HIGHER ORDER MAGNET FIELD MULTipoles
APERTURE EFFECTS, AND TRACKING STUDIES

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Review of Tracking Theory (my view)

The instabilities are non-resonant; not associated with the $y$-values going to some resonance line in $y_0 + n y_x = q$.

The instabilities are not stochastic—they happen quite fast.

No particular resonance dominates. Classical non-linear theory does not apply. Effect is complicated and probably cannot be described by simple analytical results.

Review of RHIC Results

Random $b_k$: $b_k = (k+1) b_0/R^k$

$R = 40 \text{ mm}$, $b_0 \approx 1 \times 10^{-4}$

$A_{sc} \approx 19 \text{ mm}$, random $b_k$ only

$\nu_x = \nu_y \approx 28.824$
Asl Multipole Breakdown

Single multipoles
Random bk only
\[ Y_X = 28.827 \]
\[ Y_\Sigma = 28.822 \]
Tracking Studies seem to indicate that \( A \ll \text{constant} \approx 24 \text{ mm} \) when \( K \) gets large.

Is this possible?

Is it due to point multipoles being used instead of distributed multipoles?
Point Multipoles versus Distributed Multipoles

Classical N.L. Theory $\rightarrow A_{\infty} \rightarrow \text{constant} \propto R$
for point $b_k$

$A_{\infty} \rightarrow 0$
for distributed $b_k$

Classical theory result

$y - y_{\text{res}} = \int ds e^{i \theta} \beta \left( \frac{\beta}{\beta_0} \right)^{(k-1)/2} A_{\infty}^{K-1}$

factor $\rightarrow 0$ for distributed $b_k$

Distributed Multipoles in Tracking

Point $b_k$

2 intervals

3 intervals

Equivalent to using 2nd order Runge-Kutta
Convergence as Multipole Intervals are increased

Initial $\Delta = 18 \text{ mm}$

random by only
Distributed Multiple Results

Increasing the number of intervals to describe the multiples changes results significantly. However, Asl (the stability limit) is not changed. Occasionally Asl is increased by 2 mm for some runs.

Point multiples appear to give essentially the correct result for Asl.
Asl for High order bk

Asl \rightarrow 24 \text{ mm}

In bk random case

What are the consequences?

How many bk needed to determine Asl?

I don't need up to \( k = 50 \)
I just need up to \( k \leq 6 \)

(see next slide)
Multipole breakdown of $A_{SL}$

$N_5 = 2$

$N_5 = 1$

$k = 4 \rightarrow b_2 \pm i4$ present

$k = 6 \rightarrow b_2 + b_4 + b_6$ present
Higher br. produce a wall at about 24 mm. For $k < 24$ mm, they have little effect. Thus if lower br. produce $A_B \sim 19$ mm, the higher br. do not affect $A_B$.

Higher br. limit $A_B$ to $A_B \leq 24$ mm.

Even if I correct many of the lower br. (say for $k \leq 10$), I cannot do better than $A_B \sim 24$ mm.
Systematic $b_k$

Possible problem. I expect the higher systematic $b_k$ to be larger than the higher random $b_k$.

$$b_k \approx \frac{b_0}{\Delta k^2}$$

$b_0 \approx (k+1) \times 1 \times 10^{-4}$ random $b_k$

$b_0 \approx 300 \times 10^{-4}$ not unlikely for systematic $b_k$.

$b_0 \approx 1$ is possible for systematic $b_k$. 
# Systematic bc RHIC Results

<table>
<thead>
<tr>
<th>Dipole</th>
<th>( b_k )</th>
<th>( b_k^* + R_k / 10^{-4} )</th>
<th>Quad</th>
<th>( b_k^n )</th>
<th>( b_k^* + R_k / 10^{-4} )</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>17</td>
<td>44</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>-5.9</td>
<td>-38</td>
<td>5</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
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<td>1.6</td>
<td>17</td>
<td>9</td>
<td>-</td>
<td>1.005</td>
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<td>17</td>
<td>-1.2</td>
<td>-361</td>
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<td>20</td>
<td>0.030</td>
<td>36</td>
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</table>

\[ \hat{\phi} = 1 \times (k+1) \text{ for random } b_k \]

Results for \( \{ \text{dipoles} \} \) \( k = 14 - 20 \) from H. Hahn Tech Note

\( \{ \text{quads} \} \)

Note, \( A_{sl} \sim b_k \) or large change in \( b_k \) produce small changes in \( A_{sl} \) for large \( k \)
**Measured Results**

<table>
<thead>
<tr>
<th>K</th>
<th>$b_K (10^{-2} / 10^{-4})$</th>
<th>$b_K R / 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.2</td>
<td>2.2</td>
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<tr>
<td>3</td>
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<td>-1.91</td>
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<td>4</td>
<td>-1.76</td>
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<td>-3.8</td>
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<td>-15.69</td>
<td>-40.4</td>
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<td>10</td>
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<td>-1.1</td>
<td>-14.2</td>
</tr>
<tr>
<td>14</td>
<td>1.2</td>
<td>35.</td>
</tr>
</tbody>
</table>

**FINAL**

**br, systematic**

$r = 38 \text{ mm}$
Aperture Results (Tracking Results)

$A_{SL}$ including $b_{k_{sys}}$ systematic

For $b_{k_{sys}} (k=1 \rightarrow 20) + b_{k_{ran}} (k=1 \rightarrow 10)$

$A_{SL} = 15 \text{ mm}$

For $b_{k_{sys}} (k=1 \rightarrow 20); \text{ no } b_{k_{ran}}$

$A_{SL} = 19 \text{ mm}$

Same as $b_{k_{ran}}$ random case

$b_{k_{sys}}$ systematic, multiple breakdown

(see next page)
Systematic bk

Asl breakdown

Single bk

$A_{SL} = 19$

$b_{48} = \frac{40 \times 10^{-y}}{R^k}$

$b_{16} \rightarrow 19 \text{ mm}$

$b_{4} \rightarrow 17 \text{ mm}$

$K$ (Order of Multipole, $b_k$)
For \( b_{k,sys} \ (k=10 \rightarrow 20) + b_{k,ran} \ (k=1 \rightarrow 10) \)

\[ A_{sl} = 17 \text{ mm} \]

Even if all lower \( b_{k,sys} \) for \( k=1 \rightarrow 9 \) are eliminated, one gets
\[ A_{sl} = 17 \text{ mm} \]

There is the possibility, and some indication, that proper choice of lower \( b_{k} \) (\( b_{k=0} \)) can increase \( A_{sl} \).

Non-point \( b_{k,sys} \) tests

Distributed \( b_{k,sys} \) produce only small changes in the results.
FINAL Aperture Results

brs, sys all alone → $A_{brs} = 21 \text{ mm} (\beta_x = 100)$

addition of brtran reduces $A_{brs}$ to

$A_{brs} = 19 \text{ mm} (\beta_x = 100; \gamma = 0 \text{ results})$

Tracking results of Gelfand and Willeke

Willeke says, Tevatron aperture is largely
due to brs, sys. I think both
brs, sys and brtran are important.

Note, Tevatron $A_{brs}$ for $x = y$
may be $A_{brs} \approx 15 \text{ mm}$.
Conclusions for RHIC Magnets

1) $A_{5L}=17$ mm may result from systematic $b_k$.

2) Possibility of choosing lower $b_k$ to improve $A_{5L}$.

$b_k=0$ is not necessarily optimum solution.

3) Watch out for very large higher $b_k$ ($b_{16}$ etc...).