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U. S. ATOMIC ENERGY COMMISSION
WASHINGTON, D. C. 20545

Report Number: AECOP-330

Subject Category: Isotope Separation

RESOLUTION OF A MULTICOMPONENT
CASCADE WITH TWO KEY WEIGHTS
AND MULTI-FEEDS AND WITHDRAWALS

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December 1, 1969

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A B S T R A C T

This report presents an extension of the previously reported constant key weight cascades for multicomponent isotope separation. Basic equations are given for the resolution of the productivity of a multicomponent cascade having an upper and a lower section with different key weights. Multi-feeds and withdrawals are also permitted. This cascade should prove useful in investigating the cascade management of "minor" isotopes, as well as in the separation of a "middle component" in a single cascade.

RESOLUTION OF A MULTICOMPONENT CASCADE WITH TWO KEY
WEIGHTS AND MULTI-FEEDS AND WITHDRAWALS

Introduction

Constant key weight cascades were introduced in Ref. [1] to the subject of multicomponent isotope separation in cascades. The key weight for such a cascade is the arithmetic mean of the molecular weights of two key components whose abundance ratio is everywhere matched. The key components may be dummies, with assigned molecular weights of our choosing. The separation characteristics of such a cascade depend on its key weight. For example, real components with molecular weights less than the key weight will concentrate towards the cascade top.

This report extends the previous work. Basic equations are developed for the resolution of the productivity of a multicomponent cascade having an upper and a lower section with different key weights and multi-feeds and withdrawals. This cascade should prove useful in investigating the joint effects of U-234 and U-236 from power reactor streams on the production of highly enriched uranium. It may also find use in other multicomponent separations.

Cascade to be Analyzed

Figure 1 shows the cascade to be analyzed. Here and elsewhere, the capital letter X denotes a flow of the component indicated by a subscript in the stream indicated by another subscript. For example,

$$X_{j,P} = P X_{j,P} ,$$

the latter being the notation in Ref. [1]. With few exceptions, the notation of Ref. [1] is otherwise followed.

Stage numbers

Schematic of Considered Cascade

Figure 1

The method of analysis followed is also that of Ref. [1], since this seems the simplest. From this point of view, the cascade of Fig. 1 consists of two constant M^* cascades, joined so as to maintain continuity in inter-stage flows and concentrations.

Inter-stage Flows and Concentrations

Introduce the function

$$g_j(M^*, v, n) = \left[e^{-\alpha(M^* - M_j)(v - n)} - 1 \right] / \alpha(M^* - M_j) . \quad (1)$$

Following the development under 5 in Ref. [1],^(a) we see that for the upper cascade section in Fig. 1, we have that the flow of component j in the inter-stage flow at the n th stage is given by

$$X_{j,L_n} = -X_{j,P} g_j(M_1^*, N_P, n) \text{ for } N_E < n \leq N_P , \quad (2)$$

and

$$X_{j,L_n} = -X_{j,P} g_j(M_1^*, N_P, n) + X_{j,E} g_j(M_1^*, N_E, n) \text{ for } N_I < n < N_E . \quad (3)$$

Similarly, for the bottom section,

$$X_{j,L_n} = X_{j,S} g_j(M_2^*, N_S, n) - X_{j,F} g_j(M_2^*, N_F, n) + X_{j,W} g_j(M_2^*, N_W, n) \\ \text{for } N_S < n < N_I , \quad (4)$$

$$X_{j,L_n} = -X_{j,F} g_j(M_2^*, N_F, n) + X_{j,W} g_j(M_2^*, N_W, n) \text{ for } N_F < n < N_S , \quad (5)$$

and

$$X_{j,L_n} = X_{j,W} g_j(M_2^*, N_W, n) \text{ for } N_W \leq n < N_F . \quad (6)$$

Generalization to include other side streams seems clear enough.

The inter-stage flows and component concentrations are then obtained from the above; thus:

$$L_n = \sum_j X_{j,L_n} , \quad (7)$$

and

$$x_{j,n} = X_{j,L_n} / L_n . \quad (8)$$

^(a) The attached Appendix covers this development in considerable detail.

Joining the Upper and Lower Sections

The condition is imposed that there be continuity in inter-stage flows and concentrations at $n = N_I$. This is satisfied by having X_{j,L_n} be the same at $n = N_I$ for both sections. Accordingly, from (3) and (4), evaluated at $n = N_I$, and set equal, we obtain the important relation:

$$\begin{aligned} X_{j,P} g_j(M_1^*, N_P, N_I) - X_{j,E} g_j(M_1^*, N_E, N_I) + X_{j,S} g_j(M_2^*, N_S, N_I) \\ - X_{j,F} g_j(M_2^*, N_F, N_I) + X_{j,W} g_j(M_2^*, N_W, N_I) = 0. \end{aligned} \quad (9)$$

As seen later, this is one of the "productivity equations," crucial to the resolution of a cascade.

Incidentally, in (9), the analog of the "homogeneous function" is obvious.

Relations for a Side-Withdrawal

The component concentrations of a side-withdrawal are those in the cascade at the point of withdrawal. Thus, for the side-product S at stage N_S in Fig. 1, we have from (8) that

$$x_{j,N_S} = x_{j,L_{N_S}} / L_{N_S}.$$

If S is the total flow in this stream, i.e., $S = \sum x_{j,S}$, then

$$x_{j,S} = S x_{j,N_S}.$$

Combining these two relations, we obtain

$$x_{j,S} = C_S x_{j,L_{N_S}}, \quad (10)$$

where

$$C_S = S / L_{N_S}. \quad (11)$$

We may then substitute from (5) in (10) to obtain

$$X_{j,S} = C_S \left[-X_{j,F} g_j(M_2^*, N_F, N_S) + X_{j,W} g_j(M_2^*, N_W, N_S) \right], \quad (12)$$

a particular form of (10), believed to be convenient, and applicable to the Figure 1 situation. More generally, for each side-stream, there is a relation similar to (10), and in such relations, the substitution for terms X_{j,L_n} must follow the particular situation considered.

The Productivity Situation Considered

For the cascade shown in Fig. 1, we suppose that:

- (1) The feeds E and F are completely specified; i.e., $X_{j,E}$ and $X_{j,F}$ are known.
- (2) The total flow S of the side-product is specified, so that the $X_{j,S}$ must satisfy

$$S = \sum X_{j,S}.$$

- (3) The stage numbers N_W, N_F, N_S, N_I, N_E , and N_P are given. (Note that $N_S < N_I < N_E$.)
- (4) The section key weights M_1^* and M_2^* are given.

The unknowns to be determined are: $X_{j,P}, X_{j,S}$, and $X_{j,W}$.

Resolution follows. We summarize the "productivity equations." These are Eq. (9), the component material balances, and Eq. (10), here rewritten to emphasize the linearity of the set and their component independence.

The Productivity Equations

The equations permitting resolution of the Fig. 1 cascade are:

$$\begin{aligned} X_{j,P} g_j(M_1^*, N_P, N_I) - X_{j,E} g_j(M_1^*, N_E, N_I) + X_{j,S} g_j(M_2^*, N_S, N_I) \\ - X_{j,F} g_j(M_2^*, N_F, N_I) + X_{j,W} g_j(M_2^*, N_W, N_I) = 0, \end{aligned} \quad (13)$$

$$X_{j,P} - X_{j,E} + X_{S,j} - X_{j,F} + X_{j,W} = 0 , \quad (14)$$

$$X_{j,S} = C_S \left[-X_{j,F} g_j(M_2^*, N_F, N_S) + X_{j,W} g_j(M_2^*, N_W, N_S) \right] . \quad (15)$$

Note that in this form, the productivity equations form a linear set independently for each component. Given a set of the auxiliary parameters, N_P , N_E , etc., and C_S , we may solve for the withdrawal unknowns $X_{j,P}$, $X_{j,S}$, and $X_{j,W}$, a component at a time. The auxiliary parameters are then used for iteration to satisfy imposed conditions, such as,

$$S = \sum X_{j,S} ,$$

here assumed for illustration.

Separative Work

Separative work is proportional to the sum of cascade inter-stage flows, and we obtain this sum.

It is seen from Eqs. (2) to (6) that X_{j,L_n} involve linear forms in $g(M^*, v, n)$, and that integrals of these linear forms across the sections of constant upflows for each of the key weights are needed.

Let

$$I_j(M^*, v, n) = \int g_j(M^*, v, n) dn . \quad (16)$$

From (1), we have that

$$I_j(M^*, v, n) = [g_j(M^*, v, n) - n] / \alpha(M^* - M_j) . \quad (17)$$

Now, from (2) and the above, we have that

$$\int_{N_E}^{N_P} X_{j,L_n} dn = -X_{j,P} [I_j(M_1^*, N_P, N_P) - I_j(M_1^*, N_P, N_E)] ,$$

and similarly for the remaining sections.

We obtain for the cascade,

$$\begin{aligned} \int_{N_W}^{N_P} x_{j,L_n} dn &= -x_{j,P} \left[I_j(M_1^*, N_P, N_P) - I_j(M_1^*, N_P, N_I) \right] \\ &+ x_{j,E} \left[I_j(M_1^*, N_E, N_E) - I_j(M_1^*, N_E, N_I) \right] \\ &+ x_{j,S} \left[I_j(M_2^*, N_S, N_I) - I_j(M_2^*, N_S, N_S) \right] \\ &- x_{j,F} \left[I_j(M_2^*, N_F, N_I) - I_j(M_2^*, N_F, N_F) \right] \\ &+ x_{j,W} \left[I_j(M_2^*, N_W, N_I) - I_j(M_2^*, N_W, N_W) \right]. \end{aligned}$$

Substituting from (17) in the above, we arrive at:

$$\begin{aligned} \int_{N_W}^{N_P} x_{j,L_n} dn &= x_{j,P} \left[g_j(M_1^*, N_P, N_I) + (N_P - N_I) \right] / \alpha(M_1^* - M_j) \\ &- x_{j,E} \left[g_j(M_1^*, N_E, N_I) + (N_E - N_I) \right] / \alpha(M_1^* - M_j) \\ &+ x_{j,S} \left[g_j(M_2^*, N_S, N_I) + (N_S - N_I) \right] / \alpha(M_2^* - M_j) \\ &- x_{j,F} \left[g_j(M_2^*, N_F, N_I) + (N_F - N_I) \right] / \alpha(M_2^* - M_j) \\ &+ x_{j,W} \left[g_j(M_2^*, N_W, N_I) + (N_W - N_I) \right] / \alpha(M_2^* - M_j). \end{aligned} \quad (18)$$

Finally, from (7) and the above, this sum of cascade inter-stage flows is given by

$$\int_{N_W}^{N_P} L_n dn = \sum_j \int_{N_W}^{N_P} x_{j,L_n} dn. \quad (19)$$

Needless to say, analogs of the "value function" are obvious.

Separative work by definition is the product of the above sum and $\psi_{5/8}^2/4$ where $\psi_{5/8}$ is the separation factor assumed for the binary separation of $U^{235}F_6$ from $U^{238}F_6$. As shown in Ref. [1], α is defined so that the separation factor for the binary separation of component q from component j is $\alpha(M_q - M_j)$. Accordingly, the separation factor for $U^{235}F_6$ and $U^{238}F_6$ would be 3α , the same mass range being assumed. Cascade separative work is then given by

$$\left(\frac{3\alpha}{2}\right)^2 \int_{N_W}^{N_P} L_n dn . \quad (20)$$

Generalizations

All basic formulae have been written so that side streams, feeds or withdrawals, may be included as needed, provided the considered cascade has two sections with constant key weights. More than two sections require another joint. The general treatment of these cascades -- and for that matter, the two-section cascades here considered -- is given in Ref. [2], but no specific formulae are developed. This reference also discusses possibilities in not maintaining inter-stage flow continuity and in not maintaining zero reflux at the cascade ends. But an advantage of the cascades considered in this paper is that they are fairly readily "computable."

References

- [1] de la Garza, A., "A Generalization of the Matched Abundance Ratio Cascade for Multicomponent Isotope Separation," Chem. Eng. Sci., 18, pp. 73-82 (1963).
- [2] de la Garza, A., The Variable Key-Weight Cascade for Multicomponent Isotope Separation, K-1571, Union Carbide Corporation-Nuclear Division, ORGDP, Oak Ridge, Tennessee, July 1, 1963.

Acknowledgment

The reviews made of the presented material by W. T. McCarley (Operations Analysis and Long Range Planning), Houston G. Wood (Operations Analysis and Long Range Planning), and S. R. Blum (Computing Technology Center) are appreciated.

APPENDIX

1. Homogeneous Functions

Consider a cascade in which the abundance ratio R of two key components is matched. A "homogeneous function" for this cascade is

$$x_j R^{-(2k_j - 1)} \quad (1)$$

where

R denotes the abundance ratio of the two key components (light to heavy), and

$$k_j = (M_b - M_j) / (M_b - M_a) \quad (2)$$

M_a and M_b being the molecular weights (light, heavy) of the two key components.

In this "matched- R " cascade, the R vs. n gradient is

$$dR_n/dn = R_n \alpha (M_b - M_a)/2, \quad \alpha > 0,$$

whence,

$$R_n = R_o e^{\alpha (M_b - M_a) (n - n_o)/2} \quad (3)$$

From (2) and (3),

$$R_n^{-(2k_j - 1)} = R_o^{-(2k_j - 1)} e^{-\alpha (M^* - M_j) (n - n_o)} \quad (4)$$

where $M^* = (M_a + M_b)/2$, called the "key weight." In use of "H" functions in H-balances, the factor

$$R_o^{-(2k_j - 1)} e^{-\alpha (M^* - M_j) n_o}$$

in (4) is irrelevant. We may then adopt

$$x_{j,n} e^{-\alpha (M^* - M_j) n} = x_{j,n} G_j(n) \quad (5)$$

as an H-function for use in H-balances associated with the j th component.

2. Sectional H-balances (jth component)

The H-upflow in the considered cascade is shown below:

The stage number $(n+1)$ is associated with x_{j,L_n} inasmuch as the R abundance ratio in this stream equals R_{n+1} , the abundance ratio in the inter-stage downflow from stage $(n+1)$. The H-upflow then is

$$H_{j,\text{up}} = x_{j,L_n} G_j(n+1) - (x_{j,L_n} - x_{j,U}) G_j(n) ,$$

which reduces to

$$H_{j,\text{up}} = x_{j,L_n} [G_j(n+1) - G_j(n)] + x_{j,U} G_j(n) . \quad (6)$$

The H-downflow is then also easily obtained:

$$H_{j,\text{down}} = -H_{j,\text{up}} = x_{j,L_n} [G_j(n) - G_j(n+1)] + x_{j,D} G_j(n) , \quad (7)$$

where $x_{j,D} = -x_{j,U}$ is the downflow of the jth component.

With the above results, section H-balances around the cascade top and around the cascade bottom are easily made. The purpose of these balances is to obtain expressions for $L_n x_{j,n}$, such as text Eqs. (2) to (6), from which the inter-stage flows L_n and concentrations $x_{j,n}$ may be obtained. Illustrations follow.

For the cascade section from stage n to the top stage N_P , $N_I < n < N_E$, in text Figure 1, the H-balance is:

$$H_j^{up} = X_{j,P} G_j(N_P) - X_{j,E} G_j(N_E) ,$$

and substituting from (6) with

$$X_{j,U} = X_{j,P} - X_{j,E} ,$$

we obtain

$$X_{j,L_n} = \frac{X_{j,P} [G_j(N_P) - G_j(n)] - X_{j,E} [G_j(N_E) - G_j(n)]}{G_j(n+1) - G_j(n)} ,$$

reducing to

$$X_{j,L_n} = X_{j,P} \frac{[G_j(N_P)/G_j(n)] - 1}{[G_j(n+1)/G_j(n)] - 1} - X_{j,E} \frac{[G_j(N_E)/G_j(n)] - 1}{[G_j(n+1)/G_j(n)] - 1} . \quad (8)$$

For small α ,

$$[G_j(n+1)/G_j(n)] = e^{-\alpha(M^* - M_j)} = 1 - \alpha(M^* - M_j) ,$$

so that with

$$g_j(M^*, v, n) = \left[e^{-\alpha(M^* - M_j)(v - n)} - 1 \right] / \alpha(M^* - M_j) ,$$

given by text Eq. (1), the above (8) becomes

$$X_{j,L_n} = -X_{j,P} g_j(M^*, N_P, n) + X_{j,E} g_j(M^*, N_E, n) ,$$

which, with $M^* = M_1^*$, is text Eq. (3).

Similarly, for the cascade section from stage n to the bottom stage

$N_W, N_F < n < N_S$, the H-balance is

$$H_j^{down} = X_{j,W} G_j(N_W) - X_{j,F} G_j(N_F) ,$$

and substituting from (7) with

$$X_{j,D} = X_{j,W} - X_{j,F} ,$$

we obtain

$$X_{j,L_n} = \frac{X_{j,W} [G_j(N_W) - G_j(n)] - X_{j,F} [G_j(N_F) - G_j(n)]}{G_j(n) - G_j(n+1)},$$

which reduces to

$$X_{j,L_n} = X_{j,W} \frac{[G_j(N_W)/G_j(n)] - 1}{1 - [G_j(n+1)/G_j(n)]} - X_{j,F} \frac{[G_j(N_F)/G_j(n)] - 1}{1 - [G_j(n+1)/G_j(n)]}.$$

With further indicated substitution, we obtain for small α ,

$$X_{j,L_n} = X_{j,W} g(M^*, N_W, n) - X_{j,F} g(M^*, N_F, n),$$

which, with $M^* = M_2^*$, is text Eq. (5).