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Theory and Use of the
General-Geometry TWOTRAN Program

by

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149

CONTENTS

	Page
Abstract	1
I. INTRODUCTION	1
II. THEORY	2
A. Analytic Forms of the Transport Equation	2
1. Particular Expressions for $\nabla \cdot \Omega \psi$	2
a. (x,y) geometry	2
b. (r, θ) geometry	3
c. (r,z) geometry	3
2. Spherical Harmonic Expansion of the Source Terms	3
B. Multigroup Difference Equations	4
1. Energy Group Representation	4
2. Differencing of $\nabla \cdot (\Omega \psi)$	4
3. Consistency and Truncation Error of the Difference Equations for $\nabla \cdot (\Omega \psi)$	6
4. Definition of the Source Term	7
C. Solution of the Difference Equations	8
1. Boundary Conditions	8
a. Vacuum Boundary Condition	8
b. Reflective Boundary Condition	8
c. White Boundary Condition	8
d. Periodic Boundary Condition	8
e. Boundary Sources	9
2. Arithmetic Mean (Diamond) Difference Approximation	9
3. Negative Flux Control	9
4. Convergence Acceleration	10
a. Inner Iteration Coarse-Mesh Rebalancing	10
b. Outer Iteration Coarse-Mesh Rebalancing	12
5. Adjoint Problems	12
III. PROGRAM FUNCTIONAL DESCRIPTION	12
A. Program Flow	12
1. Overlay Structure	13
2. Program TWOTRAN (Overlay 0.0)	13
3. Program INPUT11 (Overlay 1.1)	13
4. Program INPUT12 (Overlay 1.2)	13
5. Program INPUT13 (Overlay 1.3)	14
6. Program GRIND2 (Overlay 2.0, Fig. 6)	16
7. Subroutine INITIAL (Fig. 7)	16
8. Subroutine FISCAL (Fig. 8)	18
9. Subroutine OUTER (Fig. 9)	18
10. Subroutine INNER (Fig. 10)	20
11. Subroutine SETBC (Fig. 12)	23
12. Subroutine REBAL (Fig. 13)	23
13. Subroutine TESTS (Fig. 14)	23

(CONTENTS - continued)	Page
14. TWOTRAN Service Routines	24
15. TWOTRAN System Subroutines	29
a. ECS System Routines	29
b. General System Routines	29
c. SC-HO2O Plot Routines	29
B. Iterative Cycles and Convergence Tests	29
C. Progression Through the Space-Angle Mesh	30
1. S_n Constants and Angular Orientation	30
2. Sweeps of the Space-Angle Mesh	30
D. Data Storage	33
1. Allocation of Core Storage	33
2. Allocation of Extended Core Storage	33
3. Overstorage of Data in Core	43
E. Assignment of Files	44
IV. GENERAL USAGE OF THE TWOTRAN PROGRAM	44
A. Cross Sections	44
1. Input Formats	44
2. Cross-Section Mixing	45
3. Anisotropic Cross Sections	46
4. Adjoint Cross Sections	46
5. Cross-Section Checking	47
B. Geometry and Boundary Condition Specification	47
C. Source Options	48
D. Source and Flux Input Options	48
1. Source Input Options	48
2. Flux Input Options	49
3. Flux Guesses from Tape	50
a. Restart Dump Flux Guess	50
b. Scalar Flux Tape	50
E. Flux Dumps and Problem Restart	51
1. Restart Procedures	51
2. Restart Tape Composition	51
F. Flux Plotting on Film	52
V. USE OF SPECIFIC TWOTRAN OPTIONS	52
A. Coarse-Mesh Rebalancing	52
B. Eigenvalue Searches	52
C. Adjoint Computations	54
D. Edit Options	54
VI. DETAILED INPUT SPECIFICATION	55
A. Special Input and Output Routines	55
B. Job Title Cards	56
C. Input of Control Numbers	56
D. Input of Remaining Data	59
VII. EXECUTED TEST PROBLEM	60
References	80

FIGURES

	Page
Fig. 1. Coordinates in (x,y) geometry.	2
Fig. 2. Coordinates in (r, θ) geometry.	2
Fig. 3. Coordinates in (r,z) geometry.	3
Fig. 4. Flow Chart of Subroutine READQF.	14
Fig. 5. Flow Chart of Subroutine PNGEN.	15
Fig. 6. Flow Chart of Subroutine GRIND2.	16
Fig. 7. Flow Chart of Subroutine INITAL.	17
Fig. 8. Flow Chart of Subroutine FISCAL.	19
Fig. 9. Flow Chart of Subroutine OUTER.	20
Fig. 10. Flow Chart of Subroutine INNER.	21
Fig. 11. Flow Chart of Subroutine FLXUP.	22
Fig. 12. Flow Chart of Subroutine SUTBC.	24
Fig. 13. Flow Chart of Subroutine REBAL.	25
Fig. 14. Flow Chart of Subroutine TESTS.	26
Fig. 15. Ordering of S_n directions.	31
Fig. 16. Orientation of S_6 directions in (x,y) geometry.	31
Fig. 17. Orientation of S_6 directions in (r,z) geometry.	32
Fig. 18. Orientation of S_6 directions in (r, θ) geometry.	32
Fig. 19. Variation of λ during a hypothetical eigenvalue search.	53
Fig. 20. Input data sheet.	57
Fig. 21. Flux contour plot.	78
Fig. 22. Flux projective plot.	79

TABLES

	Page
TABLE I. Area and Volume Elements	6
TABLE II. Structure of the TWOTRAN Program	13
TABLE III. Geometric Functions	18
TABLE IV. Contents of Blank Common Block 1A	34
TABLE V. Contents of Named Common Block FWBGN1	39
TABLE VI. Contents of Named Common Block FWBGN2	41
TABLE VII. Contents of Named Common Block LOCAL	42
TABLE VIII. ECS Storage	43
TABLE IX. File Name and Unit Requirement Table	44
TABLE X. Ordering of Anisotropic Distributed Source Components	45
TABLE XI. Options for Special Read Formats	55

THEORY AND USE OF THE GENERAL-GEOMETRY TWOTRAN PROGRAM

by

K. D. Lathrop and F. W. Brinkley

ABSTRACT

Finite difference equations are derived for the Boltzmann transport equation in (x,y) rectangular, (r,θ) cylindrical, and (r,z) cylindrical geometries. These equations are shown to be consistent with the analytic forms of the transport equation, and when used with the arithmetic mean difference assumption, to have second-order truncation error. The TWOTRAN program to solve the multigroup approximation to the transport equation using these difference equations is described. The theory of a coarse-mesh rebalancing procedure which accelerates all the iterative processes of the program is derived. Detailed input and usage instructions for the program are given, including flow charts of the major subroutines. Functional descriptions of the major sweeps through the space-angle mesh are provided. A sample problem is described.

I. INTRODUCTION

The general-geometry TWOTRAN program is designed to solve the two-dimensional transport equation in the multigroup discrete ordinates approximation in (x,y) , (r,θ) , and (r,z) geometries. The (x,y) and cylindrical (r,θ) geometries are planar and apply to media infinite in the direction transverse to the plane (although a buckling can be used), whereas the (r,z) cylindrical geometry is finite. A version of TWOTRAN is available for finite spherical (r,θ) geometry, but is different enough to require separate documentation.¹

The general-geometry program is a major revision of the TWOTRAN (x,y) program.^{2,3} In the revision, the difference scheme, the convergence acceleration procedures, the progression through the space-angle mesh, and the storage of data were all changed. In addition, the general-geometry program is optimized for production use and contains editing and plotting features. In the material below, no familiarity with other transport programs is assumed; indeed, we have made an effort to include

everything necessary to understand and use the program.

The options and features of TWOTRAN include:

1. Direct or adjoint calculations.
2. General order scattering anisotropy (limited only by fast memory storage capability).
3. "Group-at-a-time" solution so that storage requirements are almost independent of group size.
4. Vacuum, specular reflection, isotropic return, and periodic boundary conditions (in meaningful combinations).
5. Inhomogeneous source or k_{eff} , time-absorption, nuclide concentration, and zone thicknesses eigenvalue calculations. Fission is allowed in inhomogeneous source problems.
6. Coarse-mesh problem description.
7. Coarse-mesh rebalancing for acceleration of both inner and outer iteration processes.

8. Input of cross sections from either cards or disk file.
9. Built-in S_n constants through order S_{16} .
10. Flexible restart procedures.
11. Flexible input procedures for flux guesses and inhomogeneous sources.
12. Input of isotropic or anisotropic inhomogeneous distributed sources, and right and/or top and/or bottom boundary sources (incoming angular fluxes).
13. Detailed editing capability.
14. Extensive use of Extended Core Storage.

II. THEORY

A. Analytic Forms of the Transport Equation

The time-independent transport equation is written:

$$\nabla \cdot (\underline{\Omega} \psi) + \sigma_t \psi(\underline{r}, E, \underline{\Omega}) =$$

$$\iiint dE' d\Omega' \psi(\underline{r}, E', \underline{\Omega}') \sigma_s(E' \rightarrow E, \underline{\Omega}' \cdot \underline{\Omega}) \quad (1)$$

$$+ \nu(E) \iiint dE' d\Omega' \psi(\underline{r}, E', \underline{\Omega}') \nu \sigma_f(E') / 4\pi + Q(\underline{r}, E, \underline{\Omega}),$$

in which ψ is the particle flux (number density of particles times their speed) defined such that $\psi dV dE d\Omega$ is the flux of particles in the volume element dV about $\underline{r}$, in the element of solid angle $d\Omega$ about $\underline{\Omega}$, in the energy range dE about E . Similarly, $Q dV dE d\Omega$ is the number of particles in the same element of phase space emitted by sources independent of ψ . The macroscopic total interaction cross section is denoted by σ_t , the macroscopic scattering transfer probability (from energy E' to E through an angle with cosine $\underline{\Omega}' \cdot \underline{\Omega}$) by σ_s , and the macroscopic fission cross section by σ_f . All of these quantities may depend on $\underline{r}$, but we have omitted this argument for simplicity. The number of particles emitted isotropically ($1/4\pi$) per fission is ν , and the fraction of these liberated in the range dE about E is $\chi(E)$. This fraction may actually depend on both $\underline{r}$ and the fissioning species, but such possibilities are not admitted in the TWOTRAN program.

1. Particular Expressions for $\nabla \cdot \underline{\Omega} \psi$

The form of $\nabla \cdot \underline{\Omega} \psi \equiv \underline{\Omega} \cdot \nabla \psi$ may be determined either by writing the term as the derivative along the direction of particle motion and expressing this derivative in the variables of the coordinate

system of interest or by the more elegant method of Drawbaugh.⁴ For the coordinate systems considered here (shown in Figs. 1, 2, and 3), $\nabla \cdot (\underline{\Omega} \psi)$ is given by

$$a. \quad (x, y) \text{ geometry} \quad \psi = \psi(x, y, \mu, \eta)$$

$$\mu = \hat{e}_x \cdot \underline{\Omega} \quad \eta = \hat{e}_y \cdot \underline{\Omega}$$

$$\mu \frac{\partial \psi}{\partial x} + \eta \frac{\partial \psi}{\partial y} = \nabla \cdot \underline{\Omega} \psi \quad (2)$$

Fig. 1. Coordinates in (x, y) geometry.

Fig. 2. Coordinates in (r, θ) geometry.

the expansion coefficients are given by

$$a_n^k = \frac{1}{2\pi} \int_{-1}^1 d\mu \int_0^\pi d\varphi R_n^k(\mu, \varphi) / 2\pi \quad (10)$$

Using this formula, we can rewrite (1) as

$$\nabla \cdot (\underline{\Omega} \psi) + \sigma_t \psi =$$

$$\int_0^\infty dE' \sum_{n=0}^{ISCT} (2n+1) \sigma_{sn}(E' \rightarrow E) \sum_{k=0}^n R_n^k(\mu, \varphi) \psi_n^k + \nu(E) \int_0^\infty dE' \omega_{fn}(E') \psi_{oh}^0 + \sum_{n=0}^{IQAN} (2n+1) \sum_{k=0}^n R_n^k \psi_n^k \quad (11)$$

As implied by the sum in this equation, we have assumed that the source Q is representable by $IQAN+1$ terms of an expansion like (9).

B. Multigroup Difference Equations

1. Energy Group Representation

The energy domain of interest is assumed to be partitioned into IGM intervals of width ΔE_g , $g = 1, 2, \dots, IGM$. By convention, increasing g represents decreasing energy. If we multiply (11) by ΔE_g and integrate we can write

$$\nabla \cdot (\underline{\Omega} \psi_g) + \sigma_{tg} \psi_g =$$

$$\sum_{h=1}^{IGM} \sum_{n=0}^{ISCT} (2n+1) \sigma_{snh-g} \sum_{k=0}^n R_n^k \psi_{nh}^k + \nu_g \sum_{h=1}^{IGM} \omega_{fnh} \psi_{oh}^0 + \sum_{n=0}^{IQAN} (2n+1) \sum_{k=0}^n R_n^k \psi_{ng}^k \quad (12)$$

Here, the flux for group g ,

$$\psi_g = \int_{\Delta E_g} \psi dE \quad (13)$$

is no longer a distribution in energy, but is the total number of particles in the energy interval. For this reason, when group structures are changed, the effect on results must be evaluated by comparing $\psi_g / \Delta E_g$. Because of (13), energy integrals in TWO-TRAN are evaluated by simple sums.

The cross sections subscripted with g are averages, e.g.,

$$\sigma_{tg} = \int_{\Delta E_g} \sigma_t \psi dE / \int_{\Delta E_g} \psi dE \quad (14)$$

but, of course, ψ is not known and must be approximated by some means. If in (14) the angular dependence of ψ is nonseparable, then σ_{tg} will depend on angle. No provision for such dependence is made in TWO-TRAN. Recipes for taking this dependence into account, as well as for improving the averages σ_{snh-g} when scattering is severely anisotropic, are given by Bell and Hansen.⁵

2. Differencing of $\nabla \cdot (\underline{\Omega} \psi)$

To concentrate on the finite difference representation of the streaming operator, we write (12) as

$$\nabla \cdot \underline{\Omega} \psi + \sigma_t \psi = S \quad (12a)$$

omitting the group subscript. Below, whenever we specialize to a particular geometry, we use (r, θ) cylindrical geometry. All of the results follow in the other geometries when the appropriate variables and definitions are used.

We first partition phase space into IT intervals in the r (or x) direction, JT intervals in the θ (or z) direction, and MT solid angle intervals such that

$$\begin{aligned} r_{i-1/2} &< r_i < r_{i+1/2} & i = 1, 2, \dots, IT \\ \theta_{j-1/2} &< \theta_j < \theta_{j+1/2} & j = 1, 2, \dots, JT \\ \Omega_{m-1/2} &< \Omega_m < \Omega_{m+1/2} & m = 1, 2, \dots, MT \end{aligned} \quad (15)$$

In this notation, all cell boundaries are indicated with a $\pm 1/2$ subscript, but we deliberately refrain from specifying Ω -boundaries. We also use $d\Omega = d\varphi d\mu / 2\pi$ or $d\Omega = d\mu d\varphi / 2\pi$, whichever is convenient. Here, the 2π arises from our convention of measuring solid angle in units of 2π so that angular integrals are normalized to unity.

We orient our geometries so that x , r , and r are in the i -direction while y , θ and z are in the j -direction. We refer to $j = JT$ as the top of the system, to $j = 1$ as the bottom, to $i = 1$ as the inside or left side, and to $i = IT$ as the outside or right side of the system. Although not necessary in (x, y) geometry, in the cylindrical geometries we must have $r_1 = 0$, because we have made

no provision for a cavity boundary condition.

We next multiply (12a) by the phase space volume ($dVd\Omega$) and integrate over one cell of our partitioning. For all geometries we represent the result by

$$\begin{aligned} w_m \mu_m (A_{i+\frac{1}{2},j} N_{i+\frac{1}{2},j,m} - A_{i-\frac{1}{2},j} N_{i-\frac{1}{2},j,m}) \\ + (A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j}) (\alpha_{m+\frac{1}{2}} N_{i,j,m+\frac{1}{2}} - \alpha_{m-\frac{1}{2}} N_{i,j,m-\frac{1}{2}}) \\ + w_m \eta_m (B_{i,j+\frac{1}{2}} N_{i,j+\frac{1}{2},m} - B_{i,j-\frac{1}{2}} N_{i,j-\frac{1}{2},m}) \\ + \sigma_t w_m V_{ij} N_{ijm} = w_m V_{ij} S_{ijm} \end{aligned} \quad (16)$$

and define

$$\begin{aligned} \text{volume } V_{ij} &= \iint_{ij} dV = \iint_{ij} dA dB, \\ \text{i-direction surface area } A_{i+\frac{1}{2},j} &= \int_j dA_{i+\frac{1}{2}}, \\ \text{j-direction surface area } B_{i,j+\frac{1}{2}} &= \int_i dB_{j+\frac{1}{2}}, \\ \text{quadrature weight } w_m &= \iint_m d\Omega. \end{aligned} \quad (17)$$

The N 's that appear in (16) are certain averages of ψ over the phase space cell or its boundaries. We illustrate these averages in (r, θ) geometry where the result of multiplying (12a) by $dVd\Omega \equiv r dr d\theta d\Omega \equiv r dr d\theta \frac{d\zeta d\omega}{2\pi}$ (measuring solid angle in units of 2π) is

$$\begin{aligned} r dr d\theta d\Omega \left[\mu \frac{\partial(r\psi)}{\partial r} \right] + r dr d\theta \frac{d\zeta d\omega}{2\pi} \left[\frac{\partial(-\eta\psi)}{\partial \omega} \right] \\ + r dr d\theta d\Omega \left(\eta \frac{\partial\psi}{\partial \theta} \right) + dVd\Omega (\sigma_t \psi) = dVd\Omega (S). \end{aligned} \quad (18)$$

Now when this equation is integrated over a cell and compared with (16), we have

$$w_m \mu_m A_{i+\frac{1}{2},j} N_{i+\frac{1}{2},j,m} = \iiint_{j,m} d\theta d\Omega r_{i+\frac{1}{2}} \mu \psi_{i+\frac{1}{2}}, \quad (19)$$

$$(A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j}) \alpha_{m+\frac{1}{2}} N_{i,j,m+\frac{1}{2}} = \iiint_{i,j} \frac{dr d\theta d\zeta (-\eta\psi)_{m+\frac{1}{2}}}{2\pi}, \quad (20)$$

$$w_m \eta_m B_{i,j+\frac{1}{2}} N_{i,j+\frac{1}{2},m} = \iiint_{i,m} dr d\Omega \eta \psi_{j+\frac{1}{2}}, \quad (21)$$

$$w_m V_{ij} N_{ijm} = \iiint_{i,j,m} dV d\Omega \psi, \quad (22)$$

$$w_m V_{ij} S_{ijm} = \iiint_{i,j,m} dV d\Omega S. \quad (23)$$

In the last two averages, we assume that cross sections are constant over a cell. In the other averages, the fluxes ψ with subscripts denote the flux evaluated at a cell boundary; e.g., $\psi_{i+\frac{1}{2}} = \psi(r_{i+\frac{1}{2}}, \theta, \zeta, \omega)$. From (17) we have

$$\begin{aligned} V_{ij} &= \int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} r dr d\theta = \\ &= \pi (r_{i+\frac{1}{2}}^2 - r_{i-\frac{1}{2}}^2) (\theta_{j+\frac{1}{2}} - \theta_{j-\frac{1}{2}}), \end{aligned} \quad (24)$$

$$A_{i+\frac{1}{2},j} = \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} r_{i+\frac{1}{2}} d\theta = 2\pi r_{i+\frac{1}{2}} (\theta_{j+\frac{1}{2}} - \theta_{j-\frac{1}{2}}), \quad (25)$$

$$B_{i,j+\frac{1}{2}} = \int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} dr = (r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}), \quad (26)$$

$$w_m = \iint_{\Delta\Omega_m} d\zeta d\omega / 2\pi, \quad (27)$$

where we measure θ in units of revolutions. Using these values in (22), we get

$$N_{ijm} = \frac{\int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} r dr \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} d\theta \iint_{\Delta\Omega_m} d\zeta d\omega \psi / 2\pi}{\int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} r dr \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} d\theta \iint_{\Delta\Omega_m} d\zeta d\omega / 2\pi}. \quad (28)$$

We then approximate the right-hand side of (19) by

$$\iiint_{j,m} d\theta d\Omega r_{i+\frac{1}{2}} \mu \psi_{i+\frac{1}{2}} \approx \mu_m \iiint_{j,m} d\theta d\Omega r_{i+\frac{1}{2}} \psi_{i+\frac{1}{2}}, \quad (29)$$

which, by the mean value theorem, is true for some μ in the angular interval, provided ψ is positive. By assuming that μ_m is independent of i we introduce the ray effect.⁶ This approximation and a similar one for η_m lead to the average flux definitions

TABLE I

Area and Volume Elements

Geometry	dV	dA	dB	$A_{i+\frac{1}{2},j}$	$B_{i,j+\frac{1}{2}}$	V_{ij}	$\frac{A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j}}{V_{ij}}$	$\frac{A_{i+\frac{1}{2},j} + A_{i-\frac{1}{2},j}}{V_{ij}}$	$\frac{B_{i,j+\frac{1}{2}} + B_{i,j-\frac{1}{2}}}{V_{ij}}$
(x,y)	dx dy	dy	dx	Δy_j	Δx_i	$\Delta x_i \Delta y_j$	0	$2/\Delta x_i$	$2/\Delta y_j$
(r, θ) ^a	$r d\theta dr$	$r d\theta$	dr	$2\pi r_{i+\frac{1}{2}} \Delta\theta_j$	Δr_i	$2\pi \bar{r} \Delta r_i \Delta\theta_j$	$1/\bar{r}$	$2/\Delta r_i$	$2/2\pi \bar{r} \Delta\theta_j$
(r,z)	$2\pi r dr dz$	$2\pi r dz$	$2\pi r dr$	$2\pi r_{i+\frac{1}{2}} \Delta z_j$	$2\pi \bar{r} \Delta r_i$	$2\pi \bar{r} \Delta r_i \Delta z_j$	$1/\bar{r}$	$2/\Delta r_i$	$2/\Delta z_j$

^a θ measured in revolutions

$$\bar{r} \equiv (r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}})/2.$$

$$N_{i+\frac{1}{2},j,m} = \frac{\int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} d\theta \int_{\Delta\Omega_m} d\Omega \psi_{i+\frac{1}{2}}}{2\pi\theta_{j+\frac{1}{2}} - 2\pi\theta_{j-\frac{1}{2}}}, \quad (30)$$

$$N_{i,j+\frac{1}{2},m} = \frac{\int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} dr \int_{\Delta\Omega_m} d\Omega \psi_{j+\frac{1}{2}}}{r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}}.$$

Without defining $\alpha_{m+\frac{1}{2}}$, we cannot complete the definition of $N_{i,j,m+\frac{1}{2}}$ further than

$$N_{i,j,m+\frac{1}{2}} = \frac{\int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} dr \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} d\theta \int d\xi (-\eta\psi)_{m+\frac{1}{2}}/2\pi}{\alpha_{m+\frac{1}{2}} \int_{r_{i-\frac{1}{2}}}^{r_{i+\frac{1}{2}}} dr \int_{2\pi\theta_{j-\frac{1}{2}}}^{2\pi\theta_{j+\frac{1}{2}}} d\theta}. \quad (31)$$

By $(-\eta\psi)_{m+\frac{1}{2}}$ we mean the value of $(-\eta\psi)$ evaluated at the w corresponding to Ω_m . As we will see below, we can relate α to the ξ integral of $(-\eta\psi)_{m+\frac{1}{2}}$ in the limit of small intervals, but we do not do this. Rather, we define α in a way that insures consistency of the difference equations.

For all three geometries the area, volume, and phase space elements are given in Table I. Note that in (x,y) geometry in which there is no angular

derivative, $A_{i+\frac{1}{2},j} = A_{i-\frac{1}{2},j}$ so that the term involving α vanishes from (16).

3. Consistency and Truncation Error of the Difference Equations for $\nabla \cdot (\Omega\psi)$

Let us expand the fluxes in (16) about the point $(r_i, \theta_j, \Omega_m)$ assumed to be located in the center of the interval and take $N_{i+\frac{1}{2},j,m}$ to be $\psi(r_{i+\frac{1}{2}}, \theta_j, \Omega_m)$, etc., so that

$$N_{i+\frac{1}{2},j,m} = \psi \pm \frac{\Delta r}{2} \frac{\partial \psi}{\partial r} + \frac{(\Delta r)^2}{8} \frac{\partial^2 \psi}{\partial r^2} \pm \dots, \quad (32)$$

$$N_{i,j+\frac{1}{2},m} = \psi \pm \frac{\Delta\theta}{2} \frac{\partial \psi}{\partial \theta} + \frac{(\Delta\theta)^2}{8} \frac{\partial^2 \psi}{\partial \theta^2} \pm \dots, \quad (33)$$

$$N_{i,j,m+\frac{1}{2}} = \psi \pm \frac{\Delta w}{2} \frac{\partial \psi}{\partial w} + \frac{(\Delta w)^2}{8} \frac{\partial^2 \psi}{\partial w^2} \pm \dots, \quad (34)$$

$$N_{i,j,m} = \psi = \psi(r_i, \theta_j, \Omega_m). \quad (34a)$$

Dividing the equation by $w_m V_{ij}$ and using these relations gives

$$\begin{aligned} & \left[\frac{w_m \mu_m (A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j})}{2 V_{ij}} \right. \\ & \quad + (A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j}) (\alpha_{m+\frac{1}{2}} - \alpha_{m-\frac{1}{2}}) \\ & \quad \left. w_m \eta_m (B_{i,j+\frac{1}{2}} - B_{i,j-\frac{1}{2}}) \right] \psi / w_m V_{ij} \\ & + \mu_m \frac{(A_{i+\frac{1}{2},j} + A_{i-\frac{1}{2},j}) \Delta r}{2 V_{ij}} \frac{\partial \psi}{\partial r} \\ & + \eta_m \frac{(B_{i,j+\frac{1}{2}} + B_{i,j-\frac{1}{2}}) \Delta \theta}{2 V_{ij}} \frac{\partial \psi}{\partial \theta} \end{aligned}$$

$$\begin{aligned}
& + \frac{A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j}}{V_{i,j}} \frac{(\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}}) \Delta w}{2 w_m} \frac{\partial \psi}{\partial w} \\
& + \mu_m \frac{(A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j})}{V_{i,j}} \frac{(\Delta r)^2}{8} \frac{\partial^2 \psi}{\partial r^2} \\
& + \frac{(A_{i+\frac{1}{2},j} - A_{i-\frac{1}{2},j})}{w_m V_{i,j}} (\alpha_{m+\frac{1}{2}} - \alpha_{m-\frac{1}{2}}) \frac{(\Delta w)^2}{8} \frac{\partial^2 \psi}{\partial w^2} \\
& + \eta_m \frac{(B_{i,j+\frac{1}{2}} - B_{i,j-\frac{1}{2}})}{V_{i,j}} \frac{(\Delta \theta)^2}{8} \frac{\partial^2 \psi}{\partial \theta^2} + \dots \sigma_t \psi = S. \quad (35)
\end{aligned}$$

In order to approximate the transport equation for small intervals, the first term in ψ in this equation must vanish as $w_m V_{i,j} \rightarrow 0$. We choose to make it vanish identically. Remembering that $B_{i,j+\frac{1}{2}} = B_{i,j-\frac{1}{2}}$, we can accomplish this by demanding that

$$\alpha_{m+\frac{1}{2}} - \alpha_{m-\frac{1}{2}} = -w_m \mu_m \quad (36)$$

Using this relationship and the values of A , B , and V (see Table I), we have

$$\mu \frac{\partial \psi}{\partial r} + \frac{(\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}}) \Delta w}{2 w_m} \frac{\partial \psi}{\partial w} + \frac{1}{r} \frac{\partial \psi}{\partial (2\pi\theta)} + \sigma_t \psi = \quad (37)$$

$$S + O(\Delta r^2) + O(\Delta w^2) + O(\Delta \theta^2).$$

The $O(\Delta \theta^2)$ terms here are proportional to $\partial^3 \psi / \partial \theta^3$ since the sum $(B_{i,j+\frac{1}{2}} + B_{i,j-\frac{1}{2}}) / V_{i,j} \sim 1 / \Delta \theta$. In this equation, as in Table I, $\bar{r} = (r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}}) / 2$.

Using our definition (36) of α , we have in the limit of small Δw (letting $w_m = \Delta w \Delta \xi$)*

$$\frac{d\alpha}{dw} = -\mu \Delta \xi \quad (38)$$

Integrating, we find

$$\alpha = -\eta \Delta \xi + C \quad (39)$$

Noting, from (3), the form of the angular derivative, we choose $C = 0$. This choice of C implies that α vanishes when $\eta = 0$, i.e., when $w = 0$ or $w = \pi$. We shall use this to insist that first and last values of $\alpha_{m+\frac{1}{2}}$ defined by (36) vanish.

Using (39) and (38), and writing

$$\alpha_{m+\frac{1}{2}} = \alpha + \frac{\Delta w}{2} \frac{\partial \alpha}{\partial w} + \frac{(\Delta w)^2}{8} \frac{\partial^2 \alpha}{\partial w^2} \dots \quad (40)$$

we can sum the series to find

$$\alpha_{m+\frac{1}{2}} = -\Delta \xi (\eta \cos \frac{\Delta w}{2} + \mu \sin \frac{\Delta w}{2}) \quad (41)$$

This leads to

$$\begin{aligned}
\frac{(\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}}) \Delta w}{2 w_m} &= -\eta \cos \frac{\Delta w}{2} \\
&= -\eta [1 + O(\Delta w^2)] \quad (42)
\end{aligned}$$

Thus, in the limit of small intervals, (37) approaches the analytic transport equation,

$$\mu \frac{\partial \psi}{\partial r} - \frac{\eta}{r} \frac{\partial \psi}{\partial w} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} + \sigma_t \psi = S \quad (43)$$

with second-order error.

With the condition that beginning α values vanish on each η level, we insure that a very important property of the analytic equations is maintained. In the sum of (16) over m , the terms in α vanish. That is, there is no net particle loss due to angular redistribution.

The conventions we have noted in this section include:

1. The angle theta is measured in revolutions,
2. Solid angles are measured in units of 2π ,
3. Direction cosines are independent of space,
4. The α coefficients are defined recursively such that $\alpha_{m+\frac{1}{2}} - \alpha_{m-\frac{1}{2}} = -w_m \mu_m$ with first and last α values vanishing on each η level,
5. Cross sections are constant in each space cell. We actually assume that cross sections are regionwise constant, but a region may be $\geq$ a single cell.

4. Definition of the Source Term

On the right-hand side of (12) appears the term

$$\psi_{nh}^k = \int_{-1}^1 d\mu \int_0^\pi d\varphi R_n^k(\mu, \varphi) \psi_n / 2\pi \quad (44)$$

For each group we approximate this integral by the sum

*For convenience here we measure Δw in units of 2π .

$$\lim_{\substack{\eta \rightarrow 0 \\ \mu \rightarrow 0}} \frac{d\mu d\eta}{2\pi} R_n^k(\mu, \eta) \doteq N_{nhij}^k = \sum_{m=1}^{MT} w_m R_n^k(\mu_m, \eta_m) N_{nhijm}, \quad (45)$$

where η_m is defined in the program by

$$\begin{aligned} \eta_m &= \tan^{-1} \left[(1 - \mu_m^2 - \eta_m^2)^{\frac{1}{2}} / \eta_m \right] \quad \eta_m > 0, \\ &= \tan^{-1} \left[(1 - \mu_m^2 - \eta_m^2)^{\frac{1}{2}} / \eta_m \right] + \pi \quad \eta_m < 0. \end{aligned} \quad (46)$$

Because we customarily deal with only one quadrant of directions, the program uses four functions for R_n^j :

$$\begin{aligned} R_n^j(\mu_m, \eta_m) &= R_{nm}^j = P1_{nm}^j \quad \mu < 0, \eta < 0, \\ &= P2_{nm}^j \quad \mu > 0, \eta < 0, \\ &= P3_{nm}^j \quad \mu < 0, \eta > 0, \\ &= P4_{nm}^j \quad \mu > 0, \eta > 0. \end{aligned} \quad (47)$$

With this notation we have the right side of (16), S_{gijm} , given by the sum of

$$(SS)_{gijm} = (\text{Scatter Source})_{gijm} =$$

$$\sum_{h=1}^{IGM} \sum_{n=0}^{ISCT} (2n+1) \sigma_{snh-g} \sum_{j=0}^n R_{nm}^k N_{nhij}^k, \quad (48)$$

$$(FS)_{gijm} = (\text{Fission Source})_{gijm} =$$

$$v_g \sum_{h=1}^{IGM} w_g f_n N_{ohij}^0, \quad (49)$$

$$(IS)_{gijm} = (\text{Inhomogeneous Source})_{gijm} =$$

$$\sum_{n=0}^{IQAN} (2n+1) \sum_{k=0}^n R_{nm}^k Q_{ngij}^k. \quad (50)$$

C. Solution of the Difference Equations

In the following material we adopt the convention that centered subscripts (i, j, m, g) are omitted except to indicate sums or to clarify equations. We write our finite difference equations for group g as

$$\begin{aligned} &\mu (A_{i+\frac{1}{2}, j+\frac{1}{2}} N_{i+\frac{1}{2}, j+\frac{1}{2}} - A_{i-\frac{1}{2}, j-\frac{1}{2}} N_{i-\frac{1}{2}, j-\frac{1}{2}}) \\ &+ (A_{i+\frac{1}{2}, j} - A_{i-\frac{1}{2}, j}) (\alpha_{m+\frac{1}{2}, j+\frac{1}{2}} N_{m+\frac{1}{2}, j+\frac{1}{2}} - \alpha_{m-\frac{1}{2}, j-\frac{1}{2}} N_{m-\frac{1}{2}, j-\frac{1}{2}}) / w \\ &+ \eta B (N_{j+\frac{1}{2}} - N_{j-\frac{1}{2}}) + \sigma_t V N = V(S + FS + IS). \end{aligned} \quad (51)$$

Here we have indicated that B is independent of j in the three geometries considered. This equation is to be solved for MT directions defined by ordered pairs (μ_m, η_m) , for $IT \times JT$ space cells, and for IGM energy groups.

1. Boundary Conditions

Equation (51) contains seven N 's. Information about boundary flux values can be specified by the TWOTRAN user by designating the following options:

a. Vacuum Boundary Condition

The value of the angular flux N on the boundary is set to zero for all incoming directions.

b. Reflective Boundary Condition

The value of the flux on the boundary for incoming directions is set equal, in detail, to the value of the outgoing flux in the direction corresponding to specular reflection. For example, at an i -direction outer boundary,

$$\psi_{\text{incoming}}(-\mu, \eta) = \psi_{\text{outgoing}}(\mu, \eta) \quad \mu > 0. \quad (52)$$

c. White Boundary Condition

The values of incoming boundary fluxes are all set equal to the same value. This value is the average of the outgoing fluxes such that the net flow through the boundary is zero. In particular, for an i -direction boundary, this value is given by

$$\psi_{\text{incoming}} = \frac{\sum_m w_m \mu_m \psi(\mu_m, \eta_m)_{\text{outgoing}}}{\sum_m w_m \mu_m}, \quad (53)$$

where the sums range over outgoing directions. This condition is meaningful for cell calculations in cylindrical geometries where it is applied to the outside radial boundary.

d. Periodic Boundary Condition

The values of incoming fluxes at a boundary are set equal, in detail, to the values of the outgoing fluxes on the opposite boundary. This condition is allowed only on the top and bottom

(j-direction) boundaries. Then, the incoming flux on the bottom is set equal to outgoing flux on the top, and the incoming flux at the top to the outgoing flux on the bottom. This condition must be used to represent any full circle in (r,θ) geometry in which the system is not homogeneous in θ.

e. Boundary Sources

On the top, bottom, and right sides of the domain, the user may specify the incoming angular flux in each direction. These inflows represent a source of particles and are treated as such by the code. For example, the number of particles incoming from the right-boundary source, QR_{mj} , is given by

$$\text{Number} = \sum_{m,j} w_m \mu_m QR_{mj} A_{m+\frac{1}{2},j} \quad (54)$$

where the sum is over incoming directions.

2. Arithmetic Mean (Diamond) Difference Approximation

To solve (51) we need additional relations among the fluxes. To second-order accuracy, we have from (32), (33), and (34)

$$2N = N_{i+\frac{1}{2}} + N_{i-\frac{1}{2}} \quad (55)$$

$$2N = N_{j+\frac{1}{2}} + N_{j-\frac{1}{2}} \quad (56)$$

$$2N = N_{m+\frac{1}{2}} + N_{m-\frac{1}{2}} \quad (57)$$

These relations are usually called the "Diamond" difference equations. If we assume that in (51) we know three fluxes, say $N_{i+\frac{1}{2}}$, $N_{j+\frac{1}{2}}$, and $N_{m-\frac{1}{2}}$ and that we want to calculate $N_{i-\frac{1}{2}}$, $N_{j-\frac{1}{2}}$, and $N_{m+\frac{1}{2}}$ [implying we are moving inward ($\mu < 0$), downward ($\eta < 0$), and in order of increasing angle subscript], we can use (55), (56), and (57) to eliminate the fluxes we are going to solve for from (51), giving a single equation for N,

$$N = \frac{|\mu| (A_{i+\frac{1}{2}} + A_{i-\frac{1}{2}}) N_{i+\frac{1}{2}} + 2|\eta| B N_{j+\frac{1}{2}} + (A_{i+\frac{1}{2}} - A_{i-\frac{1}{2}}) (\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}}) w / N_{m-\frac{1}{2}} + SV}{|\mu| (A_{i+\frac{1}{2}} + A_{i-\frac{1}{2}}) + 2|\eta| B + (A_{i+\frac{1}{2}} - A_{i-\frac{1}{2}}) (\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}}) / w + \sigma_t V} \quad (58)$$

In writing this equation we used (36) to arrange the denominator so that the coefficients are the same as those of the numerator. Using this result

we can use (55), (56), and (57) to solve for $N_{i-\frac{1}{2}}$, $N_{j-\frac{1}{2}}$, and $N_{m+\frac{1}{2}}$. Equation (58) applies for outward progress as well if $N_{i+\frac{1}{2}}$ is replaced with $N_{i-\frac{1}{2}}$.

Similarly, when moving upward, we use $N_{j-\frac{1}{2}}$ instead of $N_{j+\frac{1}{2}}$. We always move in order of increasing angle subscript. Thus (58) can always be used when we know three fluxes on the boundary of our phase space cell. To initialize the spatial boundary fluxes, we can start applying (58) at the outer boundaries for incoming directions. To initialize $N_{m-\frac{1}{2}}$, we use the step function approximation for the first incoming μ direction on each η level, setting $N_{m+\frac{1}{2}} = N$. This assumption leads to an equation for N exactly like (58) if we assume that $(\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}})/w$

is zero. That is, these terms do not appear in the formula. This is an important difference of the

TWOTRAN program compared to existing programs:

TWOTRAN does not use special zero-weighted directions to initialize the angular flux $N_{m-\frac{1}{2}}$. Therefore, for a given S_n order, the TWOTRAN program uses fewer directions.

The actual progression through the space-angle mesh is described in Section III.C below. There we give details of the angular and space coordinates and the manipulations involved in using (58).

3. Negative Flux Control

Although all the coefficients of (58) are positive, extrapolations of the form $N_{i+\frac{1}{2}} = 2N - N_{i-\frac{1}{2}}$ can lead to negative values for fluxes. When sources are positive, we attempt to prevent negative fluxes by using a "set-flux-to-zero-and-correct" recipe. After much experimentation with other difference schemes and recipes,⁷ including their use as fixups, we have decided that the set-to-zero scheme is the most accurate while being computationally no more expensive. It is most accurate because it is the minimum derivation from the second-order accuracy of the diamond difference equation.

All linear difference schemes which lead to positive coefficients in the extrapolations like (55) - (57) are first-order accurate,⁷ and all other

than the set-to-zero scheme lead to positive extrapolated fluxes and hence, to a greater deviation from the negative diamond result. Secondly, the set-to-zero scheme is more selective, not only because it is used only when a flux is negative, unlike a weighted diamond scheme, but also because it is applied to the extrapolations individually, unlike a scheme in which all fluxes are recomputed with, say, a step function approximation.

The logic of the set-to-zero fixup is as follows. If any one flux is negative, it is set to zero and the cell-centered flux, N , is recomputed assuming that particular flux is zero. (See formulas below). If this N should be negative, the fixup attempt is aborted. If not, the other two fluxes are re-extrapolated. If either one of these is negative, N is recomputed assuming two fluxes are negative. If this N is positive, the remaining flux is extrapolated. If it should be negative, then N is recomputed assuming all three boundary fluxes are zero (provided the total cross section is not zero). If any center N is negative (implying a negative source or boundary flux) or if any fixup leads to a division by zero (e.g., in voids), the fixup attempt is aborted. One cannot exclude negative sources because the finite Legendre polynomial expansion of anisotropic scattering may produce negative numbers in particular directions.

Formulas for fixup are derived as follows. Suppose we are attempting to calculate $N_{i-\frac{1}{2}}$ and find it negative. Then we use (56) and (57) in (51) where in (51) $N_{i-\frac{1}{2}}$ is taken to be zero. We have then

$$N = \mu |A_{i+\frac{1}{2}} N_{i+\frac{1}{2}} + 2|\eta|BN_{j+\frac{1}{2}} +$$

$$\frac{(A_{i+\frac{1}{2}} - A_{i-\frac{1}{2}})(\alpha_{m+\frac{1}{2}} + \alpha_{m-\frac{1}{2}})N_{m-\frac{1}{2}}/w + SV}{2|\eta|B + 2(A_{i+\frac{1}{2}} - A_{i-\frac{1}{2}})\alpha_{m+\frac{1}{2}}/w + \sigma_t V} \quad (59)$$

This formula, of course, lacks the symmetry of (58). If we had been trying to compute $N_{i+\frac{1}{2}}$ ($\mu > 0$) and found it negative, then (59) would be altered by using $A_{i-\frac{1}{2}}N_{i-\frac{1}{2}}$ instead of $A_{i+\frac{1}{2}}N_{i+\frac{1}{2}}$. In either case the negative flux is set to zero, and, provided N itself is positive, the remaining two fluxes, $N_{j-\frac{1}{2}}$ and $N_{m+\frac{1}{2}}$ are computed from (56) and (57). All other formulas needed (six more) can be generated in the

same fashion. These appear explicitly in subroutine FIXUP in the program.

Controlling negative fluxes in this fashion seems to be more compatible with the coarse-mesh rebalancing acceleration procedure (see the next section below) than any other fixup we examined.

4. Convergence Acceleration

The source in (51) usually depends on the fluxes being computed, through the within-group scattering and/or through fission or upscattering. We use an iterative process in which the source is updated when new flux information is available. Within each energy group we update the portion of the source due to within-group scattering (σ_{sng-g}) after each pass through the space-angle mesh. This iterative process, including iteration for implied boundary conditions (e.g., a reflective outer boundary condition), is called inner iteration and is performed by subroutine INNER. The iterative process involving the updating of sources due to fission and upscatter after a progression through all groups is called outer iteration and is performed by subroutine OUTER. Both of these processes can be accelerated by coarse-mesh rebalancing. In this section we describe the theory of rebalancing; in Section III.A.10 we describe the detailed application of rebalancing.

a. Inner Iteration Coarse-Mesh Rebalancing

When (51) is multiplied by w and summed over all directions, the resulting equation,

$$A_{i+\frac{1}{2}}I_{i+\frac{1}{2}} - A_{i-\frac{1}{2}}I_{i-\frac{1}{2}} + B(J_{j+\frac{1}{2}} - J_{j-\frac{1}{2}}) + \sigma_t V \phi = V \sigma_{sng-g} \phi^D + VQ \quad (60)$$

is a statement of particle balance. In this equation the i -direction currents, I , are given by

$$I_{i+\frac{1}{2},j} = \sum_{m=1}^{MT} w_m \mu_m N_{i+\frac{1}{2},j,m} = N_{i+\frac{1}{2},j}^0 \quad (61)$$

and the j -direction currents, J , by

$$J_{i,j+\frac{1}{2}} = \sum_{m=1}^{MT} w_m \eta_m N_{i,j+\frac{1}{2},m} = N_{i,j+\frac{1}{2}}^1 \quad (62)$$

When multiplied by the surface areas as in (60), these terms account for the net flow through the cell surface. The angle-integrated, or scalar, flux, ϕ , is defined by

$$\phi_{ij} = \sum_{m=1}^{MT} w_m N_{ijm} \approx N_{Oij}^0 \quad (63)$$

On the right side of (60) we have split the angle-integrated portion of VS into a part due to isotropic within-group scattering ($\sigma_{\text{sog-g}}$) and a remainder, Q. All anisotropic sources vanish when the m sum is performed, because of the orthogonality of the spherical harmonic functions R_n^k , provided that the quadrature set used correctly integrates these functions.

On the right of (60) we have indicated that the scattering source depends on the flux from a previous iteration by labeling this flux with a superscript p. The particle balance equation is satisfied only when $\phi = \phi^p$. It has long been realized⁸ that enforcing balance accelerates iterative convergence. DTF-IV⁹ and earlier Los Alamos codes use a system-wide within-group renormalization. Here we describe a variation of the technique which enforces particle balance in each zone of a coarse mesh.

The object of the technique is to find a factor by which all fluxes in a zone may be multiplied to insure that leakage plus absorption equals sources in the zone. Usually, application of the factor quickly brings all flux amplitudes within the zone close to their final amplitude, and subsequent iteration refines the flux shapes in the zone.

To describe the terms entering the rebalance equation, we superpose a coarse mesh on the fine space mesh defined by (15), letting [again using (r, θ) geometry to illustrate]

$$r_{k-\frac{1}{2}} < r_i < r_{k+\frac{1}{2}} \quad k = 1, 2, \dots, IM \quad (64)$$

$$\theta_{l-\frac{1}{2}} < \theta_j < \theta_{l+\frac{1}{2}} \quad l = 1, 2, \dots, JM \quad (65)$$

define the coarse mesh.

For each zone (k, l) we compute the upward, downward, rightward, and leftward flows,

$$FU_{k, l+\frac{1}{2}} = B_{l+\frac{1}{2}} \sum_{i \in k} \sum_{m > 0} w_m \eta_m N_{i, l+\frac{1}{2}, m} \quad (66)$$

$$FD_{k, l+\frac{1}{2}} = B_{l+\frac{1}{2}} \sum_{i \in k} \sum_{m < 0} w_m \eta_m N_{i, l+\frac{1}{2}, m} \quad (67)$$

$$FR_{k+\frac{1}{2}, l} = \sum_{j \in l} \sum_{m > 0} w_m \mu_m A_{k+\frac{1}{2}, j} N_{k+\frac{1}{2}, j, m} \quad (68)$$

$$FL_{k+\frac{1}{2}, l} = \sum_{j \in l} \sum_{m < 0} w_m \mu_m A_{k+\frac{1}{2}, j} N_{k+\frac{1}{2}, j, m} \quad (69)$$

the zone effective absorption,

$$AB_{kl} = \sum_{i \in k} \sum_{j \in l} V_{ij} (\sigma_t - \sigma_{\text{sog-g}}) \sum_{m=1}^{MT} w_m N_{ijm} \quad (70)$$

and the zone source,

$$QQ_{kl} = \sum_{i \in k} \sum_{j \in l} V_{ij} Q_{ij} \quad (71)$$

where Q is the nonself-scatter portion of S used in (60), i.e., the portion that does not change during iteration. If boundary-flux sources occur, they are placed in QQ_{kl} in the zones along the boundaries.

If now all fluxes are multiplied by the factor f_{kl} , we obtain the equation (written with our subscripting convention)

$$\begin{aligned} f_{kl} (FR_{k+\frac{1}{2}} + FL_{k-\frac{1}{2}} + FU_{l+\frac{1}{2}} + FD_{l-\frac{1}{2}} + AB) = \\ QQ + f_{k+1, l} FL_{k+\frac{1}{2}} + f_{k-1, l} FR_{k-\frac{1}{2}} \\ + f_{k, l+1} FD_{l+\frac{1}{2}} + f_{k, l-1} FU_{l-\frac{1}{2}} \end{aligned} \quad (72)$$

by equating losses in the cell (outflows plus absorption) to the sources (true source plus inflows from adjoining cells). This equation may also be deduced from a variational method using discontinuous (zone step functions) trial functions. Other trial functions lead to more complicated equations.¹⁰

With boundary conditions, (72) can be solved for the scale factors, f_{kl} . If the outer boundary condition is a vacuum condition, then the corresponding incoming flow is zero, eliminating the unknown f from the equation. For reflecting and white boundary conditions, we set the factor outside the boundary equal the factor just inside the boundary. For example, suppose the right boundary is reflecting. Then, at the boundary we set $f_{k+1, l} = f_{kl}$. Then, subtracting $f_{kl} FL_{k+\frac{1}{2}}$ from both sides, we have

$$f_{kl}(FR_{k+\frac{1}{2}} - FL_{k+\frac{1}{2}} + FL_{k-\frac{1}{2}} + FU_{l+\frac{1}{2}} + FD_{l-\frac{1}{2}} + AB) \\ = QQ + f_{k-1,l}FR_{k-\frac{1}{2}} + f_{k,l+1}FD_{l+\frac{1}{2}} \\ + f_{k,l-1}FU_{l-\frac{1}{2}} \quad (73)$$

The term $FR_{k+\frac{1}{2}} - FL_{k+\frac{1}{2}}$ is the net flow through the boundary and should vanish when the reflecting condition is satisfied. Such conditions are identically satisfied at the nonimplicit left and bottom boundaries. With a periodic boundary condition, the outgoing flux on, say, the bottom is used as the incoming flux on the top. Thus the factor $f_{k,l+1}$ for the top boundary zone is replaced with f_{kl} from the bottom boundary zone. A similar replacement is made for the bottom periodic boundary condition.

Equation (72) is solved iteratively, using a forward elimination, backward substitution on each k level. Iteration, including that for any periodic boundary condition, is performed in the l direction. If the iteration fails to converge after 200 iterations, if any factor becomes negative, or, if any denominator becomes zero, we set all f_{kl} equal to the single-system factor given by

$$f = \frac{\sum_{kl} QQ_{kl}}{NL + \sum_{kl} AB_{kl}} \quad (74)$$

where NL is the system net leakage determined from the boundary net flows. All of these operations are performed by subroutine REBAL.

b. Outer Iteration Coarse-Mesh Rebalancing

To accelerate the convergence of the outer iteration process, we determine a different set of scale factors, f . We collapse the entire group structure to a single group, accumulating the coarse-mesh flows from each group. We also calculate the total absorption,

$$ABT = \sum_{ick} \sum_{jcl} \sum_{g=1}^{IGM} \sigma_{ag} V_{ij} \sum_{m=1}^{MT} w_m N_{gijm} \quad (75)$$

To insure consistency, we insist that σ_{ag} be defined from the input cross sections by

$$\sigma_{ag} = \sigma_{tg} - \sum_{h=1}^{IGM} \sigma_{sog-h} \quad (76)$$

When we are performing an adjoint calculation we must, because of the cross-section transposition, calculate an effective absorption,

$$(\sigma_{ag})_{eff} = \sigma_{tg} - \sum_{h=1}^{IGM} \sigma_{sog-h} \quad (77)$$

The source for over-all-group rebalance consists of the group sum of the inhomogeneous source (if any) and the group sum of the fission source (if any). If there is a fission source, we perform a source iteration to determine the rebalance factors. If there is no inhomogeneous source, this iteration can also be used to estimate an eigenvalue, say k_{eff} . This can be visualized by replacing QQ in (73) by $f_{kl} FT_{kl}/k_{eff}$ where FT_{kl} is the total fission source. We choose to do this only for the first two outer iterations, thereafter considering the fission source to be a known inhomogeneous source. All of these iterations are controlled by subroutine TESTS.

The outer iteration rebalance process is advantageous because it accelerates all types of problems, e.g., inhomogeneous source problems with upscatter and/or fission, or eigenvalue problems with or without upscatter.

In both the within-group and over-all-group rebalancing calculations, the scale factors all usually approach unity in a few inner or outer iterations.

5. Adjoint Problems

The TWOTRAN program solves the adjoint transport equation by transposing the scattering and fission matrices and inverting the group order of the problem. The solution of the resulting problem in the direction $\underline{\Omega}$ is then identified with the adjoint solution in direction $-\underline{\Omega}$.¹¹ Transposition of the scattering matrix converts the normal, predominantly downscattering problem to an upscattering problem. Group order inversion restores this downscattering dominance and eliminates unnecessary upscattering iteration.

III. PROGRAM FUNCTIONAL DESCRIPTION

A. Program Flow

The normal flow of TWOTRAN consists of input, initialization, computation of the fission source, computation of the scattering source to a group,

TABLE II
Structure of the TWOTRAN Program

<u>OVERLAY (0,0)</u>	<u>OVERLAY (1,0)</u>	<u>OVERLAY (2,0)</u>	<u>OVERLAY (3,0)</u>	<u>OVERLAY (4,0)</u>
<u>TWOTRAN</u>	<u>INPUT1</u>	<u>GRIND?</u>	<u>OUTPUT3</u>	<u>DUMP4</u>
1. MONITR	1. READ	1. INITAL	1. FINAL	1. SPFDMP
2. ERROR	2. READI	2. INITQ	2. EDCALL	2. SP DDMP
3. CLEAR	3. REDUCE	3. FISCAL	3. EDITOR	
4. WRITE	4. LOAD	4. OUTER	4. PLOTS	
5. ECLAD	<u>INPUT11</u>	5. INNER	5. PLGTP	
6. ECDUMP		6. FIXUP	6. PLGTC	
7. SPREAD	<u>INPUT12</u>	7. SETBC	7. CONTRB	
8. UNITWR	1. CSPREP	8. REBAL	8. EDMAP	
9. JUDECS	2. READQF	9. GSUMS	9. CHMFLX	
10. MANAGR	3. SNCQN	10. TESTS		
	4. PNGEN	11. NEWPAR		
	5. LAXS			
	<u>INPUT13</u>			
	1. MAPPER			

computation of the scattering source in a group, setting of boundary conditions, solving for the group fluxes by sweeping the space-angle mesh, iterating on the within-group scattering source and boundary conditions if necessary, iterating on the fission or group-to-group scattering source, if necessary, and output. In this section we describe the functions of the subroutines that guide this flow. In subsequent sections we discuss particular aspects of the flow in detail.

1. Overlay Structure

TWOTRAN is divided into five segments. Program TWOTRAN itself calls INPUT1 (overlay 1), GRIND? (overlay 2), OUTPUT3 (overlay 3), and DUMP4 (overlay 4). INPUT1 is the only overlay which is divided, being composed of three sections. DUMP4 is a special section which prepares a decimal dump of TWOTRAN storage whenever a CDC mode error occurs. Periodic or final dumps of problem data are handled by subroutine MANAGR called from program TWOTRAN. The overlay structure itself and a list of the subroutines in each section are given in Table II. Below we describe and give flow charts for the most important routines of TWOTRAN. The service routines (e.g., MANAGR, LAXS, WRITE, etc.) are lumped together in a single section where their function is briefly stated.

2. Program TWOTRAN (overlay 0.0)

The program begins by identifying I/O systems and setting many parameters which are normally constant, but are treated as variables so that if they are changed, the only change necessary is in this routine. For example, the number of contour intervals for contour flux plotting is set to 25. Also this is the only routine in which the A vector is dimensioned. This program controls the progression through the overlay sections, of which there are four.

3. Program INPUT11 (overlay 1.1)

This program reads, checks, and prints the control data for the problem, including the number of fine-mesh intervals in each coarse-mesh interval. These include all the numbers necessary to allocate storage, which this program does by computing the first word address of all the storage arrays required. The program also computes the location of all arrays to be stored in Extended Core Storage (ECS).

4. Program INPUT12 (overlay 1.2)

This program continues reading and checking input, including cross sections (CSPREP) from cards or disk file (LAXS), sources (READQF), and fluxes (READQF). See Fig. 4 for a diagram of READQF. Program INPUT12 also reads S_n constants and generates

READQF

Fig. 4. Flow Chart of Subroutine READQF.

the spherical harmonics functions R_{3m}^k (PNGEN, see Fig. 5). These functions are computed from recursion relations in an algorithm very similar to that of the DTF-IV program for cylindrical geometry. There are a total of $NM = ((ISCT+1)*(ISCT+2))/2$ such functions in two-dimensional geometry, and the double sum,

$$\sum_{n=0}^{ISCT} \sum_{k=0}^n,$$

is always accomplished as a single sum,

$$\sum_{N=1}^{NM}.$$

Four sets of spherical harmonics (47) are computed, given μ_m and η_m as defined in (46).

5. Program INPUT13 (overlay 1.3)

This program completes the reading of the problem-dependent input data, including that needed for edit. A picture of the system is also drawn (MAPPER).

PNGEN

Fig. 5. Flow Chart of Subroutine PNGEN.

Fig. 6. Flow Chart of Subroutine GRIND2.

6. Program GRIND2 (overlay 2.0, Fig. 6)

This routine controls the flow through the major computational routines, calling INITIAL, FISCAL, OUTER, and TESTS. FISCAL is called before and after OUTER, but switching is used to avoid repeating the long call list.

7. Subroutine INITIAL (Fig. 7)

This subroutine computes all initial values

needed in the computation. Its rather fragmented organization is necessary to allow different types of eigenvalue searches. For example, the total cross-section array cannot be generated until after cross sections have been mixed, remixed (in a concentration search), or altered by a time absorption or a buckling.

First, the spherical harmonic functions gen-

Fig. 7. Flow Chart of Subroutine INITIAL.

TABLE III
Geometric Functions

Array Name and Dimension	(x,y)	Geometry (r,θ)	(r,z)
XH(IM)	Δx_i	Δr_i	Δr_i
YH(JM)	Δy_j	$\Delta \theta_j$	Δz_j
A3(IT)	Δx_i	Δr_i	$\pi(r_{i+\frac{1}{2}}^2 - r_{i-\frac{1}{2}}^2)$
A4(IT+1)	1.0	$2\pi r_{i-\frac{1}{2}}$	$2\pi r_{i-\frac{1}{2}}$
A5(IT)	Δx_i	$\pi(r_{i+\frac{1}{2}}^2 - r_{i-\frac{1}{2}}^2)$	$\pi(r_{i+\frac{1}{2}}^2 - r_{i-\frac{1}{2}}^2)$
A1(IT)	2.0	$2\pi(r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}})$	$2\pi(r_{i+\frac{1}{2}} + r_{i-\frac{1}{2}})$
A2(IT)	0.0	$2\pi(r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}})$	$2\pi(r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}})$
B1(JT)	$1/\Delta y_j$	$1/\Delta \theta_j$	$1/\Delta z_j$

$$\text{Volume, } V_{ij} = A5(I) * YH(J)$$

$$\text{Area, } A_{i+\frac{1}{2},j} = A4(I+1) * YH(J)$$

$$\text{Area, } B_i = A3(I)$$

erated in PNCEN are printed, and then the coefficients $\alpha_{m+\frac{1}{2}}$ are generated. TWOTRAN is organized on the assumption that the direction set is symmetric in μ and η so that no more than $MM = MT/4$ directions are used where $MT = (ISN)*(ISN+2)/2$ and ISN is the order (even) of the S_n quadrature set.

Next in INITIAL, group orders of the fission spectrum and velocities are inverted (cross sections are inverted and transposed in CSPREP) if the adjoint problem is to be solved, and then cross-section mixtures are made if any are required. If IEVT=4, either the coarse mesh radial or axial mesh boundaries, or both are shifted. Next, those geometric functions independent of cross sections are prepared, the fission spectrum is initialized, sources are processed (INITQ), and the fission source is initialized.

Geometric functions computed are summarized in Table III.

Because YH appears as a factor in both V and A, we divide the numerator and denominator of the recursion in (58) by YH. This leads to a factor of B1 in the area coefficient, B. We avoid storing any

two-dimensional arrays by using the factorizations of Table III. In INITIAL we also find it convenient to compute and store an array, CTOT(I,J), which is the product of $\sigma_{tg} * A5$. We compute this array here to add the buckling and/or time absorptions which may be present.

8. Subroutine FISCAL (Fig. 8)

This routine computes the volume integral of the fission source, λ (the ratio of all sources in one iteration to those in the previous), and performs normalizations of all fluxes. If IEVT=0 (a source calculation), λ is calculated, but normalizations are skipped (sources are normalized in INITIAL); and if IEVT=1, λ is used to reduce (or increase) the fission source to achieve criticality. The routine FISCAL is called at the beginning of the calculation and after every outer iteration.

9. Subroutine OUTER (Fig. 9)

The fundamental function of OUTER is to compute the total source to the group. Once this source is computed the problem is reduced to a one-group problem which is solved in INNER. The source to the group includes the fission source, distrib-

FISCAL

Fig. 8. Flow Chart of Subroutine FISCAL.

uted sources independent of the flux, and all isotropic and anisotropic scattering sources except the within-group scattering source. These sources are summed in the array $Q(N, I, J)$ which is also used for distributed sources if there are any. For this reason, the N dimension of this array is NM even if the source array size, NMQ , is less than NM . Also, the nondistributed-source portion of the array is zeroed in OUTER.

The calculation of the scattering source requires the use of the fluxes from all groups, and

these are stored in ECS units. In TWOTRAN, the dimensioning of the flux arrays allows a completely general anisotropic scattering source. In the course of generating the source, the cross sections for the groups are needed. These, as well as distributed sources and boundary sources, are read from ECS for each group.

The flow in OUTER begins with the first group ($G=1$). When the source is computed in this group, INNER is called (unless there is no source, in which case fluxes are zeroed and INNER is skipped) to

OUTER

Fig. 9. Flow Chart of Subroutine OUTER.

generate the new flux in this group. After return from INNER, sums of quantities over groups are accumulated, both in OUTER and GSUMS. After the first group is finished, the source to the next group is generated using the new first-group flux, but the fission source and the upscattering source are not based on new flux information. A complete group traverse in OUTER completes an outer iteration. If there are no fission or upscattering sources, no more iterations are required (except for printing nonstored quantities such as the angular flux). The decision to make more outer iterations is made in TESTS (which is called by MAIN after calling FISCAL once more).

After the group traverse in OUTER, a new fission rate (in direct problems, the group sum of the product of the angle-integrated flux and fission cross section) and the coarse-mesh flow totals are computed. Each of these computations is made at

this point to save storage space. The new fission rate is stored in the same location as the old, and the flow totals are stored in the within-group flow locations.

In outer, provision is made for taking periodic dumps or time-limit dumps after completing a calculation in any group. Periodic dumps are taken every 15 minutes (a variable set in TWOTRAN), while a time-limit dump is taken after a user input time.

10. Subroutine INNER (Fig. 10)

This subroutine is the heart of TWOTRAN. It consists of four parts:

1. The total source calculation,
2. The calculation of the flux,
3. The rebalancing algorithm, and
4. Convergence tests.

The total source computation consists of adding the within-group scattering source (based on latest available flux components) to the source computed

Fig. 10. Flow Chart of Subroutine INNER.

FIXUP

IF ANY COMPUTED F (CELL-CENTERED FLUX) IS NEGATIVE, EXIT IS THROUGH (80)

Fig. 11. Flow Chart of Subroutine FIXUP.

in OUTER. Given the source, the flux is calculated in INNER by making four sweeps, one for each quadrant of directions.

The four sweeps in INNER each use an algorithm which sets the boundary values of the flux (SETBC), and traverses all points of the fine-space mesh for a quadrant of directions to produce the boundary fluxes (in these directions) on the other side of the system. This algorithm could be coded as a subroutine, but is written explicitly for each sweep. In contrast to diffusion theory, transport theory gives rise to an initial value problem in that boundary fluxes do not need to be specified on all sides of the system simultaneously. For example, given the distributed source and the inward-downward directed flux on the top and right boundaries of a rectangle, the out-going flux on the left and bottom boundaries can be calculated without knowledge of other boundary fluxes. This is the first or in-down sweep in INNER. The result of the inward-downward sweep is to produce the left and bottom

boundary angular fluxes. These fluxes are stored in the same arrays (BT1 and BR1) used initially for the right and top boundary fluxes. At the time coarse-mesh boundaries are crossed in the sweep, portions of the partial currents passing through the coarse-mesh boundaries are accumulated. The total partial currents are used in the rebalancing algorithm. Crossing a coarse-mesh boundary is detected by the change in the index quantities IDX or IDYA which determine which coarse-mesh zone a fine-mesh interval is in.

The other three sweeps proceed similarly for the out-down, in-up, and out-up directions. If a negative flux is encountered in any sweeps, an attempt is made to correct it by calling FIXUP (Fig. 11) where the "set-to-zero-and-correct" algorithm described in Section II.C.3 is applied.

In each sweep, the angular source is generated by multiplying the appropriate flux component by the spherical harmonic function for the quadrant. Similarly, these spherical harmonic functions are

used to generate the needed angular quadratures of the angular flux, producing the new flux components.

After all four sweeps are made, rebalance factors are determined by calling REBAL. Input to REBAL includes the leakage (partial current multiplied by surface area) out of each face of each coarse-mesh cell, the removal rate (difference between total collision and within-group scatter) in each coarse-mesh cell, and the total source in the cell independent of the flux being calculated. Output from REBAL is the rebalance factors which are used to adjust the amplitude of all fluxes within each coarse-mesh cell. REBAL is called every other iteration in INNER. After LIMIT iterations, REBAL is used to compute a single factor for the whole system.

After rebalancing, the maximum change (over the whole system) in the scalar flux (summed for four adjacent space cells) relative to the same scalar flux in the previous iteration is compared to EPSI. If the maximum change is smaller, control returns to OUTER; but if it is larger, another inner iteration is performed starting with the recomputation of the within-group flux. If the system is only one interval wide in either direction, a pointwise maximum change test is used. If a denominator is zero, the ratio of the previous flux to the total source to the group is added to the error measure.

Convergence in INNER may also be stopped after an input-prescribed number of iterations. If this happens, all the coarse-mesh flows are multiplied by the latest coarse-mesh scale factors to insure that their amplitudes are consistent with the group flux amplitudes.

11. Subroutine SETBC (Fig. 12)

This routine sets the boundary conditions for the sweeps in INNER. The boundary flux arrays to be set are arguments of the subroutine. For example, before beginning any sweeps, both top-boundary arrays, BT1 (in-down) and BT2 (out-down), are set and both right-boundary arrays, BR1 (in-down) and BR2 (in-up), are set. If a vacuum condition is applied, the array is zeroed. If a boundary source is indicated, the value of the input boundary source (allowed on the right, top, or bottom) is entered as the boundary flux. The angular directions and angular traverses are so arranged that no

action is required to set the reflective boundary condition; the appropriate flux is already in the array. For the white boundary condition, the integral of (53) is performed, and the periodic condition is set by transferring the appropriate fluxes. Periodic conditions require the storage of additional boundary flux arrays, and are not allowed except at the top and bottom boundaries. The left boundary condition is set in INNER. Only vacuum and reflective conditions are allowed on the left, but, with the exception of periodic in (x,y) geometry, these are the only reasonable conditions on the left.

12. Subroutine REBAL (Fig. 13)

This routine performs the iteration required to solve the coarse-mesh rebalance equations derived in Section II.C.4.a. above. In addition to the situations in which a lack of convergence, negative scale factors, etc., cause switching to computation of a single factor for the whole system, a flag (set in INNER after LIMIT iterations) is used to bypass the coarse-mesh factor computation for the single-factor computation.

13. Subroutine TESTS (Fig. 14)

This subroutine controls the outer iteration termination, calculates an eigenvalue if IEVT=1, performs the coarse-mesh rebalance, and calls NEWPAR which guides the search for any parametric eigenvalue (mesh boundary location, material concentration, or time absorption).

In TESTS, if λ , the ratio of the total source in one outer iteration to that of the previous iteration, is unity to within an input precision, and if all coarse-mesh rebalance factors calculated using latest values of fluxes are unity to within a separate precision, then a convergence trigger is set and one final outer iteration is made to print or store group-dependent quantities not stored during the iteration process. For this pass, only one inner iteration is allowed in each group. Presently, only the coarse-mesh balance tables are printed during this pass.

The λ test will fail whenever the fission source is not converged. The coarse-mesh rebalance test will fail in any situation where the flux is not converged, including inhomogeneous source problems with upscatter or those in which the number of inner iterations is artificially limited.

SETBC

CALLING SEQUENCE	A	B	P	Q	W	MM	L	IB	IQ	T	C	D
TOP BOUNDARY	BT1	BT2	QT1	QT2	WETA	MM	IT	IBT	IQT	SUMETA	BT3	BT4
RIGHT BOUNDARY	BR1	BR2	QR1	QR2	WMU	MM	JT	IBR	QQR	SUMMU	BT3	BT4
BOTTOM BOUNDARY	BT1	BT2	QB1	QB2	WETA	MM	IT	IBB	IQB	SUMETA	BT3	BT4

Fig. 12. Flow Chart of Subroutine SETBC.

The coarse-mesh rebalance factors are computed using both fission and inhomogeneous sources for zone sources. For the first two outer iterations, if there is no inhomogeneous source, an eigenvalue associated with the coarse-mesh equations is also calculated. After two outer iterations this eigenvalue iteration is not performed. This eigenvalue computation seems to dramatically accelerate most k_{eff} calculations in their early stages. In every type of problem the coarse mesh acceleration factors are applied to the fluxes.

If IEVT > 1 (parametric eigenvalue searches), control is passed to NEWPAR⁹ as soon as two successive values of λ differ by an input precision.

NEWPAR selects another value of the eigenvalue and returns program control to INITIAL.

14. TWOTRAN Service Routines

OVERLAY (0,0)

- MONITR Prints the summary line of an outer iteration.
- ERROR Prints messages, and error messages, and services the error indicators.
- CLEAR Sets a given area of core to a constant.
- WRITE Prints generalized vectors, matrices and arrays, partial sections of the cross-section array, flux and Q-source arrays.
- ECLOAD Computes the needed addresses for ECS fetches and stores.
- ECDUMP Processes restart dump requests.

REBAL

Fig. 13. Flow Chart of Subroutine REBAL.

SUBROUTINE TESTS

Fig. 14. Flow Chart of Subroutine TESTS.

Fig. 14. (Cont.)

Fig. 14. (Cont.)

SPREAD Processes ECS requests for fetches and stores where the entries of the vectors are nonconsecutive.

UNITWR Processes vector binary read/write requests and checks for end of file.

JCDECS Obtains the ECS block request length from the system job card.

MANAGR Acts as ECS main processor and error processor.

OVERLAY (1,0)

READ Acts as dummy routine for floating-point input data block processor.

READI Acts as dummy routine for integer input data block processor.

LOAD Processes both integer and floating-point input data blocks.

REDUCE Checks both ECS and core storage requirements. May reduce ECS storage and adjust (increase or decrease) core request.

OVERLAY (1,2)

CSPREP Checks the basic nuclide cross-section data for neutron consistency, performs adjointing, and enters cross sections into ECS by group.

READQF Obtains the basic data for both the flux guess and Q source and enters this data into ECS by group.

SNCON Obtains the S_n constants from the library.

LAXS Reads the basic cross-section information from cards and from the library.

OVERLAY (1,3)

MAPPER Prints the material specifications of the case in a pictorial form.

OVERLAY (3,0)

EDCALL Reassigns origins and saves values required for the edit.

EDITOR Performs edit computations.

PLOTS Controls both contour and projection plot routines.

PLOTP Plots projection of the flux for a group.

PLOTG Initializes contour plot material for a group.

CONTRB Plots contour of the flux for a group.

EDMAP Prints the edit specifications of the case in a pictorial form.

CHNFLX Changes the arrangement of the flux array for use in the edit.

OVERLAY (4,0)

- DUMPH The dump taken here is an error dump taken when an arithmetic mode error occurs.
- SPFDMP Obtains the mode error information and controls the dumping of the common blocks.
- MODDMP Prints a vector in fixed or floating form with repeat skipping.

15. TWOTRAN System Subroutines

TWOTRAN requires the following system routines.

a. ECS System Routines

1. ECRD Fetches a vector from ECS.
2. ECWR Stores a vector in ECS.
3. ECFL Reduces the ECS block requirement.
4. CPAREA Dumps the CP area into core.

b. General System Routines

1. CFL Obtains the current field length of a job. May be used to adjust the field length.
2. SECOND Obtains the current time.
3. DATE1 Obtains the current date in an A10 format.
4. REMARK Places a remark found in an A10 format in the day file.
5. SHIFT Shifts a number.
6. MEPEP Initializes the arithmetic mode error return.

c. SC-4020 Plot Routines

1. ADV Advances the plot film frame.
2. DRA Defines the plot area.
3. DRV Draws a line between two points.
4. FRAME Draws a frame around the plot area.
5. LINCNT Sets the position on film for a straight hollerith write command.
6. PLOJB Plots the points of an independent and a dependent variable vector set as well as labeling the graph.
7. PLOT Plots the points of an independent and a dependent variable vector set as given by a count.
8. PLT Plots a single character as given by the coordinates.
9. TCP Types a current point (hollerith entry with no specification of coordinates needed).
10. TRICJB Plots a contour.
11. TSP Types a specified point (hollerith entry).

12. WLCH Writes large characters horizontally.

B. Iterative Cycles and Convergence Tests

There are three levels of iterative processes in the TWOTRAN program: (1) the inner iteration, in which the within-group scattering source or the boundary flux at an implicit boundary (right or top) changes, (2) the outer iteration in which the fission or upscattering source changes or which is caused by artificial inner iteration limitation (usually in inhomogeneous source problems), and (3) the parametric eigenvalue search iteration in which, after a converged outer iteration, the value of coarse-mesh boundary, a material concentration, or a time absorption is changed. Two additional iterations are required for the calculation of coarse-mesh rebalance factors, one for the factors themselves and one for the coarse-mesh rebalance eigenvalue.

Two convergence precisions are input: EPS and XLAX. In a parametric eigenvalue search, two values of r indicates iteration number)

$$\lambda = \frac{\text{Fission Source}^r + \text{Inhomogeneous Source}}{\text{Fission Source}^{r-1} + \text{Inhomogeneous Source}} \quad (76)$$

are required to differ by less than XLAX before a new eigenvalue guess is computed. All other processes are tested against precisions derived from EPS. These precisions are

- EPS ϕ = EPS outer iteration convergence
 EPSI = EPS inner iteration convergence
 EPSR = EPS/2 coarse-mesh rebalance numbers
 EPSX = EPS collapsed group coarse-mesh convergence, both factors (5*EPSX) and eigenvalue (EPSX).

In the inner iteration process, we require

$$\max_{ij} \left| 1 - \frac{(\phi_{ij}^{r-1} + \phi_{i-1,j}^{r-1} + \phi_{i,j-1}^{r-1} + \phi_{i-1,j-1}^{r-1})}{(\phi_{ij}^r + \phi_{i-1,j}^r + \phi_{i,j-1}^r + \phi_{i-1,j-1}^r)} \right| < \text{EPSI} \quad (79)$$

for the r^{th} iteration where ϕ is the scalar flux after application of rebalance factors. If the system is one interval wide in either direction, we require

$$\max_{ij} \left| 1 - \phi_{ij}^{r-1} / \phi_{ij}^r \right| < \text{EPSI} \quad , \quad (80)$$

instead. If the denominator in either (79) or (80) is zero, we increment the quantity which is compared to EPSI by the numerator divided by TS, the total volume integrated source to the group. This allows both the numerator and denominator to be zero. If the number of iterations is greater than IML, we stop inner iteration even if (78) or (80) are not satisfied.

For the iterative computation of the coarse-mesh scale factors in the inner iteration, we require

$$\max_{k,l} |1 - F_{kl}^r / F_{kl}^{r-1}| < EPSR \quad (81)$$

If F_{kl}^{r-1} is zero we compute, noniteratively, a single factor, avoiding (81). No iteration is required if the system is only one coarse-mesh interval high.

In the outer process, if both

$$|\lambda - 1| < EPS\phi \text{ and } \max_{kl} |1 - F_{kl}| < 5 * EPSX \quad (82)$$

the problem is terminated after one final outer iteration. In the coarse-mesh iterative process (denoted with iterative superscript r) we require

$$|(\lambda)_{\text{Coarse Mesh}} - 1| < EPSX \quad (83)$$

if IEVT=1 (k_{eff} calculations) and

$$|1 - (\lambda)_{\text{CM}}^r / (\lambda)_{\text{CM}}^{r-1}| < EPSX \quad (84)$$

if IEVT $\neq$ 1 (parametric eigenvalue or inhomogeneous source plus fission problems).

C. Progression Through the Space-Angle Mesh

In several previous sections we have sketched the process of inner iteration in which the recursion of (58) is used to generate the angular flux for every space-angle cell. In this section we give a detailed explanation of this process. We begin by describing the angular mesh used in TWOTRAN.

1. S_n Constants and Angular Orientation

For a quadrature order ISN, TWOTRAN requires $MM = (ISN) * (ISN+2) / 8$ positive pairs (μ, η) of direction cosines in all three geometries. These may be supplied by the user or, for ISN up to 16, obtained from the routine SMCON. If input by the

user, the numbers should be supplied first for the largest η (one value of μ), then for the next largest η (two values of μ , the largest first), then for the third largest η (three values of μ , in order of decreasing magnitude), etc. Only one quadrant of directions is used, and a 90° rotational symmetry is assumed to perform calculations in the other three quadrants.

In Fig. 15 we show the ordering of the direction cosines for ISN=4 and ISN=8. The numbers in parentheses for ISN=4 indicate the actual numbering 1, 2, ..., MM, of the direction cosines. The other numbers indicate the order of progression through the angular mesh both for ISN=4 and ISN=8. In the figure we have shown directions lying on η levels, although this is not strictly necessary. It does, however, reduce the w derivative to a purely μ derivative in (r, z) and (r, θ) geometries. Actually, there are no restrictions on the direction sets other than the number of directions. For example, for ISN=8 one could have for the quadrant, say, a 1,1,2,6 arrangement on η levels rather than the 1,2,3,4 arrangement shown. We do require ISN/2 η levels, however. Because of (46), the user should avoid the use of precisely zero values of η . The program does also check that the quadrature weights associated with the directions sum to one-fourth (because only one quadrant of directions is entered).

In Figs. 16, 17, and 18 we have attempted to show the orientation of the hemisphere of directions in each of the geometries. In these figures, as in the projection of Fig. 15, the directions in which the step function scheme is used in the angle extrapolation (Section II.C.2) are marked with a cross.

2. Sweeps of the Space-Angle Mesh

In INNER, the NM group-total source components are calculated in the array $SOURCE(N, I, J)$, and the NM components of the angular flux are stored in $FLUX(N, I, J)$. No angular flux is ever stored in its entirety; instead, local values of the angular fluxes are stored in the arrays:

$N_{i \pm \frac{1}{2}, j, m}$ in BR1(J, M) in-down and out-down sweeps
in BR2(J, M) in-up and cut-up sweeps
in TI temporarily

Fig. 15. Ordering of S_n directions.

$N_{i,j \pm \frac{1}{2},m}$ in BT1(I,M) in-down and in-up sweeps
in BT2(I,M) out-down and out-up sweeps
in TJ temporarily

$N_{i,j,m \pm \frac{1}{2}}$ in ALFL(K,I) all sweeps
in TM temporarily

where the range of subscripts is

$I = 1, 2, \dots, IT$

$J = 1, 2, \dots, JT$

$M = 1, 2, \dots, MM = MT/4$

$K = 1, 2, \dots, ISN/2$

$N = 1, 2, \dots, NM = (ISCT+1)*(ISCT+2)/2$

Computation begins for the $\mu < 0, \eta < 0$ quadrant of directions at the top, right corner of the system. We refer to the traverse through the entire space mesh for these directions as Sweep 1. The

Fig. 16. Orientation of S_6 directions in (x,y) geometry. Directions in the negative quadrants are not shown in the upper sketch.

sweep begins with the storing of the right boundary condition in BR1 for all J and M and in BT1 for all I and M. (Because it is convenient, we also set the boundary condition in BT2 and BR2 for use in other sweeps.) After the boundary condition is set, the coarse-mesh flow through the boundary is computed. For these directions, this is FL and FD. Then Sweep 1 begins first for $J = JT$, i.e., along the top boundary of the system. For this value of J, I runs from the right: $IT, IT-1, \dots, 1$. For each I, M runs in the sequence shown in Fig. 15 before the next value of I is taken. For each M, before the flux calculation, the angular source is generated by performing the sum

Fig. 17. Orientation of S_6 directions in (r,z) geometry.

$$QT_{ijm} = \sum_{N=1}^{NM} \text{SOURCE}(N, I, J) * P1(N, M) \quad , \quad (85)$$

where $P1$ is used because we are in Sweep 1. Given the source, QT , we compute N_{ijm} (stored in T) from

$$T = \frac{AA * BRL(J, M) + BB * BT1(I, M) + CC * ALFL(K, I) + QT}{AA + BB + CC + CT} \quad , \quad (86)$$

where CT is the total cross section for the cell times the I -dependent portion of the volume. (The I -dependent portion of the volume is included in $SOURCE$ before any sweeps begin.) AA , BB , and CC are the products of angle and areas shown in (58) except, as noted earlier, we find it convenient to divide numerator and denominator of (58) by the J -dependent portion of the volume.

Fig. 18. Orientation of S_6 directions in (r,θ) geometry. Directions in the $\mu < 0$, $\eta < 0$ quadrants are not shown in the upper sketch.

If we use (86) we extrapolate

$$\begin{aligned} TI &= 2T - BRL(J, M) \quad , \\ TJ &= 2T - BT1(I, M) \quad , \\ TM &= 2T - ALFL(K, I) \quad . \end{aligned} \quad (87)$$

If we are in a direction for which we have no value for $ALFL(K, I)$ (the directions indicated with crosses in Fig. 15), we use (86) with $CC = 0$ and set $TM = T$. If any of the T values in (87) are negative, we call $FIXUP$ and compute new T values, otherwise we store TI in BRL , TJ in $BT1$, and TM in $ALFL$. We then use T to accumulate the sum

$$FLUX(N,I,J) = \sum_{M=1}^{NM} WGT(M) * P1(N,M) * T \quad (28)$$

This same sum accumulates in all sweeps with P1 changing to P2, etc. If we cross a coarse-mesh boundary, we also compute the contribution to the partial flow. We decide whether we have crossed a coarse-mesh boundary by testing for a change in the $IDX(I)$ and $IDXA(J)$ arrays which contain the number of the coarse-mesh i or j zone for each fine-mesh interval. These arrays are surrounded by zeroes in core storage so that we can also identify the first and last coarse-mesh zone.

When the flux computation has been completed for all values of M , we move to the next I level to the left, for the same J and repeat the process. The flux extrapolated in the previous cell is now properly in ERI , just as if we were at the right boundary. When we reach the inside boundary, we have computed the flux for all downward pointing incoming directions along the top interval of the system. We have saved the value of the extrapolated flux $ALFL$ for each space cell for each η level ($K = 1, 2, \dots, ISN$) corresponding to the $\mu = 0$ value. These values initialize $ALFL$ for Sweep 2 which now begins for the same (top) J level. The left boundary condition is set (in ERI), and Sweep 2 proceeds along the top interval of the system, from left to right for all downward-outward directions. When we reach the right boundary we begin Sweep 1 for the next lower J interval. We now have in ERI the fluxes calculated in the cell above (for outward directions, the cell top edge fluxes are in $ET2$), and computation proceeds just as it did for the top interval. We thus proceed inward, then outward, for each J level for decreasing values of J until we reach the bottom of the system. We then set the bottom boundary condition (in ERI and $ET2$) and start moving upward in J , first inward (Sweep 3), then outward (Sweep 4), on each J level.

When we reach the top, we have generated all the new flux components, $FLUX(N,I,J)$, and partial flows needed. At this point we have completed the space-angle traverse. We next rebalance the fluxes, perform convergence tests, and, if they fail, recompute the source for the next traverse.

D. Data Storage

1. Allocation of Core Storage

All of the data of TWOTRAN in fast memory resides in two vectors, IA and A , which are in blank COMMON. Both input integers and input floating-point control numbers, as well as commonly used data and array first word addresses are stored in IA . All arrays required, i.e., $ERI(J,M)$, $FLUX(N,I,J)$, etc., are stored compactly in the A vector. This allows great flexibility in the treatment of the dimensioned variables and permits maximum use of core storage. The amount of core required is determined in INPUT for each problem during the calculation of first word addresses. (In the IASL environment, unneeded core is actually returned for use by other programs. In a sequence of problems, larger problems acquire more core as needed.) While arrays are stored as vectors in A , they can be treated as variably dimensioned arrays in subroutines. For example, LFL is the location of the first word of $FLUX$ in A . The call to $OUTER$ from $GRIND2$ contains the entry $A(LFL)$ and the arguments NM , IT , and JT . In $OUTER$, the entry in the subroutine list corresponding to $A(LFL)$ is called $FLUX$, and in this subroutine the array is dimensioned as $FLUX(NM,IT,JT)$. If it were desirable, this array could be used as a two-dimensional array in some other subroutine, say as $FLUX(NM,ITJT)$ where $ITJT = IT*JT$.

Another use made of the A array is to pass commonly used variables through COMMON. We usually accomplish this by an equivalence of the variable to the location in A or IA . For example, IT is always stored in $IA(64)$. Although this leads to long equivalence lists in some routines, it avoids repeating a very long common statement in every routine.

To make effective use of the IA and A arrays, one needs a table of contents for IA . This is given in Table IV. In the table we give the name, dimensions, and definitions of all arrays in TWOTRAN. The contents of the other common blocks of TWOTRAN are given in Tables V, VI, and VII.

2. Allocation of Extended Core Storage

Extended core storage is also allocated compactly in much the same manner as core storage. Most of the group-dependent arrays are stored in

TABLE IV
Contents of Blank Common Block 1A

Position	Name	Pointer for Array	Remarks
1	ITH		Theory
2	ISCT		Scattering order
3	ISN		Order of S_n
4	IGM		Number of groups
5	IM		Number of radial coarse-mesh intervals
6	JM		Number of axial coarse-mesh intervals
7	IBL		Left boundary specification
8	IBR		Right boundary specification
9	IBB		Bottom boundary specification
10	IBT		Top boundary specification
11	LEVT		Eigenvalue type specification
12	ISTART		Flux input option
13	MT		Total number of materials
14	MIN		Total number of input nuclides from both library and cards
15	MS		Number of mixture instructions
16	IHT		Position in table of total cross section
17	IHS		Position in table of self-scatter cross section
18	IBM		Cross-section table length
19	IQOPT		Source input options
20	IQAN		Distributed source anisotropy order
21	IQB		Bottom boundary source indicator
22			Not used
23	IPVT		Parametric eigenvalue indicator
24	IANG		Must be zero (Not presently used)
25			Not used
26	IITL		Maximum number of inner iterations
27			Not used
28	IREM		Maximum number of rebalance iterations which is set in INPUT11
29	DOM		Radial-mesh modification indicator
30	IYM		Axial-mesh modification indicator
31	IEDOPT		Edit options
32	IGEOM		Geometry type
33	IQR		Right boundary source indicator
34	IQT		Top boundary source indicator
35			Not used
36			Not used
37	EV		Eigenvalue guess
38	EVM		Eigenvalue modifier
39	PVM		Parametric value of k_{eff}
40	XLAL		Search lambda lower limit
41	XLAH		Search lambda upper limit
42	XLAX		Fine-mesh search precision

TABLE IV (Cont.)

Position	Name	Pointer for Array	Remarks
43	EPS(EPS0)		Convergence precision and outer convergence precision
44	EPSI		Inner convergence precision = EPS0
45	EPSR		Within-group rebalance convergence precision = EPS0/2
46	EPSX		Whole-system rebalance convergence precision = EPS0
47			Not used
48	NORM		Normalization amplitude
49	POD		New parameter modifier
50	BHGT		Buckling height
51	IUP		IHS-IHT-1 (up scatter indicator)
52	IHF		IHT-1 (position of w_f - in cross-section table)
53	IHA		IHT-2 (position of σ_a - in cross-section table)
54	IHTR		IHT-3 (position of σ_{tr} - transport cross section - in cross-section table if present)
55	IHRN		IHT-4 (position of $\sigma_{n,2n}$ - cross section in cross-section table if present)
56	IMJM		Product IM*JM
57	MM		(ISN*(ISK+2))/8, number of directions per octant
58	RM		((ISCT+1)*(ISCT+2))/2, number of anisotropic components of flux
59	RMQ		((IQAN+1)*(IQAN+2))/2, number of anisotropic source components
60	IP		Sum IM+1
61	JP		Sum JM+1
62	IGP		Sum IGM+1
63	IJMM		Product IM*JM*MM
64	IT		Total number of radial fine-mesh intervals
65	JT		Total number of axial fine-mesh intervals
66	ITJT		Product IT*JT
67	ITMM		Product IT*MM
68	JTMM		Product JT*MM
69	NMLJ		Product MM*IT*JT
70	NMM		Product MM*MM
71	ISPANC		Last word address of cross-section and Q-source block
72	IHMT		Product IHM*MT
73	ISPANF		Last word address of flux block
74	ISCP		Sum ISCT+1
75	IMJP		Product IM*JP
76	IPJM		Product IP*JM
77	ITP		Sum IT+1
78	JTP		Sum JT+1
79	ICLIM		Length of cross-section portion of cross-section and Q-source block
80	LIHX	IHX(IM)	Number of radial fine-mesh intervals per coarse interval
81	LIHY	IHY(JM)	Number of axial fine-mesh intervals per coarse interval
82	LC	C(IHM,MT)	Cross sections for a group
83	LA1	A1(ITP)	$A4(I+1)+A4(I)$
84	LA2	A2(ITP)	$A4(I+1)-A4(I)$

TABLE IV (Cont.)

Position	Name	Pointer for Array	Remarks
85	LA3	A3(ITP)	Axial direction surface area
86	LA4	A4(ITP)	Radial direction surface area = $A4*YH$
87	LA5	A5(ITP)	Volume = $A5*YH$
88	LQ	Q(NM,IT,JT)	Distributed source
89	LQR1	QR1(JT,MM)	Input right boundary source for in and down directions (conditional on IQR.EQ.1)
90	LQR2	QR2(JT,MM)	Input right boundary source for in and up directions (conditional on IQR.EQ.1)
91	LQT1	QT1(IT,MM)	Input top boundary source for in and down directions (conditional on IQT.EQ.1)
92	LQT2	QT2(IT,MM)	Input top boundary source for out and down directions (conditional on IQT.EQ.1)
93	LBR1	BR1(JT,MM)	Right boundary flux (in-down and out-down)
94	LBR2	BR2(JT,MM)	Right boundary flux (in-up and out-up)
95	LBT1	BT1(IT,MM)	Top boundary flux (in-down and in-up)
96	LBT2	BT2(IT,MM)	Top boundary flux (out-down and out-up)
97			Not used
98			Not used
99			Not used
100			Not used
101			Not used
102			Not used
103			Not used
104			Not used
105	LFL	FLUX(NM,IT,JT)	Flux components
106	LFLA	FLUXA(IT,JT)	Scalar flux from previous inner iteration
107			Not used
108	LFIS	FISS(IT,JT)	Same origin as FISSA
109	LFISA	FISSA(IT,JT)	Fission source
110			Not used
111	LXC	IDCS(IM,JM)	Cross-section zone identification integers
112	LXR	XRA(IP)	Input radial coarse-mesh boundaries
113	LYR	YRA(JP)	Input axial coarse-mesh boundaries
114	LDX	IDX(IT)	Radial direction indicators showing which radial coarse-mesh interval a fine radial mesh interval belongs to
115	LDY	IDY(JT)	Axial direction zone index multiples
116	LDYA	IDYA(JT)	Axial direction indicators showing which axial coarse-mesh interval a fine axial mesh interval belongs to
117	LXH	XH(JM)	Radial fine-mesh spacing
118	LYH	YH(JM)	Axial fine-mesh spacing
119	LW	WGT(MM)	Direction weights
120	LCM	COSMU(MM)	Radial direction cosines
121	LCE	COSETA(MM)	Axial direction cosines
122	LWM	WMT(MM)	Product WGT*COSMU

TABLE IV (Cont.)

Position	Name	Pointer for Array	Remarks
123	LWE	WETA(MM)	Product $WGT * COSETA$
124	LP1	P1(NM,MM)	Spherical harmonic functions for in-down sweep
125	LP2	P2(NM,MM)	Spherical harmonic functions for out-down sweep
126	LP3	P3(NM,MM)	Spherical harmonic functions for in-up sweep
127	LP4	P4(NM,MM)	Spherical harmonic functions for out-up sweep
128	LMN	MIXNUM(MS)	Input mixture numbers (conditional on MS.GT.0)
129	LMC	MIXCOM(MS)	Input mixture instructions (conditional on MS.GT.0)
130	LMD	MIXDEN(MS)	Input mixture densities (conditional on MS.GT.0)
131	LF	F(IM,JM)	Coarse-mesh rebalance factors
132	LFU	FU(IM,JP)	Coarse-mesh upward partial current
133	LFD	FD(IM,JP)	Coarse-mesh downward partial current
134	LFR	FR(IP,JM)	Coarse-mesh rightward partial current
135	LFL	FL(IP,JM)	Coarse-mesh leftward partial current
136	LAB	AB(IM,JM)	Coarse-mesh absorption removal rate
137	LQQ	QQ(IM,JM)	Coarse-mesh source
138	LQQG	QQG(IM,JM)	Coarse-mesh source over all groups (conditional on IEVT.EQ.0)
139	LFUT	FUT(IM,JP)	Sum of FU over all groups (same origin as FU)
140	LFDI	FDI(IM,JP)	Sum of FD over all groups (same origin as FD)
141	LFRJ	FRJ(IP,JM)	Sum of FR over all groups (same origin as FR)
142	LFLI	FLI(IP,JM)	Sum of FL over all groups (same origin as FL)
143	LABT	ABT(IM,JM)	Sum of absorption rate over all groups
144	LHA	HA(IM)	Used in subroutine REBAL for inversion
145	LGA	GA(IM)	Used in subroutine REBAL for inversion
146	LQG	QG(IGP)	Space integral of Q
147	LFG	FG(IGP)	Space integral of FISSA
148	LSIN	SIN(IGP)	Space integral of group inscatter source
149	LSS	SS(IGP)	Space integral of group selfscatter source
150	LSOUT	SOUT(IGP)	Space integral of group outscatter source
151	LHL	HL(IGP)	Group horizontal leakage
152	LVL	VL(IGP)	Group vertical leakage
153	LTL	TL(IGP)	Group total leakage
154	LNL	NL(IGP)	Group net leakage
155	LRL	RL(IGP)	Group right leakage
156	LABG	ABG(IGP)	Space integral of group absorption rate
157	LBAL	BAL(IGP)	Group neutron balance
158	LCHI	CHI(IGP)	Input fission spectrum
159	LCHIA	CHIA(IGP)	Fission spectrum used in the calculation
160	LVEL	VEL(IGP)	Group velocities
161	LXM	XM(JM)	Input axial direction modifiers (conditional on IEVT.EQ.4 and IYM.EQ.1)
162	LXM	XM(IM)	Input radial direction modifiers (conditional on IEVT.EQ.4 and IYM.EQ.1)
163	XRRA	XRADA(IP)	Modified coarse-mesh radial boundaries
164	LYRA	YRADA(JP)	Modified coarse-mesh axial boundaries

TABLE IV (Cont.)

Position	Name	Pointer for Array	Remarks
165	LSOU	SOURCE(NM,IT,JT)	Total source in a group (same origin as Q)
166	LANF		
167	LAA	AAJ(MT)	Adjoint absorption (conditional on ITH.EQ.1)
168	LBL	BL(JT)	1.0/axial delta - used to multiply all areas and volumes in major recursion formulas
169	LAL1	AL1(MM)	α coefficient $(MM+1/2)/WGT$
170	LAL2	AL2(MM)	α coefficient $(MM-1/2)/WGT$
171	LALF	ALFL(MN,IT)	α flux due to curvature streaming
172	IQB1	QB1(IT,MM)	Input bottom boundary source for in and up directions (conditional on IQB.EQ.1)
173	IQB2	QB2(IT,MM)	Input bottom boundary source for out and up directions (conditional on IQB.EQ.1)
174	LCTOT	CTOT(IT,JT)	Effective total cross section
175	JCONV		Final convergence indicator
176	TN2N		N ₂ N reaction term used in balance equations
177	XLAPP		Value of lambda from sequence of outer iterations previous to that of XLAP
178	XLAP		Value of lambda from previous sequence of outer iterations
179	ICNT		Iteration trigger used in NEWPAR
180	E2		Temporary storage
181	E1		Temporary storage
182	EVPP		Eigenvalue from cycle of outer iteration previous to that of EVP
183	EVP		Eigenvalue from previous sequence of outer iterations
184	E4		Temporary storage
185	NGO		Return indicator set in NEWPAR
186	ALAR		Value of lambda from previous iteration
187			Not used
188	IITNO		Inner iteration number
189	TS		Total integrated source to a group
190	G		Number of current group (Integer)
191	ICONV		Secondary convergence indicator
192	NGOTO		Return indicator set in TESTS
193	E3		Temporary storage
194	EVS		Slope used in eigenvalue search
195	IITOT		Total number of inner iterations
196	ALA		Parameter lambda
197	TIN		Time
198	FTP		Previous fission total
199	IFN		Fission calculation indicator set in INITIAL
200	OITMO		Outer iteration number
201	ZZ		Radial geometric function used in FIXUP
202	BB		Axial geometric function used in FIXUP
203	CC		Angular function used in FIXUP
204	DD		Angular function used in FIXUP

TABLE IV (Cont.)

Position	Name	Pointer for Array	Remarks
205	T		Cell-centered flux used in FIDUP
206	S		Source used in FIDUP
207	CT		Total cross section used in FIDUP
208	SUMMU		MM $\sum \text{COSMU}(M) * \text{WGT}(M)$ M=1
209	SUMETA		MM $\sum \text{COSETA}(M) * \text{WGT}(M)$ M=1
210	NN		ISN/2
211	AA		Radial geometric function used in FIDUP
212	TI		Temporary i-flux used in FIDUP
213	TJ		Temporary j-flux used in FIDUP
214	TM		Temporary x-flux used in FIDUP
215			Not used
216	ERR		Scalar flux error from comparison with previous flux
217			Not used
218			Not used
219			Not used
220	LBT3	BT3(IT,MM)	Top boundary flux (conditional on IET.EQ.3)
221			Not used
222			Not used
223	LBT4	BT4(IT,MM)	Top boundary flux (conditional on IET.EQ.3)
224			Not used
225			Not used
226	IFLAG		Whole-system rebalance indicator
227 thru			Not used
248			
249	NOSCUP		Sigma up included in cross-sections indicator
250	IOLTCS		Overlay indicator as two-octal digit number

TABLE V

Contents of Named Common Block FWBGN1

The named common block FWBGN1 contains the information required to continue the processing of the current problem if it is restarted.

Name	Contents and Remarks
IDUSE	Last title card. This is a vector 12 words in length which is used for the title of the plot routines. The words are A6 in format.
ECPTFX	ECS pointer for the flux block for the first group. The flux block contains the three-dimensional flux array as well as boundary arrays stored consecutively.

TABLE V (Cont.)

Name	Contents and Remarks
ECLTFX	Length of the flux block for a group.
ECLXFX	Length of the three-dimensional flux array.
ECSPTCQ	ECS pointer for the cross-section and Q-source block for the first group. The cross-section and Q-source block contains the cross-section array, the adjoint absorption vector (when problem is adjoint), the spatial total cross-section matrix, and the various Q sources.
ECLTCQ	Length of the cross-section and Q-source block for a group.
ECLCCQ	Combined length of the cross-section array and the adjoint absorption vector for a group.
ECLBCQ	Combined length of the cross-section array, the adjoint absorption vector and the total cross-section array, CTOT(IT, JT), for a group.
ECLACQ	Length of the cross-section matrix for a group (excluding the adjoint cross sections, and the CTOT array).
ECLACQ	Length of the Q-source block.
ECSACQ	Used to locate the total cross-section array, CTOT, in the ECS cross-section and source block.
ECSQCQ	Used to locate the Q-source block in the ECS cross-section and source block.
ECSPTFL	ECS pointer for the flow block for the first group. The flow block contains the coarse-mesh partial-currents matrices for the upward, downward, rightward, and leftward directions.
ECLTFL	Length of the flow block for a group.
ECSPTSQ	ECS pointer for the source to the group block. This is the source to the group calculated by OUTER and used in INNER.
ECLTSQ	Length of the source to the group entry.
ECLAST	ECS pointer for the next block to be assigned.
NOFILM	Plot type and plot storage indicator. -1/0/1 = no plot/plot/plot storage error. An error indication is given if temporary storage required for plot exceeds the core available.
LPTFRX	Core pointer of fine radial mesh for plot.
LPTFRY	Core pointer of fine axial mesh for plot.
LPTFMA	Core pointer of first temporary block for plot. The length of this block is the maximum of the total number of fine radial mesh intervals, the total number of fine axial mesh intervals, or 21.
LPTFMB	Core pointer of second temporary block for plot. The length of this block is the same as that of the first temporary plot block.
LPTFMC	Core pointer of third temporary block for plot. The length of this block is the same as that of the first temporary plot block.
LPTFMD	Core pointer of fourth temporary block for plot. The length of this block is the same as that of the first temporary plot block.
LPTFLX	Core pointer of scalar-flux block for plot.
TIMACC	Accumulated problem running time.
IGCDMP	Group number of restart dump.
LENDBA	Length of A data block. Used to return the A vector to core in a restart.

TABLE VI

Contents of Named Common Block FWBGN2

The named common block FWBGN2 consists primarily of those indicators which are used by the program but are not vital to restart. Parameters which define options are set in program TWOTRAN.

Name	Contents and Remarks
IFNOVY	Overlay file name. File name is given in a hollerith form and is not entered into the file name table of the program card. This name must be the same as that of the overlay specification card.
IRCOVY	Recall overlay indicator. When the overlay is in core, it will be recalled from the overlay file only if the indicator is set properly (0/6 HRECALL = no/yes).
IDUMOM	Unused. This is a vector of four words which is unused.
LENCCA	Length of the current-adjusted main data block. Used in core adjustment.
IFLCDC	Current-adjusted field length of the problem. Used in core adjustment.
LENXCA	Maximum length of the main data block. Used in core adjustment. Length of the common block called A.
IFLCDX	Maximum field length of core. Used in core adjustment.
INRCOR	Core adjustment indicator. (0/1 = no/yes). Must be zero in overlay version of TWOTRAN.
LENCIA	Length of the common parameter block IA.
LENXEC	Number of ECS blocks requested for the problem. Each block is equal to 512 words. This is initially set in program TWOTRAN to the number of blocks requested on the job card.
INRECS	Reduce ECS storage indicator. (0/1 = no/yes). If more than one case is to be run and the indicator is on, the cases must be in order of decreasing ECS size.
LENMCB	Length of the named common block called FWBGN1.
LEIMCB	Length of the named common block called FWBGN2.
LENERB	Length of the named common block called LOCAL.
LENTAP	Length of the permanent file common block. This block is the first of the blank common blocks (NINP, ..., NSCRAT).
NCARDX	Number of fixed-point parameter cards of a case. Used by fixed dimension version. (A version of TWOTRAN is available in which the variable-dimensioned arrays are converted automatically to fixed-dimensioned arrays for the problem at hand. While this procedure is complicated, it saves up to 40% of the execution time on long problems.)
NCARDL	Number of floating-point parameter cards of a case. Used by fixed-dimension version.
NCARDT	Current number of cards read for the input of a case. Used by fixed-dimension version.
INTAPE	File unit number for the input of the packed-card format. Used by fixed-dimension version.
IOTAPE	File unit number for the output of the single-card format. Used by fixed-dimension version.
ILTNOP	List all nonoperation cards indicator. (0/1 = no/yes). Used by fixed-dimension version.
ILTPOP	List all tape division operation cards indicator. (0/1 = no/yes). Used by fixed-dimension version.
ILTSOP	List all parameter substitution operation cards indicator. (0/1 = no/yes). Used by fixed-dimension version.
ILCSOP	List all resulting cards of parameter substitution operation indicator. (0/1 = no/yes). Used by fixed-dimension version.
IECSPI	Initial ECS pointer for block assignment. The first ECS block of a problem will be given this pointer and the other blocks will be assigned consecutive locations. This number is usually one.
IFILM	File unit number for SC-4020 plot film output.
IWFILM	Flux storage indicator. (0/1 = ECS/NFLUX).

TABLE VI (Cont.)

Name	Contents and Remarks
TIMEDF	Minimum time between periodic dumps. (0/F = no periodic dumps/second).
TIMSLD	Elapsed time since last dump.
TIMOFF	Floating-point form of the input fixed-point parameter ITLIM.
IPRLCD	Print data block input cards as read indicator. (0/1 = no/yes). When the indicator is on, each card of a data block is printed as it is read. Usually zero.
IPRLDT	Print loaded data block vectors in full indicator. (0/1 = yes/no). When the indicator is on, the entire contents of the block will be printed. When the indicator is off, only the title of the block will be printed. Usually zero.
IPRXCD	Print cross-section card input indicator. (0/1 = no/yes). When the indicator is on, each cross-section card is printed as it is read. Usually zero.
LENMAP	Length of the storage map vector.
ISDMAP	Storage map request indicator. (0/1 = no/yes). This section of the program is not operational.
INDPLT	SC-4020 film output indicator. (0/1/2/3 = none/contour/projection/both contour and projection).
NOCLNS	Number of contour lines desired in the plot. This must be less than 51.
MODEER	Mode error information. This is the mode error by-pass vector of two words. The first word contains the transfer address to be taken in the event of a mode error. The second word at the time of a mode error will be filled with the error information.

TABLE VII

Contents of Named Common Block LOCAL

The named common block LOCAL contains information that is passed from overlay to overlay for a problem but is not needed in restart.

Name	Contents and Remarks
NERROR	Parameter input-error indicator.
ITLIM	Fixed-point time problem removal value. (0/N = no/seconds).
LENCLR	Length of the partial block to be cleared during input.
ISNT	S _n library request indicator.
LIMIT	Storage length required by the cross-section phase of input.
LAST	Storage length required by the problem. This is a temporary value.
MCR	Number of nuclides requested from cards.
MTP	Number of nuclides requested from library.
LMTP	The core pointer for the library ID request table.
IERRST	Storage-error indicator.
IERRRT	Restart-error indicator.
MAXLEN	Total storage length to be cleared. This is a temporary storage.
NEDIT	File unit number upon which to place edit input information.
IEDIT	Edit information. This is a vector of two words which contains the edit values of NZ and NORMZ.
NEXTER	Fetch next case indicator. If factor occurs after all input is successfully read, next problem is fetched.
ICPILOG	SC-4020 contour flux log plot indicator. (0/1 = plot of flux/plot of log flux).
NEXTRA	File unit number of a scratch file.

ECS so the dimensionality is IGM times the core requirement of the array. For example, there are $IGM \times NM \times IT \times JT$ ECS locations required for $FLUX(NM, IT, JT)$.

Certain blocks of data are stored contiguously in core so that they may be read in and out of ECS in a single stream. For example, the flux block includes $FLUX(NM, IT, JT)$, $BR1(JT, MM)$, and $BR2(JT, MM)$ when the right boundary is implicit, $BT1(IT, MM)$ and $BT2(IT, MM)$ when the bottom boundary is implicit, and, if a periodic boundary condition is used on the top and bottom boundaries, $BT3(IT, MM)$ and $BT4(IT, MM)$. The first word of this block is LFL , and the last word is $ISPANF$. The cross-section and source block includes the cross sections $C(IHM, MT)$, the adjoint absorption $AA(MT)$ (conditionally), the total cross section $CT(T, IT, JT)$, and the conditional source arrays $Q(NM, IT, JT)$, $QR1(JT, MM)$, $QR2(JT, MM)$, $QT1(IT, MM)$, $QT2(IT, MM)$, $QB1(IT, MM)$, and $QB2(IT, MM)$. This block extends from LC to $ISPANF$. A complete list of ECS storage is given in Table VIII.

3. Overstorage of Data in Core

In TWOTRAN, a certain amount of overstorage is used to reduce the total amount of main memory required; i.e., more than one array may reside in the same main memory location as the problem progresses. The most important example of this is that the large arrays $Q(NM, IT, JT)$ and $SOURCE(NM, IT, JT)$ are stored in the same spaces. The Q array is used in OUTER to generate the total source to the group, including the inhomogeneous source, if any. This source is then written in ECS. In INNER this source is read into core after each inner iteration to initialize the $SOURCE$ array. Then, the rest of the group source is added to $SOURCE$.

The coarse-mesh total flows are stored in the same space as the group flows. That is, FUT , FDT , FRT , FLT are stored over FU , FD , FR , and FL . The group values of FU , FD , FR , and FL are stored in ECS, and, after all groups are finished, the total values are accumulated. To perform this accumulation, we make temporary use of the arrays $Q(NM, IT, JT)$, $FLUXA(IT, JT)$, $F(IM, JM)$, and $QQ(IM, JM)$. For this reason, when we allocate storage for, say, $FLUXA$ we must use the larger of $IT \times JT$ and $(IM+1) \times JM$ because both sizes of arrays are stored in the same core area. Similar precautions are necessary for the other arrays.

We have performed only the overstorage that is simplest; other savings can be made if the effort is expended. For example, other coarse-mesh arrays could be put in ECS, $BR2$ could be written over $BR1$ after sweeps 1 and 2 (if the right boundary condition were set for sweeps 1 and 2 separately from sweeps 3 and 4), or, more dramatically, by storing only one J level in core at a time, arrays like $Q(NM, IT, JT)$ could be reduced to $Q(NM, IT)$. On the other hand, if the time cost of peripheral storage is great, it may not be feasible to make the savings we have made. Even further in this vein, one can use another fission array [$FISS(IT, JT)$ for which the first word address $LFIS$ already exists but is the same as that for $FISSA$] to accumulate the new fission rate after every completed inner iteration rather than to make a special progression through the groups to calculate the new rate as is now done.

For the purpose of performing an edit, certain input is read after all other input and before cal-

TABLE VIII

ECS Storage

ECS First Word Address	Total Length	Contents
ECPTCQ	$(ISPANF+1-LC) \times IGM$	Cross-section and Q -source blocks by group.
ECPTFX	$(ISPANF+1-LC) \times IGM$	Flux blocks by group.
ECPTFL	$(IPJM+IMJP) \times 2 \times JM$	Flow blocks by group.
ECPTSO	$NM \times IT \times JT$	Source to the group not including within group scatter.

ulation, begins. This information is read temporarily into the FU (coarse-mesh flow upward) and F (coarse-mesh scale factors) arrays and then written on a peripheral unit for use in the edit. The F and FU arrays (as well as other arrays following these arrays) are initialized before use in computations. The edit is performed in overlay program OUTPUT3 after taking a final dump and after all other printing has been finished. Then core storage is rearranged to make space available for the edit arrays. Only the arrays necessary for the edit are saved and these are moved to the beginning of core. Then, depending on the type of each edit, storage is allocated for all the new arrays needed. It also is convenient to treat FLUX(1,IT,JT) as a IT*JT array in the edit, so if the first dimension of FLUX is greater than 1 we rearrange the flux array.

E. Assignment of Files

The assignment of files is described in Table IX below. These assignments are made in the TWOTRAN program.

IV. GENERAL USAGE OF THE TWOTRAN PROGRAM

A. Cross Sections

1. Input Formats

The TWOTRAN program accepts cross sections in the standard Los Alamos format, with one exception. In upscattering problems, the program does not need the special σ^{up} cross section⁶ which is required in earlier Los Alamos programs. In TWOTRAN, it is assumed that σ^{up} is present, and σ^{up} is automatically removed from both disk or card input cross-sections sets. To enter a cross-section set without σ^{up} the user must tag IHT with a minus sign.

TABLE IX

File Name and Unit Requirement Table

Name	Logical Unit	Contents	Remarks
NINP	10	Problem input decimal unit	Generally this unit is equivalenced to the system input unit. <u>In case both code and problems are contained on a tape, this cannot be done.</u>
NOUT	9	System output decimal unit	
IFILM	12	System SC-4020 plot output unit	
NCAQ	6	Cross-section library unit	The name of the library disk file must be equivalenced to this unit number in the file definition section of the program card.
NFLUX	7	First unit for restart dumps	Serves as an initial unit for flux guesses from tape as well as a dump unit.
NSCRAT	5	Second unit for restart dumps	Restart dumps are taken alternately upon NFLUX and this unit during the execution of a problem.
NEDIT	17	Storage for edit information	
NEXTRA	18	Used as a scratch file	This is used by the edit program to accumulate certain sums.
IFNOVY	-	Overlay file name	This file name is given in a hollerith form and must match that of the overlay control cards. The name is never entered into the file-definition section of the program card.

The Los Alamos cross-section format assumes that each nuclide is described by a block of cross sections of IHM rows for IGM group columns. The row position of cross sections is specified relative to the total cross section, σ_t (row IHT), and the within-group scattering cross section, $\sigma_{s,g-g}$ (row IHS). It is assumed that the row order of the cross sections is as follows:

	Row	Cross-Section Type-Group g
	IHT-4	$\sigma_{n,2n}$
	IHT-3	σ_{tr}
	IHT-2	σ_a
	IHT-1	$\nu\sigma_f$
	IHT	σ_t
IHM	IHT+1	$\sigma_{s,g+N-g}$
	.	
	.	
	IHS-2	$\sigma_{s,g+2-g}$
	IHS-1	$\sigma_{s,g+1-g}$
	IHS	$\sigma_{s,g-g}$
	IHS+1	$\sigma_{s,g-1-g}$
	IHS+2	$\sigma_{s,g-2-g}$
	.	
	.	
	IHS+M	$\sigma_{s,g-M-g}$

In this format, group g+1 corresponds to a group of lower energy than group g. The symbol $\sigma_{s,g-2-g}$ denotes the scattering transfer probability from group g-2 to group g. The format allows N groups of upscatter and M groups of downscatter; i.e., the scattering matrix need not be symmetric. However, all cross-section blocks must have the same values for IHM, IHS, and IHT. The fission cross section, σ_f , times the mean number of neutrons per fission, ν , must be located in row IHT-1, and the absorption cross section, σ_a , must be entered in row IHT-2. If a scattering matrix is used to represent (n,2n) reactions, that is, if $\sigma_{n,g-h}$ contains 2 times the (n,2n) transfer probability $\sigma_{n,2n;g-h}$, then the value,

$$\sigma_{n,2n} = \sum_{h=1}^N \sigma_{n,2n;g-h}$$

must be entered in IHT-4. The user is free to enter additional cross sections at the top of the format. These extra cross sections are not used in the calculation, but are available for reaction-rate computations after the particle flux is obtained. It is sometimes convenient to locate the transport cross section, σ_{tr} , in IHT-3 and use this cross section instead of σ_t in the calculation of buckling corrections.

2. Cross-Section Mixing

The user is free, in TWOTRAN, to enter macroscopic cross sections and bypass the mixing algorithms; specification of the input value MS = 0 is all that is required for this. If MS $\neq$ 0, the user must provide three sets of MS numbers which are stored in the vectors MIDNUM, MIXCOM, and MIXDEN. These numbers are used in the following algorithm to manipulate cross-section blocks:

```

DO 315 M = 1, MS
  N = MIDNUM(M)
  L = MIXCOM(M)
  AD = MIXDEN(M)
DØ 315 I = 1, IHM
  IF(L.EQ.0) GØ TØ 310
  IF((AD.EQ.0).AND.(IEVT.EQ.3)) GØ TØ 313
  C(I,N) = C(I,N) + AD*C(I,L)
  GØ TØ 315
313 C(I,N) = EV*C(I,N)
  GØ TØ 315
310 C(I,N) = AD*C(I,N)
315 CONTINUE

```

In this algorithm, cross-section block N is created or altered by adding multiples of block L or by multiplying the block N by a factor. Let us consider some examples.

Suppose we have entered 45 cross sections as input. Then any mixtures that are made must be given block numbers higher than 45. Suppose we enter:

MIXNUM (N)	MIXCOM (L)	MIXDEN (AD)
46	0	0.0
46	1	0.0478
46	20	0.0333
47	0	0.0
47	2	0.75
47	3	0.25
47	0	0.1179
48	0	0.0
48	15	0.0049
48	14	0.0078
48	48	0.0
49	0	0.0
49	33	0.5
49	34	0.5
49	0	0.187
49	49	0.0
49	46	0.1

For this example we have MS = 17 instructions. In the first three instructions, block 46 is cleared (set to zero) and then made up of 0.0478 parts of block 1 and 0.0333 parts of block 20. If blocks 1 and 20 are microscopic cross sections in barns, then 0.0333 and 0.0478 times 10^{24} are the atomic densities. In the second set of instructions, block 47 is cleared and then made up of 0.1179 times the result of adding three-fourths of block 2 to one-fourth of block 3. In the next set of instructions, block 48 is cleared and made up of portions of blocks 15 and 14. If IEVT (the input eigenvalue type option) is 3, then the resulting block 48 is multiplied by EV (the input eigenvalue guess). In this type of problem the program attempts to find a value of EV such that the resulting concentration of block 48 renders the system critical. If IEVT $\neq$ 3, the line of instructions 48, 48, 0.0 would not alter the composition of block 48. In the final sequence, block 49 is made up of 0.187 times one-half of block 33 and block 34, and provision is made to search for the concentration of this portion of 49 to which is always added 0.1 of the previously mixed block 46. It should be clear that there are many possibilities not covered in these examples, but by examining the Fortran in-

structions above, the user should be able to prepare his own sets of mixture instructions.

3. Anisotropic Cross Sections

In the TWOTRAN program it is assumed that the scattering transfer probability can be represented by a finite Legendre polynomial expansion; i.e., that

$$\sigma_s(E' \rightarrow E, \mu_0) = \sum_{n=0}^{ISCT} \frac{2n+1}{4\pi} P_n(\mu_0) \sigma_{sn}(E' \rightarrow E), \quad (89)$$

where ISCT is an input control integer. Thus if ISCT > 0, additional blocks of scattering transfer cross sections must be entered for those nuclides for which anisotropic scattering sources are to be computed. In these blocks, the rows 1 through IHT are zero, and $\sigma_{sn,g-h}$ (the energy average of $\sigma_{sn}(E' \rightarrow E)$ in groups g and h) is entered as for the isotropic component of the cross section. It is assumed in TWOTRAN that blocks of anisotropic cross sections which are used in the calculation have block numbers in ascending sequence, starting with the isotropic cross-section block. For example, suppose that block 50 is the isotropic cross-section block for hydrogen and that ISCT = 3. Then, block 51 must be σ_{s1} for hydrogen, block 52 must be σ_{s2} , and block 53 must be σ_{s3} . If a material is made by mixing two anisotropic scatters, then the anisotropic blocks must also be mixed with the same densities to form anisotropic blocks for the material.

Anisotropic scattering sources may be computed selectively within each zone, but in all zones in which such sources occur, the number of anisotropic scattering blocks (ISCT) must be the same. See Section B.

4. Adjoint Cross Sections

In adjoint calculations, cross sections are entered just as for a direct calculation. The program then transposes the scattering matrices and, because this usually changes a downscattering problem to an upscattering problem, reverses the group order of the blocks. Further, the effective absorption in an adjoint calculation is not simply related to σ_a . That is, the absorption is normally

$$\sigma_a = \sigma_t - \sum_{\text{all } h} \sigma_{so,g-h} \quad (90)$$

But when the scattering matrix has been transposed, the effective absorption is

$$(\sigma_a)_{\text{eff}} = \sigma_t - \sum_{\text{all } h} \sigma_{s0, h \rightarrow g} \quad (91)$$

In adjoint problems, this latter quantity is calculated and processed in mixing operations for use in the rebalancing algorithms.

5. Cross-Section Checking

As input cross sections are processed, Eq. (90) is computed and compared to the input value of σ_a . If there is any difference, the value of Eq. (90) is substituted for the input value. If the relative difference between the input total cross section and the computed total cross section exceeds EPSI (inner convergence precision), the user is so informed.

B. Geometry and Boundary Condition Specification

To specify the domain of the problem, the user supplies IM+1 coarse-mesh i-boundaries (defining IM intervals) and JM+1 coarse-mesh j-boundaries. Except for the first radius in cylindrical geometries, these sets need not begin at 0.0, but both sets must form a monotone increasing sequence. The user also supplies IM integers which indicate how many fine-mesh i intervals are in each k coarse-mesh interval and JM integers to indicate the number of fine-mesh j intervals in each l coarse-mesh interval. The fine-mesh spacing is uniform between the coarse-mesh boundaries. Finally, the user specifies the boundary condition on each of the four (left, right, bottom, top) boundaries.

The coarse-mesh boundaries define IMxJM zones. The user must supply a number for each of these zones to designate which cross-section block belongs in the zone. If anisotropic scattering is present in any zone, the block number for the zone is made negative. This indicates (as noted above) that the next ISCT blocks in numerical sequence contain the anisotropic cross sections for this zone.

All of the above information is converted by subroutine MAPPER to a pictorial description of the system. A sample is shown for (x,y) geometry.

Materials by broad zone. Origin at lower left. M is number of fine intervals/broad row (column).

Y		
ROW	M	
10.9000		10000000000000
		1 * 0
2	4	1 -5 * -7 0
		1 * 0
6.5400		1*****0
		1 * 0
1	6	1 -5 * -5 0
		1 * 0
0.0000		111111111111
X		0. 6. 10.
		0 5400 9000
M		6 4
COLUMN		1 2

In the problem pictured, the left and bottom boundaries are reflecting (1's) and the right and top boundaries are vacuum boundaries (0's). The square system extends from 0.0 to 10.9 in x with a coarse-mesh boundary at 6.54. In the first coarse x interval there are six fine intervals; in the second, four. The same spacing is used in the y direction. In three zones, cross-section block 5 is used, and, as indicated by the negative tag, anisotropic scattering is to be computed in these zones. Similarly, block 7 and anisotropic scattering are used in the remaining zone. (One can deduce that the scattering is linearly anisotropic (ISCT=1) because block 6 must be an anisotropic block and block 7 is used as an isotropic block). If one desires to assess the effect of anisotropic scattering in the zone with material 7, the only input change necessary is the change of -7 to +7.

Although zones defined by coarse-mesh boundaries normally coincide with material discontinuities, it may sometimes accelerate convergence if artificial boundaries are introduced. This is so because of the rebalancing algorithm which enforces particle conservation within each zone as the iteration proceeds.

The above example is for (x,y) geometry. The same procedures are followed in the other geometries in which r plays the role of x and θ , or z plays the role of y. In (r, θ) geometry the entire left boundary of the figure drawn by MAPPER corresponds to a single point, the origin.

C. Source Options

The TWOTRAN user may specify an anisotropic distributed source or the boundary flux at the top and/or bottom and/or right boundaries of the system. The inhomogeneous distributed source must be represented by the finite spherical harmonic expansion

$$Q_g(\underline{r}, \mu, \eta) = \sum_{n=0}^{IQAN} 2n+1 \sum_{k=0}^n R_n^k(\mu, \eta) Q_{gn}^k(\underline{r}), \quad (92)$$

in which IQAN is an input number designating the order of anisotropy of the source, and R_n^k is the spherical harmonic defined in (7) above. In these terms,

$$Q_{gn}^k = \frac{1}{4\pi} \int_{-1}^1 d\mu \int_0^{2\pi} d\eta Q_g(\underline{r}, \mu, \eta) R_n^k(\mu, \eta). \quad (93)$$

We have written the integral in this equation over the entire range of η and used a 4π normalization of the integral, but it should be clear that Q_g must be symmetric in $\eta(Q_g(\eta) = Q_g(-\eta))$, and this we have noted by writing Q_g as a function of μ and η alone.

In addition to specifying IQAN, the user must enter $((IQAN + 1)(IQAN + 2))/2$ components of Q_g multiplied by $(2n + 1)$; that is, the user must enter

$$(2n + 1)Q_{gn}^k \quad k = 0, 1, \dots, n \quad (94)$$

$$n = 0, 1, \dots, IQAN$$

in the order shown in Table X.

When using the anisotropic distributed source option, the order of anisotropic scattering, ISCT, must be at least as large as IQAN so that the requisite number of scattering coefficients R_n^k are computed.

The user is also allowed to specify the value of the incoming flux along the top and/or right and/or bottom boundaries, but only if these boundaries are specified as vacuum boundaries (IBT and/or IER and/or IEB = 0). Because of the quadrant organization of TWOTRAN, the user must supply two blocks of MM values at each point along the boundary. For example, at each point along the right boundary the user must supply MM boundary fluxes ($QR1$) for the in-down directions ($\mu < 0, \eta < 0$)

TABLE X

Ordering of Anisotropic Distributed Source Components

Component Number	n	k
1.	0	0
2.	1	0
3.	1	1
4.	2	0
5.	2	1
6.	2	2
7.	3	0
8.	3	1
9.	3	2
10.	3	3
.	.	.
etc.		

and MM values ($QR2$) for the in-up directions ($\mu < 0, \eta > 0$). The ordering of these values in direction corresponds directly to the ordering of direction cosines given in Fig. 15. The detailed input ordering is described in the next section.

An incoming flux on the boundary corresponds to a source of particles which is computed from

$$\text{Source} = \sum_{j=1}^{JT} \sum_{m=1}^{MM} w_m \mu (QR1_{m,j} + QR2_{m,j}) A_{IT+\frac{1}{2},j} \quad (95)$$

for the right boundary and from

$$\text{Source} = \sum_{i=1}^{IT} \sum_{m=1}^{MM} w_m \eta (QT1_{m,i} + QT2_{m,i}) B_{JT} \quad (96)$$

$$\text{Source} = \sum_{i=1}^{IT} \sum_{m=1}^{MM} w_m \eta (QR1_{m,i} + QR2_{m,i}) B_{JT} \quad (97)$$

for the top and bottom boundaries. The sum of the values of these sources for all groups is added to the value (if any) of the volume-energy integrated, distributed sources for the purpose of normalization of all inhomogeneous sources.

D. Source and Flux Input Options

1. Source Input Options

If a source of anisotropy IQAN is designated, the MMQ = $((IQAN+1)(IQAN+2))/2$ components of the

source must be entered for each group in the order listed above. This may be accomplished with the use of five options designated by the input value of IQOPT. For IQOPT=0, a value of zero is automatically entered for all elements of the distributed source, and then input boundary fluxes are read. In the remaining four options, the various arrays described below are supplied for each group, first for group 1, then for group 2, and so forth, ending with group IGM (the lowest energy group if the Los Alamos cross-section format is used). For each group the boundary sources (if any) are entered after the distributed source. If distributed or boundary sources are specified in any group, values must be entered in all groups. The ordering of source input within each group is

1. $((Q_{gijn}, i = 1, IT), j = 1, JT)$ for $n = 1, NM$
2. Optionally: a. $((QR1_{gjm}, j = 1, JT), m = 1, MM)$, then
b. $((QR2_{gjm}, j = 1, JT), m = 1, MM)$
3. Optionally: a. $((QB1_{gim}, i = 1, IT), m = 1, MM)$, then
b. $((QB2_{gim}, i = 1, IT), m = 1, MM)$
4. Optionally: a. $((QT1_{gim}, i = 1, IT), m = 1, MM)$, then
b. $((QT2_{gim}, i = 1, IT), m = 1, MM)$.

The above notation is that of standard Fortran. For example, the source Q is entered in NM blocks of $IT \times JT$ numbers, in a continuous stream, the first IT numbers for $j = 1$, the next IT numbers for $j = 2$, etc., all for $n = 1$. Then, for $n = 2$, IT numbers are entered for $j = 1$, and so on. For the boundary fluxes, the $QR1$ block corresponds to the in-down directions and the $QR2$ block to the in-up directions. Each of these blocks is entered by supplying JT numbers for $m = 1$, then JT numbers for $m = 2$, etc. The top boundary flux block, $QT1$, is for fluxes in the in-down direction, and the block $QT2$ is for the out-down directions. The block $QB1$ is for in-up di-

rections, and $QB2$ is for out-up directions.

In addition to $IQOPT=0$ in which Q_{gijn} is automatically set to zero, there are four other options to simplify the reading of Q_{gijn} . These options are all executed for each anisotropic component of Q , i.e., for $n = 1, 2, \dots, NM$. These options are:

- IQOPT=1 Enter an energy spectrum $(GR_{g,n}, g=1, IGM)$ for each n . Then
 $Q_{gijn} = GR_{g,n}$ for all i and j for each g and n .
- IQOPT=2 Enter the complete array Q_{gijn} as described above.
- IQOPT=3 Enter first a spectrum $(GR_{g,n}, g=1, IGM)$ as in option 1. Then enter a shape $((F_{ijn}, i = 1, IT), j = 1, JT)$ in a continuous block of $IT \times JT$ numbers. Then the source is made by
 $Q_{gijn} = GR_{g,n} F_{ijn}$ for all i and j for each g and n .
- IQOPT=4 Enter a spectrum $(GR_{g,n}, g=1, IGM)$, an i-direction spatial shape $(X_{i,n}, i = 1, IT)$, and a j-direction spatial shape $(Y_{j,n}, j = 1, JT)$. The source is then given by
 $Q_{gijn} = GR_{g,n} X_{i,n} Y_{j,n}$ for all i and j for each g and n .

2. Flux Input Options

Options for reading an input flux guess are similar to, but more varied than, those for reading an input source. If $ISCT$ is the order of anisotropic scattering, then there are $MM = (ISCT+1)(ISCT+2)/2$ spherical harmonic components of the angular flux.* Options for entering these components are selected by the input value of the integer $ISTART$ which may assume all values from -7 to 7. The options are as follows:

*These components are ordered exactly as are those of anisotropic sources. See Table X.

<u>ISTART</u>	<u>OPTION</u>
-7	Same as option 7 below.
-6	A problem restart dump is entered on unit 7, but only the isotropic component of the flux is read from it.
-5	Same as option 5 below.
-4	Same as option 4 except that only the first (isotropic) component of the NM components is entered.
-3	Same as option 3 except isotropic components only.
-2	Same as option 2 except isotropic components only.
-1	Same as option 1 except isotropic components only.
0	No flux guess required, but a fission guess (unity in every mesh cell) is automatically supplied.
1	As in source option 1, a spectrum is supplied so that $FLUX_{gijn} = GR_{gn}$
2	The entire array $FLUX_{gijn}$ is entered in blocks of ITxJT continuous numbers first for the first component, then the second, up to NM components. Then this process is repeated for the next group. Imagine that fluxes are read by: $DO\ 10\ G = 1, IGM$ $DO\ 10\ N = 1, NM$ $10\ READ\ ((FLUX(G,I,J,N),\ I = 1, IT),$ $J = 1, JT).$
3	Just as in source option 3, a spectrum and an (i,j) shape are entered. Then the flux is given by $FLUX_{gijn} = GR_{g,n}^F i j n$
4	Just as in source option 4, a spectrum, an i shape, and a j shape are read so that the flux is $FLUX_{gijn} = GR_{g,n}^X i, n^Y j, n$
5	A problem-restart dump is entered on unit 7; to run the problem, the user enters the problem title cards, the three cards of control integers (changing the starting option), a list of parameters to be changed, and any edit input. (See Section E below.)

<u>ISTART</u>	<u>OPTION</u> (Continued)
6	A problem-restart dump is entered on unit 7, but only the complete flux guess is used from it. For treatment when number of anisotropic components differs from that of problem at hand, see Section 3a.
7	A flux tape containing only the first (isotropic) component of the flux in IGM blocks of ITxJT numbers is entered on unit 7. This tape may be prepared by other programs.

3. Flux Guesses From Tape

The flux guess may be obtained from a tape which contains just the scalar flux or from a restart dump tape containing all information necessary for a problem restart. The code assumes that the flux guess will be obtained from the unit to which NFLUX is initially assigned. (See Section III.E above.) It should be noted that if two or more dumps of any kind are written, the second is written on NFLUX, destroying previous information.

a. Restart Dump Flux Guess

To use a flux guess from a restart dump tape requires that the problem for which the guess is requested have the same number of groups and the same product of the total number of fine radial mesh intervals and total number of fine axial mesh intervals as did the problem which produced the restart dump. The number of flux components need not be the same.

The scalar flux is obtained from the restart dump tape when the flux input indicator is minus six. The higher order components of the flux are set zero. The scalar flux and all available higher order components of the flux are obtained from the group-restart dump tape when the flux input indicator is plus six. When more flux components than are available are required by the problem, the code supplies all the flux components that are available from the restart dump tape and fills the remainder of the components with zero.

When fewer flux components than are available are required by the problem, the code supplies only those components needed from the dump tape.

b. Scalar Flux Tape

A scalar flux guess may be obtained from a tape prepared by another code as a flux guess when the

flux input indicator is set to seven or minus seven.

The form of the tape is assumed to be binary and to have IGM (the number of groups) records. Each record is assumed to contain the scalar flux as a two-dimensional matrix with lengths of the total number of fine radial mesh intervals and total number of fine axial mesh intervals. The order of the flux is that generated by the statement.

WRITE((FLUX(I,J), I = 1,IT), J = 1,JT).

E. Flux Dumps and Problem Restart

Three types of flux dumps are taken. The dump has the same form regardless of type. A periodic dump is taken every fifteen minutes to back up the calculation. A time limit dump is taken after a user-specified period of time. A time of five to ten minutes less than the requested total running time is suggested. A final dump is automatically taken after the successful completion of a problem. Each of these dumps contains all the information necessary for use in ISTART options 5 and 6 above. Each of these dumps is written alternately on unit 5 or 7 depending on which is free; a message is written to inform the user which unit contains the latest dump.

1. Restart Procedures

Problem execution can be continued using restart dumps. Certain input parameters can be changed at the time of problem restart. The restart dump also provides a means of changing certain input parameters. The restart dump tape must be assigned to the initial unit number of NFLUX.

The restart deck is composed of the problem input cards through the integer parameter section, the namelist section, and the edit input section. The integer parameter section must contain the absolute value of five in ISTART. The remaining values of the section are ignored. In the namelist section, the user may change certain input parameters and only those. The namelist feature is standard to Fortran and allows the user to enter only those numbers he wants to change.

The first column of namelist input cards is ignored. Columns 2 through 8 must contain \$TWOINP, and column 9 must contain a blank for the first card. The user then defines only the values which

he wishes to change. These entries are of the free-field form, name = value. Different entries are separated by commas, and the last entry must end with a dollar sign. Continuation cards are permitted provided the last entry of the preceding card ends with a comma. The integers that may be changed are

1. ITTL Maximum number of inner iterations.
2. TTLIM Time limit.
3. IEDOPT Edit options.

The floating-point numbers that may be changed are

1. EV Eigenvalue guess.
2. EVM Eigenvalue modifier.
3. PVM Parametric value of k_{eff} .
4. XLAL Search lambda lower limit.
5. XLAH Search lambda upper limit.
6. XLAX Fine-mesh search precision.
7. EPS Convergence precision.
8. NORM Normalization amplitude.
9. POD New parameter modifier.
10. BHGT Buckling height.

The edit input section is determined by the current value of IEDOPT. If the original problem had IEDOPT > 0 and if IEDOPT is not changed to zero on restart, then the edit input must be re-entered. The user may desire to enter his edit input only after several restarts have been performed, or for the edit of a final dump.

2. Restart Tape Composition

The restart dump is composed of the following records: restart parameter information, named common block FWBGN2, parameter common block IA, data common block A (only the portion used by the case), group flux blocks from ECS, group cross-section and Q-source blocks from ECS, and the group flow blocks from ECS.

The restart parameter information is a vector which contains twelve words in the following order:

1. IGM Number of groups.
2. NM Number of flux components.
3. ITJT Product of fine radial mesh total and fine axial mesh total.
4. LENMCB Length of named common block FWBGN1.

5. LENCIA Length of common block IA.
6. LENDBA Length of the main common data block A.
7. ECPTFX ECS pointer for the flux block for the first group.
8. ECLTFX Length of the flux block for a group.
9. ECPTCQ ECS pointer for the cross-section and Q-source block for the first group.
10. ECLTCQ Length of the cross-section and Q-source block for a group.
11. ECPIFL ECS pointer for the flow block for the first group.
12. ECLIFL Length of the flow block for a group.

F. Flux Plotting on Film

Fluxes generated by TWOTRAN can be plotted on film. Either a projective plot or a contour plot, or both, can be obtained for the angle-integrated flux in each group. The following indicators set in the first overlay segment (program TWOTRAN) specify the details of the plot:

1. INDPLT 0/1/2/3 = none/contour/projection/both
2. NOCLNS Number of contour lines desired (must be at least 1 and less than 51)
3. IWFILM 0/1 Flux is stored in ECS/Flux file NFIUX.

The plotting routines calculate the center of the fine-mesh cells for use as abscissae. Because θ is measured in revolutions, we scale the axial (θ) directions in (r, θ) geometry by the factor radial maximum/axial maximum to make a more presentable plot.

V. USE OF SPECIFIC TWOTRAN OPTIONS

A. Coarse-Mesh Rebalancing

All of the operations of the coarse-mesh rebalancing algorithms are automatic. The user, however, can select the zones in which particle balance is to be enforced. The zones needed to describe material discontinuities normally provide all the zones necessary for rebalance, but in homogeneous or nearly homogeneous systems it is frequently advantageous to add zones to accelerate convergence. Experience has shown that there is a problem-dependent optimum number of zones which most rapidly accelerates convergence. This number is close to the limit in which the coarse mesh

coincides with the fine mesh. However, too many coarse-mesh zones waste core storage and computing time.

The user should keep in mind that the solution of the coarse-mesh equations is iterative. A single line inversion in the i -direction is made, starting at the bottom j -level and moving to the top. If the system is a single coarse-mesh zone high, then there is no iteration, but in a large many-zoned system (particularly a thermal system) the inversion of the five-point rebalance equations may become time-consuming.

A zone-dependent balance edit is automatically made for each of the rebalance regions. Inflows, outflows, absorptions, and sources are given for each zone so that it is sometimes convenient to construct a zone simply to obtain this edit.

B. Eigenvalue Searches

It is possible in TWOTRAN to adjust nuclide concentrations, system dimensions, or the value of the time absorption to achieve a desired value of k_{eff} . This value is taken to be unity (criticality) unless the parametric eigenvalue trigger (IPVT) is set to unity. In this case, the parametric value of k_{eff} is entered as an input number (not applicable to time-absorption problems).

The modification of cross-section concentrations takes place as indicated in Section IV.A.2. This type of problem is run when the eigenvalue type indicator (IEVT) is 3. If IEVT is 2 (time absorption computation), the value EV/v_g is added to the absorption and total cross sections in each group. Here v_g is the speed associated with energy group g . If IEVT = 4, the coarse-mesh boundaries can be modified selectively. Either the i -boundaries ($IDM = 1$) or the j -boundaries ($IYM = 1$), or both, can be altered. In each case the modified boundaries (XRADA or YRADA) are calculated from the initial boundaries (XRAD or YRAD) by

$$XRADA_{k+1} = XRADA_k + (XRAD_{k+1} - XRAD_k) * (1 + EV * XM_k)$$

$$k = 1, 2, \dots, IM, \quad (98)$$

$$YRADA_{l+1} = YRADA_l + (YRAD_{l+1} - YRAD_l) * (1 + EV * YM_l)$$

$$l = 1, 2, \dots, JM. \quad (99)$$

In these equations, X_M and Y_M are zone modification coefficients that must be entered by the user. Clearly, if one of these numbers is zero, that particular zone is not altered. If all X_M are unity, the i -dimension is uniformly expanded or contracted. Many sophisticated changes can be made. For example, an interface between two zones can be moved while the rest of the system is left unchanged.

Regardless of the parameter being adjusted, the search is executed by performing a sequence of k_{eff} calculations, each for a different value of the parameter being treated as the eigenvalue. Each of the successive k_{eff} calculations is accelerated by coarse-mesh rebalancing, but the search for the desired value of k_{eff} is conducted by subroutine NEWPAR. Regardless of the nature of the problem, the search is for a value of the parameter which makes the value of λ defined in Eq. (78) unity.

In the following description of NEWPAR, it is helpful to refer to Fig. 19 in which the deviation of λ from unity is plotted against outer iteration number.

For the initial system, NEWPAR continues outer iteration until two successive values of λ differ by less than EPS_0 . For subsequent sequences of λ values, a different convergence precision, XLAX , is used. After the first converged λ sequence is obtained, the initial value of the eigenvalue (EV) is altered by EVM, an input value. If $\lambda > 1$ (multi-

plying system), the new eigenvalue is equal to $\text{EV} + \text{EVM}$; if $\lambda < 1$ (decaying system), the new value is $\text{EV} - \text{EVM}$. These alterations correspond to the addition or the subtraction of an absorption, e.g., as in a time-absorption search or a poison-concentration search. However, certain system changes act as negative absorptions, and the user must adjust the sign of EVM accordingly; otherwise the modification made will take the system away from the desired value of k_{eff} and the search will take longer. For example, if all i -dimensions are altered uniformly, an expansion acts as a negative absorption, and thus a negative value of EVM should be used.

Basically, after two values of $k_{\text{eff}} (\lambda)$ are obtained for two different system configurations, subroutine NEWPAR attempts to fit a curve through the most recent values to extrapolate or interpolate to a value of unity. Depending on the amount of information available and the size of $|1 - \lambda|$, this fit proceeds in different ways. A parabolic fit cannot be made until three converged values of λ are available, and is not attempted unless $|1 - \lambda|$ is greater than an input-search lower limit (XLAL) and less than an input-search upper limit (XLAH). If a parabolic fit is tried and the roots are imaginary, a straight-line fit is used. If the roots are not imaginary, the closest root is used as the new value of EV. Once a bracket is obtained (change of sign of $\lambda - 1$), the fit procedure is not allowed

Fig. 19. Variation of λ during a hypothetical eigenvalue search.

to move outside the region of the bracket. Should a parabolic fit select an eigenvalue outside the bracket region, this value is rejected and the new value is taken to be one-half the sum of the previous value and the value previous to that.

Whenever the parabolic fit is not used, a linear fit is used and the new eigenvalue is computed from

$$(EV)_{\text{new}} = (EV)_{\text{old}} + \text{POD} * \text{EQ} * (1 - \lambda) \quad , \quad (100)$$

where POD is an input "parameter oscillation damper" which may be used to restrict the amount of change in the eigenvalue. In Eq. (100), EQ is a measure of the slope of the curve. When $|1 - \lambda| > \text{XLAH}$, $(1 - \lambda)$ in Eq. (100) is replaced by XLAH (with the correct sign) to prevent too large a change in EV. After $|1 - \lambda| < \text{XLAH}$, the value of EQ is fixed and kept constant until convergence to prevent numerical difficulty in the approximation of the derivative when λ is close to unity.

Because parametric search problems represent sequences of k_{eff} calculations, it behooves the user to study the use of subroutine NEWPAR in order to optimize his calculations. It also behooves the user to pose soluble problems. That is, there are many problems, especially concentration searches, for which solutions are not possible, and discovering this by trial and error is the hard way. Ideally, the user will have some estimate of the critical parameter available from a lower order computation.

Convergence in time-absorption calculations is typically one-sided. That is, if EV is negative, then there is a possibility that the denominator of Eq. (11) will become negative. If this happens, the automatic search procedure fails (dramatically). For this reason $\text{POD} = 0.5$ or less is frequently used in such searches.

C. Adjoint Computations

The TWOTRAN program solves the adjoint transport equation by transposing the matrices of scattering coefficients and inverting the group order of the problem. The solution of the resulting problem in direction $\vec{n}$ is then identified with the solution of the adjoint equation in direction $-\vec{n}$.¹¹

The inversion of the group order is made be-

cause the transposition of the scattering matrices usually converts a downscattering problem to an up-scattering problem. Because of the inversion, the user must:

- (a) Enter any inhomogeneous sources, including boundary fluxes, in inverse group order,
- (b) Enter any flux guess in inverse group order, and
- (c) Remember that any output is in inverse group order, i.e., that groups labeled 1, 2, ..., are really group IGM, IGM - 1, etc.

Similarly, the output flux from an adjoint problem must be inverted before insertion in a direct problem. On the other hand, an output flux from one adjoint problem is in the proper group order for use in another adjoint problem.

The group order of the group speeds and the fission spectrum is inverted by the program.

D. Edit Options

During a final outer iteration, a balance table is prepared for each coarse-mesh zone in each group. This table lists the absorption, source, inflows, and outflows for each coarse-mesh interval. The absorption in this table is actually the removal rate used in the coarse-mesh rebalance. An identical balance table is also prepared for the sum of the quantities over all groups. In this table the group sum of the true absorption appears.

In addition to these balance tables, the user of TWOTRAN may also edit the output in a variety of ways. First of all the user may obtain a macroscopic activity edit. In this edit, the activity of cross sections 1 through IHT is computed for each group for the macroscopic cross sections used in the problem. These activities are computed for edit zones defined by the user by entering sets of numbers which identify each coarse-mesh interval with an edit zone. As many such sets of numbers may be used as desired. That is, the user may perform the edit calculation for as many zone configurations as he chooses. An edit zone is simply a collection of coarse-mesh intervals with the same zone number. The coarse-mesh intervals of a zone need not be contiguous. In addition to the macroscopic activity edit, the user is also given the zone right, left, top, bottom, and net leakage, the zone source integral, and the zone flux

integral. The zone absorption is given by the macroscopic activity for the cross section IHT-2.

The user may also obtain all the above information plus a microscopic activity edit. For this edit, the user enters the number of microscopic activities he desires and the cross-section block numbers of the microscopic cross sections he is interested in. Cross-section blocks for this edit must appear in the MIXNUM array; i.e., they must be part of the mixture specifications. If the microscopic cross section appears in several mixtures, the total density of the material appearing in the table is used in calculating the activity. To edit a material which is not actually part of the problem, the user may simply add a mixture instruction to the mixture tables (see Section IV.A.2); or, if interested in only a few cross sections, he may add these cross sections to other blocks in rows IHT-5, IHT-6, etc. (See Section IV.A.1.)

For either the microscopic or the macroscopic edit, the user may also obtain a zone relative power density. In this calculation, the group sum of the zone volume integral of the fission rate (times ν , the number of secondaries per fission) is divided by the zone volume, for all zones. These numbers are then normalized to (divided by) the value of this quantity for a user-designated zone. If the user selects zone zero, the normalization is to the whole system power density.

All of the edit calculations are performed in subroutine EDITOR. Certain rearrangements of core storage are made by subroutine EDCALL, and a picture of the edit zones is drawn by EDMAP. Edit input must be entered for any problem restart, and thus can be changed at this point. A Final Dump tape may also be edited by using it as a restart dump for the same problem.

VI. DETAILED INPUT SPECIFICATION

A. Special Input and Output Routines

Except for the control parameters, cross sections, and the special flux guesses, all floating-point numbers and integers are read into TWOTRAN in special formats. These formats are [6, (I1, I2, E9.4)] for reading floating-point numbers and [6, (I1, I2, I9)] for integers. In each word of both of these formats, the first integer field, I1, designates the options listed below. The second integer

field, I2, controls the execution of the option, and the remainder of the field, I9 or E9.4, is for the input data. All data blocks read with these formats must be ended with a 3 in the I1 field after the last word of the block. The available options are given in Table XI.

TABLE XI

Options for Special Read Formats

Value of I1	Nature of Option
0 or blank	No action
1	Repeat data word in 9 field number of times indicated in I2 field.
2	Place number of linear interpolants indicated in I2 field between data word in 9 field and data word in next 9 field. <u>Not allowed for integers.</u>
3	Terminate reading of data block. <u>A 3 must follow last data word of all blocks.</u>
4	Fill remainder of block with data word in 9 field. This operation must be followed by a terminate (3).
5	Repeat data word in 9 field 10 times the value in the I2 field.
9	Skip to the next data card.

Five illustrations of the use of the special formats are given below. These illustrate:

- 1 - Zero is repeated 47 times.
- 2 - Zero is repeated 470 times.
- 3 - Four interpolants are inserted between 0.0 and 5.0 giving six data numbers: 0.0, 1.0, 2.0, 3.0, 4.0, 5.0.
- 4 - Four interpolants are inserted between 0.0 and 5.0, two between 5.0 and 7.0, and 7.0 is repeated 10 times.
- 5 - After reading 0 and 4 we skip to next card and read 7.

I1 I2	I1 I2	I1 I2	I1 I2	
00 00	00 00	00 00	00 00	
01 00	01 00	01 00	01 00	EXAMPLE 1
02 00	02 00	02 00	02 00	EXAMPLE 2
03 00	03 00	03 00	03 00	EXAMPLE 3
04 00	04 00	04 00	04 00	EXAMPLE 4
05 00	05 00	05 00	05 00	EXAMPLE 5
06 00	06 00	06 00	06 00	EXAMPLE 6

A special routine, WRITE, is used to print most of the two- and three-dimensional arrays that occur in the program. This routine can be used for one-, two-, or three-dimensional arrays and has an option for printing a portion of an array, e.g., the mixed cross-section blocks, if any. When fluxes, sources, and fission rates are printed, they are oriented with the origin at the lower left of the page, corresponding to the picture drawn by MAPPER.

In the following pages the input data for TWO-TRAN are listed in exactly the order in which they are entered in the code. The data are divided into three categories: (1) job title cards, (2) control integers on cards 1 through 3 and control floating-

point numbers on cards 4 and 5, and (3) problem-dependent data on subsequent cards. Categories (1) and (2) can be entered on the form shown in Fig. 20.

B. Job Title Cards

The user begins by indicating on a card in an I6 format the number of title or job description cards he wants to use. He then enters the descriptive material on these cards which are read with a I2A6 format.

C. Input of Control Numbers

On cards 1 through 3, the user enters the following control integers which are read in a I2I6 format:

Number Word on Card	Name of Variable	Comments
CONTROL INTEGERS-----CARD 1		
1	ITH	0/1 (direct/adjoint) type of calculation to be performed.
2	ISCT	0/N (isotropic/Nth-order anisotropic) order of scattering calculation. There are $NM = (ISCT + 1)(ISCT + 2)/2$ spherical harmonics flux components computed. These are not used to compute a scattering source unless some zone material identification number is negative. See IDCS below.
3	ISN	S_n Order. Even integer only. If negative, user must supply his own quadrature coefficients. Otherwise (for ISN = 2 through 16) built-in constants are used.
4	IGM	Number of groups.
5	IM	Number of coarse-mesh intervals in the i-direction.
6	JM	Number of coarse-mesh intervals in the j-direction.
7	IBL	Left boundary condition: 0/1 vacuum/reflective.
8	IBR	Right boundary condition: 0/1/2 vacuum/reflective/white.
9	IBB	Bottom boundary condition: 0/1/2/3 vacuum/reflective/white/periodic.
10	IBT	Top boundary condition: 0/1/2/3 vacuum/reflective/white/periodic.
11	LEVT	Eigenvalue type - 0/1/2/3/4 - inhomogeneous source (Q)/ k_{eff} calculation/time absorption (alpha) search/nuclide concentration (C) search/zone thickness (delta) search.
12	ISTART	Input flux guess and starting options -7/-6/-5/-4/-3/-2/-1/0/1/2/3/4/5/6/7. See Section IV.D.2.

CONTROL INTEGERS-----CARD 2		
1	MT	Total number of materials (cross-section blocks including anisotropic cross sections) in the problem.
2,3	MIN	Number of input materials. This number is divided into 2I3 words, MTP and MCR, where MTP is the number of materials from the LIBRARY disk and MCR is the number of materials from the card reader.

Page _____

57

Fig. 20. Input data sheet.

4	MS	Number of mixture instructions. See Section IV.A.2 above and items MIXNUM, MIXCOM, and MIXDEN below.
5	IHT	Row of total cross section in the cross-section format. If IHT < 0, code assumes that there is no σ^{up} in cross-section table.
6	IHS	Row of within-group scattering cross section in the cross-section format.
7	IHM	Total number of rows in the cross-section format.
8	IQOPT	0/1/2/3/4 Options for input of inhomogeneous source. See Section IV.D.1 above.
9	IQAN	Order of anisotropy of inhomogeneous distributed source.
10,11,12	IQR/IQB/ IQT	Right/bottom/top boundary sources to be specified as input. 0/1 - (no/yes). This word is split into a 312 format. See items QR1, QR2, QB1, QB2, QT1, and QT2 below. These sources are the value of the incoming flux on the boundary.
13	IPVT	0/1 (no/yes) Is a parametric eigenvalue being entered? See item PV below.
14	ITTL	Maximum number of inner iterations allowed per group.
15	IDM	0/1 (no/yes) Are the i-direction zone thicknesses to be modified? See item XM below.

CONTROL INTEGERS-----CARD 3

1	IYM	0/1 (no/yes) Are the j-direction zone thicknesses to be modified? See item YM below.
2	ITLIM	0/seconds If an integer number of seconds is entered, a restart dump is taken after this number of seconds and the problem is terminated.
3	IGEDM	1/2/3 (x,y)/(r,z)/(r,t) geometry.
4	IEDOPT	0/1/2/3/4 (none/option) Edit options. Option 1 is a macroscopic edit, Option 2 is a macroscopic plus microscopic edit. Options 3 and 4 give the information of options 1 and 2 (respectively) plus a zone relative power density edit.

Cards 4 and 5 consist of control floating-point numbers read in a E12.6 format.

CONTROL FLOATING-POINT DATA-----CARD 4

1	EV	Eigenvalue guess. It is satisfactory to enter 1.0 for IEVT = 3 and 0.0 for all other problems.
2	EVM	Eigenvalue modifier used only if IEVT > 1. See Section V.B above.
3	PV	Parametric value of k_{eff} for subcritical or supercritical systems.
4	XLAL	Lambda lower limit for eigenvalue searches. See Section V.B above.
5	XLAH	Search lambda upper limit.
6	XLAX	Search lambda convergence precision for second and subsequent values of the eigenvalue.

CONTROL FLOATING-POINT DATA-----CARD 5

1	EPS	Convergence precision.
2	NORM	Normalization factor. Total number of particles in system normalized to this number if it is nonzero. No normalization if NORM is zero.
3	POD	Parameter oscillation damper used in eigenvalue searches. See Section V.B above.
4	BHGT	Buckling height (in cm if cross sections are in barns) used to simulate z-dimension of system by adding an absorption given by

$$\sigma_{a,BHGT} = \frac{\sigma_t}{3} (\pi / (BHGT * \sigma_t + 1.4209))^2$$

Here 1.4209 is twice the Milne problem extrapolation distance, and σ_t is the total cross section. Used in (r, θ) and (x,y) geometry only.

D. Input of Remaining Data

With the exception of the cross sections, all the following data are read by the special formats discussed in Section VI.A. We denote these formats by S(I) for integers and S(E) for floating-point numbers.

Block Name and Dimension	Format	Number of Entries	Comments																				
IHX(IM)	S(I)	IM	Integers defining the number of fine-mesh i-intervals in each coarse-mesh k-interval.																				
IHY(JM)	S(I)	JM	Integers defining the number of fine-mesh j-intervals in each coarse-mesh l-interval.																				
IDLIB(MTP)	S(I)	MTP	Identification numbers designating nuclides to be read from the library. Nuclides read from the library are assigned identification numbers MCR + 1, MCR + 2, etc., in the order read.																				
C(IHM,IGM,MIN) (6E12.5)			MCR blocks of IHM*IGM numbers. Cross-section blocks from cards. <u>Each block is preceded by a header (identification) card read with a 12A6 format.</u> MCR blocks are read in this fashion, and MIN = MCR + MTP.																				
Input FLUX Guess FLUX(NM, IT, JT)	S(E)		Number of entries depends on option. See Section IV.D.2. <table><tr><th>Option</th><th>Number</th></tr><tr><td>-4</td><td>IGM+IT+JT</td></tr><tr><td>-3</td><td>IGM+IT*JT</td></tr><tr><td>-2</td><td>IGM blocks of IT*JT</td></tr><tr><td>-1</td><td>IGM</td></tr><tr><td>0</td><td>None</td></tr><tr><td>1</td><td>NM sets of IGM</td></tr><tr><td>2</td><td>IGM groups of NM sets of IT*JT</td></tr><tr><td>3</td><td>NM sets of IGM+IT*JT</td></tr><tr><td>4</td><td>NM sets of IGM+IT+JT</td></tr></table>	Option	Number	-4	IGM+IT+JT	-3	IGM+IT*JT	-2	IGM blocks of IT*JT	-1	IGM	0	None	1	NM sets of IGM	2	IGM groups of NM sets of IT*JT	3	NM sets of IGM+IT*JT	4	NM sets of IGM+IT+JT
Option	Number																						
-4	IGM+IT+JT																						
-3	IGM+IT*JT																						
-2	IGM blocks of IT*JT																						
-1	IGM																						
0	None																						
1	NM sets of IGM																						
2	IGM groups of NM sets of IT*JT																						
3	NM sets of IGM+IT*JT																						
4	NM sets of IGM+IT+JT																						
Input Source Q(NMQ, IT, JT)	S(E)		Number of entries depends on option. See Section IV.D.1. <table><tr><th>Option</th><th>Number</th></tr><tr><td>0</td><td>None</td></tr><tr><td>1</td><td>NMQ sets of IGM</td></tr><tr><td>2</td><td>IGM groups of NMQ blocks of IT*JT</td></tr><tr><td>3</td><td>NMQ sets of IGM+IT*JT</td></tr><tr><td>4</td><td>NMQ sets of IGM+IT+JT</td></tr></table>	Option	Number	0	None	1	NMQ sets of IGM	2	IGM groups of NMQ blocks of IT*JT	3	NMQ sets of IGM+IT*JT	4	NMQ sets of IGM+IT+JT								
Option	Number																						
0	None																						
1	NMQ sets of IGM																						
2	IGM groups of NMQ blocks of IT*JT																						
3	NMQ sets of IGM+IT*JT																						
4	NMQ sets of IGM+IT+JT																						
QR1(JT,MM)	S(E)	JT*MM	Right boundary source (flux) in the in-down directions. <u>Do not enter unless IQR = 1.</u>																				
QR2(JT,MM)	S(E)	JT*MM	Right boundary source (flux) in the in-up directions. <u>Do not enter unless IQR = 1.</u>																				
QB1(IT,MM)	S(E)	IT*MM	Bottom boundary source (flux) in the in-up directions. <u>Do not enter unless IQB = 1.</u>																				
QB2(IT,MM)	S(E)	IT*MM	Bottom boundary source (flux) in the out-up directions. <u>Do not enter unless IQB = 1.</u>																				
QT1(IT,MM)	S(E)	IT*MM	Top boundary source (flux) in the in-down directions. <u>Do not enter unless IQT = 1.</u>																				
QT2(IT,MM)	S(E)	IT*MM	Top boundary source (flux) in the out-down directions. <u>Do not enter unless IQT = 1.</u>																				

Block Name	Format	Number of Entries	Comments
WGT(MM)	S(E)	MM	Quadrature weights. <u>Do not enter unless ISN < 0.</u>
COSMU(MM)	S(E)	MM	Quadrature μ cosines. <u>Do not enter unless ISN < 0.</u>
COSETA(MM)	S(E)	MM	Quadrature η cosines. <u>Do not enter unless ISN < 0.</u>
XRAD(IP)	S(E)	IM+1	Coarse k-mesh boundaries (cm). Must form increasing sequence.
YRAD(IP)	S(E)	JM+1	Coarse l-mesh boundaries (cm). Must form increasing sequence.
IDCS(IM,JM)	S(I)	IM*JM	Cross-sections zone identification numbers. These numbers assign a cross-section block to each zone defined by the coarse-mesh boundaries. If these numbers are negative, an anisotropic scattering source is calculated in the zone. <u>Do not use negative numbers here if ISCT = 0.</u> These numbers need not be negative, however, if ISCT > 0.
CHI(IGF)	S(E)	IGM	Fission fractions. Fraction of fission yield emerging in each group.
VEL(IGF)	S(E)	IGM	Group speeds. Used only in time absorption calculations.
MIXNUM(MS)	S(I)	MS	Numbers identifying cross-section block being mixed. See Section IV.A.2. <u>Do not enter if MS = 0.</u>
MIXCOM(MS)	S(I)	MS	Numbers controlling cross-section mixture process. See Section IV.A.2. <u>Do not enter if MS = 0.</u>
MIXDEN(MS)	S(E)	MS	Mixture densities. See Section IV.A.2. <u>Do not enter if MS = 0.</u>
XM(IM)	S(E)	IM	i-mesh modification factors. See Section V.B. <u>Do not enter unless IEVT = 4 and IXM > 0.</u>
YM(JM)	S(E)	JM	j-mesh modification factors. See Section V.B. <u>Do not enter unless IEVT = 4 and IYM > 0.</u>
NEDS	I6	1	Integer defining number of edits to be performed <u>IEDOPT > 0 only.</u>
MN	I6	1	Integer defining number of microscopic activities to be computed. <u>IEDOPT = 2 and 4 only.</u> See Section V.D.
MICID(MN)	S(I)	MN	Integers defining material blocks for which microscopic edit is to be made. <u>IEDOPT = 2 and 4 only.</u> See Section V.D.
NZ,NORMZ	2I6	2	The integer NZ is the number of edit zones. The integer NORMZ is the zone to which the power density is normalized (NORMZ is not used unless IEDOPT = 3 or 4). See Section V.D.
NEDZ(IM,JM)	S(I)	IM*JM	Integers defining which edit zone each coarse-mesh zone is in. See Section V.D.

VII. EXECUTED TEST PROBLEM

Below we reproduce most of the output from a small test problem which exercises many of the features of TWOTRAN, and, in fact, we have made many changes in what was once a design problem solely to exercise certain program options.

The first page of output contains one card of identification information, five cards of parameter input (the input for each card is separated by a space), the number of fine-mesh intervals per coarse-mesh interval, and core and ECS storage requirements. The next page lists the input cross sec-

tions. The third and fourth pages list all the remaining input, ending with the edit specifications. Note that flux input option -1 was used to put the isotropic component of the angular flux equal to unity in every space point in both groups. The S_6 constants from the library are listed in the position in which user input S_n constants would be listed.

On the fourth page, the system map indicates that this (r,z) computation will be executed for a cylinder of radius 10.9 cm and height 21.8 cm. Because the cylinder is symmetric about the central z-

plane, only 10.9 cm of the height is described and a reflecting bottom boundary condition is used. The system is divided into 16 coarse-mesh zones, and in 5 of them only isotropic scattering calculations are performed, i.e., in zones where material 7 is present.

Beginning on page 4, output computed by the code is printed. This includes the spherical harmonics (for this problem these are 1, μ , and η), the mixture specifications, the mixed cross sections, and the group fission fractions and velocities. Note that in the mixing of material 7, material 4 is introduced with a density of one, but is not present in the final mixture 7. These lines for material 4 are introduced only to obtain an edit of material 4. This edit will be taken only in the regions where material 7 is used in the calculation.

Page 6 lists the monitor of the progress of the calculation following a summary of convergence precisions. The column headed "rebalance convergence" contains the maximum deviation from one of any rebalance factor for the rebalance performed after each outer iteration. If the number of inner iterations in a group exceeds IITL, we print the error at that time immediately to the right of the number of inner iterations. In this problem this happens only once, in the second group during the first outer iteration. Then the error is 8.9×10^{-5} . Since this is less than the inner epsilon, the group actually converged on the tenth iteration.

The monitor lines display a typical sequence of events in an eigenvalue search. Here, both radial and axial meshes are modified, with all the radial coarse-mesh zones modified equally, but with only the second and fourth (from $z = 0$) axial zones modified, and these only half as much as the radial zones. We see that for zero eigenvalue, lambda converges to 0.877275498 (this is k_{eff} for the unaltered system),

then for an eigenvalue of 0.1 (the input value of EVM), lambda converges to 0.921901519, and so on until convergence at an eigenvalue of about 0.3064.

Following the monitor lines, balance tables are printed. On page 9 the final monitor line is printed followed by a summary of the original and final coarse-mesh boundaries. Then, isotropic fluxes and currents are printed for each group, and the fine-mesh fission rate is listed. This is the end of output unless there is an edit.

On page 14, the edit zones for the first edit are displayed, including the changed zone boundaries. Here, the edit zones correspond to the locations of materials 5 and 7. On page 15 in the microscopic edits, two items should be noted. First, the reaction rates for material 2 are zero. This is because material two (used to make 6, the anisotropic part of 5) does not appear as a constituent of either 5 or 7, and the edit routine therefore finds no indication that 2 is in the system. That is, only constituents of materials with numbers appearing in the system map can be edited. Second, the reaction rates for material 4 are nonzero only for zone 2 which is the only place material 7 occurs. To edit 4 over the whole system, it would have been necessary to add 4 to 5 in the same manner as it was added to 7.

After page 15, we have omitted the edits for group two and the group sums. On page 16, we show a second edit that can be performed, this time for three zones. Edits can be performed in this problem for up to 16 zones for any arrangement of zone numbers desired.

A contour plot of the isotropic component of the flux in group 1 is shown in Fig. 21, and a projective plot of the group 2 flux is shown in Fig. 22. Amplitudes of the contours are also given, but we have not reproduced these here.

Acknowledgements

The authors gratefully acknowledge the careful typing of Sue Vandervoort and the thorough editing of Kasha Thayer in the preparation of this report.

THIS CASE WAS PROCESSED BY THE XY/RZ/RT GEOMETRY TWOTRAN CODE OF 04/10/70 ON 04/15/70
 EDIT OPTION PROB DELTA CALC R,Z MODIFIED, BUT NOT EQUALLY
 TEST CASE NUMBER 15

①

```

0 ITH 0/1 DIRECT/ADJOINT
1 ISCT 0/N ISOTROPIC/NTH ORDER ANISOTROPIC
6 ISN SN ORDER
2 IGM NO. GROUPS
4 IM NO. COARSE MESH R INTERVALS
4 JM NO. COARSE MESH Z INTERVALS
1 0 1 0 LEFT/RIGHT/BOTTOM/TOP BOUNDARY CONDITION 0/1/2/3 VACUUM/REFLECTIVE/WHITE/PERIODIC
4 IEVT 0/1/2/3/4 Q/K/ALPHA/C/DELTA CALCULATION
-1 ISTART -7/-5/-5/-4/-3/-2/-1/0/1/2/3/4/5/6/7 STARTING OPTIONS ( MINUS FOR ISOTROPIC COMPONENT ONLY ) SEE MANUAL FOR DETAILS

7 MT TOTAL NO. OF MATERIALS
-0 4 LIBRARY/CARD READER INPUT MATERIALS
10 MS NO. OF MIXTURE INSTRUCTIONS
5 IHT ROW OF TOTAL CROSS SECTION
6 IHS ROW OF SELF SCATTER CROSS SECTION
7 IMM LAST ROW OF CROSS SECTION TABLE
0 IQOPT 0/1/2/3/4 NONE/SAME AS ISTART FOR SOURCE DISTRIBUTION
0 IQAN 0/N ISOTROPIC/NTH ORDER ANISOTROPIC SOURCE
0 0 0 TOR/IGB/IQT RIGHT/BOTTOM/TOP BOUNDARY SOURCE ( 0/1 NO/YES )
0 IPVT 0/1 NO/YES PARAMETRIC EIGENVALUE
10 IITL MAX NO. INNER ITERATIONS
1 IXM 0/1 NO/YES MODIFY R RADII (IEVT=4 ONLY)

1 IYM 0/1 NO/YES MODIFY Z RADII (IEVT=4 ONLY)
240 ITLIM 0/N NONE/SECONDS TIME LIMIT
2 IGEOM 1/2/3 (X,Y)/(R,Z)/(R,T) GEOMETRY
4 IEDOPT 0/1/2/3/4=NONE/N EDIT OPTIONS SEE MANUAL FOR DETAILS

0. EV EIGENVALUE GUESS
-1.000E-01 EVM EIGENVALUE MODIFIER
0. PV PARAMETRIC VALUE OF KEFF
1.000E-02 XLAL SEARCH LAMBDA LOWER LIMIT
5.000E-01 XLAM SEARCH LAMBDA UPPER LIMIT
1.000E-03 XLAX FINE MESH SEARCH PRECISION

1.000E-04 EPS CONVERGENCE PRECISION
1.000E+00 NORM NORMALIZATION AMPLITUDE
1.000E+00 FOD PARAMETER OSCILLATION DAMPER
0. BMGT TOTAL BUCKLING HEIGHT IN CM FOR (X,Y) AND (R,T) ONLY

```

```

FINE R MESH      4
                 3      3      2      2
FINE Z MESH      4
                 3      3      2      2

```

1728 WORDS OF CORE REQUIRED
 1359 WORDS OF EXTENDED CORE STORAGE REQUIRED
 000003 THOUSAND WORDS (OCTAL) OF EXTENDED CORE STORAGE REQUESTED
 25000 WORDS OF CORE ALLOWED
 000300 THOUSAND WORDS (OCTAL) OF EXTENDED CORE STORAGE ALLOWED

***** INPUT CROSS SECTIONS *****

MAT NO		CARD CROSS SECTION INPUT SECTION	
1	LOADED FROM CARDS	IX=F1 COM. MIX FOR F.E.1	P0 IHT= 5 IHS= 6 IMM= 7
2	LOADED FROM CARDS	IX=F1 COM. MIX FOR F.E.1	P1 IHT= 5 IHS= 6 IMM= 7
3	LOADED FROM CARDS	ABSORBER IHT=5	
4	LOADED FROM CARDS	SPECIAL FOR EDIT IHT=5	

INPUT CROSS SECT 1

②

	GROUP 1	GROUP 2
1	.257048E-03	0.
2	.174011E+00	.339009E+00
3	.298784E-01	.633738E-01
4	.761805E-01	.116041E+00
5	.281152E+00	.564013E+00
6	.186444E+00	.500640E+00
7	0.	.650869E-01

INPUT CROSS SECT 2

	GROUP 1	GROUP 2
1	0.	0.
2	0.	0.
3	0.	0.
4	0.	0.
5	0.	0.
6	.902235E-01	.166424E+00
7	0.	.193226E-01

INPUT CROSS SECT 3

	GROUP 1	GROUP 2
1	0.	0.
2	0.	0.
3	.180470E-01	.135048E-01
4	0.	0.
5	.180470E-01	.135048E-01
6	0.	0.
7	0.	-0.

INPUT CROSS SECT 4

	GROUP 1	GROUP 2
1	0.	0.
2	0.	0.
3	.100000E+00	.100000E+00

```

4 0. 0.
5 .100000E+01 .100000E+01
6 .500000E+00 .900000E+00
7 0. .400000E+00

```

***** INPUT FOR FLUX *****

ISOTROPIC COMPONENT

```

FLUX ESHAPE 2
1.0000E+00 1.0000E+00

```

③

S 6 CONSTANTS

	MU	ETA	WEIGHT							
1	.23009194E+00	.94557676E+00	.42361640E-01							
2	.68813432E+00	.68813432E+00	.40971693E-01							
3	.23009194E+00	.68813432E+00	.40971693E-01							
4	.94557676E+00	.23009194E+00	.42361640E-01							
5	.68813432E+00	.23009194E+00	.40971693E-01							
6	.23009194E+00	.23009194E+00	.42361640E-01							
COARSE RMESH	5									
0.	3.2700E+00	6.5400E+00	8.7200E+00	1.0900E+01						
COARSE ZMESH	5									
0.	3.2700E+00	6.5400E+00	8.7200E+00	1.0900E+01						
CROSS SEC ID	16									
-5	-5	-5	7	7	7	-5	-5	-5	-5	-5
-5	-5	-5	-5	7	7					
FISSION SPEC	2									
7.5382E-01	2.4618E-01									
VELOCITIES	2									
1.0000E+00	1.0000E+00									
MIX NUMBERS	10									
5	5	5	6	6	7	7	7	7	7	7
MIX COMMANDS	10									
0	1	3	0	2	0	4	0	1	3	
MIX DENSITY	10									
0.	9.9000E-01	9.9000E-01	0.	9.9000E-01	0.	1.0000E+00	0.	9.9000E-01	9.9000E-01	
R MESH MODS	4									
1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00							
Z MESH MODS	4									
0.	5.0000E-01	0.	5.0000E-01							

6 -2.3009194E-01 2.3009194E-01 -2.3009194E-01 2.3009194E-01

SPHERICAL HARMONIC COMPONENT 3

DIRECTION	SWEEP 1	SWEEP 2	SWEEP 3	SWEEP 4
1	-7.4517675E-01	-9.4557675E-01	9.4557676E-01	9.4557676E-01
2	-6.8813431E-01	-6.8813431E-01	6.8813432E-01	6.8813432E-01
3	-6.8813429E-01	-6.8813429E-01	6.8813432E-01	6.8813432E-01
4	-2.3009193E-01	-2.3009193E-01	2.3009194E-01	2.3009194E-01
5	-2.3009191E-01	-2.3009191E-01	2.3009194E-01	2.3009194E-01
6	-2.3009190E-01	-2.3009190E-01	2.3009194E-01	2.3009194E-01

(5)

MIXTURE NUMBER	MIXTURE	COMMAND	MATERIAL	ATOMIC DENSITY	
5	0				1
5	1		9.9000000E-01		2
5	3		9.9000000E-01		3
6	0				4
6	2		9.9000000E-01		5
7	0				6
7	4		1.0000000E+00		7
7	0				8
7	1		9.9000000E-01		9
7	3		9.9000000E-01		10

GROUP NUMBER 1

MIXED X-SECT

	MATERL 5	MATERL 6	MATERL 7
1	.254478E-03	0.	.254478E-03
2	.172271E+00	0.	.172271E+00
3	.474459E-01	0.	.474459E-01
4	.754187E-01	0.	.754187E-01
5	.296207E+00	0.	.296207E+00
6	.184580E+00	.893212E-01	.184580E+00
7	0.	0.	0.

GROUP NUMBER 2

MIXED X-SECT

	MATERL 5	MATERL 6	MATERL 7
1	0.	0.	0.
2	.335619E+00	0.	.335619E+00
3	.761090E-01	0.	.761090E-01
4	.114881E+00	0.	.114881E+00
5	.571743E+00	0.	.571743E+00
6	.495634E+00	.164760E+00	.495634E+00
7	.644360E-01	.161594E-01	.644360E-01

GROUP FISSION SPECTRUM VELOCITIES

1	7.5382000E-01	1.0000000E+00
2	2.4618300E-01	1.0000000E+00

3 1.0000030E+00 0.

OUTER EPS 1.00E-04 XLAX 1.00E-03
 INNER EPS 1.00E-04 XLAH 5.00E-01
 REBALANCE EPS 5.00E-05 XLAL 1.00E-02
 SEARCH EPS 1.00E-04 POD 1.90E+00
 NORM 1.00E+00 BHGT 0.

⑥

IITL 9

TIME IN MINUTES	OUTER ITERATIONS	INNER ITERATIONS TOTAL/RY GROUP	NEURON BALANCE	EIGENVALUE	EIGENVALUE SLOPE	LAMBDA	REBALANCE CONVERGENCE	
9.10E-03	0	0	0.	0.	0.	0.	0.	
6.56E-02		1	7					
9.38E-02		2	10	.89E-04				
1.74E-01	1	17	9.27242539E-07	0.	0.	1.20057317E+00	3.69509051E-01	
5.62E-02		1	6					
8.43E-02		2	9	9.39500794E-07	0.	0.	1.28933591E+00	3.92655047E-01
3.18E-01	2	15	1	5				
4.68E-02		2	6					
5.63E-02		1	5					
4.22E-01	3	11	4.26565170E-06	0.	0.	5.86566572E-01	5.04576635E-01	
4.69E-02		1	5					
4.70E-02		2	5					
5.16E-01	4	10	1.46898721E-06	0.	0.	8.75767314E-01	4.68537319E-03	
4.68E-02		1	5					
4.70E-02		2	5					
6.11E-01	5	10	8.74826142E-07	0.	0.	8.76635369E-01	2.17923131E-03	
4.68E-02		1	5					
4.69E-02		2	5					
7.05E-01	6	10	8.13225846E-07	0.	0.	8.77015739E-01	1.05770899E-03	
4.68E-02		1	5					
4.69E-02		2	5					
7.90E-01	7	10	7.99734735E-07	0.	0.	8.77191735E-01	5.18389013E-04	
4.68E-02		1	5					
4.69E-02		2	5					
8.94E-01	8	10	7.97445245E-07	1.00000000E-01	0.	8.77275498E-01	2.49395825E-04	
4.68E-02		1	5					
4.71E-02		2	5					
9.80E-01	9	10	9.13739697E-08	1.00000000E-01	0.	9.20110780E-01	5.09388634E-03	
4.67E-02		1	5					
4.70E-02		2	5					
1.08E+00	10	10	1.01573103E-06	1.00000000E-01	0.	9.21456609E-01	2.29387839E-03	
4.67E-02		1	5					
2.84E-02		2	3					
1.16E+00	11	8	1.00987995E-06	2.75006600E-01	2.24084511E+00	9.21901519E-01	1.11771196E-03	
4.68E-02		1	5					
6.57E-02		2	7					
1.27E+00	12	12	-5.82908115E-07	2.75006600E-01	2.24084511E+00	9.85761967E-01	7.96386319E-03	
2.82E-02		1	3					
4.70E-02		2	5					
1.35E+00	13	8	9.11548170E-10	2.75006600E-01	2.24084511E+00	9.87901226E-01	3.70878126E-03	
2.82E-02		1	3					
2.84E-02		2	3					
1.41E+00	14	6	-3.80672383E-07	3.09119328E-01	2.93872422E+00	9.88666036E-01	1.91208567E-03	

3.74E-02			1	4							
4.70E-02			2	5							
1.49E+00	15	9			4.77536842E-07	3.09119328E-01	2.93872422E+00	1.00016345E+00	1.78088765E-03		⑦
2.81E-02			1	3							
2.84E-02			2	3							
1.55E+00	16	6			-1.34660354E-06	3.07281846E-01	2.93872422E+00	1.00062527E+00	9.42504536E-04		
2.82E-02			1	3							
2.83E-02			2	3							
3.61E+00	17	6			-1.27613675E-06	3.06360483E-01	2.93872422E+00	1.00031352E+00	1.72382633E-04		
1.89E-02			1	2							
9.73E-03			2	1							
1.64E+00	18	3			1.03692250E-06	3.06360483E-01	2.93872422E+00	1.00006560E+00	4.36134237E-05		

COARSE MESH BALANCE TABLE FOR GROUP 1

ZONE	SOURCE	LEFT INFLOW	RIGHT INFLOW	BOTTOM INFLOW	TOP INFLOW	ABSORPTION	LEFT OUTFLOW	RIGHT OUTFLOW	BOTTOM OUTFLOW	TOP OUTFLOW
1	4.5018E-02	0.	2.8339E-02	2.2676E-02	1.9382E-02	3.2460E-02	0.	3.6665E-02	2.2676E-02	2.3612E-02
2	1.0944E-01	3.6665E-02	3.3764E-02	5.5219E-02	4.6322E-02	7.9113E-02	2.8339E-02	6.6999E-02	5.5219E-02	5.7741E-02
3	7.9582E-02	6.6999E-02	2.5426E-02	4.0508E-02	3.1571E-02	5.8005E-02	3.3764E-02	6.3344E-02	4.0508E-02	4.2465E-02
4	5.4347E-02	6.3344E-02	0.	2.8380E-02	2.2446E-02	4.1264E-02	2.5426E-02	4.4246E-02	2.8380E-02	2.9199E-02
5	4.3536E-02	0.	2.8173E-02	2.3612E-02	1.0310E-02	3.1497E-02	0.	3.4981E-02	1.9382E-02	1.9768E-02
6	1.0469E-01	3.4981E-02	3.1551E-02	5.7741E-02	2.5725E-02	7.5975E-02	2.8173E-02	5.6460E-02	4.6322E-02	4.7756E-02
7	7.4008E-02	5.6460E-02	2.0643E-02	4.2465E-02	1.9518E-02	5.4364E-02	3.1551E-02	6.0127E-02	3.1571E-02	3.5480E-02
8	5.1685E-02	6.0127E-02	0.	2.9199E-02	1.4014E-02	3.9560E-02	2.0643E-02	4.6809E-02	2.2446E-02	2.5567E-02
9	1.7233E-02	0.	1.1171E-02	1.9768E-02	5.6427E-03	1.2694E-02	0.	1.4259E-02	1.0310E-02	1.6550E-02
10	4.2438E-02	1.4259E-02	1.3794E-02	4.7756E-02	1.4458E-02	3.1304E-02	1.1171E-02	2.3971E-02	2.5725E-02	4.0534E-02
11	3.1439E-02	2.3971E-02	8.9950E-03	3.5480E-02	1.2453E-02	2.3471E-02	1.3794E-02	2.5753E-02	1.9518E-02	2.9973E-02
12	7.2206E-02	2.5753E-02	0.	2.5567E-02	8.5257E-03	1.7162E-02	8.9950E-03	2.0226E-02	1.4014E-02	2.1654E-02
13	1.1868E-02	0.	7.8207E-03	1.6550E-02	0.	9.0875E-03	0.	1.0013E-02	5.6427E-03	1.1496E-02
14	2.9552E-02	1.0013E-02	1.0186E-02	4.0534E-02	0.	2.2626E-02	7.8207E-03	1.7090E-02	1.4458E-02	2.8289E-02
15	2.1806E-02	1.7090E-02	6.9652E-03	2.9973E-02	0.	1.6810E-02	1.0186E-02	1.7199E-02	1.2453E-02	1.9185E-02
16	1.4794E-02	1.7199E-02	0.	2.1654E-02	0.	1.1916E-02	6.9652E-03	1.2895E-02	8.5257E-03	1.3345E-02

COARSE MESH BALANCE TABLE FOR GROUP 2

ZONE	SOURCE	LEFT INFLOW	RIGHT INFLOW	BOTTOM INFLOW	TOP INFLOW	ABSORPTION	LEFT OUTFLOW	RIGHT OUTFLOW	BOTTOM OUTFLOW	TOP OUTFLOW
1	3.3440E-02	0.	3.3785E-02	2.5668E-02	2.3700E-02	2.5037E-02	0.	3.9542E-02	2.5668E-02	2.5742E-02
2	8.1411E-02	3.9542E-02	4.3060E-02	6.2463E-02	5.5133E-02	6.0776E-02	3.3785E-02	6.2031E-02	6.2463E-02	6.2551E-02
3	5.9474E-02	6.2031E-02	3.4686E-02	4.5573E-02	3.7191E-02	4.3979E-02	4.3060E-02	6.1054E-02	4.5573E-02	4.5291E-02
4	4.1564E-02	6.1054E-02	0.	3.6316E-02	2.5043E-02	2.9295E-02	3.4686E-02	3.4217E-02	3.0316E-02	2.9464E-02
5	3.2400E-02	0.	3.3318E-02	2.5742E-02	1.3295E-02	2.4166E-02	0.	3.7563E-02	2.3100E-02	1.9921E-02
6	7.8048E-02	3.7563E-02	4.0173E-02	6.2551E-02	3.2890E-02	5.8004E-02	3.3318E-02	5.6783E-02	5.5133E-02	4.7985E-02

7	5.5552E-02	5.6703E-02	2.8389E-02	4.5201E-02	2.4452E-02	4.0709E-02	4.0173E-02	5.7154E-02	3.7191E-02	3.5243E-02
8	3.99715E-02	5.7154E-02	0.	2.9444E-02	1.971E-02	2.7718E-02	2.8389E-02	3.7633E-02	2.5043E-02	2.4090E-02
9	1.2956E-02	0.	1.2905E-02	1.9921E-02	7.7293E-03	9.4641E-03	0.	1.4931E-02	1.3295E-02	1.5819E-02
10	3.1930E-02	1.4931E-02	1.7285E-02	4.7985E-02	1.9613E-02	2.3286E-02	1.2905E-02	2.3768E-02	3.2690E-02	3.6835E-02
11	2.3872E-02	2.3768E-02	1.2188E-02	3.5243E-02	4.9578E-02	1.7274E-02	1.7235E-02	5.4194E-02	2.6452E-02	2.8504E-02
12	1.7159E-02	2.4194E-02	0.	2.4090E-02	1.9910E-02	1.1835E-02	1.2188E-02	1.6194E-02	1.9741E-02	1.9386E-02
13	9.1219E-03	0.	8.6278E-03	1.5819E-02	0.	6.5635E-03	0.	9.918E-03	7.7293E-03	9.4974E-03
14	2.2712E-02	9.9790E-03	1.2113E-02	3.1835E-02	0.	1.5845E-02	8.6278E-03	1.9741E-02	1.9613E-02	2.3413E-02
15	1.6935E-02	1.6134E-02	8.9072E-03	2.8504E-02	0.	1.1041E-02	1.2113E-02	1.5122E-02	1.6578E-02	1.4922E-02
16	1.1710E-02	1.5122E-02	0.	1.9396E-02	0.	7.6686E-03	8.9072E-03	0.11791E-03	1.0910E-02	9.5635E-03

(B)

COARSE MESH BALANCE TABLE FOR ALL GROUPS

ZONE	SOURCE	LEFT INFLOW	RIGHT INFLOW	BOTTOM INFLOW	TOP INFLOW	ABSORPTION	LEFT OUTFLOW	RIGHT OUTFLOW	BOTTOM OUTFLOW	TOP OUTFLOW
1	5.9718E-02	0.	6.2120E-02	4.8341E-02	4.2479E-02	3.8759E-02	0.	7.6202E-02	4.8341E-02	4.9351E-02
2	1.4518E-01	7.6202E-02	7.6820E-02	1.1768E-01	1.0145E-01	9.4220E-02	6.2120E-02	1.2302E-01	1.1768E-01	1.2039E-01
3	1.0557E-01	1.2302E-01	6.0109E-02	8.6075E-02	6.8736E-02	6.8500E-02	7.6820E-02	1.2339E-01	9.6075E-02	8.7751E-02
4	7.2093E-02	1.2439E-01	0.	5.6692E-02	4.7486E-02	4.6735E-02	6.0109E-02	9.8458E-02	5.8932E-02	5.8680E-02
5	5.7753E-02	0.	6.1487E-02	4.9351E-02	2.3603E-02	3.7481E-02	0.	7.2411E-02	4.2479E-02	3.9686E-02
6	1.3987E-01	7.2591E-02	7.1720E-02	1.2029E-01	5.9611E-02	9.1235E-02	6.1487E-02	1.1521E-01	1.0145E-01	9.5735E-02
7	9.8170E-02	1.1324E-01	4.9030E-02	8.7731E-02	4.3997E-02	4.3997E-02	7.1720E-02	1.1521E-01	6.8736E-02	7.6720E-02
8	6.9560E-02	1.1727E-01	0.	5.6680E-02	3.9752E-02	4.4441E-02	4.9030E-02	0.637E-02	4.7486E-02	4.9654E-02
9	2.2890E-02	0.	2.4075E-02	3.9686E-02	1.9371E-02	1.4830E-02	0.	2.9189E-02	2.3603E-02	3.2367E-02
10	5.6294E-02	2.9189E-02	3.1016E-02	7.5736E-02	3.9686E-02	3.6519E-02	2.4075E-02	4.7736E-02	5.8611E-02	7.9344E-02
11	4.1929E-02	4.7736E-02	2.1101E-02	7.0720E-02	2.9029E-02	2.7194E-02	3.1016E-02	4.9945E-02	4.3997E-02	5.8473E-02
12	2.9456E-02	4.9945E-02	0.	4.9654E-02	1.9434E-02	1.9082E-02	2.1181E-02	3.6418E-02	3.0754E-02	4.1047E-02
13	1.5744E-02	0.	1.6448E-02	3.2367E-02	0.	1.0205E-02	0.	1.9990E-02	1.3371E-02	2.0925E-02
14	3.9201E-02	1.9990E-02	2.2298E-02	7.9344E-02	0.	2.6410E-02	1.6448E-02	3.3226E-02	3.4059E-02	5.1700E-02
15	2.8926E-02	3.3226E-02	1.5871E-02	5.8473E-02	0.	1.8747E-02	3.2367E-02	2.9029E-02	2.9029E-02	3.4105E-02
16	1.9425E-02	3.2320E-02	0.	4.1047E-02	0.	1.2704E-02	1.5871E-02	4.2072E-02	1.9434E-02	2.2907E-02

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DUMP TAKEN UNIT 7 *FINAL*

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FINAL PRINTING

SOURCE		FISSION SOURCE		IN SCATTER		SELF SCATTER		OUT SCATTER		NET LEAKAGE	
1	0.		7.5381774E-01		6.3948846E-14		9.2155898E+01		3.2044224E-01		1.9649016E-01
2	0.		2.4618226E-01		3.2171278E-01		2.6900177E+00		7.1054274E-14		1.6481849E-01
3	0.		1.0000000E+00		3.2171278E-01		3.6115767E+00		3.2171278E-01		3.5150886E-01

ABSORPTION		NEUTRON BALANCE		RIGHT LEAKAGE		HORIZONTAL LEAKAGE		TOP LEAKAGE		VERTICAL LEAKAGE	
1	2.3080534E-01		6.3948846E-14		1.2417505E-01		1.2417505E-01		7.2315109E-02		7.2315109E-02
2	4.1307650E-01		-1.0658141E-13		9.7422552E-02		9.7422552E-02		5.7395961E-02		5.7395961E-02
3	6.4996190E-01		-1.4210555E-14		2.2159757E-01		2.2159757E-01		1.2971107E-01		1.2971107E-01

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TIME IN	OUTER	INNER	NEUTRON	EIGENVALUE	EIGENVALUE	LAMBDA	REBALANCE
MINUTES	ITERATIONS	ITERATIONS	BALANCE	SLOPE	SLOPE		CONVERGENCE
TOTAL/BY GROUP							
1.66E+00	19	2	-1.42108547E-14	3.06360483E-01	2.93872422E+00	1.00005605E+00	4.36134237E-05

DELTA CALCULATION FINAL RESULTS

I	MODIFIED R	ORIGINAL R
1	0.	0.
2	.42717989E+01	.32700000E+01
3	.85435974E+01	.65400000E+01
4	.11361463E+02	.87200000E+01
5	.14239329E+02	.10900000E+02
J	MODIFIED Z	ORIGINAL Z
1	0.	0.
2	.32700000E+01	.32700000E+01
3	.70408994E+01	.65400000E+01
4	.92208994E+01	.87200000E+01
5	.11734834E+02	.10900000E+02

FLUX COMPONENTS FOR GROUP 1

SECOND COMPONENT IS H DIRECTION CURRENT, THIRD IS Z DIRECTION CURRENT

COMPONENT NUMBER 1

R MESH	1	R MESH	2	R MESH	3	R MESH	4	R MESH	5	R MESH	6	R MESH	7	R MESH	8
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10	.480280E-03	.468256E-03	.448757E-03	.420705E-03	.387357E-03	.347745E-03	.295446E-03	.237501E-03
9	.701866E-03	.684896E-03	.656405E-03	.616314E-03	.566315E-03	.513118E-03	.450405E-03	.369932E-03
8	.877493E-03	.850126E-03	.820345E-03	.769426E-03	.705476E-03	.636571E-03	.557098E-03	.461549E-03
7	.102794E-02	.100257E-02	.953025E-03	.894705E-03	.817592E-03	.728700E-03	.629975E-03	.528805E-03
6	.123304E-02	.117211E-02	.111984E-02	.104395E-02	.946721E-03	.831218E-03	.713046E-03	.591943E-03
5	.133396E-02	.134916E-02	.128995E-02	.120697E-02	.108668E-02	.944104E-03	.799933E-03	.660002E-03
4	.151851E-02	.148335E-02	.145882E-02	.132900E-02	.119519E-02	.104134E-02	.882341E-03	.727060E-03
3	.159469E-02	.156715E-02	.148854E-02	.138879E-02	.126118E-02	.110823E-02	.945220E-03	.783148E-03
2	.163365E-02	.157193E-02	.150871E-02	.142004E-02	.129238E-02	.11443E-02	.985223E-03	.822277E-03
1	.164911E-02	.160681E-02	.153586E-02	.143667E-02	.131063E-02	.116340E-02	.100438E-02	.840358E-03

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COMPONENT NUMBER 2

10	.466063E-05	.137166E-04	.223204E-04	.301310E-04	.363154E-04	.410580E-04	.431175E-04	.437436E-04
9	.796315E-05	.200754E-04	.325103E-04	.434824E-04	.521952E-04	.575073E-04	.621015E-04	.671952E-04
8	.864349E-05	.257453E-04	.471180E-04	.559512E-04	.670194E-04	.752479E-04	.827169E-04	.939743E-04
7	.979717E-05	.291827E-04	.576405E-04	.645335E-04	.765543E-04	.898170E-04	.101032E-03	.112209E-03
6	.104394E-04	.316275E-04	.505002E-04	.593038E-04	.659599E-04	.740047E-03	.815501E-03	.912729E-03
5	.113995E-04	.336935E-04	.525562E-04	.648284E-04	.941633E-04	.100747E-03	.130144E-03	.143400E-03
4	.131442E-04	.392655E-04	.641819E-04	.875021E-04	.109065E-03	.129221E-03	.144775E-03	.154581E-03
3	.158497E-04	.461088E-04	.763911E-04	.103205E-03	.126580E-03	.144733E-03	.155206E-03	.159313E-03
2	.168255E-04	.501599E-04	.818015E-04	.110550E-03	.134306E-03	.150707E-03	.159027E-03	.161107E-03
1	.166441E-04	.500665E-04	.817559E-04	.109755E-03	.133052E-03	.149097E-03	.159880E-03	.163975E-03

COMPONENT NUMBER 3

10	.4492059E-04	.448441E-04	.707175E-04	.985522E-04	.117857E-03	.132390E-03	.145571E-03	.153227E-03
9	.701798E-04	.972284E-04	.115480E-03	.137031E-03	.15371E-03	.167672E-03	.182494E-03	.192494E-03
8	.985522E-04	.115480E-03	.137031E-03	.15371E-03	.167672E-03	.182494E-03	.192494E-03	.192494E-03
7	.117857E-03	.132390E-03	.145571E-03	.153227E-03	.167672E-03	.182494E-03	.192494E-03	.192494E-03
6	.132390E-03	.145571E-03	.153227E-03	.167672E-03	.182494E-03	.192494E-03	.192494E-03	.192494E-03
5	.145571E-03	.153227E-03	.167672E-03	.182494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03
4	.153227E-03	.167672E-03	.182494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03
3	.167672E-03	.182494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03
2	.182494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03
1	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03	.192494E-03

9	.204582E-03	.198781E-03	.190191E-03	.176655E-03	.161394E-03	.140582E-03	.112523E-03	.916069E-04
8	.194736E-03	.189113E-03	.180391E-03	.167005E-03	.150544E-03	.129226E-03	.106284E-03	.871643E-04
7	.181535E-03	.176630E-03	.167769E-03	.155247E-03	.138050E-03	.118074E-03	.100861E-03	.837699E-04
6	.160778E-03	.156720E-03	.148459E-03	.137995E-03	.121873E-03	.105237E-03	.942106E-04	.769699E-04
5	.130565E-03	.127019E-03	.120620E-03	.113073E-03	.100944E-03	.906640E-04	.857884E-04	.724326E-04
4	.956943E-04	.924479E-04	.885604E-04	.838715E-04	.775275E-04	.743613E-04	.730397E-04	.633929E-04
3	.645398E-04	.618172E-04	.598876E-04	.571295E-04	.549380E-04	.555922E-04	.551786E-04	.499704E-04
2	.387167E-04	.365966E-04	.359941E-04	.343315E-04	.337356E-04	.341413E-04	.338101E-04	.299866E-04
1	.131890E-04	.122235E-04	.122381E-04	.116419E-04	.114429E-04	.113723E-04	.111758E-04	.981427E-05

R MESH 9 R MESH 10

10	.724641E-04	.457976E-04
9	.708456E-04	.463494E-04
8	.675389E-04	.452653E-04
7	.636934E-04	.421950E-04
6	.586090E-04	.381413E-04
5	.522563E-04	.324674E-04
4	.446632E-04	.243050E-04
3	.335566E-04	.167980E-04
2	.199742E-04	.105974E-04
1	.664594E-05	.359096E-05

(11)

FLUX COMPONENTS FOR GROUP 2

SECOND COMPONENT IS R DIRECTION CURRENT, THIRD IS Z DIRECTION CURRENT

COMPONENT NUMBER 1

	R MESH 1	R MESH 2	R MESH 3	R MESH 4	R MESH 5	R MESH 6	R MESH 7	R MESH 8
10	.466732E-03	.456045E-03	.427607E-03	.410520E-03	.377986E-03	.338436E-03	.284203E-03	.224051E-03
9	.744890E-03	.728266E-03	.697974E-03	.655271E-03	.601639E-03	.545598E-03	.478677E-03	.388274E-03
8	.951182E-03	.929530E-03	.890387E-03	.834343E-03	.763990E-03	.688787E-03	.602086E-03	.493062E-03
7	.112952E-02	.110347E-02	.115557E-02	.986003E-03	.898454E-03	.798144E-03	.690391E-03	.566051E-03
6	.133096E-02	.130712E-02	.124914E-02	.116403E-02	.109370E-02	.919895E-03	.783771E-03	.641805E-03
5	.155832E-02	.152132E-02	.145299E-02	.135242E-02	.122050E-02	.105266E-02	.884239E-03	.719394E-03
4	.171804E-02	.167778E-02	.160216E-02	.149163E-02	.134815E-02	.116621E-02	.980203E-03	.797228E-03
3	.180718E-02	.176424E-02	.168488E-02	.157025E-02	.142265E-02	.124450E-02	.105428E-02	.864244E-03
2	.184412E-02	.180033E-02	.172020E-02	.160458E-02	.145724E-02	.128588E-02	.110106E-02	.910911E-03
1	.196222E-02	.181842E-02	.173802E-02	.162309E-02	.147714E-02	.130696E-02	.112299E-02	.931883E-03

R MESH 9 R MESH 10

10	.165358E-03	.927916E-04
9	.287714E-03	.164152E-03
8	.248200E-03	.216164E-03
7	.424835E-03	.252925E-03
6	.482021E-03	.287214E-03
5	.538335E-03	.320190E-03
4	.592665E-03	.343274E-03
3	.644240E-03	.358030E-03
2	.680603E-03	.371318E-03
1	.696294E-03	.379974E-03

COMPONENT NUMBER 2

	R MESH	1	R MESH	2	R MESH	3	K MESH	4	R MESH	5	P MESH	6	R MESH	7	R MESH	8
10	.275132E-05	.801985E-05	.128512E-04	.17691E-04	.204800E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04	.233131E-04
9	.45848E-05	.17340E-04	.270278E-04	.370278E-04	.437868E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04	.480597E-04
8	.586784E-05	.190713E-04	.339532E-04	.413395E-04	.506696E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04	.580733E-04
7	.638845E-05	.192213E-04	.33744E-04	.420233E-04	.537130E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04	.645219E-04
6	.649330E-05	.204306E-04	.374585E-04	.456639E-04	.588780E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04
5	.64572E-05	.204306E-04	.374585E-04	.456639E-04	.588780E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04	.733761E-04
4	.853079E-05	.250713E-04	.420233E-04	.561150E-04	.708125E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04	.86231E-04
3	.143338E-04	.318155E-04	.51337E-04	.77148E-04	.93776E-04	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03
2	.119359E-04	.35121E-04	.56395E-04	.77148E-04	.93776E-04	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03
1	.11844E-04	.35240E-04	.57121E-04	.77148E-04	.93776E-04	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03	.100753E-03

(12)

	R MESH	9	R MESH	10
10	.253867E-04	.276240E-04	.276240E-04	.276240E-04
9	.45813E-04	.506128E-04	.506128E-04	.506128E-04
8	.713720E-04	.745816E-04	.745816E-04	.745816E-04
7	.873165E-04	.901830E-04	.901830E-04	.901830E-04
6	.978030E-04	.101387E-03	.101387E-03	.101387E-03
5	.112269E-03	.115069E-03	.115069E-03	.115069E-03
4	.118565E-03	.120546E-03	.120546E-03	.120546E-03
3	.112042E-03	.113604E-03	.113604E-03	.113604E-03
2	.112206E-03	.114628E-03	.114628E-03	.114628E-03
1	.115866E-03	.11830E-03	.11830E-03	.11830E-03

COMPONENT NUMBER 3

	R MESH	1	R MESH	2	R MESH	3	R MESH	4	R MESH	5	R MESH	6	R MESH	7	R MESH	8
10	.16885E-03	.162743E-03	.150311E-03	.145783E-03	.134879E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03	.117933E-03
9	.154360E-03	.150343E-03	.144145E-03	.139866E-03	.128815E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03	.105518E-03
8	.142107E-03	.138340E-03	.132021E-03	.127036E-03	.109619E-03	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04	.82159E-04
7	.128936E-03	.125672E-03	.119222E-03	.114123E-03	.96861E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04
6	.110651E-03	.108100E-03	.10228E-03	.94138E-04	.82159E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04	.68074E-04
5	.85619E-04	.845719E-04	.80111E-04	.75355E-04	.68250E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04	.596568E-04
4	.69576E-04	.59285E-04	.50471E-04	.435712E-04	.497085E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04	.496438E-04
3	.396156E-04	.333618E-04	.27185E-04	.23747E-04	.350633E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04	.37476E-04
2	.23747E-04	.23747E-04	.22123E-04	.213460E-04	.218898E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04	.237116E-04
1	.787024E-03	.743506E-05	.751707E-05	.732816E-05	.757655E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05	.784191E-05

	R MESH	9	R MESH	10
10	.524574E-04	.293600E-04	.293600E-04	.293600E-04
9	.483286E-04	.286019E-04	.286019E-04	.286019E-04
8	.449086E-04	.277845E-04	.277845E-04	.277845E-04
7	.421483E-04	.255557E-04	.255557E-04	.255557E-04
6	.388779E-04	.230148E-04	.230148E-04	.230148E-04
5	.352310E-04	.195260E-04	.195260E-04	.195260E-04
4	.314611E-04	.137169E-04	.137169E-04	.137169E-04
3	.239199E-04	.90847E-05	.90847E-05	.90847E-05
2	.137053E-04	.609138E-05	.609138E-05	.609138E-05
1	.452239E-05	.609138E-05	.609138E-05	.609138E-05

FISSION EDIT

[illegible]

4 .256629E+00 .119897E+00
 5 .947115E+00 .442490E+00

MICROSCOPIC ACTIVITY EDIT FOR MATERIAL 2 GROUP 1

ACT.FOR XS 1--INT 1
 ZONE 1 ZONE 2
 1 0. 0.
 2 0. 0.
 3 0. 0.
 4 0. 0.
 5 0. 0.

(15)

MICROSCOPIC ACTIVITY EDIT FOR MATERIAL 3 GROUP 1

ACT.FOR XS 1--INT 1
 ZONE 1 ZONE 2
 1 0. 0.
 2 0. 0.
 3 .607948E-01 .284032E-01
 4 0. 0.
 5 .607948E-01 .284032E-01

MICROSCOPIC ACTIVITY EDIT FOR MATERIAL 4 GROUP 1

ACT.FOR XS 1--INT 1
 ZONE 1 ZONE 2
 1 0. 0.
 2 0. 0.
 3 0. .158975E+00
 4 0. 0.
 5 0. .158975E+01

EDIT NUMBER 1 GROUP NUMBER 2

GROUP

TEST CASE NUMBER 16

Fig. 21. Flux contour plot.

0-000 NUMBER 2

TEST CASE NUMBER 15

Fig. 22. Flux projective plot.

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