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Enumeration of Linear Graphs and Connected Linear Graphs up to P=18 Points

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by

M. L. Stein
P. R. Stein

PREFACE

The present report was completed in September 1963 but never published, owing to the relatively great bulk of the tables. Recently there have been several requests for copies of these tables; it seems likely that interest in this work will continue to grow. To our knowledge, the results are not available from any other source. For this reason we felt that a need would be served by issuing the tables at this time.

The text and tables remain unaltered from their original (1963) form; indeed there is substantially nothing we would wish to change. To be sure, the remarks on computation times are out of date; calculating machines are much faster now than they were in 1963. There is, however, nothing to be done about this short of recomputing Table I in its entirety, and--since we believe the results to be correct--this would be pointless.

Myron L. Stein
Paul R. Stein

1. INTRODUCTION

As part of a comprehensive program to test the practical computability of various combinatorial numbers, we have calculated the number of linear graphs $L_{p,k}$ and connected graphs $C_{p,k}$ with p points and k lines up to and including p = 18. The calculation was carried out using the MANIAC II electronic computer at Los Alamos. To the authors' knowledge, no such extensive computation of these numbers has heretofore been attempted. A short list, complete through p = 9, appears in Riordan's book(1), but the number of connected graphs is not given(2). Our results, which include the previously known values given in references (1) and (2), are listed in table I.

For the convenience of readers who may not be familiar with the literature, the theoretical basis of the enumeration is presented in section 2; the point of view here is different from the usual one (Pólya's theorem), although the prescription arrived at is, of course, the same. In section 3 we outline the method for obtaining the number of connected graphs $C_{p,k}$ once the $L_{p,k}$ are known. Section 4 is devoted to a discussion of some practical aspects of the calculation and includes the results of some timing studies. Finally, in Section 5, some asymptotic properties of the $L_{p,k}$ are briefly discussed.

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(1) J. Riordan: "An Introduction to Combinatorial Analysis", Wiley (1958), page 146.

(2) A complete list of $L_{p,k}$ and $C_{p,k}$ through p = 7 is given by G. W. Ford and G. E. Uhlenbeck as appendix 2 of their article in: "Studies in Statistical Mechanics", Vol. I, North-Holland (1962).

2. THE ENUMERATION FORMULA FOR $L_{p,k}$

The technique for the enumeration of linear graphs is generally attributed to G. Pólya[3]. In fact, Pólya was anticipated by J. H. Redfield[4], who, some ten years earlier, gave a precise prescription for dealing with a class of enumeration problems of which that of linear graphs is a special case. Redfield's paper was long overlooked by specialists in Combinatorial Analysis, although it was known to at least one eminent group theoretician[5]. Recently, Redfield's work has been generalized and given a more conventional group theoretical setting by Foulkes[6].

Omitting most of the details, we may describe the enumeration technique as follows. With every irreducible representation of the symmetric group Σ_p on p letters we associate a certain symmetric function ("Schur function"):

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(3) G. Pólya: Acta Math. 68 (1937), pp 583-599. The principal fruits of this work are collected in the Algorithm known as "Pólya's Theorem". An excellent presentation may be found in Riordan, ref.(1), page 129 ff.

(4) J. H. Redfield: Am. J. Math. 49 (1927), pp 433-455.

(5) D. E. Littlewood discusses some applications of Redfield's work in his book: "The Theory of Group Characters", Oxford, 1950 (2nd. edition). For an appraisal of ref.(4), see F. Harary: Pub. Hung. Acad. Sci. 5 (1960), pp 92-93. One of the present authors (P.R.S.) "discovered" Redfield's paper in the course of reading Littlewood's book and subsequently made it known to Harary.

(6) H. O. Foulkes: Can. J. Math. 15 (1963), pp 272-284.

$$\{\lambda\} = \frac{1}{p!} \sum_{(\rho)} H_\rho \chi_\rho^{(\lambda)} S_\rho \qquad 1),$$

where (λ) and (ρ) are partitions of p which characterize respectively the representation and the class of Σ_p, $\chi_\rho^{(\lambda)}$ is the corresponding simple character, H_ρ is the order of the class, and S_ρ is a product of power sums:

$$S_\rho = s_1^{\rho_1} s_2^{\rho_2} s_3^{\rho_3} \ldots \qquad 2)$$

$$s_j = \Sigma x_i^j \qquad 3),$$

the x_i being indeterminants. The sum in 1) is over all classes of Σ_p, i.e. over all partitions:

$$\Sigma i\rho_i = p \qquad 4).$$

If we choose the identity representation of Σ_p, for which $(\lambda) = p$ and $\chi_\rho^{(p)} = 1$ for all (ρ), 1) becomes:

$$h_p = \frac{1}{p!} \sum_{(\rho)} H_\rho S_\rho \qquad 5).$$

Identifying h_p as the homogeneous product sum of degree p (of the indeterminants x_i), 5) is seen to be nothing more than the well-known

Newton relation expressing h_p in terms of the symmetric functions s_j[7]. h_p has also been called the "cycle indicator"[8], since the coefficient H_ρ enumerates the number of permutations with cycle structure (ρ), i.e. the number of permutations with ρ_1 cycles of order 1, ρ_2 cycles of order 2, etc. For completeness we reproduce the well-known formula:

$$H_\rho = \frac{p!}{1^{\rho_1}\,\rho_1!\;2^{\rho_2}\,\rho_2!\;3^{\rho_3}\,\rho_3!\;\ldots}, \quad \Sigma i\rho_i = p \qquad 6).$$

The expression 5) is not of immediate use for our purpose. Linear graphs are collections of p points and k lines, and the basic[9] enumeration problem requires that, given p, we count all graphs with a fixed number of lines k which are inequivalent under the p! permutations of Σ_p on the points. Redfield[10] therefore constructs a different symmetric function g_p which is associated with the group G_p of permutations induced on the $n = p(p-1)/2$ lines of the complete graph by the corresponding permutations of the points under Σ_p. g_p is the cycle indicator of the new group G_p, the "pair group", abstractly isomorphic with Σ_p but of degree $n = p(p-1)/2$ (hence distinct as a permutation

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(7) See, e.g. P. A. MacMahon: "Combinatory Analysis", Vol. I, Cambridge (1915), page 7.

(8) Reference (1), page 68.

(9) I. E. The enumeration by number of lines. Enumeration with respect to other properties such as the degrees of the various points, etc., is in many cases an unsolved problem.

(10) Ref. (4), page 450 ff.

group)[11]. The process of constructing g_p may be carried out formally[10] or by considering in detail the cycle structure of line permutations induced by given point permutations[12]. The general result is as follows[13]. We replace each product S_ρ in 5) by a new product:

$$S_{\mu(\rho)} = s_1^{m_1} s_2^{m_2} \ldots s_p^{m_p} \cdot \prod_{1 \leq i < j \leq p} s_{M(i,j)}^{\rho_i \rho_j D(i,j)} \equiv s_1^{\mu_1} s_2^{\mu_2} \ldots s_n^{\mu_n} \qquad 7),$$

where M(i,j) and D(i,j) are respectively the least common multiple and the greatest common divisor of the integer pair (i,j). The m_j are determined by the formula:

$$m_j = \tfrac{1}{2} j \rho_j (\rho_j - 1) + [(j-1)/2]\rho_j + \rho_{2j} \qquad 8),$$

where [(j-1)/2] stands for the greatest integer $\leq$ (j-1)/2.

We now divide the n lines into two groups of k and n-k lines respectively; we may think of the k lines as those actually drawn in, while the other n-k are absent. The permutation group that transforms the k lines among themselves is the subgroup $K_p \equiv \Sigma_k \times \Sigma_{n-k}$ of Σ_n; its cycle

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(11) In group theoretical terms, this is the permutation representation of Σ_p induced by the subgroup $\Sigma_{p-2} \times \Sigma_2$. See ref. (6) p 274.

(12) See ref. (1), pp 144-145.

(13) F. Harary: Trans. Am. Math. Soc. 78 (1955) page 451. Harary attributes the formula to Polya.

indicator is clearly $h_k h_{n-k}$. From equation 5) we see that:

$$h_k h_{n-k} = \frac{1}{k!(n-k)!} \sum H_{\nu_1} H_{\nu_2} S_{\nu} \qquad 9)$$

where (ν_1), (ν_2) are cycle partitions of k and n-k respectively:

$$(\nu_1) = (1^{\alpha_1}, 2^{\beta_1}, 3^{\gamma_1} \ldots), \ \alpha_1 + 2\beta, + 3\gamma_1 + \ldots = k \qquad 10),$$

$$(\nu_2) = (1^{\alpha_2}, 2^{\beta_2}, 3^{\gamma_2}, \ \ldots \ ) \ \alpha_2 + 2\beta_2 + 3\gamma_2 + \ldots = n-k$$

and (ν) is a cycle partition of n:

$$(\nu) = (1^{\alpha}, 2^{\beta}, 3^{\gamma}, \ \ldots), \ \alpha + 2\beta + 3\gamma + \ldots = n \qquad 11).$$

The sum is to be carried out over all (ν_1), (ν_2) which are "separations" of (ν):

$$\begin{aligned} \alpha_1 + \alpha_2 &= \alpha \\ \beta_1 + \beta_2 &= \beta \\ \gamma_1 + \gamma_2 &= \gamma \\ &\vdots \end{aligned} \qquad 12).$$

The enumeration is now accomplished by Redfield's main theorem[14]. This may be stated as follows (for further details see the original paper).

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(14) Ref. (4), page 434. The theorem is established by elementary means, i.e. by counting.

Given q permutation groups $G_1, G_2 .. G_q$, of degree n, each operating on n distinct symbols, the number of transitive sets T of the $(n!)^{q-1}$ possible arrays under the combined operations of the groups $G_1, G_2, .. G_q$ is given by

$$T = \frac{\sum \left(\frac{n!}{H_\mu}\right)^{q-1} N(G_1;\mu) \cdot N(G_2;\mu) \ldots N(G_q;\mu)}{m_1 \quad m_2 \quad \ldots m_q} \qquad 13),$$

where m_i is the order of G_i and $N(G_i;\mu)$ is the number of operations of G_i which have cycle structure (μ). The sum is over all cycle partitions (μ) of n. Note that the non-vanishing contributions to 13) come only from those (μ) which are common to all the permutation groups $G_1, G_2, \ldots G_q$.

To apply this to the enumeration of linear graphs, we take $q = 2$, $G_1 = G_p$, $G_2 = K_p$. Then:

$$N(G_p;\mu) = H_\rho$$
$$N(K_p;\mu) = \Sigma H_{\nu_1} H_{\nu_2} \qquad (\mu \text{ fixed}) \qquad 14)$$

Inserting this in 13), and taking account of the relations 9) - 12), we obtain:

$$L_{p,k} = \frac{1}{p!} \sum H_\rho \binom{\mu_1}{\alpha_1} \binom{\mu_2}{\beta_1} \binom{\mu_3}{\gamma_1} \ldots \qquad 15).$$

$$(\alpha_1 + 2\beta_1 + 3\gamma_1 + .. = k)$$

If, now, in the cycle indicator of G_p:

$$g_p = \frac{1}{n!} \Sigma H_\rho S_{r(\rho)}$$

we formally replace each product $S_{\mu(\rho)} = s_1^{\mu_1} s_2^{\mu_2} \ldots s_n^{\mu_n}$ by the product $(1 + y)^{\mu_1} (1 + y^2)^{\mu_2} \ldots (1 + y^n)^{\mu_n}$, we see that 15) is just the coefficient of y^k in g_p. Thus Redfield's prescription is in fact the same as that given by Pólya's Theorem[15].

3. THE ENUMERATION OF CONNECTED GRAPHS

Once the total number of distinct linear graphs $L_{p,k}$ is known for all p, k up through some given value of p, the number of connected graphs $C_{p,k}$ is trivially[16] obtainable by recursion. The method is actually based on the evaluation of the number of disconnected graphs in terms of their connected components. Define the formal series ("counting series").

$$L(x,y) \equiv \Sigma x^p y^k L_{p,k} = \Sigma x^p L_p(y)$$
$$C(x,y) \equiv \Sigma x^p y^k C_{p,k} = \Sigma x^p C_p(y) \qquad 16).$$

Then, as shown, for example, in reference (2), it is easy to establish the relation:

---

(15) Cf. ref (1), page 145, equation (90).

(16) "trivially", that is, on an electronic computer! Hand-calculation for even moderate values of p would be incredibly tedious.

$$1 + L(x,y) = \exp \sum_{s=1}^{\infty} \frac{1}{s} C(x^s,y^s) \qquad 17).$$

Let us rewrite the argument of the exponential in the form:

$$\sum_{s=1}^{\infty} \frac{1}{s} C(x^s,y^s) = \lambda_1(y)\, x + \tfrac{1}{2}\lambda_2(y)x^2 + \tfrac{1}{3}\lambda_3(y)x^3 + \dots \qquad 18),$$

where

$$\lambda_i(y) = 1 + \sum_{(j\cdot u=n)} j\, C_j(y^u) \qquad 19).$$

$$C_j(y^u) \equiv \sum C_{j,k}y^{uk}$$

If we now expand both sides of 17) in powers of x and equate like powers, we simply get back the expression 5) for the cycle indicator of the symmetric group, with $L_p(y)$ playing the role of h_p and the $\lambda_i(y)$ standing in place of the power sums s_i. Using Newton's relations for h_p in terms of the s_i in their recurrence-relation form(17), we obtain:

$$\lambda_p(y) = pL_p(y) - \sum_{j=1}^{p-1} \lambda_j(y)L_{n-j}(y) \qquad 20).$$

This enables us to calculate the polynomials $\lambda_p(y)$ step by step. Since these are given in terms of $C_p(y)$ and lower (hence known) polynomials, the determination of the numbers $C_{p,k}$ is accomplished.

---

(17) Littlewood, ref (5), page 84.

4. CALCULATIONAL PROCEDURE

In principle the calculation is straightforward. Given a p, we first generate all partitions, storing them in cycle form i.e. in the form given by equation 4). For each partition (ρ) we then construct the corresponding monomial $S_{\mu(\rho)}$ according to equations 7) and 8). In addition, we prestore the class numbers H_ρ (equation 6)). To calculate $L_{p,k}$ we must simply evaluate the sum in equation 15) over all (ρ), finally dividing by p! . Because of the evident symmetry of $L_{p,k}$:

$$L_{p,k} = L_{p,n-k} \qquad 21),$$

we need perform the calculation only for $k \leq k_{max} = [n/4]$.

For a given value of p, the calculation of the complete set of $L_{p,k}$ may be carried out in two different ways. In the first procedure, which we call Method I, we take each monomial $S_{\mu(\rho)}$ in turn and calculate the contributions for all solutions of the equation

$$\Sigma j\mu_j = k', \quad o \leq k' \leq k_{max} \qquad 22).$$

When all partitions (ρ) have been so treated, we have all the quantities $p!\, L_{p,k}$ stored in memory, and all that remains to be done is to divide by p! and print out the answers. In Method II, we fix our attention on a particular value of k, rejecting all solutions of 22) for which $k' \neq k$. Thus the calculation of each $L_{p,k}$ requires a separate "pass" through the whole set of partitions (ρ). Method I is clearly the more efficient procedure, but there are certain technical difficulties connected with its effective application. For the larger values of p, the calculation is long by any method. If Method I is adopted, one must dump the partial results on magnetic tape at reasonably frequent intervals to avoid losing computing time through random machine errors. In addition, there is, of course, no check one can apply to verify the correctness of these partial results. For this reason we chose Method II. When the calculation had been completed it was repeated through $p = 15$ using Method I.

In applying Method II, we soon found that equation 22) ($k' =$ a fixed k) could not be treated by simple rejection; this was so inefficient that the calculation became hopelessly long from $p = 14$ on. Consequently a more refined logical loop was constructed which took account of the particular value of k and of the particular set of μ_i generated by each partition (ρ). A slightly simplified version of this loop is shown in figure 1. In this diagram, the quantities r_j, $j = 1$ to t, are the t non-vanishing values of the μ_i, and the e_j are the corresponding values of i. In other words, we omit all $\mu_i = o$, so that the monomial takes the form:

$$S_{\mu(\rho)} = \prod_{j=1}^{t} (1 + y^{e_j})^{r_j} \qquad 23)$$

L is the partial sum, accumulated over the separate partitions (ρ), of $p!\, L_{p,k}$. H_ρ is as usual the class number. All other quantities are defined within the diagram itself.

Because the integers involved in the computation are generally too large to be stored in a single MANIAC word, multiple precision arithmetic had of course to be used. Multiple precision operations are relatively slow, which makes it extremely desirable to reduce the number of multiplications wherever possible. The procedure we adopted was to "factor out" the two terms in 23) with the largest values of r_j -- subject to certain storage restrictions which need not be detailed here -- reserving multiplications by the corresponding binomial coefficients until all the subproducts had been accumulated. More refined factoring schemes were also tried, but in these schemes, as it turned out, the additional time consumed in logical operations offset the gain achieved by reducing the number of multiplications.

As mentioned above, the calculations were repeated for $p \leq 15$ using Method I. The computing times required for the two Methods are given in Table II. It appears that Method I is not significantly faster than Method II for the higher p values[18]. This may be due to the fact that nothing corresponding to the factorization scheme described above could be used in Method I[19].

The calculation could probably be considerably shortened by taking into account the specific structure of certain of the monomials

(18) The time consumed in magnetic tape input-output operations has been subtracted from the Method I totals.

(19) It would have required too much internal storage space.

$S_{\mu(\rho)}$. It turns out that a relatively small percentage of these contribute most of the solutions of equation 22) and hence presumably require the bulk of the computing time. If for a given p and k we arrange the $S_{\mu(\rho)}$ in descending order according to the number of solutions of 22) that they afford, we find that the first 20% contribute in the neighborhood of 80% of the total number of solutions -- at least for $k \cong k_{max}$. This situation is illustrated in figure 2 for p = 15, 16, 17, 18, $k = k_{max} = \left[\frac{p(p-1)}{4}\right]$. Here F_s is the cumulative fraction of the total number of solutions of 22), while F_p is the corresponding fraction of the total number of partitions of p. Continuous curves have been drawn through the points to facilitate visual comparison[20].

5. ASYMPTOTIC PROPERTIES

There exists a simple asymptotic formula for the number of linear graphs, viz:

$$L_{p,k} \sim \frac{1}{p!}\binom{\frac{1}{2}p(p-1)}{k} \qquad 24).$$

This is known to be "good" if p is sufficiently large and k is not too far from $k_{max} = \left[\frac{p(p-1)}{4}\right]$.[21] Since the binomial coefficient in 24) is

(20) The first few points of the p = 15 curve have been omitted to preserve the clarity of the graph.

(21) Cf. G. W. Ford and G. E. Uhlenbeck: Proc. Nat. Acad. Sci. 43 (1957), pp 163-167.

simply the number of point-labeled graphs, the formula merely states that -- under the given conditions -- most graphs are asymmetrical ("have symmetry number one" in the language of reference (21)). It is of interest to see how well 24) fits for the rather modest values of p considered in this paper. In table III we list to 4 figures the "percentage error":

$$E_{p,k} = 100 \left[1 - \frac{1}{p!L_{p,k}} \binom{\frac{1}{2}p(p-1)}{k} \right] \qquad 25),$$

$$25 \leq k \leq k_{max}$$

$$p = 15,\ 16,\ 17,\ 18$$

On the basis of table III one must conclude that the asymptotic formula is remarkably good for a considerable part of the k-range. Incidentally, if k_2 denotes the value of k for which the initial percentage error (at $k = k_{max}$) approximately doubles, we note that the quantity k_2/k_{max} is nearly constant for the four cases considered here.

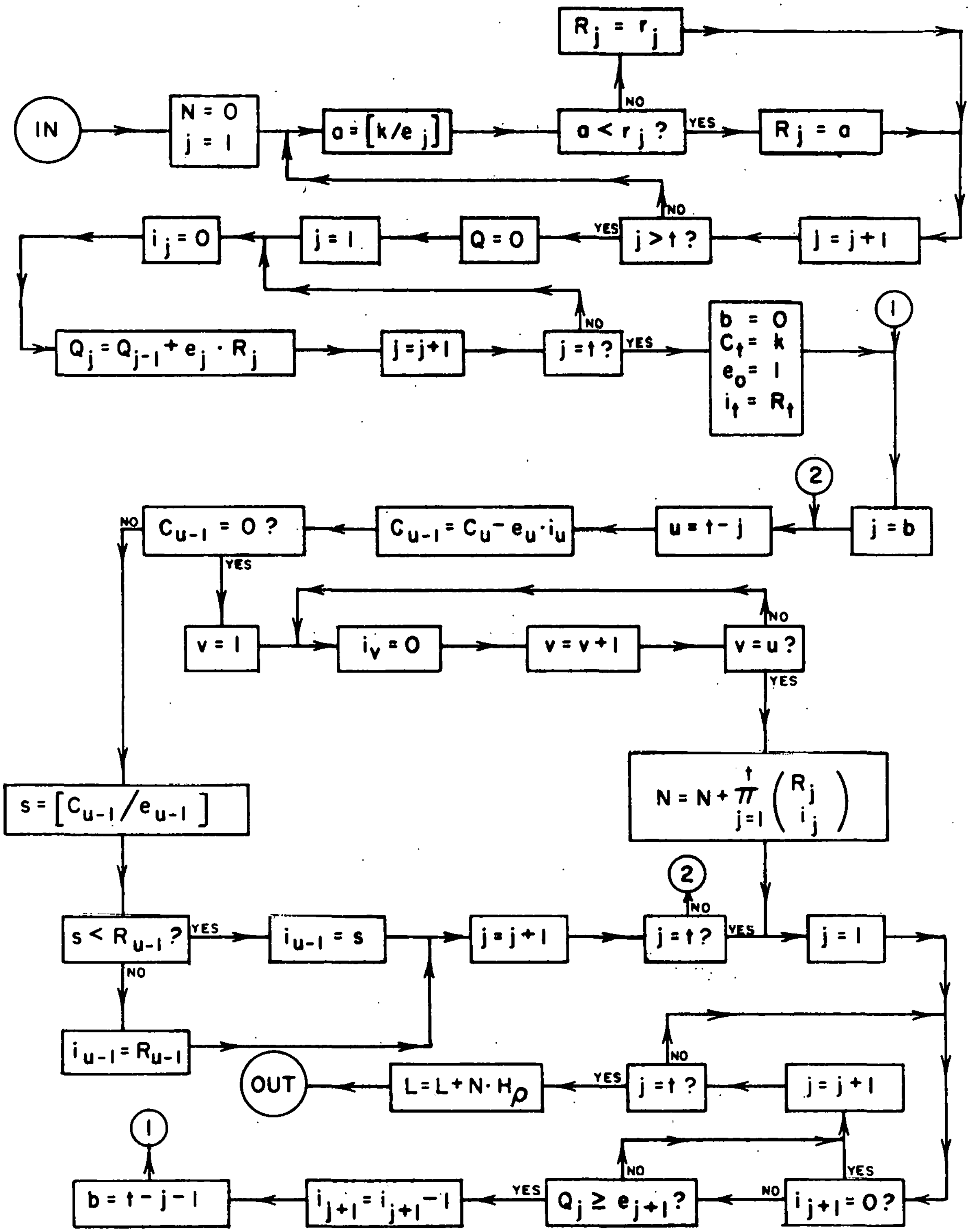
IN
N = 0
j = 1
a = [k/e j]
a < r j ?
YES
NO
R j = r j
R j = a
j = j + 1
j > t ?
Q = 0
j = 1
i j = 0
Q j = Q j-1 + e j · R j
j = j+1
j = t ?
b = 0
C t = k
e o = 1
i t = R t
1
2
j = b
u = t - j
C u-1 = C u - e u · i u
C u-1 = 0 ?
v = 1
i v = 0
v = v + 1
v = u ?
s = [C u-1 / e u-1]
s < R u-1 ?
i u-1 = s
i u-1 = R u-1
j = j + 1
j = t ?
j = 1
OUT
L = L + N · H ρ
b = t - j - 1
i j+1 = i j+1 - 1
Q j ≥ e j+1 ?
i j+1 = 0 ?

Figure 1.

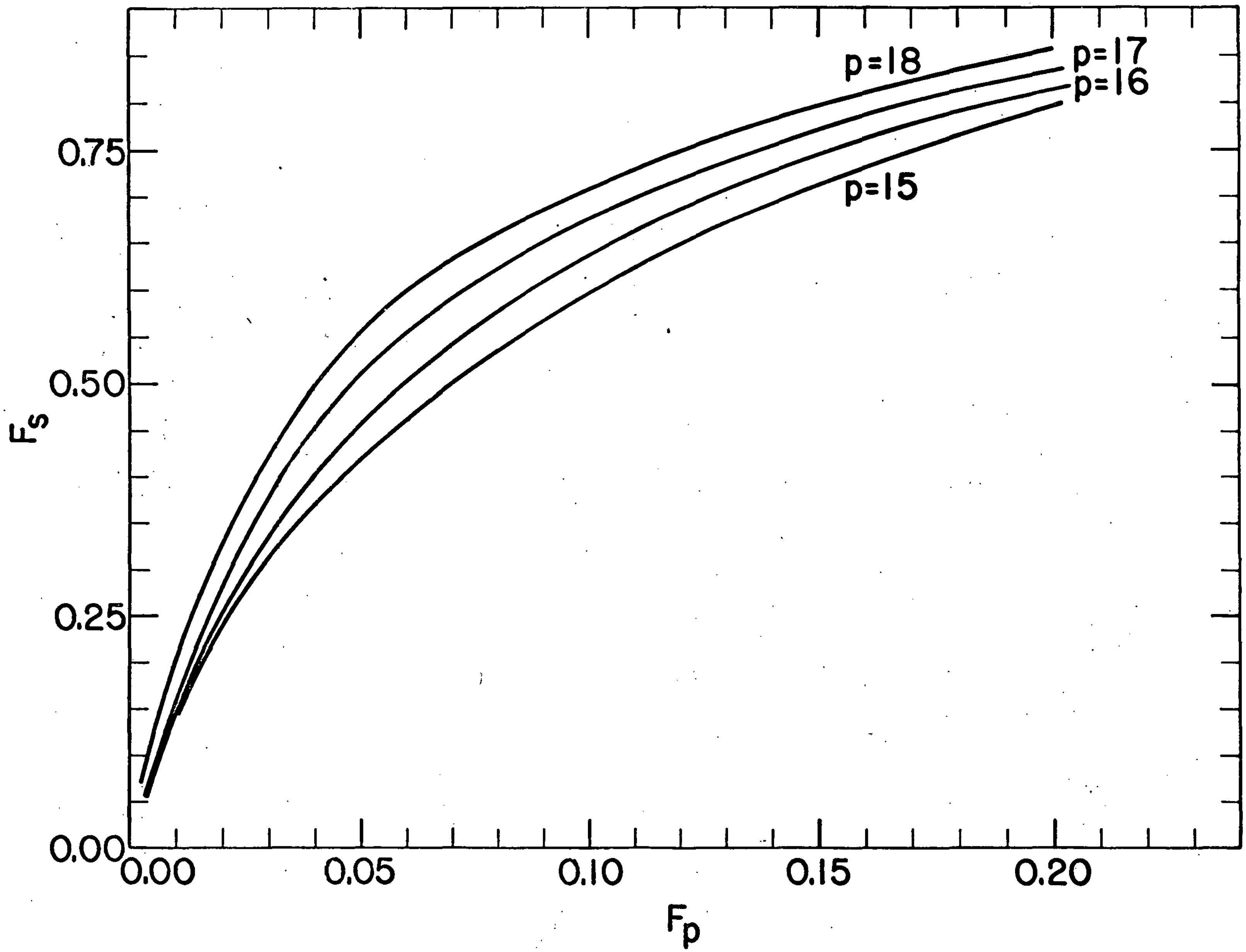

Figure 2.

Table I

$p = 4$

k	$L_{4,k}$	$C_{4,k}$
0	1	*
1	1	*
2	2	*
3	3	2
4	2	2
5	1	1
6	1	1

p = 5

k	$L_{5,k}$	$C_{5,k}$
0	1	*
1	1	*
2	2	*
3	4	*
4	6	3
5	6	5
6	6	5
7	4	4
8	2	2
9	1	1
10	1	1

p = 6

k	$L_{6,k}$	$C_{6,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	9	*
5	15	6
6	21	13
7	24	19
8	24	22
9	21	20
10	15	14
11	9	9
12	5	5
13	2	2
14	1	1
15	1	1

p = 7

k	$L_{7,k}$	$C_{7,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	10	*
5	21	*
6	41	11
7	65	33
8	97	67
9	131	107
10	148	132
11	148	138
12	131	126
13	97	95
14	65	64
15	41	40
16	21	21
17	10	10
18	5	5
19	2	2
20	1	1
21	1	1

p = 8

k	$L_{8,k}$	$C_{8,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	24	*
6	56	*
7	115	23
8	221	89
9	402	236
10	663	486
11	980	814
12	1312	1169
13	1557	1454
14	1646	1579
15	1557	1515
16	1312	1290
17	980	970
18	663	658
19	402	400

p = 8 (cont.)

k	$L_{8,k}$	$C_{8,k}$
20	221	220
21	115	114
22	56	56
23	24	24
24	11	11
25	5	5
26	2	2
27	1	1
28	1	1

p = 9

k	$L_{9,k}$	$C_{9,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	25	*
6	63	*
7	148	*
8	345	47
9	771	240
10	1637	797
11	3252	2075
12	5995	4495
13	10120	8404
14	15615	13855
15	21933	20303
16	27987	26631
17	32403	31400
18	34040	33366
19	32403	31996

p = 9 (cont.)

k	$L_{9,k}$	$C_{9,k}$
20	27987	27764
21	21933	21817
22	15615	15558
23	10120	10096
24	5995	5984
25	3252	3247
26	1637	1635
27	771	770
28	345	344
29	148	148
30	63	63
31	25	25
32	11	11
33	5	5
34	2	2
35	1	1
36	1	1

p = 10

k	$L_{10,k}$	$C_{10,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	66	*
7	165	*
8	428	*
9	1103	106
10	2769	657
11	6759	2678
12	15772	8548
13	34663	22950
14	71318	53863
15	136433	112618
16	241577	211866
17	395166	361342
18	596191	561106
19	828728	795630

p = 10 (cont.)

k	$L_{10,k}$	$C_{10,k}$
20	1061159	1032754
21	1251389	1229228
22	1358852	1343120
23	1358852	1348674
24	1251389	1245369
25	1061159	1057896
26	828728	827086
27	596191	595418
28	395166	394820
29	241577	241428
30	136433	136370
31	71318	71293
32	34663	34652
33	15772	15767
34	6759	6757
35	2769	2768
36	1103	1102
37	428	428
38	165	165
39	66	66

p = 10 (cont.)

k	$L_{10,k}$	$C_{10,k}$
40	26	26
41	11	11
42	5	5
43	2	2
44	1	1
45	1	1

$p = 11$

k	$L_{11,k}$	$C_{11,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	67	*
7	172	*
8	467	*
9	1305	*
10	3664	235
11	10250	1806
12	28259	8833
13	75415	33851
14	192788	109844
15	467807	313670
16	1069890	803905
17	2295898	1870168
18	4609179	3978187
19	8640134	7775398

p = 11 (cont.)

k	$L_{11,k}$	$C_{11,k}$
20	15108047	14013042
21	24630887	23350556
22	37433760	36052412
23	53037356	51662585
24	70065437	68803769
25	86318670	85251441
26	99187806	98355794
27	106321628	105723785
28	106321628	105925685
29	99187806	98945882
30	86318670	86182087
31	70065437	69994055
32	53037356	53002668
33	37433760	37417977
34	24630887	24624123
35	15108047	15105276
36	8640134	8639030
37	4609179	4608750
38	2295898	2295733
39	1069890	1069824

p = 11 (cont.)

k	$L_{11,k}$	$C_{11,k}$
40	467807	467781
41	192788	192777
42	75415	75410
43	28259	28257
44	10250	10249
45	3664	3663
46	1305	1305
47	467	467
48	172	172
49	67	67
50	26	26
51	11	11
52	5	5
53	2	2
54	1	1
55	1	1

p = 12

k	$L_{12,k}$	$C_{12,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	175	*
8	485	*
9	1405	*
10	4191	*
11	12763	551
12	39243	5026
13	119890	28908
14	359307	130365
15	1043774	499888
16	2911086	1694642
17	7739601	5184582
18	19515361	14484609
19	46505609	37234106

p = 12 (cont.)

k	$L_{12,k}$	$C_{12,k}$
20	104504341	88525650
21	221147351	195410521
22	440393606	401665379
23	825075506	770641945
24	1454265734	1382812173
25	2411961516	2324370686
26	3765262970	3665000607
27	5534255092	5427096867
28	7661345277	7554423219
29	9992340187	9892755100
30	12281841209	12195279971
31	14229503560	14159301249
32	15542350436	15489241571
33	16006173014	15968704512
34	15542350436	15517703745
35	14229503560	14214388740
36	12281841209	12273198300
37	9992340187	9987729903
38	7661345277	7659048949
39	5534255092	5533185036

p = 12 (cont.)

k	$L_{12,k}$	$C_{12,k}$
40	3765262970	3764795097
41	2411961516	2411768702
42	1454265734	1454190308
43	825075506	825047242
44	440393606	440383354
45	221147351	221143686
46	104504341	104503035
47	46505609	46505142
48	19515361	19515189
49	7739601	7739534
50	2911086	2911060
51	1043774	1043763
52	359307	359302
53	119890	119888
54	39243	39242
55	12763	12762
56	4191	4191
57	1405	1405
58	485	485
59	175	175

$p = 12$ (cont.)

k	$L_{12,k}$	$C_{12,k}$
60	68	68
61	26	26
62	11	11
63	5	5
64	2	2
65	1	1
66	1	1

p = 13

k	$L_{13,k}$	$C_{13,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	176	*
8	492	*
9	1446	*
10	4435	*
11	14140	*
12	46415	1301
13	154658	13999
14	517121	93569
15	1711908	489387
16	5546619	2179949
17	17422984	8610789
18	52664857	30809058
19	152339952	101093020

p = 13 (cont.)

k	$L_{13,k}$	$C_{13,k}$
20	420048805	306619793
21	1101083128	864317147
22	2739261020	2273437235
23	6461056816	5597433239
24	14441470390	12932769342
25	30583652956	28100037365
26	61372294334	57519070110
27	116724411757	111089419502
28	210474287115	202705284331
29	359954668522	349854951952
30	584089835857	571708043354
31	899632282299	885315952770
32	1315729343451	1300116617536
33	1827823498798	1811764113815
34	2412694353115	2397114478044
35	3026821673656	3012567495251
36	3609810088490	3597513119353
37	4093273437761	4083272448966
38	4413678080790	4406012122869
39	4525920859198	4520384306832

p = 13 (cont.)

k	$L_{13,k}$	$C_{13,k}$
40	4413678080790	4409911747398
41	4093273437761	4090861008231
42	3609810088490	3608355629886
43	3026821673656	3025996522703
44	2412694353115	2412253931236
45	1827823498798	1827602341191
46	1315729343451	1315624835444
47	899632282299	899585775383
48	584089835857	584070320028
49	359954668522	359946928749
50	210474287115	210471375962
51	116724411757	116723367957
52	61372294334	61371935016
53	30583652956	30583533061
54	14441470390	14441431145
55	6461056816	6461044052
56	2739261020	2739256828
57	1101083128	1101081723
58	420048805	420048320
59	152339952	152339777

p = 13 (cont.)

k	$L_{13,k}$	$C_{13,k}$
60	52664857	52664789
61	17422984	17422958
62	5546619	5546608
63	1711908	1711903
64	517121	517119
65	154658	154657
66	46415	46414
67	14140	14140
68	4435	4435
69	1446	1446
70	492	492
71	176	176
72	68	68
73	26	26
74	11	11
75	5	5
76	2	2
77	1	1
78	1	1

p = 14

k	$L_{14,k}$	$C_{14,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	177	*
8	495	*
9	1464	*
10	4541	*
11	14775	*
12	50159	*
13	175652	3159
14	628635	39260
15	2271860	300748
16	8191607	1799700
17	29148848	9181185
18	101403945	41613518
19	342260293	171301235

p = 14 (cont.)

k	$L_{14,k}$	$C_{14,k}$
20	1114246152	648723984
21	3483731123	2278745633
22	10428156463	7466604072
23	29823515196	22916781627
24	81379048741	66099464855
25	211701728866	179638437584
26	524838288774	461011580013
27	1239859474951	1119306632640
28	2791316427401	2575222889571
29	5990156004406	5622435575500
30	12257461922574	11663261941318
31	23925444193310	23013407529653
32	44564782413424	43234705737618
33	79245514741538	77402043968500
34	134581443007315	132152655846345
35	218370131994476	215327701580998
36	338656477955759	335032391116513
37	502149047941551	498043461420246
38	712113254298926	707689564672453
39	966119418676935	961585825489930

p = 14 (cont.)

k	$L_{14,k}$	$C_{14,k}$
40	1254253893785795	1249834675602570
41	1558495757031738	1554398715428131
42	1853824841346885	1850212617950038
43	2111235531267396	2108207254742072
44	2302269388519749	2299855868850890
45	2404130854745735	2402302590759788
46	2404130854745735	2402814904220039
47	2302269388519749	2301369651720480
48	2111235531267396	2110651394921418
49	1853824841346885	1853464867161397
50	1558495757031738	1558285275004448
51	1254253893785795	1254137166462738
52	966119418676935	966058045338745
53	712113254298926	712082670286631
54	502149047941551	502134606351257
55	338656477955759	338650016859694
56	218370131994476	218367392720691
57	134581443007315	134580341919994
58	79245514741538	79245094691327
59	44564782413424	44564630072987

p = 14 (cont.)

k	$L_{14,k}$	$C_{14,k}$
60	23925444193310	23925391528278
61	12257461922574	12257444499522
62	5990156004406	5990150457761
63	2791316427401	2791314715482
64	1239859474951	1239858957825
65	524838288774	524838134114
66	211701728866	211701682450
67	81379048741	81379034600
68	29823515196	29823510761
69	10428156463	10428155017
70	3483731123	3483730631
71	1114246152	1114245976
72	342260293	342260225
73	101403945	101403919
74	29148848	29148837
75	8191607	8191602
76	2271860	2271858
77	628635	628634
78	175652	175651
79	50159	50159

p = 14 (cont.)

k	$L_{14,k}$	$C_{14,k}$
80	14775	14775
81	4541	4541
82	1464	1464
83	495	495
84	177	177
85	68	68
86	26	26
87	11	11
88	5	5
89	2	2
90	1	1
91	1	1

p = 15

k	$L_{15,k}$	$C_{15,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	177	*
8	496	*
9	1471	*
10	4583	*
11	15036	*
12	51814	*
13	185987	*
14	691001	7741
15	2632420	110381
16	10176660	959374
17	39500169	6499706
18	152374465	37539466
19	578891716	192842712

p = 15 (cont.)

k	$L_{15,k}$	$C_{15,k}$
20	2149523582	901377310
21	7753406889	3885009655
22	27040032015	15572583105
23	90859878747	58393017190
24	293429720936	205715634053
25	909199479603	683177510788
26	2699941862354	2144541235919
27	7678976881470	6377475240399
28	20910536197366	18001615191104
29	54513628209893	48311050541875
30	136070590191317	123449595358013
31	325264684039708	300749354421503
32	744823014518211	699350108423184
33	1634428386201309	1553855733393286
34	3438285431507061	3301861994911062
35	6936720574597423	6715921394433155
36	13426937977152944	13085237079085229
37	24944913304587039	24439136746521945
38	44497508446112065	43781285251726544
39	76242525660396354	75271977853717106

p = 15 (cont.)

k	$L_{15,k}$	$C_{15,k}$
40	125521270445592815	124262478451794760
41	198626349980261157	197063431080433042
42	302194587531738164	300336662510870107
43	442167645425634116	440052795351963640
44	622367640359865212	620062341104086439
45	842878760010352632	840472214620329758
46	1098571996084797125	1096166036361399038
47	1378203771448328912	1375900185773096860
48	1664504614806849040	1662392479364026361
49	1935518921253340732	1933664512190616574
50	2167174862664381524	2165616006894383011
51	2336721756356087856	2335467291963665033
52	2426376196165902704	2425409960013204929
53	2426376196165902704	2425664021535713098
54	2336721756356087856	2336219576723209859
55	2167174862664381524	2166836191744515302
56	1935518921253340732	1935300544660143022
57	1664504614806849040	1664370030624532870
58	1378203771448328912	1378124524832488687
59	1098571996084797125	1098527430882329780

p = 15 (cont.)

k	$L_{15,k}$	$C_{15,k}$
60	842878760010352632	842854834413817653
61	622367640359865212	622355382845277188
62	442167645425634116	442161655252206509
63	302194587531738164	302191796209764061
64	198626349980261157	198625110119074266
65	125521270445592815	125520745606786906
66	76242525660396354	76242313958512824
67	44497508446112065	44497427067016907
68	24944913304587039	24944883481057701
69	13426937977152944	13426927548992045
70	6936720574597423	6936717090864854
71	3438285431507061	3438284317260417
72	1634428386201309	1634428043940840
73	744823014518211	744822913114198
74	325264684039708	325264654890834
75	136070590191317	136070581999699
76	54513628209893	54513625938028
77	20910536197366	20910535568729
78	7678976881470	7678976705817
79	2699941862354	2699941812194

p = 15 (cont.)

k	$L_{15,k}$	$C_{15,k}$
80	909199479603	909199464828
81	293429720936	293429716395
82	90859878747	90859877283
83	27040032015	27040031520
84	7753406889	7753406712
85	2149523582	2149523514
86	578891716	578891690
87	152374465	152374454
88	39500169	39500164
89	10176660	10176658
90	2632420	2632419
91	691001	691000
92	185987	185987
93	51814	51814
94	15036	15036
95	4583	4583
96	1471	1471
97	496	496
98	177	177
99	68	68

p = 15 (cont.)

k	$L_{15,k}$	$C_{15,k}$
100	26	26
101	11	11
102	5	5
103	2	2
104	1	1
105	1	1

p = 16

k	$L_{16,k}$	$C_{16,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	177	*
8	497	*
9	1474	*
10	4601	*
11	15144	*
12	52496	*
13	190443	*
14	720298	*
15	2821116	19320
16	11353457	311465
17	46541024	3042808
18	192525021	23118465
19	796277250	149685670

p = 16 (cont.)

k	$L_{16,k}$	$C_{16,k}$
20	3264731685	862572395
21	13169762612	4528544658
22	51944608863	21961480198
23	199325942738	99251914621
24	741262597319	420560727420
25	2663871179403	1678281686360
26	9231717579186	6329077092453
27	30807136431804	22618117867415
28	98902152234053	76773365086100
29	305278273828736	247999842702735
30	905723378795687	763685360909770
31	2582678933141963	2245154532231552
32	7078696345634785	6309879915290263
33	18651996404491557	16972802610729232
34	47260851628555164	43742890748220477
35	115191335532066321	108119250899529124
36	270167167501687087	256520586246582461
37	609958885797372810	584673421992357274
38	1326136343400463716	1281134076961638346
39	2777537605930232398	2700579614533070386

p = 16 (cont.)

k	$L_{16,k}$	$C_{16,k}$
40	560632125608383223O	5479829827952799857
41	10909380006413889417	10709494827604086670
42	20472851567540820478	20169093598306723982
43	37064510246977549534	36620483849338454029
44	64755703757207605426	64131220183281379663
45	109211416604009007814	108366231335871500301
46	177848696079000172225	176747716330872745639
47	279730709445943481878	278350098613104210378
48	425052270572963469105	423385461441565247736
49	624101038803773627548	622163407013949073453
50	885660423533434300260	883491393721262596160
51	1214956017367798514471	1212617736381684350878
52	1611417137591950662088	1608989506687991413608
53	2066686548593529103442	2064259206012034697756
54	2563407506160249980411	2561070072143092759372
55	3075292698315733707376	3073125021226493922716
56	3568803573245228168766	3566867715628928049061
57	4006453983295094401806	4004789260291990621810
58	4351363173523456284236	4349984835162465146681
59	4572323361987395717504	4571224710742375695904

p = 16 (cont.)

k	$L_{16,k}$	$C_{16,k}$
60	4648429222263945620900	4647586298937784001491
61	4572323361987395717504	4571700970421071914980
62	4351363173523456284236	4350920993620381139035
63	4006453983295094401806	4006151782717342115727
64	3568803573245228168766	3568604944103910221700
65	3075292698315733707376	3075167175805421897902
66	2563407506160249980411	2563331263109749220793
67	2066686548593529103442	2066642050873380637099
68	1611417137591950662088	1611392192597266839355
69	1214956017367798514471	1214942590399997790201
70	885660423533434300260	885653486802431529240
71	624101038803773627548	624097600514858383973
72	425052270572963469105	425050636143463019881
73	279730709445943481878	279729964622586702774
74	177848696079000172225	177848370814214728354
75	109211416604009007814	109211280533389667566
76	64755703757207605426	64755649243571203894
77	37064510246977549534	37064489336439080294
78	20472851567540820478	20472843888563310367
79	10909380006413889417	10909377306471851409

p = 16 (cont.)

k	$L_{16,k}$	$C_{16,k}$
80	5606321256083832230	5606320346884302466
81	2777537605930232398	2777537312500496686
82	1326136343400463716	1326136252540580428
83	609958885797372810	609958858757339331
84	270167167501687087	270167159748279703
85	115191335532066321	115191333382542562
86	47260851628555164	47260851049663380
87	18651996404491557	18651996252117066
88	7078696345634785	7078696306134605
89	2582678933141963	2582678922965298
90	905723378795687	905723376163265
91	305278273828736	305278273137734
92	98902152234053	98902152048065
93	30807136431804	30807136379990
94	9231717579186	9231717564150
95	2663871179403	2663871174820
96	741262597319	741262595848
97	199325942738	199325942242
98	51944608863	51944608686
99	13169762612	13169762544

p = 16 (cont.)

k	$L_{16,k}$	$C_{16,k}$
100	3264731685	3264731659
101	796277250	796277239
102	192525021	192525016
103	46541024	46541022
104	11353457	11353456
105	2821116	2821115
106	720298	720298
107	190443	190443
108	52496	52496
109	15144	15144
110	4601	4601
111	1474	1474
112	497	497
113	177	177
114	68	68
115	26	26
116	11	11
117	5	5
118	2	2
119	1	1
120	1	1

p = 17

k	$L_{17,k}$	$C_{17,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	177	*
8	497	*
9	1475	*
10	4608	*
11	15186	*
12	52763	*
13	192218	*
14	732472	*
15	2905512	*
16	11932174	48629
17	50411413	880840
18	217511951	9597679
19	950868860	81134475

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
20	4177279665	584070235
21	18296525771	3743001968
22	79325102628	21872619648
23	338296078276	118219705904
24	1411765943421	596418502890
25	5741051079728	2826245703364
26	22676300605294	12637403418903
27	86784366393764	53508871585831
28	321234417272839	215149072500451
29	1148593987918546	823402894660149
30	3963769077757287	3005410655080960
31	13195353648075411	10479890111027882
32	42362864281584321	34964179546197292
33	131148370035442310	111759155893835862
34	391539812045990580	342653895232471165
35	1127433671455288204	1008812491101019168
36	3131891884686364176	2854788877514025792
37	8395457287624530088	7772051747899351567
38	21724015569868084675	20372871858428135419
39	54280006695385411809	51457832868217433374

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
40	131007119329183389752	125324294848760195159
41	305533163531920156821	294497823353305550364
42	688784125211424981966	668111968577834234653
43	1501481098415531841466	1464113416605849309120
44	3166034784548145937607	3100835593668327754902
45	6459711714000551101633	6349876249144888287729
46	12757029589816163785517	12578335989179680499455
47	24392513824970383789517	24111682226859005541488
48	45170996006605836763143	44744563014255916056629
49	81036030181360694500427	80410262033241321563285
50	140872080022355281834324	139984481513018036868703
51	237359406149123114746239	236142228054515252894941 5
52	387724075281462985272675	386110319260510311513279
53	614140260304462130385026	612071145530742434044913
54	943470250426526060723404	940904415034488076173270
55	1406001649040108855147043	1402924018443117934495312
56	2032897246483081019191586	2029326274858900979978955
57	2852239339913852728731860	2848230949788953643852653
58	3883832284936436998365716	3879479256835728968412608
59	5133293720541433657068816	5128720018701909152525162

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
60	6586342935767421267463627	6581693407803871994261460
61	8204477211622181835575404	8199904045281537904481391
62	9923297786436063850375120	9918945800838662488792886
63	11654518975210565207321726	11650512079029430075911365
64	13292146986845604951480332	13288577881062315200410221
65	14722522059385159911201458	14719446568052952340567857
66	15837042684113299970813096	15834479151082362855242764
67	16545654197751211495418728	16543587434958539283125523
68	16788801124652327714275292	16787189663016572148130262
69	16545654197751211495418728	16544439216788667132483821
70	15837042684113299970813096	15836157010262727784177955
71	14722522059385159911201458	14721897951409598794803546
72	13292146986845604951480332	13291721931136733758792325
73	11654518975210565207321726	11654239242866686621899761
74	9923297786436063850375120	9923119936995160480575206
75	8204477211622181835575404	8204367999880312728010158
76	6586342935767421267463627	6586278179927593347304878
77	5133293720541433657068816	5133256655976673016240492
78	3883832284930430990305716	3883011812063958911512921
79	2852239339913852728731860	2852228430526167335236082

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
80	2032897246483081019191586	2032891640159124992742854
81	1406001649040108855147043	1405998871501593725223996
82	943470250426526060723404	943468924289889230478354
83	614140260304462130385026	614139650345485473115615
84	387724075281462985272675	387723805114268443548062
85	237359406149123114746239	237359290957779829271247
86	140872080022355281834324	140872032761501503754974
87	81036030181360694500427	81036011529363711116935
88	45170996006605836763143	45170988927909338753810
89	24392513824970383789517	24392511242291411147353
90	12757029589816163785517	12757028684092774813156
91	6459711714000551101633	6459711408722274640471
92	3166034784548145937607	3166034685645993012551
93	1501481098415531841466	1501481067608395223673
94	688784125211424981966	688784115979707350965
95	305533163531920156821	305533160868048962382
96	131007119329183389752	131007118587920787850
97	54280006695385411809	54280006496059467600
98	21724015569868084675	21724015517923475316
99	8395457287624530088	8395457274454767299

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
100	31318918846863641 76	31318918814216324 23
101	1127433671455288204	1127433670659010928
102	391539812045990580	391539811853465548
103	13114837003544231 0	131148369988901281
104	4236286428158432 1	4236286427023086 2
105	1319535364807541 1	1319535364525429 4
106	396376907775728 7	396376907703698 8
107	114859398791854 6	114859398772810 3
108	32123441727283 9	32123441722034 3
109	86784366393764	86784366378620
110	22676300605294	22676300600693
111	5741051079728	5741051078254
112	1411765943421	1411765942924
113	338296078276	338296078099
114	79325102628	79325102560
115	18296525771	18296525745
116	4177279665	4177279654
117	950868860	950868855
118	217511951	217511949
119	50411413	50411412

p = 17 (cont.)

k	$L_{17,k}$	$C_{17,k}$
120	11932174	11932173
121	2905512	2905512
122	732472	732472
123	192218	192218
124	52763	52763
125	15186	15186
126	4608	4608
127	1475	1475
128	497	497
129	177	177
130	68	68
131	26	26
132	11	11
133	5	5
134	2	2
135	1	1
136	1	1

$p = 18$

k	$L_{18,k}$	$C_{18,k}$
0	1	*
1	1	*
2	2	*
3	5	*
4	11	*
5	26	*
6	68	*
7	177	*
8	497	*
9	1476	*
10	4611	*
11	15204	*
12	52872	*
13	192917	*
14	737248	*
15	2939612	*
16	12180208	*
17	52211412	123867
18	230341716	2497405
19	1039651295	30134509

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
20	4769060224	281454170
21	22077090175	2236622385
22	102405806817	15822140019
23	472752601530	102120272749
24	2158737372439	610230786880
25	9698621668842	3407994560102
26	42681362095598	17903411689892
27	183329413930393	88889957079123
28	766437448902278	418617683869058
29	3112054241524242	1875340614436735
30	12253485516048893	8010675872441072
31	46733208420738751	32692889274350103
32	172509020264405007	127697179143259259
33	616037183140022935	478089202959492921
34	2127620200311537848	1717979587035188459
35	7106062209729616481	5932393907656280767
36	22952094444965640286	19706802208869010859
37	71701132617245623768	63038411161244806000
38	216681841003250716320	194352165367024105109
39	633601653376111363593	578001133691694667563

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
40	17931942648548951 21738	16594155655907060 71239
41	49134440675194445 52289	46023080265064451 07155
42	130385469264175247 50191	123388482682049023 01871
43	335194745634892451 24430	319974957713942737 74356
44	835086595562583799 30576	803054976630490360 15397
45	2016847119170014774 00777	1951601173662343030 60584
46	4723462837522806758 96235	4594798146995019493 99327
47	10730748983155811983 38137	10485041615277004559 09377
48	23654503989789141271 34164	23199991003905335029 79973
49	50610424961273263384 02467	49795805920308399564 58947
50	105131270079549857221 98178	103716297099562927676 47251
51	212084165467479903713 35840	209701700376220356243 90539
52	415606756831101420943 29965	411717348765884109156 04130
53	791340696807226289260 98492	785183159173150221966 58928
54	1464381893812909656213 52409	1454926501002099648541 24206
55	2634228429963467280191 05213	2620142754175855881528 16325
56	4607379146052707362992 73298	4587019394652052440382 69097
57	7836896167821663141976 84989	7808338060664897050680 28205
58	12966031994950329780996 04792	12927153583151190763071 84560
59	20869945279725482656509 69136	20818568806650369572598 64130

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
60	32685892690655622673168819690	32619983518609629335244155335
61	49818783544909252396280449799	49736692272153225832451401404
62	73906720766140109387839541146	73807442051845975281585041182
63	106731499367082648448605063348	106614910653464873018723072884
64	150063321978243103437451614559	149930360436209627839148461996
65	205438033244501382427633155579	205290772330413477367648126008
66	273878738529406013712503010085	273720337345884403682372222206
67	355590071108861128051996269998	355424588930329257912893089913
68	449669391716575115505908277769	449501483036890327665834186637
69	553890529463328958878471320012	553725056806222067521530790037
70	664617849796019913364709394473	664459467219058429451806011660
71	776898205764946932889787452235	776750971687434485462033653385
72	884754976541860967067834337666	884622048830812865652566968007
73	981674622914420192889288673353	981558073474057974387070135007
74	1061238094661591033895505377749	1061138858886376299409615261198
75	1117815401448182774091920754667	1117733354897559051452581569954
76	1147220353118516202716161024401	1147154488597035022238764335517
77	1147220353118516202716161024401	1147169019533749682574251110039
78	1117815401448182774091920754667	1117776562754686610510260667748
79	1061238094661591033895505377749	1061209572063462702239448776881

Table I

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
80	98167462291442019288928867353	98165429383286130311882231739
81	88475497654186096706783433766	88474091646930725872113343676
82	77689820576494693288978745235	77688877103466725841272881081
83	66461784979601991336470939473	66461170838015549419169854037
84	55389052946332895887847132012	55388665221647655181814660393
85	44966939171657511550590827769	44966701811981195149827815572
86	35559007110886112805199626998	35558866238690899081741788464
87	27387873852940601371250301085	27387792816863159149603199633
88	20543803324450138242763315579	20543758153435479637946017690
89	15006332197824310343745161459	15006307805303406676323970322
90	10673149936708264844860506348	10673137179676092349581880591
91	7390672076614010938783954146	7390665616901391214807012298
92	4981878354490925239628049799	4981875188455835413196165329
93	3268589269065562267316819690	3268587767584364949629606717
94	2086994527972548265650969136	2086993839188392247088730182
95	1296603199495032978099604792	1296602893961860214461646015
96	783689616782166314197684989	783689485775044321143053565
97	460737914605270736299273298	460737860325263299651246021
98	263422842996346728019105213	263422821272330958825072242
99	146438189381290965621352409	146438180985833626052211668

Table I

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
100	79134069680722628926098492	79134066548830731069971099
101	41560675683110142094329965	41560674555676467374309857
102	21208416546747990371335840	21208416155208177529067927
103	10513127007954985722198178	10513126876806615494230815
104	5061042496127326338402467	5061042453764462010277108
105	2365450398978914127134164	2365450385783560467705290
106	1073074898315581198338137	1073074894351812117759732
107	472346283752280675896235	472346282603686687257389
108	201684711917001477400777	201684711595767059937494
109	83508659556258379930576	83508659469474013484316
110	33519474563489245124430	33519474540812944503992
111	13038546926417524750191	13038546920676473665862
112	4913444067519444552289	4913444066107678607394
113	1793194264854895121738	1793194264516599042965
114	633601653376111363593	633601653296786260788
115	216681841003250716320	216681840984954190481
116	71701132617245623768	71701132613068344077
117	22952094444965640286	22952094444014771415
118	7106062209729616481	7106062209512104525
119	2127620200311537848	2127620200261126433

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
120	6160371831400229355	6160371831280907600
121	17250902026440500 7	17250902026149949 4
122	4673320842073875 1	4673320842000627 9
123	122534855160488 93	122534855158566 75
124	31120542415242 42	31120542414714 79
125	7664374489022 78	7664374488870 92
126	1833294139303 93	1833294139257 85
127	426813620955 98	426813620941 23
128	96986216688 42	96986216683 45
129	21587373724 39	21587373722 62
130	4727526015 30	4727526014 62
131	1024058068 17	1024058067 91
132	220770901 75	220770901 64
133	47690602 24	47690602 19
134	10396512 95	10396512 93
135	2303417 16	2303417 15
136	522114 12	522114 11
137	121802 08	121802 08
138	29396 12	29396 12
139	7372 48	7372 48

p = 18 (cont.)

k	$L_{18,k}$	$C_{18,k}$
140	192917	192917
141	52872	52872
142	15204	15204
143	4611	4611
144	1476	1476
145	497	497
146	177	177
147	68	68
148	26	26
149	11	11
150	5	5
151	2	2
152	1	1
153	1	1

TABLE II

p	Time: Method I	Time: Method II
9	0.5 Minutes	1.0 Minutes
10	1.2 Minutes	2.0 Minutes
11	3.1 Minutes	4.7 Minutes
12	8.0 Minutes	11.1 Minutes
13	19.4 Minutes	25.8 Minutes
14	52.0 Minutes	1.0 Hours
15	2.2 Hours	2.4 Hours
16	5.7 Hours	6.0 Hours
17	---	14.7 Hours
18	---	35.5 Hours

TABLE III

k	$E_{18,k}$	$E_{17,k}$	$E_{16,k}$	$E_{15,k}$
76	.2125			
75	.2135			
74	.2154			
73	.2184			
72	.2225			
71	.2276			
70	.2340			
69	.2416			
68	.2506	.3761		
67	.2611	.3771		
66	.2733	.3802		
65	.2874	.3854		
64	.3036	.3927		
63	.3221	.4024		
62	.3433	.4145		
61	.3674	.4293		
60	.3950	.4470	.6613	
59	.4265	.4679	.6635	
58	.4626	.4924	.6699	
57	.5044	.5209	.6807	
56	.5513	.5540	.6962	
55	.6056	.5924	.7166	
54	.6682	.6366	.7423	
53	.7397	.6878	.7738	
52	.8236	.7470	.8118	1.155
51	.9200	.8153	.8571	1.164
50	1.032	.8945	.9106	1.182
49	1.162	.9863	.9736	1.210
48	1.314	1.093	1.047	1.248
47	1.491	1.217	1.134	1.296
46	1.699	1.362	1.235	1.358
45	1.942	1.532	1.353	1.433
44	2.229	1.730	1.492	1.523
43	2.568	1.964	1.655	1.632
42	2.968	2.240	1.846	1.761
41	3.441	2.566	2.072	1.915
40	4.004	2.952	2.338	2.098
39	4.672	3.410	2.654	2.315
38	5.467	3.956	3.030	2.573
37	6.416	4.608	3.477	2.880
36	7.547	5.386	4.011	3.247
35	8.896	6.319	4.650	3.685
34	10.51	7.438	5.418	4.211
33	12.43	8.780	6.342	4.844
32	14.71	10.39	7.456	5.607
31	17.42	12.33	8.801	6.529
30	20.62	14.64	10.43	7.647
29	24.38	17.41	12.39	9.005
28	28.77	20.71	14.76	10.66
27	33.82	24.60	17.61	12.67
26	39.57	29.17	21.02	15.11
25	45.98	34.47	25.09	18.07