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with Controlled Error (TRICE)
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SUMMARY

An IBM-704 subroutine, TRICE, has been written in FORTRAN which evaluates a single integral by means of the trapezoidal rule approximation. In the calculational procedure, the integration step is halved until the desired degree of convergence is reached or until a preset maximum number of steps is exceeded. A maximum of ten different integrands may be used in any one calling program.

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AN IBM 704 SUBROUTINE FOR TRAPEZOIDAL INTEGRATION
WITH CONTROLLED ERROR (TRICE)

G. W. Nestor

Method

The trapezoidal rule approximation T_N to the definite integral of a function $f(x)$ between limits a and b is of the form

$$T_N = \frac{b-a}{N} \left\{ \frac{1}{2} [f(a) + f(b)] + \sum_{j=2}^N f(x_j) \right\} \quad (1)$$

where the interval $a \leq x \leq b$ is divided into N equal subintervals by the points

$$x_j = \frac{(b-a)(j-1)}{N} + a, \quad j = 1, 2, \dots, N+1.$$

Consider an interval divided into eight subintervals as shown in Fig. 1.

$a =$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	$= b$	
	x_1				x_5				x_9		T_2
	.				.				.		
	x_1		x_3		x_5		x_7		x_9		
	.	.	.	.	.	.	.	.	.		T_4

Fig. 1. Integration Interval

Then

$$\begin{cases} T_1 = (b-a) [f(a) + f(b)] \\ T_2 = \frac{(b-a)}{2} [f(a) + f(b) + f(x_5)] \\ T_4 = \frac{(b-a)}{4} [f(a) + f(b) + f(x_5) + f(x_3) + f(x_7)] \\ T_8 = \frac{(b-a)}{8} [f(a) + f(b) + f(x_5) + f(x_3) + f(x_7) + f(x_2) + f(x_4) + f(x_6) + f(x_8)] \end{cases} \quad (2)$$

and in general

$$T_{2N} = \frac{1}{2} T_N + \frac{b-a}{2N} \sum f(x_k). \quad (3)$$

In the program the maximum number of intervals is 2^{IMAX} ; the number of intervals $2N$ is 2^I where $I = 1, 2, \dots, \text{IMAX}$; the lower limit LL of the sum in (3) is

$$LL = 2^{\text{IMAX}-I} + 1.$$

The increment ID of the index k is

$$ID = 2(LL-1).$$

The upper limit LU of the sum is

$$LU = LL + (2^{I-1} - 1) ID.$$

The approximation T_{2N} is evaluated for successive values of N until either

$$\left| \frac{T_{2N} - T_N}{T_{2N}} \right| < \epsilon \quad (4)$$

or until $2N = 2^{\text{IMAX}}$; in the latter case an indicator is set to signal nonconvergence.

Use of the Subroutine

The subroutine is entered by the standard CALL statement:

CALL TRICE (A, B, EPSI, IFUN, IMAX, ANS, ICEN)

where A = lower limit

 B = upper limit

 EPSI = the desired convergence criterion

 IFUN = integer designating the integrand ($1 \leq \text{IFUN} \leq 10$)

 IMAX = integer designating the maximum number of intervals 2^{IMAX} ;

on return { ANS = result

 ICEN = convergence indicator; +1 if (4) is satisfied, -1 if not.

Ten functions must be provided for the ten possible integrands; if less than ten integrands are to be used in the calling program the unused functions may be dummy routines of the form

FUNCTION FNAME(X)

FNAME = 0.0*X

RETURN

END ().

The function names used by the TRICE subroutine are

IFUN:

1	ONE(X)
2	TWO(X)
3	THREE(X)
4	FOUR(X)
5	FIVE(X)

IFUN:

6	SIX(X)
7	SEVEN(X)
8	EIGHT(X)
9	NINE(X)
10	TEN(X)

Error Detection

If a value for IFUN other than $1 \leq \text{IFUN} \leq 10$ is specified in the call statement, a message to this effect will be printed on line and control will be returned to the main program at the normal return, with the result (ANS) set equal to -10^6 . The difference of the largest memory address minus the address of the entry into TRICE will also be printed on line as a decimal integer.

Length and Timing

The TRICE subroutine is 503 words long; in addition the following subroutines are required:

EXP(2	(FIL)
EXP(1	(BDC)
(RTN)	(DBC)
(LEV)	(IØH)I
(SPH)	(IØH)Ø .

Also, ten additional function subprograms for the integrands are required. The timing is variable, dependent upon the function integrated.

TABLE 1. FORTRAN PROGRAM

C1834	B	CWN TRAPEZOIDAL INTEGRATION WITH CONTROLLED ERROR	18340101
		SUBROUTINE TRICE(A,B,EPS,IFUN,IMAX,ANS,ICON)	18340103
		IF(IFUN)1,1,2	18340105
1		PRINT 105	18340107
105		FORMAT(36H ILLEGITIMATE FUNCTION SPECIFICATION)	18340109
		SXD *+2, 4	18340111
		TRA *+2	18340113
		HPR 0	18340115
		PRINT 50,LOC	18340117
50		FORMAT(18H ABSOLUTE LOCATION,18)	18340119
		FIN2=-1.0E6	18340121
		GO TO 104	18340123
2		IF(IFUN-10)3,3,1	18340125
3		GO TO (4,5,6,7,8,9,10,11,12,13),IFUN	18340201
4		XU=ONE(B)+ONE(A)	18340203
		GO TO 99	18340205
5		XU=TWO(B)+TWO(A)	18340207
		GO TO 99	18340209
6		XU=THREE(B)+THREE(A)	18340211
		GO TO 99	18340213
7		XU=FOUR(B)+FOUR(A)	18340215
		GO TO 99	18340217
8		XU=FIVE(B)+FIVE(A)	18340219
		GO TO 99	18340221
9		XU=SIX(B)+SIX(A)	18340223
		GO TO 99	18340225
10		XU=SEVEN(B)+SEVEN(A)	18340301
		GO TO 99	18340303
11		XU=EIGHT(B)+EIGHT(A)	18340305
		GO TO 99	18340307
12		XU=FNINE(B)+FNINE(A)	18340309
		GO TO 99	18340311
13		XU=TEN(B)+TEN(A)	18340313
99		FIN=(B-A)*0.5*XU	18340315
		FMAX=2.0**IMAX	18340106
		DO 102 I=1,IMAX	18340107
		NINT=2**I	18340109
		LL=2**((IMAX-I)+1)	18340111
		ID=2*(LL-1)	18340113
		LU=LL+(2**((I-1)-1))*ID	18340115
		SUM=0.0	18340117
		DO 100 J=LL,LU,ID	18340119
		X=A+(FLOATF(J-1))*(B-A)/FMAX	18340121
		GO TO (14,15,16,17,18,19,20,21,22,23),IFUN	18340317
14		ARG=ONE(X)	18340319
		GO TO 100	18340321
15		ARG=TWO(X)	18340323
		GO TO 100	18340325
16		ARG=THREE(X)	18340401
		GO TO 100	18340403
17		ARG=FOUR(X)	18340405
		GO TO 100	18340407
18		ARG=FIVE(X)	18340409
		GO TO 100	18340411
19		ARG=SIX(X)	18340413
		GO TO 100	18340415

Table 1 - cont'd

20	ARG=SEVEN(X)	18340417
	GO TO 100	18340419
21	ARG=EIGHT(X)	18340421
	GO TO 100	18340423
22	ARG=FNINE(X)	18340425
	GO TO 100	18340501
23	ARG=TEN(X)	18340503
100	SUM=SUM+ARG	18340505
	FN=NINT	18340125
	FIN2=0.5*FIN+(B-A)*SUM/FN	18340201
	IF(ABS((FIN-FIN2)/FIN2)-EPS)103,101,101	18340101
101	FIN=FIN2	18340205
102	CONTINUE	18340207
	ICON=-1	18340209
	GO TO 104	18340211
103	ICON=1	18340213
104	ANS=FIN2	18340215
	RETURN	18340217

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