PHENOMENOLOGY OF NEUTRINO REACTIONS

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MASTER

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1. INTRODUCTION

Neutrino experiments are done with two objectives: to discover the fundamental nature of the weak interaction, and to obtain insight into the structure of hadronic states. Before such experiments came into their own, our knowledge of the weak interactions had been derived from the study of decay processes, and had led to the evolution of a framework known as the conventional theory. (1) According to this theory, all commonly observed weak decays were the first order effects of an interaction Lagrangian

$$\mathcal{L} = -\frac{1}{2} \frac{G}{\sqrt{2}} [f_a^+ f_a + f_a f_a^+] \quad (1.1)$$

constructed from a charged ($\Delta Q = 1$) current f_a and its hermitian conjugate f_a^+ . The current f_a was supposed to have two parts

$$f_a = l_a + J_a, \quad (1.2)$$

the lepton part l_a being given by

$$l_a = \bar{e} \gamma_c (1 + \gamma_5) \nu_e + \bar{\mu} \gamma_c (1 + \gamma_5) \nu_\mu, \quad (1.3)$$

and the hadron part J_a possessing the Cabibbo structure

$$J_a = \cos \theta_c [V_a^{1-i2} + A_a^{1-i2}] + \sin \theta_c [V_a^{4-i5} + A_a^{4-i5}] \quad (1.4)$$

The phenomenological success of this hypothesis is well-known and well-documented. (2) There was, of course, the awareness that the decay reactions which formed the basis of the above theory involved only the lowest states of the particle spectrum, and were limited in energy release and momentum transfer. This left room for speculation that there might exist interactions outside the above framework which remained concealed for purely incidental reasons. There was also some disquiet over the fact that the higher order effects of the interaction (1.1) were not calculable, the theory not being renormalizable. It was hoped that the advent of neutrino beams would enable a more incisive test

of the above hypothesis, and perhaps reveal departures which would be clues to a more complete theory. It was also envisaged that at high energies, neutrinos (along with electrons) would act as probes of nucleon targets, and would enhance our understanding of hadronic structure.

Recent developments in neutrino physics have, in fact, fulfilled these expectations. First of all, a class of phenomena ("neutral currents") has been uncovered for which no hint existed in the decay processes. The new interactions appear to be as fundamental a part of the weak interactions as those described by Eq. (1.1) and thus the subject has acquired a whole new dimension. Secondly, the scattering of neutrinos (and of electrons) from matter at high energies has been found to exhibit a remarkable regularity ("scaling") that suggests strongly an interpretation of hadronic structure in terms of elementary constituents. Thirdly, there has been an emergence of theoretical ideas ("gauge theories") that not only suggest a solution to the problem of renormalization of weak interactions, but predict a diversity of new phenomena, including neutral currents. Neutrino experiments have become a vital testing ground for such ideas. Finally, evidence is accumulating for the existence of new particles, and possibly new quantum numbers, at high energy. Undoubtedly, neutrino interactions will play a role in unravelling the nature of these particles and their place in the scheme of weak interactions.

In these lectures, we attempt to survey the most important neutrino reactions from a phenomenological point of view. We assume that the underlying interaction has the form

$$\mathcal{L} = \mathcal{L}^{\text{CC}} + \mathcal{L}^{\text{NC}}, \quad (1.5)$$

where $\mathcal{L}^{\text{CC}}$ and $\mathcal{L}^{\text{NC}}$ have the current-current structure

$$\begin{aligned} \mathcal{L}^{\text{CC}} &= -\frac{1}{2} \frac{G}{\sqrt{2}} [f_a^{(\text{C})} f_a^{(\text{C})\dagger} + f_a^{(\text{C})\dagger} f_a^{(\text{C})}] \\ \mathcal{L}^{\text{NC}} &= -\frac{1}{2} \frac{G}{\sqrt{2}} [f_a^{(\text{N})} f_a^{(\text{N})}] \end{aligned} \quad (1.6)$$

The charged current $f_a^{(C)}$ will be assumed to contain the conventional current f_a given by Eqs. (1.2) - (1.4), but possibly some new pieces as well. The neutral current $f_a^{(N)}$ will be assumed to be hermitian and to contain vector and axial vector parts. (More general possibilities for the neutral current will also be considered briefly at certain points.) The possibility that these interactions are mediated by massive bosons will be kept in mind; it will be assumed that where necessary such effects can be represented by the inclusion of a propagator in the amplitude.

Our interest is in neutrino-induced reactions, and so we focus on that part of the interaction which contains neutrino currents. Such currents occur in $f_a^{(C)}$ in the form

$$f_a^{(C)} = \bar{e} \gamma_a (1 + \gamma_5) \nu_e + \bar{\mu} \gamma_a (1 + \gamma_5) \nu_\mu + \dots \quad (1.7)$$

and we shall assume that they appear in $f_a^{(N)}$ as

$$f_a^{(N)} = \eta^{(e)} (\bar{\nu}_e \gamma_a (1 + \gamma_5) \nu_e) + \eta^{(\mu)} (\bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu) + \dots \quad (1.8)$$

Note that currents of the form $\bar{\nu} \gamma_a (1 - \gamma_5) \nu$ have been excluded from Eq. (1.8). This exclusion is obligatory if neutrinos are massless two-component objects, but even if such currents are allowed, they will not influence reactions induced by existing neutrino beams that are derived from charged currents of the form (1.7). The coefficients $\eta^{(e)}$ and $\eta^{(\mu)}$ in (1.8) allow for the possibility that μ - e universality might break down in neutral current reactions. With the above assumptions, the effective interaction for neutrino-induced processes becomes

$$\begin{aligned} \mathcal{L} = & -\frac{G}{\sqrt{2}} [\bar{e} \gamma_a (1 + \gamma_5) \nu_e + \bar{\mu} \gamma_a (1 + \gamma_5) \nu_\mu] f_a^{(C)\dagger} + \text{h. c.} \\ & -\frac{G}{\sqrt{2}} [\eta^{(e)} \bar{\nu}_e \gamma_a (1 + \gamma_5) \nu_e + \eta^{(\mu)} \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu] f_a^{(N)} \end{aligned} \quad (1.9)$$

Our basic concern in these lectures is to see how the properties of the currents $f_a^{(C)}$ and $f_a^{(N)}$ may be deduced from measurements of various

reactions, and how the results of high energy neutrino scattering may be interpreted in terms of models of nucleon structure. A recurring theme will be the comparison of neutral current and charged current reactions, and the comparison of weak processes with electromagnetic ones. Reference to special models of weak interactions will be made occasionally, but for purposes of illustration only. The topics that we shall discuss are: the scattering of neutrinos by electrons (Chapter 2), by nucleons in exclusive channels (Chapter 3) and by nucleons in inclusive and semi-inclusive reactions (Chapter 4). Chapter 5 will discuss the parton model interpretation of deep inelastic phenomena.

2. SCATTERING OF NEUTRINOS BY ELECTRONS

We begin with a discussion of the simplest of all weak interaction processes: the scattering of a neutrino by an electron. The four reactions

$$\nu + e^- \rightarrow \nu + e^- \quad , \quad \nu = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu \quad , \quad (2.1)$$

currently the objects of intense investigation, ⁽³⁾ will provide decisive information about the nature of the weak currents (charged as well as neutral) and will be a touchstone for any theory of weak interactions. Some of the results obtained in this chapter will also be useful in other contexts where neutrino scattering occurs from targets with point-like character, such as the partons of Chapter 5.

2.1 The Conventional Theory

We recall that in the conventional theory, the Lagrangian for neutrino-electron scattering, contained in Eq. (1.1), is

$$\mathcal{L} = -\frac{G}{\sqrt{2}} (\bar{\nu}_e \gamma_\alpha (1 + \gamma_5) e) (\bar{e} \gamma_\alpha (1 + \gamma_5) \nu_e) \quad , \quad (2.2)$$

which allows the reactions $\nu_e (\bar{\nu}_e) + e^- \rightarrow \nu_e (\bar{\nu}_e) + e^-$ but forbids the reactions $\nu_\mu (\bar{\nu}_\mu) + e^- \rightarrow \nu_\mu (\bar{\nu}_\mu) + e^-$. The differential cross sections following from this Lagrangian are

$$\frac{d\sigma}{dQ^2} (\nu_e e^- \rightarrow \nu_e e^-) = \frac{G^2}{\pi} \quad (2.3)$$

$$\frac{d\sigma}{dQ^2} (\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-) = \frac{G^2}{\pi} \left[1 - \frac{Q^2}{s-m^2} \right]^2.$$

Here $Q^2 (= -t)$ is the modulus of the four-momentum transfer squared, s the square of the center-of-mass energy, and m the electron mass. It is useful to rewrite (2.3) in terms of energies measured in the lab system, viz. the energy of the incident neutrino E_ν , and the kinetic energy of the recoil electron E_e . Defining

$$y = \frac{E_e}{E_\nu} = \frac{Q^2}{s - m^2} \quad (2.4)$$

and noting that $s = 2m E_\nu + m^2$, we obtain

$$\frac{d\sigma}{dy} (\nu_e e^- \rightarrow \nu_e e^-) = 2 \frac{G^2 m E_\nu}{\pi} \quad (2.5)$$

$$\frac{d\sigma}{dy} (\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-) = 2 \frac{G^2 m E_\nu}{\pi} (1-y)^2.$$

Integration of (2.3) or (2.5) gives the total cross sections

$$\sigma (\nu_e e^- \rightarrow \nu_e e^-) = 2 \frac{G^2 m E_\nu}{\pi} \left(1 - \frac{m^2}{s} \right) \quad (2.6)$$

$$\sigma (\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-) = \frac{2}{3} \frac{G^2 m E_\nu}{\pi} \left[1 - \left(\frac{m^2}{s} \right)^3 \right],$$

where we have used the fact that Q^2 ranges from 0 to $(s-m^2)^2/s$ (and correspondingly y from 0 to $(1-m^2/s)$).

The following features deserve emphasis.

- (i) If the electron mass is neglected, the ratio of neutrino and anti-neutrino cross sections is

$$\frac{\sigma (\nu_e e)}{\sigma (\bar{\nu}_e e)} = 3. \quad (2.7)$$

This is a direct consequence of the $\gamma_a(1 + \gamma_5)$ character of the currents appearing in (2. 2).

- (ii) In the limit of zero electron mass, the ratio $Q^2/(s-m^2)$ reduces to $\sin^2(\theta/2)$, θ being the center-of-mass scattering angle. Eq. (2. 3) then shows that while $\nu_e e^-$ scattering is isotropic in the center-of-mass frame, $\bar{\nu}_e e^-$ scattering has the distribution $(1 + \cos \theta)^2$. This is again a consequence of the Lagrangian (2. 2) which forces scattering to occur in a state of equal helicities (hence $J = 0$) in the case of $\nu_e e^-$, but opposite helicities (hence $J = 1$) in the case of $\bar{\nu}_e e^-$.
- (iii) Both $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ become proportional to E_ν or s in the limit $m = 0$, a result following from dimensional analysis, and an elementary example of scale invariance.
- (iv) If the cross sections are decomposed into vector, axial vector and interference parts, they take the form

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} = V + A \pm I \quad (2. 8)$$

where

$$V = A = \frac{1}{4} \frac{G^2}{\pi} [1 + (1 - y)^2], \quad (2. 9)$$

$$I = \frac{G^2}{\pi} y(1 - \frac{1}{2} y).$$

The difference of the ν_e and $\bar{\nu}_e$ cross sections isolates the interference term,

$$\frac{d\sigma}{dQ^2}(\nu_e e^-) - \frac{d\sigma}{dQ^2}(\bar{\nu}_e e^-) = 2 \frac{G^2}{\pi} y(1 - \frac{1}{2} y) \quad (2. 10)$$

which vanishes in the forward direction ($y = 0$).

- (v) The cross sections may also be written in the form

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} = \frac{G^2}{4\pi} \frac{1}{(s-m^2)^2} [Q^4 \pm 2Q^2(s-u) + (s-u)^2] \quad (2. 11)$$

($u \equiv 2m^2 + Q^2 - s$) where the factor in parentheses is essentially $|m_{\nu, \nu}^2|$, the square of the invariant matrix element. The fact that $|m_{\nu}^2|_{s, t, u} = |m_{\nu}^2|_{u, t, s}$, evident from (2.11), is a consequence of crossing symmetry.

For experimental purposes, it is important to note that the recoil angle of the electron (in the lab) is given by

$$\cos \theta_e = 1 - m \left(\frac{1}{E_e} - \frac{1}{E_\nu} \right) + O(m^2). \quad (2.12)$$

At accelerator energies, this reduces approximately to

$$\theta_e \approx \left[2m \left(\frac{1}{E_e} - \frac{1}{E_\nu} \right) \right]^{1/2}, \quad (2.13)$$

which is always a small angle. Thus the characteristic signature of neutrino electron scattering is a solitary electron recoiling close to the forward direction.

The fact that neutral currents have been observed indicates that the conventional theory requires modification. The presence of such currents will not only permit reactions such as $\nu (\bar{\nu}) + e^- \rightarrow \nu (\bar{\nu}) + e^-$ to occur, but will affect the cross sections for $\nu_e (\bar{\nu}_e) + e^- \rightarrow \nu_e (\bar{\nu}_e) + e^-$ as well. The results we have obtained above will, however, be useful for obtaining predictions in the more general framework, which we discuss next.

2.2 General Theory with V, A Currents

We assume now that the reactions (2.1) are governed by a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}^{CC} + \mathcal{L}^{NC}, \quad (2.14)$$

where $\mathcal{L}^{CC}$ is as given in Eq. (2.2), and $\mathcal{L}^{NC}$ is an additional interaction of the form

$$\begin{aligned} \mathcal{L}^{\text{NC}} = & -\frac{G}{\sqrt{2}} [(\bar{\nu}_\mu \gamma_\alpha (1+\gamma_5) \nu_\mu) (\bar{e} \gamma_\alpha (C_V + C_A \gamma_5) e) \\ & + \eta (\bar{\nu}_e \gamma_\alpha (1+\gamma_5) \nu_e) (\bar{e} \gamma_\alpha (C_V + C_A \gamma_5) e)] , \end{aligned} \quad (2.15)$$

where C_V , C_A and η are real constants, and η is unity if μ -e universality is valid. A Fierz transformation in Eq. (2.2) allows us to combine $\mathcal{L}^{\text{CC}}$ and $\mathcal{L}^{\text{NC}}$ giving

$$\begin{aligned} \mathcal{L} = & -\frac{G}{\sqrt{2}} [(\bar{\nu}_\mu \gamma_\alpha (1+\gamma_5) \nu_\mu) (\bar{e} \gamma_\alpha (C_V + C_A \gamma_5) e) \\ & + (\nu_e \gamma_\alpha (1+\gamma_5) \nu_e) (\bar{e} \gamma_\alpha (C'_V + C'_A \gamma_5) e)] \end{aligned} \quad (2.16)$$

where

$$C'_V = 1 + \eta C_V , \quad C'_A = 1 + \eta C_A . \quad (2.17)$$

The cross sections for neutrino electron scattering resulting from the interaction (2.16) are⁽⁴⁾

$$\frac{d\sigma(r)}{dy} = 2 \frac{G^2 mE}{\pi} \nu [A(r) + B(r) (1-y)^2 - C(r) \frac{m}{E} y] , \quad (2.18)$$

$$r = \nu_\mu e, \bar{\nu}_\mu e, \nu_e e, \bar{\nu}_e e$$

where the coefficients A , B and C are as listed below.

Table 1

Reaction (r)	A(r)	B(r)	C(r)
$\nu_\mu e \rightarrow \nu_\mu e$	$\frac{1}{4} (C_V + C_A)^2$	$\frac{1}{4} (C_V - C_A)^2$	$\frac{1}{4} (C_V^2 - C_A^2)$
$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$	$\frac{1}{4} (C_V - C_A)^2$	$\frac{1}{4} (C_V + C_A)^2$	$\frac{1}{4} (C_V^2 - C_A^2)$
$\nu_e e \rightarrow \nu_e e$	$\frac{1}{4} (C'_V + C'_A)^2$	$\frac{1}{4} (C'_V - C'_A)^2$	$\frac{1}{4} (C_V'^2 - C_A'^2)$
$\bar{\nu}_e e \rightarrow \bar{\nu}_e e$	$\frac{1}{4} (C'_V - C'_A)^2$	$\frac{1}{4} (C'_V + C'_A)^2$	$\frac{1}{4} (C_V'^2 - C_A'^2)$

The result (2.18) is easily comprehended in the limit when the electron mass m (and therefore the term proportional to $C(r)$) is neglected. Consider, for instance, the reaction $\nu_\mu e^- \rightarrow \nu_\mu e^-$. The electron current $\bar{e} \gamma_\alpha (C_V + C_A \gamma_5) e$ may be written

$$\left(\frac{C_V + C_A}{2}\right) \bar{e} \gamma_\alpha (1 + \gamma_5) e + \left(\frac{C_V - C_A}{2}\right) \bar{e} \gamma_\alpha (1 - \gamma_5) e, \quad (2.19)$$

where the first and second parts scatter left-handed and right-handed electrons respectively. Since the two helicity states of the electron are distinct when $m = 0$, no interference can occur, and the cross section is simply the weighted sum of the two cross sections given in Eq. (2.5).

The table above shows a close correlation between the neutrino and anti-neutrino processes. Thus,

$$\begin{aligned} A(\nu e) &= B(\bar{\nu} e) \\ B(\nu e) &= A(\bar{\nu} e) \\ C(\nu e) &= C(\bar{\nu} e) \end{aligned} \quad (2.20)$$

for each type of neutrino. Further, the coefficient C is, in all four cases, related to A and B by

$$C^2 = AB. \quad (2.21)$$

The total cross sections resulting from (2.18) are, in the $m = 0$ limit,

$$\sigma(r) = 2 \frac{G^2 m E_\nu}{\pi} \left[A(r) + \frac{1}{3} B(r) \right]. \quad (2.22)$$

Combining with (2.16) gives the bounds

$$\frac{1}{3} \leq \left[\frac{\sigma(\nu_e e)}{\sigma(\bar{\nu}_e e)}, \frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} \right] \leq 3. \quad (2.23)$$

Useful constraints can also be derived on the average energy of electron recoil. Assuming a monochromatic beam of energy E_ν , we have

$$\frac{\langle E_e \rangle}{E_\nu} = \langle y \rangle = \frac{\int dy \frac{d\sigma}{dy} y}{\int \frac{d\sigma}{dy} dy} = \frac{1}{2} \frac{A + \frac{1}{6} B}{A + \frac{1}{3} B} \quad (2.24)$$

yielding the limits

$$\frac{1}{4} \leq \frac{\langle E_e \rangle}{E_\nu} \leq \frac{1}{2} . \quad (2.25)$$

If neutrino and antineutrino scattering are compared at the same energy, we find, from (2.22) and (2.24)

$$\frac{\langle E_e \rangle_{\nu e}}{\langle E_e \rangle_{\bar{\nu} e}} = \frac{1 - \frac{3}{17} \frac{\sigma(\bar{\nu} e)}{\sigma(\nu e)}}{1 - \frac{3}{17} \frac{\sigma(\nu e)}{\sigma(\bar{\nu} e)}} , \quad (2.26)$$

for either type of neutrino. (It should be remarked, however, that results such as (2.25) and (2.26) are not immediately applicable to actual experiments which employ wide band beams and select only electrons within certain energy cuts.)

Relations among the μ -type and e -type neutrino processes arise if one makes the further assumption of μ - e universality, placing $\eta = 1$ in Eq. (2.17)⁽⁵⁾. The interaction (2.16) then involves only two unknown parameters, C_V and C_A , and a number of sum rules and inequalities are generated. The coefficients A and B for the two classes of reaction now satisfy the additional constraints

$$\begin{aligned} \left| \sqrt{A(\nu_e e)} \pm \sqrt{A(\nu_\mu e)} \right| &= 1 \\ \left| \sqrt{B(\bar{\nu}_e e)} \pm \sqrt{B(\bar{\nu}_\mu e)} \right| &= 1 \end{aligned} \quad (2.27)$$

$$A(\bar{\nu}_e e) = A(\bar{\nu}_\mu e)$$

$$B(\nu_e e) = B(\nu_\mu e) .$$

These, in turn, provide the following four relations among the total cross sections, (where we have expressed these in units of $G^2 m E_\nu / \pi$):

$$\begin{aligned}
\sigma(\bar{\nu}_e e) + \sigma(\bar{\nu}_\mu e) &\geq \sqrt{2} \\
\sigma(\bar{\nu}_e e) + \sigma(\bar{\nu}_\mu e) &\geq \sqrt{\frac{2}{3}} \\
\left| [\sigma(\nu_e e) - \frac{1}{3} \sigma(\bar{\nu}_e e)]^{1/2} \pm [\sigma(\nu_\mu e) - \frac{1}{3} \sigma(\bar{\nu}_\mu e)]^{1/2} \right| &= \frac{4}{3} \\
\sigma(\nu_e e) - \sigma(\nu_\mu e) &= 3 [\sigma(\bar{\nu}_e e) - \sigma(\bar{\nu}_\mu e)]
\end{aligned} \tag{2.28}$$

2.3 Models and Data

Theoretical models have been proposed for the neutral lepton current, which specify or limit the unknown parameters C_V and C_A occurring in Eq. (2.16). (Most such models maintain μ -e universality, so that $C_{V,A}^1 = 1 + C_{V,A}$.) A sample of such models and their predictions for $\nu_e e$ and $\bar{\nu}_\mu e$ scattering are shown in Table 2. Among these, the model of Sakurai⁽⁶⁾ identifies the neutral current with the current of the fermion number (which is pure vector). Similarly, Cheng and Lo⁽⁷⁾ argue in favour of a pure axial vector current. Both of these models predict $\sigma(\nu_e e) = \sigma(\bar{\nu}_\mu e)$. By contrast models with a V-A structure (such as that discussed by Adler and Tuan⁽⁸⁾) lead to $\sigma(\nu_e e) = 3\sigma(\bar{\nu}_\mu e)$, and those with a V+A structure give $\sigma(\nu_e e) = \frac{1}{3}\sigma(\bar{\nu}_\mu e)$. In the model of Weinberg⁽⁹⁾ and Salam⁽¹⁰⁾, the structure of the neutral current is determined by a symmetry $SU(2) \times U(1)$ imposed on the complete Lagrangian of the weak and electromagnetic interactions. The resulting interaction for neutrino electron scattering is defined by the coupling constants $C_A = -\frac{1}{2}$, $C_V = -\frac{1}{2} + 2\sin^2\theta$, where $\sin^2\theta$ is an unknown parameter between 0 and 1. The model predicts a correlation between $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_\mu e)$ given by

$$\bar{\sigma} = \left(\sigma + \frac{1}{6} \right) \pm \sqrt{\frac{2\sigma}{3} + \frac{1}{12}}, \tag{2.29}$$

where cross sections are in units $G^2 m E_\nu^2 / \pi$. This relation is plotted in Fig. 1.

Table 2

Hypothesis for Neutral Current	C_V, C_A	$\sigma(\nu_\mu e), \sigma(\bar{\nu}_\mu e)$	Reference
Pure V	$C_A = 0$ C_V arbitrary	$\sigma(\nu_\mu e) = \sigma(\bar{\nu}_\mu e)$	Sakurai ⁽⁶⁾
Pure A	$C_V = 0$ C_A arbitrary	$\sigma(\nu_\mu e) = \sigma(\bar{\nu}_\mu e)$	Cheng & Lo ⁽⁷⁾
V - A	$C_V = C_A$	$\sigma(\nu_\mu e) = 3\sigma(\bar{\nu}_\mu e)$	Adler & Tuan ⁽⁸⁾
V + A	$C_V = -C_A$	$\sigma(\nu_\mu e) = \frac{1}{3}\sigma(\bar{\nu}_\mu e)$	
Salam-Weinberg Model	$C_A = -\frac{1}{2}$ $C_V = -\frac{1}{2} + 2\sin^2\theta$	Eq. (2.29) and Fig. 1	Weinberg ⁽⁹⁾ Salam ⁽¹⁰⁾

Experiments are currently providing the first glimpse of neutrino electron scattering. Events of the type $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ have been recorded in the Gargamelle bubble chamber.⁽¹¹⁾ A hint of a signal has been obtained in the reactor experiment searching for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$.⁽¹²⁾ A spark chamber experiment is currently being analyzed for $\nu_\mu e$ as well as $\bar{\nu}_\mu e$ scattering.⁽¹³⁾ The existing data, expressed as 90% confidence limits, are⁽³⁾

$$\begin{aligned}
 \sigma(\nu_\mu e) &\leq 0.30 \frac{G^2 mE_\nu}{\pi} \\
 \sigma(\bar{\nu}_\mu e) &\leq 0.39 \frac{G^2 mE_\nu}{\pi} \\
 \sigma(\bar{\nu}_e e) &\leq 1.77 \frac{G^2 mE_\nu}{\pi} .
 \end{aligned}
 \tag{2.30}$$

The constraints that these numbers impose on the coupling constants C_V and C_A are shown in Fig. 2. (It should be recognized, of course, that the limits

given in (2.30) are tentative, since the data are limited, and the evaluation of background, detection efficiencies, etc. beset with uncertainties.)

2.4 Neutrino Identity

It has been instinctively assumed that in neutral current reactions, the "neutrino" in the final state is identical to the incident neutrino. This assumption is made explicitly in writing the neutral neutrino currents in the form (1.9). It is of interest to consider the possibility that the final lepton in such processes is a distinct particle, and that the currents involved have the form $\bar{\nu}'_e \gamma_a (1 + \gamma_5) \nu_e$ and $\bar{\nu}'_\mu \gamma_a (1 + \gamma_5) \nu_\mu$, where $\nu'_e (\nu'_\mu)$ may not be the same as $\nu_e (\nu_\mu)$.

Interestingly, neutrino electron scattering offers a hope to determine the identity of ν'_e relative to ν_e .⁽¹⁴⁾ As seen in Sec. 2.3, the scattering of ν_e (or $\bar{\nu}_e$) by an electron is sensitive to both $\mathcal{L}^{CC}$ and $\mathcal{L}^{NC}$, and is a unique process in this respect. Consider, now, $\bar{\nu}_e$ scattering from an electron. The interaction $\mathcal{L}^{CC}$ will induce the reaction $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ while the interaction $\mathcal{L}^{NC}$ will induce $\bar{\nu}_e e^- \rightarrow \bar{\nu}'_e e^-$. If $\bar{\nu}_e$ and $\bar{\nu}'_e$ are the same, the final states are identical, and the two amplitudes add coherently. If $\bar{\nu}_e$ and $\bar{\nu}'_e$ are distinct, the final states differ, and the amplitudes add incoherently. This difference in the two cases reflects itself in the energy spectrum of the recoil electron. If this spectrum is written in the general form (2.18), one can show that

$$C^2 = AB \quad , \quad \text{if } \bar{\nu}_e, \bar{\nu}'_e \text{ are identical} \quad (2.31)$$

$$C^2 = A(B-1) \quad , \quad \text{if } \bar{\nu}_e, \bar{\nu}'_e \text{ are distinct} \quad (2.32)$$

This constitutes an identity test for $\bar{\nu}'_e$. If the reaction studied is $\nu_e e$ rather than $\bar{\nu}_e e$, (2.32) is replaced by $C^2 = (A-1)B$.

The test requires a measurement of C , which is possible only in a low energy experiment, such as with $\bar{\nu}_e$ from a reactor. However, interesting information may be obtained even at high energies. Thus, a measurement

$B < 1$ in the reaction $\bar{\nu}_e e$, or a measurement $A < 1$ in the reaction $\nu_e e$, would suffice to rule out the "distinct neutrino" hypothesis.

2.5 General Four-Fermi Interaction

Speculations have been made that the neutral current interaction may involve couplings of the type S, P and T in addition to, or instead of, the V and A coupling considered so far.⁽¹⁵⁾ A general local four-fermi Lagrangian for $\nu_e e$ interaction may be written as

$$\begin{aligned} \mathcal{L} = \frac{G}{\sqrt{2}} [& \{ \bar{\nu}_\mu \gamma_\alpha (1 + \gamma_5) \nu_\mu \} \{ \bar{e} \gamma_\alpha (g_V + g_A \gamma_5) e \} \\ & + \{ \bar{\nu}_\mu (1 + \gamma_5) \nu_\mu \} \{ \bar{e} (g_S + g_P \gamma_5) e \} \\ & + \{ \bar{\nu}_\mu \sigma_{\alpha\beta} (1 + \gamma_5) \nu_\mu \} \{ \bar{e} g_T \sigma_{\alpha\beta} e \}] \end{aligned} \quad (2.33)$$

The cross section for $\nu_\mu e \rightarrow \nu_\mu e$ resulting from this is

$$\begin{aligned} \frac{d\sigma}{dy} = \frac{G^2 mE}{4\pi} \nu [& 2(g_V + g_A)^2 + 2(g_V - g_A)^2 (1 - y^2) \\ & + (|g_S|^2 + |g_P|^2) y^2 + 32 |g_T|^2 (1 - \frac{1}{2} y)^2 \\ & - 8 \text{Re} g_T (g_S + g_P)^* y (1 - \frac{1}{2} y)] \quad , \end{aligned} \quad (2.34)$$

the corresponding result for $\bar{\nu}_\mu e$ scattering being obtained by $g_A \rightarrow -g_A$, $g_T \rightarrow -g_T$. A feature of (2.34) is the absence of interference between V, A terms on the one hand, and S, P, T terms on the other. This is a consequence of the fact that while the V and A currents preserve the helicity of the neutrino, S, P and T currents cause it to be flipped.

The following are some interesting results following from the general expression (2.34).

- (i) The ratio of the total cross sections $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$ is bounded by^(16, 17)

$$\frac{\sqrt{7}-2}{\sqrt{7}+2} \leq \frac{\sigma(\nu_{\mu} e)}{\sigma(\bar{\nu}_{\mu} e)} \leq \frac{\sqrt{7}+2}{\sqrt{7}-2} \quad (2.35)$$

(ii) The difference of the y distributions for $\nu_{\mu} e$ and $\bar{\nu}_{\mu} e$ satisfies

$$\frac{d\sigma}{dy}(\nu_{\mu} e) - \frac{d\sigma}{dy}(\bar{\nu}_{\mu} e) = \text{const. } E_{\nu} y \left(1 - \frac{1}{2} y\right) \quad (2.36)$$

(iii) For an interaction of the S and P type, $\sigma(\nu_{\mu} e) = \sigma(\bar{\nu}_{\mu} e)$, and $\frac{d\sigma}{dy}$ is proportional to y^2 . A tensor interaction gives also $\sigma(\nu_{\mu} e) = \sigma(\bar{\nu}_{\mu} e)$, but a y distribution of the form $(1 - \frac{1}{2} y)^2$.

A decision on the form of the neutral current interaction will probably be easier in the realm of semileptonic reactions. While the issue is still an open one, we shall base our subsequent discussion on the assumption of V and A currents only.

3. SCATTERING OF NEUTRINOS BY NUCLEONS. EXCLUSIVE REACTIONS.

We discuss in this chapter reactions that occur prominently when neutrinos of energy ≤ 1 GeV interact with nucleons.⁽¹⁸⁾ These are the processes of elastic scattering

$$\nu + n \rightarrow \mu^{-} + p \quad (3.1a)$$

$$\nu + N \rightarrow \nu + N \quad (3.1b)$$

and weak pion production

$$\nu + N \rightarrow \mu^{-} + N' + \pi \quad (3.2a)$$

$$\nu + N \rightarrow \nu + N' + \pi \quad (3.2b)$$

(where N, N' denote nucleons). The charged current reactions (3.1a) and (3.2a) have traditionally been used as probes of the charged current structure and of πN dynamics at low energy. In the present context, they have the added importance of serving as reference standards in the analysis of the neutral current reactions (3.1b) and (3.2b). The comparative study of the neutral and charged current processes will be a focal point of our discussion.

3.1 The Reaction $\nu + n \rightarrow \mu^- + p$

The effective interaction for this process may be written as

$$\mathcal{L}^{CC} = -\frac{G}{\sqrt{2}} \bar{\mu} \gamma_\alpha (1 + \gamma_5) \nu_\mu J_\alpha^\dagger + \text{h. c.} \quad (3.3)$$

If the momenta in $\nu + n \rightarrow \mu^- + p$ are labelled as $k + p \rightarrow k' + p'$, the invariant amplitude is

$$\mathcal{M} = -\frac{G}{\sqrt{2}} \bar{u}(k') \gamma_\alpha (1 + \gamma_5) u(k) \bar{u}(p') \Gamma_\alpha u(p) \quad (3.4)$$

where Γ_α has the general form

$$\begin{aligned} \Gamma_\alpha = & \left[\gamma_\alpha F_V + i\sigma_{\alpha\beta} q_\beta \frac{F_M}{2M} + q_\alpha F_S \right] \\ & + \left[\gamma_\alpha F_A + i\sigma_{\alpha\beta} q_\beta \frac{F_T}{2M} + q_\alpha F_P \right] \gamma_5 \end{aligned} \quad (3.5)$$

Here $q = k - k'$ is the momentum transfer and the F 's are form factors depending on Q^2 ($= -q^2$). (M stands for the nucleon mass.) Ideally, one would wish experiments to provide information on all of the six form factors. Practical limitations, however, make it expedient to curtail the form factors by requiring that the current J_α possess at least some of the properties abstracted from the study of decay processes. ⁽¹⁹⁾ Thus the assumption that the vector current is conserved leads to $F_S = 0$, while the assumption that J_α is a current of the first class gives, in addition, $F_T = 0$. An estimate of F_P may be obtained from the PCAC hypothesis, which suggests $F_P(Q^2) \approx -\sqrt{2} g_{NN\pi} f_\pi / (Q^2 + m_\pi^2)$, for small Q^2 . Note that the contributions of F_S and F_P to the cross section are, in any case, zero in the limit that the muon mass is neglected. Likewise, the contribution of F_T tends to be suppressed kinematically. ⁽¹⁹⁾ Thus the neglect of F_S , F_P and F_T may be a good approximation even in the case that the preceding hypotheses are not valid.

When only the form factors F_V , F_A and F_M are retained, (and the muon mass neglected) the differential cross section for $\nu + n \rightarrow \mu^- + p$ (and $\bar{\nu} + p \rightarrow \mu^+ + n$) is given by ⁽¹⁸⁻²⁰⁾

$$\begin{aligned}
\frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} = & \frac{G^2}{2\pi} \frac{1}{(s-M^2)^2} [(F_V^2 + F_A^2) \{(s-M^2)^2 - \frac{1}{2} Q^2 (s-u)\} \\
& - M^2 Q^2 (F_V^2 - F_A^2) - \frac{Q^2}{4M^2} (su - 2M^2 Q^2 - M^4) F_M^2 \\
& + Q^4 F_V F_M \pm Q^2 (s-u) (F_V + F_M) F_A] \quad (3.6)
\end{aligned}$$

Introducing the variable $y = Q^2 / (s - M^2) = E/E_\nu$, where E is the kinetic energy of the recoil nucleon, and E_ν the energy of the incident neutrino, we may rewrite (3.6) as

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dy} = 2 \frac{G^2 M E_\nu}{\pi} \left[\left(\frac{F_V \pm F_A}{2} \right)^2 + \left(\frac{F_V \mp F_A}{2} \right)^2 (1-y)^2 \right. \quad (3.7)$$

$$\left. - \frac{1}{4} (F_V^2 - F_A^2) \frac{M}{E_\nu} y + \frac{1}{2} F_M y \{ F_M (1-y) \frac{E_\nu}{2M} + y (F_M + F_V \mp F_A) \pm 2F_A \} \right]$$

Notice that in the absence of F_M , this resembles the point-like cross section derived in Eq. (2.18) (except that F_V and F_A are now functions of Q^2). In the same way, one can derive from Eq. (3.6) the Rosenbluth formula for electron proton scattering, by putting $F_A = 0$ and replacing $G^2/2\pi$ by $4\pi\alpha^2/Q^4$ (α = fine structure constant).

The following are some notable consequences of Eq. (3.6).

- (i) In the forward direction, the neutrino and antineutrino cross sections approach a common limit

$$\left. \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} \right|_{Q^2=0} = \frac{G^2}{2\pi} (F_V(0)^2 + F_A(0)^2), \quad (3.8)$$

which is independent of the energy E_ν . Since $F_V(0)$ and $F_A(0)$ are known from β -decay, this provides a normalization point for measurements of $\nu n \rightarrow \mu^- p$ and $\bar{\nu} p \rightarrow \mu^+ n$.

- (ii) The difference of the ν and $\bar{\nu}$ cross sections is

$$\begin{aligned} \frac{d\sigma^\nu}{dQ^2} - \frac{d\bar{\sigma}^\nu}{dQ^2} &= \frac{G^2}{\pi} \frac{1}{(s-M^2)^2} Q^2 (s-u) (F_V + F_M) F_A \\ &= \frac{G^2}{\pi} \frac{Q^2}{ME_\nu} \left(1 - \frac{Q^2}{4ME_\nu}\right) (F_V + F_M) F_A, \end{aligned} \quad (3.9)$$

which is proportional to F_A , and for fixed Q^2 , decreases as $\frac{1}{E_\nu}$ at large energies.

(iii) The limiting form of $d\sigma^\nu, \bar{\sigma}^\nu/dQ^2$ at large energies is

$$\left. \frac{d\sigma^\nu, \bar{\sigma}^\nu}{dQ^2} \right|_{E_\nu \rightarrow \infty} = \frac{G^2}{2\pi} [F_V^2 + F_A^2 + \frac{Q^2}{4M^2} F_M^2] \quad (3.10)$$

If the form factors decrease at large Q^2 faster than $1/Q^2$, Eq.

(3.10) leads to total cross sections $\sigma^\nu, \bar{\sigma}^\nu$ which saturate at a common constant value at high energies.

Current experimental effort is directed mainly at the determination of the axial vector form factor $F_A(Q^2)$, assuming that the current J_a is as given by the conventional theory:

$$J_a = \cos \theta_c (V_a^{1-i2} + A_a^{1-i2}) \quad (3.11)$$

This hypothesis permits us to relate the form factors F_V and F_M to the electromagnetic form factors of the proton and neutron:

$$\begin{aligned} F_V(Q^2) &= F_1^p(Q^2) - F_2^n(Q^2) \\ F_M(Q^2) &= [K_p F_2^p(Q^2) - K_n F_2^n(Q^2)] \end{aligned} \quad (3.12)$$

F_1 and F_2 being the Dirac and Pauli form factors, and K_p (K_n) the anomalous magnetic moment of proton (neutron). Assuming F_A to be parameterized as $F_A(Q^2) = (1 + Q^2/M_A^2)^{-2} F_A(0)$, experiments have determined $M_A = 0.89 \pm 0.08$ GeV.

Some insight into the physical meaning of the axial vector form factor may be gained by adopting a constituent model for the nucleon. In the same

sense that the electric form factor $F_E(Q^2)$ is a measure of the spatial distribution of electric charge in a proton, $F_A(Q^2)$ may be interpreted as a measure of the "axial charge" distribution. The values at $Q^2 = 0$ are the total "charges," which in the quark parton model have the expression^(21, 22)

$$\begin{aligned} F_E(Q^2 = 0) &= \frac{2}{3} (N_u - \bar{N}_u) - \frac{1}{3} (N_d - \bar{N}_d) - \frac{1}{3} (N_s - \bar{N}_s) \\ F_A(Q^2 = 0) &= (N_u^\dagger - N_u^\dagger) + (\bar{N}_u^\dagger - \bar{N}_u^\dagger) - (N_d^\dagger - N_d^\dagger) - (\bar{N}_d^\dagger - \bar{N}_d^\dagger) \end{aligned} \quad (3.13)$$

Here $N_i, (\bar{N}_i)$ are the probable numbers of quarks (antiquarks) of the type i in a proton, and $N_i^\dagger, (\bar{N}_i^\dagger)$ denote the numbers which have positive (negative) helicity, assuming the proton to be in a positive helicity state. (Note that $N_i = N_i^\dagger + N_i^\dagger$.) If we now imagine the various quark types to be distributed in space with densities $\rho_u^\dagger(\vec{r}), \rho_u^\dagger(\vec{r})$, etc., the form factors $F_E(Q^2)$ and $F_A(Q^2)$ may be regarded, crudely, as Fourier transforms of linear combinations of such densities, as specified by Eq. (3.13). With this interpretation, it becomes clear that the axial vector form factor $F_A(Q^2)$ could, in principle, have a very different behaviour from a vector form factor such as $F_E(Q^2)$, especially since the former depends on distributions of the form $\rho^\dagger(\vec{r}) - \rho^\dagger(\vec{r})$, while the latter depends on $\rho^\dagger(\vec{r}) + \rho^\dagger(\vec{r})$. Thus, a careful determination of $F_A(Q^2)$, that does not presume any similarity with vector form factors, would be of great interest.

3.2 The Reaction $\nu + N \rightarrow \nu + N$

The analysis of $\nu + N \rightarrow \nu + N$ may be made in close analogy with that of $\nu + n \rightarrow \mu^- + p$. Assuming the neutral current Lagrangian to be

$$\mathcal{L}^{NC} = - \frac{G}{\sqrt{2}} \bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu J_\alpha^{(N)}, \quad (3.14)$$

the amplitude can be written as

$$\mathcal{M} = - \frac{G}{\sqrt{2}} \bar{u}(k') \gamma_\alpha (1 + \gamma_5) u(k) \bar{u}(p') \Gamma_\alpha^{(N)} u(p) \quad (3.15)$$

with $\Gamma_\alpha^{(N)}$ having the general form

$$\begin{aligned} \Gamma_a^{(N)} = & \left[\gamma_a F_V^{(N)} + i\sigma_{\alpha\beta} q_\beta \frac{F_M^{(N)}}{2M} + q_a F_S^{(N)} \right] \\ & + \left[\gamma_a F_A^{(N)} + i\sigma_{\alpha\beta} q_\beta \frac{F_T^{(N)}}{2M} + q_a F_P^{(N)} \right] \gamma_5 \end{aligned} \quad (3.16)$$

In the present case, the form factors $F_S^{(N)}$ and $F_P^{(N)}$ play no role, since the corresponding terms give zero when contracted with the lepton current. Further, T-invariance and the hermiticity of $J_a^{(N)}$ ensure that $F_T^{(N)} = 0$. (Observe that the only way that $J_a^{(N)}$ can be nonhermitian is for the lepton current to be non-diagonal, e. g. to have the form $\bar{\nu}' \gamma_a (1 + \gamma_5) \nu$ instead of $\bar{\nu} \gamma_a (1 + \gamma_5) \nu$. Such a situation would occur if the final "neutrino" were distinct from the initial one. Thus, as suggested by Wolfenstein,⁽²³⁾ the existence of the form factor $F_T^{(N)}$ can be used as a test for the identity of the final lepton.)

With only the three form factors $F_V^{(N)}$, $F_A^{(N)}$, $F_M^{(N)}$ operative, the differential cross section has exactly the form given in Eqs. (3.6) and (3.7), with the F 's replaced by $F^{(N)}$'s. The results given in Eqs. (3.8), (3.9) and (3.10) hold equally for the neutral current process.

The following tests can help to determine some of the qualitative features of the neutral current.⁽²⁴⁾

- (i) A difference between $\nu + p \rightarrow \nu + p$ and $\bar{\nu} + p \rightarrow \bar{\nu} + p$ would indicate the presence of both V and A currents in $J_a^{(N)}$, since such a difference can come about only via V/A interference.
- (ii) A difference between $\nu + p \rightarrow \nu + p$ and $\nu + n \rightarrow \nu + n$ (hard to measure!) would indicate that the current $J_a^{(N)}$ contains both isoscalar and isovector components.
- (iii) A nonvanishing cross section for $\nu + d \rightarrow \nu + d$ (d = deuteron) would prove the existence of an isoscalar part in $J_a^{(N)}$. A difference between $\nu + d \rightarrow \nu + d$ and $\bar{\nu} + d \rightarrow \bar{\nu} + d$ would reveal that the isoscalar part has both V and A components.

- (iv) Observation of coherent elastic scattering $\nu + T \rightarrow \nu + T$ from a nuclear target T with spin zero (such as He^4 or C^{12}) would prove that $J_a^{(N)}$ has a vector part.

An important indicator of the neutral current will be the ratio of $\nu + p \rightarrow \nu + p$ to $\nu + n \rightarrow \mu^- + p$. Comparing these in the forward direction, we have, from Eq. (3.3),

$$R_{\text{eff}} \equiv \frac{d\sigma(\nu + p \rightarrow \nu + p)/dq^2}{d\sigma(\nu + n \rightarrow \mu^- + p)/dq^2} \bigg|_{q^2=0} = \frac{F_V^{(N)}(0)^2 + F_A^{(N)}(0)^2}{F_V(0)^2 + F_A(0)^2} \quad (3.17)$$

Theoretical models for the neutral current can almost all be represented as linear combinations of the neutral members $V_a^{(3)}$, $V_a^{(8)}$, $A_a^{(3)}$, $A_a^{(8)}$ of the vector and axial vector octets, plus possibly the $SU(3)$ singlet $V_a^{(0)}$ (the baryon number current) and its axial analogue $A_a^{(0)}$. Writing the current as⁽²⁵⁾

$$J_a^{(N)} = G_V^{(3)} V_a^{(3)} + G_V^{(8)} V_a^{(8)} + G_V^{(0)} V_a^{(0)} + G_A^{(3)} A_a^{(3)} + G_A^{(8)} A_a^{(8)} + G_A^{(0)} A_a^{(0)} \quad (3.18)$$

where the G 's are coupling constants, we obtain

$$F_V^{(N)}(0) = \sum_{i=0,3,8} G_V^{(i)} \rho_V^{(i)}, \quad (3.19)$$

$$F_A^{(N)}(0) = \sum_{i=0,3,8} G_A^{(i)} \rho_A^{(i)}.$$

In (3.19) $\rho_V^{(i)}$, $\rho_A^{(i)}$ stand for the expectation values (for a proton) of the charges associated with $V_a^{(i)}$, $A_a^{(i)}$. These constants are easy to determine for the vector currents, since $V_a^{(3)}$, $V_a^{(8)}$ and $V_a^{(0)}$ are related to the conserved charges I_3 , Y and B (baryon number). Thus

$$\rho_V^{(3)} = \frac{1}{2}, \quad \rho_V^{(8)} = \frac{\sqrt{3}}{2}, \quad \rho_V^{(0)} = 1 \quad (3.20)$$

For the axial currents, $\rho_A^{(3)}$ and $\rho_A^{(8)}$ are determined by the constants F and D of hyperon β -decay ($F \approx 0.45$, $D \approx 0.80$) while $\rho_A^{(0)}$ can be related to $\rho_A^{(3)}$

with the help of the quark model. (26) The result is

$$\begin{aligned} \rho_A^{(3)} &= \frac{1}{2} (F + D) , \quad \rho_A^{(8)} = \frac{1}{2\sqrt{3}} (3F - D) , \\ \rho_A^{(0)} &= \frac{1}{5} \sqrt{\frac{3}{2}} (F + D) . \end{aligned} \quad (3.21)$$

(Note that the charged current constants in Eq. (3.17) are $F_V(0) \approx 1$, $F_A(0) = F + D \approx 1.25$.) Thus, given a model which specifies the coupling constants $G_V^{(i)}$ and $G_A^{(i)}$, the forward cross section ratio R_{ef} can be predicted. Table 3 shows the predictions for R_{ef} of some simple models of the neutral current.

Table 3

Hypothesis for Neutral Current	$R_{ef} = \frac{d\sigma(\nu + p \rightarrow \nu + p)/dq^2}{d\sigma(\nu + n \rightarrow \mu^- + p)/dq^2} \Big _{q^2=0}$	Ref.
$J_a^{(N)} = \epsilon(V_a^{(3)} + A_a^{(3)})$	$\frac{1}{4} \epsilon^2$	Adler and Tuan(8)
$J_a^{(N)} = V_a^{(3)} + A_a^{(3)} - \xi V_a^{em}$	$\frac{1}{4} \frac{(1 - 2\xi)^2 + (1.25)^2}{1 + (1.25)^2} \geq 0.15$	Weinberg(27)
$J_a^{(N)} = \xi V_a^{(0)}$	$\xi^2 / (1 + (1.25)^2)$	Sakurai(6)
$J_a^{(N)} = G_V V_a^{em} + G_A A_a^{em}$	$(G_V^2 + G_A^2 (0.66)^2) / (1 + (1.25)^2)$	Mathur, Okubo, and Kim(28)

The forward cross section for $\nu + p \rightarrow \nu + p$ is not easily accessible, especially when the reaction $n + p \rightarrow n + p$ forms a background. Experiments more often measure the ratio of integrated cross sections

$$R'_{ef} \equiv \frac{\int_{Q^2 > Q_0^2} \frac{d\sigma}{dQ^2} (\nu + p \rightarrow \nu + p) dQ^2}{\int_{Q^2 > Q_0^2} \frac{d\sigma}{dQ^2} (\nu + n \rightarrow \mu^- + p) dQ^2} \quad (3.22)$$

where Q_0^2 is some convenient cut-off (e. g. $Q_0^2 = 0.3 \text{ GeV}^2$). To predict the ratio R'_{eff} from a model of the neutral current such as (3. 18) one needs to know not merely the coupling constants $F_V^{(N)}(0)$ and $F_A^{(N)}(0)$ given in (3. 19) but rather the complete form factors $F_V^{(N)}(Q^2)$, $F_M^{(N)}(Q^2)$ and $F_A^{(N)}(Q^2)$. A reasonable procedure is to assume for $F_V^{(N)}$ and $F_M^{(N)}$ the dependence $(1 + Q^2/M_V^2)^{-2}$, $M_V^2 \approx 0.71 \text{ GeV}^2$, and for $F_A^{(N)}$ the dependence $(1 + Q^2/M_A^2)^{-2}$, $M_A^2 \approx 0.80 \text{ GeV}^2$, which appear to characterize the electromagnetic and charged currents. The normalization of F_M is given by

$$F_M^{(N)}(0) = \sum_{i=0,3,8} G_V^{(i)} K^{(i)} \quad (3. 23)$$

$$\text{where } K^{(3)} = \frac{1}{2} (K_p - K_n), \quad K^{(8)} = \frac{\sqrt{3}}{2} (K_p + K_n), \quad K^{(0)} \approx (K_p + K_n), \quad (3. 24)$$

K_p , K_n being the anomalous magnetic moments of the proton, neutron, and the result for $K^{(0)}$ being a quark model estimate.⁽²⁶⁾ (Numerically, $K_p + K_n \approx 0$, so that for many purposes, it may be adequate to retain just the term $i = 3$ in (3. 23).) Note that R'_{eff} will, in general, depend on the incoming neutrino energy, and in the case of a wide band beam, on the neutrino spectrum. Sample calculations in some typical models are presented in Refs. (28) and (29).

3. 3 Pion Production by Charged Currents

a. Isospin Analysis

Single pion production by the charged currents occurs via three reactions

$$\nu + p \rightarrow \mu^- + p + \pi^+ \quad (3. 25a)$$

$$\nu + n \rightarrow \mu^- + p + \pi^0 \quad (3. 25b)$$

$$\nu + n \rightarrow \mu^- + n + \pi^+ \quad (3. 25c)$$

Assuming the $\Delta S = 0$ charged current $J_\alpha^{(C)}$ to transform as an isovector (as is the case in the conventional theory) the amplitudes for (3. 25) can be expressed in terms of two "reduced" amplitudes M_1 and M_3 corresponding to the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ states of the πN system. Thus

$$\begin{aligned}
\text{Amp } (\nu p \rightarrow \mu^- \bar{p} \pi^+) &= \sqrt{2} M_3 \\
\text{Amp } (\nu n \rightarrow \mu^- \bar{p} \pi^0) &= \frac{2}{3} (M_3 - M_1) \\
\text{Amp } (\nu n \rightarrow \mu^- \bar{n} \pi^+) &= \frac{\sqrt{2}}{3} (M_3 + 2M_1)
\end{aligned} \tag{3.26}$$

which yields the relation

$$\begin{aligned}
\text{Amp } (\nu p \rightarrow \mu^- \bar{p} \pi^+) &= \sqrt{2} \text{Amp } (\nu n \rightarrow \mu^- \bar{p} \pi^0) \\
&+ \text{Amp } (\nu n \rightarrow \mu^- \bar{n} \pi^+) \quad .
\end{aligned} \tag{3.27}$$

This implies that $\sqrt{\sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+)}$, $\sqrt{2\sigma(\nu n \rightarrow \mu^- \bar{p} \pi^0)}$ and $\sqrt{\sigma(\nu n \rightarrow \mu^- \bar{n} \pi^+)}$ satisfy a set of three triangular inequalities, consisting of

$$\sqrt{2\sigma(\nu n \rightarrow \mu^- \bar{p} \pi^0)} \leq \sqrt{\sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+)} + \sqrt{\sigma(\nu n \rightarrow \mu^- \bar{n} \pi^+)} \tag{3.28}$$

and its permutations.

An important feature of the πN system at low energies is the existence of the $\Delta(1236)$ resonance in the $I = 3/2$ channel. In the neighborhood of this resonance, it may be a good approximation to assume $M_3 \gg M_1$.

Eq. (3.26) then leads to the cross section ratios

$$\begin{aligned}
\sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+) : \sigma(\nu n \rightarrow \mu^- \bar{p} \pi^0) : \sigma(\nu n \rightarrow \mu^- \bar{n} \pi^+) &= 1 : \frac{2}{9} : \frac{1}{9} \\
\frac{\pi^+}{\pi^0} &= 5 \quad ,
\end{aligned} \tag{3.29}$$

where the π^+/π^0 ratio refers to an average nucleon (or deuteron) target.

More generally, a measurement of the three cross sections (into a given kinematical region) allows us to obtain the average values of $|M_1|^2$, $|M_3|^2$ and $\text{Re } M_3^* M_1$ in that region. Thus

$$\begin{aligned}
\langle |M_1|^2 \rangle &= \frac{3}{4} [\sigma(\nu n \rightarrow \mu^- \bar{p} \pi^0) + \sigma(\nu n \rightarrow \mu^- \bar{n} \pi^+) - \frac{1}{3} \sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+)] \\
\langle |M_3|^2 \rangle &= \frac{1}{2} \sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+)
\end{aligned} \tag{3.30}$$

$$\langle \text{Re } M_3^* M_1 \rangle = \frac{9}{32} [\sigma(\nu n \rightarrow \mu^- \bar{n} \pi^+) - 2\sigma(\nu n \rightarrow \mu^- \bar{p} \pi^0) + \frac{1}{3} \sigma(\nu p \rightarrow \mu^- \bar{p} \pi^+)]$$

Experiments at ANL, ⁽³⁰⁾ using a wide band ν beam with a peak energy of 0.5 GeV, have found 120 ± 13 , 38 ± 9 and 37 ± 7 events of the types $\nu p \rightarrow \mu^- p \pi^+$, $\nu n \rightarrow \mu^- p \pi^0$ and $\nu n \rightarrow \mu^- n \pi^+$, respectively. One concludes from (3.30) that for the conditions of the ANL experiment

$$\left[\frac{\langle |M_1|^2 \rangle}{\langle |M_3|^2 \rangle} \right]^{1/2} = 0.66 \pm 0.13 \quad (3.31)$$

$$\frac{\langle \text{Re } M_3^* M_1 \rangle}{\langle |M_3|^2 \rangle^{1/2} \langle |M_1|^2 \rangle^{1/2}} = 0.01$$

The first of these results implies a substantial presence of $I = \frac{1}{2}$ amplitudes in the final πN state, a feature also evident from a comparison of the results with the predictions of $I = \frac{3}{2}$ dominance (Eq. (3.25)). The second result indicates that the M_3 and M_1 amplitudes are, on average, almost 90° out of phase. This fact can be qualitatively understood when one notes that the experiment spans the region around the $(3, 3)$ resonance, in which the phase of M_3 varies from 0 to π (being $\pi/2$ at the resonance position) while that of the nonresonant amplitude M_1 remains close to zero.

b. Charge Symmetry

Relations between ν and $\bar{\nu}$ induced reactions are obtained if the $(\Delta S = 0)$ current $J_a^{(C)}$ is assumed to be charge symmetric, i. e. to satisfy

$$e^{i\pi I_2} J_a^{(C)} e^{-i\pi I_2} = -J_a^{(C)} \quad (3.32)$$

If this is the case, a reaction such as $\nu + I_3^I \rightarrow \mu^- + I_3^F$ may be related to the reaction $\bar{\nu} + (-I_3^I) \rightarrow \mu^+ + (-I_3^F)$, ⁽¹⁹⁾ where we have labeled the initial and final hadronic systems by their I_3 values. Decomposing the cross section for the first reaction into vector, axial vector and interference parts,

$$\sigma(\nu + I_3^I - \mu^- + I_3^{II}) = \sigma_V + \sigma_A + \sigma_I \quad (3.33a)$$

the cross section for the second is obtained by changing the sign of the interference term, i. e.

$$\sigma(\bar{\nu} + (-I_3^I) - \mu^+ + (-I_3^{II})) = \sigma_V + \sigma_A - \sigma_I \quad (3.33b)$$

A combination of charge symmetry with the isospin analysis given above can help to isolate the V and A parts of the amplitudes M_1 and M_3 and thus provide some insight into the dynamics of pion production.

3.4 Pion Production by Neutral Currents

a. Isospin and Hermiticity Constraints

Neutral currents can induce single pion production via the following four reactions:

$$\nu + p \rightarrow \nu + p + \pi^0 \quad (3.34a)$$

$$\nu + n \rightarrow \nu + n + \pi^0 \quad (3.34b)$$

$$\nu + p \rightarrow \nu + n + \pi^+ \quad (3.34c)$$

$$\nu + n \rightarrow \nu + p + \pi^- \quad (3.34d)$$

Assuming the neutral current to contain only isoscalar and isovector parts (i. e. $\Delta I < 2$), the amplitudes can be parameterized as

$$\begin{aligned} A(\nu p \rightarrow \nu n \pi^+) &= \frac{\sqrt{2}}{3} (M_3 - M_1) + \sqrt{\frac{2}{3}} S \\ A(\nu n \rightarrow \nu p \pi^-) &= \frac{\sqrt{2}}{3} (M_3 - M_1) - \sqrt{\frac{2}{3}} S \\ A(\nu p \rightarrow \nu p \pi^0) &= \frac{2}{3} (M_3 + \frac{1}{2} M_1) - \sqrt{\frac{1}{3}} S \\ A(\nu n \rightarrow \nu n \pi^0) &= \frac{2}{3} (M_3 + \frac{1}{2} M_1) + \sqrt{\frac{1}{3}} S \end{aligned} \quad (3.35)$$

Here M_3 and M_1 are the reduced matrix elements for production of $I = 3/2$ and $I = 1/2$ states by the isovector current, and S the reduced matrix element of the isoscalar current (which induces only a transition to

$I = 1/2$). Eq. (3.35) gives a sum rule

$$A(\nu p - \nu n \pi^+) - A(\nu n - \nu p \pi^-) = -\sqrt{2} [A(\nu p - \nu p \pi^0) - A(\nu n - \nu n \pi^0)] \quad (3.36)$$

which implies a set of four "quadrangular" inequalities,

$$-\sqrt{\sigma}(\nu p - \nu n \pi^+) \leq -\sqrt{\sigma}(\nu n - \nu p \pi^-) + \sqrt{2\sigma}(\nu p - \nu p \pi^0) + \sqrt{2\sigma}(\nu n - \nu n \pi^0) \quad (3.37)$$

and its permutations.

If the neutral current is "pure" with respect to isospin (i. e. either $\Delta I = 0$ or $\Delta I = 1$), we have

$$\begin{aligned} \sigma(\nu p - \nu n \pi^+) &= \sigma(\nu n - \nu p \pi^-) \\ \sigma(\nu p - \nu p \pi^0) &= \sigma(\nu n - \nu n \pi^0) \end{aligned} \quad (3.38)$$

Thus, any difference between π^+ and π^- production in an experiment using, e. g., a deuterium target would be immediate evidence that the neutral current has "mixed" isospin. For a pure isoscalar current and a deuterium target, an additional consequence is

$$\pi^+ : \pi^- : \pi^0 = 1 : 1 : 1 \quad (3.39)$$

If the final πN system is dominated by the $I = 3/2$ amplitude M_3 , the following ratios are predicted:

$$\begin{aligned} \sigma(\nu p - \nu n \pi^+) : \sigma(\nu n - \nu p \pi^-) : \sigma(\nu p - \nu p \pi^0) : \sigma(\nu n - \nu n \pi^0) \\ = 1 : 1 : 2 : 2 \\ \frac{\pi^+ + \pi^-}{\pi^0} = \frac{1}{2} \end{aligned} \quad (3.40)$$

For an arbitrary neutral current $J_a^{(N)}$, the antineutrino process $\bar{\nu} + I_3^I - \bar{\nu} + I_3^{II}$ is related to the neutrino process $\nu + I_3^I - \nu + I_3^{II}$ by a change in the sign of the V-A interference term. This result depends only on the hermiticity of $J_a^{(N)}$, which is guaranteed as long as the initial and final "neutrinos" are the same. A failure of this relation could show up as a difference between the ν and $\bar{\nu}$ reactions in the forward lepton configuration, in which the V-A interference term is absent.

b. Neutral Current/Charged Current Comparison

A comparison of the reactions (3.34) with (3.25) can give important clues to the structure of the neutral current $J_a^{(N)}$. A ratio that is of special importance in this respect is

$$R_0 \equiv \frac{\sigma(\nu p \rightarrow \nu p \pi^0) + \sigma(\nu n \rightarrow \nu n \pi^0)}{2\sigma(\nu n \rightarrow \mu^- p \pi^0)} \quad (3.41)$$

In general, a prediction for R_0 requires, not only knowledge of the neutral current structure (Eq. (3.18)), but also a dynamical model for pion production. As an illustration of how a measurement of R_0 can constrain the parameters of the neutral current, we will consider here a particularly simple situation, one in which all three reactions in (3.41) are dominated by the $I = 3/2$ final state. Writing the current $J_a^{(N)}$ as

$$J_a^{(N)} = G_V^{(3)} V_a^{(3)} + G_A^{(3)} A_a^{(3)} + \text{isoscalar} \quad (3.42)$$

and recalling that

$$J_a^{(C)} = V_a^{1+i2} + A_a^{1+i2},$$

we obtain (with $\sigma(\nu n \rightarrow \mu^- p \pi^0) = \sigma_V + \sigma_A + \sigma_I$),

$$R_0 = \frac{G_V^{(3)^2} \sigma_V + G_A^{(3)^2} \sigma_A + G_V^{(3)} G_A^{(3)} \sigma_I}{\sigma_V + \sigma_A + \sigma_I} \quad (3.43)$$

(Observe that the isoscalar part of $J_a^{(N)}$ plays no role when the final state has $I = 3/2$). In a static model calculation of N^* production (assuming an incident energy of 1 GeV), B. W. Lee estimated⁽³¹⁾

$$\sigma_V : \sigma_A : \sigma_I = 0.263 : 0.202 : 0.235, \quad (3.44)$$

which leads to

$$R_0 = 0.376 G_V^{(3)^2} + 0.289 G_A^{(3)^2} + 0.336 G_V^{(3)} G_A^{(3)} \quad (3.45)$$

The same model, when applied to π^0 production by antineutrinos, gives

$$\begin{aligned}
\bar{R}_0 &\equiv \frac{\sigma(\bar{\nu}p \rightarrow \bar{\nu}p\pi^0) + \sigma(\bar{\nu}n \rightarrow \bar{\nu}n\pi^0)}{2\sigma(\bar{\nu}p \rightarrow \mu^+\pi^0)} \\
&= \frac{G_V^{(3)^2} \sigma_V + G_A^{(3)^2} \sigma_A - G_V^{(3)} G_A^{(3)} \sigma_I}{\sigma_V + \sigma_A - \sigma_I} \quad (3.46) \\
&= 1.14 G_V^{(3)^2} + 0.878 G_A^{(3)^2} - 1.02 G_V^{(3)} G_A^{(3)}.
\end{aligned}$$

As one example, the Weinberg model with $G_A^{(3)} = 1$, $G_V^{(3)} = 1 - 2 \sin^2 \theta$, would give, for $\sin^2 \theta = 0.35$, $R_0 = 0.42$, $\bar{R}_0 = 0.67$.

More refined calculations of single pion production have been done in a model developed by Adler, which incorporates the effects of $I = 1/2$ amplitudes. We refer to a recent paper of Adler,⁽²⁵⁾ that gives detailed predictions for all single pion channels (charged as well as neutral current). Using the Weinberg model (with $\sin^2 \theta = 0.35$) as an illustration, the calculations indicate a strong suppression of the effects of the isoscalar current, the relations (3.38) being nearly satisfied. The values predicted for R_0 are close to the static model result in the limit that the pion is treated as "soft" ($\vec{q}_\pi = 0$), but increase significantly when $O(\vec{q}_\pi)$ effects are included. In view of the strong model dependence of theoretical calculations of R_0 , measurements of this quantity will have to be interpreted with caution.

3.5 Nuclear Corrections to Pion Production

Much of the information on single pion production comes from targets of complex nuclei. Because of rescattering, the observed ratios of π^+ , π^0 and π^- can be different from those that would be produced off a free (average) nucleon (or, equivalently, a deuteron). A simple model of nuclear effects has been proposed,⁽³²⁾ based on a two-step mechanism: pion production from a constituent nucleon, followed by the passage of the pion through the nuclear medium (assumed isoscalar). Only elastic scattering and absorption are

assumed to occur in the second step. If σ_+ , σ_0 , σ_- are the cross sections for π^+ , π^0 , π^- produced off a free nucleon, the cross sections (per nucleon) resulting from a nucleus will be σ'_+ , σ'_0 , σ'_- , where

$$\begin{pmatrix} \sigma'_+ \\ \sigma'_0 \\ \sigma'_- \end{pmatrix} = AM \begin{pmatrix} \sigma_+ \\ \sigma_0 \\ \sigma_- \end{pmatrix} \quad (3.47)$$

Here M denotes a 3×3 "charge exchange" matrix and A is an absorption factor that is unity if no absorption occurs. The structure of M may be determined by isospin arguments to be

$$M = \begin{pmatrix} 1-c-d & d & c \\ d & 1-2d & d \\ c & d & 1-c-d \end{pmatrix} \quad (3.48)$$

(To see this, observe that charge independence requires $M_{+-} = M_{-+}$, $M_{++} = M_{--}$, $M_{0+} = M_{0-} = M_{+0} = M_{-0}$. In addition conservation of pion number, in the absence of absorption, requires each column to add up to unity). We can derive from M a 2×2 submatrix N that describes nuclear effects on the ratio $(\sigma_+ + \sigma_-) : \sigma_0$. This is given by

$$N = \begin{pmatrix} 1-d & 2d \\ d & 1-2d \end{pmatrix} \quad (3.49)$$

Let us consider the effects of nuclear corrections on the $(\pi^+ + \pi^-)/\pi^0$ ratio in reactions of the class (3.25). Using (3.49), we have

$$\left(\frac{\pi^+ + \pi^-}{\pi^0} \right)_{\text{Nucleus}} = \frac{(1-d) \left(\frac{\pi^+ + \pi^-}{\pi^0} \right)_{\text{Nucleon}} + 2d}{d \left(\frac{\pi^+ + \pi^-}{\pi^0} \right)_{\text{Nucleon}} + (1-2d)} \quad (3.50)$$

In the resonance region, the ratio $(\pi^+ + \pi^-/\pi^0)_{\text{Nucleon}}$ should be close to 5 (the ANL data⁽³⁰⁾ give a ratio of about 4). On the other hand, early measurements at CERN indicated that the ratio in freon was 2.0 ± 0.4 . Taking these numbers as a guide, we would conclude from Eq. (3.50) that the parameter d is in the range $\frac{1}{4}$ to $\frac{1}{2}$. This value can now be used to estimate the nuclear corrections to the ratio R_0 defined in Eq. (3.41). Application of the matrix N gives

$$[R_0]_{\text{Nucleus}} = [R_0]_{\text{Nucleon}} \frac{d \frac{\pi^+ + \pi^-}{\pi^0} \nu \rightarrow \nu + (1-2d)}{d \frac{\pi^+ + \pi^-}{\pi^0} \nu \rightarrow \mu^- + (1-2d)} \quad (3.51)$$

where $(\pi^+ + \pi^-/\pi^0) \nu \rightarrow \nu$ and $(\pi^+ + \pi^-/\pi^0) \nu \rightarrow \mu^-$ refer to reactions (3.25) and (3.34). In the limit of $I = 3/2$ dominance, these two ratios are $1/2$ and 5 , respectively, so that the correction factor in (3.51) is in the range $1/3$ to $1/10$!

While the above discussion is no more than illustrative, it does indicate the importance, emphasized by Perkins,⁽³³⁾ of nuclear corrections to ratios such as R_0 . Note, however, that these corrections should be similar for ν and $\bar{\nu}$ induced reactions, so that quantities such as $R_0/\bar{R}_0$ should be relatively insensitive to nuclear effects.

4. SCATTERING OF NEUTRINOS BY NUCLEONS. INCLUSIVE AND SEMI-INCLUSIVE REACTIONS

We discuss in this chapter reactions that occur when neutrinos of high energy ($E_\nu \gg 1 \text{ GeV}$) collide with nucleons. Such reactions are characterized by the production of many particles, and account for the major part of the cross section at accelerator energies. The analysis of each separate channel under such conditions is a demanding task experimentally, and not very illuminating from the theoretical viewpoint. Instead attention has focussed on

the inclusive reactions

$$\begin{aligned}\nu + N &\rightarrow \mu^- + \text{anything} \\ \nu + N &\rightarrow \nu + \text{anything} \quad ,\end{aligned}\tag{4.1}$$

where all possible hadronic states and their configurations are summed over. It was conjectured that such reactions would show a strikingly simple behavior, an expectation that has been vindicated by experiment. Another class of reactions that is of interest is semi-inclusive scattering

$$\begin{aligned}\nu + N &\rightarrow \mu^- + h + \text{anything} \\ \nu + N &\rightarrow \nu + h + \text{anything} \quad ,\end{aligned}\tag{4.2}$$

where h is a specific hadron. Such studies yield the average multiplicity $\langle N_h \rangle$ of different types of hadrons, which is an important index of the dynamics of these reactions. In both the classes (4.1) and (4.2), the comparison of the charged current and the neutral current process provides valuable information on the structure of the neutral current.

4.1 Kinematics of Inclusive Reactions

Consider the reaction

$$\nu(k) + N(p) \rightarrow \mu^-(k') + \Gamma(p') \quad ,\tag{4.3}$$

where Γ is an arbitrary hadronic system (containing prescribed numbers of prescribed types of particles). We are interested in the cross section when the nucleon N is unpolarized and all the configurations of Γ (i. e. internal momenta and spins) are summed over. Such a cross section is a function of only two invariants, which we choose to be $Q^2 = -(k-k')^2$ and $\nu = p \cdot q/M$ ($q = k-k'$), and may be written as⁽¹⁹⁾ (E = incident energy)

$$\left[\frac{d^2 \sigma^\nu}{dQ^2 d\nu} \right]_\Gamma = \frac{G^2}{32\pi ME^2} L_{\mu\nu}^\nu [W_{\mu\nu}^\nu]_\Gamma\tag{4.4}$$

where

$$\begin{aligned}
L_{\mu\nu}^{\nu} &= \sum_{\text{lepton spins}} \bar{u}(k') \gamma_{\mu} (1 + \gamma_5) u(k) \bar{u}(k) \gamma_{\nu} (1 + \gamma_5) u(k') \\
&= 2 \text{Tr} [K^{\mu} \gamma_{\mu} K \gamma_{\nu} (1 + \gamma_5)] \\
&= 8 [k_{\mu} k_{\nu} + k_{\mu} k'_{\nu} - k_{\mu\nu} k k' - i \epsilon_{\mu\nu\rho\sigma} k'_{\rho} k_{\sigma}]
\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}
[W_{\mu\nu}^{\nu}]_{\Gamma} &= \frac{1}{2} \sum_{N \text{ spins}} \sum_{\Gamma \text{ spins and internal momenta}} \langle N | J_{\nu} | \Gamma \rangle \langle \Gamma | J_{\mu}^{\dagger} | N \rangle \cdot \\
&\quad \cdot (2\pi)^3 \delta^4(k + p - k' - p')
\end{aligned} \tag{4.6}$$

The general structure of $[W_{\mu\nu}^{\nu}]_{\Gamma}$ is

$$\begin{aligned}
[W_{\mu\nu}^{\nu}]_{\Gamma} &= [-g_{\mu\nu} W_1^{\nu} + \frac{p_{\mu} p_{\nu}}{M^2} W_2^{\nu} - i \epsilon_{\mu\nu\rho\sigma} \frac{p_{\rho} q_{\sigma}}{2M^2} W_3^{\nu} + \frac{q_{\mu} q_{\nu}}{M^2} W_4^{\nu} \\
&\quad + \frac{p_{\mu} q_{\nu} + p_{\nu} q_{\mu}}{2M^2} W_5^{\nu} + i \frac{p_{\mu} q_{\nu} - p_{\nu} q_{\mu}}{2M^2} W_6^{\nu}]_{\Gamma}
\end{aligned} \tag{4.7}$$

where the W 's are functions of Q^2 and ν . The last three terms do not contribute when the muon mass is neglected, in which limit we obtain

$$\left[\frac{d^2 \sigma^{\nu}}{dQ^2 d\nu} \right]_{\Gamma} = \frac{G^2}{2\pi M} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} W_2^{\nu} + 2 \sin^2 \frac{\theta}{2} W_1^{\nu} - \frac{E + E'}{M} \sin^2 \frac{\theta}{2} W_3^{\nu} \right]_{\Gamma} \tag{4.8}$$

E' and θ being the energy and angle of the final lepton in the lab. (Note that $\sin^2 \frac{\theta}{2} = Q^2/4EE'$, $\cos^2 \frac{\theta}{2} = 1 - Q^2/4EE'$, $E' = E - \nu$.)

In the case of an antineutrino reaction

$$\bar{\nu}(k) + N(p) \rightarrow \mu^+(k') + \Gamma'(p') \quad , \tag{4.9}$$

the cross section may be written as

$$\left[\frac{d^2 \sigma^{\bar{\nu}}}{dQ^2 d\nu} \right]_{\Gamma'} = \frac{G^2}{32\pi ME^2} L_{\mu\nu}^{\bar{\nu}} [W_{\mu\nu}^{\bar{\nu}}]_{\Gamma'} \quad (4.10)$$

where $L_{\mu\nu}^{\bar{\nu}}$ differs from the expression in (4.5) only in the sign of the last term, and $[W_{\mu\nu}^{\bar{\nu}}]_{\Gamma'}$ may be written as in (4.7), with the change of index $\nu \rightarrow \bar{\nu}$ and $\Gamma \rightarrow \Gamma'$. Thus

$$\left[\frac{d^2 \sigma^{\bar{\nu}}}{dQ^2 d\nu} \right]_{\Gamma'} = \frac{G^2}{2\pi M} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} W_2^{\bar{\nu}} + 2 \sin^2 \frac{\theta}{2} W_1^{\bar{\nu}} + \frac{E+E'}{M} \sin^2 \frac{\theta}{2} W_3^{\bar{\nu}} \right]_{\Gamma'} \quad (4.11)$$

The results (4.8) and (4.11) hold equally for the neutral current reactions

$$\begin{aligned} \nu + N &\rightarrow \nu + \Gamma_0 \\ \bar{\nu} + N &\rightarrow \bar{\nu} + \Gamma_0 \end{aligned} \quad (4.12)$$

provided we replace the W_i^ν and $W_i^{\bar{\nu}}$ by analogous functions $W_i^{(N)\nu}$ and $W_i^{(N)\bar{\nu}}$ derived from (4.6) with $J_\mu \rightarrow J_\mu^{(N)}$.

Since Eq. (4.8) is valid for an arbitrary hadronic state Γ , it is also valid in the special case when Γ is a nucleon, i. e. when Eq. (4.3) is the elastic scattering process. Comparison of Eq. (4.8) with Eq. (3.6) then shows that⁽¹⁹⁾

$$\begin{aligned} [W_i]_{\text{elastic}} &= M\omega_i \delta(\nu - Q^2/2M) \quad , \\ \omega_1 &= F_A^2 + \frac{Q^2}{4M^2} [F_A^2 + (F_V + F_M)^2] \\ \omega_2 &= F_A^2 + F_V^2 + \frac{Q^2}{4M^2} F_M^2 \\ \omega_3 &= 2F_A(F_V + F_M) \quad . \end{aligned} \quad (4.13)$$

If the measurements include not only all configurations of the state Γ' , but all possible hadronic states Γ as well, we obtain the inclusive cross section

$d\sigma/dQ^2 d\nu$ which may be written as in Eq. (4.8), with the index Γ omitted. The functions W_i^ν , in this case, are the structure functions for inclusive neutrino nucleon scattering.

If the charged current J_α possesses charge symmetry, we obtain a relation between the reactions

$$\nu + N \rightarrow \mu^- + \Gamma \quad (4.14a)$$

$$\text{and} \quad \bar{\nu} + \tilde{N} \rightarrow \mu^+ + \tilde{\Gamma} \quad (4.14b)$$

where $\tilde{N}$ and $\tilde{\Gamma}$ denote the isospin reflections ($I_3 \rightarrow -I_3$) of N and Γ . The relation is $[W_i^{\nu N}]_\Gamma = [W_i^{\bar{\nu} \tilde{N}}]_{\tilde{\Gamma}}$. In particular, for the inclusive structure functions, we have the result

$$W_i^{\nu p} = W_i^{\bar{\nu} p} \quad , \quad W_i^{\bar{\nu} p} = W_i^{\nu n} \quad . \quad (4.15)$$

The neutral current reactions

$$\nu + N \rightarrow \nu + \Gamma_0 \quad (4.16a)$$

$$\bar{\nu} + N \rightarrow \bar{\nu} + \Gamma_0 \quad (4.16b)$$

are related to each other as a consequence of the hermiticity of $J_\mu^{(N)}$, which implies $W_i^{(N)\nu} = W_i^{(N)\bar{\nu}}$. Thus the cross sections for (4.16a) and (4.16b) differ only in the sign of the interference term (proportional to W_3). Any deviation from this behavior would cast doubt on the hermiticity of $J_\mu^{(N)}$ and suggest that the initial and final leptons in the neutral current are distinct. ⁽²³⁾

The functions W_1 , W_2 , W_3 are subject to certain constraints arising from the positivity of the cross sections, which requires $\epsilon_\mu^* \epsilon_\nu W_{\mu\nu} \geq 0$ for arbitrary ϵ . Choosing for ϵ the three orthonormal vectors $\epsilon_\mu^S = 1/\sqrt{Q^2}(q_3, 0, 0, q_0)$, $\epsilon_\mu^R = 1/\sqrt{2}(0, 1, i, 0)$ and $\epsilon_\mu^L = 1/\sqrt{2}(0, i, 1, 0)$, satisfying $\epsilon \cdot q = 0$, we obtain

$$\begin{aligned}
\sigma_S &\equiv K[W_2 (\frac{\nu^2 + Q^2}{Q^2}) - W_1] \geq 0 \\
\sigma_R &\equiv K[W_1 + \frac{1}{2} \sqrt{\frac{\nu^2 + Q^2}{M^2}} W_3] \geq 0 \\
\sigma_L &\equiv K[W_1 - \frac{1}{2} \sqrt{\frac{\nu^2 + Q^2}{M^2}} W_3] \geq 0
\end{aligned} \tag{4.17}$$

(K being an arbitrary factor > 0) which together imply

$$(1 + \frac{\nu^2}{Q^2}) W_2 \geq W_1 \geq \frac{1}{2} \sqrt{\frac{\nu^2 + Q^2}{M^2}} |W_3| \geq 0 \tag{4.18}$$

4.2 The Scaling Hypothesis

Let us rewrite the inclusive cross section $d\sigma/dQ^2 d\nu$ in terms of dimensionless variables

$$x \equiv \frac{Q^2}{2M\nu} \quad y \equiv \frac{\nu}{E} \quad 0 < x, y < 1 \tag{4.19}$$

If we define

$$\begin{aligned}
W_1(Q^2, \nu) &\equiv G_1(x, \frac{Q^2}{M^2}) \\
\frac{\nu}{M} W_2(Q^2, \nu) &\equiv G_2(x, \frac{Q^2}{M^2}) \\
\frac{\nu}{M} W_3(Q^2, \nu) &\equiv G_3(x, \frac{Q^2}{M^2})
\end{aligned} \tag{4.20}$$

we obtain

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G^2 ME}{\pi} [(1 - y - \frac{1}{2} xy \frac{M}{E}) G_2^{\nu, \bar{\nu}} + \frac{1}{2} y^2 (2x G_1^{\nu, \bar{\nu}} + y(1 - \frac{1}{2} y)(x G_3^{\nu, \bar{\nu}}))] \tag{4.21}$$

It was suggested by Bjorken⁽³⁴⁾ that when ν and Q^2 became very large with $x = Q^2/2M\nu$ fixed, the functions G_1, G_2, G_3 would approach nontrivial limits

$$G_1(x, \frac{Q^2}{M^2}) \xrightarrow[Q^2 \rightarrow \infty, \nu \rightarrow \infty]{x \text{ fixed}} F_1(x) \quad (4.22)$$

Since large Q^2 and ν necessarily require large incident energy E , the inclusive cross section (4.21) becomes, in the Bjorken limit

$$\frac{d\sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G^2 ME}{\pi} [(1-y) F_2^{\nu, \bar{\nu}}(x) + \frac{1}{2} y^2 (2xF_1^{\nu, \bar{\nu}}(x)) + y(1 - \frac{1}{2}y)(xF_3^{\nu, \bar{\nu}}(x))] \quad (4.23)$$

The following are some immediate consequences of the scaling hypothesis.

(i) The total cross sections, according to (4.23), are

$$\sigma^{\nu, \bar{\nu}} = \frac{G^2 ME}{\pi} \int_0^1 dx \left[\frac{1}{2} F_2^{\nu, \bar{\nu}}(x) + \frac{1}{3} x F_1^{\nu, \bar{\nu}}(x) + \frac{1}{3} x F_3^{\nu, \bar{\nu}}(x) \right] \quad (4.24)$$

Thus σ^{ν} and $\sigma^{\bar{\nu}}$ should be proportional to E at high energies. This prediction appears to be in spectacular agreement with experiments in the range 1 - 200 GeV (Fig. 3a).⁽³⁵⁾ Nevertheless, deviations of the order of 30% at the higher energies cannot be ruled out.

(ii) If $f(x, y)$ is any function of x and y , its average value, defined as

$$\langle f(x, y) \rangle = \frac{\int f(x, y) \frac{d\sigma}{dx dy} dx dy}{\int \frac{d\sigma}{dx dy} dx dy} \quad (4.25)$$

should be independent of E . This has been tested for simple choices such as $f = x$, $f = y$ and $f = xy$, and found to be true for energies $E \geq 5$ GeV. There are some indications that $\langle y \rangle$ in antineutrino data above 30 GeV is higher than its value at lower energies, suggesting a possible breakdown of scaling in this energy region (Fig. 3b, c).

(iii) Eq. (4.23) shows that the y -distribution

$$\frac{d\sigma}{dy} = \int_D dx \frac{d\sigma}{dx dy} \quad (4.26)$$

obtained by integrating $d\sigma/dx dy$ over any domain D of x has to be a quadratic function of y . Furthermore as a consequence of the positivity conditions (see Eq. (4. 18))

$$F_2(x) \geq 2x F_1(x) , \quad (4. 27)$$

$$F_1(x) + \frac{1}{2} F_3(x) \geq 0 ,$$

the normalized y -distribution

$$\rho(y) = \frac{D \int \frac{d\sigma}{dx dy} dx}{\int_0^1 dy \int_D dx \frac{d\sigma}{dx dy}} \quad (4. 28)$$

must lie within the region bounded by $\rho = 1$, $\rho = 2(1-y)$ and $\rho = 3(1-y)^2$, a result due to Korthals Altes, Perrottet and de Rafael.⁽³⁷⁾ This region is shown in Fig. 3d. The existing data are not inconsistent with this constraint.

In summary, neutrino data show scaling behavior over a large energy region, starting as low as a few GeV. At the same time, there are hints of a transition in this behavior, especially in $\bar{\nu}$ data, at energies above 30 GeV. By and large, the existence of scaling (even if approximate) is the most important fact to have emerged from the study of high energy scattering, and is the principal inspiration of the parton model, discussed in the next chapter.

4. 3 Analysis of Charged Current Data

Experiments have so far studied inclusive ν and $\bar{\nu}$ cross sections off targets containing nearly equal numbers of protons and neutrons. In a first analysis of the data, it is convenient to assume charge symmetry for the current J_α , an assumption that is true in the conventional theory as long as $\Delta S = 1$ transitions are neglected (or corrected for). Then, in the scaling limit, we have

$$\frac{d\sigma^{\nu, \bar{\nu}N}}{dx dy} = \frac{G^2 ME}{\pi} \left[(1-y) F_2^{\nu N}(x) + \frac{1}{2} y^2 (2xF_1^{\nu N}) + y(1-\frac{1}{2}y)(xF_3^{\nu N}(x)) \right] \quad (4. 29)$$

where

$$F_i^{\nu N} \equiv \frac{1}{2} (F_i^{\nu p} + F_i^{\nu n}) \quad (4.30)$$

a. Neutrino/Antineutrino Comparison

Consider, first, the implications of the total cross section data σ^ν and $\sigma^{\bar{\nu}}$. Defining⁽³³⁾

$$A = \frac{\int 2xF_1^{\nu N} dx}{\int F_2^{\nu N} dx}, \quad B = - \frac{\int xF_3^{\nu N} dx}{\int F_2^{\nu N} dx} \quad (4.31)$$

we have, from (4.29)

$$\sigma^\nu, \bar{\sigma}^\nu = \frac{G^2 ME^2}{\pi} \left[\frac{1}{2} + \frac{1}{6} A \pm \frac{1}{3} B \right] \quad (4.32)$$

and

$$r \equiv \frac{\bar{\sigma}^\nu}{\sigma^\nu} = \frac{3+A-2B}{3+A+2B} \quad (4.33)$$

In the region $1 < E < 10$ GeV, this ratio has been measured to be 0.38 ± 0.02 , a result also consistent with measurements at higher energies. This constrains A and B to the limits

$$\begin{aligned} 1 \geq A \geq \frac{3(1-r)}{3r-1} &= 0.87 \pm 0.05 \\ 0.90 \pm 0.04 = \frac{2(1-r)}{1+r} \geq B \geq \frac{3(1-r)}{3r+1} &= 0.87 \pm 0.05 \end{aligned} \quad (4.34)$$

Remarkably, although A and B could, in general, have any values within the (positivity) domain $|B| \leq A \leq 1$, the data suggest the extreme values $A \approx B \approx 1$ (a consequence of the fact that r is close to 1/3). The result $A \approx 1$ is consistent with the theoretical relation

$$F_2(x) = 2xF_1(x) \quad (4.35)$$

suggested by Callan and Gross, a relation that appears to be fulfilled in inclusive electron scattering as well. This has encouraged the analysis of neutrino data under the assumption of (4.35) as an exact relation.

Eq. (4.29) then reduces to

$$\frac{d\sigma^{\nu, \bar{\nu}N}}{dx dy} = \frac{G^2 ME}{\pi} [F_2^{\nu N}(x) (1 - y + \frac{1}{2} y^2) + x F_3^{\nu N}(x) y (1 - \frac{1}{2} y)] \quad (4.36)$$

This result has two consequences. First, the y -distributions are now given by

$$\frac{d\sigma^{\nu, \bar{\nu}N}}{dy} = \frac{G^2 ME}{\pi} \left(\int_0^1 F_2^{\nu N}(x) dx \right) [1 - (1+B)y(1 - \frac{1}{2} y)] \quad (4.37)$$

where B is completely specified by Eq. (4.33) as a result of the assumption $A = 1 : B = 2(1-r)/(1+r) = 0.90 \pm 0.04$. A knowledge of the total cross section ratio r therefore fixes the y -distribution of ν and $\bar{\nu}$ scattering. The data are consistent with this expectation. Secondly, the x -distributions

$$\frac{d\sigma^{\nu, \bar{\nu}N}}{dx} = \frac{G^2 ME}{\pi} \left[\frac{2}{3} F_2^{\nu N}(x) + \frac{1}{3} x F_3^{\nu N}(x) \right] \quad (4.38)$$

now enable a separation of the structure functions $F_2^{\nu N}(x)$ and $x F_3^{\nu N}(x)$. These functions, as extracted from Gargamelle data, are shown in Fig. 4. (38) A knowledge of these structure functions permits a test of various hypotheses, some of which will be considered in Chapter 5.

Experiments at the higher energies of FNAL have found significant deviations from the pattern discussed above. These deviations arise partly from an apparent breakdown of scaling, which is one of the assumptions used in the above analysis. It is possible that a violation of charge symmetry also takes place, which would upset the relation between the ν and $\bar{\nu}$ distributions derived above. We refer to Ref. (36) for details of the experimental results, which are still a challenge to theoretical interpretation.

b. Neutrino/Electron Comparison

In the conventional theory, the $\Delta S = 0$ charged current belongs to the same isotriplet as the isovector part of the electromagnetic current.

As a consequence

$$[F_2^{\nu N}(x)]_V = 2[F_2^{eN}(x)]_{\text{isovector}} \quad (4.39)$$

where F_2^{eN} is the average of the electron scattering functions F_2^{ep} and F_2^{en} , and the subscript V indicates the vector part. In addition, there are theoretical suggestions that at high energies, the vector and axial vector parts of $F_2^{\nu N}(x)$ should be approximately equal, as a consequence of chiral symmetry. We therefore expect⁽³³⁾

$$F_2^{\nu N}(x) \approx 4[F_2^{eN}(x)]_{\text{isovector}} \quad (4.40)$$

Finally, experience with photoproduction suggests that the isoscalar contribution to $F_2^{eN}(x)$ should be only about 10%, which means that

$$F_2^{\nu N}(x) \approx 4 \times 0.9[F_2^{eN}(x)] = 3.6 F_2^{eN}(x) \quad (4.41)$$

(Amusingly, the coefficient $3.6 = \frac{18}{5}$ arises naturally in parton model considerations.) As can be seen in Fig. 4, this is strikingly well satisfied.

4.4 Analysis of Neutral Current Data

The simultaneous study of neutral current and charged current inclusive reactions can provide important constraints on the parameters of the neutral current. We shall assume for $J_\mu^{(N)}$ the general form given in (3.18), and consider $\nu, \bar{\nu}$ scattering from isoscalar matter (i.e. a target of equal proton and neutron numbers). Denoting by $\sigma_-, \sigma_+, \sigma_0, \bar{\sigma}_0$ the cross sections for the reactions $\nu \rightarrow \mu^-, \bar{\nu} \rightarrow \mu^+, \nu \rightarrow \nu, \bar{\nu} \rightarrow \bar{\nu}$, we have^(39,40)

$$\begin{aligned} \sigma_- &= A + V + I \\ \sigma_+ &= A + V - I \\ \sigma_0 &= \frac{1}{2} [G_A^{(3)^2} A + G_V^{(3)^2} V + G_A^{(3)} G_V^{(3)} I] + S \\ \bar{\sigma}_0 &= \frac{1}{2} [G_A^{(3)^2} A + G_V^{(3)^2} V - G_A^{(3)} G_V^{(3)} I] + \bar{S} \end{aligned} \quad (4.42)$$

Here A , V , and I are the axial vector, vector and interference parts of the inclusive cross section for $\nu \rightarrow \mu^-$ (we follow the notation of Ref. (39)) and S , $\bar{S}$ the contribution of the isoscalar neutral current to $\nu \rightarrow \nu$, $\bar{\nu} \rightarrow \bar{\nu}$. The essential feature is the absence of any interference terms between the isoscalar and isovector neutral currents, a consequence of the fact that the target is isoscalar and that all charge configurations of the final hadronic system are summed over. This observation is due to Pais and Treiman.⁽⁴⁰⁾ As was emphasized by Paschos and Wolfenstein,⁽³⁹⁾ the decomposition in (4.42) is also valid for cross sections to a selected class of final states, as long as all possible charge configurations are included. Bounds on the cross sections σ_0 and $\bar{\sigma}_0$ then follow from the observation that

$$\begin{aligned} S, \bar{S} &\geq 0 \\ I^2 &\leq 4AV \end{aligned} \quad (4.43)$$

We consider here some examples of such bounds. If we define the ratios

$$\begin{aligned} r &= \frac{\sigma_+}{\sigma_-}, \quad R = \frac{\sigma_0}{\sigma_-}, \quad \bar{R} = \frac{\bar{\sigma}_0}{\sigma_+} \\ \rho &= \frac{1+r - |1-r|}{1+r + |1-r|} \end{aligned} \quad (4.44)$$

the following lower bounds are satisfied by R and $\bar{R}$:

$$\begin{aligned} \underline{\text{For}} \quad |G_A| &\geq |G_V| \\ R &\geq \frac{1}{4} (1+r) [G_A^{(3)^2} - (G_A^{(3)})^2 - G_V^{(3)^2} \frac{(1+\sqrt{\rho})^2}{2(1+\rho)} + G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \\ \bar{R} &\geq \frac{1}{4} \frac{1+r}{r} [G_A^{(3)^2} - (G_A^{(3)})^2 - G_V^{(3)^2} \frac{(1+\sqrt{\rho})^2}{2(1+\rho)} - G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \end{aligned} \quad (4.45a)$$

$$\text{For } |G_A| \leq |G_V|$$

$$\begin{aligned} R &\geq \frac{1}{4} (1+r) [G_V^{(3)^2} - (G_V^{(3)^2} - G_A^{(3)^2}) \frac{(1+\sqrt{\rho})^2}{2(1+\rho)} + G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \\ \bar{R} &\geq \frac{1}{4} \frac{1+r}{r} [G_V^{(3)^2} - (G_V^{(3)^2} - G_A^{(3)^2}) \frac{(1+\sqrt{\rho})^2}{2(1+\rho)} - G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \end{aligned} \quad (4.45b)$$

If all four cross sections in Eq. (4.42) are known, the following general inequality should hold (a result due to Rajasekaran and Sarma):⁽⁴¹⁾

$$[2\Sigma_0 - G_A^{(3)^2} \Sigma - (G_V^{(3)^2} - G_A^{(3)^2}) V_m] \pm 2(2\Delta_0 - G_V^{(3)} G_A^{(3)} \Delta) \geq 0 \quad (4.46)$$

where

$$\begin{aligned} \Sigma &= \frac{1}{2} (\sigma_+ - \sigma_-), \quad \Sigma_0 = \frac{1}{2} (\sigma_0 + \bar{\sigma}_0), \quad \Delta = \frac{1}{2} (\sigma_+ - \sigma_-), \quad \Delta_0 = \frac{1}{2} (\sigma_0 - \bar{\sigma}_0) \\ \text{and } V_m &= \frac{1}{2} \Sigma [1 \pm \sqrt{1 - 4\Delta^2/\Sigma^2}], \text{ for } |G_A^{(3)}|^2 > |G_V^{(3)}|^2. \end{aligned} \quad (4.47)$$

Finally, in cases where $|G_A| \geq |G_V|$, bounds can be derived that make use of electroproduction data. A well-known result is⁽³⁹⁾

$$\begin{aligned} R &\geq \frac{G_A^2}{2} \left[1 - \left(1 - \frac{G_V}{G_A}\right) \left(\frac{v_{\sigma_-}^{\text{em}}}{\sigma_-}\right)^{1/2} \right]^2 \\ \bar{R} &\geq \frac{G_A^2}{2} \left[1 - \left(1 + \frac{G_V}{G_A}\right) \left(\frac{v_{\sigma_-}^{\text{em}}}{\sigma_-}\right)^{1/2} \right]^2 \end{aligned} \quad (4.48)$$

where

$$\frac{v_{\sigma_-}^{\text{em}}}{\sigma_-} = \frac{G^2}{\pi} \frac{1}{4\pi a^2} \frac{\int Q^4 (d\sigma/dQ^2 dv)_{\text{em}} dQ^2 dv}{\int (d\sigma/dQ^2 dv)_{\nu-\mu} dQ^2 dv} \quad (4.49)$$

Improvements in all of the above bounds are possible if the data are obtained in known regions of Q^2 and ν , since in this case one can replace the Schwartz inequality in (4.43) by a stronger form⁽³⁹⁾

$$\begin{aligned} I^2 &\leq K^2 A V, \\ K &= \frac{4(E+E')^2 (\nu^2 + Q^2)^{1/2}}{(E+E')^2 + \nu^2 + Q^2} \end{aligned} \quad (4.50)$$

Considerably more stringent constraints on $G_V^{(3)}$ and $G_A^{(3)}$ are obtained if we assume $V \approx A$ in Eq. (4.42). (As was seen in Sec. (4.3), this assumption is not inconsistent with the inclusive charged current data.) It then follows that

$$A : V : I = 1 : 1 : 2 \left(\frac{1-r}{1+r} \right)$$

where $r = \sigma_- / \sigma_+$, and we obtain from (4.42) the bounds

$$\begin{aligned} R &\geq \frac{1+r}{8} [G_V^{(3)^2} + G_A^{(3)^2} + 2G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \\ \bar{R} &\geq \frac{1+r}{8r} [G_V^{(3)^2} + G_A^{(3)^2} - 2G_V^{(3)} G_A^{(3)} \frac{1-r}{1+r}] \end{aligned} \quad (4.51)$$

It should be noted that these bounds do not depend on the assumption of scaling. (It is possible, however, that the assumption $V = A$ is valid only at energies where scaling also sets in.) We may therefore use the comparatively low energy data of the Gargamelle experiment to evaluate these bounds. The data are⁽⁴²⁾

$$\begin{aligned} R &= 0.22 \pm 0.03 \\ \bar{R} &= 0.43 \pm 0.12 \\ r &= 0.26 \pm 0.03 \end{aligned} \quad (4.52)$$

where all four cross sections in (4.42) are measured under the same kinematical conditions, which include a cut at $\nu = 1$ GeV. The resulting constraints on $G_V^{(3)}$ and $G_A^{(3)}$ are shown in Fig. 5.

At higher energies, the inclusive data show scaling behavior and are also consistent with a ratio $r \approx 1/3$ for the total inclusive cross sections. Under such conditions, the assumption $V = A$ implies $A : V : I = 1 : 1 : 1$, and we obtain for the ratios R and $\bar{R}$ the simple bounds

$$\begin{aligned}
 R &\geq \frac{1}{6} [G_A^{(3)^2} + G_V^{(3)^2} + G_A^{(3)} G_V^{(3)}] \\
 \bar{R} &\geq \frac{1}{2} [G_A^{(3)^2} + G_V^{(3)^2} - G_A^{(3)} G_V^{(3)}]
 \end{aligned}
 \tag{4.53}$$

Measurements of R and $\bar{R}$ at the higher energies are broadly consistent with (4.52), and yield constraints on $G_A^{(3)}$ and $G_V^{(3)}$ similar to those in Fig. 5.

4.5 Semi-Inclusive Pion Production

Reactions of the type

$$\begin{aligned}
 \nu + N &\rightarrow \mu^- + h + \text{anything} \\
 \nu + N &\rightarrow \nu + h + \text{anything}
 \end{aligned}
 \tag{4.54}$$

where h is a hadron, are characterized by a cross section that depends on the variables x and y introduced in Eq. (4.19) as well as variables pertaining to h . For a detailed discussion of the kinematics, we refer to Ref. (43). A simplified description based on the parton model will be given in the next chapter.

We shall obtain here some consequences that follow purely from isospin arguments. Consider the case when the target N is isoscalar (i. e. an average nucleon or an $I = 0$ nucleus) and h is a pion. For the charged current reactions in (4.43), the current is an isovector and the final hadronic state $|F\rangle$ has the quantum numbers $I = 1$, $I_3 = 1$. This implies that the unobserved system X accompanying the observed pion has $I = 0, 1$ or 2 , so that the cross section will be a sum of three positive contributions, one for each value of I . Expanding the state $|F\rangle$ in terms of the product states $|\pi\rangle |X; I(X), I_3(X)\rangle$, we have

$$\begin{aligned}
 |F\rangle = & |\pi^+\rangle \left(|0,0\rangle + \sqrt{\frac{1}{2}} |1,0\rangle + \sqrt{\frac{1}{10}} |2,0\rangle \right) \\
 & + |\pi^0\rangle \left(-\sqrt{\frac{1}{2}} |1,1\rangle + \sqrt{\frac{3}{10}} |2,1\rangle \right) \\
 & + |\pi^-\rangle \left(\sqrt{\frac{3}{5}} |2,2\rangle \right)
 \end{aligned}
 \tag{4.55}$$

Denoting by $\sigma_{-}^{\pi^i}$ the cross section for producing a π^i in the reaction $\nu - \mu^{-}$, we thus have

$$\begin{aligned}\sigma_{-}^{\pi^{+}} &= U_{-}^{(0)} + \frac{1}{2} U_{-}^{(1)} + \frac{1}{10} U_{-}^{(2)} \\ \sigma_{-}^{\pi^{0}} &= \frac{1}{2} U_{-}^{(1)} + \frac{3}{10} U_{-}^{(2)} \\ \sigma_{-}^{\pi^{-}} &= \frac{3}{5} U_{-}^{(2)}\end{aligned}\quad (4.56)$$

where $U_{-}^{(I)}$ is the cross section for producing X with isospin I. In the case of the neutral current reactions, the current may be an isoscalar-isovector mixture so that the final hadronic system can be $|F'\rangle = |I=1, I_3=0\rangle$ or $|F''\rangle = |I=0, I_3=0\rangle$. Expanding as before, we have

$$\begin{aligned}|F'\rangle &= |\pi^{+}\rangle \left(-\frac{\sqrt{1}}{2} |1, -1\rangle + \sqrt{\frac{3}{10}} |2, -1\rangle\right) \\ &\quad + |\pi^{0}\rangle \left(|0, 0\rangle - \sqrt{\frac{2}{5}} |2, 0\rangle\right) \\ &\quad + |\pi^{-}\rangle \left(-\frac{\sqrt{1}}{2} |1, 1\rangle + \sqrt{\frac{3}{10}} |2, 1\rangle\right) \\ |F''\rangle &= \sqrt{\frac{1}{3}} (|\pi^{+}\rangle |1, -1\rangle - |\pi^{0}\rangle |1, 0\rangle + |\pi^{-}\rangle |1, 1\rangle)\end{aligned}\quad (4.57)$$

and the cross sections $\sigma_0^{\pi^i}$ for producing a π^i in $\nu - \nu$ are

$$\begin{aligned}\sigma_0^{\pi^{\pm}} &= \left[\frac{1}{3} S_0^{(1)} \pm 2\sqrt{\frac{1}{6}} M_0^{(1)} + \frac{1}{2} U_0^{(1)}\right] + \frac{3}{10} U_0^{(2)} \\ \sigma_0^{\pi^0} &= \frac{1}{3} S_0^{(1)} + U_0^{(0)} + \frac{2}{5} U_0^{(2)}.\end{aligned}\quad (4.58)$$

Note that a state X with $I=1$ can now be produced by the isovector as well as isoscalar currents (terms $U_0^{(1)}$ and $S_0^{(1)}$) and that there will be an interference term (labelled $M_0^{(1)}$). (The results (4.56) and (4.58) are due to Albright et al. ⁽⁴⁴⁾) Note especially that any difference between π^{+} and π^{-} cross sections in the neutral current reactions can arise only from isoscalar-isovector interference and would be evidence for a current with mixed isospin properties. (One can show, similarly, that if the detected hadron h is a

nucleon, any asymmetry between proton and neutron production is indicative of an isospin mixture in the neutral current.) A pure isoscalar current would give $\sigma_0^{\pi^+} = \sigma_0^{\pi^-} = \sigma_0^{\pi^0}$.

The above analysis also helps to derive bounds on the ratio of π production by neutral and charged currents. For theories characterized by $|G_A^{(3)}| \geq |G_V^{(3)}|$, such bounds were derived in Ref. (44). We quote here two of the interesting results.

$$R_3 \equiv \frac{\sigma_0^{\text{ch}}}{2\sigma_-^{\text{ch}}} \geq \frac{1}{4} \left[r^{-1/2} G_A^{(3)} - (G_A^{(3)} - G_V^{(3)}) \left(\frac{V_{\text{em}}^{\text{ch}}}{2\sigma_-^{\text{ch}}} \right)^{1/2} \right]^2 \quad (4.59)$$

$$R_4 \equiv \frac{\sigma_0^{\pi^0}}{2\sigma_-^{\pi^0}} \geq \frac{1}{4} \left[(r-1)^{1/2} G_A^{(3)} - (G_A^{(3)} - G_V^{(3)}) \left(\frac{V_{\text{em}}^{\pi^0}}{\sigma_-^{\pi^0}} \right)^{1/2} \right]^2$$

where

$$\sigma^{\text{ch}} = \sigma^{\pi^+} + \sigma^{\pi^-}$$

$$r \equiv \sigma_-^{\text{ch}} / \sigma_-^{\pi^0} \quad (4.60)$$

and

$$V_{\text{em}}^i \equiv \frac{G^2}{\pi} \frac{1}{4\pi\alpha} \int q^4 \left(\frac{d\sigma^i}{dq^2} \right)_{\text{em}} dq^2$$

These bounds can be evaluated provided measurements of electroproduction and neutrino-production are available in identical kinematical conditions. (The restriction to $|G_A^{(3)}| > |G_V^{(3)}|$ must, however, be kept in mind.)

5. NEUTRINOS AND PARTONS

The parton model was originally invented⁽⁴⁵⁾ to provide an intuitive basis for scaling behavior. It has since then acquired a power far transcending its original purpose and, has become one of the most pervasive and successful concepts of high energy physics.

In its most rudimentary form, the parton model reduces the calculation of deep inelastic reactions to the fine art of counting. (For many purposes, it is a distinct advantage if one can count only to three.) Its simplicity and its success are awesome when one realizes that the theoretical basis of the model is only vaguely defined and perhaps not even consistent.

It is likely that the model, in the form that we shall use it, is only an approximation. What is undeniable is that it is very useful: an aid to thinking, and an effective medium for dialogue between theory and experiment.

5.1 The Parton Hypothesis

The scaling phenomenon discussed in the last chapter is suggestive of scattering by a point-like target, and is the principal motivation for the parton model. Thus, the linearly rising inclusive cross sections for neutrino scattering are reminiscent of neutrino scattering by an electron (Sec. 2.1). The parton hypothesis is essentially the assumption (i) that a hadron is a bound state of elementary constituents (partons) and (ii) that in the kinematical conditions known as the Bjorken limit, inelastic cross sections are the incoherent sum of scatterings by partons. The first of these assumptions may be accepted if one believes in a field theory basis for the strong interactions. The partons may then be identified with the quanta of the bare fields. The second assumption requires, for its validity, that the interaction among the partons be of a special nature: in a hadron at rest, it must limit the parton momenta to finite values.

With these assumptions, it is possible to picture a fast moving proton with momentum $p = (P + 0 (M^2/P), 0, 0, P)$, $P \rightarrow \infty$, as a collection of free partons, which (apart from limited transverse momenta) possess momentum vectors $\xi_i p$, with $0 < \xi_i < 1$ and $\sum_i \xi_i = 1$.

To describe deep inelastic scattering, we consider a frame in which the proton momentum is $p = (P + 0(M^2/P), 0, 0, P)$, $P \rightarrow \infty$, while the momentum transfer is $q = (0, 0, 0, -2Px)$, x being the Bjorken scaling variable. (Note that these vectors satisfy $Q^2 \rightarrow \infty$, $Q^2/2p \cdot q = x$, when $P \rightarrow \infty$.) Energy-momentum conservation dictates that q will be absorbed by a parton whose four-momentum is $xp \approx (xP, 0, 0, xP)$. If the cross section for this elementary process is denoted by $(d\sigma/dx dy)_0$, and $f(x)$ is the average number of constituents with a fractional momentum x , the total inelastic cross section is given by the prescription

$$\frac{d\sigma}{dx dy} = f(x) \left(\frac{d\sigma}{dx dy} \right)_0 \quad (5.1)$$

When several types of partons j are present, we have instead

$$\frac{d\sigma}{dx dy} = \sum_j f_j(x) \left(\frac{d\sigma}{dx dy} \right)_0^j \quad (5.2)$$

5.2 Neutrino Cross Sections in the Quark Parton Model

The most prevalent and successful version of the parton model is one in which the partons which interact with neutrinos (or electrons) are identified with the fractionally charged quarks p , n , λ and their antiparticles $\bar{p}$, $\bar{n}$, $\bar{\lambda}$. In this case all deep inelastic cross sections can be expressed in terms of six quark densities $f_j(x)$. We shall denote by $u(x)$, $d(x)$, $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$ the average density of p , n , λ , $\bar{p}$, $\bar{n}$, $\bar{\lambda}$ quarks in a proton, possessing a fraction x of the total proton momentum. The corresponding densities in a neutron are then given by $u(x) \rightarrow d(x)$, $\bar{u}(x) \rightarrow \bar{d}(x)$.

To calculate cross sections, we need only to specify the interaction in terms of the elementary quarks. For the charged current process, the conventional Lagrangian is

$$\mathcal{L}^{CC} = - \frac{G}{\sqrt{2}} (\bar{\nu} \gamma_\alpha (1 + \gamma_5) \mu) (\bar{n}_C \gamma_\alpha (1 + \gamma_5) p) + \text{h. c.} \quad (5.3)$$

where $n_C = \cos \theta_C n + \sin \theta_C \lambda$. For the neutral current reactions, the interaction may be written as

$$\begin{aligned} \mathcal{L}^{NC} = & -\frac{G}{\sqrt{2}} (\nu \gamma_\alpha (1 + \gamma_5) \nu) [\bar{p} \gamma_\alpha (C_V + C_A \gamma_5) p + \bar{n} \gamma_\alpha (C'_V + C'_A \gamma_5) n \\ & + \bar{\lambda} \gamma_\alpha (C''_V + C''_A \gamma_5) \lambda] . \end{aligned} \quad (5.4)$$

(The six coupling constants appearing here are related to the six constants $G_V^{(i)}$, $G_A^{(i)}$, $i = 0, 3, 8$ defined in Eq. (3. 18) by: $C_V = \frac{1}{3} G_V^{(0)} + \frac{1}{2} G_V^{(3)} + \frac{1}{2\sqrt{3}} G_V^{(8)}$, $C'_V = \frac{1}{3} G_V^{(0)} - \frac{1}{2} G_V^{(3)} + \frac{1}{2\sqrt{3}} G_V^{(8)}$, $C''_V = \frac{1}{3} G_V^{(0)} - \frac{1}{\sqrt{3}} G_V^{(8)}$, and similar relations with $V \rightarrow A$.) Finally, for completeness, we write the "effective" electromagnetic interaction of the quarks with electrons as

$$\mathcal{L}^{EM} = \frac{e^2}{Q} (\bar{e} \gamma_\alpha e) \left[\frac{2}{3} \bar{p} \gamma_\alpha p - \frac{1}{3} \bar{n} \gamma_\alpha n - \frac{1}{3} \bar{\lambda} \gamma_\alpha \lambda \right] . \quad (5.5)$$

The elementary processes induced by these interactions and the corresponding cross sections $(\frac{d\sigma}{dx dy})_0$ are tabulated below. (Note that for each process listed in Table 4, there is a mirror process (every particle replaced by its antiparticle) with an equal cross section.)

Table 4

Elementary Process	Elementary Cross Section ($\frac{d\sigma}{dx dy}$) ₀
CC	$\nu n \rightarrow \mu^- p$
	$(G^2 ME/\pi) 2x \cos^2 \theta_c$
	$\bar{\nu} p \rightarrow \mu^+ n$
	$(G^2 ME/\pi) 2x (1-y)^2 \cos^2 \theta_c$
	$\nu \lambda \rightarrow \mu^- p$
	$(G^2 ME/\pi) 2x \sin^2 \theta_c$
	$\bar{\nu} p \rightarrow \mu^+ \lambda$
	$(G^2 ME/\pi) 2x (1-y)^2 \sin^2 \theta_c$
etc.	
etc.	
NC	$\nu p \rightarrow \nu p$
	$(G^2 ME/\pi) 2x [\frac{1}{4}(C_V + C_A)^2 + \frac{1}{4}(C_V - C_A)^2 (1-y)^2]$
	$\bar{\nu} p \rightarrow \bar{\nu} p$
	$(G^2 ME/\pi) 2x [\frac{1}{4}(C_V + C_A)^2 (1-y)^2 + \frac{1}{4}(C_V - C_A)^2]$
etc.	
etc.	

Table 4 Continued

	ep → ep	$(\frac{4\pi a^2}{q^4} \text{ ME}) \frac{4}{9} \times [1 + (1-y)^2]$
EM	$\bar{e}p \rightarrow \bar{e}p$	$(\frac{4\pi a^2}{q^4} \text{ ME}) \frac{4}{9} \times [1 + (1-y)^2]$
	etc.	etc.

The above table allows us to write down the inclusive cross section for any reaction. We give below a few results that are valid for a target with equal proton and neutron numbers.

$$\text{CC:} \quad \frac{d\sigma^{\nu N}}{dx dy} = \frac{G^2 \text{ME}}{\pi} \times [(u+d) + (\bar{u} + \bar{d})(1-y)^2] \quad (5.6)$$

$$\frac{d\sigma^{\bar{\nu} N}}{dx dy} = \frac{G^2 \text{ME}}{\pi} \times [(u+d)(1-y)^2 + (\bar{u} + \bar{d})] \quad (5.7)$$

$$\text{NC:} \quad \frac{d\sigma^{(N)\nu N}}{dx dy} = \frac{G^2 \text{ME}}{\pi} \times [(u+d)(f+f') + (\bar{u} + \bar{d})(\bar{f} + \bar{f}') + 2sf'' + 2\bar{s}\bar{f}''] \quad (5.8)$$

$$\frac{d\sigma^{(N)\bar{\nu} N}}{dx dy} = \frac{G^2 \text{ME}}{\pi} \times [(u+d)(\bar{f} + \bar{f}') + (\bar{u} + \bar{d})(f+f') + 2s\bar{f}'' + 2\bar{s}f''] \quad (5.9)$$

(In Eq. (5.6) and Eq. (5.7), we have neglected $\sin^2 \theta_c$. The results are exact if $d\sigma/dx dy$ is interpreted as the cross section into $S = 0$ final states and G is replaced by $G \cos \theta_c$.) The f 's appearing in (5.8) and (5.9) are functions of y , a typical pair of which is

$$\begin{aligned} f(y) &= \frac{1}{4} (C_V + C_A)^2 + \frac{1}{4} (C_V - C_A)^2 (1-y)^2 \\ \bar{f}(y) &= \frac{1}{4} (C_V + C_A)^2 (1-y)^2 + \frac{1}{4} (C_V - C_A)^2 \end{aligned} \quad (5.10)$$

Finally, we quote the cross section for electron scattering from an average nucleon.

$$\text{EM: } \frac{d\sigma^{eN}}{dx dy} = \left(\frac{4\pi\alpha^2}{Q^4} \text{ME} \right) (1 + (1-y)^2) x \left[\frac{5}{18} (u+d+\bar{u}+\bar{d}) + \frac{1}{9} (s+\bar{s}) \right] \quad (5.11)$$

(Note that unlike in neutrino scattering, the x and y dependence in electron scattering factorizes.) Some immediate consequences of Table 4 and Eqs.

(5.6) - (5.11) are:

$$\begin{aligned} \text{(i)} \quad & \frac{1}{3} \leq \frac{d\sigma^{\nu N}}{dx} / \frac{d\sigma^{\bar{\nu} N}}{dx} \leq 3, \quad 0 < x < 1 \\ \text{(ii)} \quad & \frac{1}{3} \leq \frac{d\sigma^{(N)\nu N}}{dx} / \frac{d\sigma^{(N)\bar{\nu} N}}{dx} \leq 3, \quad 0 < x < 1 \\ \text{(iii)} \quad & \frac{1}{4} \leq \frac{d\sigma^{ep}}{dx dy} / \frac{d\sigma^{en}}{dx dy} \leq 4, \quad 0 < x < 1 \\ & \quad \quad \quad 0 < y < 1 \end{aligned} \quad (5.12)$$

(In (i) and (ii), $\frac{d\sigma}{dx} \equiv \int_0^1 dy \frac{d\sigma}{dx dy}$.) Further consequences will be obtained in the following section.

5.3 Structure Function Relations and Sum Rules

All neutrino cross sections in the quark parton model have the structure

$$\frac{d\sigma}{dx dy} = \frac{G^2 \text{ME}}{\pi} [A(x) + B(x) (1-y)^2], \quad A, B \geq 0 \quad (5.13)$$

On the other hand, the standard parameterization in terms of structure functions is

$$\begin{aligned} \frac{d\sigma^{\nu, \bar{\nu}}}{dx dy} &= \frac{G^2 \text{ME}}{\pi} \left[(1-y) F_2^{\nu, \bar{\nu}}(x) + \frac{1}{2} y^2 (2xF_1^{\nu, \bar{\nu}}(x)) + y(1 - \frac{1}{2}y)(xF_3^{\nu, \bar{\nu}}(x)) \right] \\ &= \frac{G^2 \text{ME}}{\pi} \left[\frac{1}{2} (2xF_1^{\nu, \nu}(x) + xF_3^{\nu, \bar{\nu}}(x)) + (1-y)(F_2^{\nu, \bar{\nu}}(x) - 2xF_1^{\nu, \bar{\nu}}(x)) \right. \\ & \quad \left. + \frac{1}{2} (1-y)^2 (2xF_1^{\nu, \bar{\nu}}(x) \pm xF_3^{\nu, \bar{\nu}}(x)) \right] \end{aligned} \quad (5.14)$$

Comparison of (5.13) and (5.14) shows that

$$\begin{aligned} A(x) &= \frac{1}{2} [2xF_1^{\nu, \bar{\nu}}(x) + xF_3^{\nu, \bar{\nu}}(x)] \geq 0 \\ B(x) &= \frac{1}{2} [2xF_1^{\nu, \bar{\nu}}(x) \pm xF_3^{\nu, \bar{\nu}}(x)] \geq 0 \\ F_2^{\nu, \bar{\nu}}(x) - 2xF_1^{\nu, \bar{\nu}}(x) &= 0 \end{aligned} \quad (5.15)$$

The last of these equations is the Callan-Gross relation,⁽⁴⁶⁾ the origin of which is the spin- $\frac{1}{2}$ nature of the partons in the quark-parton model. Partons of any other spin, interacting with neutrinos, would contribute a term proportional to $(1-y)$ in Eq. (5.13) and lead to $F_2(x) \geq 2xF_1(x)$. Further, the first two equations in Eq. (5.15) yield the positivity condition

$$F_2(x) \geq |xF_3(x)| \quad (5.16)$$

already noted in Eq. (4.27). Notice that since $A(x)$ and $B(x)$ are proportional to a combination of parton densities (as seen in the preceding section), so are the structure functions

$$\begin{aligned} F_2(x) &= A(x) + B(x) \\ xF_3(x) &= A(x) - B(x) \end{aligned} \quad (5.17)$$

Thus constraints on the parton densities imply corresponding constraints on the structure functions of different neutrino processes.

The parton densities are subject to the conditions

$$\begin{aligned} \int (u(x) - \bar{u}(x)) dx &= 2 \\ \int (d(x) - \bar{d}(x)) dx &= 1 \\ \int (s(x) - \bar{s}(x)) dx &= 0 \end{aligned} \quad (5.18)$$

expressing the fact that the proton has quantum numbers $Q = 1$, $Y = 1$, $B = 1$. These constraints may be translated into relations among structure functions by reference to Table 5, which lists the independent functions $F_2(x)$ and $xF_3(x)$ for the conventional reactions of ν , $\bar{\nu}$ and e scattering by protons and neutrons.

Table 5

Reaction	$F_2(x)$	$-xF_3(x)$
$\nu p \rightarrow \mu^- + \text{anything } (\Delta S = 0)$	$2x(d + \bar{u}) \cos^2 \theta_c$	$2x(d - \bar{u}) \cos^2 \theta_c$
$\bar{\nu} n \rightarrow \mu^+ + \text{anything } (\Delta S = 0)$	$2x(d + \bar{u}) \cos^2 \theta_c$	$2x(d - \bar{u}) \cos^2 \theta_c$
$\nu n \rightarrow \mu^- + \text{anything } (\Delta S = 0)$	$2x(u + \bar{d}) \cos^2 \theta_c$	$2x(u - \bar{d}) \cos^2 \theta_c$
$\bar{\nu} p \rightarrow \mu^+ + \text{anything } (\Delta S = 0)$	$2x(u + \bar{d}) \cos^2 \theta_c$	$2x(u - \bar{d}) \cos^2 \theta_c$
$\nu p \rightarrow \mu^- + \text{anything } (\Delta S = 1)$	$2x(s + \bar{u}) \sin^2 \theta_c$	$2x(s - \bar{u}) \sin^2 \theta_c$
$\bar{\nu} p \rightarrow \mu^+ + \text{anything } (\Delta S = 1)$	$2x(\bar{s} + u) \sin^2 \theta_c$	$-2x(\bar{s} - u) \sin^2 \theta_c$
$\nu n \rightarrow \mu^- + \text{anything } (\Delta S = 1)$	$2x(s + \bar{d}) \sin^2 \theta_c$	$2x(s - \bar{d}) \sin^2 \theta_c$
$\bar{\nu} n \rightarrow \mu^+ + \text{anything } (\Delta S = 1)$	$2x(\bar{s} + d) \sin^2 \theta_c$	$-2x(\bar{s} - d) \sin^2 \theta_c$
$ep \rightarrow e + \text{anything}$	$x[\frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s})]$	0
$en \rightarrow e + \text{anything}$	$x[4/9(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s})]$	0

Some important sum rules emerging from Table 5 and Eq. (5.18) are the following:

$$\begin{aligned}
 \text{(i)} \quad \int \frac{dx}{x} (F_2^{\bar{\nu}p} - F_2^{\nu p})_{\Delta S=0} &= \int \frac{dx}{x} (F_2^{\nu n} - F_2^{\bar{\nu}n})_{\Delta S=0} \\
 &= \int \frac{dx}{x} (F_2^{\bar{\nu}p} - F_2^{\bar{\nu}n})_{\Delta S=0} = \int \frac{dx}{x} (F_2^{\nu n} - F_2^{\bar{\nu}n})_{\Delta S=0} = 2 \cos^2 \theta_c
 \end{aligned} \quad (5.19)$$

$$\text{(ii)} \quad \int \frac{dx}{x} (F_2^{\bar{\nu}p} - F_2^{\nu p})_{\Delta S=1} = 2 \int \frac{dx}{x} (F_2^{\bar{\nu}n} - F_2^{\nu n})_{\Delta S=1} = 4 \sin^2 \theta_c \quad (5.20)$$

$$\begin{aligned}
 \text{(iii)} \quad \int \frac{dx}{x} (F_2^{\bar{\nu}p} - F_2^{\nu p})_{\text{total}} &= 2(2 - \cos^2 \theta_c) \\
 \int \frac{dx}{x} (F_2^{\bar{\nu}n} - F_2^{\nu n})_{\text{total}} &= 2(1 - \cos^2 \theta_c)
 \end{aligned} \quad (5.21)$$

$$(iv) \quad \int dx (F_3^{\nu P} + F_3^{\nu n})_{\Delta S=0} = \int dx (F_3^{\bar{\nu} P} + F_3^{\bar{\nu} n})_{\Delta S=0} = 0 \quad (5.22)$$

$$= \int dx (F_3^{\nu P} + F_3^{\bar{\nu} P})_{\Delta S=0} = \int dx (F_3^{\bar{\nu} P} + F_3^{\nu P})_{\Delta S=0} = -6 \cos^2 \theta_c$$

$$(v) \quad \int dx (F_3^{\nu P} + F_3^{\bar{\nu} P})_{\Delta S=1} = 2 \int dx (F_3^{\nu n} + F_3^{\bar{\nu} n})_{\Delta S=1} = -4 \sin^2 \theta_c \quad (5.23)$$

$$(vi) \quad \int dx (F_3^{\nu P} + F_3^{\bar{\nu} P})_{\text{total}} = -2(2 + \cos^2 \theta_c) \quad (5.24)$$

$$\int dx (F_3^{\nu n} + F_3^{\bar{\nu} n})_{\text{total}} = -2(1 + 2 \cos^2 \theta_c) .$$

The results (i) - (iii) are special cases of the Adler sum rule,⁽⁴⁷⁾ which may be stated as follows: If $F_2^{\nu T}$, $F_2^{\bar{\nu} T}$ are the structure functions for ν , $\bar{\nu}$ scattering by a target T, then

$$\int \frac{dx}{x} (F_2^{\bar{\nu} T} - F_2^{\nu T}) = 2 < T | [Q^+, Q^-] | T > . \quad (5.25)$$

Here $Q^\pm$ are the "charges" associated with the charged current $J_a^{(C)}$, that is,

$$Q^+ = \int J_0^{(C)+} (x_0=0, \vec{x}) d^3x \quad (5.26)$$

$$Q^- = (Q^+)^+ .$$

The results (5.19) - (5.21) follow when one recognizes that in the conventional theory,

$$Q^\pm = \cos \theta_c (I_1 \pm iI_2) + \sin \theta_c (V_1 \pm iV_2) , \quad (5.27)$$

$$[Q^+, Q^-] = \cos^2 \theta_c (2I_3) + \sin^2 \theta_c (\frac{3}{2}Y + I_3) .$$

The remaining results (iv) - (vi) are various forms of the Gross-Llewellyn Smith sum rule.⁽⁴⁸⁾ They essentially express the fact that the net number of constituents (particles minus antiparticles) in a nucleon is three, and are therefore tests of the quark model with fractional quantum numbers. (For a preliminary evaluation of the Adler and Gross-Llewellyn Smith sum rules using experimental data, see Ref. (38).)

In addition to the sum rules (5. 19) - (5. 24), a number of local relations among structure functions may be derived from Table 5. Some of the interesting results are:

$$(vii) \quad (F_2^{\nu P} + F_2^{\nu n})_{\Delta S=0} \leq \frac{18}{5} (F_2^{ep} + F_2^{en}) \cos^2 \theta_c \quad (5. 27)$$

$$(viii) \quad x(F_3^{\nu P} - F_3^{\nu n})_{\Delta S=0} = 6 (F_2^{ep} - F_2^{en}) \cos^2 \theta_c \quad (5. 28)$$

$$(ix) \quad [(F_2^{\nu P} + F_2^{\bar{\nu} P}) - (F_2^{\nu n} + F_2^{\bar{\nu} n})]_{\Delta S=1} = 6 (F_2^{ep} - F_2^{en}) \sin^2 \theta_c \quad (5. 29)$$

$$(x) \quad x[(F_3^{\nu P} - F_3^{\nu n}) - (F_3^{\bar{\nu} P} - F_3^{\bar{\nu} n})]_{\Delta S=1} = 6 (F_2^{ep} - F_2^{en}) \sin^2 \theta_c \quad (5. 30)$$

Of these, the result (5. 27) appears to be approximately satisfied with the equality sign (see Fig. 4 and Eq. (4. 41)).

5. 4 Momentum Composition of the Proton

Measurements of deep inelastic neutrino and electron scattering, when interpreted in terms of the parton model, provide insight into the manner in which the momentum of a fast moving nucleon is shared among its constituents. Consider, for example, the cross sections $\frac{d\sigma}{dx}$ for $\nu, \bar{\nu}$ scattering from a proton target. The parton model gives

$$(\frac{d\sigma^{\nu P}}{dx})_{\Delta S=0} = \frac{G^2 ME}{\pi} 2x (d(x) + \frac{1}{3} \bar{u}(x)) \cos^2 \theta_c \quad (5. 31a)$$

$$(\frac{d\sigma^{\nu P}}{dx})_{\Delta S=1} = \frac{G^2 ME}{\pi} 2x (\bar{s}(x) + \frac{1}{3} \bar{u}(x)) \sin^2 \theta_c \quad (5. 31b)$$

$$(\frac{d\sigma^{\bar{\nu} P}}{dx})_{\Delta S=0} = \frac{G^2 ME}{\pi} 2x (\bar{d}(x) + \frac{1}{3} u(x)) \cos^2 \theta_c \quad (5. 31c)$$

$$(\frac{d\sigma^{\bar{\nu} P}}{dx})_{\Delta S=1} = \frac{G^2 ME}{\pi} 2x (\bar{s}(x) + \frac{1}{3} u(x)) \sin^2 \theta_c \quad (5. 31d)$$

Similarly, for an average nucleon target $N = \frac{1}{2} (p + n)$, we have (c. f. Eqs. (5. 6)

and (5. 7))

$$\left(\frac{d\sigma}{dx}\right)_{\Delta S=0}^{\nu N} = \frac{G^2 ME}{\pi} \times [u(x) + d(x) + \frac{1}{3}(\bar{u}(x) + \bar{d}(x))] \cos^2 \theta_c \quad (5. 31e)$$

$$\left(\frac{d\sigma}{dx}\right)_{\Delta S=0}^{\bar{\nu} N} = \frac{G^2 ME}{\pi} \times [\bar{u}(x) + \bar{d}(x) + \frac{1}{3}(u(x) + d(x))] \cos^2 \theta_c \quad (5. 31f)$$

The above six differential cross sections suffice, in principle, to determine the six parton densities $u, \bar{u}, d, \bar{d}, s, \bar{s}$ that describe the distribution of momentum within the proton.

Existing measurements are not sufficiently detailed to determine the parton densities completely, but, nevertheless, provide interesting information. If we denote by $\sigma, \bar{\sigma}$ the integrals of Eqs. (5. 31e), (5. 31f), and express cross sections in units $G^2 ME/\pi$, we have (neglecting θ_c)⁽⁴⁹⁾

$$\int x(u + d) dx = \frac{3}{8} (3\sigma - \bar{\sigma}) \quad (5. 32)$$

$$\int x(\bar{u} + \bar{d}) dx = \frac{3}{8} (3\bar{\sigma} - \sigma) \quad (5. 33)$$

Experimentally, ^(33, 35) $\sigma = 0.50 \pm 0.05$, $\bar{\sigma} = 0.19 \pm 0.02$, giving

$$\int x(u + d) dx = 0.49 \pm 0.06 \quad (5. 34)$$

$$\int x(\bar{u} + \bar{d}) dx = 0.02 \pm 0.03 \quad (5. 35)$$

Remarkably, the total momentum carried by the antiquarks $\bar{p}, \bar{n}$ is small compared to that carried by the quarks p, n , a reflection of the fact that $\bar{\sigma}/\sigma$ is close to 1/3. Information on the momentum carried by the strange quarks $\lambda, \bar{\lambda}$ is obtained by combining σ and $\bar{\sigma}$ with the electron structure functions F_2^{ep} and F_2^{en} . We find, by reference to Table 5,

$$\int x(s + \bar{s}) dx = \frac{9}{2} \left[\int (F_2^{ep} + F_2^{en}) dx - \frac{5}{12} (\sigma + \bar{\sigma}) \right] \quad (5. 36)$$

Experimentally, $\int F_2^{ep} dx = 0.16 \pm 0.02$, $\int F_2^{en} dx = 0.12 \pm 0.02$, so that

$$\int x(s + \bar{s}) dx = \frac{9}{2} (0.01 \pm 0.04) \quad , \quad (5. 37)$$

which is compatible with being small, albeit with large uncertainty.

If we picture the proton as a set of "valence" quarks together with an indefinite number of quark-antiquark pairs, the above analysis suggests that the valence quarks carry a much larger share of the proton's momentum than the sea of pairs. The existence of $q\bar{q}$ pairs is, of course, unavoidable, and we can only surmise that such pairs occur predominantly at small x values. This is borne out by measurements of $d\sigma^{\nu N}/dx$ and $d\sigma^{\bar{\nu} N}/dx$, (38) which yield the distributions $x(u+d)$ and $x(\bar{u}+\bar{d})$ shown in Fig. 6. It is clear from this figure that $x(\bar{u}+\bar{d})$ is insignificant for $x > 0.2$, and one may reasonably suppose that the same will be true of the strange parton distribution $x(s+\bar{s})$. Excluding the region $x \leq 0.2$, therefore, all cross sections in the parton model are determined by just two density functions: $u(x)$ and $d(x)$. The approximation of retaining only the functions $u(x)$ and $d(x)$ in parton model calculations has been termed "valence quark dominance,"⁽⁵⁰⁾ and provides a remarkably simple guideline for many purposes. A typical consequence of this approximation is the total cross section ratio

$$\sigma^{\nu n} : \sigma^{\nu p} : \sigma^{\bar{\nu} p} : \sigma^{\bar{\nu} n} = 1 : \eta : \frac{1}{3} : \frac{1}{3} \eta \quad (5.38)$$

where $\eta = \int x d(x) dx / \int x u(x) dx = \int (4F_2^{\text{en}} - F_2^{\text{ep}}) dx / \int (4F_2^{\text{ep}} - F_2^{\text{en}}) dx = 0.6 \pm 0.1$.

An empirical feature of great interest is that $d(x)/u(x) \rightarrow 0$ as $x \rightarrow 1$, suggesting that the "leading" quark in a proton is a p-quark. This has the implication that $d\sigma^{\nu p}/dx/d\sigma^{\nu n}/dx \rightarrow 0$ as $x \rightarrow 1$, but $d\sigma^{\bar{\nu} p}/dx/d\sigma^{\bar{\nu} n}/dx \rightarrow \infty$ in the same limit. An experimental check of this prediction would be of great interest as a test of the parton model.

Finally, we observe that Eqs. (5.34), (5.35), and (5.37) together give

$$\int x(u + d + \bar{u} + \bar{d} + s + \bar{s}) dx \approx 0.5 \quad (5.39)$$

indicating that quarks and antiquarks account for only about 50% of the momentum of a proton. The remainder must be attributed to a component of the proton (termed gluon) that does not interact with electrons or with neutrinos (at least in charged current reactions). It is gratifying that the sum

in (5.39) does not exceed unity. That would have been a deathblow to the quark-parton model.

5.5 Angular Momentum Composition of the Proton

Just as deep inelastic scattering from an unpolarized nucleon analyses its momentum composition, we can show that scattering from a polarized nucleon would reveal its angular momentum composition. We demonstrate this below, not because neutrino experiments with polarized targets are imminent, but as an interesting illustration of the power and simplicity of the parton model.

Consider a +ve helicity proton moving with high momentum in the +z-direction ($J_z = \frac{1}{2}$) and let $u_{\pm}$, $d_{\pm}$, $s_{\pm}$, $\bar{u}_{\pm}$, $\bar{d}_{\pm}$, $\bar{s}_{\pm}$ denote the densities of the various partons, the subscripts +(-) denoting partons of +ve (-ve) helicity. (These densities are related to the densities in an unpolarized proton by $u = u_+ + u_-$, $d = d_+ + d_-$, etc.) A neutrino colliding with such a proton would scatter only from left-handed n quarks or right-handed $\bar{p}$ quarks, with elementary cross sections

$$\frac{d\sigma(\nu n_- \rightarrow \mu^- \bar{p}_-)}{dx dy} = \frac{G^2 ME}{\pi} \cdot 4x \quad (5.40)$$

$$\frac{d\sigma(\nu \bar{p}_+ \rightarrow \mu^- \bar{n}_+)}{dx dy} = \frac{G^2 ME}{\pi} \cdot 4x(1-y)^2,$$

(we neglect θ_c). By contrast, an antineutrino would "see" only left-handed p quarks and right-handed $\bar{n}$ quarks, the cross sections being

$$\frac{d\sigma(\bar{\nu} p_- \rightarrow \mu^+ n_-)}{dx dy} = \frac{G^2 ME}{\pi} \cdot 4x(1-y)^2 \quad (5.41)$$

$$\frac{d\sigma(\bar{\nu} \bar{n}_+ \rightarrow \mu^+ \bar{p}_+)}{dx dy} = \frac{G^2 ME}{\pi} \cdot 4x.$$

(Note that these results are consistent with the unpolarized cross sections in Table 4, which are averages over the two helicity states of the parton. Since helicity cross sections other than those given in Eqs. (5.40 and (5.41) vanish, the unpolarized parton cross section is just one-half the polarized one.) Knowing the parton cross sections, we can immediately write the deep inelastic cross sections for a polarized proton:

$$\begin{aligned} \frac{d\sigma}{dx dy} (\nu P_+ \rightarrow \mu^- + \text{anything}) &= \frac{G^2_{ME}}{\pi} 4x [d_-(x) + \bar{u}_+(x)(1-y)^2] \\ \frac{d\sigma}{dx dy} (\bar{\nu} P_+ \rightarrow \mu^+ + \text{anything}) &= \frac{G^2_{ME}}{\pi} 4x [\bar{d}_+(x) + u_-(x)(1-y)^2] \end{aligned} \quad (5.42)$$

If the polarization of the proton is reversed, the cross sections are (replace $+$ $\rightarrow$ $-$):

$$\begin{aligned} \frac{d\sigma}{dx dy} (\nu P_- \rightarrow \mu^- + \text{anything}) &= \frac{G^2_{ME}}{\pi} 4x [d_+(x) + \bar{u}_-(x)(1-y)^2] \\ \frac{d\sigma}{dx dy} (\bar{\nu} P_- \rightarrow \mu^+ + \text{anything}) &= \frac{G^2_{ME}}{\pi} 4x [\bar{d}_-(x) + u_+(x)(1-y)^2] \end{aligned} \quad (5.43)$$

Thus, measurements of the above four cross sections can, in principle, isolate the eight functions $u_{\pm}(x)$, $d_{\pm}(x)$, $\bar{u}_{\pm}(x)$, $\bar{d}_{\pm}(x)$.

We now observe that the angular momentum of a polarized proton is built up from the spins and orbital momenta of its parton constituents, and so may be written as⁽⁵¹⁾

$$J_z = \langle L_z \rangle + \langle S_z \rangle = \frac{1}{2} \quad (5.44)$$

(Note that $\langle L_z \rangle$ need not be zero, since partons do have (limited) transverse momenta over and above their longitudinal motion.) The contribution of a parton species q to $\langle S_z \rangle$ is

$$\langle S_z \rangle_q = \frac{1}{2} \int dx (q_+(x) - q_-(x)) \quad (5.45)$$

where q_+ , q_- are the densities of such partons with +ve(-ve) helicity. It is then clear that a knowledge of the functions $u_{\pm}$, $d_{\pm}$, $\bar{u}_{\pm}$, $\bar{d}_{\pm}$, gained from

neutrino scattering by a polarized proton, would determine the quantities $\langle S_z \rangle_u$, $\langle S_z \rangle_d$, $\langle S_z \rangle_{\bar{u}}$, $\langle S_z \rangle_{\bar{d}}$ and thus illuminate the structure of the proton with respect to its spin angular momentum.

Remarkably, the parton model also helps to relate $\langle L_z \rangle$ and $\langle S_z \rangle$ to weak interaction parameters that are already known from low energy experiments.⁽⁵¹⁾ One can show that the ratio G_A/G_V of coupling constants in neutron β -decay can be expressed as^(21, 22)

$$\left(\frac{G_A}{G_V}\right)_{n \rightarrow p} = \int [(u_+ - u_-) + (\bar{u}_+ - \bar{u}_-) - (d_+ - d_-) - (\bar{d}_+ - \bar{d}_-)] dx \quad (5.46)$$

(This relation was mentioned earlier in Eq. (3.13).) By SU(3) symmetry, the corresponding ratio for Ξ decay is

$$\left(\frac{G_A}{G_V}\right)_{\Xi^- \rightarrow \Xi^0} = \int [(d_+ - d_-) + (\bar{d}_+ - \bar{d}_-) - (s_+ - s_-) - (\bar{s}_+ - \bar{s}_-)] dx \quad (5.47)$$

Assuming that the contribution of strange partons is negligible, and noting that

$$\langle S_z \rangle = \frac{1}{2} \int [(u_+ - u_-) + (\bar{u}_+ - \bar{u}_-) + (d_+ - d_-) + (\bar{d}_+ - \bar{d}_-)] dx ,$$

we obtain⁽⁵¹⁾

$$\begin{aligned} \langle S_z \rangle &= \frac{1}{2} (3F - D) \\ \langle L_z \rangle &= \frac{1}{2} - \langle S_z \rangle = \frac{1}{2} (1 - 3F + D) , \end{aligned} \quad (5.48)$$

(where we have used the octet model results $(G_A/G_V)_{n \rightarrow p} = F + D$, $(G_A/G_V)_{\Xi^- \rightarrow \Xi^0} = F - D$). Numerically, $F \approx 0.45$, $D \approx 0.80$, so that $\langle S_z \rangle \approx 0.3$, $\langle L_z \rangle \approx 0.2$. We thus discover that a significant part of the angular momentum of a proton comes from the orbital motion of its constituents (a nontrivial result, considering that the naive quark model gives $\langle S_z \rangle = \frac{1}{2}$, $\langle L_z \rangle = 0$).

5.6 Analysis of Neutral Currents

The application of the parton model facilitates the analysis of neutral current data in the deep inelastic region. If the neutral current is specified

(i. e. if the coupling constants $C_V, C_A, C_V^i, C_A^i, C_V^{ii}, C_A^{ii}$ in Eq. (5.6) are known), one has only to refer to Table 4 to write down the deep inelastic cross sections. For the case of an average nucleon target, these cross sections were given in Eqs. (5.8) and (5.9) (see also Eq. (5.11)). For a first orientation, we consider below the approximation in which only the valence partons p and n contribute to the scattering, an approximation that was shown in Sec. 5.4 to be reasonable. The charged current and neutral current cross sections off an average nucleon are then⁽⁵²⁾

$$\begin{aligned}
 \frac{d\sigma(\nu \rightarrow \mu^-)}{dx dy} &= \frac{G^2 ME}{\pi} x(u+d) \\
 \frac{d\sigma(\bar{\nu} \rightarrow \mu^+)}{dx dy} &= \frac{G^2 ME}{\pi} x(u+d)(1-y)^2 \\
 \frac{d\sigma(\nu \rightarrow \nu)}{dx dy} &= \frac{G^2 ME}{\pi} x(u+d)(f(y) + f'(y)) \\
 \frac{d\sigma(\bar{\nu} \rightarrow \bar{\nu})}{dx dy} &= \frac{G^2 ME}{\pi} x(u+d)(\bar{f}(y) + \bar{f}'(y))
 \end{aligned} \tag{5.49}$$

where

$$\begin{aligned}
 f(y) &= \frac{1}{4} (C_V + C_A)^2 + \frac{1}{4} (C_V - C_A)^2 (1-y)^2 \\
 \bar{f}(y) &= \frac{1}{4} (C_V + C_A)^2 (1-y)^2 + \frac{1}{4} (C_V - C_A)^2
 \end{aligned} \tag{5.50}$$

and $f'(y), \bar{f}'(y)$ are given by $C_{V,A}^i \rightarrow C_{V,A}^{i'}$. Interestingly, the x and y dependence factorizes in this approximation, the x dependence being common to all four reactions. This has the consequence that the ratio of neutral current and charged currents can be predicted entirely in terms of the coupling constants $C_{V,A}$ and $C_{V,A}^i$. We find, for the total cross section ratios,

$$\begin{aligned}
 R &= \frac{\sigma(\nu \rightarrow \nu)}{\sigma(\nu \rightarrow \mu^-)} = \frac{1}{4} [(C_V + C_A)^2 + (C_V^i + C_A^i)^2 + \frac{1}{3} (C_V - C_A)^2 + \frac{1}{3} (C_V^i - C_A^i)^2] \\
 \bar{R} &= \frac{\sigma(\bar{\nu} \rightarrow \bar{\nu})}{\sigma(\bar{\nu} \rightarrow \mu^+)} = \frac{3}{4} [\frac{1}{3} (C_V + C_A)^2 + \frac{1}{3} (C_V^i + C_A^i)^2 + (C_V - C_A)^2 + (C_V^i - C_A^i)^2]
 \end{aligned} \tag{5.51}$$

Special models of the neutral current may now be examined. Those which postulate a pure vector or pure axial vector current predict $\sigma(\nu - \bar{\nu}) = \sigma(\bar{\nu} - \bar{\nu})$, or $R = \frac{1}{3} \bar{R}$. Those in which the neutral current is chirally pure predict $R = \bar{R}$ when the current is left-handed (i. e. $C_V = C_A$, $C'_V = C'_A$) and $R = \frac{1}{9} \bar{R}$ when the current is right-handed (i. e. $C_V = -C_A$, $C'_V = -C'_A$). An example of a one-parameter model is the model of Weinberg and Salam, in which the coupling constants are

$$\begin{aligned} C_A = C'_A = \frac{1}{2} , \\ C_V = \frac{1}{2} (1 - \frac{8}{3} \sin^2 \theta) , \quad C'_V = \frac{1}{2} (1 - \frac{4}{3} \sin^2 \theta) , \end{aligned} \quad (5.52)$$

(θ = Weinberg angle). For this model, we obtain the prediction

$$\begin{aligned} R &= \frac{1}{2} - \sin^2 \theta + \frac{20}{27} \sin^4 \theta \\ \bar{R} &= \frac{1}{2} - \sin^2 \theta + \frac{20}{9} \sin^4 \theta . \end{aligned} \quad (5.53)$$

Measurements of R and $\bar{R}$ in the scaling region are available, subject, however, to corrections for certain kinematical cuts and biases. As an example, the uncorrected results of the Caltech-FNAL experiment are⁽⁵³⁾ $R \approx 0.21$, $\bar{R} \approx 0.43$. (These numbers are remarkably close to the ratios measured at lower energies (see Eq. (4.52).) As a qualitative conclusion, one can probably rule out the "chirally pure" theories mentioned above. The pure V or pure A cases may be marginally consistent, as is the Weinberg-Salam model with a parameter $\sin^2 \theta$ between 0.3 and 0.4.⁽⁵²⁾

5.7 Parton Fragmentation Hypothesis

The parton model suggests certain distinctive features⁽⁵⁴⁾ for the distribution of a hadron h produced in semi-inclusive reactions of the type

$$l + N \rightarrow l' + h + \text{anything} , \quad (5.54)$$

where l, l' are leptons and N a nucleon. In the deep inelastic domain, we may view the scattering in a frame in which the nucleon momentum p and the momentum transfer q have the components

$$\begin{aligned}
 p &= (P + 0 \left(\frac{M^2}{P} \right), 0, 0, P) \\
 q &= (0, 0, 0, -2Px)
 \end{aligned}
 , \quad P \rightarrow \infty \quad (5.55)$$

with $x = -q^2/2p \cdot q$. The momentum q is then absorbed by a parton whose four-momentum is

$$xp \approx (xP, 0, 0, xP) ,$$

the scattered parton emerging in the negative z direction with momentum $(xP, 0, 0, -xP)$. Hadrons in the final state are supposed to fall into two groups: the fragments of the left-moving parton and the fragments of the right-moving residual nucleon. The former group is assumed to manifest itself as a jet of hadrons produced in the forward hemisphere (relative to the vector $\vec{q}$). It is with such parton- (or current-) fragments that the following discussion is concerned.

The fragmentation hypothesis is the statement that if h is a parton fragment (of four-momentum h), its distribution will depend only on the variable $z = h \cdot p/q \cdot p$. In the reference frame considered above, this variable is the energy of the hadron divided by the energy of the scattered parton. Essentially, therefore, one assumes that the distribution of a hadron h emanating from the parton depends only on the fractional momentum carried by h , in much the same way that the distribution of a parton contained in a hadron depends only on the fractional momentum ξ .

The implication of the above hypothesis is that if h is a parton fragment (as defined above), the semi-inclusive cross section for the reaction (5.54) is⁽⁵⁵⁾

$$\frac{d\sigma^h}{dx dy dz} = \sum_j \left(\frac{d\sigma}{dx dy} \right)_j D_j^h(z) \quad (5.56)$$

where $(d\sigma/dx dy)_j$ is the total cross section corresponding to a (final) scattered parton of type j , and $D_j^h(z)$ is a "fragmentation function" expressing the probability for such a parton to produce a hadron h . Recall that the inclusive cross section for the process $l + N \rightarrow l' + \text{anything}$ is given, in the

parton model by

$$\frac{d\sigma}{dx dy} = \sum_j \left(\frac{d\sigma}{dx dy} \right)_j \quad (5.57)$$

Hence, the average multiplicity of h is

$$\langle N^h \rangle \equiv \frac{d\sigma^h / dx dy dz}{d\sigma / dx dy} = \frac{\sum_j \left(\frac{d\sigma}{dx dy} \right)_j D_j^h(z)}{\sum_j \left(\frac{d\sigma}{dx dy} \right)_j} \quad (5.58)$$

Clearly, relations can be expected between hadron multiplicities in different reactions (neutrino-induced or electron-induced), since all such multiplicities are determined by the same quantities: the set of fragmentation functions $D_j(z)$, and the set of parton densities $f_j(x)$. One needs only to consult Table 4 to write down the multiplicities explicitly.

5.8 Application to Pion Production

The model described above is especially convenient for predicting pion multiplicities in neutrino-induced reactions. Isospin and charge-conjugation arguments give

$$\begin{aligned} D_u^{\pi^+} &= D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}, \quad D_d^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} \\ D_u^{\pi^0} &= D_d^{\pi^0} = \frac{1}{2} (D_u^{\pi^+} + D_d^{\pi^+}) \\ D_s^{\pi^0} &= D_s^{\pi^+} = D_s^{\pi^-} \end{aligned} \quad (5.59)$$

so that there are just three independent fragmentation functions: $D_u^{\pi^+}$, $D_d^{\pi^+}$, $D_s^{\pi^0}$. The last of these is unimportant for most considerations. The first two may be extracted from electron scattering measurements and are displayed in Fig. 7. (56, 57)

For the charged current reactions

$$\begin{aligned} \nu + N &\rightarrow \mu^- + \pi + \text{anything} \\ \bar{\nu} + N &\rightarrow \mu^+ + \pi + \text{anything} \end{aligned} \quad (5.60)$$

the pion multiplicities have an extraordinarily simple form. We recall that when θ_c is neglected, a neutrino can produce only a p-quark or an $\bar{n}$ -quark in the final state which, by Eq. (5.59), have the same probability for producing a given pion. Thus the semi-inclusive cross section takes the factorized form

$$\left(\frac{d\sigma^\pi}{dx dy dz}\right)_{\nu \rightarrow \mu^-} = D_u^\pi(z) \sum_j \left(\frac{d\sigma}{dx dy}\right)_j \quad (5.61)$$

and the multiplicity is simply

$$\langle N^\pi \rangle_{\nu \rightarrow \mu^-} = D_u^\pi(z) \quad (5.62)$$

By a parallel argument, the multiplicity in the $\bar{\nu}$ reaction is

$$\langle N^\pi \rangle_{\bar{\nu} \rightarrow \mu^+} = D_d^\pi(z) \quad (5.63)$$

Remarkably, the multiplicities (5.62) and (5.63) are independent of x and y as well as of the target. Since the multiplicities are identical to the fragmentation functions, they may be read directly off Fig. 7. These consequences of the fragmentation hypothesis find some support in preliminary measurements,^(35, 56) and would be extremely interesting to check in the future.

The model may similarly be used to obtain pion multiplicities $\langle N^\pi \rangle_{\nu \rightarrow \nu}$ and $\langle N^\pi \rangle_{\bar{\nu} \rightarrow \bar{\nu}}$ in the neutral current reactions. A discussion is given in Ref. (56), including detailed predictions for the case of the Weinberg model.

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Figure Captions

- Fig. 1 Relation between $\sigma(\nu_e \mu)$ and $\sigma(\bar{\nu}_\mu e)$ in the Weinberg-Salam model.
- Fig. 2 Constraints on coupling constants of the neutral electron current $\bar{e} \gamma_\alpha (C_V + C_A \gamma_5) e$ imposed by limits on neutrino-electron scattering (90% C. L.). (Data from Ref. 3.) The dotted line is the predicted domain of the Weinberg model.
- Fig. 3 Tests of scaling. (a) Total $\nu, \bar{\nu}$ cross sections as functions of energy. (b) Average value of x as function of energy. (c) Average value of y as function of energy. (d) Allowed domain for the normalized y -distributions (Ref. 37).
- Fig. 4 Structure functions $F_2^{\nu N}(x')$ and $x'F_3^{\nu N}(x')$ extracted from Gargamelle data (Refs. 38 and 42). Variable $x' = Q^2 / (2M\nu + M^2)$ is an empirical scaling variable which $\rightarrow x$ for large Q^2 and ν . Continuous curve is from electroproduction.
- Fig. 5 Constraints on the isovector coupling constants of the neutral current from measurements of R and $\bar{R}$ (see Eq. 4.52). Allowed region is common interior of the elliptical bands (thickness of bands corresponds to one standard deviation). If isoscalar current is negligible, $G_V^{(3)}$ and $G_A^{(3)}$ should lie in the region of intersection.
- Fig. 6 Distribution of quark and antiquark momenta in a proton, as extracted from deep inelastic data (Refs. 38 and 42). The curves are $x'(u+d)$ and $x'(\bar{u}+\bar{d})$.
- Fig. 7 Fragmentation functions $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$ extracted from electroproduction measurements (Ref. 56). Multiplicities in neutrino reactions are predicted to be $\langle N^{\pi^+} \rangle_{\nu \rightarrow \mu^-} = \langle N^{\pi^-} \rangle_{\bar{\nu} \rightarrow \mu^+} = D_u^{\pi^+}$, $\langle N^{\pi^-} \rangle_{\nu \rightarrow \mu^+} = \langle N^{\pi^+} \rangle_{\bar{\nu} \rightarrow \mu^-} = D_d^{\pi^+}$ (current fragments).

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 3c

Fig. 3d

$F_2^{\nu N}$ AND $x' F_3^{\nu N}$

GARGAMELLE DATA

2674 ν events
1104 $\bar{\nu}$ events } $E > 1\text{GeV}$

Fig. 4

Fig. 5

Fractional nucleon momentum carried by
quarks(antiquarks) per unit interval of x'

Fig. 6

Fig. 7