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EVALUATION OF POTENTIAL TUBE VIBRATION
IN EBR-II STEAM SUPERHEATERS
AND EVAPORATORS AT FULL POWER

by

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NOMENCLATURE

<u>Symbol</u>	<u>Definition</u>	<u>Symbol</u>	<u>Definition</u>
a	Cross-sectional area of tube	N_s	Strouhal number for vortex-shedding
d, d_o	Outside diameter of tube	$(N_s)_{str}$	Strouhal number for tube
d_i	Inside diameter of tube	N_R	Reynolds number
e	Elongation due to temperature change	P	Pitch
f_n	Tube natural frequency	S	Stress
f_s	Fundamental natural frequency of tube	T	Transverse spacing ratio
f_v	Vortex shedding frequency	T_s	Average shell temperature
h	Amount shell is strained	T_t	Average tube temperature
k	Decimal-fraction change in natural frequency caused by axial loading	U	Cross-flow velocity
l	Tube length	U_c	Mean cross-flow velocity through minimum cross section
m	Virtual mass per unit length	U_{cr}	Critical flow velocity
n	Number of tubes	W	Mass flowrate through shell
p	Axial force (positive in tension)	α	Coefficient of thermal expansion for shell and tube material
t	Time	ϵ	Strain
u	Nondimensional flow velocity	λ, λ_n	Eigenvalue
u_{cr}	Critical nondimensional flow velocity	ν	Forcing frequency
x, y	Distance measured along tube	ρ	Density of shell and tube material
x_l	Longitudinal spacing ratio	ρ_f	Fluid density
x_t	Transverse spacing ratio	ρ_{Na}	Sodium density
A	Cross-sectional area of shell	σ	Stress
A_B	Cross-sectional area of displaced water	ϕ	Mode shape
A_C	Cross-sectional area of core tube	ψ	Shape function representing velocity distribution
A_D	Cross-sectional area of duplex tube	ω	Frequency
A_x	Average cross-flow area within tube bundles between baffle windows	ω_n	Tube natural frequency
C_L	Lift coefficient	ω_v	Vortex-shedding frequency
D_B	Outer diameter of tube bundle	Ω	Nondimensional frequency
E	Young's modulus of elasticity	<u>Subscripts</u>	
I	Moment of inertia of tube	1	Shell
Im	Imaginary part	2	Tube
K	Constant	<u>Superscripts</u>	
L	Longitudinal spacing	i	Initial load or preload
L_B	Length of tube span between baffles	T	Loading due to thermal expansion
M	Added mass of fluid per unit length	I, II	Tube-support configurations

EVALUATION OF POTENTIAL TUBE VIBRATION IN EBR-II STEAM SUPERHEATERS AND EVAPORATORS AT FULL POWER

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ABSTRACT

An analytical evaluation of the EBR-II steam superheaters and evaporators is performed to determine the potential for detrimental flow-induced tube vibrations at current operating power (45 MWt) and at design power (62.5 MWt). Based upon state-of-the-art technology, the evaluation includes analyses and discussions of resonant vibration induced by cross flow; hydroelastic instabilities due to parallel flow; baffle design and orientation in the flow field; and tube-baffle impact and its effect on vibrational characteristics and response of the tubes. Although some insight is gained into the mechanisms involved, limitations of the technology and lack of experiments preclude any conclusions regarding reliability of the EBR-II steam superheaters and evaporators.

I. INTRODUCTION

Heat-exchanger tubes, being subjected to a dynamic fluid environment, may experience vibrations even during normal operation. These vibrations can be classed as (1) those caused by resonant conditions and/or instability, and (2) those that are force (nonresonant)-excited. Resonance and instability imply large-amplitude vibrations, generally leading to a rapid structural failure. Vibrations that are force-excited by a broad-band random loading (predominantly at frequencies other than those of resonances) imply small-amplitude vibrations with the potential for wear and eventual high-cycle, low-strain fatigue failure.

In Experimental Breeder Reactor II (EBR-II), a 62.5-MWt fast breeder, the reactor heat is removed by the primary sodium-coolant system to the intermediate heat exchanger in the primary-system tank. Here, the heat is transferred by the secondary sodium system to the steam generator. The steam generator consists of a natural-circulation evaporating section, a conventional steam drum, and a once-through superheating section. The evaporating section comprises eight identical shell-and-tube heat exchangers connected in parallel on the tube side to a horizontal, overhead steam drum with conventional moisture-separation internals. Dry and saturated steam

flows from the top of the steam drum, down through two identical vertical shell-and-tube superheaters (modified evaporators), and into the turbine-generator unit. To render the evaporators useful as superheaters, the evaporator tubes were cored with 0.812-in.-OD tubes to restrict the flow channel to a 1/8-in. annulus and effect the requisite steam velocity.

Table I and Fig. 1 give the pertinent design details and full-power operating characteristics of the evaporators and superheaters. All units are constructed entirely of 2 $\frac{1}{4}$ % Cr-1% Mo material (Croloy) and use seamless, double-walled tubes to minimize the potential for a sodium-water reaction. In addition, as shown in Fig. 1, the duplex tubes at the sodium inlets to all units are covered with shock tubes for protection against thermal shock and loading caused by impingement of hot sodium.

TABLE I. Design Data: EBR-II Steam Superheaters and Evaporators

	Superheater	Evaporator
Type	Shell and tube; segmented baffles; multipass cross flow	
Shell side	Sodium	Sodium
Tube side	Steam	Water
Design temperature, °F		
Shell side	1,000	1,000
Tube side	900	700
Design pressure, psi		
Shell side	150	150
Tube side	1,500	1,500
Shell ID, in.	19.225	19.225
Sodium flowrate, lb/hr	1,250,000	312,500
Sodium temperature, °F		
Inlet	870	803
Outlet	790	582
Feedwater inlet temperature, °F	-	549
Steam temperature, °F		
Inlet	580	-
Outlet	840	579
Number of tubes	73	73
Tube material	Croloy	Croloy
Overall tube length, ft	25.125	25.125
Tube pitch, in.	1.938 (triangular)	1.938 (triangular)
Duplex-tube ID, in.	1.065	1.065
OD, in.	1.438	1.438
Core-tube OD, in.	0.812	-
Baffle hole diameter, in.	1.500	1.500

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Fig. 1. Details of EBR-II Superheater and Evaporator

No specific tube-vibration analysis was performed before fabrication of these units. However, the guidelines suggested in the TEMA* standards were followed. In addition, each duplex tube was inspected as a single-wall tube and again as a duplex tube. Finally, the units were tested to the equivalent of 250,000 cycles of tube vibration at flowrates in excess of those associated with full-power operation.

Since December 1962, the evaporators and superheaters have accumulated 17,500 hr of operation at power levels up to 45 MWt without serious failure. This would indicate that tube vibrations, at least relatively large-amplitude vibrations, are not a problem. However, there is concern that the effective cross-flow velocity associated with the secondary sodium flowrate ($\sim 10^6$ lb/hr) through each superheater at 45 MWt may be close to critical, and that a proportional increase in flowrate on approaching design power (62.5 MWt) may promote potentially damaging tube vibrations.

Support for this concern over tube vibrations was supplied by failures in the Advanced Test Reactor (ATR), Hallam, and Fermi heat exchangers, and reenforced by the results of a recent survey by Nelms and Segaser¹ of primary heat exchangers in 16 operating reactor plants for which the USAEC (DRDT) has technical responsibility. Results of this survey showed that nine of the plants (including ATR and Hallam) experienced tube-vibration problems in the heat exchangers.

In view of these findings, and at the request of the USAEC (DRDT), a preliminary evaluation was made of the potential for detrimental flow-induced tube vibrations in the EBR-II steam superheaters and evaporators. As in Ref. 1, the evaluation was limited to an analytical study, using state-of-the-art technology; no funds were allocated for additional experimental or development work. The immediate objective of the study was to provide the basis for a more comprehensive engineering evaluation. Such an evaluation, of necessity, requires consideration of specific parameters such as fatigue, wear, elastic stability, creep, tolerances, and thermoelasticity.

In general, the state-of-the-art technology permits only analytical prediction of conditions relating to resonance and instability. But, even in these cases, the analysis requires empirical factors and results based on laboratory tests. However, despite the limitations of the technology, Nelms and Segaser¹ concluded that, in all cases, if heat-exchanger designs had been critically analyzed before fabrication, the proximity of the natural frequencies of tube vibration to a calculated characteristic frequency of the flow would have been justification for design modification.

The present study identifies and analyzes several important aspects of vibration-inducing mechanisms to the extent permitted by state-of-the-art

*Tubular Exchanger Manufacturers Association.

techniques. Primary emphasis is focused on the vortex-shedding phenomenon. Consideration is given to the calculation of tube natural frequencies, including effects of axial loads and creep. Hydroelastic instabilities associated with parallel components of both shell-side and tube-side flows are also considered. The design of the baffle plate and its effect on vortex shedding caused by flow orientation are studied. Tube-baffle impact and nonuniform distribution of cross-flow velocity are considered preliminarily to determine the order of magnitude of these effects. Pump pulsations in the flow and external sources of excitation, such as structural-borne vibrations from pump installations or other adjacent equipment, were excluded since they are not considered a serious problem. Other important variables and areas of study not adequately treated in the state-of-the-art technology or requiring integrated analytical-experimental development are identified and discussed briefly.

In short, the present study is not an in-depth evaluation of the EBR-II superheaters and evaporators. It has provided some insight into the mechanisms that could lead to damaging tube vibrations. More importantly, it emphasizes the need for a comprehensive design and experimental program to establish an effective analytical technique for evaluating tube vibrations. Only with such a technique can the reliability of current and future heat exchangers be predicted with a reasonable degree of confidence.

II. STATE-OF-THE-ART EVALUATION

A. Superheater

1. Tube Natural Frequencies

Knowledge of structural natural frequencies is basic to any vibration analysis and fatigue study. In the EBR-II steam superheater, the baffle plates restrict lateral motion of the tubes within the limits provided by the tube-baffle clearance. Therefore, calculation of tube natural frequencies is prefaced by the initial assumption that the baffles act as internal supports, restricting the tube to no motion at the locations of these supports. The duplex tube can then be approximated as a uniform beam (this neglects the influence of the shock tube) of length $\ell = 25.125$ ft, with three interior supports located a_0 , b_0 , and c_0 from the tube sheet at the sodium inlet. When the shell is considered in halves, two support configurations result from the different baffle spacings in each half; they are defined as:

Configuration I: $a_0 = 9.25$ ft; $b_0 = 15.75$ ft; $c_0 = 22.25$ ft.

Configuration II: $a_0 = 6.0$ ft; $b_0 = 12.5$ ft; $c_0 = 19.0$ ft.

The natural frequencies of free vibration of a uniform beam can be expressed by

$$\omega_n^2 = \frac{EI}{\rho a} \lambda_n^4. \quad (1)$$

For the EBR-II superheater tubes (see Appendix A),

$$\omega_n = 506 \lambda_n^2, \quad (2)$$

where the eigenvalues have the units ft^{-1} . In Appendix B, the frequency equation is derived as the determinant of a three-by-three matrix of interior receptances of the beam. These receptances are available² for any combination of free, clamped, pinned, or sliding-end conditions.

A computer code was written and used to evaluate the eigenvalues. Table II lists the first three natural frequencies for both support configurations. These frequencies are used in subsequent analyses for resonant phenomena.

TABLE II. Eigenvalues and Natural Frequencies of Superheater Tubes

n	Configuration I		Configuration II	
	$\lambda_n, \text{ft}^{-1}$	f_n, Hz	$\lambda_n, \text{ft}^{-1}$	f_n, Hz
1	0.452	16.4	0.528	22.4
2	0.563	25.5	0.622	31.2
3	0.669	36.0	0.726	42.5

In an attempt to improve on the state-of-the-art analytical prediction of tube natural frequencies, the effects of initial preload, axial loads under operating conditions, and creep due to operational cycling were considered. Theoretically, a significant change in natural frequency caused by one of these mechanisms could result in a resonant condition if the new frequency coincides with the characteristic frequency of the flow.

a. Effect of Axial Loads. To minimize stresses at tube-to-sodium-tubesheet welds caused by differential expansion between the shell and the tubes at operating temperature, cold-springing* was used to produce a compressive stress in the tubes and tensile stress in the shell at room temperature. This induced axial load on the tubes is of interest because it affects the natural frequencies of the tubes. Therefore, the percent change in tube natural frequency due to initial preload and at operating temperatures was calculated.

*The shell was heated and elongated approximately 1/8 in. while the tubes were being welded to tubesheets.

The fundamental natural frequency of transverse vibrations of a bar loaded by an axial force can be calculated from⁷

$$\omega^2 = \frac{EI \int_0^{\ell} (\phi'')^2 dx + p \int_0^{\ell} (\phi')^2 dx}{\rho a \int_0^{\ell} (\phi)^2 dx} \quad (3)$$

Since we are interested in the decimal-change in frequency, we rewrite Eq. 3 as

$$\omega = (1 + k) \Omega, \quad (4)$$

where Ω is the natural frequency of the tube with no axial force applied, and k is the decimal-fraction change due to axial loading. From Eq. 3,

$$k = \left[1 + \frac{p}{EI} f(\phi) \right]^{1/2} - 1, \quad (5)$$

where

$$f(\phi) = \frac{\int_0^{\ell} (\phi')^2 dx}{\int_0^{\ell} (\phi'')^2 dx} \quad (6)$$

From elementary strength of materials and thermoelasticity we obtain (see Appendix C) as the expression for the axial force in the tubes due to the combined preload and axial load

$$p = - \frac{Ea[h - \alpha(T_s - T_t) \ell]}{\left(1 + n \frac{a}{A}\right) \ell} \quad (7)$$

As an approximation, assume a fundamental-mode shape for the tube of the form

$$\phi(x) = \sin(4\pi x / \ell). \quad (8)$$

From Eq. 6, and using $\ell = 301.5$ in.,

$$f(\phi) = (\ell / 4\pi)^2 = 576 \text{ in.}^2.$$

For the geometry and material of the superheater,

$$k(h, T_s, T_t) = \{1 - 2.87[h - 0.0025(T_s - T_t)]\}^{1/2} - 1. \quad (9)$$

In the superheater, cold-springing was accomplished by straining the shell $1/16 \pm 1/64$ in. The average shell-side (sodium) operating temperature is 830°F ; the average tubeside (steam) operating temperature is 710°F . Substitution of these values in Eq. 9 gives the range of percent change in fundamental frequency for conditions corresponding to equilibrium (or room temperature) and to maximum and mean temperature differentials.

The results in Table III illustrate that, under operating conditions, the natural frequencies can range from 10 to 30% greater than those computed with the assumption of no axial loading.

TABLE III. Percent Change in Fundamental Frequency due to Axial Loading [$k(h, T_s, T_t) \times 100, \%$]

T_s, T_t	$h = 3/64$ in.	$h = 1/16$ in.	$h = 5/64$ in.
$T_s = T_t$	-6.9	-9.4	-12.0
830, 710	30.5	29.8	27.9
830, 770	14.0	11.8	9.8

b. Creep Effects. Operational cycling (e.g., startup and shut-down) of the superheater was considered for its significance in terms of long-term creep effects on altering the vibrational characteristics of the tubes. For example, at elevated operating temperatures ($\sim 800^\circ\text{F}$), creep relaxation of the tensile stresses will result in a decrease in axial load and an accompanying decrease in tube natural frequency.

Creep data⁸ for $2\frac{1}{4}\%$ Cr, 1% Mo steel (Croloy) at 1112°F , as characterized by the secondary creep rate, can be represented by

$$\tau \frac{d\epsilon}{dt} = \left(\frac{\sigma}{30.9} \right)^{7.6} \quad (\tau \equiv \text{one hour}), \quad (10)$$

where $d\epsilon/dt$ is the creep rate in strains/hour, and σ is the stress in tons/in.².

At design operating conditions (temperature difference of $\sim 60^\circ\text{F}$ between shell and tube) and with a $1/16$ -in. strain of the shell, the tensile stress in the tubes is 2240 psi. On the assumption that Eq. 10 holds also at approximately 800°F , the creep rate at the design stress is 1.12×10^{-11} strain/hr. Even at a constant stress of 2240 psi, the strain per 10,000 hr (approximately one year) is only 0.00001%. Therefore, within the limits of this study and using available creep data, we can conclude that creep will have a negligible effect on altering the vibrational characteristics of the tubes.

2. Resonant Vibrations Induced by Cross Flow

a. General Discussion. Baffles are used in a heat exchanger to cause the shell-side fluid to flow at right angles to the axes of the tubes. This maintains a state of turbulence and results in higher heat-transfer coefficients. However, when fluid flows across a bluff body, vortices are shed alternately from each side of the body, inducing a periodic force perpendicular to the flow. If the vortex-shedding frequency is close to a natural frequency of the body, a resonance condition can occur. Therefore, designing to avoid detrimental flow-induced vibrations means keeping the fundamental natural frequency of the tubes sufficiently greater than the maximum expected vortex-shedding frequency.

The vortex-shedding frequency is a function of the geometry of the body, the transverse and longitudinal spacing between adjacent bodies, and the flow velocity. It is represented by the Strouhal number, a nondimensional number that is a function of the Reynolds number, given by

$$N_s = f_v d / U_c$$

$$= N_s(N_R), \quad (11)$$

where U_c is the average velocity through the minimum cross section.

Considerable work has been done on vortex shedding from a single cylinder.^{3,4} As shown in Fig. 2, over a range of Reynolds numbers ($300 < N_R < 2 \times 10^5$) the Strouhal number is approximately a constant equal to 0.2. Although the mechanism is much more complicated in a tube bank, Chen⁵

measured the vortex-shedding frequency in staggered finned and unfinned tube banks and showed that in the range $1.5 \times 10^4 < N_R < 6 \times 10^4$ (the limits of his experiment), the Strouhal number is constant for each arrangement of tubes. Chen presented his results as a family of curves of Strouhal number versus transverse spacing ratio, with longitudinal spacing ratio as a parameter. His curves for unfinned tubes in staggered tube banks are reproduced in Fig. 3.

Fig. 2. Strouhal Number for Vortex-shedding Frequency as a Function of Reynolds Number for a Single Cylinder (reproduced from Ref. 5)

Fig. 3. Strouhal Number vs Transverse-spacing Ratio, with Longitudinal-spacing Ratio ($X_L = L/d$) as a Parameter for Staggered Tube Banks (reproduced from Ref. 5)

$$(N_s)_{str} = f_s d / U_c$$

(12)

To assess the possibility of a resonant condition, the tube Strouhal numbers were compared with the one for the flow. A suggested rule of thumb is that the tube Strouhal number should be at least two to three times greater than the one for vortex shedding to avoid detrimental cross-flow-induced vibration.¹

The Strouhal numbers for the flow given in Figs. 3 and 4 are based on the mean cross-flow velocity through the minimum cross-sectional area. This velocity is determined by calculating a cross-flow area as the product of the number of tubes in a longitudinally sliced section, the minimum clearance between adjacent tubes (pitch minus tube diameter), and the length between baffles (baffle spacing); and then dividing this area into the average volume rate of flow. (A mean cross-flow velocity calculated in this manner for the ATR primary heat exchanger compared favorably with experimentally measured velocities from an independent study.)

Heat-exchanger tubes, as in the EBR-II steam superheaters, are often mounted on an equilateral triangular pitch. Thus two extreme cases of flow direction can be defined. With reference to the inset of Fig. 4, subscripts 1 and 2 represent the two cases, T is the spacing transverse to the flow direction, and L is the longitudinal spacing. Geometrical considerations give $T_1 = 0.577L_1$ and $T_2 = 1.732L_2$. Based on this information, the curves in Fig. 4 were sketched from Chen's results (Fig. 3) as an aid in applying them to heat-exchanger tubes mounted on a triangular pitch.

To characterize the potential for the tube to be excited by a vortex-shedding phenomenon, the Strouhal number for the structure was defined by

Fig. 4. Strouhal Number vs Transverse Spacing Ratio for a Tube Bank Mounted on an Equilateral Triangular Pitch

For the EBR-II superheater, a minimum cross-flow area of 1.22 ft^2 is calculated in Appendix D. At full reactor power, with sodium flowing at $1,250,000 \text{ lb/hr}$, the mean cross-flow velocity is 5.26 ft/sec . This velocity, along with information in Table II, was used to compute the Strouhal numbers for tubes in both support configurations. Even in the "worst" case considered, i.e., full-power flow and tube-support configuration I, Table IV shows that the Strouhal numbers for the tubes are at least twice the Strouhal number associated with flow across a single cylinder ($N_s = 0.2$); this would imply a safe design from the standpoint of this resonance mechanism.

TABLE IV. Strouhal Numbers
for Superheater Tubes

Percent of Full-power Flow ^a	Tube-support Configuration	
	I	II
60	0.62	0.85
74 (45 MWt)	0.50	0.69
80	0.47	0.64
100 (62.5 MWt)	0.38	0.51

^aFull-power flow = $1,250,000 \text{ lb/hr}$;
 $U_c = 5.26 \text{ ft/sec}$.

However, as discussed earlier in this section, tube spacing can have a significant effect on vortex-shedding frequency and hence on Strouhal number for the flow. For example, the baffle plates in the EBR-II

Fig. 5. Baffle Plate for EBR-II Superheaters and Evaporators

superheater (shown in Fig. 5) orient the shell-side flow pattern in the "2" direction, as illustrated in Fig. 4. The 1.438-in.-dia tubes mounted on a 1.938-in. triangular pitch gives $X_t = T_2/d = 2.34$. From the corresponding curve of Fig. 4, we read 0.66 as the Strouhal number for vortex shedding. On comparing this value with the tube Strouhal numbers in Table IV, we observe that it is, in general, greater than those for the tubes; in fact, at the current operating power (45 MWt), it is very close to the tube Strouhal number associated with tube-support configuration II. This would indicate an unsafe design, since a resonant condition, as indicated by the coincidence of

these frequencies, would lead to relatively rapid failure of the superheater. But, as mentioned in the introduction, the superheaters have operated trouble-free for several years. Thus, the validity of the analysis is suspect.

Although Chen's work is generally accepted and used, his results are not in complete agreement with those published by König and Gregorig,⁶ particularly for transverse-spacing ratios less than 1.75 (see Fig. 6).

Fig. 6. Comparison of Strouhal Numbers as a Function of Transverse and Longitudinal Spacing

The reason may be that Chen's results for small transverse-spacing ratios are extrapolations; the smallest ratio used in his measurements was 1.735. The deviations for ratios greater than 1.75 are not fundamental; interpolation of the curves of König and Gregorig in Fig. 6 yields a flow Strouhal number of approximately 0.33, or one-half that obtained from Chen's curves. Moreover, comparison with values in Table IV reveals that, at the current EBR-II power, the tube Strouhal numbers are sufficiently greater, indicating a safe design, as borne out by the successful operation of the superheater. However, at full-power flow, the proximity of the tube and the flow Strouhal

numbers for tube-support configuration No. I indicates that a vibration problem might be expected.

b. Effect of Axial Loads. To take into account the effects of preload and axial loads at operating conditions, we recalculated the tube Strouhal numbers, using the results in Table II. As shown in Table V, the overall effect at operating conditions is an increase in tube natural frequency, and hence in the Strouhal number. As before, comparison with published Strouhal numbers associated with vortex shedding (0.2 for a single tube; 0.66 for a tube bank;⁵ 0.33 for a tube bank⁶) indicates a potential vibration problem or, at best, a marginally safe design. These results also call attention to a further uncertainty in predicting tube natural frequencies and support the use of a "safety factor" of from 2 to 3 between Strouhal numbers of tubes and flow.

TABLE V. Range of Strouhal Number for Superheater Tubes at Operating Conditions

Percent of Full-power Flow	Tube-support Configuration	
	I	II
60	0.68-0.81	0.93-1.11
74 (45 MWt)	0.55-0.65	0.76-0.90
80	0.52-0.61	0.70-0.83
100 (62.5 MWt)	0.42-0.50	0.56-0.66

3. Limitations of Strouhal-number Comparisons

In general, Strouhal-number comparisons are good only for qualitative evaluation of the potential for cross-flow-induced resonant vibrations in heat exchangers. Among the various reasons are:

a. Measured vortex-shedding frequencies are often determined from single-cylinder and tube-bank models using rigid cylinders. In the works of Chen⁵ and König and Gregorig,⁶ a single tube in the bank was allowed to oscillate; however, even in these cases, the interaction effects between adjacent vibrating tubes are neglected.

b. In theoretical studies, idealized cross flow with near-uniform velocity distribution is assumed, whereas in reality (i.e., within a heat exchanger) the flow pattern is not one of ideal cross flow, and a nonuniform velocity distribution is expected.

c. Influence of the parallel component of flow is neglected; its effect may be to break up and damp out vortex formation.

d. Accurately predicting the natural frequencies of tube vibrations is often difficult because of tube geometry (U-tubes, sine-wave bends, etc.), mounting methods, and baffle-tube clearances.

e. Most of the data are derived from wind-tunnel tests; little work has been done in water tunnels.

4. Hydroelastic Instabilities

a. External Flow. Although the secondary-fluid flow pattern is primarily one of cross flow (due to the presence of baffles), a strong parallel-flow component can be expected in the vicinity of the tubes in the outer rows (nearest the shell). Paidoussis⁹ derived an expression for computing a dimensionless complex frequency of vibration ω of a slender cylinder in an axial flow field. The response is stable or unstable accordingly as the imaginary part of ω is positive or negative; $\text{Im}(\omega) = 0 \rightarrow$ neutral stability.

In addition to being a function of other nondimensional parameters, the dimensionless frequency is a function of a dimensionless flow velocity defined by

$$u = \left(\frac{M}{EI} \right)^{1/2} U \ell. \quad (13)$$

If u_{cr} is defined as the value of u at which $\text{Im}(\omega) = 0$, a critical flow velocity is defined by

$$U_{cr} = U_c (u = u_{cr}).$$

Paidoussis also plotted the dimensionless complex frequency of the three lowest modes of pinned-pinned, clamped-clamped, and clamped-free cylinders as a function of u for a set of nondimensional parameters closely approximating those associated with the superheater tubes. His results are summarized in Table VI. For the superheater tubes,

$$U_{cr} = 169u_{cr} \text{ ft/sec.} \quad (14)$$

Inspection of Eq. 14 and the values in Table VI leads to the conclusion that fluid elastic instability due to external parallel flow is not a problem, because the critical flow velocities are impractical.

TABLE VI. Critical Dimensionless Flow Velocity (u_{cr})

Mode	Pinned-pinned	Clamped-clamped	Clamped-free
1	3.13 ^a	-	2.62 ^a
2	5.55	8.4	4.7
3	8.25	11.2	7.4

^aBuckling instability.

b. Internal Flow. Housner¹⁰ showed that unless the fluid inside the tube is very dense and is flowing at very high velocities, it does not appreciably affect the vibration or stability of the tube. Therefore, for steam flowing in the EBR-II superheater tubes (low density and velocity), no problem is anticipated from this means of vibration.

5. Baffle Design and Orientation in Flow Field

For tubes mounted on an equilateral triangular pitch, the orientation of the flow (whether in the 1 or 2 direction as illustrated in Fig. 4) is defined by the baffle design--more specifically, by the diameter on which the baffle is cut. As shown in Figs. 3 and 4, the spacing ratios can have a significant effect on the flow Strouhal number. Since the spacing ratios are determined, in part, by the flow orientation, baffle design will influence the vortex-shedding frequency and, therefore, could be the determining factor in establishing a resonant condition.

For example, when Chen's results⁵ are applied to the EBR-II baffle design (shown in Fig. 5), a potential resonant problem is predicted in the superheater. However, if the baffle were cut at right angles to the diameter on which it was designed to be cut, the flow pattern through the heat exchanger would be oriented in the 1 direction rather than in the 2 direction. For such a flow orientation, the transverse spacing ratio is 1.35. On the appropriate curve of Fig. 4, the Strouhal number for the flow reads approximately 0.18. This is close to the value of 0.2 for vortex

shedding from a stationary single cylinder. As discussed in Section 2.a above, based on a Strouhal number of 0.2, the EBR-II superheater design is safe by a factor of 2.5 at the current operating power. Hence, according to Chen's results and this analysis, the design or "cut" of the baffle is the difference between a conservatively safe and a potentially unsafe system from the standpoint of a resonant condition between vortex shedding and tube natural frequencies.

6. Tube-Baffle Impact

Because of the relatively small clearances between the tubes and the baffle, it is realistic to assume that if the tubes vibrate at all, some contact, or impact, will occur. Among other things, the impact will affect the vibrational characteristics and response of the tubes; for example, motion between stops effectively increases the frequency of free vibrations. Also, the impact phenomenon and impact forces and velocities are important because they relate to increased bending stress, contact stress, and wear.

To gain insight into the impact phenomenon, Chen et al.¹¹ theoretically studied a force-excited beam supported at the ends, with motion-limiting stops at the midpoint, and developed a method for solution. Briefly, the method involves determining the "gross" steady-state motion from a single-mode approximation in which the system is reduced to a bilinear vibrating model. Results from the approximation serve as initial conditions in a classical modal analysis in which many modes are included to describe the dynamic stresses and forces resulting from impacting. Finally, a computer code is used to calculate displacement, shearing force, and bending stress of a beam with motion-constraint stops at the midpoint. This code was used to gain insight into the tube-baffle impact effects in the EBR-II superheater.

The distributed transverse force associated with vortex shedding can be represented by¹²

$$f(x,t) = C_L \left(\frac{1}{2} \rho_f U_c^2 \right) d \sin(\omega_v t), \quad (15)$$

where C_L is the lift coefficient. The forcing function given by Eq. 15 is highly idealized: The distribution of flow velocity along a length of tube between baffles is not expected to be uniform; therefore, both u and ω_v , which is related to u through the Strouhal number, are functions of distance measured along the tube. Of equal importance, at least for small amplitudes of tube motion, the phasing of the vortex shedding is coherent only over a short length. There is also uncertainty in selecting a suitable value for the lift coefficient; values suggested in the literature range from 0.2 to 2.0, the problem being further complicated for flexible tubes in tube banks.

In the present study, the mean of the reported values was used ($C_L = 1$). Also, it was assumed the vortices are shed in phase at a constant vortex-shedding frequency determined from the mean cross-flow velocity. Thus the calculation can be considered conservative. To investigate the effect of a nonuniform velocity distribution, we express the cross-flow velocity by

$$U = U(x) = U_c \psi(x), \quad (16)$$

where U_c is the mean cross-flow velocity, and $\psi(x)$ is a shape function normalized such that

$$\int_0^{\ell} \psi(x) dx = \ell.$$

Equation 15 can now be written as

$$f(x,t) = \alpha_1 \psi^2(x) \sin(\omega_v t), \quad (17)$$

where

$$\alpha_1 = 0.5 C_L \rho_f U_c^2 d.$$

For the superheater, $\alpha_1 = 2.78 \text{ lb/ft}$, $\omega_v = 55.2 \text{ rad/sec}$ for $N_s = 0.2$ (single-tube value), $\omega_v = 182.2 \text{ rad/sec}$ for $N_s = 0.66$ (tube-bank value).⁵

Two velocity distributions were considered:

$$\psi^2(x) = \begin{cases} 1 \\ \frac{\pi}{2} \sin \frac{\pi x}{\ell} \end{cases} \quad (18)$$

In their analysis, Chen et al.¹¹ used a nondimensional frequency given by

$$\Omega = \left(\frac{m}{EI} \right)^{1/2} \omega_1 \ell^2, \quad (19)$$

where ω_1 is the fundamental frequency of tube vibration without the influence of stops. On application to the EBR-II superheater, Eq. 19 yields $\Omega = 16.0$ for $N_s = 0.2$ (single-tube value) and $\Omega = 52.8$ for $N_s = 0.66$ (tube-bank value) for the two velocity distributions. Figure 7 shows the corresponding amplitude-response curves (obtained from a single-mode analysis) for the two velocity distributions. The procedure is to enter the curves of Fig. 7 with the calculated value of Ω and read off the corresponding steady-state, nondimensional amplitude value. This value is then used in a multimode analysis to predict dynamic bending stresses and shear forces.

Fig. 7. Amplitude-response Curves for Tube-baffle Impact in EBR-II Superheater (reproduced from Ref. 11)

The resulting maximum-response distributions of deflection, shearing force, and bending stress are given in Figs. 8 and 9 for the uniform and half-sine velocity distributions, respectively. For a Strouhal number of 0.2 ($\Omega = 16.0$), the maximum bending stresses are less than 1000 psi. On the other hand, for a Strouhal number of 0.66 ($\Omega = 52.8$), the stresses are significantly higher because the tube is vibrating near a

Fig. 8. Dynamic Response of Superheater Tube Excited by Uniform Cross Flow (reproduced from Ref. 11)

Fig. 9. Dynamic Response of Superheater Tube Excited by Nonuniform Cross Flow (reproduced from Ref. 11)

resonant condition (operating point 2 on Fig. 7). However, even this situation is less severe than it may seem. First of all, due to the particular nonlinearity of the problem, a jump phenomenon is exhibited at a Strouhal number of 0.66: The tube can vibrate at operating points 2 or 3. If it vibrates at point 3, the tube does not contact the stop, and the stress is very low. Because operation is close to an instability region, if for some instant the tube vibrates at point 2, it can be expected to jump back to point 3. Second, the results are based on the assumption of pinned supports. As long as there is some rotational constraint, the resonant frequency will be higher and consequently the response will be much smaller. Third, the response will be smaller if the damping factor (ratio of effective viscous damping coefficient to the critical damping associated with the first mode) is greater than the assumed value of 0.01.

In summary, a comparison of response curves for the two different velocity distributions (Figs. 8 and 9) shows relatively little difference for a Strouhal number of 0.66. For a Strouhal number of 0.2, the curves for a uniform velocity distribution are slightly greater in magnitude. Although the results are limited, they lend support to the contention that variations in cross-flow velocity distribution occurring in a heat exchanger might not strongly affect the dynamic response of the tubes and that the uniform distribution might represent the conservative case.

B. Evaporator

As with the superheater, state-of-the-art technology was used to evaluate the potential for detrimental conditions of resonance and instability in the evaporator. Again, within the limitations of this technology, the results indicate that tube failures due to these mechanisms are not to be expected. The pertinent findings are summarized below.

Except for the absence of core tubes, the evaporator is structurally identical to the superheater. However, the environmental conditions are less severe: Operating temperatures and temperature differentials are lower and the sodium flowrate is one-fourth that through the superheater (as shown in Table I).

TABLE VII. Natural Frequencies
of Evaporator Tubes

n	f_n^I , Hz	f_n^{II} , Hz
1	16.1	21.9
2	24.9	30.4
3	35.2	41.5

1. Tube Natural Frequencies

Because of the structural similarity, the same eigenvalues were used in Eq. 1 to compute the tube natural frequencies. Table VII lists the first three natural frequencies for both tube-support configurations.

Similar calculations were made to determine the changes in natural frequency incurred by axial loading and creep relaxation. Table VIII lists the percent change due to axial loading, as given by Eq. 11. Long-term creep effects and operational cycling have a negligible influence on the natural frequencies of the tubes.

TABLE VIII. Percent Change in Fundamental Frequency due to Axial Loading^a

T_s, T_t	$h = 3/64$ in.	$h = 1/16$ in.	$h = 5/64$ in.
$T_s = T_t$	-6.9	-9.4	-12.0
677, 615	14.4	12.7	10.5
677, 646	4.4	2.2	-0.1

^a $h \equiv$ amount the shell is strained; $T_s \equiv$ shell temperature;
 $T_t \equiv$ tube temperature

2. Resonant Vibrations due to Vortex Shedding

For the baffle design and resultant flow-field orientation in the evaporator, the Strouhal number for vortex shedding is 0.66 from Fig. 3, or 0.33 from Fig. 6. At operating conditions, the tube Strouhal numbers (listed in Table IX) can be expected to increase according to the percentages given in Table VIII.

TABLE IX. Strouhal Numbers for Evaporator Tubes

Percent of Full-power Flow ^a	Support Configuration	
	No. 1	No. 2
60	2.44	3.29
74 (45 MWt)	1.97	2.67
80	1.83	2.46
100 (62.5 MWt)	1.46	1.97

^aFull-power flow = 312,500 lb/hr; $U_c = 1.32$ ft/sec.

For all flows and both tube-support configurations (including the possible changes caused by axial loading), the tube Strouhal numbers are at least double the flow Strouhal number for vortex shedding. Thus vortex-shedding-induced resonant vibrations are not anticipated.

3. Instabilities Caused by Parallel Flow

As for the superheater, fluid velocities through the shell side and tube side of the evaporator are sufficiently less than the critical values so that no problem is anticipated from an instability caused by parallel-flowing fluid.

4. Baffle Design and Orientation in Flow Field

With regard to baffle design and tube-baffle impact, the observations relative to the superheater also apply to the evaporator.

III. DISCUSSION

The foregoing evaluation emphasized a vibration mechanism involving the vortex-shedding phenomenon. Based on the experimental results of Chen⁵ for vortex shedding in tube banks, a vibration problem in the EBR-II superheater is indicated by the near coincidence of tube natural frequencies and vortex-shedding frequency. On the other hand, more recent results,⁶ if used as the basis for evaluation, indicate a reasonably safe design at current operating power and a marginally safe design at full power. Because of various limitations and idealizations in acquisition of data on vortex shedding in tube banks and the uncertainties in computing natural frequencies and mean cross-flow velocity, such comparisons are good only for qualitative evaluation. Nevertheless, this is perhaps the best calculation that can be performed today. In fact, Nelms and Segaser¹ found it to be a reliable calculation in their survey, since in all the units that experienced vibration problems, there was a close agreement between the natural frequency of tube vibration and the vortex-shedding frequency.

Although Nelms and Segaser used Chen's results⁵ to determine the flow Strouhal numbers in the heat exchangers of their survey, the spacing ratios were such that in all cases the Strouhal number for flow through the tube bank was calculated to be approximately 0.2, the value for a single cylinder. With regard to the EBR-II superheaters and evaporators, application of Chen's results appears to be only the second instance reported in which the spacing ratio is such that a Strouhal number significantly different from that of a single cylinder was encountered.* In both instances, confrontations between prediction and operating performance are inconsistent with the general findings of Nelms and Segaser, since the indicated vibration problems have not been experienced. This leads one to question the validity of the analysis and, more specifically, the mechanism of vibration.

In his consideration of the buffeting excitation of boiler-tube vibration, Owen¹³ observed that although the vortex-shedding mechanism is an attractive one, it relies heavily on the assumption of a regular periodic flow pattern. He went on to state that, "Deep within a tube-bank, the cumulative growth of random irregularities in the labyrinthine-like, high-Reynolds-number flow must lead to a state of almost complete incoherence on which it is difficult to imagine any superposed regular pattern (apart from a general drift through the bank) to be discernible." I agree with Owen's observation

*Using Chen's correlations, Nelms and Segaser obtained a value of 0.5 for the flow Strouhal number in the steam superheater of the Piqua Nuclear Power Facility.

and, along with Owen, feel that the tubes are force-excited by the random pressure fluctuations in the turbulent, shell-side flow. Owen developed a hypothesis based on the idea of a dominant frequency associated with the turbulent fluctuations. He assumed this frequency to be identifiable with the dominant frequency of the fluctuating force on the tubes. Should this assumption be correct, it is not difficult to imagine the onset of a "resonant" condition when the dominant frequency coincides with the natural-vibration frequency of the tube.

Owen found that the dominant frequency is proportional to the average flow velocity through the tube bank. In particular, theoretical considerations led him to the equation

$$\frac{\nu L}{U} \frac{T}{d} = K(1 - d/T)^2, \quad (20)$$

where K is a constant. Subsequent comparison of the theoretical relationship with (rather limited) experimental data yielded a K equal to 3.05.

Further considerations enabled Owen to propose the following rule for estimating ν : "The dominant frequency of the vibrations in a bank of tubes, for which the ratio of the diameter to the lateral spacing lies between 0.2 and 0.6, is equal to the interstitial gas velocity divided by twice the distance between successive rows," i.e.,

$$\nu = \frac{U}{2L}. \quad (21)$$

In terms of a Strouhal number,

$$\frac{\nu d}{U} = N_s = \frac{1}{2} \frac{d}{L}. \quad (22)$$

For the flow orientation in the EBR-II superheater and evaporator, as determined from Fig. 5, L_2/d equals 1.35. Accordingly, from Eq. 22,

$$N_s = 0.37. \quad (23)$$

A comparison of Eq. 23 with the tube Strouhal numbers given in Table V indicates a safe design at current operating power (45 MWt) and a marginal design at full power (62.5 MWt).

Once again, if the baffle had been cut along the "short" diameter, the longitudinal spacing ratio would be L_1/d and equal to 2.34. In this case, Eq. 22 gives

$$N_s = 0.21. \quad (24)$$

Hence, Owen's analysis also illustrates the desirability of having the baffle designed to be cut along the "short" diameter rather than the "long" diameter.

Owen included a typical experimentally determined one-dimensional turbulent-energy spectrum; this is reproduced in Fig. 10. The transverse and longitudinal spacing ratios are $T/d = 1.5$ and $L/d = 1.3$, respectively. Using Owen's empirical relationship given by Eq. 21,

$$\frac{\nu L}{U} \frac{T}{d} = \frac{1}{2} \frac{T}{d}. \quad (25)$$

Accordingly, for $T/d = 1.5$, the peak in the turbulent-energy spectrum should occur at $(\nu L/U)(T/d) = 0.75$, rather than at 0.4, as shown in Fig. 10.

Fig. 10. Typical One-dimensional Turbulent-energy Spectrum Determined Experimentally (reproduced from Ref. 13)

The actual peak location can be translated into a Strouhal number of 0.21, which again corresponds to the number associated with vortex shedding from a single tube. This correspondence may be a coincidence, or it may be that the peak in the turbulent-energy spectrum for cross flow through tube banks occurs at a constant Strouhal number of 0.2 (single-tube value for vortex shedding). If the latter is true, a mechanism for tube vibration based on

buffeting can be postulated which agrees with the vibration problems that have occurred and that would have been "predicted" by the proximity of the tube Strouhal numbers to a flow Strouhal number of approximately 0.2.

This mechanism can be described as follows:

1. There is a dominant frequency associated with the turbulent-energy spectrum for flow velocity which can be characterized by a Strouhal number $(\nu d/U)$ of 0.2.

2. If the flow velocity through the tube bank is such that the dominant frequency of turbulent fluctuations corresponds to a natural frequency of tube vibrations (i.e., if the tube Strouhal number is close to 0.2), a type of resonance occurs. This resonance is sharpened by the selectivity of the tube in responding to that group of eddies whose dimensions are comparable with the tube diameter.¹³

3. The resulting increased amplitude of vibration caused by the "resonant" condition generates vortices at the frequency of oscillation (by virtue of the tube's motion through the fluid) and the phasing* necessary to continue the vibration, giving rise to a type of vibration self-excitation.¹⁴

4. The process is reenforcing, leading to large-amplitude destructive vibrations and eventual tube failure.

The concept is intuitively appealing. Owen's work has provided insight and initial experimental evidence to support the hypothesis. More work, particularly of an experimental nature, is required.

IV. RECOMMENDATIONS FOR FUTURE WORK

Observations made in the preceding section lead to the consensus that (1) analytical treatment of flow-induced vibrations in heat-exchanger tubes involves many complexities, and (2) the state-of-the-art technology is insufficiently developed to cope with these complexities; hence, (3) numerous problem areas still exist and must be resolved before reliability of a given heat-exchanger design can be predicted with a reasonable degree of confidence. Despite these complexities, however, answers to questions regarding reliability, and new analytical or experimental techniques to provide these answers, are needed today.

Accordingly, the recommended areas of future work can be divided into two classes:

1. Applied studies of an experimental nature, with the objective of developing correlations among a few basic design parameters over which the heat-exchanger designer has some control.

2. Basic or fundamental studies of an integrated theoretical-experimental nature. These would be designed to obtain insight into, and understanding of, the mechanism of flow-induced vibration, the characteristics of flow through tube banks, the measurement and characterization of system damping, and the forcing function. The first class of work would be relatively "short-ranged," the purpose being to satisfy the immediate needs and concerns of heat-exchanger designers and plant operators. The second class of work could be considered "long-range," since its purpose is to advance the state of the art.

*Phase coherence, or the distribution of phase, across the span of tubes is an important consideration in this mechanism because it determines the effectiveness of the fluctuating force in inducing and sustaining mechanical vibrations. An increased vibration amplitude can be expected to increase the correlation length or the length of tube over which the force is coherent with respect to phase.

Nelms and Segaser¹ addressed their recommendations to the first class of work. More specifically, they suggested that the experimental effort be concentrated on cross-flow heat exchangers containing U-tube bundles. They also suggested that this effort include (1) the effects of velocity distribution as a function of tube size and spacing, baffle spacing and openings, and bypass areas; (2) fluid conditions at the entrances and exits; and (3) structural and fluid damping parameters, along with such factors as baffle thickness and tube-to-baffle clearances that affect the vibrational frequency of tubes. As an experimental setup, they suggested (1) a fluid-circulation system capable of attaining high fluid velocities over a bank of tubes within a bundle diameter of about 24 in., and (2) an assembly equivalent to a heat exchanger about 12 ft long to provide flexibility in baffle spacing and support arrangement, tube pattern and spacing, and baffle clearance and shape. In general, I agree with their recommendations.

A more detailed and in-depth study is implied in the second class of work in which Argonne is currently involved.* Of immediate concern is the cause of the vibration mechanism (i.e., whether by periodic vortex-shedding, turbulent buffeting, a fluid-elastic phenomenon, or some combination of these) and the range of flow velocities (in terms of Reynolds number or Strouhal number) over which each mechanism is prevalent.

The experimental phase of study will include (1) single-tube, (2) tube-bank, and (3) heat-exchanger models. Single-tube experiments have application in developing and testing an instrumented test element (for use in tube-bank tests), and in verifying the ability to reproduce certain basic results of earlier investigators. Experiments involving tube-bank models deserve major emphasis. A 5 x 5 tube bank, made up of spring-mounted rigid tubes, with variable tube spacing is considered appropriate for measurements of (1) tube displacements, (2) temporal and spatial variation of surface pressures, (3) lift and drag forces, and (4) flow velocity and its distribution. Various interaction effects (such as phasing relationships and the effect of displacement amplitude on correlation lengths, virtual mass of a tube vibrating within a bundle, and frequency synchronization between vortex-shedding and tube-vibration frequencies) also can be studied.

Tube-baffle clearance, baffle-plate thickness, and baffle spacing are parameters that can markedly affect the vibrational characteristics and response of heat-exchanger tubes. These factors have received only limited study,¹⁵ the results of which do not lend themselves to evaluation of a given design. A theoretical analysis relating to tube-baffle impact has been performed,¹¹ and associated experimental work is planned. However, there is a need for fundamental theoretical and experimental work in studying the effects of variations of these parameters.

*Work sponsored by the Special Technology Branch, Division of Reactor Development and Technology, U.S. Atomic Energy Commission; USAEC Budget Activity No. 04-40-03-01-1.

Prediction of root-mean-square tube displacement and knowledge of its probability law are necessary to determine if tubes will impact with adjacent tube. Also, such information as rms stress will be required when appropriate fatigue and wear data become available. Small-amplitude tube displacement is supposedly caused by randomly fluctuating forces imposed on the tubes by the turbulent eddies in the flow. In such a broad-band random process, the tube acts as a mechanical filter in extracting energy in a frequency band near its natural frequency. Prediction of rms response will require a study and mathematical characterization of the spectrum of the fluctuating forces as well as knowledge of system damping and its dependence on mean axial flow velocity.

When sufficient insight and understanding have been gained from the fundamental studies, the applicability of the results to the geometrical configurations and flow patterns of heat exchangers will have to be confirmed. This will necessitate instrumented flow tests with model heat exchangers. As a final step, the results will have to be put into a form (equation or graph) that is readily usable by designers.

APPENDIX A

Calculation of Constant in Frequency Equation

The free-vibration natural frequency of a uniform beam is proportional to the square of the eigenvalue of the frequency equation. The constant of proportionality is

$$\left(\frac{EI}{\rho A}\right)^{1/2}$$

The virtual mass of a body is the "effective inertia" of the body, which is greater when the body is accelerated in a fluid than in a vacuum. The excess of virtual mass over the actual mass is the added mass caused by the surrounding fluid. This added mass depends on the size and shape of the body, the direction of acceleration, and the density of the fluid. Expressing the added mass as a fraction of the mass of fluid displaced by the body, we can write

$$m_{\text{added}} = K\rho_{\text{fluid}}A_{\text{body}} \quad (\text{A.1})$$

For a long circular cylinder accelerated transversely, analysis and experiment give $K = 1.0$.¹⁶ Therefore, the virtual mass of the superheater tube can be written

$$\rho A = \rho A_D + K\rho_{\text{Na}}A_B + \rho A_C, \quad (\text{A.2})$$

where ρA_D is the mass per unit length of the duplex tube, $K\rho_{\text{Na}}A_B$ is the added mass per unit length due to the sodium environment, and ρA_C is the mass per unit length of the core tube. In consistent units,

$$\begin{aligned} \rho A &= (0.283)(0.734) + (1.0)(0.0312)(1.624) + (0.283)(0.069) \\ &= 0.278 \text{ lb/in.} \end{aligned} \quad (\text{A.3})$$

In computing the flexural rigidity, we neglect the effect of the core tube, and the moment of inertia of the cross-sectional area is given by

$$\begin{aligned} I &= \frac{\pi}{64} (d_o^4 - d_i^4) \\ &= 0.147 \text{ in.}^4 \end{aligned} \quad (\text{A.4})$$

Young's modulus for Croloy at 765°F is approximately 26×10^6 psi; hence,

$$EI = 3.82 \times 10^6 \text{ lb-in.}^2 \quad (\text{A.5})$$

From Eqs. A.3 and A.4,

$$\left(\frac{EI}{\rho A}\right)^{1/2} = 506 \text{ ft}^2/\text{sec} \quad (\text{A.6})$$

and

$$\omega_n = 506 \lambda_n^2. \quad (\text{A.7})$$

APPENDIX B

Derivation of Frequency Equation for Tube

Figure B.1 shows simplified model of the EBR-II superheater. Considering the unit in halves, there exist two basic support configurations. The following assumptions are made: (1) The tubes are fixed (clamped) at the tubesheet interface; (2) the baffles restrict the tubes to no motion at the baffle locations; and (3) the tubes can be considered as uniform beams. With these assumptions, the problem of computing the natural frequencies of tube vibration reduces to that of determining the eigenvalues for a uniform beam with three internal supports.

Fig. B.1
Simplified Model of EBR-II
Steam Superheater

The method of receptances¹⁷ is well-suited for determining the natural frequencies of such a beam-support system. A receptance, α_{xy} , is defined as the displacement at coordinate x due to a harmonic force at coordinate y . Similarly,

$\alpha_{x'y} \equiv$ slope at x due to harmonic force at y ,

$\alpha_{xy'} \equiv$ deflection at x due to harmonic moment at y ,

and

$\alpha_{x'y'} \equiv$ slope at x due to harmonic moment at y .

The beam-support system can be represented as two subsystems linked by displacement coordinates as shown in Fig. B.2(a). A block diagram of the composite system is shown in Fig. B.2(b). The frequency equation for such a composite system can be written as the determinantal equation¹⁸

$$\begin{vmatrix} \beta_{11} + \gamma_{11} & \beta_{12} + \gamma_{12} & \beta_{13} + \gamma_{13} \\ \beta_{21} + \gamma_{21} & \beta_{22} + \gamma_{22} & \beta_{23} + \gamma_{23} \\ \beta_{31} + \gamma_{31} & \beta_{32} + \gamma_{32} & \beta_{33} + \gamma_{33} \end{vmatrix} = 0, \quad (\text{B.1})$$

where

$\beta_{ij} \equiv$ receptances for subsystem B,

and

$\gamma_{ij} \equiv$ receptances for subsystem C.

Fig. B.2. Beam Support System Used to Derive Frequency Equation for Tube

Subsystem C consists of a series of three rigid supports; therefore,

$$\lambda_{ij} = 0 \text{ (all } i \text{ and } j). \quad (\text{B.2})$$

On expanding Eq. B.1 with Eq. B.2 and using the reciprocity of receptances, i.e., $\alpha_{xy} = \alpha_{yx}$, we obtain

$$\beta_{11}\beta_{22}\beta_{33} - \beta_{11}\beta_{23}^2 - \beta_{22}\beta_{13}^2 - \beta_{33}\beta_{12}^2 + 2\beta_{12}\beta_{13}\beta_{23} = 0. \quad (\text{B.3})$$

The β_{ij} correspond to interior receptances of the beam. From Ref. 17, for a clamped-clamped beam and $0 \leq x \leq y$,

$$\begin{aligned} \beta_{xy} = & \{-F_7(\lambda\ell)F_8(\lambda x)F_8[\lambda(\ell - y)] - F_{10}(\lambda\ell)F_{10}(\lambda x)F_8[\lambda(\ell - y)] \\ & - F_{10}(\lambda\ell)F_8(\lambda x)F_{10}[\lambda(\ell - y)] \\ & + F_8(\lambda\ell)F_{10}(\lambda x)F_{10}[\lambda(\ell - y)]\} / 4EI\lambda^3 F_3(\lambda\ell), \end{aligned} \quad (\text{B.4})$$

where

$$F_3(\lambda x) = \cos \lambda x \cosh \lambda x - 1,$$

$$F_7(\lambda x) = \sin \lambda x + \sinh \lambda x,$$

$$F_8(\lambda x) = \sin \lambda x - \sinh \lambda x,$$

and

$$F_{10}(\lambda x) = \cos \lambda x - \cosh \lambda x.$$

A computer code was used to solve Eq. B.3 for the eigenvalues (λ_n); these are given in Table II.

APPENDIX C

Derivation of Axial Load due to Preload and Thermal Expansion

An initial preload is induced by a cold springing, which is accomplished by maintaining a desired temperature differential between the shell and the tubes while the tubes are being welded to the sodium tube-sheet. (A wraparound heater is used to heat the shell; the tubes are cooled with circulating water.) The desired differential expansion between the shell and the tubes is $1/16 \pm 1/64$ in., which is given by

$$h = \epsilon_1^i \ell_1 - \epsilon_2^i \ell_2. \quad (C.1)$$

At room temperature, equilibrium requires

$$\sum p = 0, \quad p_1 + np_2 = 0, \quad (C.2)$$

where n is the number of tubes. The resultant force is

$$p_1 = S_1 A = E_1 \epsilon_1^i A \quad (C.3)$$

in the shell, and

$$p_2 = S_2 a = E_2 \epsilon_2^i a \quad (C.4)$$

in a tube. Equations C.1-C.4 can be combined to give

$$p_2^i = -h / (\ell_2 / E_2 a + n \ell_1 / E_1 A) \quad (C.5)$$

for the initial preload on the tubes,

$$S_2^i = -h / (\ell_2 / E_2 + n \ell_1 a / E_1 A) \quad (C.6)$$

for the initial prestress in the tubes, and

$$S_1^i = h / (\ell_2 A / n E_2 a + \ell_1 / E_1) \quad (C.7)$$

for the initial prestress in the shell.

To calculate the instantaneous loads and stresses due to the operating temperature difference between shell and tubes, an initial equilibrium position is assumed as described by

$$p_1^i + np_2^i = 0. \quad (C.8)$$

The free elongation due to a temperature change ΔT is

$$e = \alpha(\Delta T)\ell. \quad (C.9)$$

At the equilibrium position,

$$e_1 + \epsilon_1^T \ell = e_2 + \epsilon_2^T \ell \quad (C.10)$$

and

$$\sum p = p_1^i + p_1^T + n(p_2^i + p_2^T) = 0. \quad (C.11)$$

From Eqs. C.11 and C.8,

$$E_1 \epsilon_1^T A + n E_2 \epsilon_2^T a = 0. \quad (C.12)$$

Substituting Eq. C.10 into Eq. C.12 gives

$$\epsilon_2^T = \frac{-E_1 A (e_2 - e_1)}{(E_1 A + n E_2 a) \ell}. \quad (C.13)$$

The axial load due to the combined preload and thermal load is given by

$$p_2 = S_2 a = E_2 a (\epsilon_2^i + \epsilon_2^T). \quad (C.14)$$

Therefore, if we let $e_2 = \alpha_2(\Delta T)_2 \ell$ and $e_1 = \alpha_1(\Delta T)_1 \ell$, and assume $E_1 = E_2 = E$, $\alpha_1 = \alpha_2 = \alpha$, $T_2^i = T_1^i$, and $\ell_1 = \ell_2 = \ell$, we obtain

$$p_2 = - \frac{E a [h - \alpha(T_1 - T_2)\ell]}{(1 + na/A) \ell}. \quad (C.15)$$

APPENDIX D

Calculation of Cross-flow Area

Calculation of the Strouhal numbers for the tubes requires a value for the mean cross-flow velocity. This velocity can be obtained by dividing the shell-side volumetric flowrate by the cross-flow area. Therefore, the problem reduces to that of computing a cross-flow area.

In an analysis of tube vibration in the ATR primary heat exchanger, an average cross-flow velocity was computed by calculating a cross-flow area as the product of the number of tubes in a "longitudinally sliced" section, the clearance between adjacent tubes, and the length between baffles (baffle spacing), and dividing this area into the average volume rate of flow. This yielded an average cross-flow velocity of 6.05 ft/sec, which compared favorably with the 5.5 ft/sec determined experimentally. (Experimental values varied from 3 to 7 ft/sec.)

The same approach was used in the EBR-II superheater evaluation. Taking the number of tubes at a midsection to be 9, with a clearance of 0.5 in. between adjacent tubes and an average baffle spacing of 3.25 ft, the cross-flow area is calculated as

$$A_x = 9 \cdot \frac{0.5}{12} \cdot 3.25 = 1.22 \text{ ft}^2. \quad (\text{D.1})$$

The area between the tube bundle and shell is neglected; also the relationship is good only near the midplane of the tube bundle.

Nelms and Segaser¹ presented relationships listed by Tinker¹⁹ for determining average shell-side velocities for use in rating commercial heat exchangers. The basic relationship is

$$U_c = W / [\rho_f A_x (3.6 \times 10^3)], \quad (\text{D.2})$$

where U_c is the average shell-side velocity (ft/sec), W is the mass flowrate through the heat-exchanger shell (lb/hr), ρ_f is the density of the shell-side fluid (lb/ft³), and A_x is the average cross-flow area within the tube bundle between baffle windows (ft²). For tubes mounted on a triangular pitch, Tinker recommended

$$A_x = \frac{C_w}{144} \left(\frac{P - d_o}{P} \right) D_B L_B, \quad (\text{D.3})$$

where C_w is that portion of the tube-bundle diameter representing the mean bundle width between baffle windows (Tinker suggested a value of 0.97); all length dimensions are in inches.

As a check, we compute for the EBR-II superheater

$$A_x = \frac{0.97}{144} \left(\frac{1.938 - 1.438}{1.938} \right) (17.6)(39) = 1.19 \text{ ft}^2. \quad (\text{D.4})$$

This value compares favorably with the value of 1.22 ft^2 given by Eq. D.1 and used in the evaluation. Similarly, the average cross-flow velocity of 5.40 ft/sec computed from Eq. D.3 compares with the value of 5.26 ft/sec used in the analysis.

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