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FINAL REPORT

Project

COWBOY

Winnfield, Louisiana

DECEMBER 1959 - MARCH 1960

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STATIC STRESS DETERMINATIONS IN SALT

U. S. BUREAU OF MINES
APPLIED PHYSICS RESEARCH LABORATORY
COLLEGE PARK, MD.

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STATIC STRESS DETERMINATIONS IN SALT
SITE COWBOY

by

Robert H. Merrill

July 29, 1960

STATIC STRESS DETERMINATIONS IN SALT
SITE COWBOY ^{1/}

by
Robert H. Merrill ^{2/}

INTRODUCTION

This report presents the static stress determinations made at site COWBOY. ^{3/} This site is in the Carey Salt Mine near Winnfield, Louisiana. The purpose of this investigation was to determine the stress field in the salt dome and the stress concentrations on the surface of the small (12-foot diameter) sphere.

For the most part, the apparatus and techniques used in this investigation are new and have not been described in other reports. Therefore, the theory, concepts, apparatus, and some of the tests made to determine the reproducibility of the apparatus are described briefly.

GEOLOGY AND DESCRIPTION OF MINE OPENINGS

Although the geology of the salt formation is not within the scope of this report, some reference is made to certain geologic features. Hence, the following brief description is presented:

The salt formation at site COWBOY is a dome surrounded on the sides and top by anhydrite or other sedimentary rock types. The salt consists of grains or crystals that are about 1/4 to 3/4 inch long. The grains or crystals are moderately well bonded.

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^{1/} Work on manuscript completed July 29, 1960.

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^{3/} This work was done for the Atomic Energy Commission under Memorandum of Understanding No. AT(29-2)-914, Appendix A, Schedule 4, Rock Mechanics Studies. The work was in cooperation with the Lawrence Radiation Laboratory and other contractors.

The color of the salt is usually white to gray; darker bands, 1 to 2 inches wide, intersect the openings along a strike about E-W and dip about 80 degrees (from the horizontal). The darker bands contain varying percentages of anhydrite. The mine openings are about 800 feet below the surface. The formation above the openings consists mostly of salt and anhydrite. The estimated thickness of the salt above the openings is 300 feet. The specific gravity of samples of the salt and anhydrite is 2.2 and 2.8, respectively.

Near the shot site a regularly spaced, room-and-pillar system was used to mine the salt. In some areas, square pillars were used; in other areas, long, narrow pillars were used. The room widths are usually 50 feet. In the area selected for these tests, the room-and-pillars are relatively free of fractures or other signs of failure or deterioration. The cross section of the rooms are rectangles with slightly rounded corners. The salt drills easily and unbroken lengths of 6-inch core, 2-feet long, were diamond-drilled consistently. Also, unbroken lengths of NX-size core 4 to 5-feet long were recovered in many instances.

GENERAL CONSIDERATIONS

The stress in the salt both near and at a distance from the surface of underground openings was determined 1.) by measuring the change in the deformation of borehole before and after stress relieving the hole by over drilling, and 2.) by calculating the stress from the relationship between measured borehole deformation and the applied stress. In the derivation of this relationship it is assumed that the medium is perfectly elastic, homogeneous, and isotropic. The derivations for the cases of plane stress and plane strain are presented in Appendix I, part A. The conditions for plane stress and for uniaxial and biaxial stress fields are used in this investigation.

For a uniaxial stress field and plane stress, the change in the diameter, that is, the deformation, of the borehole, U , is related to the stress in the medium by (see fig. 1, in which $T = 0$)

$$U = \frac{dS}{E} (1 + 2 \cos 2\theta) \quad (1)$$

where

U = deformation (change in diameter) of hole,

d = diameter of hole,

S = applied uniaxial stress,

θ = angle between and direction of deformation and the x axis,

E = modulus of elasticity.

For a biaxial stress field and plane stress, the deformation of the borehole is related to the mutually perpendicular applied stresses, S and T , by (see fig. 1 and Appendix I, part A),

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta] \quad (2)$$

The two dimensional stress fields can be resolved into two perpendicular components, P and Q , called the principal stresses. P and Q are characterized by having the directions of the algebraically greatest and algebraically smallest stress. The shear stress is zero in the direction of P and Q (1).^{4/}

If the borehole deformation across three diameters is measured and the modulus of elasticity of the medium is known, the magnitude and direction of the principal stresses can be computed. If the measured deformations are 60 degrees apart (that is, a 60-degree-deformation-rosette), the relationships between principal stress and deformation are (see fig. 2):

$$P + Q = \frac{E}{3d} (U_1 + U_2 + U_3) \quad (3)$$

$$P - Q = \frac{2E}{6d} [(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2} \quad (4)$$

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^{4/} Underlined numbers in parenthesis refer to items in the bibliography at the end of this report.

where

U_1, U_2, U_3 = deformations across diameters 60 degrees apart, and the angle, θ_1 , between the direction at which U_1 is measured and the principal stress, P , is

$$\tan 2\theta_1 = - \frac{\sqrt{3} (U_2 - U_3)}{2U_1 - U_2 - U_3} \quad (5)$$

The deviations for equations 3 through 6 are presented in Appendix I, part B, together with the deviation for the 45-degree-deformation-rosette.

INSTRUMENTATION AND PROCEDURE

The instrumentation used in the field measurement of stress consists of a borehole gage, a strain gage indicator and a diamond-drill. The drill serves two purposes, namely, to drill a hole in which the gage is inserted and to concentrically core around the borehole to produce stress relief.^{5/}

The transducing element in the gage is a strip of beryllium copper on which four resistance-type strain gages are bonded to form a 4-arm-bridge, (see fig. 3) (3). The strip is actuated by a piston which senses the diameter of the hole. The strain in the strip changes proportionally to the diametral change, and this strain is measured by a conventional strain gage indicator. A sample calibration curve relating the piston displacement to strain is shown in figure 4. This curve was obtained by displacing the piston in known increments (measured by a micrometer screw) and recording the corresponding strain.

The gage, together with the bridge, has a sensitivity of about 30 micro-inches, which, for a rock with a modulus of elasticity of 3×10^6 p.s.i., corresponds to a change in stress of about 20 p.s.i.

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^{5/} A similar method of concentric coring around a borehole was developed by Nils Hast of Sweden (2).

FIGURE 1. - Cross-Section of a Hole in a Biaxial Stress Field.

FIGURE 2. - Cross-Section of a 60-Degree-Deformation-Rosette.

FIGURE 3. - Sections Through the Borehole Gage.

FIGURE 4. - Calibration Curve for the Borehole Gage.

The principle of stress-relief (sometimes called strain-relief) and equations 3 and 4 are used to determine the stress in the rock. The procedure is as follows: An EX-size diamond drill hole (1.5 inch diameter) is drilled to a depth of about 20 feet. The borehole gage is inserted at a depth of about 6-inches, or deeper, in the hole and oriented at 30 degrees with respect to the horizontal. The rock surrounding the gage is stress relieved by over-drilling concentrically around the EX hole with a 6-inch bit, see fig. 5; the over-drilling is stopped when no further deformation of the EX-hole is indicated (that is, when the rock is stress relieved). Stress relief is accomplished when the gage is over-drilled 1.5 to 2-inches past the gage contacts. Next, the gage is moved 4- to 6-inches past the core depth resulting from the first stress-relief, oriented at 90 degrees (vertically) and the stress relief repeated. This procedure is again repeated for a gage orientation of 150 degrees. This cycle of measurement at 30-, 90-, and 150-degree-orientation is continued at greater depths in the hole until the measured deformation show no further change. At this depth the stress in the rock is not influenced by the boundary of the opening from which gage hole was drilled, and hence, only the stress field, that is, the stress before mining, is being measured. Photographs of the drill, gage, and associated equipment are shown in figures 6 and 7.

Solid 6-inch diameter cores were taken at each stress-relief site to provide specimens for a laboratory determination of the modulus of elasticity. To prepare specimens for the laboratory test the cores were cut into cylinders 6-inches long and a 1-1/2 inch diameter (EX) hole was drilled through the core. A borehole gage was inserted in the hole and the specimen loaded in a compression machine. A photograph of a prepared specimen under test is presented in fig. 8.

The borehole deformation and the applied stress were recorded at increasing and decreasing stress increments of 100 p.s.i. The modulus of elasticity, E , was calculated from equation 1 using the deformation obtained from the decreasing load portion of either the second or third cycle. The decreasing load slope was used because the stress determinations in the field are made by stress-relief, that is, for a decreasing stress.

For comparative purposes, the modulus of elasticity was also determined on samples of 6-inch core without a borehole. The previous procedure was employed except that a Tuckerman gage was used for strain measurements.

The data from these laboratory tests are given graphically in figures 9 and 10. The results of the tests show that the modulus of elasticity computed from stress-strain and stress-deformation measurements are essentially the same.

The magnitude and direction of the in situ principal stresses in the plane normal to the axis of the hole were calculated from the borehole deformation measurements and the modulus of elasticity using equations 3, 4, and 5. To obtain the borehole measurements at a given depth, the following procedure was employed. An average line was drawn through the borehole deformations for each of the three angles of measurement. The borehole deformation at a given depth was read off from these curves. Sample data are presented in fig. 11. The deformations U_1 , U_2 , U_3 , at any depth are obtained from the 30-, 90-, and 150-degree curves. For example, at 30 inches these values are, $U_1 = 850 \mu \text{ in.}$, $U_2 = 1400 \mu \text{ in.}$, and $U_3 = 1150 \mu \text{ in.}$

Because hole depths of only 6 inches were permitted in the 12-foot-diameter sphere, the magnitude and direction of the principal stresses were calculated by assuming that the stress distribution around the sphere at any given latitude is symmetric. A single deformation measurement at 30-degrees was made in a 6-inch deep hole, on the same latitude a measurement at 90-degrees was made in a second hole, and at 150-degrees in a third hole. These measurements gave the values for U_1 , U_2 , and U_3 , from which the magnitude and direction of the principal stresses were obtained from equations 3, 4, and 5. The average modulus of elasticity determined from samples taken from Carey Drift No. 4 were used in these computations.

IN SITU MEASUREMENTS

Eleven boreholes were drilled and stress relieved in Carey Drift No. 4, see fig. 12. Boreholes 1, 2, and 3, were used to evaluate the reproducibility of the method; boreholes 5 and 7 were used to determine the stresses in the vertical East-West plane; boreholes 4 and 6 were used to determine the stress in the vertical

FIGURE 5. - Section of the Gage and Core Barrel in a Borehole.

FIGURE 6. - Photograph of the Concentric Drill.

FIGURE 7. - Photograph of the Core Barrel and Gage.

FIGURE 8. - Photograph of Laboratory Set Up for Measuring Borehole Deformation.

FIGURE 9. - Stress Versus Deformation Curves for Sample Taken Near Hole 5.

FIGURE 10. - Stress Versus Strain Curves for Sample Taken Near Hole 5.

FIGURE 11. - Representative Curves for Deformation Versus Distance From Collar of Borehole.

FIGURE 12. - Plan of North End of Carey Drift No. 4 Showing Location of Boreholes.

North-South plane; and boreholes 8, 9, 10, and 11, were used to determine the stresses in the horizontal plane.

Twelve boreholes were stress-relieved in the small sphere. All holes were in the South half of the sphere, that is, the side opposite to the portal of the sphere. The location of the holes are shown in fig. 13. (The point, 0° latitude, 0° longitude (hole 10) was opposite to, and on the line of center with the portal to the sphere.)

The 6-inch cores for laboratory evaluation of the modulus of elasticity were obtained from the salt at the back of the boreholes in the rib and from points near the boreholes in the floor.

DATA

The deformation data from the reproducibility tests are summarized graphically in fig. 14, from the EW planes (holes 5 and 7) in figures 15 and 16; from the NS planes (holes 4 and 6) in figures 17 and 18; and from the floor (holes 8, 9, 10, 11) in table 1. The data from the sphere are presented in table 2. The stress-deformation curves from third-cycle (decreasing load) are given in fig. 19. In fig. 19 the samples are identified by the hole number from which they were drilled.

TABLE 1. - Data from floor reliefs, Carey Drift No. 4.

<u>Hole No.</u>	<u>Depth in Floor</u> (inches)	<u>Orientation</u> (degrees)	<u>Deformation</u> (micro-inches)
8	6	0	280
8	11	45	422
8	165	90	NG
8	21.5	90	616
9	4	90	830
9	11	30	1240
9	19	150	730
10	7	0	619
10	14	45	1027
10	23	90	884
10	30	0	782
10	40	135	--
11	7	0	796
11	18	90	809
11	25	45	816
11	33	90	966
11	40	0	598
11	49	135	639

FIGURE 14. - Deformation Versus Distance From Collar of Boreholes 1, 2, and 3.

FIGURE 15. - Deformation Versus Distance From Collar of Borehole 5.

FIGURE 16. - Deformation Versus Distance From Hole 7.

FIGURE 17. - Deformation Versus Distance From Hole 4.

FIGURE 18. - Deformation Versus Distance From Hole 6.

FIGURE 19. - Stress Versus Deformation for 3d-Cycle, Decreasing Load.

TABLE 2. - Data from stress reliefs in sphere. (All gage depths are 6-inches).

<u>Hole No.</u>	<u>Latitude</u> (degrees)	<u>Orientation</u> (degrees)	<u>Deformation</u> (micro-inches)
9	+30	150	1130
1	+30	90	1310
6	+30	30	1285
12	0	150	1260
2	0	90	1210
10	0	30	1035
7	-30	150	965
3	-30	90	1110
11	-30	30	1325
7	-30	150	965
8	+60	90	1035
6	+30	30	1285
4	-60	90	940
5	0	0	800

RESULTS AND CONCLUSIONS

The data from the reproducibility tests given in fig. 14 show a good agreement. However, deformations as high as 3000 micro-inches (obtained at a hole depth of 8 inches in holes 1, 2, and 3) were not measured in any of the succeeding tests. The measurements in holes 1, 2, and 3, at a depth of 10 inches or more were about the same as those measured in other tests in Carey Drift No. 4. The results of stress calculations made from data taken at various depths in holes 4 through 7 are presented in table 3; in holes 8 through 11 in table 4, and for the surface stresses on the sphere in table 5.

TABLE 3. - Computed stresses, Carey Drift No. 4.

<u>Borehole</u>	<u>Depth</u> (inches)	<u>P</u> (p. s. i.)	<u>Q</u> (p. s. i.)	<u>θ</u> (degrees)
4	40	1200	960	52
4	60	1050	930	40
4	80	1140	940	43
5	45	993	850	49
5	50	938	833	46
5	60	913	787	48
5	80	915	801	51
5	100	840	772	60
5	120	918	838	67
5	140	962	854	47
5	160	968	878	41
6	35	782	676	165
6	40	746	731	74
6	50	808	705	173
6	60	802	710	6
6	70	803	701	16
6	80	813	699	17
6	90	799	696	16
6	100	799	751	35
6	120	843	806	8
7	40	1042	813	52
7	60	1189	954	169
7	80	1081	909	73
7	100	1086	861	67
7	120	1125	973	44
7	140	1127	1026	29
7	160	1071	953	33

TABLE 4. - Computed stresses, floor reliefs,
Carey Drift No. 4.

<u>Hole No.</u>	<u>P</u> (p. s. i.)	<u>Q</u> (p. s. i.)	<u>θ</u> Degrees
8	not calculated - insufficient data		
9	1120	800	E79S(from South)
10	1050	705	E42S
11	900	820	E60S

TABLE 5. - Computed stresses, 12-foot sphere.

<u>Latitude</u> (degrees)	<u>P</u> (p. s. i.)	<u>Q</u> (p. s. i.)	<u>θ</u> (degrees)
+30	1290	1180	15
0	1230	1090	25
-30	1230	1020	22
+30, -30, -60 (longitude +90)	1180	990	24

The results from these tests indicate that the salt at this site is not elastic, but plastic to the degree that the stress field, that is, the stress before mining is almost hydrostatic. This conclusion is substantiated by two observations. First, if the salt were elastic the approximate stress distribution around a theoretical opening simulating the opening at the Drift No. 4 test site would be as shown by the solid curve in fig. 20. This curve was calculated assuming a uniform, vertical stress field having a magnitude equal to the dead weight of the overburden, which, from the density and thickness of the cover at this site is 880 p. s. i. However, the average stress was 900 p. s. i. and independent of the distance from the surface of the opening, as shown by the dashed curve in fig. 20. Although the calculated and measured values of the stress field are in good agreement, the lack of stress concentrations at the surface could occur only in plastic or semi-plastic materials.

Second, the magnitude of the vertical and horizontal stress, P and Q, measured at the Drift No. 4 site, were, on the average, almost equal. This could occur in a plastic material having a Poisson's ratio of approximately 0.5. Also, the stress measurement

in the floor indicates a hydrostatic condition. The average of the floor stresses (P and Q, see table 4) is also about 900 p.s.i. If the floor were subjected to a stress field other than hydrostatic, and the floor were not plastic, this stress would be smaller.

The fact that most of the P and Q stresses are about equal makes the error in the calculated direction of the stress field large. For example, if P is equal to Q, U_1 , U_2 , and U_3 , are equal and equation 5 is indeterminate.

The anhydrite bands in the salt must have about the same physical properties as the salt. Otherwise, the stress calculations along the length of a hole would have varied significantly. For example, if the modulus of elasticity of the anhydrite bands were significantly higher, or lower, than the salt, the magnitude of the stress calculated from equations 3 and 4 would vary correspondingly from point to point in the hole. This was not the case, see table 3.

The average of the principal stresses at the edge of the sphere is about 1200 p. s. i. If the salt on the surface of the sphere were as plastic as that in Drift No. 4, a stress of about 900 p. s. i. would have been expected. On the other hand, had the salt on the surface of the sphere been elastic, the surface stress should have been 1.5 times the residual stress field (1), or about 1350 p. s. i. Because of this difference it is inferred that the degree of plasticity of the salt is dependent upon the age of the opening.

FIGURE 20. - Theoretical and Measured Stress Concentrations.

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APPENDIX I

The equations for the deformation of holes in an infinite plate that is perfectly elastic, homogeneous, and isotropic, are presented in Part A of this appendix. The cases for uniaxial and biaxial stress fields in plane stress and plane strain are considered.

The equations relating the principal stresses in a medium to the deformation of a hole as measured along three diameters 45- or 60-degrees apart, are derived in Part B of this appendix. These equations are called the 45- and 60-degree-deformation-rosette-equations.

The symbols used in Part A are as follows:

e_r = radial strain,

e_θ = tangential strain,

$\lambda_{r\theta}$ = shearing strain,

σ_r = radial stress,

σ_θ = tangential stress,

$\tau_{r\theta}$ = shearing stress,

θ = angle measured from direction of applied stress (S) to the x-axis,

u = radial displacement,

v = tangential displacement,

E = Young's modulus,

ν = Poisson's ratio,

a = radius of hole,

d = diameter of hole,

r = radial distance,

S = applied stress,

T = applied stress,

U = deformation (change in length of the diameter) of the hole.

Part A, Case 1 - Deformation of a hole in a plate subjected to a unidirection stress field under the condition of plane stress.

To obtain the deformation of a hole in a plate under the condition for plane stress the Kirsch equation for the stress distribution around a hole are substituted in Hookes' law equations for plane stress and integrated.

Relationship for the strain components and displacements in polar coordinates are

$$e_r = \frac{\partial u}{\partial r}, \quad (1)$$

$$e_\theta = \frac{u}{r} + \frac{\partial v}{r \partial \theta}, \quad (2)$$

$$\lambda_{r\theta} = \frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}. \quad (3)$$

Hookes' law equations for plane stress are

$$e_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (4)$$

$$e_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (5)$$

$$\lambda_{r\theta} = \frac{2(1 + \nu)}{E} \tau_{r\theta}. \quad (6)$$

The Kirsch equations for the stress distributions around a circular hole (unidirectional stress field, $T = 0$) are

$$\sigma_r = \frac{S}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta, \quad (7)$$

$$\sigma_\theta = \frac{S}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta, \quad (8)$$

$$\tau_{r\theta} = -\frac{S}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) \sin 2\theta, \quad (9)$$

Combining equations 1 and 4, 2 and 5, 3 and 6 yields

$$\frac{\partial u}{\partial r} = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (10)$$

$$\frac{u}{r} + \frac{\partial v}{r \partial \theta} = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (11)$$

$$\frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = \frac{2(1+\nu)}{E} \tau_{r\theta}. \quad (12)$$

Substituting equations 7 and 8 into 10 gives

$$\begin{aligned} \frac{\partial u}{\partial r} = \frac{1}{E} & \left[\frac{S}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \right] \end{aligned} \quad (13)$$

Integrating equation 13 gives

$$\begin{aligned} u = \frac{1}{E} & \left[\frac{S}{2} \left(r + \frac{a^2}{r}\right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3}\right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r}\right) - \frac{S}{2} \left(r - \frac{a^4}{r^3}\right) \cos 2\theta \right] + g_1(\theta), \end{aligned} \quad (14)$$

where $g_1(\theta)$ is an arbitrary function of θ only. Substituting equations 7 and 8 into equation 11 and combining terms gives

$$\begin{aligned} \frac{\partial v}{\partial \theta} &= \frac{1}{E} \left[-\frac{S}{2} \left(2r + \frac{4a^2}{r} + \frac{2a^4}{r^3} \right) \cos 2\theta \right] \\ &- \frac{v}{E} \left[\frac{S}{2} \left(2r - \frac{4a^2}{r} + \frac{2a^4}{r^3} \right) \cos 2\theta \right] - g_1(\theta). \end{aligned} \quad (15)$$

Integrating equation 15 gives

$$\begin{aligned} v &= \frac{1}{E} \left[-\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ &- \frac{v}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] - \int g_1(\theta) d\theta + g_2(r), \end{aligned} \quad (16)$$

where $g_2(r)$ is a function of r only.

Differentiating equation 14 with respect to θ , gives

$$\frac{\partial u}{\partial \theta} = \frac{1}{E} \left[-S \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \sin 2\theta \right] - \frac{vS}{E} \left(r - \frac{a^4}{r^3} \right) \sin 2\theta + \frac{\partial g_1(\theta)}{\partial \theta}. \quad (17)$$

Differentiating equation 16 with respect to r , gives

$$\begin{aligned} \frac{\partial v}{\partial r} &= \frac{1}{E} \left[-\frac{S}{2} \left(1 - \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] \\ &- \frac{v}{E} \left[\frac{S}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] + \frac{\partial g_2(r)}{\partial r}. \end{aligned} \quad (18)$$

Substituting equations 9, 16, 17, and 18 into equation 12 and collecting terms gives

$$\frac{\partial g_1(\theta)}{\partial \theta} + \frac{r \partial g_2(r)}{\partial r} + \int g_1(\theta) d\theta - g_2(r) = 0 \quad (19)$$

Since $g_1(\theta)$ is a function of θ only, and $g_2(r)$ is a function of r only, the terms containing θ and the terms containing r must be equal to zero, thus

$$\frac{r \, dg_2(r)}{dr} - g_2(r) = 0, \quad (20)$$

$$\frac{d^2 g_1(\theta)}{d\theta^2} + g_1(\theta) = 0, \quad (21)$$

where total derivatives are used because g_2 is a function of r only, and g_1 is function of θ only. Integrating equation 20 gives

$$g_2(r) = Cr. \quad (22)$$

Integrating equation 21 gives

$$g_1(\theta) = A \sin \theta + B \cos \theta. \quad (23)$$

As there can be no tangential displacements on either the x or y axis the boundary conditions are

$$v = 0 \text{ when } \theta = 0 \text{ for all values of } r;$$

$$v = 0 \text{ when } \theta = \frac{\pi}{2} \text{ for all values of } r.$$

Substituting equations 22 and 23 and the boundary condition into equation 16 gives

$$A = 0, B = 0, C = 0.$$

Because the constants in equations 22 and 23 are zero, the arbitrary functions $g_1(\theta)$ and $g_2(r)$ are zero; thus, the displacements u and v , from equations 14 and 16, are

$$u = \frac{1}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] - \frac{v}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right], \quad (24)$$

$$v = -\frac{1}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] - \frac{v}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right]. \quad (25)$$

Equations 24 and 25 describe radial and tangential displacements anywhere in the medium. To obtain the change in diameter, U , of the hole, (deformation) substitute $r = a = \frac{d}{2}$, into equation 24, thus

$$U = \frac{Sd}{E} (1 + 2 \cos 2\theta) . \quad (26)$$

Part A, Case 2 - Deformation of a hole in a plate subjected to a unidirectional stress under the condition of plane strain.

To obtain the deformation of a hole in a plate in plane strain the Kirsch equations (equations 7, 8, and 9) are substituted in the Hookes' law equations for plane strain which are

$$e_r = \frac{1}{E} [(1 - \nu^2) \sigma_r - \nu(1 + \nu) \sigma_\theta] , \quad (27)$$

$$e_\theta = \frac{1}{E} [(1 - \nu^2) \sigma_\theta - \nu(1 + \nu) \sigma_r] , \quad (28)$$

$$\lambda_{r\theta} = \frac{2(1 + \nu)}{E} \lambda_{r\theta} . \quad (29)$$

Equations 1, 2, 3, 7, 8, and 9, are valid under conditions of both plane stress or plane strain. Hookes' law equations for plane strain differ from the plane stress equations only by the terms $(1 - \nu^2)$ and $(1 + \nu)$. Therefore, the method of solution and the boundary conditions are valid for both cases and the radial and tangential displacements anywhere in the medium are

$$u = \frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] \\ - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right] , \quad (30)$$

$$v = - \frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] . \quad (31)$$

To obtain U , $r = a = d/2$ is substituted into equation 30, thus

$$U = \frac{Sd}{E} (1 - \nu^2) (1 + 2 \cos 2\theta) .$$

Part A, Case 3 - deformation of a hole in a plate subjected to a biaxial stress field and for the condition of plane stress.

The equations for the stress distribution around a hole are obtained by superposition and equations 7, 8, and 9, become

$$\sigma_r = \frac{S+T}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{S-T}{2} \left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right) \cos 2\theta , \quad (32)$$

$$\sigma_\theta = \frac{S+T}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{S-T}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta , \quad (33)$$

$$\tau_{r\theta} = - \frac{S-T}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right) . \quad (34)$$

The solution for the displacements is obtained by the same method as used for Case 1; the displacements are

$$u = \frac{1}{E} \left[\frac{S+T}{2} \left(r + \frac{a^2}{r}\right) + \frac{S-T}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3}\right) \cos 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S+T}{2} \left(r - \frac{a^2}{r}\right) - \frac{S-T}{2} \left(r - \frac{a^4}{r^3}\right) \cos 2\theta \right] , \quad (35)$$

$$v = - \frac{1}{E} \left[\frac{S-T}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right) \sin 2\theta \right] \\ - \frac{\nu}{E} \left[\frac{S-T}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right) \sin 2\theta \right] . \quad (36)$$

To obtain the deformation of the hole, $r = a = d/2$ is substituted in equation 35, thus

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta] . \quad (37)$$

Part A, Case 4 - deformation of a hole in a plate subjected to biaxial stress and for the conditions for plane strain.

The equations for the stress distributions around a hole in a plate subjected to a biaxial stress field, (equations 32, 33, and 34,) are substituted in equations 27, 28, and 29. Then, the procedures outlined under Case 1 are followed to obtain

$$U = \frac{(1 - \nu^2)d}{E} [(S + T) + 2(S - T) \cos 2\theta] . \quad (38)$$

Part B, Case 1 - 60-degree-deformation-rosette-equations.

The equations for the deformation of a hole in a plate in biaxial stress fields are used to relate the stresses in the plate to the deformations measured across three diameters of the hole. It is assumed that the principal stresses P and Q can be substituted for the stresses S and T in equations 37 and 38.

The symbols for Part A are the same in Part B; the following symbols are also used:

U_1 = deformation measured across a diameter in the hole,

U_2 = deformation at angle 60 degrees from U_1 ,

U_3 = deformation at angle 120 degrees from U_1 ,

θ_1 = angular direction of U_1 with respect to the maximum principal stress P, ^{4/}

K = constant,

A = constant,

B = constant.

---

^{4/} See figure 3 for a 60-degree-deformation-rosette.

The equations for the deformation of a hole in a plate in a biaxial stress field is (see equation 37)

$$U = \frac{d}{E} [(S + T) + 2(S - T)\cos 2\theta] .$$

If we let $K = d/E$, $A = P + Q$, $B = P - Q$, and substitute P and Q for S and T , equation 37 becomes

$$U = KA + 2KB \cos 2\theta . \quad (39)$$

To obtain U_1 , U_2 , and U_3 ,

$$U_1 = KA + 2KB \cos 2\theta_1 , \quad (40)$$

$$U_2 = KA + 2KB \cos 2\theta_2 , \quad (41)$$

$$U_3 = KA + 2KB \cos 2\theta_3 , \quad (42)$$

For a 60-degree-rosette, $\theta_2 = \theta_1 + 60^\circ$ and $\theta_3 = \theta_1 + 120^\circ$; and equations 40, 41, and 42, become

$$U_1 = KA + 2KB \cos 2\theta_1 , \quad (43)$$

$$U_2 = KA + 2KB \cos 2(\theta_1 + 60^\circ) , \quad (44)$$

$$U_3 = KA + 2KB \cos 2(\theta_1 + 120^\circ) , \quad (45)$$

The cosine of the sum of the angles in equations 44 and 45 is

$$\cos 2\theta_1 + 120^\circ = -\frac{1}{2} \cos 2\theta_1 - \frac{\sqrt{3}}{2} \sin 2\theta_1 , \quad (46)$$

$$\cos 2\theta_1 + 240^\circ = -\frac{1}{2} \cos 2\theta_1 + \frac{\sqrt{3}}{2} \sin 2\theta_1 . \quad (47)$$

Substituting equations 46 and 47 in equations 44 and 45 and solving for A and B gives

$$A = \frac{U_1 + U_2 + U_3}{3K} , \quad (48)$$

$$B = \frac{\sqrt{2}}{6K} [U_1 - U_2]^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2} . \quad (49)$$

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Substituting equations 46 and 47 in equations 43, 44, and 45, and solving for θ_1 gives

$$\tan 2\theta_1 = \frac{-\sqrt{3}(U_2 - U_3)}{2U_1 - U_2 - U_3} \quad (50)$$

Because $A = P + Q$ and $B = P - Q$, the sum and difference of the principal stresses are

$$P + Q = \frac{E}{3d} (U_1 + U_2 + U_3), \quad (51)$$

$$P - Q = \frac{\sqrt{2}E}{6d} [(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2]^{1/2} \quad (52)$$

Hence, the principal stresses in the medium can be obtained from a measurement of U_1 , U_2 , and U_3 . The angle between the principal stress P and the angle at which U_1 is measured is obtained from equation 50.

The conditions for plane strain can be obtained by dividing equations 51 and 52 by the term $(1 - \nu^2)$.

Part B, Case 2 - 45-degree-deformation-rosette-equations.

The solution for the 45-degree-deformation-rosette is obtained by the same steps as were used for the solution of the 60-degree-deformation-rosette. The solution for the sum and difference of the principal stresses are

$$P + Q = \frac{E}{2d} (U_1 + U_3), \quad (53)$$

$$P - Q = \frac{E}{2\sqrt{2}d} [(U_1 - U_2)^2 + (U_2 - U_3)^2]^{1/2}, \quad (54)$$

and the tangent of twice the angle between the principal stress P and the deformation U_1 is

$$\tan 2\theta_1 = \frac{2U_2 - U_1 - U_3}{U_1 - U_3} \quad (55)$$

Again, for plane strain, equations 53 and 54 can be used by dividing by the term $(1 - \nu^2)$.

SUMMARY OF SHOT DATA-PROJECT COWBOY

<u>SHOT NO.</u>	<u>DATE</u>	<u>TIME (CST)</u>	<u>YIELD (lbs)</u>	<u>STATION</u>	<u>TYPE</u>
1	17 Dec.	0015	20	1.2	Coupled- 45' hole
2	17 Dec.	0045	20	1.1	Decoupled- 12' diam. sphere
3	19 Dec.	0000	100	1.1	Decoupled- 12' diam. sphere
4	19 Dec.	0015	100	1.3	Coupled- 45' hole
5	23 Jan.	0000:00.113	198.35	2.1	Decoupled- 30' diam. sphere
6	30 Jan.	0001:00.112	200.0	2.1	Decoupled- 30' diam. sphere
7	30 Jan.	0101:00.112	199.65	2.2	Coupled- 110' hole
8	6 Feb.	0001:00.115	477.4	2.1	Decoupled- 30' diam. sphere
9	6 Feb.	0101:00.113	499.7	2.3	Coupled- 110' hole
10	13 Feb.	1901:00.113	954.0	2.1	Decoupled- 30' diam. sphere
11	13 Feb.	2001:00.114	1003.0	2.4	Coupled- 110' hole
12	20 Feb.	0001:00.112	929.0	1.1	Decoupled- 12' diam. sphere
13	20 Feb.	0100:59.614	987.6	2.5	Coupled- 110' hole
14	27 Feb.	0001:00.127	1902.4	1.1	Decoupled- 12' diam. sphere
15	28 Feb.	0401:00.131	936.2	2.6	Coupled- 110' hole
16	3 Mar.	2301:00.128	199.5	1.4	Coupled- 45' hole
17	4 Mar.	0001:00.130	199.8	1.3-1	Coupled- 45' hole

NOTE 1) All times for shots 5 through 17 are derived from comparisons with WWV. Accuracies are ± 0.001 sec, except for shot 15, which is ± 0.003 sec.

2) All yields include "Nitramon" booster and detonator weights of either 2 or 3 pounds.

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