, a location having less weight in determining reactivity changes than the centrally located unorificed elements. On the basis of this reasoning, the effects of changes in mass rate on kinetic behavior are omitted in the flow diagram shown in Figure 9.

IX. MODERATOR-LEVEL CONTROL

Moderator-level control with a positive void coefficient can be effected on the basis of an error signal proportional to the difference between reactor pressure and a set pressure,

$$\frac{dL}{dt} = C_1 (P - P_o) \quad , \quad (59)$$

in which C_1 must be negative. Preliminary analog work with the positive void coefficient of reactivity has shown this control to be stable; following a change in steam demand, the pressure is returned to the set point without excessive oscillations.

X. SYSTEM DIAGRAM

A reactor kinetics system flow diagram that summarizes the relations derived in the preceding sections is presented in Figure 9. Variations in coolant mass rate are omitted for the reasons cited in section VIII. Variations in feedwater rate are not shown because changes in core inlet temperature produced by varying only the feedwater temperature are sufficient for initial stability studies of the reactor.

The kinetics model described by Figure 9 has been transferred to an electronic analog computer, and preliminary stability studies have been completed for the case of a positive void coefficient. Table V presents the definitions of the constants shown in Figure 9 and gives the reference values of the constants used in the preliminary stability study. It should be noted that the symbol Δ' indicates a fractional change in the quantity operated upon, e.g., $\Delta'G = \frac{G - G_o}{G_o}$. The analog computer diagram is shown in Figure 10.

TABLE V
CONSTANT DEFINITIONS FOR FIGURE 9

Constant	Definition	Units	Reference Value
α_1	$-\partial R / \partial T_f$	$\$/^\circ\text{F}$	0.00133
α_2	$\partial R / \partial L$	$\$/\text{ft}$	7.42
α_3	$\partial R / \partial \rho$	$\$/\text{lb}/\text{ft}^3$	-0.0553
α_6	$\partial \rho / \partial P$	$(\text{lb}/\text{ft}^3)/\text{psi}$	0.06
α_7	$\partial \rho / \partial \Delta'Q$	lb/ft^3	-10
α_8	$\partial \rho / \partial T_{in}$	$(\text{lb}/\text{ft}^3)/^\circ\text{F}$	-0.45
α_{10}	$(1 - \frac{G_o}{m_o} \frac{dT_{sat}}{dP})$	$^\circ\text{F}/\text{psi}$	0.113
α_{11}	G_o / m_o	--	0.07
α_{13}	$1 + \frac{m_o}{G_o} \frac{c_p}{\Delta h} (T_{sat} - T_{in,o})$	--	1.171
α_{15}	$\frac{c_p m_o}{\Delta h G_o}$	$1/^\circ\text{F}$	0.0284
α_{16}	$\frac{\Delta h G_o}{3600 c_p W_o (dT_{sat}/dP)}$	psi/sec	46
τ_1	(see eq. 20)	sec	0.4
τ_2	(see eq. 20)	sec	0.4
τ_4	(see eq. 43)	sec	3.0
τ_5	ξ_1 (eq. 30)	sec	9.5
C_3	$-(dL/dt) / (P - P_o)$	$\text{ft}/\text{sec}-\text{psi}$	1×10^{-6}
K	Dowhcomer Transport Delay	sec	8

REACTOR KINETICS SYSTEM FLOW DIAGRAM

FIGURE 9

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[illegible]

REACTOR KINETICS MODEL
WIRING DIAGRAM FOR
ELECTRONIC ASSOCIATES ANALOG COMPUTER
MODEL 221 K

FIGURE 10

ATTACHMENT II

BOCH
A PROGRAM FOR COMPUTING THE
HYDRODYNAMIC PERFORMANCE
OF BOILING WATER REACTORS

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INTRODUCTION

Predictions of the hydrodynamic characteristics of boiling water reactors are required, in order to determine the relative merits of natural and forced recirculation and to help establish optimum configurations for various power levels, operating pressures, and other design variables. Since the necessary calculational procedures are involved and tedious for hand computation, a program for such computations has been developed for use on a digital computer. This program has been coded and used with an IBM 650 to calculate all of the hydrodynamic information required for the VMR project.

The program is restricted in application to boiling water reactors operating at pressures from 250 to 2500 psia and has been written for natural circulation of the water. However, since forced circulation can be considered as a special case of natural circulation, no difficulty is involved in applying the program to forced-circulation reactors.

PROBLEM ANALYSIS

A. General Scheme of Steady-State Performance Calculation

The quantities presumed fixed for the reactor are: 1) geometry, 2) steam pressure, and 3) feedwater temperature.

The sequence of calculations required to determine recirculation rate, void distribution, and pressure differences is outlined below.

1. Calculate the steam rate, S (lb/hr).

$$S = \frac{Q_c}{\Delta h_{fg} + c_p (T_{sat} - T_{feed})}$$

where Δh_{fg} , c_p , and T_{sat} are functions of steam pressure.

2. Assume subcooling, ΔT_p , in plenum chamber below core.

3. Calculate total mass rate, M_c (lb/hr), of flow through reactor core, corresponding to the assumed ΔT_p .

$$M_c = \frac{S(T_{sat} - T_{feed})}{\Delta T_p - K_4(P)H}$$

where $K_4(P)$ is the rate of change of saturation temperature with height of water at the nominal saturation temperature, and H (ft) is the distance between the bottom of the core and the liquid-free surface above the core.

$$K_4(P) = 0.1244 - 0.1182 \left(\frac{P}{1000} \right) + 0.0323 \left(\frac{P}{1000} \right)^2$$

4. Assume the value of static pressure difference, ΔP_{ca} (lb/ft²), between the plenum and the bottom of riser. (The following calculation procedure is based on the postulate that this static pressure difference is the same for all fuel channels.)

5. Calculate the mass flow rate, m_i , through each fuel channel, which will give the assumed ΔP_{ca} . This calculation will necessarily be reiterative for each channel.

6. Sum, over all channels, the mass flow calculated for each channel and compare with the total mass flow rate calculated in step 3 above. If the sum agrees within 2%, proceed with step 7. If the sum is less than the value calculated in 3, then assume a larger ΔP_{ca} than assumed in 4 and repeat calculations 4 through 6. Similarly, if the sum is greater by more than 2% than the value calculated in 3, then assume a smaller ΔP_{ca} , etc. That is, if

$$\frac{\sum m_i N_i - M_c}{M_c} < 0.02, \text{ assume a larger } \Delta P_{ca} ;$$

$$\text{if } \frac{\sum m_i N_i - M_c}{M_c} > 0.02, \text{ assume a smaller } \Delta P_{ca}$$

N_i is the number of fuel elements in the i^{th} annular region of the core.

7. Calculate the net change in static pressure for the entire free-convection circuit.

If this sum of static pressure changes is less than 2% of ΔP_{ca} , then the conditions leading to natural convection have been correctly assumed. If the sum is positive, then the mass rate of the circulating water is probably, too high. In this case, the procedure is to return to step 2, assume a larger ΔT_p , and repeat all calculations.

In the case of a forced-convection reactor, the steps are identical. For forced convection, however, the pressure head supplied by the pump will be $\int \Delta P$ if $\int \Delta P > 0$, and the mass rate pumped will be $M_c - S$.

A flow diagram representing the sequence of steps outlined above is presented in Figure 1.

B. Calculation of Local Quality and Void Fraction

Let $p(Z)$ be a variable proportional to local heat input to the water coolant. Then the ratio between the local heat input per unit axial length of the fuel element and the length-average heat input is expressible as:

$$\frac{\frac{q}{L}(Z)}{(\frac{q}{L})_{\text{avg}}} = \frac{p(Z)}{\frac{1}{L} \int_0^L p(Z) dZ}$$

The non-boiling length, L_{nb} , of the element (i.e., the length of the element traversed by the water before the water is brought to the saturation temperature) is given by simple heat balance as

$$\frac{GC_p A \Delta T_{\text{sub}} L}{Q} = \frac{\int_0^{L_{nb}} p(Z) dZ}{\frac{1}{L} \int_0^L p(Z) dZ}$$

FIGURE 1

where G = mass rate per unit flow area in element (lb/hr-ft^2), A_1 = flow area in element (ft^2), L = total length of the element (ft), C_p = unit heat capacity of saturated liquid water at the nominal reactor pressure, Q = total heat output of the element (Btu/hr).

$$G = \frac{m}{A}$$

The local quality, $\chi(Z)$, of the liquid-vapor mixture is given, by a similar heat balance, as

$$\frac{\chi(Z) \Delta h_{fg} \text{ GAL}}{Q} = \frac{\int_0^Z p(Z) dZ}{\frac{1}{L} \int_0^L p(Z) dZ}$$

where Δh_{fg} (Btu/lb) is the heat vaporization of steam at the nominal reactor pressure.

The local void fraction $\alpha(Z)$, is calculated by the equation:

$$\alpha(Z) = \frac{\chi(Z)}{\chi(Z) + [1 - \chi(Z)] \psi}$$

where ψ^* is a function of G and the nominal reactor pressure.

$$\psi = K_o(P) \left(\frac{G}{10^6} \right)^{-0.15}$$

$$K_o(P) = 0.0044 + 0.0532 \left(\frac{P}{1000} \right) + 0.0173 \left(\frac{P}{1000} \right)^2$$

In order to program the IBM 650 electronic computer, the following approximations are made. The reactor core is divided into concentric annuli with the axis of the annuli parallel to the length of the fuel elements. Within each annulus, the fuel elements are identical and are subjected to identical conditions; heat input, mass rate of coolant, etc. The number of fuel elements in the i^{th} annular region is N_i .

The value $p(Z)$, a variable proportional to local heat input to the water coolant, is expressed by p_{ij} ; subscript i indicates the annular region, and j indicates the number of axial mesh points of the fuel element. Then

$$\int_0^L p(Z) dZ \cong \Delta Z \sum_{j=1}^J p_{ij} = \bar{p}_i$$

* See Appendix A for evaluation of ψ .

The total heat input, Q_i , to an element of the i^{th} annular region, is expressed by

$$\frac{Q_i}{Q_c} = \frac{\bar{p}_i}{\sum_{i=1}^I \bar{p}_i N_i}$$

The local quality of steam and void fraction can be assumed as zero until ΔT_{sf} , which is found by the equation below, becomes larger than ΔT_p .

ΔT_{sf} of the first mesh point, $j = 1$, is computed by

$$\Delta T_{sf} = \frac{p_{ij}}{\bar{p}_i} \Delta Z \frac{Q_i}{m_i C_p}$$

The successive mesh points are taken, and a new ΔT_{sf} is added to the previous ΔT_{sf} until ΔT_{sf} becomes larger than ΔT_p .

In order to find the non-boiling length, L_{nb_i} of the elements in the i^{th} region, the following interpolation is made.

$$Z - \Delta Z \frac{\Delta T_{sf} - \Delta T_p}{\Delta T_{sf} - \Delta T'_{sf}} = L_{nb_i}$$

where $\Delta T'_{sf}$ is the previous value of new ΔT_{sf} , ΔZ is the length of a unit mesh, and Z is the sum of ΔZ up to j where ΔT_{sf} becomes larger than ΔT_p .

After finding L_{nb_i} , the first local quality is computed from the equation

$$(\Delta T_{sf} - \Delta T_p) \frac{C_p}{\Delta h_{fg}} = x_{ij}$$

Although the first void fraction can also be computed at this stage, for the sake of simplification it is computed at the next mesh point, namely $j+1$.

The second local quality, x_{ij} , is computed by the equation

$$x_{i,j-1} + \Delta Z \frac{p_{i,j}}{\bar{p}_i} \frac{Q_i}{m_i \Delta h_{fg}} = x_{ij}$$

Then the first void fraction, α_{ij} , is computed by the equation

$$\frac{x'_{ij}}{x_{ij} + (1 - x_{ij}) \psi_i} = \alpha_{ij}$$

The above calculations are repeated until $j=J$, and all values are stored in the locations and printed out later.

The flow diagram of Figure 2 shows more precisely each step, and it is recommended that the reader follow the diagram concurrently with this text.

C. Static-Pressure Drop from Plenum to Bottom of Riser

The static-pressure difference between the plenum chamber below the core and the bottom of the riser is postulated to be the same for all fuel channels. This static-pressure difference is computed for any fuel channel as the sum of the five components summarized below. The subscript i is neglected, since most of the calculations are identical.

1. Static-Pressure Difference Due to Wall Friction

The static-pressure drop due to wall friction is calculated by equation

$$\Delta P_1 = \frac{dP}{dZ} \int_0^L R(P, \chi) dZ,$$

where $\frac{dP}{dZ}$ = single-phase pressure gradient that would exist with no boiling (lb/ft³),
 L = total length of the fuel element (ft), $R(p, \chi)$ = local two-phase friction multiplier, * and
 Z = distance from the bottom of the fuel element (ft).

$$\frac{dP}{dZ} = K_1(P) \left(\frac{G}{10^6} \right)^{1.8} \frac{1}{D_e^{1.2}}.$$

$$K_1(P) = 0.253 + 0.0230 \left(\frac{P}{1000} \right) + 0.00561 \left(\frac{P}{1000} \right)^2.$$

$$R(P, \chi) = 1 + F K_2(P) \chi^{0.75}.$$

$$K_2(P) = 139.932 - 262.89 \left(\frac{P}{1000} \right) + 202.86 \left(\frac{P}{1000} \right)^2 - 51.612 \left(\frac{P}{1000} \right)^3.$$

G = mass flow rate per unit flow area in the fuel element (lb/hr-ft²);

χ = local quality of steam-water mixture;

P = pressure of the steam in the reactor;

F is a constant and normally 1.0.

In order to program the IBM 650, the following approximations are made:

$$\Delta P_1 = \frac{dP}{dZ} \int_0^L R(P, \chi) dZ.$$

* See Appendix B for derivation of R and dP/dZ relations.

$$\cong K_1(P) \left(\frac{G}{10^6}\right)^{1.8} \frac{1}{D_e^{1.2}} \Delta Z \sum_{j=1}^J R_j,$$

where $R_j = 1 + F K_2(P) \chi_j^{0.75}$.

2. Static-Pressure Change Due to Acceleration

$$\Delta P_2 = \left(\frac{G}{10^5}\right) \frac{2400}{\rho_l} \left[\frac{\chi_J}{\alpha_J} \frac{\rho_l}{\rho_v} + \frac{(1 - \chi_J)^2}{(1 - \alpha_J)} - 1 \right],$$

where χ_J and α_J are the quality and void fraction at the top end of fuel element, and ρ_l and ρ_v (lb/ft³) are the densities of the liquid water and vapor at the saturation temperature.

3. Static-Pressure Difference Between the Top End of the Fuel Element and the Bottom of the Riser*

$$\Delta P_3 = \frac{K m_i}{g} \left\{ \frac{\chi_J}{A_2 \rho_v} \left[\frac{1}{\alpha_J^2 A_1^2} - \frac{1}{\alpha_J A_1} \right] + \frac{(1 - \chi_J)^2}{A_2 \rho_l} \left[\frac{1}{A_2 (1 - \alpha_J)^2} - \frac{1}{A_1 (1 - \alpha_J)} \right] \right\} \\ + \frac{1}{g} \frac{(K - 1) m^2}{\frac{\chi_J}{\rho_v} + \frac{1 - \chi_J}{\rho_l}} \left\{ \frac{\chi_J^3}{2 \rho_v} \left[\frac{1}{\alpha_J^2 A_1^2} - \frac{1}{\alpha_J A_1} \right] + \frac{(1 - \chi_J)^3}{2 \rho_l} \left[\frac{1}{(1 - \alpha_J)^2 A_1^2} - \frac{1}{(1 - \alpha_J) A_1} \right] \right\},$$

where χ_J = the quality at the top of the fuel element,

α_J and α_2 = void fractions at top of fuel element and at bottom of riser respectively,

A_1 = flow area in the element and A_2 = flow area of the riser associated with the element (if there are n fuel elements and the total riser area is A_r , then

$$A_2 = A_r/n).$$

The multiplier K is unity for sudden expansion. The relation between α_J and α_2 is given by the following equation:

$$\left(\frac{1}{\alpha_2} - 1\right) = \left(\frac{1}{\alpha_J} - 1\right) \left(\frac{A_2}{A_1}\right)^{0.2} \\ \alpha_2 = \frac{1}{\left(\frac{1}{\alpha_J} - 1\right) \left(\frac{A_2}{A_1}\right)^{0.2} + 1}$$

It should be noted that ΔP_3 is a negative number.

* See Appendix C for derivation.

4. Static-Pressure Difference Between the Plenum and the Bottom of the Fuel Element

$$\Delta P_4 = \left(\frac{G}{10^6} \right)^2 \frac{1200}{\rho_l} (1 + K_{5_i})$$

A nominal value for K_5 for an unorificed element is 0.5. For orificed fuel element channels, the value can be very much larger.

It should be noted that K_5 differs from channel to channel; subscript i is added especially for that purpose. The different K_{5_i} 's are read in as the part of input as shown in Figure 2.

5. The Hydrostatic Pressure Difference

The static pressure difference attributable to the weight density of the fluid contained between the bottom of the fuel element and the bottom of the riser is given by the equation:

$$\Delta P_5 = \int_0^{L_1} \bar{\rho} dZ + \bar{\rho}_{ex} L_2$$

where $\bar{\rho}$ = local weight density of the liquid-vapor mixture (lb/ft³),

$\bar{\rho}_{ex}$ = density at top of the element,

L_1 = total length of the fuel element (ft),

L_2 = distance from the top of the element to the fuel channel exit, i. e., bottom of riser (ft).

The local mixture density is given by

$$\bar{\rho}(Z) = \alpha(Z) \rho_v + [1 - \alpha(Z)] \rho_f$$

and $\bar{\rho}_{ex} = \bar{\rho}(L_1)$

For programming purposes, the above equation is written

$$\Delta P_5 = \Delta Z \sum_{j=1}^J \bar{\rho}_{ij} + \bar{\rho}_{ij} L_2$$

D. Static-Pressure Difference In Remainder of Circuit

Static-pressure differences through the remainder of the flow circuit, that is, the riser and the downcomer with attendant turning losses, depend on the total fluid flow rate through all coolant channels of the core. The individual contributions to the pressure differences in the circuit are considered separately.

1. Static-Pressure Difference Due to Riser Wall Friction

As in the case of the core coolant channel, the pressure difference is determined by multiplying the pressure drop that would develop if all the fluid were liquid by two-phase friction multiplier, R . Slip velocity ratio and steam quality are assumed to be constant throughout the volume of the riser; consequently, R is constant.

$$\Delta P_r = f R_r \frac{L_r}{D_r} \frac{(\sum m_i N_i)^2}{2 g \rho_l A_r^2}$$

$$f = \frac{0.316}{R_e^{0.2}} = 0.316 \left(\frac{\mu A_r}{D_r \sum m_i N_i} \right)^{0.2}$$

$$R_r = 1 + F K_2(P) \chi_r^{0.75}$$

L_r = length of riser,

D_r = equivalent diameter of riser,

$\sum m_i N_i$ = total mass flow rate of coolant,

A_r = cross-sectional area of riser.

R_r is a function of steam quality. The steam quality in the riser, χ_r , is given by

$$\chi_r = \frac{\sum m_i \chi_{ij} N_i}{\sum m_i N_i}$$

The summations are over all coolant channels in the core. For the sake of convenience, ΔP_r is rewritten in the following:

$$\begin{aligned} \Delta P_r &= f R_r \frac{L_r}{D_r} \frac{(\sum m_i N_i)^2}{2 g_c \rho_l A_r^2} = \frac{0.3 \mu^{0.2}}{G_r^{0.2} D_r^{0.2}} \frac{L_r}{D_r} \frac{R_r G_r^2}{2 g_c \rho_l} \\ &= K_1(P) \frac{L_r}{D_r^{1.2}} G_r^{1.8} R_r \end{aligned}$$

where $R_r = 1 + FK_2(P)\chi_r^{0.75}$;

$$K_1(P) = \frac{0.3 \mu^{0.2}}{2g_c \rho_l} \quad [K_1(P) \text{ was expressed by the polynomial form as described before.}]$$

2. Static-Pressure Difference Due to Turning

A pressure loss is occasioned by the excess turbulence created when the liquid steam is turned from an upward velocity in the riser to a downward velocity in the downcomer. This static pressure difference is assumed to be equal to the sum of the kinetic energy of the liquid portion of the fluid in riser plus the kinetic energy of the coolant at the downcomer entrance.

$$\Delta P_{T(\text{upper})} = \frac{\rho_l V_{lr}^2}{2g_c} + \frac{\rho_l V_{de}^2}{2g_c}$$

$$V_{lr} = \frac{(1 - \chi_r) \sum m_i N_i}{\rho_l (1 - \alpha_r) A_r}$$

$$V_{de} = \frac{\sum m_i N_i}{\rho_l A_d}$$

Substituting:

$$\Delta P_{T(\text{upper})} = \frac{\sum m_i N_i}{2g_c \rho_l} \left[\frac{(1 - \chi_r)^2}{(1 - \alpha_r)^2} \frac{1}{A_r^2} + \frac{1}{A_d^2} \right]$$

where $2g_c = 64.4 \times 3600^2 = 4.17 \times 10^8 \text{ lb-ft/hr}^2\text{-lb.}$

3. Static-Pressure Difference Due to Downcomer Wall Friction

The fluid flowing through the downcomer is liquid. The feedwater return is assumed to be at the top of the downcomer; consequently, the mass flow rate through the downcomer is the total mass flow rate of coolant, $m_i N_i$.

$$\Delta P_d = f \frac{L_d}{D_d} \frac{\rho_l V_d^2}{2g}$$

$$\begin{aligned}
&= \frac{0.316}{\text{Re}_d^{0.2}} \left(\frac{L_d}{D_d} \right) \frac{(\sum m_i N_i)^2}{2g\rho_\ell A_d^2} \\
&= \frac{0.3 \mu^{0.2}}{G_d^{0.2} D_d^{0.2}} \left(\frac{L_d}{D_d} \right) \frac{(\sum m_i N_i)^2}{2g\rho_\ell A_d^2} \\
&\Rightarrow K_1(P) \frac{L_d}{D_d^{1.2}} \left(\frac{\sum m_i N_i}{10^6 A_d} \right)^2
\end{aligned}$$

where $K_1(P) = \frac{0.3 \mu^{0.2}}{2g\rho_\ell}$ and expressible by the polynomial equation mentioned before;

$$G_d = \frac{m_i N_i}{A_d}$$

4. Lower Turning Losses

As at the upper end of the downcomer, the static-pressure difference due to turning at the lower end is assumed to be one downcomer velocity head plus one lower plenum velocity head, which is assumed to be negligible.

$$\Delta P_{T \text{ (lower)}} = \frac{(\sum m_i N_i)^2}{2g_c \rho_\ell A_d^2}$$

5. Hydrostatic-Pressure Difference Through Riser

The static pressure difference between the core and the top of the riser is given by:

$$\Delta P_{hr} = \bar{\rho} L_r$$

where $\bar{\rho} = (1 - \alpha_r) \rho_\ell + \alpha_r \rho_v$, the average fluid density in the riser.

Substituting:

$$\Delta P_{hr} = [(1 - \alpha_r) \rho_\ell + \alpha_r \rho_v] L_r$$

6. Hydrostatic-Pressure Difference Through Downcomer

The hydrostatic pressure difference between the top and bottom of the downcomer is given by:

$$\Delta P_{hd} = -\rho_d (L_r + L_c) \cdot \quad (L_c = L_1 + L_2)$$

Here, ρ_d is the density of the liquid in the downcomer, which is subcooled to temperature T_p . T_p is determined from

$$T_p = \chi_r T_{feed} + (1 - \chi_r) T_{sat}$$

since

$$\rho_d \cong \rho_l$$

$$\Delta P_{hd} \cong -\rho_l (L_r + L_c) = -\rho_l L_d$$

7. Static-Pressure Change Due to Changes in Kinetic Energy

The change in static pressure resulting from changes in the cross-sectional flow area at a given mass flow rate are already accounted for as part of the assumed turning losses.

E. The Net Static-Pressure Difference

The net static-pressure difference between the core entrance above the lower plenum and the core exit at the lower end of the riser is obtained by summing, with due regard for algebraic signs, the individual pressure-difference contributions.

$$\Delta P_{remain} = -\Delta P_{hd} + \Delta P_{hr} + \Delta P_r + \Delta P_{t(upper)} + \Delta P_d + P_{t(lower)}$$

In order to achieve the natural circulation, ΔP_{remain} is also the net static-pressure difference across the core that is derived and discussed in section C.

F. "Convergent" Equation

The static-pressure difference across the core is assumed to be ΔP_{ca} . The mass flow through each element must be selected by a reiterative procedure to give the assumed ΔP_{ca} .

1. For the first trial, the mass flow is selected by the equation

$$1_{m_i} = \frac{Q_i}{X_e \Delta h_{fg} + c_p \Delta T_p}$$

where Q_i is the total rate of heat generation by the i^{th} element, and X_e is a selected value, i. e., one the operator can preselect. In the absence of preselection of X_e , the value can be assigned: $X_e = 0.025$.

2. The pressure $1_{\Delta P_c}$ corresponding to 1_{m_i} is now calculated and compared to ΔP_{ca} :

$$\frac{{}^1\Delta P_c - \Delta P_{ca}}{\Delta P_{ca}} : \epsilon_1$$

Say that $\epsilon_1 = 0.02$.

If $\left| \frac{{}^1\Delta P_c - \Delta P_{ca}}{\Delta P_{ca}} \right| < \epsilon_1$, then the correct 1m_i has been selected and the machine can proceed to $i + 1$. If the inequality is not satisfied, then a new mass flow 2m_i is selected. The new 2m_i is selected by the following procedure.

$${}^2m_i = {}^1m_i + \Delta m_i,$$

where Δm_i is derived from the following treatment.

ΔP_c can be considered as the sum of hydrostatic head ΔP_5 , which does not vary rapidly with m_i , a friction head which varies approximately as m_i^2 .

$$\Delta P_c = \Delta P_5 + Km_i^2,$$

so that $d(\Delta P_c) = 2Km_i dm_i$,

and $\Delta(\Delta P_c) = 2Km_i \Delta m_i$;

but $K = \frac{{}^1\Delta P_c - \Delta P_5}{m_i^2}$,

so $\Delta(\Delta P_c) = \frac{2}{m_i} \Delta m_i ({}^1\Delta P_c - \Delta P_5)$;

but $\Delta(\Delta P_c) = \Delta P_{ca} - {}^1\Delta P_c$,

so ${}^2m_i = {}^1m_i - \frac{{}^1m_i}{2} \left(\frac{\Delta P_c - \Delta P_{ca}}{\Delta P_c - \Delta P_5} \right)$.

3. Summing the m_i 's that give the assumed ΔP_{ca} gives the total mass rate through the core:

$$\sum {}^1m_i N_i$$

This mass rate is then compared to the mass rate, $M_c m$, calculated on the basis of the assumed ΔT_p :

$$\frac{\sum m_i N_i - M_c}{M_c} : \epsilon_2 \quad \text{Say that } \epsilon_2 = 0.02.$$

If $\left| \frac{m_i N_i - M_c}{M_c} \right| < \epsilon_2$, then the correct ΔP_{ca} has been assumed corresponding to the value of ΔT_p , and the machine can proceed to calculate ΔP 's in the remainder of the circuit.

If the inequality is not satisfied, then a new static pressure drop ΔP_{ca} across the core must be assumed. The new value is selected as follows:

$$\Delta P_{ca} = K m_i^2 + \Delta P_5,$$

and $\Delta(\Delta P_{ca}) = Z K m_i \Delta m_i ;$

but $K = \frac{\Delta P_{ca} - \Delta P_5}{m_i^2},$

so that $\Delta(\Delta P_{ca}) = \frac{Z(\Delta P_{ca} - \Delta P_5) \Delta m_i}{m_i}$

Finally ${}^{n+1}\Delta P_{ca} = {}^n\Delta P_{ca} - Z({}^n\Delta P_{ca} - \Delta P_5) \frac{(\sum {}^n m_i N_i - M_c)}{M_c}$

The above equation shows that if $\sum m_i N_i$ is too large, then smaller ΔP_{ca} is required.

4. With ΔP_{ca} and m_i values consistent with the assumed ΔT_p , the pressure drops in the remainder of the circuit are calculated. The requirement is that $\int \Delta P = 0$, where the sum is taken around the entire loop.

If $\left| \int \Delta P \right| < \epsilon_3 \quad (\text{say that } \epsilon = 20 \text{ psf}),$

then a solution of the entire problem has been achieved. If the inequality is not satisfied, then a new value of ΔT_p must be assumed and steps 1, 2, and 3 repeated. The new value of ΔT_p can be selected as follows:

If $\int \Delta P > \epsilon_3$, then increase ΔT_p by 20%.

If $\int \Delta P < -\epsilon_3$, then decrease ΔT_p by 20%.

If, following the appropriate variation in ΔT_p , the new ${}^2\Delta T_p$ does not lead to $|\int \Delta P| < \epsilon$, then a linear approximation can be used on succeeding values of ΔT_p . For the third try:

$${}^3\Delta T_p = {}^2\Delta T_p - \frac{(\int \Delta P)_2}{b},$$

where
$$b = \frac{(\int \Delta P)_1 - (\int \Delta P)_2}{{}^1\Delta T_p - {}^2\Delta T_p}.$$

For any successive tries:

$${}^n\Delta T_p = {}^{n-1}\Delta T_p - \frac{(\int \Delta P)_{n-1}}{b};$$

$$b = \frac{(\int \Delta P)_{n-2} - (\int \Delta P)_{n-1}}{{}^{n-2}\Delta T_p - {}^{n-1}\Delta T_p}.$$

If ΔT_p has been changed three times by the above formula, namely three complete iterations, the computations are stopped and all input and output results should be checked. The program involves the above stop, and machine time is therefore not wasted by poor first-guess values.

BOCH FLOW CHART
(Page 1)

FIGURE 2

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BOCH FLOW CHART
(Page 2)

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BOCH FLOW CHART
(Page 3)

APPENDIX A

TWO-PHASE FRICTION MULTIPLIER

The two-phase friction multiplier, R , is defined as the ratio of the local pressure gradient with two-phase flow to the pressure gradient that would obtain if the two-phase mixture were entirely liquid. Expressions for predicting the multiplier have been presented by Lottes, et al,¹ Martinelli and Nelson,² Sher, et al,³ and others. Comparisons by Lottes of predicted and measured two-phase pressure drops indicate that a method suggested by Sher gives reasonable agreement over a wide range of pressure level, mass flux, and quality. This method is recommended for any subsequent revisions of the BOCH program. The program as presently written incorporates the friction factor correlations presented by Martinelli. The relations required for programming both Sher and Martinelli two-phase friction multipliers are presented in this appendix.

Sher has analyzed a large body of two-phase pressure drop data taken at pressures of 1100 and 2000 psia at the Bettis laboratory. For a 2000-psia pressure level, Sher gives a table for R as a function of mass flux and quality.

For pressures other than 2000 psia, Sher recommends using the Martinelli-Nelson correlation to establish the variation of R with pressure and his own to establish the variation with mass flux, G , and quality χ . In brief, the recommended multiplier is determined by the equation

$$R(G, P, \chi) = \frac{R_m(P, \chi)}{R_m(2000, \chi)} R_s(G, 2000, \chi)$$

In this equation, R_m and R_s are, respectively, the Martinelli-Nelson and the Sher multipliers.

Both Martinelli and Sher present their multipliers in graphical or tabular form. In order to express the multipliers in equation form, it was found useful to express the multiplier in the form $R = 1 + I$.

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1. P. A. Lottes, et al, "Experimental Studies of Natural Circulation Boiling and Their Application to Boiling Reactor Performance," Argonne National Laboratory.
 2. R. C. Martinelli and D. B. Nelson, "Prediction of Pressure Drop During Forced Circulation Boiling of Water," Trans. of ASME, 70:695-702 (1948).
 3. N. C. Sher, "Estimation of Boiling and Non-Boiling Pressure Drop in Rectangular Channels at 2000 Psia," WAPD-TH-300.

Figure A-1 shows the I function for the tabulated data of Sher. The points represent the tabulated data, the lines the equation

$$I = 10.7 \frac{\chi^{0.9}}{\left(\frac{G}{10^6}\right)^{1.1}}$$

Within the limit $\chi < 0.1$, the multiplier R_s is represented by

$$1 + 10.7 \frac{\chi^{0.9}}{\left(\frac{G}{10^6}\right)^{1.1}}$$

within 5% and within 15% for $\chi < 0.20$. Except for very high pressure reactors, it can be expected that the maximum quality will be less than 0.10.

The I function for the Martinelli-Nelson friction multiplier is shown in Figure A-2. (The points are calculated from Figure A-3, which is a reproduction of Figure 2 of their paper.) The lines represent the equation

$$I = 1 + K(P) \chi^{0.75} \quad (\chi < 0.10),$$

$$\text{with } K(P) = 139.932 - 262.89 \left(\frac{P}{1000}\right) + 202.86 \left(\frac{P}{1000}\right)^2 - 51.612 \left(\frac{P}{1000}\right)^3$$

(250 < P < 2000 psia)

The multiplier R_m is represented by $1 + I$ within 5% over the range indicated.

According to Sher's suggestion previously noted, the Sher-Martinelli friction multiplier is given in terms of the foregoing equations as

$$R(G, P, \chi) = \frac{[1 + K(P) \chi^{0.75}]}{[1 + 12.696 \chi^{0.75}]} \left[1 + 10.7 \frac{\chi^{0.9}}{\left(\frac{G}{10^6}\right)^{1.1}} \right]$$

The single-phase (liquid) pressure gradient is calculated by use of the equation

$$\frac{dP}{dZ} = \frac{f}{D_e} \left(\frac{G}{10^6}\right)^2 \frac{10^{12}}{2g\rho_f}$$

Representing the friction factor, f , by

$$f = \frac{0.23}{R_e^{0.2}}$$

gives $\frac{dP}{dZ} = K_1(P) \left(\frac{G}{10^6} \right)^{1.8} \frac{1}{D_e^{1.2}}$,

with $K_1(P) = 0.253 + 0.023 \left(\frac{P}{1000} \right) + 0.00561 \left(\frac{P}{1000} \right)^2$

and $250 \leq P \leq 2000$.

REPRESENTATION OF SHER'S
TWO-PHASE FRICTION MULTIPLIER FOR WATER AT 2000 PSIA

FIGURE A-1

TWO-PHASE FRICTION MULTIPLIER FOR WATER AT VARIOUS PRESSURES,
 ACCORDING TO MARTINELLI-NELSON

FIGURE A-2

MARTINELLI-NELSON CORRELATION OF TWO-PHASE FRICTION DROP

FIGURE A-3

APPENDIX B

STATIC-PRESSURE DIFFERENCE AT FUEL-ELEMENT EXIT

The static-pressure difference caused by expansion of the liquid-vapor mixture at the fuel-element exit is derived from the energy and momentum equations. The energy equation is

$$m_g \left(\frac{P_1}{\rho_g} - \frac{P_2}{\rho_g} + \frac{V_{1g}^2}{2} - \frac{V_{2g}^2}{2} \right) + m_f \left(\frac{P_1}{\rho_f} - \frac{P_2}{\rho_f} + \frac{V_{1f}^2}{2} - \frac{V_{2f}^2}{2} \right) = m_o E.$$

Using the following definitions:

$$m_g = m_o \chi,$$

$$m_f = m_o (1 - \chi),$$

$$V_g = \frac{\chi m_o}{\alpha A \rho_g},$$

$$V_f = \frac{1 - \chi}{1 - \alpha} \frac{m_o}{A \rho_f},$$

one obtains

$$E = (P_1 - P_2) \left(\frac{\chi}{\rho_g} + \frac{1 - \chi}{\rho_f} \right) + m_o^2 \left\{ \frac{\chi^3}{2\rho_g^2} \left[\frac{1}{\alpha_1^2 A_1^2} - \frac{1}{\alpha_2^2 A_2^2} \right] + \frac{(1 - \chi)^3}{2\rho_f^2} \left[\frac{1}{(1 - \alpha_1)^2 A_1^2} - \frac{1}{(1 - \alpha_2)^2 A_2^2} \right] \right\}$$

In this equation, E is the energy loss per pound of liquid-vapor mixture, m_o is the mass flow rate of the mixture, χ is the quality at fuel-element exit, and α_1 and α_2 are the void fractions before and after expansion.

For an abrupt expansion, the static-pressure difference ($P_1 - P_2$) is obtained from the momentum equation

$$m_g V_{1g} + m_f V_{1f} + P_1 A_2 = P_2 A_2 + m_g V_{2g} + m_f V_{2f},$$

which gives

$$P_1 - P_2 = \frac{m_o^2}{A_2} \left\{ \frac{\chi^2}{\rho_g} \left[\frac{1}{\alpha_2^2 A_2} - \frac{1}{\alpha_1^2 A_1} \right] + \frac{(1 - \chi)^2}{\rho_f} \left[\frac{1}{A_2(1 - \alpha_2)} - \frac{1}{A_1(1 - \alpha_1)} \right] \right\}$$

Combining the equations gives finally

$$P_1 - P_2 = K m_o^2 \left\{ \frac{\chi^2}{A_2^2 \rho_g} \left[\frac{1}{\alpha_2^2 A_2^2} - \frac{1}{\alpha_1^2 A_1^2} \right] + \frac{(1-\chi)^2}{A_2^2 \rho_f} \left[\frac{1}{A_2^2 (1-\alpha_2)^2} - \frac{1}{A_1^2 (1-\alpha_1)^2} \right] \right\} \\ - \frac{(1-K) m_o^2}{\left(\frac{\chi}{\rho_g} + \frac{1-\chi}{\rho_f} \right)} \left\{ \frac{\chi^3}{2 \rho_g^2} \left[\frac{1}{\alpha_1^2 A_1^2} - \frac{1}{\alpha_2^2 A_2^2} \right] + \frac{(1-\chi)^3}{2 \rho_f^2} \left[\frac{1}{(1-\alpha_1)^2 A_1^2} - \frac{1}{(1-\alpha_2)^2 A_2^2} \right] \right\}$$

In this equation, K is to be taken as unity for abrupt expansion and a different value for a gradual expansion. Experimental data for evaluating K in two-phase flow for non-abrupt expansions is not available. In the absence of such data, the value of K for single-phase flow is suggested. A graph of K for single-phase flow versus the angle of the expansion core is given in "Standards of the Hydraulic Institute", 8 Edition, p 24 (Pipe Friction Section).

APPENDIX C

DERIVATION OF THE VOID FRACTION-QUALITY RELATION

The void fraction, α , and quality are related by the equation

$$\alpha = \frac{\chi}{\chi + (1 - \psi)}$$

with, by definition,

$$\psi = \frac{V_g}{V_f} \frac{\rho_g}{\rho_f}$$

The ratio V_g/V_f is the slip ratio. Experimental data show that the slip ratio is a function of the mass flux (G), quality (χ), and pressure level. However, insufficient data are available to establish the relation of the four variables and consequently, following the recommendation of Lottes, et al,¹ the slip ratio is related only to the pressure level and mass flux by the curves in Figure 5 of Lottes' paper. Figure C-1 shows the dependence of ψ on pressure and mass flux. The points represent data taken from Figure 5 of Lottes' paper. The lines represent the equation

$$\psi = K(P) \left(\frac{G}{10^6} \right)^{0.15}$$

$$\text{with } K(P) = 0.0044 + 0.0532 \left(\frac{P}{1000} \right) + 0.0173 \left(\frac{P}{1000} \right)^2$$

The accuracy of representation afforded by the above equation is consistent with the accuracy with which slip ratio is known at the present time.

-
1. P. A. Lottes, et al, "Experimental Studies of Natural Circulation Boiling and Their Application to Boiling Reactor Performance," Argonne National Laboratory.

SLIP RATIO FUNCTION, ψ , FOR DIFFERENT PRESSURES

FIGURE C-1

NOMENCLATURE

A_1	Flow area in element	ft^2
A_2	Flow area in riser associated with a single fuel element	ft^2
A_d	Flow area of downcomer	ft^2
A_r	Flow area of riser	ft^2
A_x	Flow area in a single fuel element	ft
C_p	Heat capacity at constant pressure	$\text{Btu/lb}^\circ\text{F}$
D_e	Equivalent diameter of fuel element	ft
D_d	Equivalent diameter of downcomer	ft
D_r	Equivalent diameter of riser	ft
E	Energy loss due to expansion	energy/mass
f	Fluid flow friction factor	
g	Acceleration of gravity	ft/sec^2
G	Coolant mass flow rate per unit flow area	lb/hr-ft^2
G_r	Mass flow rate per unit flow area in the riser	lb/hr
H	Distance between bottom of core and the liquid free surface	ft
$I \equiv R-1$	Definition	
K_1	An empirical function of nominal pressure (defined in text)	
K_2	An empirical function of nominal pressure (defined in text)	
K_4	Rate of change of saturation temperature with height of water at nominal saturation temperature	$^\circ\text{F/ft}$
K_5	Fuel element entrance loss coefficient	
K	Loss coefficient for two-phase flow through a channel of non-uniform area	
K_o	An empirical function of nominal pressure (defined in text)	
$K(P)$	An empirical function of nominal pressure (defined in text)	
L_1	Total length of fuel element	ft
L_2	Distance from top of the fuel element to channel exit	ft

NOMENCLATURE

(Continued)

L_c	Height of core	ft
L_d	Height of downcomer	ft
L_{nb}	Non-boiling length of the heated fuel element	
L_r	Height of riser	ft
M_c	Total mass rate of flow through reactor core	lb/hr
m_f	Mass flow rate of liquid	
m_g	Mass flow rate of steam	
m_i	Mass flow rate of coolant through a fuel-element coolant channel	lb/hr
n	Number of fuel elements in reactor core	
N_i	Number of fuel elements in i^{th} annular region	
P	Nominal pressure (the pressure in the steam dome)	lb/in ² absolute
p_{ij}	Variable proportional to heat input at location ij	
$p(Z)$	Normalized local heat input rate	
Q	Heat output of a single element (no shroud losses included)	Btu/hr
Q_c	Reactor power output	Btu/hr
Q_i	Total heat input to element of i^{th} annular region	
q/L	Local heat input rate per unit axial length of fuel element	Btu/hr-ft
q/L_{avg}	Length-average heat input	Btu/hr-ft
R	Local two-phase friction multiplier	
Re	Reynolds number	
S	Mass flow rate of steam from the reactor	lb/hr
T_{feed}	Temperature of feedwater at reactor inlet	°F
T_{sat}	Saturation temperature at nominal operating pressure	°F
V_d	Velocity in downcomer	ft/sec
V_{de}	Liquid velocity in downcomer entrance	ft/sec

NOMENCLATURE (Continued)

V_f	Velocity of liquid	
V_g	Velocity of steam	
V_{lr}	Liquid velocity in riser	ft/sec
V_o	Nominal velocity of coolant (velocity through flow area of fuel element if coolant were all saturated liquid)	ft/sec
Z	Distance from bottom of fuel element	ft
α_1	Steam void fraction at top of fuel element	
α_2	Steam void fraction at bottom of riser	
α	Steam void fraction defined as the ratio of volume of steam to total coolant volume	
$\alpha(z)$	Local void fraction	
Δh_{fg}	Latent heat of vaporization	Btu/lb
ΔP_1	Static-pressure difference due to wall friction in fuel elements	lb/ft ²
ΔP_2	Static-pressure difference due to acceleration in the coolant channel	lb/ft ²
ΔP_3	Static-pressure difference between upper end of fuel elements and bottom of riser	lb/ft ²
ΔP_4	Static-pressure difference between the plenum and the bottom of the fuel element	lb/ft ²
ΔP_5	Hydrostatic-pressure difference between bottom of fuel element and bottom of riser	lb/ft ²
ΔP_{ca}	Static-pressure difference between the plenum and the bottom of riser	lb/ft ²
ΔP_d	Static-pressure difference due to downcomer wall friction	lb/ft ²
ΔP_{hd}	Hydrostatic-pressure difference between top and bottom of downcomer	lb/ft ²
ΔP_{hr}	Hydrostatic-pressure difference between the core exit and the top of the riser	lb/ft ²
ΔP_r	Static-pressure difference due to riser wall friction	lb/ft ²

NOMENCLATURE (Concluded)

$\Delta P_{t(\text{lower})}$	Static-pressure difference due to turning from the downcomer into the lower plenum	lb/ft ²
$\Delta P_{t(\text{upper})}$	Static-pressure difference due to turning from the riser into the downcomer	lb/ft ²
ΔT_p	Subcooling of coolant in lower plenum (saturation temperature at plenum pressure minus actual temperature)	°F
ΔT_{sub}	Subcooling of coolant in lower plenum (saturation temperature at depth at which net boiling starts, minus actual temperature in plenum)	°F
ϵ	Error	
μ	Viscosity of water	lb/ft-hr
$\bar{\rho}$	Average local density of the steam water mixture	lb/ft ³
$\bar{\rho}_{\text{ex}}$	Density of steam-water mixture at top of element	lb/ft ³
ρ_f	Saturation density of liquid at nominal pressure	lb/ft ³
ρ_g	Saturation density of steam at nominal pressure	lb/ft ³
χ	Local steam quality defined as the ratio of steam mass flow rate to total coolant flow rate	
$\psi \equiv$	$\frac{V_g \rho_g}{V_f \rho_f}$ Definition	

ATTACHMENT III

'PUREE'
A METHOD AND CODE FOR
THE CALCULATION OF PHYSICS PROPERTIES OF
H₂O-MODERATED POWER REACTORS

ATL-229 Preliminary

29 February 1960

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INTRODUCTION

The purpose of this report is to describe 1) a method for calculating the reactor physics properties of H_2O -moderated power reactors, 2) the IBM 650 codes that have been developed to carry out numerical calculations utilizing the method, and 3) the cross-section library currently being used with the codes. The method, the various component codes, and the cross-section library are collectively referred to as PUREE'.

In tests of PUREE' against a variety of experimental lattices, the agreement between calculated and experimental reactivity has been excellent. However, the accuracy of the over-all reactivity appears better than the accuracy of some of the component parts of the model, such as fast effect and neutron leakage. It is thus expected that some further improvements will be made in the procedure with regard to these component parts before a final version of this report is published.

This preliminary issue contains only a discussion of the method. The final version will include a description of details of the code and cross sections.

The following sections present discussions of 1) the model in general, 2) the determination of the above-thermal spectrum, 3) the thermal spectrum, and 4) the fine-structure weighting factors, followed by a fifth section briefly describing the code. Sections II through V are all concerned with determination of the average constants for the four-group differential equations.

I. THE MODEL

Prior to the first Geneva Conference, most calculations of H_2O lattices were made using the simple model commonly referred to as the four-factor formula, i. e. , a formula of the type

$$\frac{\eta f p \epsilon e^{-\tau B^2}}{1 + L^2 B^2} = 1$$

or

$$\frac{\eta f p \epsilon}{1 + M^2 B^2} = 1$$

Kouts, et al,¹ in their work reported at the first Geneva Conference, made a vigorous attempt to correlate their experimental results with the four-factor formula. However, the formula was designed to describe the neutronic behavior of large, well-thermalized reactors of the type that have graphite or D_2O as moderator. In H_2O -moderated power reactors, the fuel elements are usually close-packed, and a significant amount of epithermal fissioning occurs. Thus, the simple model above is seriously strained. It is still possible to determine experimentally the η , f , p , and ϵ as if the reactor were thermal and then lump the effect of this over-simplification into an artificial value of M^2 . However, when conditions of a new reactor design are different from those covered by the critical experiments, the accuracy of the formula and constants is questionable. An example of this would be a reactor that has sustained some burnup and in which Pu has replaced some of the uranium.

Between the first and second Geneva Conferences, some attempts were made to redefine and modify the simple model, but none of these was very successful or satisfying. There was a growing feeling that it would be best to abandon the simple model and use a multigroup approach. Physicists at WAPD needed a reliable model for the design of the Shippingport PWR; after investigating various group structures, they decided that four groups would be the minimum number allowable to calculate adequately the various reactor physics properties of interest. The physical events in the life history of neutrons in this

1. H. Kouts, et al, "Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water," Proceedings of the International Conference on Peaceful Uses of Atomic Energy, Vol. 5, P/600 (1955).

type of reactor determine this choice as well as the choice of energy limits of the various groups. The energy limits chosen for PUREE' are very close to those used at WAPD and are shown in Table I.

Table I

<u>Group No.</u>	<u>Energy Limits</u>
1	1 Mev to ∞
2	5.53 kev to 1 Mev
3	0.625 ev to 5.53 kev
4	0 to 0.625 ev

The first group contains those neutrons causing fission of U 238. About 69% of neutrons emerging from fission (of U 235 or U 238) are born in this group. The energy range of the second group contains the unresolved resonances of U 238. The remainder of the fission neutrons (31%) emerge into this group. The third group is characterized by the resolved resonances of U 238 and U 235 and also contains an important Pu 240 resonance at about 1 ev. The fourth group encompasses what is perhaps better referred to as the "slow" neutrons rather than the "thermal" neutrons. A look at BNL 325 "Neutron Cross Sections" shows that the energy range of this group contains the important low-lying resonances of Pu 239 and Pu 241, both of which peak at about 0.3 ev. However, most of the cross sections are $1/v$ in this fourth group.

The four group equations can be written:

$$\begin{aligned}
 -D_1 \nabla^2 \phi_1(\vec{r}) + \Sigma_{a_1} \phi_1(\vec{r}) + \Sigma_{r_1} \phi_1(\vec{r}) - f_1 \Sigma_i \nu_i \Sigma_{fi} \phi_i(\vec{r}) &= 0 ; \\
 -D_2 \nabla^2 \phi_2(\vec{r}) + \Sigma_{a_2} \phi_2(\vec{r}) + \Sigma_{r_2} \phi_2(\vec{r}) - \Sigma_{r_1} \phi_1(\vec{r}) - f_2 \Sigma_i \nu_i \Sigma_{fi} \phi_i(\vec{r}) &= 0 ; \\
 -D_3 \nabla^2 \phi_3(\vec{r}) + \Sigma_{a_3} \phi_3(\vec{r}) + \Sigma_{r_3} \phi_3(\vec{r}) - \Sigma_{r_2} \phi_2(\vec{r}) &= 0 ; \\
 -D_4 \nabla^2 \phi_4(\vec{r}) + \Sigma_{a_4} \phi_4(\vec{r}) - \Sigma_{r_3} \phi_3(\vec{r}) &= 0 ;
 \end{aligned} \tag{1}$$

where D_i $\equiv$ diffusion coefficient in group i ,

$\phi_i(\vec{r})$ $\equiv$ flux in group i at point $\vec{r}$,

Σ_{ai} $\equiv$ macroscopic absorption cross section,

Σ_{ri} $\equiv$ macroscopic removal cross section for the slowing down of neutrons from group i to $i + 1$,

Σ_{fi} $\equiv$ macroscopic fission cross section,

ν_i $\equiv$ neutrons born per fission in group i ,

f_i $\equiv$ fraction of fission neutrons that emerge in group i .

More precisely, the term $\sum_i \nu_i \Sigma_{fi} \phi_i(\vec{r})$ should be written

$$\sum_i \sum_a \nu_{ia} \Sigma_{fia} \phi_i(\vec{r}),$$

where ν_{ia} is the number of neutrons born per fission of isotope "a" caused by a neutron in group i . Since ν is quite different for Pu 239 than for U 235, this is an important point in fuel-life calculations.

The quantities D , Σ_a , Σ_r , Σ_f are not shown as functions of position either because they are constant in a region (core, reflector, etc.) or because the region is divided into subregions and the quantities taken to be constant in these subregions. There is thus a set of four-group equations of the type (1) for each region. In PUREE', the code VAL PROD² is used to solve these sets of differential equations under the appropriate boundary conditions. Since VAL PROD is one-dimensional, these regions and subregions must be concentric or parallel in the usual way.

The bulk of PUREE' is concerned with evaluation of the macroscopic constants, for each region, that appear in equations 1. Although equations 1 assume that the regions are each homogeneous in composition, H₂O-moderated power reactors have in general a cell or lattice-type structure. Although the fuel-rod size and lattice pitch are not as large as in such reactors as the Calder Hall, the composition is definitely heterogeneous with respect

2. R. H. Stark and C. H. White, "VAL PROD, a Revision of the PROD II 630 Multigroup," General Electric, Atomic Power Equipment Division, GEAP 0952, January 9, 1957.

to neutron behavior. The regions of the reactor (i. e. , core, blanket, reflector, etc.) cannot in general each be considered as a homogenized mixture as is done in H_2O -moderated research reactors, e. g. , the MTR. The procedure that is followed is thus to describe, by means of equations 1, a fictitious reactor that does have homogeneous regions and that is equivalent to the actual heterogeneous reactor. Thus, as mentioned above, the macroscopic cross sections in the equations are constant over each region, and the fluxes, ϕ_i , represent "smoothed out" functions of position that do not follow the hills and valleys the true flux distributions have in each cell of the lattice.

Let $\phi^D(\vec{r}, E)$ symbolize the detailed flux distribution, as a function of space and energy, that does contain the fine structure (hills and valleys) through a cell. Then, if any product of pairs of the type Σ_i and ϕ_i of equations 1 is to give a correct reaction rate, it follows that

$$\Sigma_i \int_{V_c} \phi_i(\vec{r}) d\vec{r} = \int_{E_{i-1}}^{E_i} \int_{V_c} \Sigma(\vec{r}, E) \phi^D(\vec{r}, E) dE d\vec{r}, \quad (2)$$

where $\Sigma(\vec{r}, E)$ represents the detailed variation of cross section over energy and spatially over the various parts of a lattice cell, and the integration is carried out for some particular lattice cell of volume V_c . The energy limits of group i are denoted by E_i to E_{i-1} .

Now, suppose that $\phi^D(\vec{r}, E)$ can be represented as a function of position only, $\phi^D(\vec{r})$, times a function of energy only, $\psi(E)$:

$$\phi^D(\vec{r}, E) = \phi^D(\vec{r}) \psi(E). \quad (3)$$

This implies, of course, that the spectrum is assumed to be the same in each part of the cell, e. g. , in fuel and coolant. The right side of equation 2 can then be written as

$$\int_{E_{i-1}}^{E_i} \Sigma_1(E) \psi(E) dE \int_{V_1} \phi^D(\vec{r}) d\vec{r} + \int_{E_{i-1}}^{E_i} \Sigma_2(E) \psi(E) dE \int_{V_2} \phi^D(\vec{r}) d\vec{r} + \dots, \quad (4)$$

where 1, 2, etc. , denote the different parts of the cell. Now suppose in expression 4 that we choose a normalization on $\psi(E)$ such that

$$\int_{E_{i-1}}^{E_i} \psi(E) dE = 1; \quad (5)$$

then expression 4 becomes

$$\Sigma_{i1} \int_{V_1} \phi^D(\vec{r}) d\vec{r} + \Sigma_{i2} \int_{V_2} \phi^D(\vec{r}) d\vec{r} + \dots, \quad (6)$$

where Σ_1, Σ_2 , etc., represent averages over $\psi(E)$. This can also be written:

$$\Sigma_{i1} V_1 \bar{\phi}_{i1}^D + \Sigma_{i2} V_2 \bar{\phi}_{i2}^D + \dots, \quad (7)$$

where $\bar{\phi}_{ik}^D$ represents the average flux in part k of the cell.

Now, turning our attention to the rest of equation 2, the left side can be written

$$\Sigma_i V_c \bar{\phi}_i, \quad (8)$$

where $\bar{\phi}_i$ is the average of the smooth flux over the cell. The equality of expressions 7 and 8, results in a formula for Σ_i , i. e.,

$$\Sigma_i = \Sigma_{i1} \frac{V_1}{V_c} \frac{\bar{\phi}_{i1}^D}{\bar{\phi}_i} + \Sigma_{i2} \frac{V_2}{V_c} \frac{\bar{\phi}_{i2}^D}{\bar{\phi}_i} + \dots \quad (9)$$

All the macroscopic cross sections for use in equations 1 can be obtained in this way except the removal cross sections, Σ_{ri} . These necessitate a special treatment, as discussed below.

First, removal cross sections for the homogenized region are computed from the following formulas:

$$\Sigma_{r1}^H = \frac{0.6918 - \int_{E_0}^{E_1} \left\{ \Sigma_a(E) + B^2 D(E) \right\} \psi(E) dE}{\int_{E_0}^{E_1} \psi(E) dE}$$

$$\Sigma_{r2}^H = \frac{1.0000 - \int_{E_0}^{E_2} \left\{ \Sigma_a(E) + B^2 D(E) \right\} \psi(E) dE}{\int_{E_1}^{E_2} \psi(E) dE};$$

$$\Sigma_{r3}^H = \frac{1.0000 - \int_{E_0}^{E_3} \left\{ \Sigma_a(E) + B^2 D(E) \right\} \psi(E) dE}{\int_{E_2}^{E_3} \psi(E) dE} \quad (10)$$

These formulas are statements of the fact that the number of neutrons which can be considered to be slowing past the lower energy limit of a group is equal to the number born at energies higher than this minus the number that have been absorbed or leaked at energies also higher than this lower limit of the group.

Next, removal cross sections for the various parts of the cell, i. e., Σ_{rik} needed for equation 9 are computed by normalizing approximate expressions to the above correct result. This normalization is done by determining correction factors, K_{ri} , from equations of the type

$$\Sigma_{ri}^H = K_{ri} \left(\tilde{\Sigma}_{ri1} \frac{V_1}{V_c} + \tilde{\Sigma}_{ri2} \frac{V_2}{V_c} + \dots \right),$$

where $\tilde{\Sigma}_{rik}$ denotes approximate values, and then setting

$$\Sigma_{rik} = K_{ri} \tilde{\Sigma}_{rik}.$$

The approximate values are obtained as follows. In group 1, the fuel-removal cross section is set equal to the average inelastic scattering cross section, while the clad-coolant and shroud-moderator cross sections are set equal to the average inelastic cross section plus the average hydrogen-scattering cross section for the medium. In groups 2 and 3, it is assumed that no slowing down occurs in the fuel, and formulas of the type $\frac{(\xi \Sigma_s)_k}{(\Delta u)_i}$ are used in the clad-coolant and shroud-moderator parts of the cell.

Let us turn our attention now to procedures for obtaining the spectral function $\psi(E)$ in equation 2. We shall return later to further discussion of the quantities $\frac{\bar{\phi}_{i1}}{\bar{\phi}_i}$, etc., in equation 9, which are determined from the fine structure of flux distribution in a cell.

II. ABOVE-THERMAL SPECTRUM

As mentioned above, the spectrum (energy distribution) of neutrons can be assumed, as an approximation, to be the same in all parts of the cell, except of course for the sharp flux depression in the fuel part of the cell at U 238 or thorium resonance energies. This depression does not exist to the same extent in the adjacent coolant at these energies.

Except for a special treatment of the U 238 resonance cross section, PUREE' simply considers the contents of each region (and cells in that region) to be homogenized, and solves for the energy spectrum of flux in each region by a multigroup method similar to VAL PROD, using 19 groups. In doing this, each region is considered as an isolated, single region such that the solution can be made in the energy variable only (no position variables).

In the core, the leakage of neutrons will affect the spectrum somewhat. This effect is not great and can be handled through the use of a term DB^2 in the single-region multigroup formulation. The quantity B^2 is the estimated total buckling for the region. It is usually satisfactory to determine B^2 simply from a guess as to the size of the equivalent bare reactor, since the effect on energy distribution of flux due to leakage is quite small.

The energy limits of the 19-group structure are shown in Table II.

TABLE II
MULTIGROUP STRUCTURE

Group No.	Energy Range		Lethargy Range		Lethargy Width	Comments
1	5	- 10 Mev	0	- 0.693	0.69	Range of fast fission in U 238
2	3	- 5	0.693	- 1.203	0.51	
3	2	- 3	1.203	- 1.609	0.41	
4	1	- 2	1.609	- 2.303	0.69	
5	0.821	- 1.0	2.303	- 2.527	0.197	Unresolved U 238 resonances.
6	0.6	- 0.821	2.527	- 2.815	0.315	
7	0.3	- 0.6	2.815	- 3.506	0.69	
8	0.09	- 0.3	2.506	- 4.701	1.19	
9	0.01	- 0.09	4.701	- 6.909	2.21	
10	5.53 kev	- 0.01 Mev	6.909	- 7.502	0.59	
11	3.0	- 5.53 kev	7.502	- 8.113	0.61	
12	530	- 3.0 kev	8.113	- 9.845	1.73	Contains higher resolved U 238 resonances.
13	132	- 530 kev	9.845	- 11.237	1.39	
14	73	- 132	11.237	- 11.830	0.59	Contains 81, 90, 103 and 117 ev U 238 resonances.
15	29	- 73	11.830	- 12.753	0.92	Contains 37 and 66 ev U 238 resonances.
16	13	- 29	12.753	- 13.556	0.80	Contains 21.0 ev U 238 resonances.
17	4	- 13	13.556	- 14.735	1.18	Contains 6.7 and 10.2 ev U 238 resonances.
18	2	- 4	14.735	- 15.428	0.69	Contains the Pu 240 resonances.
19	0.625	- 2	15.428	- 16.591	1.16	

Let us turn now to a discussion of the way in which absorption is handled in preparing the input for the 19-group calculation.

The $1/v$ absorbers present no problem in the specification of Σ_{aj} for the various groups of the multigroup. The U 235 cross section, although it exhibits resonances, is so dilute that there is inappreciable self protection. The U 235 cross-section curves can be "smoothed out" and average values chosen for each group of the multigroup. The U 238 and Pu 240, however, present a special problem. Fictitious values are needed for Σ_{aj} for these isotopes, which will cause a solution of a homogenized mixture of the various materials to give a set of multigroup fluxes, ψ_j , and the associated total neutron capture rate in each group, j , which are the same in magnitude as the fluxes and capture rates in the actual heterogeneous lattice. But this is exactly what the "effective" resonance cross section, $\sigma_{a28eff}(E)$ is. It follows that in arriving at average cross sections for U 238 in the groups of the multigroup, we should obtain an average of

$$\sigma_{a28eff}(E) = a(E) + (1 - C_1 - C_2 - \dots) \frac{S}{M} b(E) \quad , \quad (11)$$

with a similar expression for Pu 240. The quantities in equation 11 have the following significance:

$\sigma_{a28eff}(E) \equiv$ effective resonance cross section.

$C_1, C_2, \dots \equiv$ Dancoff correction due to neighboring fuel rods.

$S/M \equiv$ surface-to-mass ratio of the fuel rods.

$$a(E) \equiv \frac{\sigma_{cR}^{28}(E)}{1 + \frac{\sigma_{sR}^{28}(E) + \sigma_{cR}^{28}(E)}{\Sigma_{sP}^F / N_{28}}} \quad (12)$$

$$b(E) \equiv \frac{1}{4} \frac{M}{N_{28} V_F} \left[\frac{\sigma_{cR}^{28}(E) \left\{ \sigma_{sR}^{28}(E) + \sigma_{cR}^{28}(E) \right\} P_c(E)}{\left\{ \sigma_{sR}^{28}(E) + \frac{\Sigma_{sP}^F}{N_{28}} + \sigma_{cR}^{28}(E) \right\}^2} \right] \quad (13)$$

$M \equiv$ mass of the fuel lump contained in volume V_F .

$P_c(E) \equiv$ probability that neutrons of energy E that have entered a fuel rod will suffer a collision.

$\sigma_{c,R}^{28}(E) \equiv$ resonance capture cross section of the U 238.

$\sigma_{s,R}^{28}(E) \equiv$ resonance scattering cross section of the U 238.

$\left[\frac{\sum_s^F P}{N^{28}} \right] \equiv$ total potential scattering in the fuel per U 238 atom.

Equation 14 is an example of how $\sigma_{a,28eff}(E)$ is averaged over a group of the 19-group structure for group 17 (see Table II):

$$\sigma_{a,28} = \frac{\int_{6.7}^{10.2} \sigma_{a,28eff}(E) \frac{dE}{E} + \int_{10.2}^{13} \sigma_{a,28eff}(E) \frac{dE}{E}}{\ln \frac{10}{4} - \ln \frac{10}{13}}, \quad (14)$$

where 4 and 13 ev are the energy limits of the group, and the integrals are over the 6.7 and 10.2 resonances, i. e., the two resonances included in this group.

It is necessary at this point to discuss the procedure for evaluation of

$$\int \sigma_{a,28eff}(E) \frac{dE}{E} = \left\{ \int a(E) \frac{dE}{E} + (1 - C_1 - C_2 - \dots) \frac{S}{M} \int b(E) \frac{dE}{E} \right\} \quad (15)$$

over each resonance.

First, the total integral over all resonances without the factors for Dancoff correction will be given by the published values of A and μ and the fact that

$$\int_{\text{All resonances}} \left\{ a(E) + \frac{S}{M} b(E) \right\} \frac{dE}{E} = A + A\mu \frac{S}{M}. \quad (16)$$

The distribution of the quantity into components attributable to each of the individual resonances will have to be normalized so that the total of these components equals the experimental value for $A + A\mu \frac{S}{M}$. It is important to normalize against the total of the bulk plus surface capture and not to normalize $\int a(E) \frac{dE}{E}$ against A and $\int b(E) \frac{dE}{E}$ against $A\mu$

individually. The reasons why this is so rest with the factor $P_c(E)$ in the formula for $b(E)$, which will be discussed later.

The first integral, i. e. ,

$$\int a(E) dE ,$$

is performed in PUREE' by utilizing a table lookup procedure. The tabled data is taken from some work by L. Dresner.³

Now consider the μ .

$$\mu = \frac{1}{A} \frac{M}{4N_{28}V_F} \int \frac{\sigma_{cR}^{28}(E) \left\{ \sigma_{sR}^{28}(E) + \sigma_{cR}^{28}(E) \right\} P_c(E)}{\left\{ \sigma_{sR}^{28}(E) + \frac{\Sigma_F^P(E)}{N_{28}} + \sigma_{cR}^{28}(E) \right\}^2} \frac{dE}{E} \quad (17)$$

We shall at this point pass to the less-cumbersome notation adopted by Dresner⁴ and use some of his material as well as some of Wigner's. Equation 17 becomes, in Dresner's notation:

$$\mu = \frac{1}{A} \frac{m}{4} \int \frac{(\sigma_a + \sigma_s) \sigma_a P_c}{(\sigma_a + \sigma_s + \sigma_p)^2} \frac{dE}{E} , \quad (18)$$

where $m \equiv$ the mass of fuel associated with each U 238 nucleus,

$$\sigma_a \equiv \sigma_{cR}^{28}(E) ,$$

$$\sigma_s \equiv \sigma_{sR}^{28}(E) .$$

Spherical geometry, i. e. , spherical fuel elements, will be considered because P_c takes a simpler form in this geometry. Of the neutrons passing into a cm^2 of rod surface, a fraction $2\beta d\beta$ will enter in the increment of the direction cosine between β and $d\beta$ where $\beta = \cos \theta$ and θ is the angle between the direction of entry and the normal to the surface

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3. L. Dresner, "The Effective Resonance Integrals of U-238 and Th-232," Nuclear Science and Engineering, 1, pp 68-79 (1956).
 4. L. Dresner, "Effect of Geometry on Resonance Neutron Absorption in Uranium," Nuclear Science and Engineering, 1, p 501.

(pointed upward). The distance these particles would travel in traversing the sphere is $2R\beta$, where R is the radius of the sphere. The probability that this flight would lead to a collision is

$$1 - e^{-2R\beta N(\sigma_a + \sigma_s + \sigma_p)}$$

where $N = N_{28}$. Thus equation 18 can be written

$$\mu = \frac{1}{A} \frac{m}{2} \int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)^2} \int_0^1 \left\{ 1 - e^{-2R\beta N(\sigma_a + \sigma_s + \sigma_p)} \right\} \beta d\beta \frac{dE}{E} \quad (19)$$

Carrying out the inner integral gives

$$\mu = \frac{m}{2A} \int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)^2} \left[\frac{1}{2} - \frac{1}{4R^2 N^2 (\sigma_a + \sigma_s + \sigma_p)^2} + e^{-2RN(\sigma_a + \sigma_s + \sigma_p)} \left\{ \frac{1}{4R^2 N^2 (\sigma_a + \sigma_s + \sigma_p)^2} + \frac{1}{2RN(\sigma_a + \sigma_s + \sigma_p)} \right\} \right] \frac{dE}{E} \quad (20)$$

The quantity μ could be evaluated from equation 20, using again the Breit-Wigner formulas for the resonance cross section σ_a and σ_s . It is clear that the evaluation of equation 20 in the exact form shown is a very laborious chore. However, Drsner has evaluated an approximation to it that was suggested by Wigner, which will be given in a moment. First though, from the form of equation 20, it is possible to make some interesting observations.

1. If $RN(\sigma_a + \sigma_s + \sigma_p) \gg 1$ over the resonance, then equation 20 reduces to

$$\mu = \frac{m}{4A} \int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)^2} \frac{dE}{E} \quad (21)$$

This was widely quoted as the proper formula for calculating μ until Wigner pointed out in 1955 the importance of the P_c in equation 18 and thus the R -dependence of μ .

2. Equation 20 for μA contains terms in $\frac{1}{R^2}$, $\frac{1}{R^2} e^{-Rf(E)}$, and $\frac{1}{R} e^{-Rf(E)}$, where $f(E) = 2N(\sigma_a + \sigma_s + \sigma_p)$. When μA is multiplied by S , which is proportional to R^2 , in the formula

$$\int (\sigma_a)_{\text{eff}} \frac{dE}{E} = A + A\mu \frac{S}{M},$$

it is clear that part of the "surface term," $A\mu \frac{S}{M}$, will be independent of the surface area of the lump and that another part has a complicated dependence on lump size. Thus the attempt to represent experimental values of $\int (\sigma_a)_{\text{eff}} \frac{dE}{E}$ by the simple formula $A + A\mu \frac{S}{M}$, with A and μ being pure constants, independent of lump geometry, is not likely to be completely successful. Indeed this has proved to be so, and more complicated empirical expressions than $A + A\mu \frac{S}{M}$ have been used to fit the experimental data when a wide range of lump sizes is under study. However, $A + A\mu \frac{S}{M}$, with A and μ as pure constants, is capable of being fitted to the data for the range of larger rod sizes used in most reactors. When A and μ are determined by fitting $A + \mu \frac{S}{M}$ to the experimental data, it is clear that part of what is referred to theoretically as "surface" capture (equation 20), will show up experimentally as part of the "volume" term. It is thus best (in the opinion of the authors) to seek agreement of only the total capture integral, $\int \sigma_a \frac{dE}{E}$, when comparing theory and experiment, rather than trying to correlate volume and surface terms independently.

Now let us return to equation 20 and present the Wigner-Dresner work regarding the evaluation of μ from this equation. Wigner introduced the following brilliant approximation into equation 20 to make it integrable:

$$\left[\frac{1}{2} - \frac{1}{4R^2 N^2 (\sigma_a + \sigma_s + \sigma_p)^2} + e^{-2RN(\sigma_a + \sigma_s + \sigma_p)} \left\{ \frac{1}{4R^2 N^2 (\sigma_a + \sigma_s + \sigma_p)^2} + \frac{1}{2RN(\sigma_a + \sigma_s + \sigma_p)} \right\} \right] \approx \frac{2RN(\sigma_a + \sigma_s + \sigma_p)}{3 + 4RN(\sigma_a + \sigma_s + \sigma_p)} \quad (22)$$

Equation 20 becomes, after substituting 22,

$$\mu = \frac{m}{2A} \int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)} \frac{2RN}{[3 + 4RN(\sigma_a + \sigma_s + \sigma_p)]} \frac{dE}{E} \quad (23)$$

In the notation of Dresner,³

$$\psi(\xi, x) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp - \left\{ \xi^2 (x - y)^2 / 4 \right\}}{1 + y^2} dy$$

However, we shall evaluate μ first for the case of $\xi = \infty$, i. e., no Doppler broadening.

In this case,

$$\psi(\xi, x) = \psi(x) = \frac{1}{1 + x^2},$$

and $\sigma_a = \sigma_o \frac{\Gamma_\gamma}{\Gamma} \psi(x);$

$$\sigma_s = \sigma_o \frac{\Gamma_n}{\Gamma} \psi(x).$$

Letting $E \approx E_o$, equation 23 becomes

$$\mu = \frac{2m}{2AE_o} \int_0^\infty \frac{\left\{ \sigma_o \frac{\Gamma_\gamma}{\Gamma} \left(\frac{1}{1+x^2} \right) + \sigma_o \frac{\Gamma_n}{\Gamma} \left(\frac{1}{1+x^2} \right) \right\} \sigma_o \frac{\Gamma_\gamma}{\Gamma} \left(\frac{1}{1+x^2} \right) 2RN \frac{\Gamma}{2} dx}{\left\{ \sigma_o \frac{\Gamma_\gamma}{\Gamma} \left(\frac{1}{1+x^2} \right) + \sigma_o \frac{\Gamma_n}{\Gamma} \left(\frac{1}{1+x^2} \right) + \sigma_p \right\} \left[3 + 4RN \left\{ \sigma_o \frac{\Gamma_\gamma}{\Gamma} \left(\frac{1}{1+x^2} \right) + \sigma_o \frac{\Gamma_n}{\Gamma} \left(\frac{1}{1+x^2} \right) + \sigma_p \right\} \right]} \quad (24)$$

Using $\Gamma = \Gamma_\gamma + \Gamma_n$, and $\beta \equiv \frac{\sigma_p}{\sigma_o}$, and simplifying, equation (24) becomes

$$\mu = \frac{m\Gamma_\gamma}{4AE_o} \frac{1}{\beta^2 \left(1 + \frac{3}{4RN\sigma_p} \right)} \int_0^\infty \frac{dx}{\left\{ \left(\frac{1+\beta}{\beta} \right) + x^2 \right\} \left[\left\{ \frac{1 + \beta \left(1 + \frac{3}{4RN\sigma_p} \right)}{\beta \left(1 + \frac{3}{4RN\sigma_p} \right)} \right\} + x^2 \right]} \quad (25)$$

This can be integrated to give

$$\mu = \frac{m\Gamma_\gamma}{4AE_o} \frac{1}{\beta^2 \left(1 + \frac{3}{4RN\sigma_p} \right)} \frac{1}{\left[\frac{1 + \beta \left(1 + \frac{3}{4RN\sigma_p} \right)}{\beta \left(1 + \frac{3}{4RN\sigma_p} \right)} - \frac{1 + \beta}{\beta} \right]}$$

$$\frac{\pi}{2} \left\{ \sqrt{\frac{\beta}{1 + \beta}} - \sqrt{\frac{\beta \left(1 + \frac{3}{4RN\sigma_p} \right)}{1 + \beta \left(1 + \frac{3}{4RN\sigma_p} \right)}} \right\}$$

$$\begin{aligned}
&= \frac{m\Gamma}{4AE_0} \frac{\gamma}{\beta} \frac{1}{\left(1 + \frac{3}{4RN\sigma_p}\right)} \frac{\pi}{2} \sqrt{\frac{\beta}{1+\beta}} \left\{ 1 - \sqrt{\frac{(1+\beta)\left(1 + \frac{3}{4RN\sigma_p}\right)}{1 + \beta\left(1 + \frac{3}{4RN\sigma_p}\right)}} \right\} \\
&\approx \frac{m\Gamma}{8AE_0} \frac{\gamma}{\beta} \frac{\pi}{3} \sqrt{\frac{\beta}{1+\beta}} \left\{ 1 - \sqrt{1 + \frac{3}{4RN\sigma_p}} \right\} \\
&\approx \frac{m\Gamma}{8AE_0} \frac{\gamma}{\beta} \frac{\pi}{3} \sqrt{\frac{\beta}{1+\beta}} \left\{ \frac{1}{1 + \sqrt{1 + \frac{3}{4RN\sigma_p}}} \right\}
\end{aligned} \tag{26}$$

But
$$A = \int \frac{\sigma_a \sigma_p}{\sigma_a + \sigma_s + \sigma_p} \frac{dE}{E}$$

Carrying out this integration in a similar way, it is found that

$$A = \frac{\Gamma}{E_0} \frac{\gamma}{\beta} \frac{\sigma_p}{2} \frac{\pi}{2} \sqrt{\frac{\beta}{1+\beta}} \tag{27}$$

Substituting A from equation 27 into 26, it follows that

$$\mu = \frac{m}{8\sigma_p} \left[\frac{2}{1 + \sqrt{1 + \frac{3}{4RN\sigma_p}}} \right] \tag{28}$$

The quantity in the brackets is denoted by C, i. e. ,

$$C = \frac{2}{1 + \sqrt{1 + \frac{3}{4RN\sigma_p}}}$$

and thus

$$\mu = \frac{m C}{8\sigma_p} \tag{29}$$

The above manipulations were carried through for spherical fuel elements. Dresner has observed that the quantity $\frac{3}{4RN\sigma_p}$ in C for spheres is a special case of the form

$\frac{1}{N \sigma_p \bar{\ell}}$ for any geometrical shape of the fuel lumps, where $\bar{\ell}$ is the average chord length.

For spheres, $\bar{\ell} = \frac{4R}{3}$; for cylinders, $\bar{\ell} = 2R$; and in general, $\bar{\ell} = \frac{4V}{S}$. Thus, more generally,

$$C = \frac{2}{1 + \sqrt{1 + \frac{1}{N \sigma_p \bar{\ell}}}} \quad (30)$$

Now if P_c is omitted from formula 18 for μ , then equation 21 is obtained; if the integration in equation 21 is carried out in the manner of that above, the formula obtained for μ is simply

$$\mu = \frac{m}{8 \sigma_p}$$

Thus, from equation 28, C is a correction factor to μ which arises from the introduction of the quantity P_c in equation 21. Dresner has shown that equation 29 holds approximately, even when Doppler broadening is allowed, by assuming that

$$\mu \approx \frac{mC}{4A} \int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)^2} \frac{dE}{E} \quad (31)$$

By using $\psi(\xi, x)$ instead of just $\psi(x)$, and the appropriate formulas for σ_a and σ_s , he showed that the quantity

$$H = \frac{\int \frac{(\sigma_a + \sigma_s) \sigma_a}{(\sigma_a + \sigma_s + \sigma_p)^2} \frac{dE}{E}}{\int \frac{\sigma_a + \sigma_p}{\sigma_a + \sigma_s + \sigma_p} \frac{dE}{E}} \quad (32)$$

in equation 31 has the value shown in Figure 1 as a function of β for various values of ξ .

Since typical values of ξ and β are 0.3 and 10^{-3} , it follows from Figure 1 that

$$H \approx \frac{0.5}{\sigma_p} \quad ; \quad (33)$$

thus equation 29 holds approximately even when Doppler broadening is allowed.

In conclusion then, it follows that one can evaluate

$$\int (\sigma_{a28})_{\text{eff}} \frac{dE}{E} = \int a(E) \frac{dE}{E} \left\{ 1 + (1 - C_1 - C_2 - \dots) \frac{S}{M} \frac{m}{8 \sigma_p} C \right\} \quad (34)$$

for each resonance for use in equations of the type (14). The integration of $a(E)$ is carried out as illustrated in the article by Dresner referred to previously.³ The condition must be met that

$$\int a(E) \frac{dE}{E} \left\{ 1 + \frac{S}{M} \frac{m}{8 \sigma_p} C \right\} = A \left(1 + \mu \frac{S}{M} \right) , \quad (35)$$

where A and μ are the quoted experimental values. If this does not hold, all contributions are ratioed down or up accordingly before use in equation 14.

The other cross-section data present no problem, and the materials in a region (and cells in that region) are simply homogenized to obtain macroscopic data for the multigroup determination of the spectral function, $\psi(E)$, above thermal.

From L. Dresner, "Effect of Geometry on Resonance Neutron Absorption in Uranium," Nuclear Science and Engineering, 1, 1956, p 503.

FACTOR IN EVALUATION OF μ/A

FIGURE 1

III. THERMAL SPECTRUM

Let us turn to the solution of the spectrum $\psi(E)$ for slow neutrons. Because the H_2O lattices are close-packed, and absorption is therefore strong, the simple picture of Maxwellian plus $1/E$ tail is no longer sufficiently representative of the correct spectrum of slow neutrons. It is necessary to use a method known as the Wigner-Wilkins procedure.

As in so many topics in reactor physics, the exact determination of the spectrum of slow neutrons in an H_2O -moderated reactor is an exceedingly difficult, though fascinating, subject; however, through various simplifying assumptions, considerable improvement over the assumption of a Maxwellian plus $1/E$ distribution can be achieved without great difficulty. In the exact determination, it is necessary to take into account the fact that when a slow neutron collides with a hydrogen nucleus, the effective mass of the nucleus is a function of the angle between the direction of neutron travel and the line joining the hydrogen and oxygen nucleus. Furthermore, the scattering cross section of hydrogen in the thermal range is not a constant, and this is a complicating factor.

It has therefore been customary in most H_2O power reactors to be content with the simplest possible assumptions regarding the scattering. It is assumed that 1) the hydrogen nucleus is free from any forces of attraction with the oxygen nuclei, i. e. , that there is no "binding effect"; 2) the scattering cross section of hydrogen is constant with energy; and 3) any scattering with materials other than hydrogen has no effect on the spectrum. It has been found,⁵ surprisingly enough, that the spectrum in H_2O -moderated reactors as determined experimentally agrees quite well with that calculated under these assumptions regarding the scattering. In the original analytical solution of the problem by Wigner and Wilkins,⁶ it was furthermore assumed that the absorption was $1/v$. However, Amster and Suarez⁷ have removed this restriction and have solved by numerical methods for the

-
5. H. J. Amster, "The Wigner-Wilkins Calculated Thermal Neutron Spectrum Compared with Measurements in a Water-Moderator," *Nuclear Science and Engineering*, 2, No. 3, May 1957, p 394.
 6. E. P. Wigner and J. E. Wilkins, Jr., "Effect of the Temperature of the Moderator on the Velocity Distribution of Neutrons with Numerical Calculations for H as Moderator," AECD-2275, September 1944.
 7. H. S. Amster and R. Suarez, "Distribution of Neutrons at Thermal Energies: Description of Donate Code," WAPD-PM-45, May 1946.

spectrum for arbitrary dependence of absorption upon energy. They have established an extensive table⁸ of cross sections of various isotopes averaged from 0 to 0.625 ev over the spectrum determined for this condition and for various temperatures and various magnitudes of $1/v$ absorption, U 235 content, and Pu 239 content.

Since adequate write-ups of the Wigner-Wilkins procedure are available in the literature, there is no point in discussing the matter further here.

8. H. J. Amster, "A Compendium of Thermal Neutron Cross-Section Averages Over the Spectra of Wigner and Wilkins," WAPD-185, January 1958.

IV. FINE-STRUCTURE WEIGHTING FACTORS

Referring to equation 9, we have at this point the procedure for the determination of $\Sigma_1, \Sigma_2, \dots$, the average macroscopic cross sections in the various parts of the cell. Restating this: the contents of a cell are homogenized (except for special treatment of U 238), and a flux spectrum $\psi(E)$ is determined, normalized as in equation 5. Then averages of the macroscopic cross sections are obtained by

$$\Sigma_{i1} = \int_{E_{i-1}}^{E_i} \psi(E) \Sigma_1(E) dE, \Sigma_{i2} = \int_{E_{i-1}}^{E_i} \psi(E) \Sigma_2(E) dE, \dots$$

Actually, since Amster's tables give the averages themselves for group four, it is unnecessary to carry out this flux weighting.

It should be remarked that, in the case of the diffusion coefficient, it is really the reciprocal of the macroscopic cross sections, i. e., the transport cross section, that is weighted. For example:

$$D_{i1} = \int_{E_{i-1}}^{E_i} D_1(E) \psi(E) dE, \dots$$

From equation 9 it can be seen that the various Σ_i can be determined now if the

$$\frac{\phi_{i1}}{\phi_i}, \frac{\phi_{i2}}{\phi_i}, \dots$$

can be determined. These are, it will be recalled, the average fluxes in the various parts of the cell, based on the detailed flux distribution divided by the average value of the "smooth" flux in the cell. These ratios will, in the precise determination, vary slightly from cell to cell because the smooth flux has a changing gradient.

The basic cell in the VMR, for which PUREE' was designed, consists, at least in an idealized picture, of an outer ring of moderator, a shroud of material such as stainless steel, and inside this a cluster of fuel pins with a metallic clad and water coolant. One of these fuel pins, its clad, and the coolant volume associated with it forms a subcell of the basic cell. This shall be referred to as the small cell. The basic cell can also be

considered as formed of three regions: 1) a central region consisting of a homogenized mixture of the fuel pins, cladding, and coolant, 2) the shroud, and 3) the moderator. This shall be referred to as the large cell.

The detailed flux distribution in the cell is determined as follows in the various groups:

A. Group One

Diffusion equations are used, with the constants determined as averages over $\psi(E)$ within the group. Neutrons are assumed to be born in the fuel and nowhere else. They are assumed to be removed from the group by a single inelastic scattering collision or a single collision with hydrogen. An elastic scattering collision with an element other than hydrogen is assumed to leave the neutron unchanged in energy. (It may be well to emphasize that these assumptions are made only in computing fine-structure correction factors and not in obtaining the basic unweighted constants. The latter come from the 19-group calculation.)

The clad and coolant are considered to be homogenized, and the flux distribution is determined first for the small cell by a two-region, i. e., fuel and clad-coolant, diffusion-theory-type calculation. If the average flux in the fuel is denoted as $\bar{\phi}_F$ and that in the clad-coolant as $\bar{\phi}_{CC}$, and the average over the entire small cell as $\bar{\phi}_{FCC}$, then the ratios $\bar{\phi}_F/\bar{\phi}_{FCC}$ and $\bar{\phi}_{CC}/\bar{\phi}_{FCC}$ can be determined. The fuel, clad, and coolant are next considered homogenized, the shroud and moderator are likewise homogenized, and the flux distribution is determined for this large cell, again by a diffusion-theory-type calculation. Denoting the average flux in the fuel-clad-coolant by $\bar{\phi}_{FCC}$, that in the shroud-moderator by $\bar{\phi}_{SM}$, and that over the entire large cell by $\bar{\phi}$, we can form the ratios $\bar{\phi}_{FCC}/\bar{\phi}$ and $\bar{\phi}_{SM}/\bar{\phi}$. The quantities described for use in equation 9 are

$$\frac{\bar{\phi}_F}{\bar{\phi}} = \frac{\bar{\phi}_F}{\bar{\phi}_{FCC}} \frac{\bar{\phi}_{FCC}}{\bar{\phi}}, \quad (36)$$

$$\frac{\bar{\phi}_{CC}}{\bar{\phi}} = \frac{\bar{\phi}_{CC}}{\bar{\phi}_{FCC}} \frac{\bar{\phi}_{FCC}}{\bar{\phi}}, \quad (37)$$

and
$$\frac{\bar{\phi}_{SM}}{\bar{\phi}}$$

The corresponding Σ_{ik} 's (where k refers to the part of the cell) for fuel, clad-coolant,, and shroud-moderator are then used together with these flux ratios in equation 9 to find the average Σ_i for the cell.

This procedure is used to find the Σ_{a_i} 's, the Σ_{f_c} 's, and the Σ_{r_i} 's. In finding the average D_i 's for the cell, 'PUREE' weights the $\frac{1}{D_{ik}}$'s for the different parts of the cell to find the average $1/D_i$'s for the cell, using equation 9 rather than weighting the D_{ik} 's, i. e., it weights the reciprocals rather than the quantities themselves.

B. Group Two

The solution for the various $\bar{\phi}_{ik}/\bar{\phi}_i$'s that are needed for equation 9 for group two is handled the same way as for group one, with the following exceptions.

The neutron source terms in the differential equations for the various parts of the cell are taken to be equal to the neutron removal rate as determined for group one.

It is assumed that there is no degradation of neutron energy in the fuel part. In the clad-coolant part and in the shroud-moderator part of the cell, removal cross section is taken to be

$$\frac{(\xi \Sigma_s)_k}{\Delta u_2},$$

where Δu_2 is the lethargy width of group two, and $(\xi \Sigma_s)_k$ is evaluated for part k of the cell and includes the hydrogen contribution.

C. Group Three

Group three is handled the same way as group two with the following exception.

The source terms in the differential equations for the various parts of the cell are taken to be proportional to the $(\xi \Sigma_s)_k$ evaluated for the appropriate part at an energy just above group three.

D. Group Four

Because of strong absorption in the fuel, the hills and valleys in the detailed flux distribution in the cell are much more pronounced. Since fine-structure corrections are thus much more important in group four, there are important differences in the computational procedures used in 'PUREE' for this group.

The first of these differences is that transport theory is used rather than diffusion theory. The General Electric P_3 code⁹ is employed to find the flux distributions, and the POP code¹⁰ is used to find the spatially averaged thermal constants.

Instead of treating the clad and coolant as a single homogenized part of the cell and the shroud and moderator as another homogenized part as is done with the first three groups, the small cell is computed as a three-region calculation (fuel, clad, and coolant) and the large cell also as a three-region calculation (homogenized fuel-clad-coolant, shroud, and moderator). Formulas similar to equations 36 and 37 are used to find the flux weighting ratios for use in equation 9. The Σ_{ik} that appear in 9 are now five in number, i. e. , fuel, clad, coolant, shroud, and moderator. These Σ_{ik} are obtained, as described previously, by averaging the cross sections as a function of energy over a Wigner-Wilkins distribution. (This averaging process has already been carried out, and it is necessary only to look up the averaged values in Amster's tables).

9. J. W. Weil, KAPL-1173, 10 July 1954, Confidential-Restricted Data.

10. R. L. Crowther, "POP, A P_3 Output Processing Code," GEAP-0982, 15 March 1957.

V. CODE THAT DETERMINES CONSTANTS FOR GROUPS 1, 2, AND 3

The IBM 650 code that generates constants for the first three of the four-group differential equations is described briefly below.

The three-group portion of the PUREE' analysis is accomplished by means of three IBM 650 program decks: SAFURR, PFLUGER, and FINEST. For ease of operation, only one set of input cards is assembled for each case (parameter variation). This set comprises the input deck for SAFURR, which punches as output the input deck for PFLUGER. Similarly, the output of PFLUGER is the input deck for FINEST.

A set of inputs consists primarily of a table of the 19-group cross sections for the materials present in the fuel, clad-coolant, and shroud-moderator regions, the concentrations of these materials, the dimensions of the regions, temperature, and estimated buckling.

SAFURR calculates the effective 19-group absorption cross sections for uranium-238 in the resonance region.

PFLUGER uses the resonance and non-resonance cross sections to compute the 19-group macroscopic cross sections for each region and the 19-group fluxes for the homogenized regions.

FINEST evaluates the three-group flux-weighted average cross sections and then corrects for the heterogeneity of the configuration by using the fine-structure analysis. The output consists of two parts, one of which is the three-group cross sections that will be used with the thermal group cross sections in the VAL PROD diffusion theory code. The preparation of these four-group cross sections for the VAL PROD usage is accomplished by VIPER 4. The remainder of the FINEST output is fuel-region flux information for UNPUREE', an auxiliary code for burnup calculations that computes the three-group flux-weighted average fission and absorption cross sections for uranium-235 and uranium-238.

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