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**A TECHNIQUE FOR MEASURING THE PRESSURE, VOLUME,
ENTHALPY, AND RESISTANCE OF EQUILIBRIUM
THERMODYNAMIC STATES OF LIQUID METALS
AT HIGH TEMPERATURES AND PRESSURES**

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(Ph. D. Thesis)

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A Technique for Measuring the Pressure, Volume, Enthalpy,
and Resistance of Equilibrium Thermodynamic States
of Liquid Metals at High Temperatures and Pressures

Abstract

A method for overcoming the temperature and pressure limitations of conventional techniques for measuring equilibrium thermodynamic data has been developed. It can be applied to conducting materials that can be resistively heated and that do not disassociate in the liquid phase. This makes its application to the determination of pure liquid-metal thermodynamic data of great value.

A material specimen in the form of a wire 1 mm in diameter by 25 mm long is mounted between two current leads such that it is axially concentric with a high-pressure cell. The cell is filled with helium to the desired working pressure. A current pulse of controlled magnitude and duration, from the overdamped discharge of a high-voltage capacitor bank, heats the wire to the desired state at a rate such that its expansion is nearly isobaric. The energy deposited in a segment of the sample is calculated from data obtained by measuring the current through the sample and the potential difference between two probes spaced about 5 mm apart near the center of the specimen. These data also allow calculation of the sample resistance during a major portion of the time that current flows. The thermodynamic specification of the high-temperature state is completed by measuring the expanded diameter of the sample, using flash x-radiographic techniques.

The principal unique feature of this technique that succeeds in overcoming the limitations of previous techniques is the use of a rare gas to transmit pressure to the portion of the sample measured. The technique has been applied extensively to lead, successfully reaching temperatures exceeding 5000°K, at pressures up to 2 kbar. However, a large scatter in this lead data, probably due to oxide-impurity-induced nonuniformities in deposited energy, prevents claiming the full accuracy capability of the technique. A very limited application to uranium has been made. These data are insufficient to conclusively test their thermodynamic validity, but the repeatability of the data appears better than obtained with Lead.

A Technique for Measuring the Pressure, Volume, Enthalpy, and Resistance of Equilibrium Thermodynamic States of Liquid Metals at High Temperatures and Pressures

INTRODUCTION

Greatly increased use of liquid metals in the space and nuclear reactor fields has pointed up the significant lack of available data at higher temperatures and pressures. Current engineering application relies on the use of equations-of-state that are totally unsupported by thermodynamic data for these higher temperatures and pressures. The specific goal of this work was the development of a technique capable of providing the energy, pressure, and volume data needed to support these equations-of-state. The technique that was developed is capable of measuring the variables—pressure, volume, and enthalpy (from which the energy may be obtained)—characteristic of equilibrium high-temperature, moderate-pressure states.

Classical experiments for obtaining these data are limited to states in the temperature and pressure ranges below about 2000°K and 2 kbar. This limitation results chiefly from the reduced strength and increased chemical reactivity of solid-sample containers at higher temperatures. This technique overcomes this limitation by allowing only a rare gas (helium) to contact the portion of the sample from which the data are obtained. The lack of support of the liquid-metal sample against body forces requires that the experiment be conducted in minimum time, commensurate with the ability to achieve an equilibrium state. Rapid heating is achieved by resistive energy deposition, and the equilibrium volume is measured by flash x-radiographic techniques. The technique has the capability to achieve and measure equilibrium states of metals of atomic number greater than 29 over a range of states extending from the temperature that produces a volumetric expansion factor greater than 1.1, up to the temperature at which the onset of vapor resistivity occurs, for pressures up to the structural limit of the beryllium windows in the pressure cell (about 5-7 kbar).

Relatively simple but unconventional apparatus is required to conduct the experiments. Three major components are required: a pressure cell having windows transparent to low-energy x-ray photons; a high-energy, current-limited capacitor bank equipped with the necessary switching gear

to allow accurate timing of the current pulse length; and one or more pulsed, high-intensity x-ray sources suitable for recording the shadow of the sample. Oscilloscopes record the wave shape of the current through the sample and the potential difference between the voltage probes to provide the data necessary to determine the energy deposited. Additionally, a means of timing the operation of the equipment is required, so that within a time span of less than 200 μ sec the oscilloscopes can be triggered, the capacitor bank turned on and off, and the x-ray sources pulsed.

The measured voltage record must be corrected for inductive effects to determine the resistive component of the voltage. The enthalpy is then given by the integral of the product of the current and the resistive voltage component. The ratio between the resistive voltage and the current yields the sample resistance. The pressure is constant throughout the experiment. The final sample volume is determined by measuring the diameter of the x-ray shadowgraph image by comparison with simultaneously photographed rods of known diameter. The volume is computed, based on the cylindrical symmetry of the expanded wire.

This technique has been extensively applied to lead and, to a limited extent, to uranium. Lead was chosen for use in developing the technique because its relatively high resistivity makes it easy to heat resistively, and because its relatively low estimated critical constants reduce the pressure and energy requirements imposed on the apparatus. The data taken serve to demonstrate the versatility of the technique and define the capabilities and limitations. However, the accuracy that may be ascribed to these data remains considerably less than is expected from this technique, because of a large scatter in the data that seems to be due to the inability to obtain oxide-free lead. The presence of significant quantities of lead oxide, distributed somewhat randomly throughout the material, results in sample-to-sample variation in the ability to achieve homogeneous energy deposition by resistive processes, and, consequently, variability in the sample expansion observed for a given average energy.

The work with uranium, very recently undertaken, has not yet resulted in sufficient accumulation of data to conclusively test its accuracy and repeatability. However, the few data available appear to be more repeatable than was observed with lead samples.

TECHNICAL CONCEPTS AND ANALYTICAL FOUNDATION

Background

One of the major goals of materials research over the past half century has been the complete characterization of the equilibrium thermodynamic states of elemental materials. Typically, the values sought are those of pressure, temperature, volume, and specific heat. These four alone allow determination of any variables desired, so far as the physical states of a material are concerned. Engineering application of materials often requires additional data on thermal and electrical conductivities. These data may also be of interest to theoreticians whose work often allows prediction of these variables as well as the more fundamental thermodynamic quantities.

At the relatively normal temperatures and pressures attainable in the laboratory, these data are not difficult to measure. Even the experimental exploration of the liquid and vapor phases of materials that normally exist as vapors or that have low vaporization temperatures is easily accomplished. Thus, at normal temperatures and pressures, liquids (such as water or alcohol) and most gases (such as the rare gases) have long been studied experimentally and their characterization is well defined, even in the critical region.

Mathematically, the critical point is defined by the values of pressure, temperature, and volume at which the first and second derivatives of pressure with respect to volume, taken at constant temperature, are zero, simultaneously.¹ All materials for which data are available exhibit the characteristic that, above the critical pressure, the first-order liquid-vapor phase transition, with its associated heat of vaporization, vanishes. This high-pressure region of the thermodynamic phase diagram has been called the fluid phase. Although many materials have been completely investigated throughout the solid, liquid, vapor, and even fluid phases, many other materials, such as the metals, have proved resistant to investigation of their thermodynamic properties, particularly in the liquid and vapor states. The principal reason for this resistance to investigation is the much higher temperatures and pressures required because of the higher interatomic bond strengths of these materials. Techniques successfully

applied to investigate materials at more moderate temperatures and pressures are unacceptable for the more difficult, normally solid materials.

The conventional experiments make use of a pressure cell within a furnace, or, conversely, a furnace within a pressure cell, to produce the desired values of temperature and pressure in the material being studied.^{2,3} Since these experiments are designed to measure equilibrium data, the states reached are maintained over long times to assure that equilibrium has been achieved. This means that the sample has to be supported against the action of body forces such as gravity; therefore, some type of container is necessary.

Application of various modifications of this basic experimental technique has resulted in the accumulation of a vast body of data characterizing many materials throughout their solid phase and often their low-pressure liquid phase.^{4,5} However, two major effects limit the utility of this approach at the higher values of temperature and pressure characteristic of the critical region of most metals. First, as higher and higher temperatures are reached, the chemical reactivity of both the sample material and the containment material increases. Above 1000°K, it becomes increasingly difficult to find a containment material that doesn't react chemically with the sample to a significant extent. Such a reaction, of course, precludes the achievement of the desired state of thermodynamic equilibrium. Secondly, and limiting the available choice of containment materials, is the requirement that such containment remain solid and relatively rigid at the temperatures and pressures desired. Above about 2000°K and perhaps 2 kbar, these two requirements become essentially impossible to fulfill.

Over the past 20 yr, one class of normally solid material has become of engineering importance in the liquid state at elevated pressure. This is the group of liquid metals whose properties are needed in the aerospace and nuclear power reactor fields. Because of this practical application of thermodynamic data on liquid metals (extending in some cases up to the critical region), research has been strongly directed to devising suitable techniques to measure the data.^{6,7,8} In the absence of true measured data, several people have applied various procedures to estimate data based on the only liquid metal data available: liquid mercury.^{9,10,11} These estimation methods rely on the determination of parameters of two or more materials that may theoretically be expected to be related. Thus data for one

material are used to predict the behavior of another, similar material. Perhaps the most powerful of these techniques relies on the Law of Corresponding States. This has been applied to nearly all materials of interest, to predict their critical constants and the liquid-vapor coexistence boundary. Useful as these estimates are from the engineering standpoint, it must be kept in mind that there are many factors in materials structure that may invalidate them. It is still of great importance that these values be measured experimentally. Experimental data are also required for comparison with a number of theories of liquid metals that have been proposed.^{12,13,14}

Within the last decade, at least cesium¹⁵ and perhaps rubidium have yielded to sophisticated experimental techniques, and their critical constants have been measured. Also, the work of Franck in Germany continues to improve the available data on mercury.¹⁶ These are the notable experiments that seek to measure the required data from a sample that is indeed in thermodynamic equilibrium. Shock-wave physics, although chiefly oriented toward the solid-state characterization of materials, nevertheless yields some data on the liquid state. Care must be used in the interpretation of these data, however, because they generally do not reflect the characterization of true thermodynamic equilibrium except by application of a mathematical model (equation of state) of the material behavior. A similar objection may be raised to data obtained from exploding-wire research.¹⁷ There is no doubt that valuable data on the very high-temperature liquid and compressed vapor phases are available from exploding-wire experiments¹⁸; but generally these cannot be claimed to be measurements of equilibrium states, and the interpretation of the data in such terms requires a model of expected material behavior.

Since more than 75 of the elements may be classed as metallic, it is obvious that the complete thermodynamic characterization of at most three of them leaves a considerable gap in the knowledge of liquid-metal states. From an engineering application standpoint it is extremely desirable to measure the properties of at least the elements lithium, beryllium, carbon, sodium, aluminum, potassium, titanium, iron, copper, zinc, silver, gold, tungsten, lead, bismuth, and uranium. Data on these materials, with the data on mercury, cesium, and rubidium, would meet most of the current engineering need. It would also be sufficient to allow the thorough testing

of the various comparison techniques that have been used and to improve their accuracy in application to any other metals for which information should be subsequently required.

Concepts of the New Technique

The thermal range of the classical technique is limited principally by chemical or physical effects of the temperature and pressure on the material chosen to support the sample. The development of a technique capable of significantly increasing the thermal range of the experiment depends, therefore, on a suitable choice of sample containment. The least reactive of all materials are the rare gases. Because of their spherical atomic structure they resist the effects of temperatures and pressures that would severely degrade other materials. The fundamental idea developed by this new technique is the use of a rare gas (helium) as a nonreactive, nondegradable container for the liquid sample, in a high-temperature, high-pressure, thermodynamic equilibrium state.

The use of a gaseous container places some unique restrictions on other aspects of the technique. The duration of the experiment must be sufficiently short that body forces cannot cause motion or deformation of the sample as a whole or at its surface. Gravity is such a force, but it is relatively weak and requires milliseconds to move the sample detectably. However, submillisecond experiment times require a rapid means of uniformly heating the sample. Resistive heating of wires is routinely used in exploding-wire phenomena research and can be adapted to this new technique very readily.^{19,20}

Resistive heating may be designed to provide a homogeneous deposition of the energy. Furthermore, measurement of the current passing through the sample and the resistive voltage drop along the sample allows the energy deposited to be measured. Since the process occurs at constant pressure, the deposited electrical energy is equivalent to the enthalpy change in the sample. A pulsed current source is readily constructed such that the energy deposition rate can be controlled. This control is necessary, since the only other independent variable in the experiment is the pressure, which is held constant. Energy deposition control allows the technique to produce and examine selected states along an isobar.

A rapid, accurate means of data acquisition is required. The parameter that may be of most interest is the material density or, equivalently, the volume of the sample in the high-temperature state. Direct measurement of the density does not seem to be feasible, but there are techniques by which the volume may be measured. It is useful to make as much use of sample symmetry as possible, to reduce the number of data that must be taken. Spherical geometry would, of course, require only a measure of the diameter to determine the volume, but homogeneous energy deposition by resistive heating, in a spherical sample, is not feasible. If the sample length is fixed, cylindrical symmetry requires only a measurement of the diameter to determine the volume. This geometry is compatible with uniform deposition of energy from resistive heating.

The sample length is determined by the voltage probe separation, not by the total length between the current leads. The current leads are rigid and do not allow the sample to expand along its axis. If the heating rate is made sufficiently slow that low-pressure waves can travel along the sample axis, converting the attempted longitudinal expansion into radial expansion, then there will be no change in separation of the voltage probes during the expansion. The expanded-state equilibrium volume can therefore be found by measuring the expanded diameter of the sample.

Photographic techniques are applicable to determination of the sample diameter on the very short time scales required. In the initial design of the experimental procedure, optical photographic processes were considered. However, they were rejected in favor of flash x-ray techniques because optical distortion effects in the gas might have introduced significant error into the measurement. Data are not available to determine the temperature gradients in the high-pressure gas resulting from contact with a sample heated to several thousand degrees.

The use of higher-energy x-ray photons, which are unaffected by pressure and temperature gradients in the gas and by distortional effects induced by stress in the pressure-cell windows, eliminates the possible problems of the optical process.

The selection of an x-ray photographic process to make the diameter measurement introduces restrictions and requirements of its own. First, the source must produce photons of energy capable of penetrating the pressure-cell window material, but not the sample material. Second, the

flux must be of sufficient intensity to withstand the transmission attenuation of the pressure cell and still produce acceptable image contrast on the film. Third, the sample expansion rate requires that the x-ray pulse width be less than $1\ \mu\text{sec}$ to allow measurement of sample diameters preceding equilibrium. However, only a pulsed x-ray source can meet the intensity requirements of these short exposure times for the low photon energy required. The photon energy must be in the range from 10 keV to 100 keV to allow penetration of the beryllium windows but not place such stringent requirements on the absorptive properties of the sample that the materials that can be studied are severely limited.

To provide maximum transmission of this spectrum through the pressure cell, the use of beryllium windows and helium gas is required. Still, the combination of limited source intensity and the attenuation of the pressure cell requires the use of zinc-sulfide fluorescent screens to provide acceptable image contrast. These screens broaden the image because of their finite thickness and complicate the determination of the sample diameter from its film image.

The thermodynamic data characterizing the high-temperature state consists of the enthalpy, determined from the current and resistive voltage records, and the volume, determined from a measure of the final diameter of a longitudinally constrained cylindrical sample. The only additional requirement is knowledge of the equilibrium pressure. The gas pressure is held constant over the duration of each experiment. This pressure is measured by an accurate gauge connected to the gas line leading to the cell. During heating, there is a perturbation in the actual pressure applied to the sample because of magnetic effects. For the current used to heat lead (15 kA peak magnitude), the peak magnetic pressure is calculated to be 134 bars (Appendix A). This magnetic pressure is proportional to the current squared and becomes zero when the energy deposition is complete. Consequently, it does not affect the pressure of the equilibrium state.

Pressure disturbances in the gas due to the expansion of the sample are small (a few bars) and are rapidly damped by the gas so that they, likewise, do not disturb the equilibrium pressure of the sample. The pressure of the sample at the equilibrium high-temperature state is, therefore, just the hydrostatic pressure of the gas as measured by the gauge.

Equilibrium Heating

Homogeneous energy deposition, uniform expansion of the sample to a stable equilibrium volume, and the possible instabilities inherent with a hot, dynamic, liquid-metal column supported only by hydrostatic gas pressure are three factors crucial to the success of the technique. This and the following two sections treat these subjects in detail from a theoretical viewpoint. They are, of course, not entirely unrelated topics with respect to cause and effect; that is, nonequilibrium energy deposition provokes nonequilibrium expansion, which in turn contributes to certain instabilities.

Thermal equilibration times, defined as the time required to achieve temperature equilibration between two neighboring segments of the sample having initially different temperatures, govern the care with which the energy deposition must be made uniform. Thus, if the equilibration times are short with respect to energy-deposition times, nonhomogeneous energy deposition on a reasonably large scale may be tolerable. Conversely, long equilibration times imply the requirement of precisely homogeneous energy deposition. Thermal conductivity data for sample materials at the high temperatures of interest do not exist, but an extrapolation from existing data allows the shortest expected equilibration time to be estimated. It is estimated that the time required to achieve 90% temperature equilibration in lead is of the order of $20 \mu\text{sec}$ for temperature gradients extending over distances of the order of 10^{-3} cm.

These long equilibration times require that the energy be uniformly deposited, which implies that the current must be uniformly distributed throughout the sample. Direct current flowing through a cylindrical sample will be uniformly distributed, providing the sample is of uniform resistivity on a reasonably microscopic basis. Pulsed current, even from a nonoscillatory source, is subjected to magnetic/inductive effects that give rise to a time-varying current distribution in the sample. This is the skin effect, typical of all time-varying currents. It is both a function of the conductor geometry and the pulse shape. Principal concern lies not in the current distribution itself, of course, but in the energy-deposition distribution, which varies as the current squared.

For lead, the energy deposition at the surface of a 1-mm-diam wire heated by a $10 \mu\text{sec}$ square-wave current pulse is computed to be 3% greater

than at the wire center (Appendix B). This calculation ignores the current redistribution that would result from the higher material resistivity near the surface. This redistribution would make the energy deposition even more uniform. Since pulse widths necessary to heat lead are typically of the order of 20 μ sec long, it may be seen that essentially uniform energy deposition results in the lead experiments.

As materials are heated, the resistivity increases, except for small temperature ranges. A positive thermal coefficient of resistivity produces a feedback effect on constant-current resistive heating, which results in a basically unstable situation—that is, even for constant current, the energy deposition rate increases with time. This is a desirable situation, providing the material resistivity remains uniform on a microscopic basis. However, if there are impurities in the sample that have different resistivity than the bulk material, they will cause a local nonuniformity in the current density, which will result in nonuniform energy deposition (Appendix C). Once such nonuniformities are created, the resistive instability resulting from a positive thermal coefficient will cause them to grow. It is the rate of growth that is important, and this depends on many factors in a complicated way. For example, thermal-conduction processes and resistive current redistribution both tend to slow the growth rate, while factors such as a positive temperature dependence of the thermal coefficient of resistivity and increased initial size of the nonuniformity tend to increase the growth rate. The achievement of uniform energy deposition depends on instability growth rates that are small with respect to experiment times.

This resistive instability forces consideration of the dynamics involved in phase changes, because they may locally affect the resistivity and may not occur with microscopic homogeneity. Solid-solid phase changes are crystallographic rearrangements that become energetically favorable at a given temperature and that occur in times dependent on the available driving energy. As they progress they may act either to raise or lower the resistivity of the material. It is presumed that, if homogeneous heating exists at the onset of a solid-solid phase change, the phase change will occur reasonably uniformly throughout the sample, and the uniformity of the energy distribution will not be appreciably affected. This is of no consequence with lead, but uranium exhibits two such solid-solid phase changes.

The solid-liquid phase change is of significant interest because the object of the technique is to progress through this transition with each

sample to reach the interesting high-temperature states. The possibility of introducing nonuniformities in the resistivity and energy distribution exists with this phase change because of the tendency for the liquid phase to be nucleated at a relatively small number of points within the sample.²¹ The nucleation of the liquid phase must take place at sites that have less than normal bonding energies—that is, at the surface, at grain boundaries, or at dislocations. In a material such as uranium, where the increase in resistivity between the two phases varies only slightly, there is, then, no great reason for concern. However, in materials such as lead, which increase resistivity greatly at the transition, the possibility exists of creating a mixed-phase sample that does not have microscopically homogeneous resistivity. Suppose that the primary nucleation sites are grain boundaries, and that the grains are large and of nonuniform size, as is true of pure lead at room temperature. Initially, then, melting of the grain boundaries results in a region of higher resistivity than is in the bulk solid that is not yet melted. The consequence of continued current flow through the higher-resistance liquid sets up a resistive instability that can be damped only by thermal conduction. The matrix of molten boundaries prevents resistive redistribution of current. If this condition persists and becomes extreme, it may affect the accuracy of the measured average enthalpy data. A check on this effect was made for lead by observing that lead alloyed with small amounts of silver and copper to provide small grains produced the same results as pure lead.

Much the same thing could certainly happen at the liquid-vapor transition temperature, but the nucleation sites for this transition are not clearly defined and the dynamics involved not clearly understood.²² The vapor phase resistivities are orders of magnitude higher than those in the liquid phase. Therefore, once significant inhomogeneities in the mixture of liquid and vapor phases occur, or the percentage of vapor phase begins to increase rapidly on the time scale of the experiment, a loss of control of deposited energy results, with consequent full vaporization of the sample.

The reported work on vaporization dynamics has been principally limited to the relatively slow boiling processes wherein gravity is sufficient to cause phase separation in short times relative to the addition of the heat of vaporization. In this case, nucleation occurs at points of stress in the liquid. That is, a liquid boils much more readily in contact with a rough

surface, which reduces the local bonding energy, than in contact with a smooth surface; a smooth surface has little effect on the normal surface energy of the liquid. The dynamics of vaporization in systems that have been tremendously superheated and then allowed to vaporize has been studied by exploding-wire techniques²³ and by laser techniques. But, in this experiment, the unique condition is that the energy distribution function is thought to remain Boltzmann and is merely shifted up in energy during the addition of the heat of vaporization; but no nucleation surface is provided, nor is the superheat excessive, at least up to the onset of vapor resistivity. The probability of nucleating the vapor phase other than at the surface of the sample under these conditions is negligibly small.²⁴ It is concluded that the experimental technique is capable of reaching homogeneous mixed liquid-vapor states some distance into the coexistence region.

Equilibrium Expansion

Presuming homogeneous energy deposition, it is necessary to consider the dynamics of the expansion of the sample to the final equilibrium volume associated with the temperature and pressure achieved by the experiment. The addition of an increment of energy to the sample results in a pressure within the sample that is slightly higher than the gas pressure. This pressure will be completely homogeneous without regard for sample geometry; however, the relief to equilibrium will depend on the sample geometry and restraint. A pressure-relief wave will originate at the surfaces of the sample and proceed toward the center. For the cylindrical sample with restrained ends, a radial wave will provide stress relief by propagating to the axis and expanding the material behind it. An axial wave will propagate in from both ends and interact with the radial wave in such a way as to also produce radial expansion. The speed and damping of these pressure waves will be directly dependent upon the bulk modulus of the material. Except possibly at the critical point, the sample expands by this mechanism with only a slight lag from the equilibrium volume at each of the states through which it passes in transition to the desired state. Once the energy has been turned off, the sample, which is continuing to expand from momentum, will develop pressure waves of the opposite sign that stop the expansion and return the volume to the value corresponding to the energy-pressure equilibrium. At the critical point, the velocity of these pressure

waves presumably goes to zero, and anomalous behavior of the sample expansion might be expected. Such anomalies have not been observed, but this is probably either because the critical point has not been closely approached or the rate of approach and transition of the critical region was too fast to cause observable effects.

The expansion dynamics for lead were calculated approximately by a hydrodynamic computer code, using an experimentally measured energy-deposition time history. This predicted that a 20-bar overpressure was sufficient to drive the expansion and that equilibrium volume should occur within 10-15 μ sec after energy deposition was complete. The observed time to achieve equilibrium has often been as much as 40-60 μ sec. In Appendix D, the pressure necessary to overcome the inertia of the material and produce a four-fold expansion in 20 μ sec is estimated to be 137 bars at the surface. This is probably an overestimate.

Inherent Instabilities

The basic processes of this technique—resistive heating, expansion of a dense liquid into a less dense vapor, and the achievement of a relatively long-lived high temperature equilibrium—subject the process to several sources of instability. If the technique is to be applicable and useful, these instabilities must be either counteracted or have growth rates that are slow compared with experiment times. This section treats the instability problems from a theoretical standpoint, but this approach is limited, and the real test lies in the practical application of the technique.

One concern that is readily handled analytically is the effect of the force of gravity on the liquid sample supported only by high pressure gas. In this respect, the sample is not supported at all except for the viscous force in the gas, which may be neglected in determining the gravitational effects. The body force on the sample due to gravity must not be capable of moving the sample a measurable amount in the time scale of the experiment. The gravitational effects are undetectable over times less than a few milliseconds.

Another body force on the sample exists during the current pulse, because of the asymmetry of the current leads. This force is caused by the magnetic fields that surround the current-carrying conductors. One component of the force is radially symmetric with respect to the sample. For

lead, this is estimated to be of the order of 100 bars. The asymmetric component of the force resulting from the current lead geometry is capable of producing an impulse in the sample sufficient to cause observable motion on the time scale of a few hundred microseconds. It is estimated (Appendix A) that this impulse will result in about 0.3 mm of movement in 150 μ sec, in excellent agreement with observation. The 32-kA current used to heat uranium causes observable motion within 100 μ sec. This still allows sufficient time to conclude heating and measuring of the sample, so long as the heating current need not be too great.

Resistive heating is itself subject to instability, because the increase of resistivity with temperature provides a positive feedback mechanism causing resistive inhomogeneities to grow rather than to be damped out. A nonhomogeneous energy distribution may produce nonuniform expansion observable in the shadowgraphs or may simply introduce errors in the average deposited energy measured. The lead sample material apparently produced this problem on random experiments because of lead-oxide impurity inclusions. The effects of such inclusions are discussed in Appendix E.

The expansion, or motion of a surface, between fluids of differing density, such that the more dense fluid moves into the less dense fluid, gives rise to Taylor instability. As an example of this effect, consider the action of a perfectly planar surface of a liquid such as water moving under the action of gravity into the fluid air. Statistical variations in the pressure at the interface of the two fluids produce a wave-like geometry to the interface. Under the rectilinear nature of the force, these waves grow to ultimately cause the interface to fracture, allowing interpenetration of the two fluids. The time scale for observable distortion of the interface and the wave length of this distortion are dependent on the acceleration, surface tension, and density difference between the two fluids. A reasonable estimate of this time for the case of liquid metal expanding into high-pressure helium is impossible. However, such instability has not been observed in practice with either lead or uranium.

Finally, the heated sample is not in thermal equilibrium with its surroundings, and this introduces the question of thermal stability. Accuracy of the technique rests upon making measurements on a sample that is of reasonably uniform temperature at a time when the internal energy of that state is known. Therefore, heat losses during experiment times are important, since they must be either known or negligible. Heat loss from the

sample may occur by conduction to the current leads, conduction to the voltage probes, or conduction into the ambient helium in contact with the sample, or by convection in the helium. Estimates of these various heat-loss mechanisms (Appendix F) predict that conduction to the current leads perturbs the temperature distribution only within 0.2 mm of the current lead contact. This is far from the portion of the sample considered to be in equilibrium. Similar arguments demonstrate that the perturbation of the voltage probes is negligible. However, for lead at 6000°K it is estimated that perhaps as much as 0.3% of the energy may be conducted into the helium in 200 μ sec, and as much as 1% may be radiated to the walls. Convection in the helium is presumed negligible at this pressure. These heat losses are well within the experimental error assigned to the measured energy. Heat losses by these mechanisms are concluded to be negligible on time scales that are long compared with heating and measurement times.

Since the number density of the high-pressure helium is comparable to the number density of the hot sample, concern was evidenced for the diffusion of helium into lead on time scales pertinent to the experiment. Three independent estimates indicated that no observable effect should occur.

EXPERIMENTAL APPARATUS

The Pressure Cell^{*}

During the development of this experimental technique, two different pressure cells were designed and constructed. The original, somewhat more simplified design, was used to prove feasibility, and was then replaced with a more sophisticated system allowing the use of higher pressures. Pressure limitation with the original cell stemmed from the use of an unsupported beryllium structure. This cell had a calculated capability to withstand up to 3 kbar of pressure. The pressurizing system used with it, however, utilized a high-pressure gas supply limited to a maximum pressure of slightly less than 1.7 kbar.

Figure 1 is a mechanical drawing of the details of the 3-kbar pressure cell. The construction and assembly is axial, using massive steel end caps

^{*}The high-pressure equipment reported here was designed and constructed by D. Stephens, H. Stromberg, and H. Washington, Lawrence Radiation Laboratory.

27	63-119358	NOB - C
26	63-119358	PLATE
25	63-119358	POINT
24	63-119358	PLATE
23	63-119358	CLIP
22	63-119358	NO. 100
21	63-119358	VOLTA

20	63-119358	CALIBRA
19	63-119358	CALIBRA
18	63-119358	NO. 100
17	63-119358	PLATE
16	63-119358	LOC. 10
15	63-119358	LOC. 10
14	63-119358	LOC. 10
13	63-119358	NUT, HE
12	63-119358	SEAL
11	63-119358	INSULAT
10	63-119358	INSULAT
9	63-119358	SEAL, C
8	63-119358	CALIBRA
7	63-119358	INSULAT
6	63-119358	UPPER
5	63-119358	SEAL, C
4	63-119358	LOC. 10
3	63-119358	LOC. 10
2	63-119358	LOC. 10
1	63-119358	HEAD PC

Fig. 1. Mechanical details of

and four 1-in.-diam bolts to retain the seal integrity against the gas pressure. The body of the cell is a solid beryllium cylinder 4 in. in diameter with an axial void 1.5 in. in diameter. The bottom of the cell is sealed with a steel plug that carries plumbing required to transfer the gas to and from the cell. Both top and bottom seals are standard "O" rings that float between brass wedge rings and a 45-deg bevel on the opening in the beryllium. The top of the cell is sealed with a copper plug through which three ceramic insulated copper leads are passed. These electrical feed-through leads serve to carry the current to the sample and return the voltage signals from the two voltage probes. The current return is made through the copper top plug itself which then serves as ground for the entire electrical assembly.

The sample support and current leads are detailed in Fig. 2. They are constructed of brass, which offers mechanical strength without sacrificing electrical conductivity. The top sample support is fitted with a notched end plate attached with a screw such that the sample may be securely clamped to provide good electrical contact. The bottom sample support is cantilevered from a rod attached to the top plug of the cell. It is designed in such a way that it supports the sample axially symmetrically with the pressure cell. Attached to the bottom sample support are three copper or tinned copper rods extending downward in a plane that passes through the axis of the sample. These rods are oriented with respect to the two x-ray sources so that their images are separate in the shadowgraphs. They serve as calibration rods for the determination of the unknown diameter of the expanded sample.

Both the effect of shrapnel from a ruptured pressure cell and the possibility of beryllium contamination represent hazards which must be eliminated. This is accomplished by enclosing the assembled cell in a steel cylinder which provides rupture containment. This steel cylinder has a steel bottom welded to the sides, sealed electrical and gas plumbing feed-throughs and a flanged top. The top carries a duct to a particulate filter to provide gas relief while containing the beryllium, in the event of rupture. Safety is thus provided against these hazards.

A schematic diagram of the pressure plumbing system used with the 3-kbar pressure cell is shown in Fig. 3. Gas pumping was not employed. The pressure was provided by the residual pressure in three helium reservoirs, through the appropriate valves and tubes. Pressures in the reservoirs were 1500 psi, 5500 psi, and 25,000 psi, respectively. Since the

Fig. 2. Sample support and electrical details used with lead.

Fig. 3. Plumbing system used to control helium pressure in the 3-kbar cell.

high-pressure cylinder was necessarily small the lower-pressure reservoirs were used to provide as much as possible of the gas needed to fill the cell, thereby requiring only a small gas flow from the high-pressure reservoir to reach the desired operating pressure. The two low-pressure reservoirs had pressure gauges to indicate their residual pressure. The high-pressure reservoir pressure was monitored only during the filling operation by observing the cell pressure on a Heise gauge. This pressure system operated satisfactorily at the 1-kbar fill pressure level but was very short-lived at the 1.5-kbar level. If there were no leaks in the system, two experiments at the higher pressure could be conducted before the pressure dropped below 1.5 kbar in the high-pressure cylinder. Careful planning of the arrangement of the experiments would, however, allow several more experiments at lower pressures to be made before refilling the high-pressure reservoir. This limitation was overcome in the second-generation (7 kbar) pressure cell by developing a pumping system capable of achieving cell pressures exceeding the 7-kbar limitation of the cell.

Two major improvements in cell design were required to construct the 7-kbar pressure cell. First, a method of overcoming the 3-kbar limitation placed on the cell by the tensile strength of the beryllium was necessary. Second, a method was required of providing seal preload sufficient to maintain the seal but increasing with cell pressure so that the seal load wouldn't fracture the cell. Design studies revealed that the required strength could be achieved with a beryllium cell, fully supported by a steel sleeve. One-in.-sq openings in the sleeve provide the x-ray windows. The steel sleeve was prepared with an axial opening slightly smaller than the outside diameter of the beryllium cylinder. Then the steel was heated to 300°F and the beryllium cylinder pressed into the steel. As the steel cooled, a compressive stress computed to be about the equivalent of 3 kbar internal pressure was applied to the beryllium cylinder. The beryllium is not in tension at all until operating pressures in excess of this compressive load are reached. The computed strength of this assembly is 7 kbar, and it is thought to easily be good to 5 kbar. Repeated operation has not been attempted at higher than 2 kbar yet.

The variable seal-loading was handled by constructing the cell in such a manner that it became an integral part of a hydraulic press. The seal load is then readily controlled from a remote location by controlling the oil

pressure to the press and monitoring this on a pressure gauge. Continuously variable end-loading is not required, but a stepwise increase in the seal load as the pressure is increased in the cell precludes exceeding the capability of the beryllium to withstand the stress concentration occurring at the point of application of the seal. Details of the construction of the 7-kbar system are given in Fig. 4. The containment system for protection against shrapnel and beryllium contamination are basically the same as was used with the 3-kbar cell. Likewise the pressure cell end plugs are of similar design for the two systems and indeed the topplugs are completely interchangeable.

The schematic drawing of the pumping and plumbing system constructed for use with the 7-kbar cell is shown in Fig. 5. The pump chosen for the job is an air-driven Haskel oil pump. This supplies the oil pressure required for operation of the seal-loading ram and through the use of an oil-gas separator and an intensifier raises the gas pressure from that of the supply to as high as 10 kbar. Oil and gas flow are controlled by a system of valves arranged such that the pump can be connected to the ram, separator, and intensifier separately and independently. Thus it is possible to vary the gas pressure and the seal load in incremental steps to achieve any desired pressure.

Energy Source and Control

Resistive heating requires a current source that can be controlled in magnitude, turn-on time, and pulse length. The magnitude of current required to heat a sample to the desired temperature in an appropriate time, depends in a complicated way on the material geometry, resistivity, and thermal coefficient of resistivity. For lead a pulse of between 14 and 16 kA adequately met these requirements. For uranium, the increase in resistivity with temperature is much less than for lead, and current magnitudes of the order of 30 kA are required. These currents deposit a few hundred calories per gram in the sample in from 15-40 μ sec. At least an additional 60 μ sec is left to achieve and measure the equilibrium volume, before the onset of instability.

Fig. 4. Mechanical details of the 7-kbar pump.

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Mechanical details of the 7-kbar pressure cell.

2

Fig. 5. Helium pumping and pressure-control system used with the 7-kbar pressure cell.

The current source that meets the above requirements uses a capacitor bank for energy storage. To provide a relatively flat-topped pulse, the bank discharge is heavily damped. Most of the stored energy is dissipated in the resistive ballast. Consequently, much more energy is stored than is ultimately deposited in the sample. This approach produces a relatively constant current flow through the sample, independent of the resistivity of the sample material.

Cold cathode gas discharge tubes, commonly called triggered spark-gaps, are suitable to switch the currents and voltages involved and are readily remote controlled. Current lead length and circuit geometry can be adjusted to control the inductance within reasonable limits, thereby affording some control over the pulse shape, particularly the rise and fall times.

Two different current sources were designed and constructed for use with this experiment. The details of their construction are basically similar; the major differences are the size of the capacitor bank itself and the values of the damping or current limiting resistors. The smaller, a 17-kJ, 20-kV bank was initially constructed for the work with lead and proved adequate for this purpose. A larger 45-kJ, 20-kV bank was required to adequately heat the uranium. Both banks share use of the charging and timing control equipment, but they are not operated simultaneously.

Figure 6 shows the details of the current source circuits. Spark gap S1 is triggered by a pulse from a capacitor discharge unit through its pulse transformer to initiate the current pulse through the sample. Resistors R1 and R2 in conjunction with the capacitance of the bank determine the flat top shape of the current pulse and are selected to give a discharge time of the order of milliseconds. They also limit the current to a value controlled by selection of the bank-charging voltage and the size of R1 and R2, and assure a current magnitude independent of the sample resistance. Spark gap S2 is fired by a pulse from a CDU to short-circuit the discharge current in the sample, effecting current turnoff at a preselected time. R1 prevents this switching from being a direct short circuit on the bank, which typically still has 80% of its original energy. R2 provides sufficient voltage so that S2 can be triggered. The value of the series inductance around the complete

^aTypical resistance is 0.5 Ω each.

Fig. 6. Details of current-pulse supply and control.

bank-sample loop, in conjunction with the resistance, determines the rise time of the current pulse. Similarly the inductance in the sample-S2 loop and R2 determine the fall time of the current following bank turnoff switching. A typical current pulse produced by the 17-kJ bank with $R1 = R2 = 0.5 \Omega$ and a charging voltage of about 16 kV is shown in Fig. 7. A complete and detailed electrical analysis of these circuits is given in Appendix G.

Figure 8 is a block diagram of the system-control equipment. When the experiment has been readied for operation, the safety interlocks are set, causing power to be available for the various control chassis. Operation is initiated by pressing a switch on the master control chassis. This causes a low-voltage pulse to be fed to the fire-control pulser. It sends a 200-V pulse to the pulse-delay unit. A six-channel delay unit is used, although only five of the channels are currently necessary. Appropriately delayed pulses from this unit are fed in succeeding order to trigger the oscilloscope traces, turn the current on, turn the current off, and independently trigger the Blumleins. The capacitor discharge units in series with four of these controls serve to convert from the low-voltage pulse delivered by the delay unit to pulses of about 2500 V required to reliably trigger the spark-gap switches used with the current source and Blumleins.

X-Ray Sources

Two severe requirements are placed upon any source chosen for the flash radiography required by this technique: the photon energy spectrum must be such that it is transmitted through beryllium, yet is absorbed by the sample material; and the photon intensity must be sufficient to expose film with adequate contrast after the transmission through the pressure cell. These are relatively conflicting requirements, since the efficiency of x-ray production drops off sharply as the photon energy is lowered to satisfy the first requirement. Photon energies of about 50 keV are probably optimum for this application. Over several years, the laboratory has developed and refined a flash x-ray source driven by a Blumlein transmission line. It adequately met the characteristics needed in this experiment.

The Blumlein (named for its inventor, Alan Blumlein) is a folded, three conductor transmission line whose impedance can be adjusted to match the relatively low impedance ($\sim 1 \Omega$) of a cold cathode emission x-ray

1. 17 kJ Bank source
2. Charging voltage, nominal 16 kV
3. Limiting resistors, 0.5 Ω each
4. Pulse width is nominal 20 μ sec

Fig. 7. Typical sample current from 17-kJ bank.

Fig. 8. Firing and control equipment.

tube. Thus the transmission line is capable of delivering a high-current (50 kA), relatively low-voltage (85 kV) pulse to the tube, to produce a reasonable intensity of low energy Bremstrahlung photons.

Figure 9 shows the mechanical details of Blumlein construction. A central aluminum tube is connected to the anode of the x-ray tube at one end and, through a 200- Ω resistor, to ground at the other end. On this tube is placed a coaxial layer of dielectric material, such as polyethylene, covered by another coaxial layer of aluminum foil. This foil layer is insulated from ground at both ends of the Blumlein. Over it is placed another coaxial layer of dielectric material and an outer layer of aluminum foil covered by a protective aluminum shell. The outer foil is grounded and is connected to a cathode ring structure concentric with the conical anode. Provision is made to connect the x-ray tube to a vacuum pump capable of maintaining a vacuum better than 10^{-2} Torr. At the switching end of the Blumlein, the middle conductor is connected to the negative terminal of a 50-60-kV power supply. A low-inductance switch capable of rapidly shorting the middle (charged) conductor to the outer (grounded) conductor is added.

Refer to the electrical schematic shown in Fig. 10. In operation, the center conductor is charged negatively to a potential of about 50 kV with respect to the other two (grounded) conductors. When the flash of x-ray photons is desired, the center conductor is rapidly shorted to ground at the switching end. This causes a pulsed voltage wave of 50 kV between the inner and outer conductors to be propagated toward the x-ray tube at the characteristic wave speed of the transmission line. The inner conductor is positive with respect to ground. When this voltage pulse reaches the x-ray tube it appears across the open circuit of the tube, theoretically doubling the voltage pulse. This doubled voltage is sufficient to cause the tube to conduct, producing an x-ray pulse of length equal to twice the voltage wave transit time of the line. In actual operation of the Blumlein, the anode-cathode separation requires a voltage of about 85 kV for conduction, and a pulse length approaching 50 nsec is achieved.

Originally an explosively driven switch was used to provide the low-inductance short; however, subsequent development demonstrated that an appropriately designed, coaxially mounted spark gap could be made to have sufficiently low inductance to successfully serve as the switch. This improved the repeatability of the Blumlein pulses as well as significantly

Fig. 9. Construction details of Blumlein x-ray source.

Fig. 10. Electrical operation of a Blumlein.

reducing the required maintenance. But some loss in intensity, as a result of increased inductance, was observed.

Figure 11 shows, in plan view, the relation of the two Blumlein sources to the sample and film plane. The photons tend to appear to come from a point source from this device; but this is dependent on the discharge geometry of the cathode-anode, which need not be the same each time. Therefore, a separation of 6 ft was utilized between the x-ray tube and the pressure cell to aid in assuring a reasonably repeatable beam at the sample. Since the film plane must lie behind the sample by as much as 6 in., this helps to reduce the magnification effect of the point source. Unfortunately, the x-ray intensity decreases as the reciprocal of the separation distance squared. Consequently, the intensity transmitted through the pressure cell is insufficient to expose the fast, medical x-ray film used to record the image. This necessitated the use of film packs employing zinc-sulfide fluorescent screens to intensify the image sufficiently to produce a record. Appendix H discusses in detail the construction of the film packs used and the standard development techniques applied to the exposed film.

Sample Preparation

The selection of the sample geometry depends on two factors: the minimization of skin-depth effects, and the maximization of the accuracy of radiographic determination of the diameter. The former dictates cylindrical sample symmetry of minimum diameter, and the latter dictates cylindrical sample symmetry of maximum diameter. With the lead and uranium samples, a diameter of 1 mm was chosen, which reasonably satisfied both criteria. This is near the minimum diameter that can still be accurately measured radiographically.

The soft nature of lead makes convenient preparation of samples by extrusion, through a 1-mm die, in a hydraulic press. Handling of the soft lead wire during the sample mounting requires considerable care. About 50 mm of the extruded lead wire is first clamped in the top sample support. After careful straightening and alignment with the axis of the sample holder, the bottom support clamp is added and the residual material cut away. Copper voltage probes are constructed from pieces of 0.125-mm-thick copper strips about 1.0 mm wide. They are bent such that spring tension will hold them against the sample. The probes are attached to the voltage

Fig. 11. Geometric relation of sample and film Blumleins.

feedthrough leads by clamping them under the head of a screw. These probes are cut and sanded such that a very fine point contacts the sample, providing the required voltage measurement without disturbing the dynamics of the expanding liquid sample.

Uranium samples were prepared from a billet of fine-grained, pure uranium by a machining technique. They are nominally 1 mm in diameter and 25 mm long. Because they are very rigid, their handling during assembly is not at all critical. It is necessary to first remove the oxide from the surface by acid etching or other suitable technique. It is difficult to maintain contact between the voltage probes and the hard uranium surface, because the spring tension is insufficient to withstand the stronger magnetic forces associated with the higher current needed to heat the uranium. (A probe would separate from the sample early in the current pulse and regain contact only at the termination of the current flow.) Various techniques such as use of springier probe material were tried to attempt solution of this problem, but they gave only limited improvement. It finally became necessary to use rigid vertical probe supports as shown in Fig. 12.

Data Recording

Because considerable data handling is required to arrive at the desired thermodynamic values, the data must be as accurate as possible. The time-histories of the current flowing through the sample and the simultaneous voltage developed across the portion of the sample between the voltage probes are recorded on oscilloscopes. The only problem encountered in making these measurements is the possibility of ground loops. A ground loop may exist whenever a circuit must be connected to ground at more than one point. Data errors result if a significant potential can exist between grounded points. This problem is considerably reduced by using an inductive pickup to monitor the current. This kind of transducer utilizes the magnetic field surrounding a current-carrying conductor and makes no electrical connection to the circuit itself. In this case, a Pearson Wide Band Current Transformer was chosen. It produces a signal of 0.005 V/A when a terminated lead is used. The frequency response ranges from 1 Hz to 65 MHz.

Cross-sectional View

Top and Side View of Voltage Probe

Fig. 12. Voltage probe modifications required with uranium.

Both voltages measured by the probes are with respect to a circuit ground established at the top of the pressure cell. Consequently, the desired voltage is obtained by electrically subtracting the voltage of one probe from the voltage of the other. This is conveniently done by connecting the two voltage signals to the input terminals of a differential, oscilloscope preamplifier. A complete wiring schematic of the instrumentation is given in Fig. 13. Each voltage signal and, in most cases, the difference between them will saturate the differential preamplifier unless attenuation is used. The line impedance-matching attenuators provide 100:1 attenuation without causing unwanted reflections on the signal cables. Although only one pair of current and voltage records is required, the signals were usually recorded on three Tektronix 555 oscilloscopes to provide scaling variation and duplicate records in the event one or more scope channels should fail. The Tektronix 555 dual-beam oscilloscopes have a response range from dc (non-time-varying) to 33 MHz. The Type L preamplifiers used to record the current signals have a response range from dc to 30 MHz. The type 1A5 differential preamplifiers used to measure the voltage have a response range from dc to 30 MHz at sensitivity down to 1 mV/cm deflection.

In addition to the current and voltage records, pressure, and shadowgraphs, an accurate measure of the sample diameter, voltage-probe separation, and initial resistivity of the sample is required. These data are used in reducing the other data to the desired thermodynamic values. The optically measured diameter and probe separation distance define that portion of the total sample for which data apply. The initial resistivity is used in computing the inductance of the sample so that the resistive voltage may be determined. The physical dimensions are measured after the sample is mounted in the holder, using an optical comparator with micrometer readout. They are determined within ± 0.005 mm.

Since variation in sample material resistivity from sample to sample may occur, a technique was developed to measure the resistance of the portion of the sample between the voltage probes. A 50- μ sec, 80-A square wave current pulse is passed through the sample, measured by a current pulse transformer and displayed on an oscilloscope. The voltage developed between the probes is simultaneously displayed on the scope. After the inductive voltage has become negligible, the ratio between the voltage and the current determines the sample resistance. For lead, the resistance

Fig. 13. Electrical wiring of the current/voltage instrumentation.

measured by this technique was typically of the order of $1 \text{ m}\Omega$ and was repeatable within 1-2% for a given sample. Figure 14 is an oscilloscope record typical of the data obtained by this procedure.

DATA PROCESSING PROCEDURE

The data obtained for each experiment include time histories of the current pulse and interprobe voltage, two preshot shadowgraphs of the sample and calibration rods, two postshot shadowgraphs of the sample and calibration rods, the initial sample diameter, the probe separation distance, and the initial sample resistivity. From these data, values of the resistance, resistivity, enthalpy deposition rate, deposited enthalpy, equilibrium volumetric expansion, and estimated internal energy and temperature are obtained. The pressure is read directly from a gauge, and no further reduction of this data is required. Figures 15a and b present typical data from an experiment using lead as the sample material.

Calculation of Enthalpy and Resistance

The enthalpy deposition rate is the product of the current (I) and the resistive component of the measured voltage (V_R) as functions of time. The integral of this product over the duration of the energy pulse (t_p) yields the deposited enthalpy:

$$\Delta H = H(T) - H(298) = \int_0^{t_p} I(t) V_R(t) dt. \quad (1)$$

The ratio of the resistive component of the measured voltage to the current as a function of time yields the average resistance of the sample:

$$R(t) = V_R(t)/I(t). \quad (2)$$

The current and voltage records are not readily expressible in analytic form, so numerical application of Eqs. (1) and (2) is required. The only difficulty is determining the resistive voltage.

The measured voltage includes both resistive and inductive components. A detailed electrical schematic and complete analysis of the voltage waveform are given in Appendix G. There are two inductive components to the

Fig. 14. Data for determination of initial sample resistivity.

Typical Current Pulse
17 kJ Bank, 15 kV, 22 μsec

Fig. 15a. Typical voltage and current records for lead.

$$\Delta H = 205 \text{ cal/g} \quad \frac{V}{V_0} = 3.10 \quad P = 1.0 \text{ kbar}$$

Fig. 15b. Typical shadowgraphs of lead sample.

measured voltage: one arising from the self-inductance of the sample to the current flowing through it (L), the other arising from the coupling between the loop formed by the voltage probes and the magnetic field surrounding the sample (M). Further, the inductance is not constant with time, but decreases as the sample expands. This is expressed mathematically in Eq. (3), where the two inductances are additively combined to give L :

$$V_m(t) = I(t) R(t) + L(t) \frac{dI(t)}{dt} + I(t) \frac{dL(t)}{dt}. \quad (3)$$

The complicated and variable nature of the geometry of the current-lead and voltage-probe assembly precludes accurate calculation of the inductance. Fortunately, a method of measuring the initial total inductance affecting the voltage waveform is available. The measurement depends on the fact that the measured voltage is purely resistive at two points early in the voltage time history: first, at $t = 0$, when $V_r = 0$; second, at the peak of the current pulse, when $dI/dt = 0$ and $V_r = V_m$. Since expansion is negligible during this interval, $dL/dt = 0$. The functional dependence of the material resistivity on enthalpy must be known over the range of enthalpy deposited during the rise time of the current pulse. For lead, a linear relationship [Eq. (4)] works quite well, providing that the peak current occur before incipient sample melting. Values of the inductance measured in this manner ranged from 20-30 nH in reasonable agreement with the 5-30 nH based on a geometric considerations.

$$R[H(t)] = R_0 \left[1 + \frac{\alpha}{c_p} H(t) \right]. \quad (4)$$

For uranium, the solid-phase resistivity does not increase linearly with the enthalpy, but is complicated by solid-phase transitions as well as a power-law dependence. This complication is not important to the application of the technique so long as it is reasonably known.

An iterative technique is used to fit the known form of the resistance function to the two measured V_r data points. As functions of time, the resistive voltage obtained is subtracted from the measured voltage and the result divided by the measured rate of change of current. Digital errors occurring in the application of this procedure are reduced by averaging the resultant inductance data over the central third of its period, since this is when the measured voltage and rate of change of current should be most

accurately known. The initial value of the inductance (L_0) thus determined is used to compute the inductive component of the voltage over the remaining pulse duration. This inductive voltage $L_0 \frac{dI}{dt}$ is subtracted from the measured voltage record to obtain an initial estimate of the resistive voltage, assuming a constant inductance. The estimated resistive voltage is used to compute the energy-deposition rate, enthalpy change, and resistance as functions of time. To improve the calculation, the expansion of the sample as a function of time must be known. A direct measurement of this is not available, but an alternative is to use the estimated enthalpy to construct an enthalpy-volume isobar from which the expansion rate can be inferred.

The numerical details of these computations are very tedious for hand calculations. A computer program, EOSDTA, was written for the CDC6600 computer to improve the accuracy and speed the process of data reduction. This program is listed and discussed in Appendix I. To use it, the current and voltage data are read on a Grant film reader attached to an IBM card punch. EOSDTA accepts the digitized data, scales it appropriately, and constructs 500 element tables of current, voltage, and time that are used to make the computations described above.

After sufficient data have been taken and reduced to adequately describe the enthalpy-volume isobar, this isobar is used to improve the calculation of the resistive voltage and subsequently the resistance, resistivity, enthalpy, and internal energy. For each experiment, the correlation between the enthalpy as a function of time and the enthalpy as a function of volume is used to determine the sample diameter as a function of time. These data are used with Eq. (5) to determine the inductance as a function of time.²⁵ The geometric mean distance between conductors (GMD) is determined from the initial inductance and is assumed constant throughout the experiment. The geometric mean radius of the sample (GMR) changes with time as the sample expands:

$$L(t) = 2 \times 10^{-9} s \ln[GMD/GMR(t)] \text{ henrys,} \quad (5a)$$

where s (mm) is the interprobe separation distance, and the GMD is defined by

$$GMD = GMR(0) e^{[L(0)/2 s \times 10^{-9}]} \quad (5b)$$

This allows Eq. (5a) to be written

$$L(t) = L(0) - 2 \times 10^{-9} s \ln[GMR(t)/GMR(0)], \quad (5c)$$

from which

$$\frac{dL(t)}{dt} = [-2 \times 10^{-9} s/GMR(t)] \frac{d[GMR(t)]}{dt} \text{ henrys per sec.} \quad (5d)$$

The geometric mean radius of a conductor of circular cross section is 0.7788 times the true radius.²⁵

The inductance and the rate of change of inductance as functions of time are used to recompute the resistive component of the measured voltage data. Then, recomputed values of the enthalpy and resistance are made as before.

Knowledge of the sample radius as a function of time allows computation of additional data. The resistivity, accounting for the dimensional change of the sample, is obtained from

$$\rho(t) = R(t) [\pi r^2(t)]/s, \quad (6)$$

where s is the interprobe separation distance, ρ is the resistivity, and r is the radius of the sample. The internal energy of the sample may be estimated by

$$E = H - P(\Delta V), \quad (7)$$

where the deviation from hydrostatic pressure is neglected. If an estimate of the specific heat is available, then it may readily be used to estimate the temperature.

A second computer program (DTACOR) was written for the CDC6600, to perform the calculations described above. DTACOR is listed and discussed in Appendix J. It accepts the isobaric data from punched cards but takes the current and voltage data for each experiment from a tape prepared by EOSDTA to avoid the repetition of several calculations. Results from DTACOR show that, as expected, the correction to the resistive voltage, because of temporal inductive effects, are very small and completely negligible for expansion ratios less than twofold.

Determination of Sample Diameter

Each shadowgraph contains an image of the sample and images of three or four calibration rods of known diameter placed below the sample, but in the same photographic plane. The use of fluorescent, image-intensifier

screens to obtain adequate film exposure makes necessary the use of a comparative technique to determine the diameter of the sample from its image. The images of the sample and rods are made fuzzy at their edges by the angular distribution of the converted photons. Direct correlation between image width and sample diameter then becomes very inaccurate.

A detailed description of this comparison technique is given in Appendix H. The sample and calibration rod images on each film record are scanned by a Joyce-Lobel recording microdensitometer which produces a plot of the film density as a function of magnified scan distance. The comparison technique is then applied to these density plots. Figure 16 illustrates typical density plots. A comparison of the magnified true diameter of each calibration rod with the density plot of its image determines the depth of the sample-image density plot corresponding to the magnified sample diameter. Typically this fit occurs at a height from the bottom of between 15 and 45% of the total plot depth, depending on the level of image contrast observed. The true diameter of the sample is then obtained by dividing by the magnification factor. The use of average calibration data seems to improve the accuracy of the technique and somewhat reduces the subjective aspects of it.

An alternative technique was developed for use with a recently acquired Grant comparator, which has densitometer capability. With this machine, digitized data decks of density as a function of scan distance can be made. Appendix K describes the computer program (VOLDTA) written for the CDC6600 computer to reduce these data. Objective, numerical application of the technique described above is used.

Preshot shadowgraphs of the sample in the pressure cell were taken for use in studying the accuracy of radiographic diameter determination. They were reduced by the same procedure used to determine the expanded diameter. The sample diameter thus obtained was then compared with the sample diameter measured optically, and the resultant error was computed. These data are presented in histogram form (Fig. 17) that shows the error as a function of frequency of occurrence for 222 preshot films. There is a tendency to measure the unexpanded sample diameter too large by 3 or 4%. This appears to be due to the fact that the diameters of all the calibrations rods were greater or at least equal to the diameter of the sample being measured. The standard deviation of the error is about $\pm 5\%$.

Fig. 16. Typical densitometer plots of film images (film density vs 51 X 5 can distance).

Fig. 17. Result of error study of preshot films, comparing diameter from image with optical measure.

ERROR ANALYSIS

Accuracy considerations fall into two classes: absolute accuracy (precision) and relative accuracy. Absolute accuracy applies to errors that recur in similar amount with each experiment, therefore affecting the validity of the ultimate thermodynamic values desired. Relative accuracy applies to those errors whose magnitudes fluctuate from experiment to experiment and reduce the repeatability of the data. Relative accuracy introduces a jitter in the thermodynamic data about a constant error specified by the absolute accuracy. In the following paragraphs, error analysis is considered in terms of these two effects.

Errors in current and voltage data may be introduced by the transducers, by ground resistance effects, or by the oscilloscopes. The current transducer specifications are 0.005 V/A from 1 Hz to 65 MHz. Since the frequencies in the current pulse range from 12 kHz to an effective maximum of about 100 kHz, the accuracy of data fed to the transmission line by the transducer is better than $\pm 0.5\%$.

The transmission line (RG-58) is terminated by its 50- Ω characteristic impedance to eliminate reflection. The only ground required is at the scope, eliminating the possibility of ground loop error. There should be negligible error in transmission of these data.

The voltage probes are cut from thin copper stock, sharpened to a point approximately square (0.125 mm on a side). The interprobe spacing ranges from 4.0 to 6.0 mm measured between the probe centers on an optical comparator at 20 $\times$ power magnification. The accuracy of the comparator reading is ± 0.005 mm, but the precision with which the probe center can be determined is probably no better than ± 0.02 mm. The maximum error in interprobe separation distance is then $\pm 1.25\%$. Since this length is used to calculate specific values of the thermodynamic variables, the probe thickness does not affect the relative accuracy of the data; only the absolute accuracy is affected.

The signal from each probe, relative to the ground established at the top of the pressure cell, is transmitted to the oscilloscopes on RG-217 cable; each cable is terminated by its 50- Ω characteristic impedance. This cable is more than adequate to handle the range of voltages and frequencies involved without introducing distortion of the signal. Small errors may

occur because of excessive resistances at the contacts and because of the cable resistance itself. A complete analysis of this is given in Appendix G. If these resistances remain negligible with respect to the 50- Ω line termination, then this error is likewise negligible. Similarly, a very slight error is introduced by the current that must flow in the voltage-probe circuit. A worst-case estimate places the maximum total of these errors at much less than 1%. This affects absolute accuracy much more strongly than relative accuracy.

The voltage terminator and attenuator tolerance could introduce as much as $\pm 3\%$ error. However, measurement of these resistance values indicates that the error is less than $\pm 1\%$ with respect to the assumed values of 50 Ω and 100:1 attenuation factor.

The repeatability of the oscilloscopes is excellent; only the calibration and linearity need be considered as error sources. Therefore, care was taken to ensure the proper calibration of each oscilloscope within 1-2% of full-scale sweep, before each experiment. Use of a scope calibrator that generated a composite signal, causing the scope to trace a rectangular graticule on the screen, allowed photographic comparison of the scope calibration and linearity with the linear, etched, oscilloscope graticule. This procedure reduced the scope error to about $\pm 1\%$.

The absolute error in the current data is probably less than $\pm 2\%$. The absolute error in the voltage data may be as great as $\pm 3\%$. The relative error observed between current records from different oscilloscopes after those records had been processed by computer was never greater than 2% and usually less than 1%. The total relative error in the voltage records is expected to be less than $\pm 4\%$. It was observed that oscilloscope records of the voltage data after computer processing generally showed less than $\pm 2\%$ variation, even though the data were recorded with different voltage scale factors.

The errors in the resistance and enthalpy data also include error introduced by the extensive data handling. These errors include the digitizing error, computer scaling error, inductive correction error, and computer integration error. Digitization of the data reduces the continuous record to between 40 and 60 coordinate pairs. Because variation in the width of the trace recorded on the film occurs at points of rapid vertical beam motion, an effort was made to read only along the center line of the trace. This

results in the introduction of random error between coordinate pairs. The size of this random error varies, depending on the trace position. It is a much larger percentage for regions of low vertical displacement of the image than for regions of maximum signal. Fortunately, the multiplication and integration required to compute the enthalpy averages this error, and the enthalpy is negligibly affected by it. However, the resistance is very sensitive to these random errors, even after the data are smoothed somewhat by the curve-fitting techniques used. Likewise, the derivative of the current is extremely sensitive to these fluctuations.

The digitized coordinate pairs are in units of length from an arbitrary set of machine axes and must be appropriately converted into units of current, voltage, and time. This requires that additional coordinate pairs be supplied whose separation is known in terms of the desired units. Appendix I discusses the details of this process and the steps taken to reduce any errors involved.

The inductive correction that must be applied to the measured voltage data involves several error sources, the magnitudes of which are difficult to define. The determination of the initial inductance can be estimated analytically to be $20 \text{ nH} \pm 50\%$. It may also be measured (see Appendix G) from the current and voltage data during the current-pulse rise time. This requires an assumed functional relationship between resistivity and enthalpy over the range of enthalpy deposited during the current-pulse rise time. It further involves the rate of change of the current and the voltage data over a portion of the records strongly affected by random digitization errors. To overcome the effects of this sensitivity to random error, the inductance is computed as if it were a function of time. As such, it oscillates widely—sometimes as much as an order of magnitude or more, particularly at the extreme ends of the time range. The value chosen as the inductance is the average over the central third of the time range. Generally the data are smoothest in this area because the random digitization errors are much smaller or have less effect. The value achieved is usually in good agreement with the estimated 20 nH and is probably accurate to within $\pm 20\%$.

The determination of the temporal variation of the inductance involves errors in the initial inductance, error in the isobaric data used to determine the expansion rate, and error introduced by the assumption of isobaric

expansion dynamics. There appears to be no reasonable way to eliminate these errors; however, the variation of inductance with time is very small over the range of sample expansion usually accessible to the experiment, and its effect on the enthalpy is almost negligible. Likewise, the effect of the initial inductance on the computed enthalpy is reasonably small. Complete neglect of the inductive correction would produce an error in the enthalpy that is a function of the voltage wave shape but that would probably be less than $\pm 5\%$. With this correction, however, the enthalpy error is probably less than $\pm 1\%$.

The numerical integration is done by a simple trapezoidal approach. That this is adequate is demonstrated by estimating the maximum integration error. Equation (8) gives the maximum error²⁶:

$$\text{Absolute error} \leq n \Delta t^3 M_2 / 12, \quad (8)$$

where n is the number of integration steps, Δt is the step width and M_2 is the maximum value of the second derivative of the integrand $(d^3H/dt^3)_{\max}$. Application of this equation to particular integrations yields the fact that the integration error is always less than 0.1% .

In summary, the data-handling errors are in large part due to random digitization errors and errors in the determination of the inductance. The analysis indicates a maximum of $\pm 1\%$ error expected in the enthalpy. This comes from the expected random error of $\pm 5\%$ in the resistive voltage. The resistance is directly sensitive to the random voltage error and would be expected to show a random error of $\pm 5\%$ and an absolute error proportional to the error in determining the inductance. This analysis is substantiated by results of error tests routinely applied to the total data-reduction process.

Several data decks were prepared from the same pair of current-voltage records. Values obtained agreed within less than 1% for all computed quantities. Further, two or three pairs of current and voltage records are usually available for each experiment, from the duplication in instrumentation used. These were handled as individual experiments, and the computed values were compared. The total random error of the oscilloscopes plus data handling was found to be less than $\pm 3\%$. A consideration of all errors leads to belief that the maximum error in enthalpy should be less than $\pm 5\%$. The resistance data are so strongly affected by random errors that only the relation obtained between current maximum and shutoff can be

considered valid. Over this range the absolute error may be as great as $\pm 5\%$, due principally to erroneous inductance.

Errors in the measurement of the expanded volume may come from several sources: variation in x-ray intensity, nonlinear film response, densitometer error, and the accuracy limitation of the data-reduction technique. The variation in x-ray intensity from experiment to experiment should not significantly affect the accuracy of the end result unless this variation becomes so extreme as to produce films of poor contrast. Typically, the apparent image on the film tends to grow as the contrast is reduced. But this should not affect the measurement, providing the sensitivity of the densitometer allows accurate density traces to be produced, and providing the calibration of each film is performed. Similarly, nonlinear film response should produce negligible error, providing intimate contact between the intensifier screens and the film is maintained across the image area. Otherwise, broadening may occur over parts of the image, giving an erroneous result, even with application of the comparative analysis.

The densitometer error has been found to be negligible by repeatedly tracing a given image cross section. Small variations in a tracing thus obtained are averaged out by the comparative analysis. The structure observed in density plots (Fig. 16) is real and probably due to large grains on the film.

The analysis of the preshot photos, considering the sample diameter to be unknown, indicates that the error in measured diameter is random with a standard deviation of about $\pm 5\%$. This translates into a volumetric random error with a standard deviation of $\pm 10\%$. No correlation between this error and a particular source has been found.

The helium pressure is read from a 7-kbar Heise gauge. The reading precision is ± 5 bars, or better than 0.1%. The absolute error of the gauge is thought to be less than $\pm 1\%$. Sources of possible deviation between the gauge reading and the pressure on the sample include pressure gradients in the cell or in the plumbing between the gauge and the cell, and the magnetic pressure effects. The pressure gradients are allowed to damp out before the experiment by filling the cell and waiting a few minutes for the gas-cooling effect to result in gauge equilibrium. Cell seal leakage, if severe,

could result in an inaccurate reading; but, at the operating pressures used, seal leakage has generally been negligible.

The magnetic fields that surround a conductor carrying a current produce a pressure on the conductor. This pressure is radially symmetric and merely adds a time-varying component to the hydrostatic gas pressure. However, the current return-lead geometry introduces an asymmetric component to the magnetic field, which produces a pressure on the sample, imparting a net momentum to it. An additional magnetic pressure asymmetry results if the sample is not straight. The magnetic component that is normally axially symmetric, then, is caused to become unsymmetric in the area of sample curvature, such that the curvature is increased.

Since the magnetic pressure is proportional to the square of the current, it peaks very early in the heating sequence and falls rapidly to zero as the heating is completed. The peak symmetric component of the force is calculated to be less than 15% of the hydrostatic pressure for currents applied to lead at pressures of at least 1 kbar. This increased pressure on the sample during heating serves only to retard the expansion rate during the early heating, and it is expected that the equilibrium expansion will be negligibly affected. The observed retention of cylindrical geometry for the liquid sample supports the view that the asymmetric component of the magnetic pressure does not affect the equilibrium thermodynamic state. The pressure is probably the most accurate measurement made. The error in these data is conservatively $\pm 1\%$.

EXPERIMENTAL APPLICATION

During the development of this technique, the material chosen for experimentation was lead. Consequently, an extensive amount of data and experience with lead has been obtained, with, unfortunately, rather uncertain results. It was discovered, in the course of the work, that lead has a peculiar purity problem involving one or more of its oxides. The result of using lead with a high concentration of oxide impurity is a significant scattering of the resultant data. Recently, attention turned to application of the technique to uranium. The data are as yet insufficient to provide a conclusive test of their thermodynamic validity, but significantly reduced data scatter is expected.

The principle application of the technique is measurement of data that correlate enthalpy with volume at constant pressure for high-temperature equilibrium thermodynamic states. These data are most conveniently presented in the form of isobars in the enthalpy-volume plane. Since this plane is not conventional for presentation of thermodynamic data, Fig. 18 is included to demonstrate the thermodynamics applicable to this representation.

Note that in the enthalpy-volume plane the coexistence curve is not dome shaped as in the pressure-volume plane. This is a result of the fact that enthalpy must be added during the phase changes, even though the temperature remains constant. The total height of the coexistence curve represents the sublimation energy. Note, also, that the critical point is located on the side of the coexistence curve. At the critical pressure, the isobar is just tangent to the coexistence curve at the critical point. Isobars at lower than critical pressure lie very close to the rising portion of the coexistence curve, up to the enthalpy of incipient vaporization. At this point they cross the boundary and continue across the two-phase region with constant, positive slope. This slope is determined by the heat of vaporization divided by the difference in volume of the vapor and liquid phases at the temperature of vaporization. After leaving the two-phase region, the isobaric slope again changes to eventually match the constant slope of the perfect-gas isobar:

$$\partial H / \partial V)_P = c_p P / nR \text{ for a perfect gas.} \quad (9)$$

Several interesting and pertinent consequences arise from the measurement of enthalpy and volume at constant pressure. The isobars become closer spaced in the vicinity of the critical point, and the change in slope upon crossing the coexistence curve becomes much less pronounced. The technique is capable of producing data on states of mixed liquid-vapor phases, so long as the sample resistivity does not become dominated by the vapor-phase resistivity. The stability of the data depends upon the degree of homogeneity of the two phases and the magnitude of forces tending to separate them. The onset of vapor resistivity occurs when the expansion of the sample is sufficient to cause loss of the applicable band structure.

Figure 19 shows enthalpy-volume data obtained for lead at pressures of 1.0, 1.25, 1.5 and 2.0 kbar. Before any data were taken, the best estimate of the critical pressure of lead was 0.85-0.95 kbar (Ref. 7), prompting the

Fig. 18. Representation of the thermodynamic equilibria in the enthalpy-volume plane.

Fig. 19. Isobaric data obtained for lead. Heat of sublimation is derived from Ref. 27; existing static data are from Refs. 4 and 5.

choice of 1.0 kbar as convenient for the development work. The experimental evidence and a later estimate²⁸ infer that the critical pressure is greater than 1 kbar and possibly as high as 2 kbar.

The volumetric coordinates of the data of Fig. 19 were measured 60 μ sec after current shutoff. The energy deposition was complete 10 μ sec after current shutoff. A number of studies on the dynamics of expansion indicated that 40 μ sec were required after deposition of the energy to reach volumetric equilibrium. The predicted equilibration time was of the order of 10 μ sec. The discrepancy is thought to be connected with thermal equilibration of nonuniform energy distributions resulting from impurities in the lead.

The achievement and demonstration of volumetric equilibrium are crucial to the success of the experimental technique. Consequently, numerous data were taken specifically for the purpose of measuring the time required to obtain equilibrium. When the first data on lead at a pressure of 1 kbar were taken, shadowgraphs were made 10 μ sec after energy deposition with one x-ray source and 60 μ sec after energy deposition with the other x-ray source. It was observed that the expansion ratio at the earlier time was often smaller than the expansion ratio at the later time. Furthermore, the later data produced a smooth, self-consistent isobar whereas the earlier data were quite random. These data are shown in Fig. 20, where the isobar drawn is the same as that shown on Fig. 19. Subsequently, the presence of gross inclusions of lead-oxide in the material from which samples had been extruded was discovered.

New, zone-refined lead was purchased and a series of experiments were performed to examine the so-called late-time expansion phenomena. Data were taken at constant energy and shadowgraphs were made simultaneously with both x-ray sources at a sequence of times after energy deposition was complete. This technique eliminated the possibility of asymmetric expansion clouding the results. The energy was first selected to be about 150 cal/g to assure that only single-phase states were examined. These data are presented in Fig. 21, which demonstrates that equilibrium volume was achieved within 10 μ sec after energy deposition and persisted at least as long as 90 μ sec. A second series of data were taken at 205 cal/g. This was the maximum stable energy deposition attainable with lead at a pressure of 1 kbar. Above this value, the onset of vapor resistivity was observed to

Fig. 20. Isobaric data for lead at 1 kbar showing self-consistency of subset.

Fig. 21. Late-time expansion data at 150 cal/g and 1 kbar.

occur with resultant loss of control of the deposited energy and complete vaporization of the sample. These data are presented in Fig. 22. They indicate a late-time expansion occurring over the first 50 μ sec after energy deposition. This was not totally unexpected, since at this energy and pressure the achieved state is probably deep in the coexistence region. Here the expansion dynamics may well be a function of the vaporization dynamics involving superheating of the material and two-phase homogeneity, which are not well understood. A check point was taken at 170 cal/g, which agrees favorably with the data presented in Fig. 21.

Table 1 lists in detail the data illustrated graphically in Fig. 19. It may be observed that groups of the data appear completely self-consistent, while isolated data appear completely unrelated and uncorrelated. A thorough study of these data was made to discover the cause of the lack of repeatability. This study demonstrated that the individual sample was always to blame. Studies of the purity of the lead (Appendix E) showed the possibility of the presence of random oxide concentration of a magnitude sufficient to cause the observed variation. The self-consistency of the various subsets of data is apparently due to the examination of a series of uniformly homogeneous samples, whereas the data scatter apparently results from samples that are not homogeneous and probably contain impurities.

Figure 23 compares the results of a series of 10 experiments, for which the resistivity of lead is given as a function of enthalpy at a pressure of 1 kbar. These data are compared with existing data at standard pressure. Errors introduced by the random variation of the rate of change of current invalidate any data following the time of current shutoff. It should be kept in mind that these data are taken during the heating and expansion of the sample and are not representative of a state of complete thermodynamic equilibrium. However, the comparison indicates the capability of the technique to study material resistivity as a function of the heating of the material; the results substantially agree with existing equilibrium data.^{29,30}

Table 1 also lists three data for uranium taken at a pressure of 1 kbar. As observed, the current-pulse characteristics were constant, and repeatability in the deposited energy and resultant expansion were the parameters sought. These values are observed to be within the error tolerance ascribed to the technique. The data are, of course, insufficient to draw conclusions about the thermodynamics involved. There is still uncertainty about whether

Fig. 22. Late-time expansion data at 205 cal/g and 1 kbar.

Table 1. Tabulated isobaric data for lead and uranium.

Exp. No.	Enthalpy (cal/g)	V/V ₀	At time (μsec)	V/V ₀	At time (μsec)	Pressure (kbar)	Material
35	128	1.73	60	1.59	10	1.0	Lead
36	129	1.77	60	1.78	10		
37	113	1.73	60	1.68	10		
40	148	1.85	60	1.78	10		
41	157	2.01	60	1.83	10		
42	158	2.08	60	1.76	10		
43	173	2.42	60	2.21	10		
44	182	2.62	60	2.24	10		
45	190	2.88	60	2.13	10		
49	177	2.46	60	2.17	30		
50	215	2.42	60	2.00	10		
51	187	2.15	60	2.15	90		
52	186	2.44	60	2.44	90		
55	162	1.99	60	1.97	30		
57	157	2.58	60	2.22	30		
58	187	2.82	60	2.52	30		
74	209	2.82	60	2.15	10		
82	219	2.72	60	3.16	90	1.25	Lead
83	200	2.65	60	3.11	90		
84	187	2.20	60	2.28	90		
85	180	2.01	60	2.30	90		
89	151	1.85	60	2.10	90		
90	148	1.82	60	1.79	90		
91	133	1.57	60	1.81	90		
95	123	1.61	60	1.53	90		
96	115	1.57	60	1.79	90		
80	194	2.48	60	2.86	90	1.5	Lead
81	213	2.54	60	3.08	90		
86	217	3.21	60	4.08	90		
87	234	4.27	60	5.09	90		
93	265	6.28	60	7.33	90		
94	179	2.19	60	2.22	90		
109	140	1.61	60	1.74	60	1.0	Lead
117	165	1.64	60	1.82	60		
149	165	2.78	60	3.06	60		
150	165	—	—	2.44	60	1.5	Lead
151	165	—	—	2.04	60	2.0	Lead
133	207	2.34	30	1.96	60		
134	196	2.60	30	2.83	60		
141	230	4.07	30	4.84	60		
144	180	2.46	30	3.10	60		
145	170	2.46	30	3.06	60		
U15	320	3.37	60	3.30	60	1.0	Uranium
U16	335	2.80	60	2.90	60		
U17	325	3.37	60	3.35	60

Fig. 23. Resistivity of lead at high temperature. The bars indicate the spread of experimental data; fewer data were available at higher energies.

these data lie in the single-phase liquid region, as seems to be indicated by the experiment, or in the two-phase liquid-vapor coexistence region, as indicated by various estimates of the location of the coexistence boundary.³¹ A full experimental determination of the isobar may be required to resolve this issue. Certainly, more data will be required.

The resistance data obtained for uranium are illustrated in Fig. 24. These data were reduced by assuming a constant inductance value of 15 nH instead of by the procedure followed with lead, because of the difficulty of producing a sufficiently accurate fit to the known data. This appeared to work better for uranium after several approaches had been tried and the 15 nH figure determined to be a reasonable value. These data are in fair agreement with previously available resistance data for uranium.³¹

There is evident in Fig. 19 a large scatter in data—much greater than is indicated by the expected experimental error ($\pm 5\%$ in enthalpy, $\pm 10\%$ in volume). A full analysis of the current, voltage, and volume data indicates that sample variability in voltage per unit length as a function of time, for constant current pulse parameters is to blame for this scatter. Further investigation indicates that the variability probably results from the presence in the lead sample material of a nonhomogeneous distribution of randomly sized lead-oxide particles.

Before experimentation commenced, the lead stock from which samples were to be extruded was analyzed by vacuum fusion techniques for gaseous contaminants. A very low oxygen content (< 1 ppm) was found. The results of this analysis were believed valid, and numerous experiments were conducted with lead from this supply. Then some visible oxide inclusions were found in a piece of the lead being prepared for extrusion into wire samples. This led to the belief that the vacuum fusion technique might not be adequately reducing the compound oxygen. In fact, one of the inclusions was carefully removed and subjected to vacuum fusion analysis. The results indicated about 70 ppm oxygen where there should have been more than 1000 ppm. However, tests with reagent-grade lead-oxide in the vacuum fusion system indicated that 85-90% of the oxygen was recovered.

Neutron activation analysis was then performed on the lead. This indicated that as much as 10,000 ppm oxygen was present, and that the oxide content of pieces of the order of the sample size was variable over wide ranges. A new supply of lead that had been zone-refined to contain less

Fig. 24. Observed resistance of uranium.

than 10 ppm impurities was purchased. As in normal practice, the gas content of the material was not specified by the supplier. Experimental work with this lead soon indicated that the sample-to-sample variability problem was still present. Vacuum fusion and neutron activation analyses were again undertaken to determine the oxide content. The vacuum fusion analysis indicated only 10-70 ppm oxygen, but the neutron analysis showed oxygen concentrations of the order of 700 ppm. To further complicate matters, it recently became possible to perform gamma activation analysis on two 1-g samples; the gamma analysis indicated about 70 ppm oxygen. If this oxygen is in the form of lead oxide, multiplication by a factor of 14 yields the concentration of oxide.

There is good reason to suspect the vacuum fusion accuracy because, with this technique, one must rely on the partial pressure of carbon above the crucible to chemically reduce the lead-oxide. However, both the neutron and gamma activation techniques are believed to be accurate analyses within $\pm 10\%$. Resolution of this discrepancy may require considerable study.

Discussions were held with the material supplier, who was then beginning to examine lead for oxides for another customer. His analysis was not complete, but our neutron analysis results caused no great surprise. The conclusion was drawn that the lead contained microscopic particles of lead oxide in rather high concentration, but with quite random distribution. Studies defining the expected effect of significant oxide concentration were immediately undertaken and demonstrate that this is incompatible with the experimental technique.

Appendix E discusses two theories that help to understand the interaction between the impurities and the experimental technique. One model examines the effect of nonconducting inclusions in samples heated by resistive energy deposition. The other model is based on the chemical disassociation of the lead oxide at temperatures reached by the experiment. The result of disassociation is that free oxygen may become available at a relatively small number of microscopic sites.³² This free oxygen is then available to drive the sample expansion in a manner not at all representative of the expansion of pure liquid lead at the same temperature. Further, this may produce the nonconducting inclusions considered by the first model.

Inclusions of a nonconducting material in a sample that is resistively heated severely distort the current distribution in the region of the inclusion.

At the point of increased current density, the deposited energy is greater, creating a very nonuniform distribution of deposited energy and sample resistivity. A computer program was used to study the effect this process has on the measured energy deposition and expansion, since these measurements are strictly valid only if the energy is deposited homogeneously and the expansion is uniform. Two extreme cases are easily visualized. First, the existence of a single, relatively large inclusion will rapidly produce a cross-sectional region of the sample that is resistively unstable. This "hot spot" will expand much more than the region that is impurity-free and thus be easily visible in the shadowgraphs. At various times such behavior has observed. Second, the presence of finely dispersed, homogeneously distributed, nonconducting material will serve only to increase the resistivity of the sample material, causing more energy than expected to be deposited, but not upsetting the equilibrium of the heating or expansion processes to any observable extent. This effect, too, has been observed.

To treat the intermediate cases, the computer solution was required, so that the nonlinear aspects of the problem could be considered. The studies carried out with the program (Appendix E) include a study of the effects of particles of the order of 10^{-3} mm in diameter as a function of the concentration. A conclusive demonstration was obtained, that particles of this size, in concentrations of as little as 0.5%, do indeed upset the equilibrium heating, causing distortion of the average enthalpy as measured experimentally. Also, distortion of the average volume determined from x-ray shadowgraphs of the expanded sample will result.

Uranium was chosen as a second sample both because of interest in the liquid properties of this metal and because of the relative ease with which the apparatus could be modified to handle it. Six experiments were done, using the 17-kJ capacitor bank as a source. This demonstrated that this source was too small to deposit interesting amounts of energy in the uranium. The experiments did, however, show an interesting aspect of the behavior of the resistivity of uranium: no step in resistivity as a function of enthalpy occurs at the melt transition, as is observed with lead. Further, the thermal coefficient of resistivity for uranium was shown to be considerably less than that for lead from near the melting temperature to the maximum temperature so far achieved (6000-8000°K). Thus, although the ambient resistivity of uranium is 50% greater than lead's, the lack of increase

in resistivity during the heating required the use of higher currents than could be obtained with the small capacitor bank to allow achievement of the desired temperatures.

Completion of the 42-kJ capacitor bank has allowed the application of the technique to uranium to be restarted. The few data available indicate that operation of the experimental technique is repeatable as expected. These data are quoted with trepidation because of their preliminary nature. There are experimental indications that at this energy the value of $\partial H / \partial V)_P$ is indicative of a single-phase liquid region. Furthermore, the very slowly increasing resistivity is indicative of a liquid state. However, estimates of the coexistence boundary for uranium indicate that the state measured is deeply within the two-phase region. More data will certainly be required to resolve this issue, since it is entirely possible that the estimated coexistence boundary is incorrect.

SUMMARY AND CONCLUSIONS

To overcome the temperature and pressure limitations of the classical technique, it is necessary to contain the sample in a very inert material. The unique idea developed by this research is the use of a rare gas (helium) as the containment material.

The use of helium entails a number of other innovations to make the technique operable. Since the gas cannot support the sample against the action of body forces such as gravity, a means of rapidly achieving the test conditions and a provision for the rapid measurement of that state are required.

Resistive energy deposition is used to heat the sample in a few tens of microseconds. A current pulse of controlled shape, magnitude, and duration allows accurate control of the deposited energy. The energy is deposited uniformly in the sample at a rate such that the sample remains close to thermodynamic equilibrium during the transition between the ambient and the desired state. Pressure-cell windows transparent to x-rays allow flash x-ray photography of the sample, to obtain dimensional data.

The sample is a wire 1 mm in diameter by 25 mm long. The cylindrical symmetry of a wire is required, to minimize skin-depth effects in heating and to allow volumetric measurement to be obtained from measurement of a single dimension. Only for that portion of the sample between the voltage probes (~ 5 mm) is data taken.

The gas pressure is held constant throughout the experiment. The use of voltage probes to measure the voltage drop across a central portion of the sample avoids the effects of contact resistance and nonequilibrium thermal states at the current lead contacts. Integration of the product of the current with the resistive component of the voltage yields the enthalpy change of the sample. The diameter of the sample (obtained from the image on the x-ray shadowgraph) and the separation of the voltage probes determine the volume.

In addition to the thermodynamic data, the dynamic resistivity from sample melt to current-pulse shutoff is readily computed. However, the resistivity of the equilibrium state is not available. The thermodynamic energy of the sample is not measured directly but may be estimated from measurements of the enthalpy, pressure, and expansion ratio. Likewise, the temperature is not measured but may be estimated from the enthalpy using assumed values for the specific heat at constant pressure.

The accuracy of the volumetric data limits application of the technique to thermodynamic states having volumetric expansion ratios exceeding 1.10. The pressure limit is determined solely by pressure cell design, but that design is restricted by the requirement for transparency dictated by the volumetric measurement. The high temperature limitation of the technique is controlled by the capability to achieve stable equilibrium before the onset of instability. This capability is limited principally by the onset of significant vapor phase resistivity.

The critical constants are not directly determinable by this technique. All of the measured quantities, with the possible exception of the resistivity, are stable and continuous at the critical point. Similarly the left-hand side of the coexistence boundary is not directly observable except for pressures considerably less than the critical pressure. Low-pressure isobars show measurable change in slope upon crossing the boundary. The absence of strong forces acting to separate the liquid and vapor phases on the time scale of the experiment makes this technique capable of penetrating the coexistence region; the depth of penetration depends on the pressure and the expansion dynamics of the sample.

A maximum of $\pm 5\%$ error is predicted for the measurement of the enthalpy. The principal experimental error is in determination of the volume. An analysis of collected data obtained from preshot shadowgraphs shows

that the three-sigma error for the volumetric data is probably as great as $\pm 10\%$. The pressure is known to gauge accuracy that is better than 1%.

Two materials, lead and uranium, have been studied to demonstrate the applicability and utility of the technique. The work with lead has been quite extensive, but sample-purity problems have continually plagued this work.

The technique has been applied to uranium to a very limited extent. While uranium has a higher initial resistivity than the lead, that resistivity does not increase with temperature nearly as rapidly. The preliminary nature of the uranium studies preclude inclusion of completely verified thermodynamic data, but no difficulties of the type observed with lead have been discovered.

The conclusion is drawn that this technique for measuring equilibrium thermodynamic data on metals at very high temperatures in the liquid phase is capable of providing valid data. In this respect, it will allow the achievement of a goal currently required by materials application: the measurement of thermodynamic data for liquid metals up to and including states in the critical region. This work is expected to provide a base from which improvements in apparatus and specialized techniques may be employed to ultimately allow all liquid metals to be adequately characterized.

RECOMMENDATIONS

General

Throughout the development and application of this technique to lead and uranium a number of ideas have been formulated that, for one reason or another, we were unable to put into practice. Chief among these are the three subjects which warrant special headings in this section. Other more subtle ideas are lumped together under this general heading even though they may well not be any less important.

It is important that this work with lead and uranium be carried on to completely characterize these materials. With this in mind, Appendix L is included to give an operation procedure as it was applied to the author's equipment. Special emphasis on the lead work should be directed toward

first obtaining pure material with which to work. In addition to studying lead at higher pressures, an attempt should be made to fit data from this experiment to the existing (low-pressure) data and obtain a better estimate of the low-pressure coexistence boundary.

The technique is applicable to materials of atomic number greater than copper. A number of these materials probably may be studied without requiring major modifications of the equipment. Of course, the really good electrical conductors, such as gold and silver, may be difficult to heat with existing current sources; but tin, and possibly even rubidium and cesium, are materials that should and probably can be examined with relative ease.

The somewhat controversial topic regarding the determination of data on states within the two-phase liquid-vapor coexistence region has proved quite interesting. The unique capability of the technique to reach these states is not expected to produce data of much engineering significance; but, if properly developed, it might well provide data on various aspects of the vaporization dynamics issue. For example, is vaporization nucleated at sites throughout the sample, or is vaporization only occurring at the surface? Is the onset of vapor-phase resistivity governed only by the reduced density, or is the presence of pockets of vapor in the sample giving rise to the observed increase in resistivity? These are interesting questions that have not yet been completely answered.

As more and more data are obtained with the technique, it will become necessary to develop means of formulating these data in terms suitable for application to specific needs. This formulation may take the form of an analytical fit to the data or perhaps just numerics suitable for calculations performed by computer. In either form it will allow other thermodynamic variables than those specifically measured to be obtained if they are needed. Furthermore, such treatment of the data might allow the critical-point constants to be obtained.

Optical Volumetric Measurement

If an accurate volumetric measurement can possibly be made using optical techniques instead of the x-ray process, two major advantages would be obtained. First, the restrictions on pressure-cell transparency, sample opacity, and x-ray source intensity would be removed. Optically flat,

transparent, pressure-cell windows would, of course, be required, but this is a state-of-the-art capability for pressure cells withstanding up to at least 10 kbar. The opacity of the sample to optical photons should be adequate for all materials capable of being resistively heated. The optical source intensity could easily be made as great as required to obtain the necessary contrast on the film.

Secondly, the use of optical techniques would allow the observation and recording of the sample expansion dynamics. This means that significantly more data would be available from each experiment. Optical recording might be accomplished either through the use of a continuous light source and a shuttered camera³³ or, alternatively, a pulsed laser source and an open-shutter camera.³⁴

Although the techniques for application of this method are readily available, the assurance of successful results is not certain. Concern involves the optical properties of the high-pressure gas in intimate contact with a sample heated to several thousands of degrees, through which a small pressure disturbance is transmitted. Either thermal or pressure gradients in the gas may modify the refractive index and produce optical distortion, creating error in the resultant measurement. Reliable data on the expected magnitude or spatial extent of these disturbances is not available. Therefore, prediction of the probability of success is not possible. However, the advantages to be gained are so great that recommendation is made that efforts to develop and test this applicability be expended.

Spectrophotometric Temperature Measurement

A serious limitation to the existing technique is the inability to obtain a measure of either the specific heat or the temperature of the sample material in the high-temperature state. Therefore, the development of optical windows for the pressure cell is highly desirable so that the applicability of spectrophotometric techniques for very rapidly determining the temperature can be assessed.³⁵⁻³⁷ As with the optical volumetric measurement, a determination of the distortion introduced by pressure and temperature gradients in the high-pressure gas must be made.

Techniques for measuring temperatures within the required times exist, but are not straightforward. Some ingenuity is expected to be required to produce a technique compatible with the rest of the experiment. Another problem requiring considerable analysis is that determination of temperature from the radiative spectrum will yield a temperature characteristic of the surface of the sample. The steep temperature gradient expected to exist between the sample and the gas may extend beneath the surface of the sample for a significant depth, causing the surface temperature to be significantly different from the mean temperature of the sample. Furthermore, the emissivity of the radiating sample is probably not unity, and some means of determining what it is will be needed. Additional material that may be of value in designing an acceptable technique is found in Ref. 38-41.

Although formidable problems in application of this temperature measurement technique and in the interpretation of the results obtained appear to be present, the considerable additional thermodynamic data which would then become available makes the development effort worthwhile. An acceptably accurate measure of the temperature associated with the currently available pressure-volume-enthalpy data will allow the calculation of any other thermodynamic data desired.

Equilibrium Resistivity Data

The resistance and resistivity data available from application of this technique are obtained from the heating current and associated voltage. The large inductive voltage subtracted from the measured voltage following current shutoff involves both a relatively poor estimate of the inductance and errors made in measuring the rate of change of current. Consequently, the resistance data obtained as the sample approaches equilibrium become meaningless. Further, the resistance measured during the current pulse, although reasonably accurate, may not be representative of equilibrium thermodynamic states of known pressure and volume. Also, if nonuniform energy deposition is present, the resistance data may involve error due to the averaging process of the measurement.

An attempt was made to obtain the resistance of the sample following energy deposition for correlation with the volumetric data (Appendix M). The idea was to permit a small constant current to flow through the heated sample, thus eliminating the necessity of inductively correcting the

resultant voltage dropped across the sample. The rate of energy deposition was negligible over times of interest. It was discovered, however, that the main-bank circuits continue after shutoff to deliver a time-varying signal, comparable to the measurement current, because of resonance ringing. The recorded voltage contained such a large inductive component resulting from this oscillatory current that reasonable data could not be obtained.

It is recommended that continued effort be placed on obtaining a solution to this measurement problem. Probably, elimination of the unwanted signal can only be accomplished by mechanical switching of the connection between the main bank and the sample. Because of the long operation times required by suitable mechanical switches, a significant timing requirement is introduced to the application of this solution. However, if a mechanical switch with sufficiently low jitter is used, the timing problem can be solved electronically, and the system can be made to produce the desired data.

Over the time that this resistance data would be obtained, there is negligible energy deposition. Therefore, this data should be invaluable in accurately determining the approach to equilibrium volume and the duration of a suitably stable, high-temperature state. In addition, the correlation of equilibrium resistivity with the other thermodynamic properties of this state will prove of value for correlation with theoretical investigations of liquid-metal physics.^{12,13,14}

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APPENDIX A

Magnetic Pressures and Effects

An electric current creates a magnetic field of flux density given by Ampere's Law:

$$d\vec{B} = kI \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}. \quad (A-1)$$

If the current is time-varying, then not only is the action of establishing the magnetic field present, but the reaction force on the moving electron due to the field is also present. This interaction between current and field is the origin of skin-depth effects, inductance effects, and, ultimately, pressure effects.

If the conductor carrying the current is long, straight, and spaced sufficiently far from conductors carrying return currents, the magnetic field that surrounds it is symmetric with respect to the conductor axis. The force on the conductor from such a field is radially symmetric and directed toward the conductor axis. This is the equivalent of an additional pressure applied to the conductor material.^{A-1,A-2} The derivation of the magnitude of this pressure as a function of current is straightforward.^{A-3}

The magnitude of the magnetic pressure is

$$P_m = \frac{B^2}{8\pi}. \quad (A-2)$$

For a long, straight conductor, the Biot-Savart Law gives

$$B = \frac{2I}{cr}, \quad (A-3)$$

where I is the current, r is the radius of the conductor, and c is the speed of light. Combining these equations yields

$$P_m = \frac{I^2}{200 A} \text{ dynes/cm}^2, \quad (A-4)$$

where the final result involved a change in current units so that the pressure is given in dynes/cm² when the current is in amperes, and the area is in cm². For a given sample, this pressure is just proportional to the square of the current.

Figure A-1 shows the calculated magnetic pressure exerted on a lead sample. This calculation assumes that the magnetic pressure is completely

Fig. A-1. Radially symmetric magnetic pressure as a function of heating time for a lead sample (diameter = 1 mm).

symmetrical with respect to the sample axis. But the geometry of the conductors in the pressure cell violate the conditions necessary for such symmetry. The proximity of the return current causes the magnetic field strength to be much larger between the sample and current return lead than on the opposite side of the sample. This produces a force tending to separate the two conductors. The action of this force throughout the time of current flow imparts a net impulse to the liquid sample, which is free to move.

It is observed, experimentally, that this momentum is always sufficient to result in the ultimate impact of the sample against the side of the pressure cell, opposite the current-return lead. However, the momentum is not sufficient to cause observable motion of the sample in the experimental time of interest. With larger currents or less massive samples, this condition may not be maintained. Steps to improve the current return-lead geometry by making it more coaxial with the sample may be required.

An estimate of the magnetic force on the sample, considering the true geometry, can be made. The magnetic force may be considered to be the superposition of a symmetric component, as previously derived, and an asymmetric component that imparts momentum to the sample. The magnitude of the impulse depends on the force and time of application:

$$\text{Impulse} = \int_0^t F(t) dt. \quad (\text{A-5})$$

Assuming that the sample and return-current lead are straight, parallel conductors and ignoring end effects, the force is given by

$$\overline{F} = \frac{I^2}{c^2} \int_a^b \int_a^b \frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \overline{r})}{r^3}, \quad (\text{A-6})$$

which predicts a repulsive force between the two conductors of magnitude

$$F = \frac{I^2 l^2}{c^2 d^2}. \quad (\text{A-7})$$

The impulse imparted to the sample is just equal to the final sample momentum. The final velocity may then be calculated by

$$v = \frac{9 \times 10^{18} \ell}{\pi r^2 d^2 c^2 \rho} \int_0^{\tau} I^2(t) dt, \quad (\text{A-8})$$

where I is given in amperes, d is the distance between conductors, ℓ is the conductor length, ρ is the sample density, and r is the sample radius. Assuming that the value of the integral can be replaced with a suitably chosen value of constant current applied over time τ then the expression for the velocity becomes

$$v = \frac{9 \times 10^{18} I^2 \ell \tau}{\pi r^2 c^2 d^2 \rho}, \quad (\text{A-9})$$

where all quantities are in cgs units except the current, which is more conveniently expressed in amperes.

Putting the following realistic set of parameters into Eq. (A-9) yields the result

$$I = 12.5 \text{ kA}$$

$$\tau = 18 \mu\text{sec}$$

$$\ell = 1 \text{ cm}$$

$$d = 1 \text{ cm}$$

$$c = 3 \times 10^{10} \text{ cm/sec}$$

$$\rho = 11 \text{ g/cm}^3$$

$$r = 5 \times 10^{-2} \text{ cm}$$

$$v = 200 \text{ cm/sec} \quad (\text{A-10})$$

This implies that in $150 \mu\text{sec}$ the sample will move about 0.3 mm , which is just detectable from the shadowgraphs and is in excellent agreement with the observed results.

There are, also, magnetic forces between the current flowing through the sample and return lead, and the currents flowing in the voltage probes. Since these forces depend upon the product of the currents, and inversely upon the square of the conductor separation distance, they are much smaller than the forces on the sample. Nevertheless, the higher currents used to heat uranium did cause some problem with maintaining probe-to-sample contact, and the probe construction was changed to make them more resistant to this force.

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APPENDIX B

The Skin Effect on Deposited Energy

Since current flow in a wire is axial, coaxial filaments of the wire must be equipotential surfaces. The impedance length of any of these filaments must be the same as for all other parallel filaments. For direct current, this condition is easily satisfied because the current density is the same in all filaments. However, if the current is time-varying, there is a reactance voltage drop in addition to the resistive voltage drop. The equipotential requirement, when applied to the time-varying currents, yields

$$\frac{\partial H}{\partial t} = \frac{\rho}{\mu} \frac{\partial i}{\partial r}, \quad (\text{B-1})$$

where H is the magnetic field intensity, ρ is the resistivity, μ is the permeability, and i is the current density. Neglecting displacement currents and making use of the fact that

$$H = \frac{I}{2\pi r}, \quad (\text{B-2})$$

where I is the total current through a circle of radius r , Eq. (B-1) can be rearranged to yield

$$\frac{\partial i}{\partial t} = \frac{\rho}{\mu} \left(\frac{1}{r} \frac{\partial i}{\partial r} + \frac{\partial^2 i}{\partial r^2} \right), \quad (\text{B-3})$$

which expresses the rate of change of current with time in terms of the radial distribution of the current density.

The solution of Eq. (B-3) is expressed in terms of a Bessel function^{B-1}:

$$i = i_0 J_0(nr), \quad (\text{B-4})$$

where i_0 is the current density at the wire center, and where:

$$n = \sqrt{\frac{-j\omega\mu}{\rho}} = \frac{-1+j}{\delta}; \quad j \equiv \sqrt{-1}. \quad (\text{B-5})$$

That is, the argument of the Bessel function is complex. It may be expressed by

$$J_0(\sqrt{-j} x) = J_0(j\sqrt{j} x) = \text{ber}(x) + j \text{bei}(x) \quad (\text{B-6})$$

and evaluated by referring to a set of mathematical tables.

If the Fourier analysis of the particular time-varying current is known, Eq. (B-4) may be used to find the applicable skin depth. Here, concern is directed more to specifying the minimum pulse length acceptable in terms of the nonuniform energy deposition. Consequently, a worst-case analysis is undertaken, using the mathematical description of a square-wave current pulse. The increase in resistivity with energy deposition is neglected. Its consideration would introduce a strong nonlinearity to the problem.

A square-wave current pulse is expressed by

$$I = \sum_{n=0}^{\infty} I_{2n+1} \sin [(2n + 1)\omega t], \quad (\text{B-7})$$

where I is expressed as uniform current density, and I_{2n+1} is the maximum value of the Fourier component. For a true square wave, the coefficient I_{2n+1} varies inversely with the order of the harmonic. For application to Eq. (B-4), this becomes

$$i_o = \sum_{n=0}^{\infty} \frac{i_m(\omega_o)}{2n+1} \sin [(2n + 1)\omega t], \quad (\text{B-8})$$

where i_o is the current density at the center of the wire and i_m is the maximum value of the lowest-frequency component at the wire center.

The energy deposited at the center of the wire by each harmonic component of the current is just proportional to the square of the rms value of the current density:

$$H_c \propto \left(\frac{0.707 i_m}{2n+1} \right)^2. \quad (\text{B-9})$$

At the surface of the wire, the current density is increased because of the skin effect. The energy deposited at the surface is proportional to the square of this increased current density:

$$H_s \propto \left[\frac{0.707 i_o J_0(nr)}{2n+1} \right]^2, \quad (\text{B-10})$$

where:

$$J_0(nr) = J_0 \left[\frac{\sqrt{-j}}{\delta} r \sqrt{2} \right] \quad (\text{B-11})$$

and:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} = \frac{\delta\omega_0}{\sqrt{2n+1}}. \quad (\text{B-12})$$

In Table B-1, values for H_c are compared with H_s computed through the ninth harmonic of a square current pulse 10 μsec wide. The results are applicable to the lead samples used in the experiment. Superposition

Table B-1. Skin-depth induced nonuniform heating.

$2n+1$	$x = \frac{\sqrt{2}r}{\delta}$	$ J_0(\sqrt{-j}x) ^2$	H_c/i_o^2	H_s/i_o^2
1	0.67	1.0074	0.500	0.5030
3	1.16	1.0630	0.055	0.0390
5	1.50	1.1590	0.020	0.0230
7	1.77	1.3310	0.010	0.0136
9	2.01	1.5110	0.006	0.0093
Totals			0.591	0.6079
Percentage nonuniformity = 2.75%				

of the deposited energy is used to compute the percentage nonuniformity. For this case, the energy deposited at the outer surface of the wire is about 3% greater than that deposited at the center. The use of longer current pulses, or of pulse shapes that reduce the quantities of the higher harmonics, will result in even less nonuniformity. Further, for materials whose resistivity depends strongly on deposited energy, the increased energy deposition at the surface will cause increased surface resistance, which will cause the current to shunt to the more central lower-resistance path.

The selection of the current pulse length is controlled between limits set by achieving maximum required energy deposition in the sample without significant skin-depth effect and within the time limit imposed by the achievement of equilibrium and the onset of instability. The selection of the magnitude of the current is determined by the range of required energy deposition, commensurate with limitations imposed by the pulse-width restrictions and the relation between sample resistivity and deposited energy. A method for estimating the energy deposited in a material for a given current magnitude and pulse duration is given in Appendix C.

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APPENDIX C

Resistive Instabilities and Effects

The process of heating materials by resistive energy deposition is subject to the development of inhomogeneity in the deposited energy because of what has been called a resistive instability. The resistive instability results from the positive feedback effect of dependence of the resistivity on temperature and, consequently, deposited energy. Mathematically, it is described by Eq. (C-1):

$$\Delta H(t) = \int I^2(t) R[\Delta H(t)] dt, \quad (C-1)$$

where:

$$R[\Delta H(t)] = \frac{\rho_o l}{A} f[\Delta H(t)]; f[\Delta H(t)] \geq 1. \quad (C-2)$$

If the resistance of the material is dependent upon the deposited energy in a positive manner, then, for constant current, the solution of Eq. (C-1) tends rapidly toward infinity as time progresses. The fundamental physics of the electrical conduction phenomena dictates that, except over small temperature intervals, the material resistivity will increase with increasing energy. Electrical conduction depends on the long-range, relatively easy movement of the conduction electrons in the material. Increased energy deposition increases the vibration amplitude, or gross motion of the atoms, thereby increasing the probability of collision between conduction electrons and material atoms. This effectively shortens the mean free path of the conduction electrons, resulting in increased resistance to current flow. Consequently, to some extent, all materials are subject to this resistive instability.

Under normal experimental conditions, the resistive instability is more to be blessed than cursed. Providing that the resistivity of the material to be heated is uniform, in a microscopic sense, and that a relatively uniform current density is used, the instability merely makes the material easier to heat but does not affect the uniformity of the energy deposition nor make the process particularly difficult to control. That is to say, if the instability is uniformly applicable to the whole sample, no adverse effects are noted. It must, however, be kept in mind that the process is basically

unstable, and the slightest inhomogeneity may grow so rapidly as to result in unacceptable uniformity of energy deposition.

Two processes work against the onset of violent instability. If they did not, resistive heating would probably be completely unsuitable as a heating technique. The first of these is the thermodynamic tendency to achieve temperature equilibrium through conduction processes. Thus, if the heating rate is sufficiently slow, heat conduction will have ample time to prevent the occurrence of significantly hotter regions in the material. The second process is electrical and due to the requirement that parallel resistance elements carry current inversely proportional to their resistance. Therefore, if an element in the current path becomes of higher resistance than its neighbor, the current distribution will change in a manner tending to automatically bring their resistances back into equilibrium. These processes never completely eliminate the resistive instability; they slow the rate of growth. If the growth rate of the instability is much slower than the heating rate, effects of these instabilities will not be observed.

Experience indicates that heating rates applicable to the experimental technique may be marginal with respect to growth rates of resistively unstable regions that are initially of the order of 1% of the sample diameter in size. However, subtle factors such as the dimension of the unstable region parallel to the current direction, the geometric location within the sample, and the number of such unstable regions in the sample, may all affect the growth rate and other observable effects (see Appendix E). In any case, this instability must be considered in any resistive-energy deposition process.

If the resistance of a material is known to vary linearly with the deposited energy, or may be assumed linear over ranges of energy, then an equation can be derived to estimate the energy deposited by a given current magnitude and pulse length. Assuming a constant current magnitude, the rate of energy deposition is given from Eq. (C-1) by

$$\frac{dH}{dt} = \frac{I^2 R(H)}{4.18 D \pi r^2 l}, \quad (C-3)$$

where H is the enthalpy change in the sample (cal/g), t is the time (seconds), I is the magnitude of the current (A), r is the radius of the sample (cm), l

is the sample length (cm), and D is the initial density of the material (g/cm^3). The resistance $R(H)$ is expressed by

$$R(H) = \frac{\rho_0 \ell}{\pi r^2} (1 + bH) \text{ ohms}, \quad (\text{C-4})$$

where ρ_0 is the initial resistivity and b is a constant (g/cal). Differentiating Eq. (C-3) with respect to time gives

$$\frac{d^2 H}{dt^2} = \frac{I^2 \rho_0 b}{4.18 \pi^2 r^4 D} \frac{dH}{dt} = C_1 \frac{dH}{dt}, \quad (\text{C-5})$$

which has the solution

$$H(t_2) - H(t_1) = \frac{1}{b_{1,2}} \exp \left[\frac{I^2 b_{1,2} \rho(t_1)(t_2 - t_1)}{4.18 \pi^2 r^4 D} \right]. \quad (\text{C-6})$$

In application, it may be more convenient to assume a current and the enthalpy desired, and to estimate the pulse length required. This may be done using

$$\tau = \frac{4.18 \pi^2 r^4 D}{I^2 \rho b} \ln(b \Delta H). \quad (\text{C-7})$$

APPENDIX D

Thermal and Volumetric Equilibration Effects

The utility of this method of measuring accurate values of the thermodynamic properties of metals at high temperatures is crucially dependent on the ability to achieve and maintain a state of complete thermodynamic equilibrium while the necessary measurement is performed. The process by which the desired state is reached is described by $V = f(P, H)$, where V is the volume, P is the pressure, and H is the enthalpy. Neglecting the magnetic-pressure component, the pressure is constant during the experiment, and the high-temperature state is reached by increasing the enthalpy approximately isobarically. The path deviates from the isobar by the pressure gradient required to drive the expansion.

The expanding material acquires momentum during the transition, which will cause it to overshoot the equilibrium state defined by the final enthalpy and the hydrostatic pressure. This overshoot causes the pressure gradient to be reversed, stopping the expansion and returning the volume toward the equilibrium value. Some oscillation about the equilibrium volume may occur, but it will be damped out as the energy stored in collective motion of the atoms is dispersed by the random collisions that occur. The probability of successfully meeting the objective of the experiment is increased by minimizing the deviation from equilibrium during transition, consistent with the requirement that instabilities resulting from external forces not affect the sample.

The measurement of the volume of the high-temperature state presumes the existence of cylindrical symmetry of the expanded wire. To achieve such a condition requires that the wire be uniformly heated. This may be accomplished either by uniform deposition of the energy or by thermal equilibration of the material following the energy deposition. Thermal equilibration requires heat transfer between various parts of the sample by conduction. Data are not available on the conductivity of metals at these very high temperatures, but estimates can be made. The equation for one-dimensional conduction of heat is

$$q_k = -k A \frac{dT}{dx}, \quad (D-1)$$

where q_k is heat-flow rate, k is the thermal conductivity, A is the cross-sectional area, and dT/dx is the temperature gradient.^{D-1} At 300°K the value of the thermal conductivity for lead is about 0.004 cal/sec-cm-deg.^{D-2} An estimate of the thermal conductivity at high temperature may be obtained by application of the Wiedemann-Franz law, $k = L\sigma T$, where L is the Lorentz number (2.45×10^{-8} W Ω /deg²), σ is the electrical conductivity and T is the temperature. For lead, at 6000°K, the decrease in electrical conductivity is a factor of 15, while the increase in temperature is a factor of 20. Thus, the thermal conductivity might be expected to increase perhaps by $1.33 = 20/15$. Using this with Eq. (D-1), an expression for the thermal relaxation time τ_k can be estimated:

$$\tau_k = \frac{c_p \rho}{3k} \frac{r^2(r_1 - r)}{r_1}, \quad (D-2)$$

where a spherical region of radius r is assumed to lose heat through a shell of thickness $r_1 - r$. A realistic value to assume for the shell thickness is $r_1 = 1.5 r$, since the equation for τ_k does not account for the additional temperature gradients with time either in the hot sphere or in the cooler heat-sink region.

For lead, Eq. (D-2) becomes

$$\tau_k = \frac{25.7}{3} r^2 \text{ sec}, \quad (D-3)$$

which predicts that for a sphere of radius 10^{-3} cm, the time required to lose $1/e$ of its excess temperature is about 8 μ sec. At least 20 μ sec would be required to achieve a 90% equilibration. This result demonstrates that uniform energy deposition on a scale of microns (10^{-4} cm) is critical to the achievement of the equilibrium state.

At ambient temperature, the gas pressure in the cell is raised to the desired operating value. The internal pressure of the sample increases to the same value, accompanied by a slight decrease in volume. As energy is added to the sample, the equilibrium is disturbed. The internal pressure increases to produce expansion required to correct the equilibrium. Pressure waves travel into the sample from the surface to relieve the internal pressure and move the atoms farther apart, increasing the volume.

The velocity of these pressure waves will be the velocity of sound in the material, typically of the order of 1-5 mm/ μ sec. The sample size chosen for the experiment is 1 mm in diameter. Therefore, these pressure waves transit the sample, from surface to axis in 0.5 μ sec. The disturbance of equilibrium is minimized during the state transition if the rate of energy deposition is kept small with respect to this pressure-wave transit time.

The rate of energy deposition for the experiment is not constant, because of variation in the magnitude of the current flow and the increase in resistance with deposited energy. For the current pulse used with lead, the rate of energy deposition increased quadratically through the melt transition, then more slowly and nearly linearly to current turnoff, at which point it decreased rapidly to zero. The maximum rate of energy deposition was typically of the order of 10 cal/g- μ sec. Assuming that, on the average, the expansion lags by the pressure-wave transit time, and ignoring the inertia of the material, the disturbance from equilibrium is never greater than 5%. However, inertia does play a significant role, and its effect may be estimated.

Considering only radial expansion, an element of volume at r contains a mass $m = \rho A dr$. The force that is to move this element of mass is $F = (dP/dr) A dr$. If the volume is to expand by V/V_0 , then the mass must be moved to $r(V/V_0)^{\frac{1}{2}}$.

Now, assume that the acceleration of this mass is constant with respect to time. Then

$$a = \frac{2}{\tau^2} \left(\sqrt{\frac{V}{V_0}} - 1 \right) r \quad (D-4)$$

and

$$\frac{dP}{dr} = \frac{2\rho}{\tau^2} \left(\sqrt{\frac{V}{V_0}} - 1 \right) r, \quad (D-5)$$

from which the pressure in the cylinder, as a function of radius, is

$$P(r) - P(0) = \frac{\rho}{\tau^2} \left(\sqrt{\frac{V}{V_0}} - 1 \right) r^2. \quad (D-6)$$

The maximum overpressure predicted by Eq. (D-6), for an assumed four-fold volumetric expansion in 20 μ sec from an initial radius of 5×10^{-2} cm, is

$$\Delta P|_{\max} = 1.37 \times 10^8 \frac{\text{dynes}}{\text{cm}^2} = 137 \text{ bars.} \quad (\text{D-7})$$

Obviously, the assumption of constant acceleration is not consistent with the experimental conditions, but to handle the problem more realistically requires the assumption of an equation-of-state and the solution of coupled equations—far beyond the capability of hand calculations. This is undoubtedly an overestimate of the pressure, because the fact that the radius varies with time was ignored in performing the integration.

A similar mechanism, involving negative pressure deviation in the sample with respect to the ambient gas pressure, serves to stop the expansion after energy deposition ceases. In fact, the momentum in the sample from the motion during expansion will cause it to overshoot the equilibrium volume. It is this overshoot that causes the negative pressure gradient necessary to stop the expansion and return the sample toward the equilibrium radius. If there were no damping of the motion, the sample would continue to oscillate around the point of thermodynamic equilibrium. But ultimately the energy of collective motion of the atoms will be randomized by the interatomic collision processes, and collective motion will cease.

The energy deposition as a function of time for a lead experiment was used with a hydrodynamic computer program (KO) at the Lawrence Radiation Laboratory. This code contained no provision for viscous damping, so the predicted volume oscillated about its equilibrium value, as expected. The predicted half-period of this oscillation was about 3 μ sec. Allowing for some reduction in the sound speed in the liquid sample, an estimate of the time after heating required to achieve a stable equilibrium volume is 10-15 μ sec. Experimentally, the work with lead quite often required as much as 50-60 μ sec to reach the equilibrium volume. No overshoot was observed experimentally. However, these late-time expansion dynamics may well be attributable to the nonuniform energy distribution due to impurities, rather than to a severely overdamped expansion. Uranium studies have not been sufficiently extensive to yield data on this effect.

References

- D-1. F. Kreith, Principles of Heat Transfer (International Textbook Co., Scranton, 1958), pp. 1-324.
- D-2. C. Kittel, Introduction to Solid State Physics, 3rd ed. (John Wiley & Sons, Inc., New York, 1966), pp. 251-299.

APPENDIX E

Effect of Local, Resistive Inhomogeneities
in Resistively Heated Wires

Several problems which arose during the application of the experimental technique to lead have been studied in detail. Ultimately, the cause of these problems appeared most likely to be the presence of significant quantities of lead-oxide distributed throughout the sample material. Lead-oxide is expected to be relatively poor conductor of electricity compared to lead, since the molecular bond with the oxygen effectively localizes those electrons that would otherwise be free to contribute to the conductivity. The results of compositional analyses of various samples of the material used in the experiment are given in Table E-1. The variation in the oxide content between lead pieces of size comparable to experimental samples appears to be as great as a factor of from 10 to 100. Analytic work to examine, so far as possible, the effects of oxide concentration in resistively heated wires was performed.

The gross effects are easily visualized. If all the oxide were accumulated at one point in the wire, it is certain that the increased current density through the decreased cross section of pure material would result in locally increased energy deposition. In a material, such as lead, where the resistance depends heavily on the temperature, inhomogeneities in deposited energy result in inhomogeneities in temperature, which further increases the resistance, causing the development of an instability.

On the other hand, if the oxide were uniformly distributed throughout the material on a scale such that significant thermal gradients could not arise, the expected effect would be only an increase in the apparent material resistivity, relative to the resistivity of pure material. This would result in increased deposited specific energy from a given current pulse. The expansion of the material should be uniform, and in agreement with that expected for the measured specific energy. However, there exists the possibility that at high temperatures, dissociation of the oxide may occur, complicating the expansion dynamics.

The probability of dissociation cannot be accurately calculated because of the uncertainty in the temperature of the oxide. However, estimates indicate that, above a material temperature of about 3000°K, the

Table E-1. Oxygen analysis of various lead samples.

Oxygen (ppm)	Percent lead-oxide ^a	Notes
7900 ± 600	11.0 ± 0.8	1
5700 ± 400	7.9 ± 0.6	
5200 ± 400	6.8 ± 0.6	
4000 ± 300	4.9 ± 0.4	2
3700 ± 300	5.0 ± 0.4	
720 ± 70	0.95 ± 0.1	3
250 ± 30	0.33 ± 0.03	
560 ± 60	0.74 ± 0.07	
61 ± 6	0.085 ± 0.008	4
66 ± 7	0.092 ± 0.009	
6 ± —	—	5

NOTES

1. Neutron-activation analysis by B. T. Kenna, Sandia Laboratories, August 1970. (1-10-g samples.) Lead contained visible inclusions of material identified as PbO by electron probe.
2. Neutron-activation analysis by B. T. Kenna, Sandia Laboratories, August 1970. (1-10-g samples.) Zone-refined lead purchased July 1970.
3. Neutron-activation analysis by B. T. Kenna, Sandia Laboratories, December 1970. (About 1 g extruded wire samples.) Zone-refined lead purchased July 1970.
4. Gamma-activation analysis by J. Brownlee, Lawrence Radiation Laboratory, February 1971. (About 1 g extruded wire.) Zone-refined lead purchased July 1970.
5. Vacuum fusion analysis by J. Fischer, Lawrence Radiation Laboratory, January 1969. Initial purity analysis indicated 99.994% pure lead. Several samples were analyzed by vacuum fusion in August through November 1970. The results were not officially reported and are not included here. None of these analyses showed more than 70 ppm oxygen.

^aConversion assumes all oxygen is compound PbO.

oxide may dissociate, leaving free oxygen dispersed throughout the material. Considerable uncertainty exists regarding the effect of this free oxygen on the expansion dynamics of the sample. If it is excessively compressed in the volume in which it is released, then sample expansion in excess of that normally expected would be observed. However, if the oxygen is able to exist interstitially in equilibrium with the lead, then perhaps less overexpansion would result.

An examination of the more subtle effect of oxide inclusions in the sample material becomes somewhat difficult because of the nonlinearity introduced into the energy deposition mathematics. In a material that exhibits an increase in resistance with temperature, some nonlinearity is always present in the calculation of deposited energy, but the presence of a nonconducting inclusion causes an additional localized nonlinearity to be associated with the geometry. The energy deposited in a microscopic volume is given by the integral of the square of the current density through the volume times the local resistivity, over the time duration of the current flow. If the resistivity depends on temperature and temperature depends on deposited energy, then the resistivity becomes a function of deposited energy. This yields an equation in which the integrand depends upon the value of the integral. For the solution of such an equation, numerical techniques are required.

To evaluate the marginal effects of insulating inclusions in the resistively heated sample, a model was selected which could reasonably be handled by a computer program. The model assumes a spherical, nonconducting region in a volume of material that is otherwise completely homogeneous. To determine the initial current-density distribution, an analog is drawn to the equipotential lines around a conducting metal sphere in a uniform electric field.^{E-1} This gives a distribution defined by Eq. (E-1):

$$J = J_0 \left[1 + \frac{a^3}{r^3} (3 \cos^2 \theta - 1) \right]; \quad \begin{matrix} r \geq a \\ 0 \leq \theta \leq \pi/2 \end{matrix} \quad (E-1)$$

To limit the geometric complexity, maximum use of the symmetry involved was made. The problem was configured as shown in Fig. E-1 by passing a plane through the center of the sphere and considering only the first quadrant of this plane. Symmetry is evident in reflection through the X axis, reflection through the Y axis, and rotation about the Y axis. The dimensions of the section are proportional to the radius of the excluded circular segment corresponding to a given volumetric proportion of insulator studied.

The computer program contains a section that uses experimentally determined equation-of-state data to estimate the expansion that occurs as the sample is heated. Because of this, the name selected for the program was EXPAND. Also, the program accounts for the redistribution of current density in a semi-realistic manner. This results in as accurate an analysis

Region (b, b) is divided into $(MS)^2$ cells.

Average properties of Region (b, b) are representative of measured values for nonuniform energy distribution.

Fig. E-1. Geometry applicable to EXPAND.

of the true situation as can be reasonably made, without resorting to statistical techniques.

The initial current-density distribution is squared for each cell and used to compute the energy deposited by multiplication by the cell resistivity and the time interval. A linear function of the resistivity of the material with enthalpy is then used to predict the new value of cell resistivity. This function is derived from experimental results. Next, a two-slope linear relation between enthalpy and volume is used to estimate the expansion of the cell due to the energy deposited. This function is also derived from experimental data. Finally, before the program repeats the above calculations for the subsequent time interval, the current density is modified. This accounts in a semirealistic manner for the diversion of current flow away from high-resistance regions. The new current density is made inversely proportional to the new resistance of the cell in such a manner that the total cross-sectional current is conserved. This is consistent with the true case in which the sample resistance does not affect the total magnitude of current flowing through it.

The details of EXPAND are given in the following paragraphs. This is followed by a description of the computation results obtained.

```

PROGRAM EXPAND
COMMON/A1/X(100),CR(100,100),CSQ(100,100)R(100,100),H(100,100),
1XP(100,100),BOUND(100),HUN(100),VUN(100),VAV(100),HAV(100),
2RUN(100),RAV(100),T(100),CONI(100),DLI(100)
RIT 2,100,DLT,TP,A,B,CDNO,ROH,DEN,MS
100 FORMAT(7E10.3,I5)
WOT 3, 200,DLT,TP,A,B,CDNO,ROH,DEN,MS
200 FORMAT(1H1/23X, 74HPROGRAM EXPAND APPROX DYNAMICS OF RESIST HEATED
1 WIRE CONTAINING INCLUSIONS //5X,19HTIME INTERVAL (SEC) E12.4,24X
2 18HPULSE LENGTH (SEC) E12.4 //5X,24HRAIUS OF INCLUSION (CM) E12.
34,19X,19HSYMETRY LENGTH (CM) E12.4 //5X,31HUNIFORM CURRENT DENSITY
4 (A/CM2) E12.4,12X28HINITIAL RESISTIVITY (OHM-CM) E12.4 //5X14HDEV
5SITY (G/CC) E12.4,29X,16HMATRIX DIMENSION I5,)
C SET UP TABLE OF COORDINATES

```

The above statements initiate the program and read the required input data. A user may format the output variables as fits his requirements.

The variables DLT and TP are the time interval and total current-pulse length considered, respectively. The variables A and B are the radius of inclusion and the side length of the square area considered,

respectively. They are interrelated and related to the percentage impurity by volume, by

$$B = \left(\frac{\pi A^3}{6W} \right)^{1/3}, \quad (E-2)$$

where W is the volumetric percentage of insulator. CDNO is the uniform current density, found by dividing the desired current by the cross-sectional area of a typical sample. No time variation of the current is considered by this program; consequently, no inductive effects need be considered by this program. ROH is the initial resistivity of the material, and DEN is the normal material density. The variable MS is the number of cells along either the X or Y axis to be considered by the program.

The following statements set up the required table of coordinates of these cells in the X - Y space:

```
DLTA=B/MS
X(1)=DLTA/2.
DO 10 I = 1,MS-1
10 X(I+1) = X(I) + DLTA
```

Next, the initial current density distribution is calculated. Note that current is excluded from the circular nonconducting area.

```
DD 11 I = 1,MS
DD 11 J = 1,MS
AR = SQRT(X(I)**2 + X(J)**2)
IF (AR - A)15,16,16
15 CR(I,J) = 0.
CSQ(I,J) = 0.
GO TO 11
16 COST = (X(I)/AR)**2
CR(I,J) = CDNO*(1.+(A**3/AR**3)*(3.*COST - 1.))
CSQ(I,J) = CR(I,J)**2
11 CONTINUE
HMS = MS/2
HNS=HMS+1
WDT 3,205
205 FORMAT(1H1,5X77HTHE FOLLOWING TABLE GIVES THE CURRENT DENSITY AT C
ENTER OF EVERY FIFTH SQUARE )
WDT 3,210,(X(MS-3-J),(CR(I,MS-3-J),I=1,HMS,5),J=1,MS,5)
210 FORMAT(///(11E10.2))
WDT 3,210,(X(MS-3-J),(CR(I,MS-3-J),I=HNS,MS,5),J=1,MS,5)
WDT 3,215
215 FORMAT(1H1,5X44HTABLE OF CURRENT DENSITY SQUARED OVER MATRIX )
WDT 3,210,(X(MS+1-J),(CSQ(I,MS+1-J),I=1,HMS,5),J=1,MS,5)
WDT 3,210,(X(MS+1-J),(CSQ(I,MS+1-J),I=HNS,MS,5),J=1,MS,5)
```

The variable CSQ designates the square of the current density.

The following statements integrate the current density over planes parallel to the X axis as a check on the conservation of current:

```

      AM = 2*3.1416*DLTA*DLTA
      DO 70 J = 1,MS
      CONI(J) = 0.
      DO 71 I = 1,MS
      CONI(J) = CONI(J) + (I-0.5)*CR(I,J)
70  CONTINUE
      CONI(J) = AM * CONI(J)
70  CONTINUE
      WOT 3,245
      WOT 3,250,(CONI(J),J=1,MS)
245  FORMAT (1H1,5X32HCHECK ON CONSERVATION OF CURRENT //)
250  FORMAT(5E20.4)

```

Next, the initial resistivity distribution is established. The resistivity of each cell whose center lies outside the radius of the inclusion is assumed to have a value equal to the initial resistivity of the material.

```

      DO 12 I = 1,MS
      DO 12 J = 1,MS
12  R(I,J) = RDH

```

Then, the time is established and various constants used inside the computation loop are calculated. A decision is made regarding whether the current-density distribution is to be readjusted on the basis of resistance, or, as is true of the first time step, the program is to proceed directly to the calculation of the deposited energy and increased resistivity.

```

      TIME = 0.
      TRIT = 10.* DLT
      K = 0
      CON = DLT/(DEN*4.18)
      CONS = 9./80.
      AV = MS*MS/2.
20  TIME = TIME + DLT
      IF(K)60,60,61

```

After the first time step, the program will return to Statement 20, with K positive. Control will then be transferred to the following statements, which readjust the current density by making the change in current density inversely proportional to the cell resistivity, such that the cross-sectional current is conserved. Finally, the new current-density distribution and the distribution of the square of the current density are computed.

```

61 DO 62 J = 1,MS
   N = 0
   ROAV = 0.
   N1 = 0
   DLIS = 0.
   DO 63 I = 1,MS
     AR = SQRTF(X(I)**2 + X(J)**2)
     IF (AR - A)63,65,65
65   ROAV = ROAV + R(I,J)
     N = N + 1
63 CONTINUE
   ROAV = ROAV/N
   DO 64 I = MS+1-N,MS
     DLI(I) = CR(I,J)*((ROAV/R(I,J))-1.)
     DLIS = DLIS + DLI(I)*(I-0.5)
     IF (DLI(I))64,67,64
67   N1 = N1 + 1
64 CONTINUE
   N2 = N - N1
75 CONTINUE
   DIFF = 0.
   N3 = 0
   DO 69 I = MS+1-N,MS
     IF(DLI(I))68,69,68
68   COR = DLIS/(N2*(I-0.5))
     IF(CR(I,J)+DLI(I)-COR)74,74,69
74   DIFF = DIFF + COR
     N3 = N3 + 1
     CR(I,J) = CR(I,J) + DLI(I)
     CSQ(I,J) = CR(I,J)*CR(I,J)
     DLI(I) = 0.
69 CONTINUE
   IF (N3)76,76,77
77   DLIS = DLIS + DIFF
     N2 = N2 - N3
     GO TO 75
76 DO 78 I = MS+1-N,MS
     IF(DLI(I))79,78,79
79   CR(I,J) = CR(I,J) + DLI(I) - DLIS/(N2*(I-0.5))
     CSQ(I,J) = CR(I,J) * CR(I,J)
78 CONTINUE
62 CONTINUE

```

Using the current-density distribution appropriate for each time step, the following statements compute the deposited energy and the resulting resistivity of each cell. Note that also the average values of the enthalpy and resistivity are stored. These are proportional to the values that would be measured experimentally. The cell farthest from the center of the inclusion has properties nearly equal to those that would be calculated if no current-density distortion were present. Computed values for this cell represent the case of uniform energy deposition.

```

60 CONTINUE
  K = K + 1
  DO 25 I = 1,MS
    DO 25 J = 1,MS
      RAV(K) = RAV(K) + R(I,J)
      H(I,J) = CSQ(I,J)*R(I,J)*CON + H(I,J)
      HAV(K) = HAV(K)+H(I,J)
      R(I,J) = ROM*(1.+CONS*R(I,J))
25 CONTINUE

```

The next step is the calculation of the volumetric expansion commensurate with the deposited energy. A two-slope, straight-line fit to experimental data relating the deposited energy to the volume is used to compute the new volume required for each cell. Cylindrical symmetry is assumed. The calculation begins at $X=0$ for each plane parallel to the X axis and proceeds outward, finally arriving at a new boundary for the sample at that particular value of the Y coordinate. The Y axis is assumed to be stationary throughout this process.

```

  AN = 2.*DLTA
  DO 30 J = 1,MS
    XPO = 0.
    DO 35 I = 1,MS
      IF (H(I,J)-150.)31,31,32
31 VRT = 1.+(H(I,J)/150.)
      GO TO 33
32 VRT = ((4.*H(I,J))/50.)-10.
33 XP(I,J) = SQRT((XPO**2) + AN*VRT*X(I))
      XPO = XP(I,J)
35 CONTINUE
    BOUND(J) = XPO
    VAV(K) = VAV(K) + BOUND(J)
30 CONTINUE

```

Next, the values computed for the uniform-deposition case and the average values computed for the distorted case are filed for later output. Then the flag TRIT is checked to determine if the output is desired at this time step. If not, control returns to Statement 20 and a new time step is considered.

```

  T(K) = TIME
  HUN(K) = H(MS,MS)
  RUN(K) = R(MS,MS)
  VUN(K) = (BOUND(MS)/B)**2
  HAV(K) = HAV(K)/(MS*MS)
  RAV(K) = RAV(K)/(MS*MS)

```

```

VAV(K) = (VAV(K)/(MS*B))*2
IF (TIME-TP)40,21,21
40 IF (TIME-TRIT)20,41,41
41 TRIT = TRIT + 10.*DLT
21 CONTINUE

```

The inclusion of any desired output statements at this point in the program results in their being applied every tenth time step. The selection of these statements may be modified by the user.

```

WOT 3,205
WOT 3,210,(X(MS-3-J),(CR(I,MS-3-J),I=1,HMS,5),J=1,MS,5)
WOT 3,210,(X(MS-3-J),(CR(I,MS-3-J),I=HNS,MS,5),J=1,MS,5)
WOT 3,215
WOT 3,210,(X(MS+1-J),(CSQ(I,MS+1-J),I=1,HMS,5),J=1,MS,5)

WOT 3,210,(X(MS+1-J),(CSQ(I,MS+1-J),I=HNS,MS,5),J=1,MS,5)
DO 72 J = 1,MS
CONI(J) = 0.
DO 73 I = 1,MS
CONI(J) = CONI(J) + (I-0.5)*CR(I,J)
73 CONTINUE
CONI(J) = AM * CONI(J)
72 CONTINUE
WOT 3,245
WOT 3,250,(CONI(J),J=1,MS)
WOT 3,220
220 FORMAT(1H1,5X27HFINAL ENTHALPY DISTRIBUTION )
WOT 3,210,(X(MS-3-J),(H(I,MS-3-J),I=1,HMS,5),J=1,MS,5)
WOT 3,210,(X(MS-3-J),(H(I,MS-3-J),I=HNS,MS,5),J=1,MS,5)
WOT 3,225
225 FORMAT(1H1,5X30HFINAL RESISTIVITY DISTRIBUTION )
WOT 3,210,(X(MS-3-J),(R(I,MS-3-J),I=1,HMS,5),J=1,MS,5)
WOT 3,210,(X(MS-3-J),(R(I,MS-3-J),I=HNS,MS,5),J=1,MS,5)
WOT 3,230,(T(L),HUN(L),HAV(L),RUN(L),RAV(L),VUN(L),VAV(L),L=1,100)
230 FORMAT(1H1,7X4HTIME,19X8HENTHALPY,25X11HRESISTIVITY,21X,15HRELATIV
IE VOLUME,72X7HUNIFORM,10X7HAVERAGE,13X7HUNIFORM,10X7HAVERAGE,10X7
2HUNIFORM,10X7HAVERAGE //(7F17.4))
WOT 3,235
235 FORMAT(1H1,5X29HFINAL COORDINATE DISTRIBUTION )
WOT 3,210,(X(MS-3-J),(XP(I,MS-3-J),I=1,HMS,5),J=1,MS,5)
WOT 3,210,(X(MS-3-J),(XP(I,MS-3-J),I=HNS,MS,5),J=1,MS,5)
WOT 3,240,(X(I),X(I+1),BOUND(I),BOUND(I+1),I=1,MS,2)
240 FORMAT(1H1,5X24HINITIAL CELL COORDINATES,42X26HFINAL BOUNDARY COO
RDINATES //(5X,2E20.4,20X,2E20.4))

```

Finally, a check is made to see if the total pulse length requested has been satisfied. If no, control returns to Statement 20 for consideration of more energy deposition. If yes, the program is terminated.

IF (TIME-TP)20,43,43
43 CONTINUE
CALL EXIT
END

The results obtained from EXPAND require careful interpretation. First, they are undoubtedly worst-case predictions. The assumption is made that the predicted average values of enthalpy, resistivity, and expansion are the values that would be measured experimentally from a macroscopic sample made up of many uniformly distributed microscopic regions of the type used in the program. This ignores the fact that a geometric consideration like this excludes about 9% of the macroscopic volume. Several studies were made, assuming the resistive inclusions were of radius equal to 10^{-4} cm. When they were assumed to occupy 1% of the sample volume, the deviation between the average enthalpy-volume data predicted for the nonuniform energy distribution and the enthalpy-volume data predicted for the uniform energy distribution became significant at very low values of the energy. When the volumetric fraction of resistive material was reduced to 0.5%, the deviation between the average and uniform values decreased considerably. These results are illustrated in Fig. E-2.

Comparative data at higher energy values for the two cases are not available, but it is noted that the deviation increases at an increasing rate as the energy is increased for either case. It is concluded from these studies that the presence in the lead of anything like the 1000-ppm oxygen values observed by neutron-activation analysis will cause significant scatter in the experimental data, unless the oxide particles are smaller than 1μ (10^{-4} cm) and very homogeneously dispersed.

Reference

- E-1. J. D. Jackson, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962), pp. 132-139.

Fig. E-2. Compared predictions by EXPAND.

APPENDIX F

Heat Loss and Thermal Stability

One of the important factors related to an accurate knowledge of the enthalpy of the sample in the high-temperature state is determination of the heat lost to the surroundings of the sample during and following the heating process. Assuming that uniform energy deposition is achieved and measured, consideration is then directed to the various heat-loss mechanisms and the resultant thermal stability of the high-temperature sample.

Heat may be transferred from the sample by conduction through the contact with the current leads, by conduction through the contact with the voltage probes, by conduction to the high-pressure helium, by convection in the helium, and by radiation to the pressure-cell walls. Since reasonably good electrical contact must be maintained between the current leads and the sample, it is likely that good thermal contact exists there, also. However, the electrical resistance of this contact (1-10 m Ω) results in some energy being deposited there, helping to offset the sample heat otherwise transferred across the boundary. Uncertainty and geometrical complexity of the contact area make an accurate calculation of the magnitude of heat transferred across this boundary impossible. Since only the material located between the voltage probes is considered in determining the equilibrium properties of the high-temperature state, it is necessary to estimate only if heat loss to the current leads can affect portions of the sample removed from the contact by at least 1 mm.

The propagation rate of a temperature disturbance due to heat conduction through the end of a hot sample may be estimated. Assume the geometry is that of a hot cylindrical bar whose end is placed in contact with a heat sink at zero time. The rate that heat is conducted through the interface depends upon the temperature gradient in the bar:

$$q_k = -k A \frac{dT}{dx}, \quad (F-1)$$

where q_k is the heat-flow rate (cal/sec), k is the thermal conductivity, A is the cross-sectional area, and dT/dx is the temperature gradient. The thermal energy in the bar segment between the contact and the point x , considered to be the maximum extension of the thermal gradient into the bar, is

$$A = c_p \rho A x T, \quad (F-2)$$

where c_p is the specific heat (cal/g-deg) and ρ is the density (g/cm³). The rate of change with time of Q is just the rate at which heat is removed from the bar by conduction:

$$c_p \rho A x \frac{dT}{dt} = -k A \frac{dT}{dx}. \quad (F-3)$$

Rearranging Eq. (F-3) slightly yields an expression for the velocity of the temperature disturbance propagating into the bar:

$$x \frac{dT}{dt} \frac{dx}{dT} = x \frac{dx}{dt} = \frac{k}{c_p \rho}. \quad (F-4)$$

where the sign change accounts for the fact that the velocity is opposite to the direction of heat flow. (Note that this velocity is not dependent on the temperature.)

The solution of this equation results in a formula for computing the distance the temperature disturbance has propagated in time t :

$$x = [(2 k t) / c_p \rho]^{1/2}. \quad (F-5)$$

A time of 200 μ sec well covers the total experiment time and yields, from Eq. (F-5), the result that no end effect will be observed more than 0.022 mm from the plane of the sample contact.

The electrical contact resistance of the voltage probe needs only to be small in comparison with the 50- Ω line termination. This allows the use of probes with very small contact area, thereby minimizing thermal effect on the heated sample. The small contact area of the probes is also required to prevent perturbation of the expansion of the sample material. Voltage probes are constructed from 0.125-mm sheet metal, with a point sharpened on emery paper such that the cross section of a typical probe will not exceed 2×10^{-4} cm² area. The heat lost through such a small area will be an exceedingly small amount of the total heat available, and completely negligible in comparison. The envelopment of the tip of the probes by the expanding liquid likewise involves small contact area. Further, the previous argument regarding disturbance-propagation velocities may be applied to demonstrate that negligible heat loss will occur. Experimentally, it is observed that copper probes are not damaged significantly by contact with the lead at temperatures exceeding 5000°K.

Heat lost from the sample to the gas by mechanisms of conduction and convection presents a difficult analysis. No data are available on the expected contact coefficients applicable at these temperatures, nor is data available regarding thermal conductivity or convection in the helium at such high pressures. It is not even certain that a worst-case estimate can be made using available data, because the number density of the high-pressure helium and the hot lead are comparable, presumably increasing the conductive heat-transfer coefficient considerably in the gas and across the interface. Convective processes in the gas at this pressure, and over times of interest to the experiment, can probably be assumed negligible because the molecular mean free path is estimated to be of the order of one molecular diameter.

A very rough estimate, designed to be a considerable overestimate, can be made. Taking the propagation distance for the temperature disturbance calculated by Eq. (F-5), and assuming that half of this distance will be in the gas, allows calculation of the temperature gradient in the outer surface of the sample:

$$\frac{dT}{dr} = \frac{5700}{2 \times 3 \times 10^{-3}} = 9.5 \times 10^5 \frac{\text{deg}}{\text{cm}}. \quad (\text{F-6})$$

This should be divided by two because the gradient is not linear but is steepest near the interface. It should be again divided by two because the time required to propagate the temperature disturbance to the given depth should be averaged. Including this factor of 4 with the heat-rate equation for a cylindrical shell, and multiplying by the 200- μ sec time of interest, yields

$$q_k t = k(2\pi r l) \frac{dT}{dx} t = 0.0373 \text{ cal}. \quad (\text{F-7})$$

The energy deposited in the sample to reach the 6000°K that is assumed is about 10 cal. Thus, this estimate predicts that less than half of 1% of that energy will be lost through the surface to the gas. This is probably still an overestimate by as much as a factor of 10, so the heat loss is presumed negligible.

The heat lost by radiation must be considered.^{F-1, F-2} A worst-case analysis is assured by assuming unity emission and absorption coefficients and applying the Stefan-Boltzmann law:

$$q_r = 5.67 \times 10^{-5} (A_1 T_1^4 - A_2 T_2^4) \text{ ergs/sec}. \quad (\text{F-8})$$

Assuming a sample 0.1 cm in diameter by 1 cm long, radiating at 6000°K to walls of a cylinder 2.5 cm in diameter by 1 cm long, absorbing at a temperature of 300°K and neglecting the variation of temperature of either surface with time, gives

$$q_r = 5.52 \times 10^2 \text{ cal/sec.} \quad (\text{F-9})$$

For the same 200- μ sec time of interest used before, this gives 0.11 cal of heat lost by radiation, or about 1% of the total sample energy.

The conclusion drawn from these preceding analyses is that the pre-dominant thermal energy loss mechanism is that of radiation. Since the 1% loss of energy by radiation is well within the accuracy with which the total energy can be measured, it is considered negligible, and no correction is applied to the data. Other materials that may require the examination of states of considerably higher temperature may require a correction to the enthalpy assigned to the equilibrium state.

References

- F-1. F. Kreith, Principles of Heat Transfer (International Textbook Co., Scranton, 1958), pp. 1-324.
- F-2. C. Kittel, Introduction to Solid State Physics, 3rd ed. (John Wiley & Sons, Inc., New York, 1966), pp. 251-299.

APPENDIX G

An Analysis of the Current Pulse
and Measurement Circuitry

A complete, detailed schematic diagram of the current-pulse and measurement circuitry is given in Fig. G-1. The distributed inductance of the current-pulse control circuitry has been lumped into L_1 and L_2 . The inductance L_3 is the self-inductance of that portion of the sample that lies between the voltage signal probes. The mutual-inductance M is included to account for the magnetic coupling between the main-bank current and the voltage probes. Estimates of the values of L_3 and M involve significant uncertainty because of the complicated geometry of the current path and voltage probes. These estimates range from 5 to 15 nH for each of these inductances. The total inductance affecting the measured voltage data may range between 10 and 30 nH.

An analysis of the current-pulse circuit^{G-1} yields the analytic form of the pulse given in Eq. (G-1):

$$I_1 = I_{1A} + I_{1B} = A[e^{-\alpha t} - e^{-\beta t}][u(t) - u(t - \tau)] + A[e^{-\alpha \tau} - e^{-\beta \tau}] e^{-Rt/L} u(t - \tau), \quad (G-1)$$

where:

$$A = \frac{V_o}{2\left(\frac{R^2}{4} - \frac{L}{C}\right)}; \quad \alpha = \left(\frac{R^2}{4L^2} - \frac{1}{LC}\right)^{\frac{1}{2}} - \frac{R}{2L}; \quad \beta = -\alpha - \frac{R}{L} \quad (G-2)$$

and:

$$L = L_1 + L_2$$

$$R = R_1 + R_2.$$

A fit of this equation to an actual pulse shape, using known values of the capacitance and resistance, yielded a value for the series inductance ($L_1 + L_2$) of 0.644 μ H.

The complete expression for the voltage V_m generated across the sample and present at the transmission-line input is given in Eq. (G-3):

$$V_m = (I_1 - I_2)R_4(t) + [L_3(t) + M]\frac{d(I_1 - I_2)}{dt} + (I_1 - I_2)\frac{d[L_3(t) + M]}{dt}. \quad (G-3)$$

Symbol	Definition	Typical value	
R1,R2	Current limiting resistors	0.5	Ω
R3,R5	Sample contact resistances	0.010	Ω
R4	Sample resistance	0.001-0.030	Ω
R6,R7	Probe contact resistances	0.010	Ω
R8	Ground resistance	0.010	Ω
R9,R10,R13,R14	Line termination resistors	50.0	Ω
R11,R12	Voltage attenuation resistors	5000.0	Ω
R15	Ground isolation resistor	5000.0	Ω
L1,L2	Lumped circuit inductances	1.17	μH
L3	Sample self inductance	10.0	nH
M	Mutual coupling inductance	10.0	nH
C	Capacitor bank	84-224	μF
V_o	Bank charging voltage	16- 20	kV
I1	Bank current	14- 18	kA
I2	Measurement current	6×10^{-5}	11A
I3	Ground current	Small	
V_m	Voltage measurement desired	200-300	V
Va - Vb	Voltage measurement observed	200-300	V

Fig. G-1. Complete circuit schematic for use with circuit-response analysis.

Providing that contact resistances in the voltage measuring circuits are kept small with respect to $50\ \Omega$, and neglecting the attenuation factor, this is also the correct equation for the difference voltage observed on the scopes. The resistors R11 and R12 serve to attenuate the voltage signals 100:1 to make them compatible with scope input ranges.

The expression in Eq. (G-3) may be made somewhat more realistic by neglecting the very small terms. The currents that flow in the voltage-probe circuits are seen to be fractionally related to the main-bank current by the ratio of the sample resistance to the $50\text{-}\Omega$ termination resistance. Typically this is a factor exceeding 10^3 . Thus, the terms involving these currents or the rate of change of these currents can safely be dropped, yielding Eq. (G-4):

$$V_A - V_B = V_m = I_1(t)R(t) + L(t)\frac{dI_1(t)}{dt} + I_1(t)\frac{dL(t)}{dt}, \quad (\text{G-4})$$

where L (the sum of L_3 and M) represents the total inductance affecting the measured voltage. Figure G-2 graphically relates the various terms of this equation. The variation of sample resistance with time shown here is copied from that typical of lead data.

One problem that must be considered in the reduction of data from each experiment is the determination of the appropriate value of the inductance to use in determining the resistive component of the measured voltage. Moses and Korneff^{G-2} were successful in their experiment in using specially designed coils to cancel out the inductive voltage, but the tight geometry of the pressure cell and the sample-to-sample variation in the geometry of the voltage probes preclude application of their technique to this experiment. If the sample resistance were constant over the time of rise of the current pulse, this correction would be trivial. The appropriate resistive voltage could be subtracted from the measured voltage and the result divided by the rate of change of current to obtain the inductance. However, for most materials, this is not the case.

The problem can be solved, because data relating the resistance to the enthalpy or temperature are available, for most materials, over the range of energy deposited during the time of current rise. These data may be fit by some functional form suitable for numerical application on a digital computer. Then, since the resistive voltage is known at the peak of the

Fig. G-2. Comparative representation of the components of the measured voltage signal for lead.

current pulse, where $dI/dt = 0$, the computer may be caused to produce an iterative fit of the appropriate functional form over this time range. As above, subtraction and division then yield the desired value of the inductance. Very little resistance data are lost by application of this method. The numerical difficulty involved in making a smooth inductive correction, of the magnitude required, even if the inductance used were precisely known, would preclude accurate determination of the resistance during this part of the pulse. Most of this loss of accuracy comes from the differentiation of the current data, which accentuates the random errors introduced by the digitization of this data.

The measurement of inductance for each data record is desirable for two reasons. First, a more accurate value than can be calculated from geometric considerations is obtained. Second, the inductance is directly a function of the voltage-probe location, shape, and separation distance. Any or all of these may change a large amount from shot to shot because of the handcrafted nature of the probe design and assembly. The measured inductance reflects these changes.

A precise value for the inductance is not really required. This fortunate situation results from the fact that when the inductive voltage is large the current is typically quite small. Consequently, the reactive energy indicated by the measured voltage is a small percentage of the real energy. Further, the reactive energy is positive during the current rise time and negative during the current fall time. These tend to cancel, leaving only the contribution along the slowly decreasing top of the current pulse. In this region the inductive voltage is typically only 1-2% of the resistive voltage. The enthalpy error made by completely neglecting the inductive voltage correction would be typically about 5%. By making the correction, this is reduced to less than 1%.

The variation in the inductance with time is somewhat more difficult to obtain. Because of expansion of the sample during the heating, the inductance indeed decreases, but no direct measure of this is available. Instead, it may be estimated. A geometric expression for the inductance of a system of current-carrying conductors^{G-3} is given in Eq. (G-5):

$$L = 2 \times 10^{-7} S \ln(\text{GMD}/\text{GMR}), \quad (\text{G-5})$$

where S is the length, GMD is the geometric mean distance between conductors, and GMR is the radius of the conductor under consideration. This

expression is strictly valid only by neglecting end effects and does not directly involve the mutual-coupling effect of probe design. These restrictions are offset to some extent by using the measured $L = L_3 + M$ to calculate the GMD value, since before expansion the GMR for the sample is just $0.7788 r$, where r is the true sample radius. This results in a GMD value that to some extent reflects the perturbation due to end effects and to mutual-coupling of the circuits.

It is necessary to assume that only the GMR value changes with time as the sample expands. This approximation is valid if $GMR \ll GMD$. In the experimental situation, this inequality holds pretty well. Data relating the sample diameter to time are, of course, required before the change in inductance can be calculated. This must generally be obtained by first reducing the experimental data, assuming a constant inductance, then subsequently correcting for temporal inductive effects. The magnitudes of the temporal inductive effects are quite small and generally negligible.

The effect of contact and ground resistance in producing errors in the voltage signal delivered to the oscilloscopes has been analyzed. The contact resistance between the sample and the current leads can easily be shown to have no effect on the difference voltage, although resistance at one contact can affect the magnitude of both voltage signals, relative to ground. The ground resistance acts in series with the probe-contact resistance for each voltage signal. So long as the sum of these two resistances is small in comparison with the $50\text{-}\Omega$ cable-termination resistance, the error produced will be negligible. In fact, if both probes had a contact resistance of $0.1\ \Omega$, and the ground resistance were likewise $0.1\ \Omega$, the error in voltage produced would still be less than -0.5% .

References

- G-1. Y. H. Ku, *Transient Circuit Analysis* (D. Van Nostrand Company, Inc., Princeton, 1961), pp. 1-127, 195-214, 263-275, 284-305.
- G-2. K. G. Moses and T. Korneff, Rev. Sci. Instr. **34**, 849 (1963).
- G-3. H. H. Skilling, Electric Transmission Lines (McGraw-Hill Book Company, Inc., New York, 1951), pp. 96-168, 390-411.

APPENDIX H

Photographic and Measurement Techniques

The photographic technique utilizes a pulsed x-ray source to radiograph the pressure cell containing the expanded sample. The x-ray photon spectrum produced by the source is a typical bremsstrahlung spectrum with maximum photon energy at about 85 keV. The transmission of this spectrum through the x-ray tube window and intervening air serve to make the spectrum incident on the pressure cell approximately equivalent in effect to a monoenergetic 50 keV beam. The analysis of the photographic process utilizes this equivalence and considers the resultant transmission of 50 keV photons through the pressure cell and sample, and the resultant exposure, processing and interpretation of the image produced on the film.

All of the photons which strike the film must pass through the beryllium windows of the pressure cell and the high pressure helium inside. There must be enough of them transmitted that the portion of the film exposed, is made completely opaque after development. Those photons which strike any part of the sample or calibration rods must be absorbed and must not reach the film. The maximum image contrast which can be achieved, is required to assure reasonable accuracy in the data reduction processes.

The separation distance of 3 ft between the source and the sample was chosen to reduce the effects of source geometry (deviation from a point source) in the resultant image. This has now been found to be less crucial than originally thought; because the data reduction technique is very insensitive to effects which apply equally to both sample and calibration rod images. The separation of the film plane and sample (about 6 in.) is dictated by the geometry of the pressure cell and, the requirement for film access after assembly of the sample with the pressure cell. However, with this geometry, source intensity, and cell attenuation, the transmitted photon beam is not sufficiently intense to expose x-ray film without the use of fluorescent converter screens. These screens convert the energy in any x-ray photon which is stopped within them to optical photons of much lower energy. The process achieves a high figure of multiplication of sensitivity because the film is as sensitive to the optical photons as to the x-ray photons.

The construction of the film packs used, is shown in Fig. H-1. The zinc-sulfide screens are placed in intimate contact with both the front and back sides of double emulsion, x-ray film.* With these screens, sufficient contrast is achieved, but the increased effective thickness of the film plane and angular dispersion of the converted photons, results in a diffusing of the boundaries of the images. This loss of boundary resolution prohibits use of a direct measure of the diameter of the object photographed, and a comparison technique, using the known diameters of the calibration rods; must be employed.

Another effect which can cause loss of definition at the edges of the image is the reduced material thickness in the photon path at the edge of the cylindrical objects photographed. Using the assumption of a 50-keV mono-energetic photon beam, and assuming linear film response, this effect was computed for a number of materials.

The intensity of an x-ray beam varies with transmission distance in a material as expressed by

$$\frac{I}{I_0} = e^{-\mu \rho x}, \quad (H-1)$$

where I_0 is the intensity of the incident beam, μ (cm^2/g) the photon absorption coefficient of the material, ρ (g/cm^3) the material density, and x (cm) the penetration depth for which the value of I is desired. In this case, x is the total length of a chord passing through the edge of a cylinder, since the attenuation of the cylinder is the quantity desired (see Fig. H-2). Considering a line mutually perpendicular to the planar beam and the axis of the cylinder, it is found, that beginning with the point projected on this line by the beam just tangent to the cylinder edge, the intensity behind the cylinder will be a continuously varying function of distance. It is this function which is to be computed.

Let $r - y$ represent the distance from the point on the cylinder where the beam is just tangent. Then the depth of material the beam must penetrate, x is given by

$$x = 2(r^2 - y^2)^{\frac{1}{2}}, \quad (H-2)$$

* Kodak RB-54 medical x-ray film.

Fig. H-1. Photograph of film pack, showing layered construction. The pack is held together with spring clips to ensure intimate contact between the film and screens.

Fig. H-2. Geometry considered for analysis of cosine effect.

and Eq. (H-1) becomes

$$\ln(I/I_0) = -2\mu\rho (r^2 - y^2)^{\frac{1}{2}} \quad (\text{H-3})$$

or

$$\ln(I/I_0) = -\mu\rho d \left(1 - \frac{y^2}{r^2}\right)^{\frac{1}{2}}, \quad (\text{H-4})$$

where d is the diameter of the cylinder.

The exponential nature of the intensity behind the cylinder will make the point of departure from I_0 impossible to detect. Therefore, it is necessary to select some reasonable value of $\ln(I/I_0)$ that can be considered in determination of the wire diameter, from a knowledge of the intensity behind it. A convenient value is given by

$$I = \left(1 - \frac{1}{1}\right) I_0 = 0.634. \quad (\text{H-5})$$

Now, an apparent diameter D can be defined in terms of the true diameter by

$$\ln(0.634) = -\mu\rho d \left(1 - \frac{y^2}{r^2}\right)^{\frac{1}{2}}, \quad (\text{H-6})$$

but, since $D = 2y$,

$$\left(1 - \frac{D^2}{d^2}\right)^{\frac{1}{2}} = \frac{\ln(0.634)}{\mu\rho d}, \quad (\text{H-7})$$

from which

$$D = d \left[1 - \left(\frac{\ln(0.634)}{\mu\rho d}\right)^2\right]^{\frac{1}{2}}. \quad (\text{H-8})$$

Expanding the square root gives

$$D = d \left[1 - \frac{1}{2} \left(\frac{-0.455}{\mu\rho d}\right)^2 + \dots\right]. \quad (\text{H-9})$$

Assuming we wish to make less than 1% error in measuring the diameter, then

$$-0.01d \geq D - d \geq -\frac{d}{2} \left(\frac{0.455}{\mu\rho d}\right)^2 \quad (\text{H-10})$$

or $\mu\rho \geq 32.2$.

Table H-1 lists values of $\mu\rho$ for a number of metals at standard density for 50 keV photons. For materials having a $\mu\rho$ product exceeding 30,

Table H-1. A comparison of the $\mu\rho$ product for some common materials at standard density, for 50 keV photons.

Element	$\mu\rho \frac{\text{cm}^2}{\text{g}}$	Element	$\mu\rho \frac{\text{cm}^2}{\text{g}}$
Aluminum	1.08	Silver	120.0
Iron	18.0	Gold	160.0
Copper	28.4	Lead	114.0
Molybdenum	84.0	Bismuth	108.0

the cosine effect will produce negligible error in the determined volume. The use of copper calibration rods to reduce by comparison the diameter of an expanded lead sample from its film image is justified. In fact, the accuracy achieved should improve as the lead is measured in reduced density states.

The film development processing was carried out immediately following completion of each experiment. After the film packs were opened, the experiment index number, Blumlein source identification number, and time of exposure following current shutoff were written on the undeveloped film with a ballpoint pen. Then the films were developed in a solution compatible with the film emulsion* for a period of between 5 and 6 min. Following development, they were washed in water and placed in a fixing solution for an additional 5 or 6 min. Then they were placed in water for at least 30 min to assure hardening of the image. Finally they were air dried and filed for data reduction.

The reduction of the sample diameter data from each film required the development of as objective a comparison technique as possible. Various procedures were evaluated by comparing the initial diameter of the sample measured by the x-ray technique with the initial diameter of the sample measured optically. This comparison also gives an idea of the overall accuracy of the technique which was $\pm 5\%$.

Each image on a particular film was scanned radially by a Joyce-Lobel recording microdensitometer. This machine passes a light beam

*Kodak X-Ray Developer.

through the film and through a slit to a detector. The output of the detector is compared with a reference signal related to pen displacement. A servo-mechanism control is used to adjust the pen displacement to maintain a null comparison signal. The pen with displacement proportional to film density records this data on rectilinear graph paper on a plotting table. Horizontal motion of the table is produced by motor drive. The table is mechanically linked with the film table such that adjustable magnification of the resultant plot may be achieved. A magnification factor of 50 was used for this work. Sensitivity of the instrument is controlled by the insertion of an appropriate optical wedge in the path of the reference beam. All images on a given film must be read with the same optical wedge, however, to ensure the validity of the comparison method of diameter determination.

Typical image density traces obtained from this densitometer are shown in Fig. 15b in the main text. The determination of the unknown diameter is made by first measuring the depth H of each calibration rod image trace. Because of the "noise" in some of the traces this process becomes somewhat subjective. Next, the point at which the tracing width is just equal to the magnified value of the true diameter of the calibration rod is determined (L_1). Finally, the distance above the minimum film density value of the fit L_1 is measured (H_1). The fraction

$$F = H_1/H \quad (H-11)$$

is computed. Values of F varied widely from film to film, ranging from 0.15 to 0.45. Furthermore the values of F for the different calibration images on a given film typically varied as much as $\pm 10\%$ with no apparent correlation with the diameter of the particular calibration rod. The average of the fractions computed for each calibration rod trace F_{ave} has been found to give more accurate data than obtained by any other reasonably objective scheme, such as use of the calibration rod closest in size to the sample. Figure 16 in the main text shows the result of this study.

To reduce the unknown diameter of the sample from its image trace, the total depth H is first determined, as before. Then, the average fraction F_{ave} is multiplied by the trace depth H to give the distance above the density minimum H_{ave} required. The width of the image trace at this point L_2 is measured, and divided by the magnification factor to yield the unknown diameter. From this diameter, assuming cylindrical geometry of the sample is observed, the volume of the expanded state is readily calculated.

APPENDIX I

EOSDTA

A Computer Program for the CDC6600 to Compute
the Enthalpy and Resistance from the Current and Voltage

The non-analyticity of the current and voltage records from the experiment, coupled with the required multiplication, integration and division of these quantities necessary to obtain the enthalpy and resistance data, make numerical data reduction desirable. EOSDTA was written for this express purpose and encompasses all of the manipulation of the data which can be done, assuming that information on the dynamics of sample expansion are not yet available. This restriction dictates that the inductance of the sample be considered constant with respect to time and prohibits consideration of the material resistivity, since the appropriate diameter needed to compute this is not known. Specifically, this program produces an estimate of the enthalpy so that an isobar in the enthalpy-volume plane may be defined and used in a subsequent program (DTACOR) to account for these factors which depend on the expansion dynamics. For convenience in the subsequent application of DTACOR, EOSDTA prepares a data tape containing time correlated records of all available data for each experiment, so that re-computation is not required. The program listing in the LRL version of Fortran follows,^{I-1} interspersed with commentary expanding, in detail, the various program segments.

The following statements introduce the program and establish the required memory storage for the variables noted. Also, the later use of the cathode-ray plotter is identified by the CALL CRTID statement.

```
COMMON/A1/XI(100),YI(100),XV(100),YV(100),SIX(20),SIY(20),SVX(20),
1SVY(20),SFIY,SFVX,SFVY,N1,N2,WRGNG,WIS(20),ZIS(20),WVS(20),
2ZVS(20),NSPI,NSPV
COMMON/A2/T(500),CUR(500),DLT,NP,NINT,LOFNS
COMMON/A3/DIDT(500),VOLT(500),NOGOOD,ITFIT
COMMON/A4/V(500),DHDT(500),A(500),H(500),R(500),IT,AL,RINT,ISTRT,
1ISTOP,WGT,ITMAX
CALL CRTID (2HKH,1,1 )
```

The following statements read the necessary control information and input data from punched cards. The variable IFRSTM is a flag used for control of the taping option. If IFRSTM is zero, no preparation for taping

occurs; if one, then the tape is rewound and taping begins with the first record position on the tape; if two, then the tape is rewound and the number of records previously recorded (NREC) are skipped preparatory to taping data currently considered by the program. The variable ELEMENT stores alphanumeric information about the sample such as the material name. Also it was found handy for storing letter designations of the particular scope record being considered. The variables DBVT, DIA, DNSTY, RINT, and GASP record the voltage probe separation distance, the sample diameter, the initial density, the initial resistance and the gas pressure used for the experiment, respectively. ITMAX is the maximum number of time scale intervals which can be allowed for the inductance determination. ITFIT specifies the order of the polynomial fit to be applied to the input data in establishing the equal-interval time scales required by the numerical process. NINT specifies the number of time intervals, up to a maximum of 500 which the fitting routines are to construct. Finally ISHOT is the experiment identification number.

The current and voltage scope traces are separately prepared for input to the computer by digitizing them, using a Grant Comparator coupled to an IBM card punch. This machine gives X-Y coordinates of desired points on the film, relative to an arbitrary set of machine axes. Following the digitization of the film trace, five coordinates of a rectangular portion of the graticle are punched for scaling use. They must be the lower left hand point, upper left hand point, center, lower right hand point and upper right hand point relative to the orientation in which the film trace was read. The digitization of the trace must begin with one point at zero time, followed by as many points along the trace as desired to a maximum of 100, and ending with a point of zero ordinate, relative to the established base line of the trace. All X coordinates must increase monotonically in the direction of increasing time on the film trace but the interval size is arbitrary. Also the orientation of the film with respect to the machine axes is arbitrary and orientation to an appropriate set of coordinates is accomplished by the program. Preceding each deck of current data must be a card which specifies N1, the number of points in the digitized trace, SFIX, the real time equivalent to the distance between the X coordinates of two adjacent scale points (such as the left hand lower and center points) SFY, the real current equivalent to the separation between any two adjacent scale points, and NSPI the

number of scale points to be read (five). The variables XI and YI are the X and Y coordinates of the current trace respectively. SIX and SIY are the X and Y coordinates of the scale points from the current trace, respectively.

Likewise, the voltage data deck must be preceded by a card which specifies N2, the number of digitized data points along the voltage trace; SFVX, the real time equivalent to the difference between two adjacent scale points; SFVY, the real voltage equivalent to the difference between two adjacent scale points; and NSPV, the number of scale points to be read (five). XV and YV are the X and Y coordinates of the voltage data points along the trace and SVX and SVY are the X and Y coordinates of the voltage scale points, respectively. Finally a card carrying the value assigned MRDTA is used to control the program with respect to capability to handle stacked decks of several experiments in sequence. A typical data deck is given as an example following the program listing.

```

      RIT 2,125,IFRSTM,NREC
125  FORMAT (2I5)
      IF (IFRSTM - 1)61,62,63
      61 GO TO 12
      62 CALL REWIND (6)
      63 CALL REWIND (6)
      64 DO 64 I = 1,NREC
      CALL REDEF (6)
      64 CONTINUE
C    STATEMENT 12 MUST PRECEED ALL OTHER READ STATEMENTS
      12 CONTINUE
      RIT 2,100,ELEMT,DBVT,DIA,DNSTY,RINT,GASP,ITMAX,ITFIT,NINT,ISHOT
100  FORMAT (A8,2X,5E10.3,4I5)
      RIT 2,110,N1,SFIX,SFIY,NSPI
110  FORMAT (I5,2E10.3,I5)
      RIT 2,115,(XI(I),YI(I),I=1,N1)
115  FORMAT (10F7.3)
      RIT 2,115,(SIX(I),SIY(I),I=1,NSPI)
      RIT 2,110,N2,SFVX,SFVY,NSPV
      RIT 2,115,(XV(I),YV(I),I=1,N2)
      RIT 2,115,(SVX(I),SVY(I),I=1,NSPV)
      RIT 2,120,MRDTA
120  FORMAT (I5)

```

The following statements calculate initial sample properties useful in later application, such as the cross-sectional area, the volume, the mass, and the initial resistivity.

```

C   THE ABOVE INPUT STATEMENTS DESCRIBE THE DATA DECK FORMAT
C   THE NEXT FOUR STATEMENTS CALC INITIAL SAMPLE PROPERTIES
      AREA = (DIA**2*3.1416)/4.
C   DIAMETER GIVEN IN CM GIVES AREA IN CM SQUARED
      VOL = AREA * DBVT
C   DISTANCE BETWEEN VOLTAGE TAPS MUST BE GIVEN IN CM
      WGT = VOL * DNSTY
C   DNSTY IS INITIAL DENSITY GIVEN IN GRAMS PER CC
      RHO = (RINT * AREA)/DBVT
C   INITIAL RESISTANCE IN OHMS GIVES RHO IN OHM-CM
      CALL SCLDTA

```

The main program calls the subroutine SCLDTA which scales the input data and establishes tables of the input data in appropriate units of time, current and voltage. The following statements give that routine.

```

      SUBROUTINE SCLDTA
      COMMON/A1/XI(100),YI(100),XV(100),YV(100),SIX(20),SIY(20),SVX(20),
1      SVY(20),SFIX,SFIY,SFVX,SFVY,N1,N2,WRONG,WIS(20),ZIS(20),WVS(20),
2      ZVS(20),NSPI,NSPV
C   THE FOLLOWING ROUTINE ORIENTS AND PROPORTIONATELY SCALES THE DATA
C   FIRST MAKE SURE DATA IS PRESENTED RIGHT SIDE UP AND FORWARD
      IF(N1-N2)92,92,93
92 NG = N2
      GO TO 94
93 NG = N1
94 CONTINUE

```

It is necessary that both the current and voltage records be the same length in real time, even though for part of that time one of the records may have zero ordinate. The above statements begin the process of constructing them in that manner by first determining which record is longer in number of points and equating NG to that length. The following statements initialize a flag, WRONG which is used to communicate with the main program if trouble develops during the application of SCLDTA. Then as a diagnostic aid, the input data is printed out.

```

      WRONG = 0.  $SC THIS PRECEEDS IF STATEMENT
      WOT 3,205
205 FFORMAT (1H1, 42H CURRENT AND VOLTAGE AS READ INTO PROGRAM /)
      WOT 3,200,(I,XI(I),YI(I),XV(I),YV(I),I=1,NG)
      WOT 3,220,(SIX(I),SIY(I),SVX(I),SVY(I),I=1,5)
200 FFORMAT (/5X27HDIAGNOSTIC DATA FROM SCLDTA //9X1H1,12X5HXI(I),
1      115X,5HYI(I),15X5HXV(I),15X5HYV(I) //(110,4E20.4))
220 FFORMAT(///12X6HSIX(I),14X6HSIY(I),14X6HSVX(I),14X6HSVY(I) //(4E20.
1      14))

```

Next, the orientation of the current time axis is checked. If it is right oriented then control passes to the next test but if it is left oriented

Statement 10 inverts it. Note that if the two tested coordinates are the same value an error notation is generated.

```

      IF(XI(1)-XI(5))1,2,3
2    WOT 3,125
      WRONG = 1.
      GO TO 1
125  FORMAT (5X,34HUNABLE TO ORIENT CURRENT TIME AXIS /)
      DO 10 I=2,N1
10   XI(I) = XI(1) + (XI(1)-XI(I))
      DO 11 I = 1,NSPI
11   SIX(I) = XI(1) + (XI(1)- SIX(I))

```

Precisely analogous tests and corrective measures are applied to the current axis, the voltage time axis and the voltage axis. Note that in each case where reorientation is required, the scale data must be reoriented also. This sequence of statements insures that the ordinates and abscissas of the data traces are oriented properly with respect to a right hand set of coordinate axes and establish the first data point given for each trace as the zero—zero point of that set of axes.

```

1   IF(YI(1) - YI(5))4,5,6
5   WOT 3,130
      WRONG = 1.
      GO TO 4
130  FORMAT (5X,29HUNABLE TO ORIENT CURRENT AXIS /)
      DO 12 I=2,N1
12   YI(I) = YI(1) + (YI(1) - YI(I))
      DO 13 I = 1,NSPI
13   SIY(I) = YI(1) + (YI(1)- SIY(I))
      IF(XV(1) -XV(5))7,8,9
8   WOT 3,135
      WRONG = 1.
      GO TO 7
135  FORMAT (5X,34HUNABLE TO ORIENT VOLTAGE TIME AXIS /)
      DO 14 I=2,N2
14   XV(I)= XV(1) + (XV(1) - XV(I))
      DO 15 I = 1,NSPV
15   SVX(I) = XV(1) + (XV(1)-SVX(I))
      IF (YV(1) -YV(5))16,17,18
17   WOT 3,140
      WRONG = 1.
      GO TO 16
140  FORMAT (5X,29HUNABLE TO ORIENT VOLTAGE AXIS /)
      DO 19 I=2,N2
19   YV(I) = YV(1) + (YV(1)-YV(I))
      DO 20 I = 1,NSPV
20   SVY(I) = YV(1) + (YV(1) - SVY(I))
16  CONTINUE
      IF(WRONG)40,41,40
40  RETURN

```



```

C   NOW WITH EVERYTHING RIGHT ORIENTED, TRANSLATE THE AXES
41  CONTINUE
    DO 21 I=2,N1
      XI(I) = XI(I) - XI(1)
21  YI(I) = YI(I) - YI(1)
      DO 22 I = 1,NSP1
        SIX(I) = SIX(I) - XI(1)
22  SIY(I) = SIY(I) - YI(1)
      XI(1) = 0.
      YI(1) = 0.
      DO 23 I=2,N2
        XV(I) = XV(I) - XV(1)
23  YV(I) = YV(I) - YV(1)
      DO 24 I = 1,NSPV
        SVX(I) = SVX(I) - XV(1)
24  SVY(I) = SVY(I) - YV(1)
      XV(1) = 0.
      YV(1) = 0.
      WDT 3,210
      WDT 3,200,(I,XI(I),YI(I),XV(I),YV(I),I=1,NG)
      WDT 3,220,(SIX(I),SIY(I),SVX(I),SVY(I),I=1,5)
210  FORMAT (1H1,5X, 66HCURRENT AND VOLTAGE DATA AFTER ORIENTATION AND
      1TRANSLATION OF AXES /)

```

The translation of the axes insures the proper initial time indexing of the current data with the voltage data, but either or both may yet require rotation to bring the base line into coincidence with the positive X axis. The following statements determine the angular relation between the base line and the X axis and produce the required rotation for alignment, if it is not within the established tolerance.

```

      IF(ABSF(YI(N1))-.0005)30,30,31
31  CONTINUE
33  ETA1 = ATANF(YI(N1)/XI(N1))
34  COSETA = COSF(ETA1)
      SINETA = SINF(ETA1)
      DO 35 I=2,N1
        XPR = XI(I)*COSETA + YI(I) * SINETA
        YPR = -XI(I)*SINETA + YI(I)*COSETA
        XI(I) = XPR
        YI(I) = YPR
35  CONTINUE
      DO 36 I = 1,NSP1
        XPR = SIX(I) * COSETA + SIY(I) * SINETA
        YPR = -SIX(I) * SINETA + SIY(I) * COSETA
        SIX(I) = XPR
        SIY(I) = YPR
36  CONTINUE
30  IF(ABSF(YV(N2))-.0005)37,37,38
38  CONTINUE
53  ETAV = ATANF(YV(N2)/XV(N2))
54  COSETA = COSF(ETAV)

```

```

SINETA = SIN(ETAV)
DO 55 I=2,N2
  XPR = XV(I) * COSETA + YV(I) * SINETA
  YPR = -XV(I) * SINETA + YV(I) * COSETA
  XV(I) = XPR
  YV(I) = YPR
55 CONTINUE
DO 56 I = 1,NSPV
  XPR = SVX(I) * COSETA + SVY(I) * SINETA
  YPR = -SVX(I) * SINETA + SVY(I) * COSETA
  SVX(I) = XPR
  SVY(I) = YPR
56 CONTINUE
37 CONTINUE
  WOT 3,215
  WOT 3,200,(I,XI(I),YI(I),XV(I),YV(I),I=1,NG)
  WOT 3,220,(SIY(I),SIY(I),SVX(I),SVY(I),I=1,5)
215 FORMAT (1H1,5X, 64HCURRENT AND VOLTAGE DATA ORIENTED, ROTATED AND
  IREADY FOR SCALING /)

```

The data is now ready for scaling. Linear interpolation scaling is used in each of four quadrants determined by constructing vertical and horizontal lines through the center of the five scale points read. When this method was originally conceived, it was intended to apply to a calibration graticle traced by the scope and including scope non-linearities. This process never became operational, however, the routine was never changed and has been applied somewhat superfluously to scale points read from scope graticle lines. These are, of course, rectilinear except for small distortions introduced by stretching of the film, and single factor scaling would probably be about as accurate as the more complicated procedure applied here.

```

FIX1 = SFIX / (SIX(3) - SIX(1))
FIY1 = SFIY / (SIY(3) - SIY(1))
FIX2 = SFIX / (SIX(3) - SIX(2))
FIY2 = SFIY / (SIY(2) - SIY(3))
FIX3 = SFIX / (SIX(4) - SIX(3))
FIY3 = SFIY / (SIY(3) - SIY(4))
FIX4 = SFIX / (SIX(5) - SIX(3))
FIY4 = SFIY / (SIY(5) - SIY(3))
XA = XI(1)
YA = YI(1)
DO 70 I = 2,N1
  XC = XI(I)
  YC = YI(I)
  IF(XI(I) - SIX(3)) 61,61,62
61 J = 1
  GO TO 71

```

```

62 J = 3
71 IF(YI(I) - SIY(3))63,63,64
64 J = J + 1
63 IF(J-2)65,66,67
67 IF(J-4)68,69,69
65 XI(I) = (XI(I) - XA)*FIX1 + XI(I-1)
   YI(I) = (YI(I) - YA) * FIY1 + YI(I-1)
   GO TO 72
66 XI(I) = (XI(I) - XA)*FIX2 + XI(I-1)
   YI(I) = (YI(I) - YA) * FIY2 + YI(I-1)
   GO TO 72
68 XI(I) = (XI(I) - XA)*FIX3 + XI(I-1)
   YI(I) = (YI(I) - YA) * FIY3 + YI(I-1)
   GO TO 72
69 XI(I) = (XI(I) - XA)*FIX4 + XI(I-1)
   YI(I) = (YI(I) - YA) * FIY4 + YI(I-1)
72 XA = XC
   YA = YC
70 CONTINUE
C  VJW DO SAME FOR VOLTAGE DATA
   FVX1 = SFVX/(SVX(3) - SVX(1))
   FVY1 = SFVY/(SVY(3) - SVY(1))
   FVX2 = SFVX/(SVX(3) - SVX(2))
   FVY2 = SFVY/(SVY(2) - (SVY(3)))
   FVX3 = SFVX/(SVX(4) - SVX(3))
   FVY3 = SFVY/(SVY(3) - SVY(4))
   FVX4 = SFVX/(SVX(5) - SVX(3))
   FVY4 = SFVY/(SVY(5) - SVY(3))
   XB = XV(1)
   YB = YV(1)
   DO 90 I = 2,N2
   XD = XV(I)
   YD = YV(I)
   IF(XV(I) - SVX(3))81,81,82
81 J = 1
   GO TO 91
82 J = 3
91 IF(YV(I) - SVY(3))83,83,84
84 J = J + 1
83 IF(J-2)85,86,87
87 IF(J-4)88,89,89
85 XV(I) = (XV(I) - XB) * FVX1 + XV(I-1)
   YV(I) = (YV(I) - YB) * FVY1 + YV(I-1)
   GO TO 95
86 XV(I) = (XV(I) - XB) * FVX2 + XV(I-1)
   YV(I) = (YV(I) - YB) * FVY2 + YV(I-1)
   GO TO 95
89 XV(I) = (XV(I) - XB) * FVX4 + XV(I-1)
   YV(I) = (YV(I) - YB) * FVY3 + YV(I-1)
   GO TO 95
88 XV(I) = (XV(I) - XB) * FVX3 + XV(I-1)
   YV(I) = (YV(I) - YB) * FVY4 + YV(I-1)
95 XB = XD
   YB = YD
90 CONTINUE
   RETURN
   END

```

The final values assigned to XI, YI, XV, and YV are the equivalent of the input data except in right oriented, left indexed conventional current, voltage and time units. The RETURN statement passes control back to the main program, which first checks the error flag, then calls DTAFIT to create the required equal interval integration tables of time, current, voltage and additionally, the rate of change of current, DIDT.

```
C      SCIDTA CONVERTS SCOPE DATA IN CM TO AMPS, VOLTS, AND TIME IN SEC
      IF(WRONG)20,40,20
      40 CONTINUE
      CALL DTAFIT
```

The following statements list and discuss the operation of the subroutine DTAFIT. These first statements are concerned with making both the current and voltage records the same length. Whichever is shorter is increased in time scale to the length of the other by adding appropriate zero ordinates.

```
      SUBROUTINE DTAFIT
      COMMON/A1/XI(100),YI(100),XV(100),YV(100),SIX(20),SIY(20),SVX(20),
      1SVY(20),SFIX,SFIY,SFVX,SFVY,N1,N2,WRONG,WIS(20),ZIS(20),WVS(20),
      2ZVS(20),NSPI,NSPV
      COMMON/A2/T(500),CUR(500),DLT,NP,NINT,LOFNS
      COMMON/A3/DIDT(500),VOLT(500),NOGCCD,ITFIT
C      INCLUDE COMMON A1, A2 AND A3
C      INCLUDE ALL NECESSARY TO RUN SPLICE AND SPLINE FROM THIS ROUTINE
C      FIRST MAKE CURRENT AND VOLTAGE RECORDS THE SAME LENGTH
      IF(XI(N1)-XV(N2))10,10,11
      10 TIME = XV(N2)  %%C TIME IS TOTAL TABLE LENGTH
      GO TO 17
      11 TIME = XI(N1)
      17 IF (N1-N2)15,15,16
      15 LOFNS = N2+4%%C LOFNS IS LARGEST OF N1 OR N2      PLJS FOUR
      GO TO 12
      16 LOFNS = N1 +4
      12 DLT = TIME /(NINT)
```

Next, two zero ordinate points are added both in front of and behind the current and voltage records. This process helps to improve the fit to the data in these regions but otherwise contributes nothing to the calculated values desired. Then an appropriate time table is established to provide the basis for the equal-interval fit to be made to the data.

```
XI1 = XI(1) $ YI1 = YI(1)
XI2 = XI(2) $ YI2 = YI(2)
DO 20 I = 1,N1
XI3 = XI(I+2) $ YI3 = YI(I+2)
XI(I+2) = XI1 + 2 * DLT
```

```

      YI(I+2) = YI1
      XI1 = XI2 $ YI1 = YI2
      XI2 = XI3 $ YI2 = YI3
20  CONTINUE
      XI(1) = 0. $ YI(1) = 0.
      XI(2) = DLT $ YI(2) = 0.
      XI(N1+3) = TIME+3*DLT $ YI(N1+3) = 0.
      XI(N1+4) = TIME + 4.* DLT $ YI(N1+4) = 0.
      XV1 = XV(1) $ YV1 = YV(1)
      XV2 = XV(2) $ YV2 = YV(2)
      DO 21 I = 1, N2
      XV3 = XV(I+2) $ YV3 = YV(I+2)
      XV(I+2) = XV1 + 2 * DLT
      YV(I+2) = YV1
      XV1 = XV2 $ YV1 = YV2
      XV2 = XV3 $ YV2 = YV3
21  CONTINUE
      XV(1) = 0. $ YV(1) = 0.
      XV(2) = DLT $ YV(2) = 0.
      XV(N2+3) = TIME+3*DLT $ YV(N2+3) = 0.
      XV(N2+4) = TIME + 4. * DLT $ YV(N2+4) = 0.
C   NEXT SET UP TIME TABLE
      T(1) = 0. $ NP = NINT +4
      DO 14 I = 2,NP
      T(I) = (I-1)*DLT
14  CONTINUE
C   USING EKTERP SET UP LAGRANGIAN FIT TO CURRENT
      M= N1+4
      CALL EKTERP ( M, XI, YI, NP, T, CUR, ITFIT)

```

Both a spline fit technique and Lagrangian fit technique have been tried. The Lagrangian fit seemed to give somewhat less trouble and provides a smooth fit with the option of selecting the order of the polynomial desired. The routine given here is a standard routine available from the computer library.^{I-2}

```

SUBROUTINE EKTERP(LXY,X,Y,NP,XP,YP,NG)
C
C   N-TH DEGREE WALKING POLYNOMIAL INTERPOLATION SCHEME UTILIZING
C   LAGRANGES FORMULA FOR UNEVENLY SPACED PIVOTAL POINTS
C
C   CALLING ARGUMENTS
C   LXY = NUMBER OF INPUT DATA PAIRS
C   X = INPUT PIVOTAL POINTS
C   Y = INPUT ORDINATES
C   NP = NUMBER OF INTERPOLATED PAIRS
C   XP = INPUT INTERMEDIATE BCISSAS
C   YP = OUTPUT INTERPOLATED ORDINATES
C   NG = DEGREE OF INTERPOLATING POLYNOMIAL

```

```

57 CONTINUE
56 DO 58 I = KC,LXY
   IF (X(I)-T)22,22,58
58 CONTINUE
51 CONTINUE
   IF (T-X(1)) 16, 8, 8
8   IF (T-X(KC)) 10, 10, 14
10  DO 12 I=2, KC
   IF (X(I)-T) 12, 22, 22
12 CONTINUE
14 IF (T-X(LXY)) 18, 18, 16
16 YT = 0.
   GO TO 45

C
18 DO 20 I=KC, LXY
   IF (X(I)-T) 20, 22, 22
20 CONTINUE
22 N = I

C
C   IF T LIES NEAR EITHER END OF X, USE THE FIRST (OR LAST) NOR VALUES
C   OF ARRAY X TO CONSTRUCT THE INTERPOLATING POLYNOMIAL ... OTHERWISE
C   THE RUNNING INDICES I AND M ARE CALCULATED FOR THAT PURPOSE.
C
   IF (N-KL) 30, 30, 31
30  L = 1
   J = NOR
   GO TO 36
31  IF (KR-N) 32, 32, 34
32  L = LXY-NG
   J = LXY

C
   DIMENSION X(320),Y(320),XP(320),YP(320)

C
   NOR = NG+1
   KC = LXY/2
   KL = NOR/2 + 1
   KR = LXY - NOR + KL
   DO 45 K=1, NP
   SUM = 0.

C
C   LET T = CURRENT VALUE OF THE INPUT ARGUMENT
C   T = XP(K)

C
C   TEST TO SEE IF T IS BOUNDED IN THE UPPER OR LOWER HALF OF ARRAY X
C   WHEN T IS BOUNDED IN ARRAY X DETERMINE WHICH ELEMENT OF X LIES
C   CLOSEST TO T.
C   IF T IS NOT BOUNDED ANYWHERE IN ARRAY X, SET YT = 0.
   IF (X(KC)-X(1))52,51,51
52  IF (T-X(1))53,53,16
53  IF (T-X(KC))55,54,54
55  IF (T-X(LXY))16,56,56
54  DO 57 I = 2,KC
   IF (X(I)-T)22,22,57

```

```

      GO TO 36
34  L = N - KL + 1
      J = L+NG
36  DO 42 I=L, J
      P = Y(I)
      DO 40 M=L, J
      IF (I-M) 38, 40, 38
38  P = P * (T-X(M))/(X(I)-X(M))
40  CONTINUE
42  SUM = SUM + P
      YT = SUM
45  YP(K) = YT
      RETURN
      END

```

Control is returned to DTAFIT which then recalls EXTERP to consider the voltage data. These fitting procedures may yield negative values for the current and voltage in regions where the true values are small but positive. Such negative values can cause problems in later calculations and so are eliminated by simply setting them to zero. Then the rate of change of current is computed and control returned to the main program.

```

      M= N2+4
      CALL EKTERP ( M, XV, YV, NP, T, VOLT, ITFIT)
C    NOW ZERO ANY NEGATIVE VALUES WHICH THE FIT MAY HAVE GIVEN
      DO 35 I = 1, NP
      IF (CUR(I)) 36, 37, 37
36  CUR(I) = 0.
37  IF (VOLT(I)) 38, 35, 35
38  VOLT(I) = 0.
35  CONTINUE
C    CALC DI/DT FROM THE CURRENT POINTS
      DIDT(1) = 0. $ DIDT(NP) = 0.
      THES2 = 0.
      DO 30 I = 2, NP -1
      THES1 = (CUR(I+1) - CUR(I))/ DLT
      DIDT(I) = (THES1 + THES2)/ 2.
      THES2 = THES1
30  CONTINUE
      CONTINUE
      RETURN
      END
*    LIST8
*    FORTRAN          VOLTAGE
      CODE ANALYSIS

```

The main program calls the subroutine VOLTAGE which operates on the scaled voltage data to determine the initial inductance, the inductive component of the measured voltage and ultimately the resistive voltage needed for the calculation of resistance and deposited enthalpy.

CALL VOLTAGE

The following statements determine the peak of the current pulse by testing for the point at which the rate of change of current passes through zero.

```

SUBROUTINE VOLTAGE
COMMON/A2/T(500),CUR(500),DLT,NP,NINT,LOFNS
COMMON/A3/DIDT(500),VOLT(500),NOGOOD,ITFIT
COMMON/A4/V(500),DHDT(500),A(500),H(500),R(500),IT,AL,RINT,ISTRT,
11STOP,WGT,ITMAX
C   INCLUDE COMMON CARDS A2, A3, A4
C   FIRST FIND THE MAXIMUM IN THE CURRENT
DO 12 I = 10,NP
  IF (DIDT(I))14,14,12
12 CONTINUE
14 IT = I

```

The variable IT is the index of the time of peak current. With lead, it is necessary that the manipulation which determines the initial inductance be applied to only that portion of the voltage record from zero time to the initiation of melting. In the event that the peak of the current pulse should occur slightly later in time, the provision is included for external control of this limit through the variable ITMAX. The following statements determine the smaller of IT and ITMAX and further calculations use the value thus determined.

```

  IF (IT - ITMAX)30,30,31
31 NOT 3, 200, IT, ITMAX
200 FORMAT (// 5X3HIT= , I5,2X, 34HEXCEEDS THE GIVEN VALUE OF ITMAX= ,
115,2X25HAND IS REPLACED BY ITMAX //)
  IT = ITMAX
30 NOT 3, 210, IT
C   GO TO 17
210 FORMAT (//5X3HIT= ,I5//)

```

A full discussion of the method of computing the initial inductance followed by this portion of the program is given in Appendix G. Basically, the enthalpy and resistance of the solid material are assumed to be related by a known function suitable for use on a computer. The resistive voltage is known at zero time and at the current pulse peak ($di/dt = 0$). The enthalpy is known at zero time. An iterative technique is applied to compute the

enthalpy-time relation and the resistive voltage-time relation between zero time and the time of current peak. This resistive voltage is subtracted from the measured voltage to give the inductive component over this time range. Division by dI/dt yields the inductance. An average of the computed inductance over the central third of the time range is taken as the value used in subsequent calculations.

```

DO 15 J = 1,IT
V(J) = CUR(J) * RINT
15 CONTINUE
R(IT) = RINT
ISTRT = 2 $ ISTOP = IT $ NOGOOD = 0
CALL ENTHLP

```

The preceding statements calculate an initial estimate of the resistive voltage assuming constant sample resistance. Then the subroutine ENTHLP is called to compute the enthalpy. The following statements give this routine in complete form. It simply multiplies the current by the resistive voltage and performs simple trapezoidal integration.

```

SUBROUTINE ENTHLP
COMMON/A2/T(500),CUR(500),DLT,NP,NINT,LOFNS
COMMON/A4/V(500),DHDT(500),A(500),H(500),R(500),IT,AL,RINT,ISTRT,
1ISTOP,WGT,ITMAX
C  INCLUDE COMMON A2 AND A4
C  THIS ROUTINE CALCULATES ENERGY DEPOSITION RATE AND INTEGRATES BY
C  SIMPLE TRAPEZOIDAL FORMULA TO GET THE ENTHALPY
DHDT(1) = 0. $ H(1)=0.
DO 10 I = ISTRT, ISTOP
DHDT(I) =(V(I) * CUR(I))/(4.18 * WGT)
H(I) = H(I-1) + ((DHDT(I) + DHDT(I-1))/2.)* DLT
10 CONTINUE
RETURN
END

```

Control is then returned to the VOLTAGE routine which handles the further iteration required. A linear relation between resistance and enthalpy is assumed here. This was found to be quite adequate for the lead work, however, any required functional form could easily be used. The form used with uranium is included with comment (C) marks leading each of these statements.

```

      HINT = H(IT) $ ICOUNT = 0
C     STATEMENTS BETWEEN 19 AND 17 ITERATE UNTIL H(IT) IS UNCHANGED BY
C     FURTHER ITERATION
      WDT 3,1300
1300 FORMAT (1H1,5X, 55HDIAGNOSTIC INFORMATION FROM THE VOLTAGE ITERATI
      ION LOOP //6X,6HICOUNT,8X,5HSLOPE,16X,4HHINT,15X5HH(IT) //)
      RSTAR = VOLT(IT)/CUR(IT)
19 CONTINUE
      ICOUNT = ICOUNT + 1
      S = ((VOLT(IT)/CUR(IT)) - RINT)/ HINT
C     S = (RSTAR - RINT)/(RINT*(1. - EXPF(-0.09486 * HINT)))
      DO 16 I=1,IT
      R(I) = RINT + S*H(I)
C     R(I) = RINT + RINT * S * (1. - EXPF(-0.09486 * H(I)))
      V(I) = CUR(I) * R(I)
16 CONTINUE
      CALL ENTHLP
      WDT 3,300,ICOUNT,S,HINT,H(IT)
300 FORMAT (110,3E20.4)
      IF (ICOUNT - 50)21,21,22
22 NOGOOD = 1
      WDT 3,310
310 FORMAT (5X36HEXCEEDED ICOUNT LIMIT AND TERMINATED )
      GO TO 20
21 IF(ABSF(H(IT)-HINT)-0.005)17,17,18
18 HINT = H(IT)
      GO TO 19
17 CONTINUE
      ACT = 0.
      DO 2 I = 1,NP-1
      ACT = ACT + ((CUR(I) + CUR(I+1))/2.)*DLT
2 CONTINUE
      WDT 3,3,1SHOT,ACT
3 FORMAT(///5X,8HFOR SHOT ,15,14H THE ACTION IS ,E12.4 )
C     NOW CALC INDUCTANCE BY SUBT V(I) FROM VOLT(I) AND DIV BY DI/DT
      CALL ENDUCT

```

The iteration process above typically converged in six to eight passes. When the resistive voltage has been found to the desired accuracy, the following routine ENDUCT is called to calculate the inductance. ENDUCT is straightforward, computes the inductance as described above, and is here listed in its entirety. Note that statements are included above to compute the action integral for current pulse comparison. Also, statements preceded by a comment mark (C) have been used to allow the program to consider the data, given an arbitrary value of the inductance.

```

SUBROUTINE ENDUCT
COMMON/A3/DIDT(500),VOLT(500),NOGOOD,ITFIT
COMMON/A4/V(500),DHDT(500),A(500),H(500),R(500),IT,AL,RINT,ISTR,
1 ISTOP,WGT ,ITMAX
C   INCLUDE COMMON A3 AND A4
C   THIS CALCS THE INITIAL INDUCTANCE FIRST AS A FUNCTION OF TIME UP
C   TO IT THEN THE AVERAGE OF THE CENTER THIRD OF THIS TABLE IS USED
DO 10 I = 3,IT
  A(I) = (VOLT(I)-V(1))/DIDT(I)
10 CONTINUE
  I1 = (IT -2)/2 $ I2 = I1/3
  I3 = I1- I2 $ I4 = I1 + I2 $ C=0.
  DO 20 J = I3,I4
    C = C + A(J)
20 CONTINUE
  AL = C/(2 * I2 + 1)
  RETURN
END

```

Control is then returned to the VOLTAGE routine which corrects the remaining time history of the measured voltage data for inductance effects, assuming, of course, constant inductance. Also included is a calculation of the sample resistance. Then control is returned to the main program.

```

      ITP1 = IT+1
C     ITP1 = 1
C     AL = 1.5E-08
      DO 20 I = ITP1,NP
        V(I) = VOLT(I) - AL * DIDT(I)
        IF (CUR(I) - 1.0)40,41,41
40    R(I) = 0.
        GO TO 20
41    R(I) = V(I)/CUR(I)
20 CONTINUE
      RETURN
      END

```

The main program first checks the flag NOGOOD to determine that the iterative fit made by VOLTAGE was successful. Then it establishes the proper index control to cause ENTHLP to compute the enthalpy for the remaining time history of the data.

```

      IF (NOGOOD)30,30,20
C     VOLTAGE MAKES ITERATIVE FIT DURING CURRENT RISE, CALLS ENDUCT TO
C     CALC INDUCTANCE, CALLS ENTHLP TO CALC ENTHALPY DURING CURRENT
C     RISE, CALCULATES INDUCTIVE CORRECTION AND RESISTANCE
C     THE NEXT STATEMENTS FINISH THE ENTHALPY AND RES RATIO CALC
30    ISTR = IT + 1
C     ISTR = 1
      ISTOP = NP
      CALL ENTHLP

```

When control is returned, the main program computes the resistance ratio and begins transmitting output to the printer.

```

      DO 20 I=1,NP
      R(I) = R(I)/RINT
20  CONTINUE
C   READY FOR OUTPUT - THEN CHECK IF MORE DATA TO BE PROCESSED
      N1 = N1 + 4
      N2 = N2 + 4

```

The printer write statements are given as an indication of the data which is available for output. A program user may wish to format the output to suit his own requirements. The program also makes use of an available cathode-ray tube display device.^{I-3} The statements that control this method of data presentation are included but may be tailored to suit a users requirements.

```

      WDT 3,200
      WDT 3,210,1SHOT,ELEMT,GASP,DNSTY
      WDT 3,215,DIA,DBVT,AREA,VOL,WGT,RINT,RHO
      WDT 3,220,1SHOT,SFIX,SFIY,SFVX,SFVY,A1,N2
      WDT 3,225,(XI(I),YI(I),XV(I),YV(I),I=1,LOFNS)
      WDT 3,230,NINT,DLT,T(IT),AL,(T(I),A(I),I=2,IT)
      WDT 3,235,(T(I),VOLT(I),V(I),CUR(I),DIOT(I), I=1,NP)
      WDT 3,240,(T(I),DHDT(I),H(I),R(I),I=1,NP)
      CMAX = 0.
      VMX = 0.
      DO 1000 I = 1,NP
      IF(CMAX - CUR(I))1010,1020,1020
1010 CMAX = CUR(I)
1020 IF(VMX - V(I))1030,1000,1000
1030 VMX = V(I)
1000 CONTINUE
      CMAX = CMAX + 1000.
      VMX = VMX + 10.
      CALL FRAME
      CALL SETCRT (0.,0.,0,1)
      CALL SETCH (2.,42.,0,0,1,0,0)
      WRITE OUTPUT TAPE 100,300,1SHOT,ELEMT,GASP,DNSTY,DIA,DBVT,AREA,
      1VOL,WGT,RINT,RHO
300 FORMAT (32HEQUATION OF STATE DATA FOR SHOT ,15//10H MATERIAL ,A8,
      13X,23HGAS PRESSURE (KBARS) = ,F5.2,4X,24HINIT. DENSITY (GM/CC) = ,
      2F6.3//18HINIT. DIA. (CM) = , E12.4,5X,21HSAMPLE LENGTH (CM) = ,
      3E12.4//22HINIT. AREA (SQ.CM.) = ,E12.4,5X20HINIT. VOLUME (CC) = ,
      4E12.4//19HSAMPLE MASS (GM) = ,E12.4,//25HINIT. RESISTANCE (OHMS) =
      5E12.4,5X,29HINIT. RESISTIVITY (OHM-CM) = ,E12.4//)
      CALL SETCH (2.,28.,0,0,1,0,0)
      WRITE OUTPUT TAPE 100,305,NINT,DLT,IT,T(IT),AL
305 FORMAT (28HNO. INTEGRATION INTERVALS = ,15,//16HDELTA T (SEC) = ,
      1E12.4,5X,2HT(,15,4H) = ,E12.4, 8HSECONDS ,//31HCOMPUTED INDUCTANCE
      2 (HENRYS) = ,E12.4)
      CALL FRAME

```

```

CALL MAPG (0.,T(NP),0.,CMAX)
CALL TRACE (XI,YI,N1)
CALL POINTS (T,CUR,NP)
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,310,ISHOT
310 FORMAT (50X,20HSCALED CURRENT DATA //55X,4HSHOT ,15 )
CALL SETCH (14.,7.,0,0,1,0,0)
CALL CRTBCD (32HPOINTS REPRESENT POLYNOMIAL FIT )
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (15HCURRENT (AMPS) )
CALL FRAME
CALL MAPG (0.,T(NP),0.,VMX)
CALL TRACE (XV,YV,N2)
CALL POINTS (T,VOLT,NP)
CALL SETCH (14.,7.,0,0,1,0,0)
CALL CRTBCD (32HPOINTS REPRESENT POLYNOMIAL FIT )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,315,ISHOT
315 FORMAT (50X,20HSCALED VOLTAGE DATA //55X,4HSHOT ,15 )
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (15HVOLTAGE (VOLTS) )
CALL FRAME
CALL MAPG (0.,T(NP),0.,200.)
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (15HVOLTAGE (VOLTS) )
CALL POINTS (T,VOLT,NP)
CALL TRACE (T,V,NP)
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,320,ISHOT
320 FORMAT (30X,43HUNCORRECTED VOLTAGE (POINTS) COMPARED WITH /
137X,29HINDUCTIVELY CORRECTED VOLTAGE //60X,4HSHOT ,15 )
CALL FRAME
CALL MAPG (0.,T(NP),-5.E+09,5.E+09)
CALL TRACE (T,DIDT,NP)
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,34.,0,0,1,1,0)
CALL CRTBCD (17HDI/DT (AMPS/SEC) )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,325,ISHOT
325 FORMAT (//// 20X,4HSHOT 15 )
CALL FRAME
CALL MAPG (0.,T(NP),0.,1.5E+07)
CALL TRACE (T,DHDT,NP)
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,20.,0,0,1,1,0)

```

```

CALL CRTBCD (45HENTHALPY DEPOSITION RATE DH/DT (CAL/GM-SEC) )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,325,ISHOT
CALL FRAME
HMS = H(NP) + 10.
CALL MAPG (0.,T(NP),0.,HMS)
CALL TRACE (T,H,NP)
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (11HTIME (SEC) )
CALL SETCH (1.,33.,0,0,1,1,0)
CALL CRTBCD (18HENTHALPY (CAL/GM) )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,325,ISHOT
CALL FRAME
CALL MAPG (0.,300.,1., 04.)
CALL SETCH (33.,1.,0,0,1,0,0)
CALL CRTBCD (18HENTHALPY (CAL/GM) )
CALL SETCH (1.,29.,0,0,1,1,0)
CALL CRTBCD (26HRE SISTANCE RATIO :R/R(0)) )
CALL TRACE (H,R,NP)
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,325,ISHOT

```

The following statements test IFRSTM to determine whether the data is to be recorded on tape for transmission to DTACOR. This transfer is accomplished by first equating all data to elements of a matrix AA which is then buffered onto the tape as one record.

```

IF(IFRSTM-1)31,60,60
60 CONTINUE
C SET UP MATRIX AA FOR TAPE TRANSFER TO DTACOR
AA(1) = ISHOT
AA(2) = ELEMENT
AA(3) = GASP
AA(4) = DIA
AA(5) = DBVT
AA(6) = AREA
AA(7)=VOL
AA(8)= WGT
AA(9) = RINT
AA(10) = RHO
AA(11) = NINT
AA(12) = DLT
AA(13) = IT
AA(14) = AL
AA(15) = NP
DO 50 I = 16,515
50 AA(I) = T(I-15)
DO 51 I = 516,1015
51 AA(I) = VOLT(I-515)
DO 52 I = 1016,1515

```

```

52 AA(I) = V(I-1015)
   DO 53 I = 1516,2015
53 AA(I) = CUR(I-1515)
   DO 54 I = 2016,2515
54 AA(I) = DIDT(I-2015)
   DO 55 I = 2516,3015
55 AA(I) = H(I-2515)
   DO 56 I = 3016,3515
56 AA(I) = DHDT(I-3015)
   DO 57 I = 3516,4015
57 AA(I) = R(I-3515)
   AA(4024) = MRDTA
   WDT 3,82,(AA(I)),I=1,4024)
82 FORMAT(1H1,5X42H THE FOLLOWING AA MATRIX WAS READ FROM TAPE //
   1(10E10.3))
   BUFFER OUT (6,1:(AA(1),AA(4024)))
75 IF(UNIT,6)75,65,66,67
66 WDT 3,70
70 FORMAT(//5X,40HTAPING OPERATION ENCOUNTERED EOF OR EOT //)
   CALL EXIT
67 WDT 3,71
71 FORMAT(//5X,51HTAPING OPERATION ENCOUNTERED PARITY OR COUNT ERROR)
   CALL EXIT
65 CONTINUE
   CALL WRTEOF (6)
   NREC = NREC + 1
   WDT 3,250,1SHOT,NREC
250 FORMAT(///5X,13HRECORDED SHOT ,15,15X,23HRECORD NUMBER (NREC) IS
   1 ,15 )

```

The following statements first check to see if return is required for consideration of another experiment data record. If not, then they accomplish the termination of the program.

```

31 IF(MRDTA)10,15,10
10 GO TO 12
15 IF (IFRSTM - 1)76,77,77
77 CONTINUE
   CALL UNLOAD (6)
76 CONTINUE
   CALL PLOTE
200 FORMAT (1H1,20X774T4IS PROGRAM CALCULATES THE FIRST APPROXIMATION
   1TO THE ENTHALPY FROM RAW DATA //)
210 FORMAT (5X11HSHOT NUMBER 15,5X9HMATERIAL A8,5X17HGAS PRESSURE WAS
   1F5.2,13H KBARS HELIUM,5X16HINITIAL DENSITY F6.3,6H GM/CC//)
215 FORMAT (5X,21HSAMPLE DIAMETER (CM) E12.4,5X19HSAMPLE LENGTH (CM)
   1E12.4,//5X22HINITIAL AREA (SQ.CM.) E12.4,5X20HINITIAL VOLUME (CC)
   2E12.4,5X17HSAMPLE MASS (GM) E12.4,//5X25HINITIAL RESISTANCE (OHMS)
   3E12.4,5X29HINITIAL RESISTIVITY (OHM-CM) E12.4 //)
220 FORMAT (1H1,15X85H THE FOLLOWING TABLE LISTS THE SCALED CURRENT-TIME
   1E AND VOLTAGE-TIME HISTORIES FOR SHOT 15 //2X21H THE SCALE FACTORS
   2ARE,5X4HI X= E12.4,5X4HI Y= E12.4,5X4HI V= E12.4,5X4HI V= E12.4 //2X,

```

```

310HTHERE ARE 13,21H CURRENT POINTS AND 13,16H VOLTAGE POINTS //)
225 FFORMAT (12X4HTIME,14X7HCURRENT,15X4HTIME,14X7HVOLTAGE// (4E20.4))
230 FORMAT (1H1,2X34HNUMBER OF INTEGRATION INTERVALS = 15,23H GIVING A
1 DELTA TIME OF E12.4,8H SECONDS //2X47HTHE TIME OF CURRENT MAXIMUM
2 WAS COMPUTED TO BE E12.4,9H SECONDS //2X72HTHE INDUCTANCE COMPUTED
3ED FROM THE CENTRAL THIRD OF THE FOLLOWING TABLE IS E12.4,7H HENRY
4S //12X4HTIME,13X10HINDUCTANCE // (2E20.4))
235 FORMAT (1H1,12X4HTIME,15X11HUNCORRECTED,13X9HCORRECTED,15X7HCURREN
1T,17X5HDI/DT //11X5H(SEC),17X7HVOLTAGE,16X7HVOLTAGE,17X6H(AMPS),
215X10H(AMPS/SEC) // (E20.4,3X,E20.4,3X,E20.4,3X,E20.4,3X,E20.4 ))
240 FORMAT (1H1,12X4HTIME,18X15HDEPOSITION RATE,15X8HENTHALPY,15X16HRE
1SISTANCE RATIO //12X5H(SEC),18X12H(CAL/GM-SEC),17X8H(CAL/GM),19X8H(
2R/R(0)) // (E20.4,7X,E20.4,7X,E20.4,7X,E20.4 ))
CALL EXIT
END
* LISTB
* FORTRAN SCLDTA
CODE ANALYSIS

```

Figures I-1 to I-8 illustrate the typical CRT plotter output for this data. Preceding these figures is a listing of a typical data deck.

```

AA LEAD      6.826E-01 1.002E-01 1.136E+01 1.853E-03 1.000E+00  94   2  400 109
50      1.E-05      8.E+03      5
20.166 58.246 20.547 56.375 20.982 53.471 21.307 50.706 21.803 47.975 CUR 109A
22.282 45.301 22.711 43.207 23.306 40.670 24.113 37.966 24.963 35.465 CUR 109A
25.685 33.735 26.688 31.926 27.730 30.441 29.177 28.996 30.789 28.012 CUR 109A
32.023 27.554 33.611 27.251 34.989 27.189 36.633 27.271 38.191 27.487 CUR 109A
39.807 27.738 41.283 28.012 42.719 28.308 44.446 28.672 46.213 29.123 CUR 109A
48.149 29.539 49.811 29.910 51.463 30.276 52.775 30.551 53.463 30.728 CUR 109A
53.671 31.026 54.266 32.81 55.060 35.231 55.936 37.705 56.796 39.970 CUR 109A
57.691 42.058 58.575 43.955 59.802 46.275 61.278 48.511 62.622 50.263 CUR 109A
64.350 51.995 66.487 53.586 68.278 54.600 70.566 55.652 72.813 56.492 CUR 109A
74.555 57.055 76.429 57.490 78.050 57.802 79.533 57.999 81.732 58.056 CUR 109A
34.160 60.808 34.285 27.048 51.143 43.811 68.196 60.733 67.933 26.947 CUR 109A
50      1.E-05      2.E+02      5
19.418 77.417 20.171 59.211 20.322 60.286 20.925 61.977 21.452 63.239 VLT 109A
22.056 64.686 22.951 66.402 23.803 57.846 24.774 68.874 26.068 69.996 VLT 109A
27.297 70.653 28.365 71.009 29.552 71.118 30.760 71.021 31.916 70.757 VLT 109A
32.569 70.522 33.099 70.023 33.815 68.615 34.539 66.925 34.916 66.397 VLT 109A
36.326 66.047 37.800 65.646 39.352 65.242 41.121 64.786 42.789 64.369 VLT 109A
44.492 63.918 46.507 63.417 48.288 62.944 49.739 62.567 50.910 62.245 VLT 109A
52.081 61.933 52.622 61.772 52.763 61.929 52.842 65.605 53.008 68.082 VLT 109A
53.375 69.068 54.173 70.442 55.259 71.730 56.528 73.032 57.993 74.425 VLT 109A
59.474 75.346 61.130 75.981 62.789 76.501 64.635 76.851 66.044 77.139 VLT 109A
67.344 77.305 69.061 77.402 70.749 77.472 72.476 77.472 74.751 77.472 VLT 109A
34.152 77.891 34.186 43.852 51.192 60.766 68.344 77.893 68.046 43.761 VLT 109A
1

```


EQUATION OF STATE DATA FOR SHOT 109

MATERIAL AA LEAD GAS PRESSURE (KBARS) = 1.00 INIT. DENSITY (GM/CC) = 11.360
INIT. DIA. (CM) = 1.0020E-01 SAMPLE LENGTH (CM) = 6.8260E-01
INIT. AREA (SQ.CM.) = 7.8854E-03 INIT. VOLUME (CC) = 5.3826E-03
SAMPLE MASS (GM) = 6.1146E-02
INIT. RESISTANCE (OHMS) = 1.8530E-03 INIT. RESISTIVITY (OHM-CM) = 2.1406E-05

NO. INTEGRATION INTERVALS = 400
DELTA T (SEC) = 9.1107E-08 T(94) = 8.4729E-06SECONDS
COMPUTED INDUCTANCE (HENRYS) = 4.4324E-08

Fig. I-1. CRT output data from EOSDTA.

Fig. 1-2. CRT output data from EOSDTA.

Fig. I-3. CRT output data from EOSDTA.

Fig. I-4. CRT output data from EOSDTA.

Fig. I-5. CRT output data from EOSDTA.

Fig. I-6. CRT output data from EOSDTA.

Fig. I-7. CRT output data from EOSDTA.

Fig. 1-8. CRT output data from EOSDTA.

References

- I-1. F. N. Fritsch and G. E. Theall, LRLTRAN Language Manual (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1967).
- I-2. W. Bartolini, Interpolation—A Function of Two Variables (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1966).
- I-3. J. D. Ford and M. J. Welsh, CRT Plotting Routines (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1965).

APPENDIX J

DTACOR
A Computer Program for the CDC6600
to Incorporate Sample-Expansion Effects

The computer program DTACOR is intended to be used in conjunction with EOSDTA to provide: (a) an improved calculation of the resistive component of the measured voltage, (b) a calculation of the temporal variation of the sample inductance, (c) an improved calculation of the enthalpy and resistance, (d) estimates of the internal energy and the temperature and (e) a calculation of the resistivity. The program is written in the LRL version of Fortran.^{J-1} The description given here is broken into segments, interspersed with operational descriptive comments.

```

C      PROGRAM DTACOR
C
C      PROGRAM DTACOR TAKES THE OUTPUT FROM EOSDTA AND A GIVEN ISOBAR IN
C      THE H - V PLANE AND CORRECTS THE VOLTAGE FOR EFFECT OF TEMPORAL
C      VARIATION IN INDUCTANCE.  VARIOUS OTHER PARAMETERS ARE ALSO MADE
C      AVAILABLE FOR OUTPUT
C
      DIMENSION SP(1000),NOSHCT(50),FVRTO(50)
      COMMON/A1/ENT(100),VRATO(100),T(500),VOLT(500),V(500),CUR(500),
      1DIDT(500),H(500),DHDT(500),R(500),RATIO(500),AA(4024),AIND(500),
      2DLDT(500),VC(500),DHDT(500),HC(500),RC(500),RSTVTY(500),TEMP(500)
      COMMON/A2/FV(20),HCA(20),TEMPA(20),RSTV(500,20),HCB(500,20)
      RIT 2,100,M,NSH
100  FORMAT (2I5)
      RIT 2,105,(NOSHCT(I),FVRTO(I),I = 1,NSH)
105  FORMAT (5(I5,E10.3))
      RIT 2,110,(ENT(I),VRATO(I),I=1,M)
110  FORMAT (6E12.4)
      RIT 2,120,SPHT,HMELT,HMLTD,HVAP,HVPED,TROOM
120  FORMAT      (6E10.3)

```

The variables M and NSH are respectively the number of enthalpy-expansion ratio (expanded volume/initial volume) coordinates of the particular isobar the program is to consider, and the number of experiments for which the data is to be made available. The variables NOSHCT and FVRTO are respectively, the experiment index numbers, as recorded on the data tape by EOSDTA, and the expansion ratio data obtained for these experiments. The variables ENT and VRATO are, respectively, the enthalpy and volumetric expansion ratio coordinates of the isobar to be considered. The variable SPHT is the materials specific heat at constant pressure used

to obtain an estimate of the temperature. The variables HMELT, HMLTD, HVAP, and HVPED are respectively, known or estimated values of the enthalpy at initiation of melting, the enthalpy at completion of melting, the enthalpy at initiation of vaporization and the enthalpy at completion of vaporization. These are also used in estimating the temperature of the sample. Finally, the variable TROOM is the ambient temperature at which the experiment was conducted.

```
CALL CRTID (2HKH,1,1)
CALL SPLICE (M,ENT,VRATC,SP)
```

The CRTID routine is one of a number of library routines connected with the use of the cathode-ray tube display option.^{J-2} The SPLICE routine is operated in conjunction with a SPLINE routine to produce a spline fit to a given set of coordinates and construct a digital tabulation of points along this fit at equal intervals, for easy application to numerical processing. These routines are taken from a standard collection of useful routines maintained as a library for the CDC6600.^{J-3,J-4} The listing is included here, for completeness and convenience.

SUBROUTINE SPLICE(NP,XT,YT,SP)

```
C
C  GENERATION OF THE SPLINE INTERPOLATION COEFFICIENTS
C
C  NP IS THE NUMBER OF (XT,YT) PAIRS
C  XT IS THE FWA OF TABULATED INDEPENDENT VARIABLE ARRAY
C  YT IS THE FWA OF TABULATED DEPENDENT VARIABLE ARRAY
C  SP IS THE ARRAY OF SPLINE COEFFICIENTS
C
C  PARAMETER N1 SHOULD BE SET TO MAXIMUM NP TO BE USED
C  BLOCK SSSS WILL THEN BE DIMENSIONED FOR 2N1(N1+2) WORDS
C
C  DIMENSION XT(2),YT(2),SP(2)
C  PARAMETER (N1=100)
C  COMMON/SSSS/W(N1,N1),B(N1,N1),Y(N1),S(N1),T(N1),V(N1)
C
C  NT = NP
C  W(1,1) = -1./(XT(2)-XT(1))
C  W(1,2) = -W(1,1)+1./(XT(3)-XT(2))
C  W(1,3) = -1./(XT(3)-XT(2))
C  W(NT,NT-2) = -1./(XT(NT-1)-XT(NT-2))
C  W(NT,NT-1) = 1./(XT(NT-1)-XT(NT-2))+1./(XT(NT)-XT(NT-1))
C  W(NT,NT) = -1./(XT(NT)-XT(NT-1))
C  Y(1) = 0.
C  Y(NT) = 0.
```

```

DO 30 K=2,NT-1
W(K,K-1) = (XT(K)-XT(K-1))/6.
W(K,K) = (XT(K+1)-XT(K-1))/3.
W(K,K+1) = (XT(K+1)-XT(K))/6.
30 Y(K) = (YT(K+1)-YT(K))/(XT(K+1)-XT(K))-(YT(K)-YT(K-1))/(XT(K)-
1 XT(K-1))
CALL MLR(N1,NT,NT,W,Y,SP,B,S,T,V)
RETURN
END

```

The MLR routine is a library function routine which accomplishes the matrix inversion. Return of control is, of course, back to the main program DTACOR which proceeds to read the requested data from the tape prepared by EOSDTA. The following statements rewind the tape to assure indexing and then read each experiment data record in succession. After each record is read, a test is made to determine if that experiment index number is one of those to be considered. If yes, then control passes to the operation portion of the program; if no, the next record is read.

```

      ICNT = 0
11 CONTINUE
C   BUFFER IN AA
      CALL REWIND (6)
      GO TO 65
66 CALL REDEF (6)
65 BUFFER IN (6,1) (AA(1),AA(4024))
60 IF (UNIT,6)60,63,61,62
61 WOT 3,80
      WOT 3,82,(AA(I),I=1,4024)
82 FORMAT(1H1,5X42HTHE FOLLOWING AA MATRIX WAS READ FROM TAPE //
1(10E10.3))
80 FORMAT(//5X,40HTAPING OPERATION ENCOUNTERED EOF OR EQT //)
      CALL EXIT
62 WOT 3,81
      WOT 3,82,(AA(I),I=1,4024)
81 FORMAT(//5X,50HTAPING OPERATION ENCOUNTERED PARITY OR COUNT ERROR)
      CALL EXIT
63 CONTINUE
C   ABOVE ROUTINE MARKS TIME SO FIRST ELEMENT AA WILL BE STORED
      ISHOT = AA(1)
      DO 10 I = 1,NSH
      IF (ISHOT-NOSHOT(I))10,12,10
12 ICNT = ICNT + 1
      FV(ICNT) = FVRTD(I)
      GO TO 13
10 CONTINUE
C   BUFFER IN NEXT AA RECORD
64 IF (UNIT,6)64,66,61,62
13 CONTINUE

```

When a record is found which has been listed as one for which computation is desired, it is convenient to assign appropriate names to the matrix elements read from the tape. A series of statements which are the exact inverse of the statements in EOSDTA are required to do this.

```

      WOT 3,82,(AA(I),I=1,4024)
      NP = AA(15)
      ELEMENT = AA(2)
      GASP = AA(3)
      DIA = AA(4)
      DBVT = AA(5)
      AREA = AA(6)
      VOL = AA(7)
      WGT = AA(8)
      RINT = AA(9)
      RHO = AA(10)
      NINT = AA(11)
      DLT = AA(12)
      IT = AA(13)
      AL = AA(14)
      DO 50 I = 16,515
50    T(I-15) = AA(I)
      DO 51 I = 516,1015
51    VOLT(I-515) = AA(I)
      DO 52 I = 1016,1515
52    V(I-1015) = AA(I)
      DO 53 I = 1516,2015
53    CUR(I-1515) = AA(I)
      DO 54 I = 2016,2515
54    DIDT(I-2015) = AA(I)
      DO 55 I = 2516,3015
55    H(I-2515) = AA(I)
      DO 56 I = 3016,3515
56    DHDT(I-3015) = AA(I)
      DO 57 I = 3516,4015
57    R(I-3515) = AA(I)
C    NOW READY TO CALCULATE VARIOUS PERTINENT QUANTITIES
      CALL SPLINE (M,ENT,VRATO,SP,H,RATIC,NP)

```

Note, that MRDTA was not converted from the matrix.

The previous call to SPLICE computed the constants SP required by the SPLINE routine. SPLINE must be called for each data record considered to establish the required number of equal-interval enthalpy-volume coordinate pairs. However, SPLICE need only be recalled when a different isobar is considered. RATIO is the variable which correlates the expansion dynamics with the enthalpy deposition dynamics.

```

* LIST8
* CARDS COLUMN
* FORTRAN
CODE ANALYSIS
  SUBROUTINE SPLINE (NA,XT,YT,SP,XP,YP,NB)
C
C SIX-POINT SPLINE INTERPCLATION/DIFFERENTIATION SUBROUTINE
C
C NA IS THE NUMBER OF (XT,YT) PAIRS
C XT IS THE FWA OF TABULATED INDEPENDENT VARIABLE ARRAY
C YT IS THE FWA OF TABULATED DEPENDENT VARIABLE ARRAY
C SP IS THE ARRAY OF SPLINE COEFFICIENTS
C NB IS THE NUMBER INTERPOLATION ARGUMENTS
C XP IS THE FNA OF INTERPCLATION ARGUMENTS
C YP IS THE FWA OF INTERPOLATED VALUES
C
C DIMENSION XT(2),YT(2),SP(2),XP(2),YP(2)
C
C NT = NA
C NP = NB
C JT = 1
5 N = 2
DO 90 I=1,NP
DO 20 L=N,NT-1
IF (XT(L+1).GE.XP(I)) 30,
20 CONTINUE
L = L-1
30 A = XT(L+1)-XP(I)
B = XP(I)-XT(L)
C = XT(L+1)-XT(L)
GO TO (40,50,60),JT
C
C CALCULATE YP AT XP BY SPLINE INTERPOLATION
C
40 YP(I) = (A*A*A/C-A*C)*SP(L)/6.+(B*B*B/C-B*C)*SP(L+1)/6.+(YT(L)*A+
1 YT(L+1)*B)/C
GO TO 90
C
C CALCULATE FIRST DERIVATIVE YP AT XP
C
50 YP(I) = (C/6.-A*A/(2.*C))*SP(L)+(B*B/(2.*C)-C/6.)*SP(L+1)
1 +(YT(L+1)-YT(L))/C
GO TO 90
C
C CALCULATE SECOND DERIVATIVE YP AT XP
C
60 YP(I) = SP(L)*A/C+SP(L+1)*B/C
90 N = L
RETURN
C
ENTRY DYDX1 (NT,XT,YT,SP,XP,YP,NP)
JT = 2
GO TO 5
C
ENTRY DYDX2 (NT,XT,YT,SP,XP,YP,NP)
JT = 3
GO TO 5
END

```

It is possible for SPLINE to produce a fit which may have values for RATIO less than unity. Since this is not realistic, the following statements set any such numbers equal to one. The write statements are included as a diagnostic aid in the event of trouble later in the program. Only the independent variables will be given with other than zero value.

```

WOT 3,200,1SHOT,ELEMT,3ASP,DIA,DBVT,AREA,VOL,WGT
WOT 3,205,RINT,RHO,DLT,AL,HC(NP),ENRG,SPMT,TROOM
WOT 3,215,(ENT(I),VRATC(I),I=1,M)
DO 85 I = 1,NP
IF (RATIO(I) - 1.)86,85,85
86 RATIO(I) = 1.
85 CONTINUE
WOT 3,220,(IT(I),CUR(I),V(I),VC(I),AIND(I),RSTVTV(I),RATIO(I),H(I),
LHC(I),TEMP(I),I=1,NP)

```

The following statements calculate the inductance AIND and the rate of change of inductance DLDT for that portion of the heating time from incipient melt through the remaining current and voltage records. See Appendix G for a detailed discussion of the physics and mathematics applied.

```

GMRO = 0.3894 * DIA
CON = 2.E-09 * DBVT
GMD = GMRO * EXPF(AL/CON)
C NOW READY TO CALC INDUCTANCE AND DLOT AS FTNS OF TIME
DO 20 I = 1,NP
AIND(I) = CON * LOGF(GMD/(GMRO * RATIO(I)))
20 CONTINUE
DO 21 I = 1+1,NP-1
DLDT(I) = (AIND(I+1) - AIND(I-1))/(2.*DLT)
21 CONTINUE

```

Since the portion of each data record between zero time and the earlier of either the initiation of melt or the peak of the current pulse, is used to determine a constant value of the initial inductance, the values of all data over this time period are unchanged by DTACOR. The following statements merely serve to equate the variables to be calculated by DTACOR with those data produced by EOSDTA over the initial period of the current pulse.

```

DO 30 I = 1,IT
VC(I) = V(I)
AIND(I) = AA(14)
HC(I) = H(I)
DHDT(I) = DHDT(I)
RC(I) = R(I)
30 CONTINUE

```

The next step then, is to correct the measured voltage record for time varying inductance effects to obtain a better measure of the resistive component of the voltage than was obtained from EOSDTA. Both the current and the inductance are assumed to vary with time so that not only the $L(dI/dt)$ term is subtracted, but also the $I(dL/dt)$ term. In the same operation loop, the program calculates new values for the enthalpy deposition rate DHDTC, the deposited enthalpy HC, and the resistance RC. Note, that care is taken to avoid the problems which arise if the current should become zero. This can occur at the end of the current time record because of the approach taken in EOSDTA to insure that the integral range is the same for both the current and voltage records. Note also, that the resistance ratio is the computed quantity. This is convenient when converting to resistivity, since the volumetric data is carried as a ratio with the initial volume.

```

OLDT(IT) = 0.
OLDT(NP) = OLDT(NP-1)
C NEXT CORRECT THE VOLTAGE FOR THE INDUCTANCE EFFECT
DO 22 I = IT,NP
VC(I) = VOLT(I) - AIND(I) *OLDT(I) - CUR(I) * OLDT(I)
DHDTC(I) = (VC(I) * CUR(I))/(4.18 * WGT)
DHDTC(I) = (VC(I) * CUR(I))/(4.18 * WGT)
HC(I) = HC(I-1) + ((DHDTC(I) + DHDTC(I-1))/2.)*DLT
IF (CUR(I))32,33,32
33 RC(I) = 1.
GO TO 22
32 CONTINUE
RC(I) = VC(I) / (CUR(I) * RINT)

```

The calculation of the resistivity proceeds in the following statements by simple multiplication of the resistance ratio by the ratio of the final diameter to the initial diameter, considered as a function of time, **RATIO**. The internal energy of the sample is computed only for the final state by subtracting the PAV work required to reach the expanded state.

```

DO 24 I = 1,NP
RSTVTY(I) = RC(I) * RATIO(I)
24 CONTINUE
ENRG = HC(NP) - (GASP*VCL*(RATIO(NP) -1)*100./(4.18*WGT))

```

The following statements produce a computed estimate of the sample temperature as a function of time. From the phase transition enthalpy data given, the heats of fusion and vaporization are computed. The various

branch points are included to establish the appropriate enthalpy-temperature relationship for the condition of phase corresponding to the considered enthalpy.

```

DHMLT = HMLTD - HMELT
DHVAP = HVPED - HVAP
DO 40 I = 1,NP
  IF (H(I) - HMELT)41,42,42
41 TEMP(I) = H(I)/SPHT + TROOM
  GO TO 40
42 IF (H(I) - HMLTD)43,44,44
43 TEMP(I) = HMELT/SPHT + TROOM
  GO TO 40
44 IF (H(I) - HVAP)45,46,46
45 HTMP = H(I) - DHMLT
  TEMP(I) = HTMP/SPHT + TROOM
  TEMP(I) = HTMP/SPHT + TROOM
  GO TO 40
46 IF (H(I) - HVPED)47,48,48
47 HTMP = HVAP - DHMLT
  TEMP(I) = HTMP/SPHT + TROOM
  GO TO 40
48 HTMP = H(I) - DHMLT - DHVAP
  TEMP(I) = HTMP/SPHT + TROOM
40 CONTINUE

```

The calculations of DTACOR are complete and all that remains to be accomplished is to print them out. The write statements are given to indicate the data which is available. The format may be modified to meet the needs of the user.

```

200 FORMAT (1H1,5X,63HEQUATION OF STATE DATA CORRECTED FOR TEMPORAL IN
  DUCTIVE EFFECTS //5X11HSHOT NUMBER,39X,15,5X8HMATERIAL,34X,A8 //
  25X,16HPPRESSURE (KBARS),27X,E12.4,5X13HDIAMETER (CM),25X,E12.4 //
  35X,11HLENGTH (CM),32X,E12.4,5X,13HAREA (SQ.CM.),25X,E12.4 //5X,
  41HVOLUME(CG),32X,E12.4,5X,9HMASS (GM),29X,E12.4 //)
205 FORMAT (5X,17HRESISTANCE (OHMS),26X,E12.4,5X,20HRESISTIVITY (OHM-C
  M),18X,E12.4//5X,19HTIME INTERVAL (SEC)24XE12.4,5X19HINDUCTANCE (H
  2ENRYS),19X,E12.4//5X,17HENTHALPY (CAL/GM),26X,E12.4,5X,15HENERGY (
  3CAL/GM),23X,E12.4 //5X,26HSPECIFIC HEAT (CAL/GM-DEG),17X,E12.4,
  45X,24HROOM TEMPERATURE (DEG-K),14X,E12.4 //)
  WDT 3,200,1SHOT,ELEMT,GASP,DIA,DBVT,AREA,VOL,WGT
  WDT 3,205,RINT,RHO,DLT,AL,HC(NP),ENRG,SPHT,TROOM
  WDT 3,210,HMELT,HMLTD,HVAP,HVPED,DHMLT,DHVAP
210 FORMAT (5X,19HSTART MELT (CAL/GM),24X,E12.4,5X,22HCOMPLETE MELT (C
  1AL/GM),16X,E12.4 //5X,20HSTART VAPOR (CAL/GM),23X,E12.4,5X23HCOMPL
  3ETE VAPOR (CAL/GM),15X,E12.4//5X,23HHEAT OF FUSION (CAL/GM),20X,
  3E12.4,5X,22HHEAT OF VAPOR (CAL/GM),16X,E12.4 //)
215 FORMAT (1H1,30HDATA FOR H - V ISOBAR AS INPUT //10X8HENTHALPY,11X,
  110HVOL. RATIO,/10X8H(CAL/GM,13X,6H(V/V0) //(2E20.4))
  WDT 3,220,(T(I),CUR(I),V(I),VC(I),AIND(I),RSTVY(I),RATIO(I),H(I),

```

```

1HC(I),TEMP(I),I=1,NP)
220 FORMAT(1H1,8X,4HTIME,6X7HCURRENT,5X,4HV(1),7X,5HVC(1),5X6HINDUCT,
15X,6HRSTVTY,6X4HV/VO,7X4HH(I),6X5HHC(I),7X4HTEMP //15X10E11.4)

```

The final enthalpy and temperature records for each experiment are stored for use in recreating the corrected isobaric plot. Also the corrected resistivity and enthalpy records of each experiment as tabulated data, are stored for comparison of these data over all the experiment records which were considered. A check is made to determine if the program has considered all of the experiment records required by the input. If not, then control returns to the buffer statement to read another record. If all experiments have been considered, then the program is terminated.

```

HCA(ICNT) = HC(NP)
TEMPA(ICNT) = TEMP(NP)
DO 400 I = 1,NP
RSTV(I,ICNT) = RSTVTY(I)
HCB(I,ICNT) = HC(I)
400 CONTINUE
CSF = 0.
DO 410 I = 1,NP
IF(CUR(I)-CSF)410,410,412
412 CSF = CUR(I)
410 CONTINUE
VSF = 0.
DO 415 I = 1,NP
IF(VC(I)-VSF)415,415,416
416 VSF = VC(I)
415 CONTINUE
RSF = 0.
DO 420 I = 1,NP
IF(RC(I)-RSF)420,420,421
421 RSF = RC(I)
420 CONTINUE
RSF = RSF/3.
ASF = 0.
DO 425 I = 1,NP
IF(AIND(I)-ASF)425,425,426
426 ASF = AIND(I)
425 CONTINUE
ASF = 2.*ASF
DO 430 I = 1,NP
CUR(I) = CUR(I)/CSF
VC(I) = VC(I)/VSF
VOLT(I) = VOLT(I)/VSF
RC(I) = RC(I)/RSF
AIND(I) = AIND(I)/ASF

```

```

430 CONTINUE
  CALL SETCRT (0.,0.,0,1)
  CALL FRAME
  CALL SETCH (2.,42.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,500,1SHOT,ELEMT,GASP,DIA,DBVT,AREA,VOL,WGT
500 FORMAT(15X,63HEQUATION OF STATE DATA CORRECTED FOR TEMPORAL INDUCTI
  1VE EFFECTS //2X11HSHOT NUMBER,22X,15,5X8H MATERIAL,20X,A8//2X16HPRE
  2SSURE (KBARS),10X,E12.4,5X13HDIAMETER (CM),11X,E12.4 //2X11HLENGTH
  3 (CM),15X,E12.4,5X13HAREA (SQ.CM.),11X,E12.4 //2X11HVOLUME (CC),
  415X,E12.4,5X9HMASS (GM),15X,E12.4 )
  CALL SETCH (2.,30.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,505,RINT,RHO,DLT,AL,HC(NP),ENRG,SPHT,TROOM
505 FORMAT(2X17HRESISTANCE (OHMS),9X,E12.4,5X20HRESISTIVITY (OHM-CM),
  14X,E12.4 //2X19H TIME INTERVAL (SEC),7X,E12.4,5X19HINDUCTANCE (HENR
  2YS),5X,E12.4 //2X17HENTHALPY (CAL/GM),9X,E12.4,5X15HENERGY (CAL/GM
  3),9X,E12.4 //2X21HSP. HEAT (CAL/GM-DEG),5X,E12.4,5X17HROOM TEMP (D
  4EG-K),7X,E12.4 )
  CALL SETCH (2.,22.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,510,HMELT,HMLTD,HVAP,HVPED,OHMLT,OHVAP
510 FORMAT(2X19HSTART MELT (CAL/GM),7X,E12.4,5X19HCOMPL MELT (CAL/GM),
  15X,E12.4 //2X20HSTART VAPOR (CAL/GM),6X,E12.4,5X20HCOMPL VAPOR (CA
  2L/GM),4X,E12.4 //2X20HHEAT FUSION (CAL/GM),6X,E12.4,5X19HHEAT VAP
  3R (CAL/GM),5X,E12.4 )
  CALL FRAME
  CALL MAPG (0.,T(NP),0.,1.)
  CALL TRACE (T,CUR,NP)
  CALL TRACE (T,VC,NP)
  CALL TRACE (T,AIND,NP)
  CALL POINTS (T,VOLT,NP)
  CALL POINTS (T,RC,NP)
  CALL SETCH (37.,1.,0,0,1,0,0)
  CALL CRTBCD (11H TIME (SEC) )
  CALL SETCH (7.,40.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,515,1SHOT
515 FORMAT(52HNORMALIZED CURRENT VOLTAGE RESISTANCE AND INDUCTANCE //
  1 BHFOR SHOT, 15 )
  CALL SETCH (40.,12.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,520,CSF,VSF,ASF,RSF
520 FORMAT(13HSCALE FACTORS //14HCURRENT (AMPS)9X,E12.4 /15HVOLTAGE (V
  10LTS),8X,E12.4 / 19HINDUCTANCE (HENRYS),4X,E12.4 /18HRESISTANCE (R
  2ATIO),5X,E12.4 )
  CALL FRAME
  CALL MAPG (0.,TEMP(NP),0.,50.)
  CALL TRACE (TEMP,RSTVITY,NP)
  CALL SETCH (32.,1.,0,0,1,0,0)
  CALL CRTBCD (19HTEMPERATURE (DEG-K) )
  CALL SETCH (1.,35.,0,0,1,1,0)
  CALL CRTBCD (17HRESISTIVITY RATIO )
  CALL SETCH (5.,42.,0,0,1,0,0)
  WRITE OUTPUT TAPE 100,525,1SHOT
525 FORMAT(8X11HSHOT NUMBER ,15 )
  CALL FRAME
  CALL MAPG (0.,HC(NP),0.,50.)

```

```

CALL TRACE (HC,RSTVTV,NP)
CALL SETCH (35.,1.,0,0,1,0,0)
CALL CRTBCD (17HENTHALPY (CAL/GM) )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (17HRESISTIVITY RATIO )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,525,ISHOT
CALL FRAME
CALL MAPG (1.,RATIO(NP),0.,HC(NP) )
CALL TRACE (RATIO,HC,NP)
CALL SETCH (38.,1.,0,0,1,0,0)
CALL CRTBCD (12HVOLUME RATIO )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (17HENTHALPY (CAL/GM) )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,525,ISHOT
CALL FRAME
CALL MAPG (1.,RATIO(NP),0.,TEMP(NP))
CALL TRACE (RATIO,TEMP,NP)
CALL SETCH (38.,1.,0,0,1,0,0)
CALL CRTBCD (12HVOLUME RATIO )
CALL SETCH (1.,32.,0,0,1,1,0)
CALL CRTBCD (19HTEMPERATURE (DEG-K) )
CALL SETCH (5.,42.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,525,ISHOT
IF(ICNT-NSH)66,90,90
90 CONTINUE
CALL UNLOAD (6)
CALL SETPCH (0,0,2,0,1)
CALL FRAME
CALL MAPG (1.,VRATO(M),0.,ENT(M))
CALL TRACE (VRATO,ENT,M)
CALL PDINTC (1H0,FV,HCA,NSH)
CALL SETCH (38.,1.,0,0,1,0,0)
CALL CRTBCD (12HVOLUME RATIO )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (17HENTHALPY (CAL/GM) )
CALL SETCH (40.,20.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,530,GASP
530 FORMAT(8X33HISOBAR WITH CORRECTED DATA POINTS //
110X12HGAS PRESSURE,E12.4 )
CALL SETCH (45.,16.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,535,(NOSHOT(I),I=1,NSH)
535 FORMAT(5X15HDATA FROM SHOTS /(10X4I5))
CALL FRAME
CALL MAPG (1.,VRATO(M),0.,6000.)
CALL PDINTC (1HX,FV,TEMPA,NSH)
CALL SETCH (38.,1.,0,0,1,0,0)
CALL CRTBCD (12HVOLUME RATIO )
CALL SETCH (1.,35.,0,0,1,1,0)
CALL CRTBCD (19HTEMPERATURE (DEG-K) )
CALL SETCH (45.,16.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,535,(NOSHOT(I),I=1,NSH)

```

```
CALL FRAME
CALL MAPG (0.,200.,0.,50.)
DO 540 I = 1,NSH
CALL TRACE (HCB(1,I),RSTV(1,I),NP)
540 CONTINUE
CALL SETCH (37.,1.,0,0,1,0,0)
CALL CRTBCD (17*ENTHALPY (CAL/GM) )
CALL SETCH (1.,32.,0,0,1,1,0)
CALL CRTBCD (17*RESISTIVITY RATIO )
CALL SETCH (10.,36.,0,0,1,0,0)
WRITE OUTPUT TAPE 100,535,(NOSHNT(I),I=1,NSH)
CALL PLOTE
CALL EXIT
END
```

DTACOR makes use of a cathode-ray tube plotter for display of the data. These statements are given above but may be modified to suit user needs. Figures J-1 to J-9 illustrate the plotted data which was available from DTACOR.

EQUATION OF STATE DATA CORRECTED FOR TEMPORAL INDUCTIVE EFFECTS

SHOT NUMBER	109	MATERIAL	CC LEAD
PRESSURE (KBARS)	1.0000E+00	DIAMETER (CM)	1.0020E-01
LENGTH (CM)	6.8260E-01	AREA (SQ.CM.)	7.8854E-03
VOLUME (CC)	5.3826E-03	MASS (GM)	6.1146E-02
RESISTANCE (OHMS)	1.8530E-03	RESISTIVITY (OHM-CM)	2.1406E-05
TIME INTERVAL (SEC)	8.9426E-08	INDUCTANCE (HENRYS)	4.5612E-08
ENTHALPY (CAL/GM)	1.3910E+02	ENERGY (CAL/GM)	1.3738E+02
SP. HEAT (CAL/GM-DEG)	3.5200E-02	ROOM TEMP (DEG-K)	2.9800E+02
START MELT (CAL/GM)	9.9800E+00	COMPL MELT (CAL/GM)	1.5500E+01
START VAPOR (CAL/GM)	1.5600E+02	COMPL VAPOR (CAL/GM)	2.3400E+02
HEAT FUSION (CAL/GM)	5.5200E+00	HEAT VAPOR (CAL/GM)	7.8000E+01

Fig. J-1. CRT output data from DTACOR.

Fig. J-2. CRT output data from DTACOR.

Fig. J-3. CRT output data from DTACOR.

Fig. J-4. CRT output data from DTACOR.

Fig. J-5. CRT output data from DTACOR.

Fig. J-6. CRT output data from DTACOR.

Fig. J-7. CRT output data from DTACOR.

Fig. J-8. CRT output data from DTACOR.

Fig. J-9. CRT output data from DTACOR.

References

- J-1. F. N. Fritsch and G. E. Theall, LRLTRAN Language Manual (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1967).
- J-2. J. D. Ford and M. J. Welsh, CRT Plotting Routines (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1965).
- J-3. T. C. Varljen, Generation of Spline Fit Coefficients (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1966).
- J-4. T. C. Varljen, Third Degree Spline Interpolation and Differentiation (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1966).

APPENDIX K

VOLDTA

A Computer Program for the CDC6600
to Analyze the Volumetric Data

In anticipation of the availability of a Grant Comparator with intensity capability, a computer program was written to assist in the reduction of the volumetric data from the x-ray shadowgraphs. The objective was to be able to prepare card decks of film density as a function of scan distance for the images of the sample and the calibration rods, and, by a totally objective method, make the comparison necessary to deduce from these data the diameter of the sample. The procedure followed is essentially that used for hand reduction of the densitometer plots produced by the Joyce-Lobel densitometer, except that no subjective option is available.

The Grant densitometer did not become operable in time for application of this process to any of the lead data. Application to some of the uranium data has recently been made. The accuracy of this method appears comparable to the accuracy achieved with hand reduction. However, the preparation of the data decks from the shadowgraphs requires more operator time than the total hand reduction process does. Consequently, use of this program was suspended until such time that the digitization of the data can be made more rapid.

The program listing follows, K-1 interspersed with comments regarding the detailed operation and application.

```

C      PROGRAM VOLDTA
C
C      THIS PROGRAM REDUCES THE VOLUMETRIC DATA FROM SHADOWGRAPHS
C      IT REQUIRES THAT THE CAL RODS AND SAMPLE BE REDUCED TO TABLES OF
C      INTENSITY-Y VERSUS SCAN DISTANCE-X
      COMMON/AL/DOP(10),FMAG(10),NP(10),X(100,10),Y(100,10),DOPS(5),FMAG
15(5),NPS(5),XS(100,5),YS(100,5),AX(500,10),AY(500,10),YMIN(10),YMA
2X(10),YMAX2(10),J1(10),J2(10),J3(10),H(10),WCAL(10),H1(10),F(11),
3AXS(500,5),AYS(500,5),YMS(5),YMS1(5),YMS2(5),JS1(5),JS2(5),JS3(5)
4,MS(5),DXRA(11,5),XRAD(11,5),ERR(5),VRATO(5),XMIN(10),XMAX(110),
5XMAX2(10),XMS(5),XMS1(5),XMS2(5)
      DIMENSION SP(1000)
12 CONTINUE
      RIT 2,100,1SHOT,1BLUM,1TIME,0BVI,DIA,NCAL,NSAM,1SETJP
100 FORMAT (3I5,2E10.3,3I5)
      DO 10 I = 1,NCAL
        RIT 2,110,DOP(I),FMAG(I),NP(I)
110 FORMAT (2E10.3,15)

```



```

      RIT 2,120,(X(J,I),Y(J,I),J=1,NP(I))
120 FORMAT (10F7.3)
10 CONTINUE
   DO 11 I = 1,NSAM
      RIT 2,110,DOPS(I),FMAGS(I),NPS(I)
      RIT 2,120,(XS(J,I),YS(J,I),J=1,NPS(I))
11 CONTINUE
      RIT 2,130,MRDTA
130 FORMAT (15)

```

The variables ISHOT, IBLUM, and ITIME are used to identify the particular film record under consideration. They are respectively the experiment index number, the Blumlein identification number, and the number of microseconds following current pulse shutoff before the picture was taken. The variables DBVT and DIA are the distance between the voltage probes and the initial sample diameter, respectively. NCAL and NSAM give the number of calibration rod tracings and sample tracings which were made, while ISETUP is a flag identifying whether the record is a preshot or postshot film. These are handled slightly differently.

The variables DOP, FMAG, and NP give the measured diameter of the calibration rod, the magnification factor from the densitometer and the number of density-distance data for each calibration rod image scanned. X and Y are the density-distance data for the calibration rods. Likewise, DOPS, FMAGS, NPS, X, and Y are the true diameter, the magnification factor, the number of data to be read and the data for each sample image considered. Finally, MRDTA is a flag used to indicate whether additional film records are to be read and analyzed.

The following statements write out the input data for use in correcting program trouble if it should occur. Then a 500 point distance scale is established for each record preparatory to constructing a spline fit to the data. The SPLICE and SPLINE routines make this spline fit (see Appendix J and Refs. K-2 and K-3).

```

      WOT 3,205
205 FORMAT (1H1,10X,34HREDUCTION OF DATA FROM SHADOWGRAPH //)
      IF (ISETUP)206,207,206
206 WOT 3,208,ISHOT
208 FORMAT (40X,5HSETUP ,15 //)
      GO TO 209
207 WOT 3,210,ISHOT
210 FORMAT (40X,5H SHOT , 15 //)
209 WOT 3,211,IBLUM,ITIME,DIA,DBVT,NCAL,NSAM

```

```

211 FORMAT(37X84BLUMLEIN,I5, //26X19HTIME AFTER BANK OFF,I5// 5X16HINI
    1TIAL DIAMETER,E12.4,27X29HDISTANCE BETWEEN VOLTAGE TAPS,E12.4 //
    25X22HNO. CALIBRATION TRACES,I5,28X17HNO. SAMPLE TRACES,I5 )
C PASS SMOOTH CURVES THROUGH THE DATA POINTS
DO 20 I = 1,NCAL
    WOT 3,214,I,DOP(I),FMAS(I)
    WOT 3,217
    WOT 3,215,(J,X(J,I),Y(J,I),J=1,NP(I))
214 FORMAT(1H1,40X17HCALEBRATION TRACE,I5 //5X16HOPTICAL DIAMETER, E12
    1.4,27X,20HMAGNIFICATION FACTOR,E12.4)
215 FORMAT (4(I5,E12.4,E12.4))
    DLTX = ABSF(X(NP(I),I)-X(1,I))/500.

    AX(1,I) = X(1,I)
    DO 21 J = 2,500
    AX(J,I) = AX(J-1,I) + DLTX
21 CONTINUE
    CALL SPLICE (NP(I),X(1,I),Y(1,I),SP)
    CALL SPLINE (NP(I),X(1,I),Y(1,I),SP,AX(1,I),AY(1,I),500)

```

Assuming that the value of Y is minimum for the transparent portion of the film record and maximum for the opaque portion, the following statements determine the coordinates of the two maxima and the minimum of each record considered. A straight line is drawn between these two maxima and the height of the tracing is determined by subtracting the minimum Y value from the value of Y predicted along the line at the X value of the minimum.

```

7 CONTINUE
    YMIN(I) = 100.
    DO 22 J = 1,500
    IF (AY(J,I)-YMIN(I))23,22,22
23 YMIN(I) = AY(J,I)
    XMIN(I) = AX(J,I)
    J1(I) = J
22 CONTINUE
    YMAX1(I) = 0.
    DO 24 J = 1,250
    IF (AY(J,I)-YMAX1(I))24,24,25
25 YMAX1(I) = AY(J,I)
    XMAX1(I) = AX(J,I)
    J2(I) = J
24 CONTINUE
    IF (AX(10,I)-AX(1,I))19,15,15
19 DO 9 J = 1,500
    9 AX(J,I) = AX(500,I) - AX(J,I).
    GO TO ?
15 IF (AY(1,I)-AY(250,I))16,8,8
16 DO 17 J = 1,500

```

```

17 AY(J,I) = 10. - AY(J,I)
   GO TO 7
8 CONTINUE
   YMAX2(I) = 0.
   DO 26 J = J1(I),500
   IF (AY(J,I)-YMAX2(I))26,26,27
27 YMAX2(I) = AY(J,I)
   XMAX2(I) = AX(J,I)
   J3(I) = J
26 CONTINUE
   AM = (YMAX1(I)-YMAX2(I))/(XMAX1(I)-XMAX2(I))
   B = (XMAX1(I)*YMAX2(I)-XMAX2(I)*YMAX1(I))/(XMAX1(I)-XMAX2(I))
   YH = AM * XMIN(I) + B
   H(I) = YH - YMIN(I)

```

The following statements relate the true diameter of the calibration rod to the scaled data under consideration by multiplication by the magnification factor. The program then finds the distance above the minimum Y at which the scaled diameter just fits the horizontal width of the data. This depth H1 is divided by the total depth of the data H to give a fractional depth F. This is done for each calibration rod record.

```

WCAL(I) = DOP(I) * FMAG(I)
DO 30 J = J2(I),J1(I)
IF (AY(J,I)-(0.6*H(I) + YMIN(I)))31,31,30
31 L = J
   IF (J - J2(I))40,40,32
30 CONTINUE
32 DO 35 J = 1,J3(I)-J1(I)
   IF (AY(J3(I)-J,I)-AY(L,I))36,36,35
36 L1 = J3(I) - J
   L2 = L1 + 1
   IF (J-1)40,40,38
35 CONTINUE
38 DX = ((AY(L,I)-AY(L1,I))/(AY(L2,I)-AY(L1,I)))*(AX(L2,I)-AX(L1,I))
   AWX = DX + AX(L1,I) - AX(L,I)

   IF (AWX - WCAL(I))40,40,39
40 WDT 3,200
200 FORMAT ( ////14HDATA NOT GOOD ////)
39 CONTINUE
   L = L+1
43 CONTINUE
   IF (AY(L1,I) - AY(L,I))41,41,42
42 L1 = L1 - 1
   L2 = L2 - 1
   GO TO 43
41 DX = ((AY(L,I)-AY(L1,I))/(AY(L2,I)-AY(L1,I)))*(AX(L2,I)-AX(L1,I))
   AWX1 = AWX
   AWX = DX + AX(L1,I) - AX(L,I)
   IF (AWX - WCAL(I))44,45,39

```

```

44 DW = WCAL(I) - AWX
   DW1 = AWX1 - AWX
   R = DW / DW1
   H1(I) = AY(L,I) + R*(AY(L-1,I)-AY(L,I)) - YMIN(I)
   GO TO 47
45 H1(I) = AY(L,I)
47 F(I) = H1(I)/H(I)
   WOT 3,216,I,J2(I),J1(I),J3(I),H(I),WCAL(I),H1(I),F(I)
216 FORMAT(1H1,30X27H500 POINT CALIBRATION TRACE,I5 //5X16HLEFT HAND M
1AX AT ,I5,14X10HMINIMUM AT,I5,25X17HRIGHT HAND MAX AT,I5 //5X14HDE
2PTH OF TRACE,E12.4,29X,15HMAGNIFIED WIDTH,E12.4 //5X13HHEIGHT OF F
3IT E12.4,30X12HRATIO FACTOR,E12.4)
   WOT 3,217
217 FORMAT (//4X1H16X1HX11X1HY9X1H16X1HX11X1HY9X1H16X1HX11X1HY9X1H16X
11HX11X1HY //)
   WOT 3,215,(J,AX(J,I),AY(J,I),J=1,500)
20 CONTINUE

```

The above statements complete the consideration of the calibration rod data except that it may be written out for later review, if desired.

The next step is to prepare the density-distance data given for the sample tracings, for analysis. This is handled in exactly the same manner as with the calibration rod data. The following statements create a 500 point equal-interval distance scale for use by the spline fit routines.

```

DO 50 I = 1,NSAM
   WOT 3,218,I,DOPS(I),FMAGS(I)
218 FORMAT(1H1,40X12HSAMPLE TRACE,I5 //5X16HOPTICAL DIAMETER,E12.4,
127X20HMAGNIFICATION FACTOR,E12.4)
   WOT 3,217
   WOT 3,215,(J,XS(J,I),YS(J,I),J=1,NPS(I))
   DLTx = ABSF(XS(NPS(I),I)-XS(1,I))/500.
   AXS(1,I) = XS(1,I)
   DO 51 J = 2,500
   AXS(J,I) = AXS(J-1,I) + DLTx
51 CONTINUE
   CALL SPLICE (NPS(I),XS(1,I),YS(1,I),SP)
   CALL SPLINE(NPS(I),XS(1,I),YS(1,I),SP,AXS(1,I),AYS(1,I),500)

```

The total depth of the tracing must be determined. This is accomplished by the following statements which fit a straight line across the trace from the left-hand maximum to the right-hand maximum. Then the Y value of the minimum in the curve is subtracted from the Y value of the straight line evaluated at the X value of the minimum of the curve. The result is called HS.

```

1007 CONTINUE
      YMNS(I) = 1000.
      DO 52 J = 1,500
        IF (AYS(J,I)-YMNS(I))53,52,52
53      YMNS(I) = AYS(J,I)
      XMNS(I) = AXS(J,I)
      JS1(I) = J
52 CONTINUE

      YMS1(I) = 0.
      DO 54 J = 1,250
        IF (AYS(J,I)-YMS1(I))54,54,55
55      YMS1(I) = AYS(J,I)
      XMS1(I) = AXS(J,I)
      JS2(I) = J
54 CONTINUE
      IF (AXS(10,I)-AXS(1,I))1019,1015,1015
1019 DO 1009 J = 1,500
1009 AXS(J,I) = AXS(500,I) - AXS(J,I)
      GO TO 1007
1015 IF (AYS(1,I)-AYS(250,I))1016,1008,1008
1016 DO 1017 J = 1,500
1017 AYS(J,I) = 10. - AYS(J,I)
      GO TO 1007
1008 CONTINUE
      YMS2(I) = 0.
      DO 56 J = JS1(I),500
        IF (AYS(J,I)-YMS2(I))56,56,57
57      YMS2(I) = AYS(J,I)
      XMS2(I) = AXS(J,I)
      JS3(I) = J
56 CONTINUE
      AMS = (YMS1(I) - YMS2(I)) / (XMS1(I) - XMS2(I))
      BS = (XMS1(I)*YMS2(I)-XMS2(I)*YMS1(I)) / (XMS1(I)-XMS2(I))
      YHS = AMS * XMNS(I) + BS
      HS(I) = YHS - YMNS(I)

```

Next the average fractional depth of all the calibration traces is computed and appropriately stored.

```

      FAVE = 0.
      DO 58 J = 1,NCAL
        FAVE = FAVE + F(J)
58 CONTINUE
      F(NCAL+1) = FAVE / NCAL

```

Then the diameter of the sample traces are determined by computing the width of the sample trace at various heights above the minimum specified by the calibration trace fractions F. This width DXRA is divided by the magnification factor to properly scale it with respect to the experiments.

```

DD 60 L = 1, NCAL + 1
YTR = YMNS(I) + F(L) * HS(I)
DD 61 J = JS2(I), JS1(I)
IF (AYS(J,I)-YTR)62,62,61
62 M = J
GO TO 63
61 CONTINUE
63 DD 64 J = 1, JS3(I)-JS1(I)
IF (AYS(JS3(I)-J ,I)-YTR)65,65,64
65 M1 = JS3(I)-J
GO TO 66
64 CONTINUE
66 XTR=(YTR-AYS(M,I))/(AYS(M-1,I)-AYS(M,I))*(AXS(M-1,I)-AXS(M,I))
XTR = XTR + AXS(M,I)
XTRP = (YTR-AYS(M1,I))/(AYS(M1-1,I)-AYS(M1,I))
XTRP = (XTRP*(AXS(M1-1,I)-AXS(M1,I)) + AXS(M1,I)
DXRA(L,I) = XTRP - XTR
XRAD(L,I) = DXRA(L,I)/FMAGS(I)
60 CONTINUE
WDT 3,219,I,JS2(I),JS1(I),JS3(I),HS(I),F(NCAL+1)
219 FORMAT(1H1,40X22H500 POINT SAMPLE TRACE,I5, //5X16HLEFT HAND MAX AT
115,14X10HMINIMUM AT,I5,25X17HRIGHT HAND MAX AT,I5, //5X14HDEPTH OF
2TRACE,E12.4,29X20HAVERAGE RATIO FACTOR,E12.4 )

WDT 3,217
WDT 3,215,(J,AXS(J,I),AYS(J,I),J=1,500)
WDT 3:220,(L,F(L),DXRA(L,I),XRAD(L,I),L=1,NCAL+1)
220 FORMAT(1H1,5X96HTHE FOLLOWING DATA WAS OBTAINED FOR THIS SHOT TRAC
1E. LAST LINE RESULTS USE AVERAGE RATIO FACTOR //5X 7HCAL ROD,14X,
26HFACTOR,4X18HMAGNIFIED DIAMETER,4X16HREDUCED DIAMETER //
3(5X,15,3E20.4))
50 CONTINUE

```

If the experiment record read is not a postshot film but instead is a preshot film for which we know the sample diameter, then some error calculations can be made. These are handled by the following statements.

```

IF (ISETUP)80,70,80
80 CONTINUE
C WE NOW TREAT THE SETUP FOR ERROR ANALYSIS DATA
C FOR PRESENT JUST COMPUTE ERROR IN READING SAMPLE LATER MAY EXPAND
C THIS SECTION TO INCLUDE TREATING ERROR IN READING CAL RODS USING
C SAMPLE AS ONE OF THE CALIBRATION CURVES
DD 81 J = 1, NSAM
ERR(J) =(XRAD(NCAL+1,J)-DOPS(J))/(.01*DOPS(J))
81 CONTINUE
WDT 3,221,1SHOT,1BLUM,1TIME,(I,ERR(I),I=1,NSAM)
221 FORMAT(////5X83HUSING AVERAGE RATIO FACTOR THE PERCENTAGE DEVIATIO
IN FROM OPTICAL DIAMETER FOR SETUP,I5,1H/,I5,1H/,I5,3H IS //5X,
26HSAMPLE,5X16HPERCENTAGE ERROR //(3X,I5,12X,E12.4))
GO TO 90

```

```

70 DO 71 I = 1, NSAM
   VRATO(I) = (XRA)(NCAL+1, I)**2 / (DOPS(I)**2)
71 CONTINUE
   WDT 3, 222, ISHOT, IBLUM, ITIME, (I, VRATO(I), I=1, NSAM)
222 FORMAT(///5X66HUSING AVERAGE RATIO FACTOR THE VOLUMETRIC EXPANSI
   IN RATIO FOR SHOT, I5, 1H/, I5, 1H/, I5, 3H IS //5X6HSAMPLE, 5X16HVOLUME, 4
   2IC RATIO // (3X, I5, 10X, E12.4))
90 CONTINUE

```

That completes the calculations required of the program. Beside the input data, the important data available for output is F(I), XRAD(J,I), ERR(K), and VRATO(I). A number of other parameters may be printed for diagnostic use as required. The output statements follow but may be modified to suit the needs of the user.

```

C   THIS IS THE OUTPUT SECTION OF THE PROGRAM INCLUDING CRT ROUTINES
   CALL CRTID (2HKH, 1, 1)
   CALL SETCRT (0., 0., 0, 1)
   DO 300 I = 1, NCAL
   CALL FRAME
   X1 = AX(1, I)
   X2 = X1 + 5.
   Y1 = YMIN(I) - 0.5
   Y2 = Y1 + 8.
   CALL MAPG (X1, X2, Y1, Y2)
   CALL TRACE (AX(1, I), AY(1, I), 500)
   CALL LINEP (XMAX1(I), YMAX1(I), XMAX2(I), YMAX2(I))
   Y3 = YMIN(I) + H(I)
   CALL LINEP (XMIN(I), YMIN(I), XMIN(I), Y3)
   X4 = XMIN(I) - 0.5
   X5 = X4 + 1.
   CALL LINEP (X4, H1(I), X5, H1(I))
   CALL SETCH (10., 40., 0, 0, 1, 0, 0)
   WRITE OUTPUT TAPE 100, 301, DOP(I), I, ISHOT, IBLUM, ITIME, F(I)
301 FORMAT(E12.4, 11H MM CAL ROD, I5, 4H FOR, I5, 1H/, I5, 1H/, I5, //10X2HF=,
   1E12.4)
300 CONTINUE
   IF (ISETUP) 302, 303, 302

303 DO 304 I = 1, NSAM
   CALL FRAME
   X1 = AXS(1, I)
   X2 = X1 + 5.
   Y1 = YMNS(I) - 0.5
   Y2 = Y1 + 8.
   CALL MAPG (X1, X2, Y1, Y2)
   CALL TRACE (AXS(1, I), AYS(1, I), 500)
   CALL LINEP (XMS1(I), YMS1(I), XMS2(I), YMS2(I))
   Y3 = YMNS(I) + HS(I)
   CALL LINEP (XMNS(I), YMNS(I), XMNS(I), Y3)
   CALL SETCH (10., 40., 0, 0, 1, 0, 0)

```

```

      WRITE OUTPUT TAPE 100,305,I,ISHT,IBLUM,ITIME,F(NCAL+1),XRAD(NCAL+
11,I),VRATO(I)
305 FORMAT(12HSAMPLE TRACE,I5,4H FOR,I5,1H/,I5,1H/,I5,1H,11HAVERAGE F =
1E12.4,5X10HDIAMETER =,E12.4 //14HVOLUME RATIO =,E12.4)
304 CONTINUE
      GO TO 310
302 DD 306 I = 1,NSAM
      CALL FRAME
      X1 = AXS(1,I)
      X2=X1+5.
      Y1 = YMNS(I) - 0.5
      Y2=Y1+8.
      CALL MAPG (X1,X2,Y1,Y2)
      CALL TRACE (AXS(1,I),AYS(1,I),500)
      CALL LINEP (XMS1(I),YMS1(I),XMS2(I),YMS2(I))
      Y3 = YMNS(I) + HS(I)
      CALL LINEP (XMNS(I),YMNS(I),XMNS(I),Y3)
      CALL SETCH (10.,40.,0,0,1,0,0)
      WRITE OUTPUT TAPE 100,307,DOPS(I),I,ISHT,IBLUM,ITIME,F(NCAL+1),
1XRAD(NCAL+1,I),ERR(I)
307 FORMAT(E12.4,15H MM SETUP. TRACE,I5,4H FOR,I5,1H/,I5,1H/,I5,1H,11HAVERAGE F =,E12.4,5X10HDIAMETER =,E12.4,18HPERCENTAGE ER4JR =
2E12.4)
306 CONTINUE
310 CONTINUE

```

Extensive use of the cathode-ray tube plotter was made with this program.^{K-4} Figures K-1 to K-6 illustrate the presentation of data that was obtained.

The final program statements are given below.

```

      IF(MRDTA)12,91,12
91 CONTINUE
      CALL PLOTE
      CALL EXIT
      END

```

Fig. K-1. CRT output data from VOLDTA.

Fig. K-2. CRT output data from VOLDTA.

Fig. K-3. CRT output data from VOLDTA.

Fig. K-4. CRT output data from VOLDTA.

Fig. K-5. CRT output data from VOLDTA.

Fig. K-6. CRT output data from VOLDTA.

References

- K-1. F. N. Fritsch and G. E. Theall, LRLTRAN Language Manual (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1967).
 - K-2. T. C. Varljen, Generation of Spline Fit Coefficients (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1966).
 - K-3. T. C. Varljen, Third Degree Spline Interpolation and Differentiation (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1966).
 - K-4. J. D. Ford and M. J. Welsh, CRT Plotting Routines (Computation Division, Lawrence Radiation Laboratory, Livermore, California, 1965).
- 4

APPENDIX L

Operational Procedure and Check List

The operational procedure described here is applicable to the seven kilobar pressure system. However it may serve as a guide to the reader who is interested in constructing and operating similar equipment.

1. Prepare straight, clean sample.
2. Remove top sample support from top of pressure cell plug.
3. Clean new "O" ring seal with alcohol and install.
4. Carefully clamp sample in top sample support.
5. Remove sample clamp from bottom sample support.
6. Reinstall top sample support and sample on pressure cell top plug.
7. Align sample, and reclamp bottom sample support.
8. Trim off excess material and install calibration rod bracket.
9. Fashion two voltage probes as required.
10. Attach voltage probes to voltage leads and assure contact with sample.
11. Measure sample diameter and distance between voltage probes on the optical comparator.
12. Pulse sample with current pulse of about 50 A, 50 μ sec.
13. Record current through sample and voltage probe signal. These data are later used to determine the initial resistivity.
14. Beware of ground loop problems in the above resistance measurement.
15. Clean the pressure cell components with alcohol.
16. Clean new "O" ring with alcohol and install bottom pressure cell seal.
17. Check that steel bottom plug is tightly connected to pressure tubing.
18. Install beryllium cylinder, making sure windows are properly aligned.
19. Insert appropriate alignment tools to facilitate installation of the top of pressure cell with sample.
20. Attach alignment ring to top of pressure cell.
21. Install top of pressure cell such that voltage probes do not interfere with x-ray paths.

22. Remove alignment tools.
23. Guide leads up center hole as end-loading nut is carefully lowered in place.
24. Tighten end-loading nut to within $5/8$ - $3/4$ in. of flush. This should produce no seal loading.
25. Connect current and voltage leads. Observe correct polarity.
26. Make sure x-ray window alignment tool is correctly in place.
27. Increase the end-load to 1500 psi ram pressure as follows.
28. Close all panel valves.
29. Open master air source valve.
30. Open V8 and V10 ram-up control valves.
31. Open pump air valve to operate pump until ram oil pressure reaches 1500 psi.
32. Close V8 and V10.
33. Remove x-ray window alignment tool.
34. Install film packets for preshot shadowgraph.
35. Latch doors, clear area and set all interlocks.
36. Insert Blumlein Key, Master Key, and CDU Key.
37. Set Blumlein power switch to off and voltage control to zero.
38. Start safety red-flashing light. Switch on master control panel.
39. Check that time intervals are as desired for the experiment.
40. Turn Blumlein and CDU keys on.
41. Turn Master key on and listen for contactor pick-up. Check to see that run-safe light and interlock lights are all lit.
42. Press CDU charge switch on master control panel.
43. Press Blumlein charge switch on Blumlein control panel.
44. Check voltage on the CDU heads and leave switch in null position.
45. Switch Blumlein power on and rotate voltage control clockwise.
46. Charge Blumleins to 50 kV (full scale on the meter).
47. When Blumleins have charged, press permissive fire switch.
48. When fire light appears (5 sec) press fire switch.
49. Reset Blumlein controls to off position.
50. Turn off all keys and remove master key before leaving control area.
51. Turn off safety light.
52. Unlock doors and turn warning lights off.

53. Remove film packs and replace with fresh film packs for shot.
54. Attach top lid of containment cylinder.
55. Remove grounding hooks from capacitor bank.
56. Latch doors and reset interlocks.
57. Check all scopes for proper operation, calibration, and settings.
58. Check film in all cameras.
59. Set scopes for single trace operation.
60. Check continuity of voltage signal leads using ohmmeter provided.
61. Insert all keys and turn Blumlein, CDU and capacitor bank keys on.
62. Turn safety light on. Pressurize as follows.
63. Open V7, Haskel helium supply valve.
64. Open V1, V2, V3, and V4.
65. Slowly open V5 and fill system with helium to pressure of supply.
66. Close V7.
67. Slowly open V6 and vent all but about 25 bars from the system. This procedure reduces the oxygen level in the system.
68. Close V6 and V5.
69. Reopen V7 and slowly open V5 to refill system with helium.
70. Close V5, V7, V3, and V4.
71. Open pump air control to apply oil pressure to separator.
Caution: Do not exceed 1 kbar cell pressure before pausing for leaks and increase end-loading on seals.
Caution: Monitor oil pressure gauge closely. If it should rapidly increase turn off pump air control. This will occur if oil control valves are not set correctly or if the separator travel reaches its maximum extent (e.g. low helium supply pressure may not allow reaching 1 kbar without exceeding separator travel limitation).
72. At 1 kbar the ram pressure must be increased. Close V1.
73. Slowly open V8 and V10. The line oil pressure will exceed the ram oil pressure and as V8 and V10 are opened they will equilibrate causing the ram oil pressure to increase slightly.
74. Open pump air slowly to increase ram oil pressure to 2500 psi.
75. Close V8 and V10.
76. If separator capability has not been exceeded, re-open V1.
77. Pump to separator capability or 1.5 kbar, whichever occurs first.

78. Close V1 and V2. If 1.5 kbar is reached open V8 and V10 and increase ram pressure to 3200 psi. If 1.5 kbar was not reached open V4 and continue to pump on the intensifier until 1.5 kbar is reached.

Note: When using the intensifier a slightly different procedure is required for increasing the ram pressure. The following instructions illustrate.

79. Close V4.
 80. Slowly open pump air control to raise pump oil pressure to about 20,000 psi.
 81. Then slowly open V8 and V10 and increase ram oil pressure as required.
 82. Close V8 and V10 and re-open V4 to continue pressurizing.

Note: The following table lists the required ram pressure for ranges of cell pressure. This must be followed to avoid either breaking the cell with seal load or loss of seal integrity from excessive cell pressure.

P (bars)	Ram pres. (psi)
0-1000	1500
1000-1500	2500
1500-2000	3200
2000-2500	4000
2500-3000	4400
3000-3500	4800
3500-4000	5300
4000-4500	5800

Above 4500 bars increase ram pressure 500 psi for each additional 500 bars.

83. When desired pressure is reached close V4.
 84. Turn on master key.
 85. Press CDU switch on master control and check CDU voltages.
 86. Press Blumlein charge switch on Blumlein panel.
 87. Press bank charge switch on master control panel.
 88. Check scope ready lights.

89. Check pulse repetition switch on tail- , this must be single pulse.
90. Reconnect voltage signal cables to scopes.
91. Capacitor bank and Blumleins should be charged at about the same rate.
92. When both are charged press the permissive fire switch.
93. When fire light ready lights, open shutters and press fire switch.
94. Turn off master key and reset control to off position.
95. Pull film on all cameras.
96. Reduce the pressure in the pressure cell as follows.
Note: Always maintain a seal end-load consistent with the range of cell pressure as described in the above table.
97. Open V4.
98. Slowly open V3 letting gas pressure push the oil out of the intensifier.
99. When end load is to be reduced close V3 and V4, slowly open V9 and V13, reduce ram oil to next lowest setting.
100. When intensifier travel has been initialized, close V3 and V4.
101. Open V1 and V2 and slowly open V3 bleeding oil from separator.
102. When separator is initialized close V1 and V3.
103. Open V5 and slowly open V6 to vent remaining helium to atmosphere.
104. Open V9 and V13 to relieve ram pressure and then pump 8 strokes to back ram enough to free seals in pressure cell.
105. Close V9 and V13 and close main air supply valve.
106. Remove all keys.
107. Turn off safety light.
108. Unlatch doors and switch warning lights off.
109. Put grounding hooks on bank.
110. Remove lid from containment cylinder.
111. Insert x-ray window alignment tool before disassembly.
112. Disconnect electrical leads.
113. Bolt to nut extension in place and unscrew top nut.
114. Disassemble pressure cell.
115. Remove film packs and develop film.

116. Take scope film records with appropriate notation to Grant comparator and make digital decks of the data for computer input.
117. Either read developed shadowgraphs on a Grant densitometer or on a Joyce-Lobel recording densitometer and reduce the volumetric data.

APPENDIX M

Measurement of Equilibrium Resistivity

One of the significant limitations of this technique is the inability to obtain resistivity data for the equilibrium, high-temperature state. Resistivity data can be readily obtained throughout the portion of the current pulse from peak current to the point of current shutoff. Thereafter, the accuracy with which the inductive correction can be applied becomes so low that the resistance data computed for times following current turnoff is virtually worthless. Further, the establishment of equilibrium volume may not occur until after the current has become very small.

The resistivity obtained during the heating of the sample is not necessarily characteristic of a known, equilibrium thermodynamic state. Non-equilibrium processes involving the pressure are known to occur during this time. Inhomogeneities in the resistance of the sample or the deposition of energy, may not affect the determination of the final state enthalpy, but may give misleading information regarding the resistivity as a function of enthalpy or volume.

Aside from the obvious value of high temperature resistivity data on known equilibrium thermodynamic states, a more immediate problem can be solved if this measurement is taken. The x-ray photography methods allow examination of the volume to be made at only one select time for each x-ray source. Therefore, data on expansion dynamics and the approach to equilibrium must rely on the repeatability of the experiment, to allow the appropriate time range to be examined. Since the main heating current is essentially zero a few microseconds after current shutoff the resistivity of the sample should be a function only of the volume, for the late times. A method for recording this resistance as a function of time, would produce data which could be correlated with the volume measurements and which would, in effect, serve as a sensitive measure of the volume expansion dynamics.

In the hope of making such a technique a working part of the experiment, a small, severely overdamped capacitor bank was connected in parallel with the main, heating capacitor bank, through a spark-gap switch. The output from the small bank was initiated 20 μ sec after turnoff of the main

bank, and it rapidly established in the sample, an effectively direct current flow of about 100 A. This current level results in the production of readable data without depositing significant energy in the sample during the 100 μ sec or so of remaining interest. Relatively intricate, signal-clamping circuitry was required to allow an oscilloscope to record these late-time signals (of the order of 1 V in magnitude), within microseconds after main bank signals (of the order of several hundred volts) were passed down the cables.

The experiment failed, because the main bank current does not go to zero following operation of the crowbar switch. Instead it tends to oscillate at a frequency of about 16 kHz throughout the time of interest. Signals resulting from this main bank current overshadowed signals produced by the small bank. It was found that the inductance caused the voltage signal to lag the current signal by several degrees during the oscillatory portion. This phase difference prevented obtaining meaningful data. Figure M-1 shows the resultant late time signals. At this point the experiment was abandoned until such time as a better plan could be formulated.

It is conceivable that a switching technique could be applied to the output from the main bank to disconnect it from the circuit. Probably this would involve the application of a mechanically operated switch. Such a switch typically has operation times as long as milliseconds, but if the jitter in operation time could be reduced to the order of 5 μ sec, then timing of the switch operation becomes merely a troublesome electronics problem. Time was not available for any further work toward institution of such a system, but its value is great, and it is included as a recommendation for future consideration.

Fig. M-1. Equilibrium resistance data.