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ABSTRACT

The origin of the axial current observed in Scyllac (a high beta stellarator experiment) is discussed. A shaped coil and/or helical winding produce rotational transform which links magnetic lines of force to the plasma column and the axial current is induced electromagnetically. This phenomenon is inherent in a pulsed high-beta stellarator. The rotational transform produced by the induced axial current is much smaller than that associated with the $\ell = 1, 0$ equilibrium fields. The effect of the axial current on the equilibrium and stability of the plasma column is thus small. It is also shown that the magnetic field shear near a plasma surface is very strong.

I. INTRODUCTION

An axial plasma current of $1 \sim 1.5$ kA peak amplitude is observed in the Scyllac full torus experiments.¹ The amplitude of the current exceeds the Kruskal - Shafranov limit at a radius of 1 cm (~ 500 A). Therefore, the kink instability is possible. However, the gross behavior of the plasma column appears to be unchanged, irrespective of whether the net axial current is reduced to zero by means of an externally induced axial current. This effect is not inconsistent with the small theoretical growth rate of the kink instability.

In this report, the origin of the axial current is discussed in terms of rotational transform. Rotational transform is produced with a shaped compression coil and/or helical windings. Magnetic lines of force link the toroidal plasma column. If the plasma is produced in a pulsed fashion, linkage of the magnetic lines of force leads to electromagnetic induction along the axis. The magnitude of the linkage can be obtained from the rotational transform, and determines the magnitude of the axial current.

II. AXIAL CURRENT

Rotational transform t is defined as²

$$t(r) = 2\pi \frac{d\chi}{d\phi} \quad (1)$$

where χ is the azimuthal magnetic flux and ϕ is the longitudinal flux. We assume that the longitudinal component of the magnetic field is constant in the plasma and in the surrounding vacuum and is much larger than the azimuthal component. Then the longitudinal flux ϕ is, in lowest order,

$$\phi = \pi a^2 B_1 + \pi(r^2 - a^2) B_0 \quad (2)$$

where B_1 and B_0 are the magnetic field in the plasma and in the vacuum, respectively, and a is the average radius of the plasma column. The azimuthal flux χ outside the plasma column is expressed as

$$\chi(r) \approx B_0 \int_a^r t(r) dr \quad (3)$$

The azimuthal flux χ is the magnetic flux per unit length which links the plasma column and produces electromagnetic induction along the axis if the magnetic field is applied abruptly. The resultant induced axial current I has the form

$$I \approx \frac{\chi(b)}{R/\mu \ln b/a} = \frac{B_0}{\mu \ln b/a} \int_a^b t(r) dr \quad (4)$$

where $2\pi/h$ is the wavelength of the helical $\ell = 1$ magnetic field and b is the average wall radius of the shaped compression coil.³ The magnitude of the axial current is proportional to the rotational transform i , and is induced in such a manner as to reduce the original rotational transform produced with the shaped coil.

III. ROTATIONAL TRANSFORM

The magnetic field in the vacuum region is calculated from

$$\vec{B} = \vec{\nabla}\phi \quad (5)$$

$$\Delta\phi = 0$$

with boundary conditions

$$\vec{\nabla}f_1 \cdot \vec{\nabla}\phi = 0 \quad (6)$$

$$\vec{\nabla}f_2 \cdot \vec{\nabla}\phi = 0$$

The quantities f_1 and f_2 describe the inner surface of the shaped compression coil and the outer surface of the plasma column respectively:

$$f_1(r, \theta, z) = r - b - \Delta_{\ell=1} \cos(\theta + hz) - \Delta_{\ell=0} \cos(hz) = 0 \quad (7)$$

$$f_2(r, \theta, z) = r - a - \delta_{\ell=1} \cos(\theta + hz) - \delta_{\ell=0} \cos(hz) = 0$$

The last terms of f_1 and f_2 do not contribute to the following discussion and are omitted hereafter. Toroidal effects are also ignored. For simplicity, we further assume that

$$\Delta_{\ell=1}/b \ll 1 \quad (8)$$

$$\delta_{\ell=1}/a \ll 1$$

The first assumption in Eq. (8) is satisfied experimentally in Scyllac but the second is not. The solution of Eq. (5) is, in lowest order,

$$\phi = B_0 z + \frac{1}{h} [CI_1(hr) + DK_1(hr)] \sin(\theta + hz) \quad (9)$$

where

$$C = hB_0 \frac{\delta_{\ell=1} K_1'(hb) - \Delta_{\ell=1} K_1'(ha)}{I_1'(hb) K_1'(ha) - I_1'(ha) K_1'(hb)} \quad (10)$$

$$\approx -2hB_0 \frac{b^2 \Delta_{\ell=1} - a^2 \delta_{\ell=1}}{b^2 - a^2}$$

$$D = hB_0 \frac{\Delta_{\ell=1} I_1'(ha) - \delta_{\ell=1} I_1'(hb)}{I_1'(hb) K_1'(ha) - I_1'(ha) K_1'(hb)}$$

$$\approx -hB_0 \left(\frac{h^2 a^2 b^2}{b^2 - a^2} \right) (\Delta_{\ell=1} - \delta_{\ell=1})$$

for $ha, hb \ll 1$. The rotational transform per $\ell = 1$ wavelength is calculated from the method of averaging⁴ and is, in lowest order,

$$i(r) = \frac{\pi}{B_0^2} \left\{ C^2 \left(\frac{I_1'(x)}{x} \right)^2 [1 - 2I^1 + (1 + x^2) (I^1)^2] \right.$$

$$+ 2CD \frac{I_1'(x)}{x} \frac{K_1'(x)}{x} [1 - I^1 - K^1 + (1 + x^2) I^1 K^1]$$

$$\left. + D^2 \left(\frac{K_1'(x)}{x} \right)^2 [1 - 2K^1 + (1 + x^2) (K^1)^2] \right\} \quad (11)$$

where

$$I^1 = \frac{I_1(x)}{x I_1'(x)}$$

$$K^1 = \frac{K_1(x)}{x K_1'(x)}$$

$$x = hr$$

For $ha, hb \ll 1$, Eq. (11) reduces to

$$i(r) \approx \frac{\pi h^2 \Delta_{\ell=1}}{1 - a^2/b^2} \left\{ \left(1 - \frac{a^2}{b^2} \frac{\delta_{\ell=1}}{\Delta_{\ell=1}} \right)^2 + \left(1 - \frac{a^1}{b^2} \frac{\delta_{\ell=1}}{\Delta_{\ell=1}} \right) \right.$$

$$\left. \times \frac{a^2}{r^2} + \frac{4}{h^2 a^2} \left(1 - \frac{\delta_{\ell=1}}{\Delta_{\ell=1}} \right)^2 \frac{a^6}{r^6} \right\} \quad (12)$$

The first term in Eq. (12) is the rotational transform of the vacuum magnetic field and the other terms are those resulted from the presence of the plasma. When the beta value of the plasma is large and $\Delta_{\ell=1}$ is not equal to $\delta_{\ell=1}$, the last term is dominant and the rotational transform with the plasma is much larger than that without the plasma. Substitution of Eq. (12) into Eq. (4) gives the magnitude of the induced axial current as

$$\begin{aligned}
I &\approx \frac{nB_0}{\mu f n b/a} \frac{h^3 \Delta_{\ell=1}^2 b^2}{\left(1 - \frac{a^2}{b^2}\right)^2} \left\{ \frac{1}{2} \left(1 - \frac{a^2}{b^2} \frac{\delta_{\ell=1}}{\Delta_{\ell=1}}\right)^2 \right. \\
&\quad \times \left(1 - \frac{a^2}{b^2}\right) + \frac{a^2}{b^2} \left(1 - \frac{a^2}{b^2} \frac{\delta_{\ell=1}}{\Delta_{\ell=1}}\right) f n b/a + \frac{1}{h^2 b^2} \\
&\quad \times \left(1 - \frac{\delta_{\ell=1}}{\Delta_{\ell=1}}\right)^2 \left(1 - \frac{a^4}{b^4}\right) \left. \right\} \\
&\approx \frac{nB_0 \Delta_{\ell=1}^2}{\mu f n b/a} \left(1 - \frac{\delta_{\ell=1}}{\Delta_{\ell=1}}\right)^2 \quad (13)
\end{aligned}$$

for $\delta_{\ell=1} \neq \Delta_{\ell=1}$. For the parameters $B_0 = 40$ kG, $h = 15$ m⁻¹, $\Delta_{\ell=1} = 0.43$ cm, $\delta_{\ell=1} = 0$ cm, $f n b/a = 2$, we find $I \approx 1.4$ kA. The value is compatible with the experimental value.¹ The time evolution and more exact magnitude of I are obtained by taking into consideration that the plasma radius a and distortion of the plasma column $\delta_{\ell=1}$ are time dependent quantities. As we have noted above, the direction of the axial current is oriented in such a manner as to reduce the original rotational transform and dependent on whether the helical $\ell = 1$ magnetic field rotates clockwise or counter-clockwise along the line of force. If the helical $\ell = 1$ field rotates clockwise as in Scyllac, the axial current is induced in antiparallel with the line of force.

Now we examine to what extent the axial current affects the original magnetic field. This is conveniently carried out in terms of rotational transform. The rotational transform $i_c(r)$ associated with the axial current is given by

$$i_c(r) \approx \frac{4\pi I}{hr^2 \delta_0} \approx \frac{1}{2 \ell n b/a} \frac{\Delta_{\ell=1}^2}{r^2} \quad (14)$$

at radius r . The ratio of the original rotational transform $i(a)$ to the current produced rotational transform $i_c(a)$ on the plasma surface indicates the influence of the axial current:

$$\frac{i_c(a)}{i(a)} \approx \frac{1}{4 \ell n b/a} \sim 10\% \quad (15)$$

for the present Scyllac parameters. Therefore, we conclude that the induced axial current has little

influence on the original magnetic field. Equation (15) is easily understood from the fact that linkage of the magnetic field to the plasma column is localized near the plasma surface, as seen in Eq. (12). However, the axial current observed in Scyllac exceeds the Kruskal-Shafranov limiting current at the radius of 1 cm. This is explained by the fact that the total rotational transform in Scyllac is much larger than 2π . From Eq. (12), we obtain

$$i(a) \approx \frac{4\pi \Delta_{\ell=1}^2}{a^2} \left(1 - \frac{\delta_{\ell=1}}{\Delta_{\ell=1}}\right)^2 \approx \frac{\pi \beta^2 \delta_{\ell=1}^2}{a^2} \quad (16)$$

where we have used the equilibrium condition

$$\frac{\delta_{\ell=1}}{\Delta_{\ell=1}} \approx \frac{1}{1 - \beta/2 (1 - a^2/b^2)} \approx \frac{1}{1 - \beta/2} \quad (17)$$

The quantity β is the plasma beta in our sharp boundary model. The plasma radius a in the Scyllac experiments is comparable to the distortion of the plasma column $\delta_{\ell=1}$. Then the rotational transform $i(a)$ per the wavelength of the helical $\ell = 1$ magnetic field is of the order of 1 in the presence of a high beta plasma, while the vacuum rotational transform is of the order of $(h\Delta_{\ell=1})^2$.

We find from Eq. (12) that the magnetic field shear near the plasma surface is very strong. The shear parameter θ is given as

$$\theta = \frac{di}{dr} \frac{hr^2}{2r} \approx - \frac{12\Delta_{\ell=1}^2 a^4 h}{r^5} \sim -1. \quad (18)$$

IV. CONCLUDING REMARKS

We have shown that the axial current in Scyllac results from rotational transform. Therefore, any procedure which varies rotational transform, such as dynamic stabilization, lead to the additional current induction. However, the induced axial current has little effect on the original (not vacuum!) rotational transform.

Rotational transform and magnetic field shear in Scyllac are also calculated. The strong shear calculated near the plasma surface suggests that flute-like instabilities with $m > 2$ are suppressed for Scyllac parameters, even in the cases where finite Larmor radius (FLR) effects are weak, such as in a staged theta pinch.

In this study, we have ignored the effect of the magnetic field inside the plasma column. This effect does not seem to be important for a high beta plasma.

G. Miller has obtained a result similar to Eq. (13) by considering the motion of the plasma column during the setting-up phase.⁵

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