WHAT IS THE EMITTANCE OF THE INJECTED BEAM?

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The usual statement is that the emittance of the injected beam is $\varepsilon_x = \varepsilon_y = 10$ (normalized) for heavy ions in RHIC.

Tracking studies require a more precise statement. Tracking requires that a 4-dimensional surface be specified in $x, x', y, y'$ space that contains 95% of the beam. The tracking studies can then investigate the stability of the particles inside this surface.

A simple way to specify this surface is by

$$\varepsilon_T = \varepsilon_x (x, x') + \varepsilon_y (y, y') = C,$$

where $C$ is a constant chosen so that this surface contains 95% of the beam. One reason for using this expression is that $\varepsilon_T$ is roughly a constant of the motion.
It will be argued below that the proper choice of $C$ is

$$C = 10 \left( \frac{5}{3} \right) = 16.7$$

Corresponding to the statement that $\bar{x} = \bar{y} = 10$.

Assuming for the moment that $C = 16.7$ is the proper choice of $C$, then in tracking studies where runs are done with $\bar{x} = \bar{y}$, then the proper starting emittance is

$$\bar{x} = \bar{x} = 8.33$$

If runs are done with $\bar{y} = 0$, then the starting $\bar{x}$ is

$$\bar{x} = 16.7$$

$\bar{y} = 0$

These two points are on the surface $\varepsilon_T = \text{constant}$ that contains 95% of the beam.
I assume that the statement \( \exists x = \exists y = 10 \) means that for the projection of the particles on the \( x, x' \) plane, 95% of the particles have an \( \exists x \) which is smaller than \( \exists x = 10 \), and a similar statement applies to the \( y, y' \) plane.

I assume that the distribution \( p(x, x', y, y') \) is Gaussian with the form

\[
p(x, x', y, y') \sim \exp \left( - \frac{(\exists x(x, x') + \exists y(x, y'))}{\Sigma} \right)
\]

The projection on the \( xx' \) plane has the distribution

\[
p(x, x') = \int dy \int dy' p(x, x', y, y')
\sim \exp \left( - \frac{\exists x(x, x')}{\Sigma} \right).
\]

In order for 95% of the particles to have an \( \exists x \) which is smaller than \( \exists x = 10 \), then

\[
\overline{\Sigma} = 10/3
\]
The fraction of the particles that have a total emittance, \( \varepsilon_T = \varepsilon_x + \varepsilon_y \), which is smaller than \( \varepsilon_T \) is given by

\[
F(\varepsilon_T) = 1 - \exp \left(-\frac{\varepsilon_T}{\varepsilon}\right) \left(1 + \frac{\varepsilon_T}{\varepsilon}\right).
\]

This may be derived from Eq. (1) for \( P(x, x', y, y') \). The choice \( \varepsilon_T \) that includes 95% of the particles is \( \varepsilon_T = 5 \bar{\varepsilon} \) (the actual answer is closer to 4.8 \( \bar{\varepsilon} \)).

Thus, the \( \varepsilon_T \) that contains 95% of the particles is

\[
\varepsilon_T = 5 \bar{\varepsilon} \\
\varepsilon_T = 5 \times 5 \left(\frac{10}{3}\right) \\
\varepsilon_T = 16.67
\]
I wish to thank Harald Bohr for his suggestion and for the meaning of the statement that \( E_x = 10 \).