THE RADIOACTIVE DECAY OF ^{168}Ho AND ^{169}Ho

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Peter Eugene Haustein

Gamma rays accompanying the β^- decay of ^{168}Ho (3.3 min) and ^{169}Ho (4.8 min) were investigated using Ge(Li) detectors. Sources were prepared by the reactions $^{170}\text{Er}(\gamma, np)$ ^{168}Ho and $^{170}\text{Er}(\gamma, p)$ ^{169}Ho from targets of isotopically enriched $^{170}\text{Er}_2\text{O}_3$ (>98%) and from erbium metal of natural isotopic distribution. Eight gamma-ray transitions were observed in the ^{168}Ho decay and 21 transitions in the ^{169}Ho decay. Several additional weak, short lived gamma rays (>1 MeV) were observed which could not be definitely assigned to either activity due to their similar half-lives. Energies and relative intensities are reported for these transitions. Gamma-ray decay schemes are proposed for these two activities based on previously deduced energy level structure in ^{168}Er and ^{169}Er . The decay of ^{168}Ho occurs primarily to the 2^+ and 3^+ members of the $K = 2$, γ -vibrational band of ^{168}Er . The spin of odd-odd ^{168}Ho was deduced to be 3^+ by the method of Gallagher and Moskowsky. The decay of the ^{169}Ho ground state assigned to the $7/2^-(523)$ Nilsson state occurs primarily to the first two members of the $5/2^-(523)$ rotational band in ^{169}Er . The γ -ray decay pattern of the $5/2^-(523)$ rotational states to lower lying bands ($1/2^-(521)$, $5/2^-(512)$, and $7/2^+(633)$) is discussed. This decay pattern is shown to be consistent with the Nilsson model predictions for nuclei in this mass range.

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INTRODUCTION

A number of nuclear models have been developed for descriptions of deformed nuclei such as ^{168}Ho and ^{169}Ho and the erbium nuclei to which they decay (1,2,3). In these models the deformed nuclear surface is expanded in terms of some convenient set of functions, usually spherical harmonics of order $l = 2$ or 3 . The properties of the nuclear states of even-even nuclei are then explained in terms of rotational and vibrational excitation of this deformed core. For odd-A nuclei the properties of the nuclear states can usually be ascribed to the motion of the unpaired nucleon in the potential created by the deformed even-even core. Single particle wave functions for the odd nucleon were first presented by Nilsson (3). Solutions are characterized by the asymptotic quantum numbers $K^\pi(N, n_3, \Lambda)$ where K is the projection of the nuclear spin on the 3-axis (the body-fixed z-axis), π is the parity, N is the oscillator shell number, n_3 is the 3-axis oscillator quantum number, and Λ is the component of the odd nucleon orbital angular momentum on the 3-axis. Some of these quantities are shown in Figure 1. The Nilsson states energy level diagram for the odd protons of ^{169}Ho and the odd neutrons of ^{169}Er is shown in Figure 2, as a guide to the reader in understanding the discussions which will follow. Core excitations couple to these single particle states to produce rotational and

Figure 1. Vector coupling diagram for an odd-A axially symmetric deformed nucleus

Figure 2. Nilsson energy level diagram for the odd protons of ^{169}Ho and the odd neutrons of ^{169}Er

vibrational bands of energy levels. Davidson (1) has presented a number of selection rules governing beta- and gamma-ray transitions occurring in these nuclei. Extensions of the Nilsson model to odd-odd nuclei involve the description of the interactions of the two odd particles with each other and with the deformed even-even core. For the small number of deformed odd-odd nuclei with sufficiently well characterized excited state spectra this model appears to successfully explain the properties of these nuclei.

Two reports of the decay of ^{168}Ho to ^{168}Er have appeared in the literature. Wille and Fink (4) first produced ^{168}Ho by the $^{168}\text{Er}(n,p)^{168}\text{Ho}$ reaction with 14 MeV neutrons and isotopically enriched ^{168}Er . They observed a 3.3 ± 0.5 min activity with a gamma ray at 850 keV which they assigned to ^{168}Ho . No decay scheme was proposed. Takahashi et al. (5) also studied ^{168}Ho produced by the same reaction. They observed the same 850 keV gamma-ray transition and measured the beta-ray end-point energy to be 2.2 MeV. No decay scheme was given. The energy level structure of ^{168}Er has been deduced from ^{168}Tm decay (6), and is shown in Figure 3 for levels below 1 MeV.

The first report of the decay of ^{169}Ho into levels of ^{169}Er was that of Miyano and Kuroyanagi (7). Sources produced by the reaction $^{170}\text{Er}(\gamma,p)^{169}\text{Ho}$ with isotopically enriched ^{170}Er (87%) and bremsstrahlung from 21 MeV elec-

Figure 3. Energy level structure of ^{168}Er below 1 MeV

trons, were counted using beta- and gamma-ray scintillation detectors. The half-life of this activity was found to be 4.8 ± 0.1 min. Two beta-ray components with energies of 1.95 and 1.20 MeV, having $\log ft$ values of 6.1 and 4.7 respectively, were observed. Six gamma rays were observed and placed in a decay scheme shown in Figure 4a. The excited states of ^{169}Er deduced from these measurements were interpreted in terms of the Nilsson model (3). The ground state of ^{169}Er was assigned to the $1/2^-(521)$ Nilsson state with rotational members of $3/2^-$ at 65 keV and $5/2^-$ at 75 keV. A $5/2^-(512)$ state was placed at 150 keV along with $7/2^+(633)$ and $5/2^-(523)$ states at 230 and 915 keV respectively. A $5/2^-$ state at 990 keV was also observed. The spin and parity of ^{169}Ho was deduced to be $7/2^-$; the ground state being assigned to the $7/2^-(523)$ Nilsson state based on its strong beta decay to the $5/2^-(523)$ state in ^{169}Er .

A second study of the ^{169}Ho decay was reported by Funke et al. (8). Sources of ^{169}Ho were prepared from targets of Er_2O_3 , enriched to 97% in ^{170}Er , by the (γ, p) reaction with 30 MeV bremsstrahlung. Singles and coincidence gamma-ray counting were employed using scintillation and Ge(Li) detectors. The half-life of ^{169}Ho was measured to be 4.4 ± 0.2 min. Thirteen gamma rays were observed to follow this half-life. Energy levels of ^{169}Er were deduced to lie at the following excitation energies: the ground

DECAY SCHEME OF ^{169}Ho
PROPOSED BY MIYANO AND KUROYANAGI

(a)

DECAY SCHEME OF ^{169}Ho
PROPOSED BY FUNKE et al.

(b)

PARTIAL LEVEL SCHEME OF ^{169}Er
PROPOSED BY MULLIGAN et al.

(c)

Figure 4. Previous decay schemes for ^{169}Ho and the energy level structure of ^{169}Er

state band ($1/2^-(521)$) with members at 0, 65, and 75 keV as proposed by Miyano and Kuroyanagi (7); the $5/2^-$ and $7/2^-$ members of the $5/2^-(512)$ band at 150 and 222 keV respectively; and the $5/2^-$ and $7/2^-$ members of the $5/2^-(523)$ band at 850 and 920 keV respectively. Inconclusive evidence for a state at 91 keV was also reported. Beta rays from the ground state of ^{169}Ho which was assigned to the $7/2^-(523)$ Nilsson state, appeared to feed levels in ^{169}Er at 920, 850, and 150 keV. This decay scheme is shown in Figure 4b.

Excited states of ^{169}Er have also been studied by (d,p) and (d,t) reactions on ^{168}Er and ^{170}Er and by the $^{168}\text{Er}(n,\gamma)^{169}\text{Er}$ reactions. Harlan and Sheline (9) observed 23 excited states of ^{169}Er below 1 MeV of excitation in their study of the $^{168}\text{Er}(d,p)^{169}\text{Er}$ reaction. Tjøm and Elbek (10) have reported similar results for ^{169}Er based on the reactions $^{168}\text{Er}(d,p)^{169}\text{Er}$ and $^{170}\text{Er}(d,t)^{169}\text{Er}$. A recent study by Mulligan et al. (11) of the ^{169}Er levels deduced by the $^{168}\text{Er}(d,p)^{169}\text{Er}$, $^{170}\text{Er}(d,t)^{169}\text{Er}$, and $^{168}\text{Er}(n,\gamma)^{169}\text{Er}$ reactions reports extensive results for 40 ^{169}Er states below 1.5 MeV. Most of these states have been assigned to rotational bands within the framework of the Nilsson model (3). Assignments for some of these states were reported as follows: the ground state, $1/2^-(521)$, with a rotational band to $11/2^-$; $5/2^-(512)$ at 92.2 keV

with a band to $11/2^-$; $7/2^+(633)$ at 243.7 keV with a band to $13/2^+$; $1/2^-(510)$ at 562.1 keV with a band to $7/2^-$; $3/2^-(521)$ at 714.5 keV with a band to $11/2^-$; $7/2^-(514)$ at 823 keV with a band to $9/2^-$; and $5/2^-(523)$ at 850 keV with a band to $11/2^-$. Vibrational character was also observed in some of these bands. Detailed theoretical discussions of these results were also included. A partial energy level scheme based on this work is shown in Figure 4c.

One measurement of nuclear lifetimes of excited states in ^{169}Er has been reported (12). Using a pulsed deuteron beam on targets of ^{168}Er , delayed gamma rays following the production of ^{169}Er were observed. By this technique, lifetimes of 2.85×10^{-6} sec and 2.00×10^{-6} sec were measured for the $5/2^-$ state at 91.5 keV and the $7/2^+$ state at 241 keV. The gamma-ray decay pattern of these two states was also reported.

The apparent disagreement between the two previously published decay schemes for ^{169}Ho and the recently reported energy level structure of ^{169}Er suggested that a reinvestigation of the ^{169}Ho decay would be useful. Production of ^{169}Ho using 70 MeV bremsstrahlung would also yield ^{168}Ho from erbium targets. High resolution Ge(Li) detectors could aid in obtaining significantly improved decay scheme information for these two activities, and provide new data

for tests of the theoretical models that have been developed
for these nuclei.

EXPERIMENTAL TECHNIQUES

Sources of ^{168}Ho and ^{169}Ho were prepared by synchrotron irradiation of erbium metal of natural isotopic distribution and of small quantities of erbium oxide (Er_2O_3) isotopically enriched to greater than 98% in ^{170}Er . The source production reactions were $^{170}\text{Er}(\gamma, np)^{168}\text{Ho}$ and $^{170}\text{Er}(\gamma, p)^{169}\text{Ho}$. Bremsstrahlung with a maximum energy of 70 MeV was allowed to impinge on targets placed at the internal probe position of the ISU synchrotron. A description of the synchrotron and its associated equipment has been published (13,14,15). For irradiation of the erbium, small chips of erbium metal (approximately 3 mm thick and roughly circular with a diameter of 8 mm) were fastened on the end of a Synthane probe. The powdered $^{170}\text{Er}_2\text{O}_3$ targets were irradiated in small aluminum cans which could be positioned to intercept the bremsstrahlung beam along their length.

Following the irradiation, the targets were pneumatically transferred from the synchrotron building to a counting room at the Ames Laboratory Research Reactor, a distance of about 1/5 mile. Timing was performed with an electric clock which started automatically when the synchrotron bombardment ended. The targets could typically be ready for counting approximately 90 secs after the end of the bombardment. The powdered targets were counted in small plastic test tubes placed near a detector. The metal chips were placed on

electrical tape and fastened to the detector housing.

Counting equipment consisted of three commercial Ge(Li) detectors with nominal volumes of 6, 30, and 55 cc, their associated preamplifiers and amplifiers and a 4096-channel pulse height analyzer. Singles gamma-ray spectra were accumulated in either full 4096-channel spectra or in the four 1024-channel subgroups of the analyzer memory. The 6 cc detector, from which the bulk of the gamma-ray energy and intensity measurements were obtained, had a resolution of 1.8 keV at 100 keV and 3 keV at 1 MeV. A plot of the relative efficiency of this detector is shown in Figure 5. It was measured using sources of ^{110m}Ag and ^{182}Ta , for which gamma-ray relative intensities and energies are known (16).

Data from the analyzer were punched onto paper tape which could be read and recorded onto 7-track magnetic tape using a facility of the ISU Cyclone Computer Lab. These data were then transferred and permanently stored on 9-track tape at the ISU Computation Center. Various gamma-ray photopeak fitting programs were used to analyze the data on the 9-track tape. These programs find least-squares fits of a Gaussian function for the spectrum photopeaks and calculate the centroid, height, width, area, and standard deviations of these quantities. Descriptions of the performance of these programs have been given elsewhere (17).

Figure 5. Efficiency curve for the 6 cc detector

GAMMA-RAY SPECTRA OF ^{168}Ho AND ^{169}Ho

A preliminary search of gamma rays from ^{168}Ho and ^{169}Ho indicated that all the strong transitions of these two activities were below 1 MeV in energy. A complex cluster of photopeaks in the energy range of 600-900 keV was observed, with several additional peaks below 200 keV. A spectrum from the enriched $^{170}\text{Er}_2\text{O}_3$ target of the 600-900 keV region is shown in Figure 6. The peaks at 741, 816, and 821 keV were observed to decay more rapidly than the other peaks in this energy range. This fact coupled with the existence of identical transitions seen in the decay of ^{168}Er levels populated by the ^{168}Tm decay (6), indicated the 741, 816, and 821 keV resulted from the decay of ^{168}Ho , which has a half-life reported to be shorter than ^{169}Ho (7, 8). Accurate energy measurement of the remaining peaks due to ^{169}Ho was made using the three ^{168}Ho lines as internal calibration standards. The work of Keller et al. (6) on the decay of ^{168}Tm measured the energy of these calibration transitions to a hundredth of a keV. The energy nonlinearity of the spectrometer was measured to be ≤ 0.1 keV over the energy range used in the internal calibration procedure.

Observation of the lower energy peaks was hampered by the intense 2.1 min ^{150}Eu activity induced in the targets of $^{170}\text{Er}_2\text{O}_3$. The Compton distribution of pulses associated

Figure 6. Gamma-ray spectrum of the synchrotron irradiated $^{170}\text{Er}_2\text{O}_3$ target in the range of 600 to 900 keV

with the 511 keV positron annihilation peak of ^{150}O tended to obscure these lower energy lines. Irradiation of erbium metal, despite the production of other unwanted activities (principally ^{161}Er , ^{164}Ho , and ^{167}Ho) proved to yield better sources for the observation of these lower energy lines. Peaks with energies of 65, 67, 74, 80, ~ 150, 159, 167, and 184 keV shown in Figure 7 were observed to decay with half-lives of the order of 3-5 minutes. Based on the ^{168}Tm decay (6), the peaks at 80 and 184 keV were assigned to the ^{168}Ho decay and the rest to ^{169}Ho decay. Accurate measurements of the energies of these lines were again made using the internal calibration standards based on the ^{168}Tm decay reported by Keller et al. (6).

Computer fits to the photopeaks of the gamma-ray spectrum accumulated with the 6 cc detector and an erbium metal target (Figure 7) provided peak area and peak centroid values for all the strong transitions in the decay of ^{168}Ho and ^{169}Ho . Table 1 gives these energies and relative transition intensities for both activities. Transitions decaying with the appropriate half-life for ^{168}Ho or ^{169}Ho , but which could not be assigned to either activity are also listed. The isotope assignments are based on the observed half-life characteristics of the photopeaks or on the basis of placement in the two proposed decay

Figure 7. (Continued)

Table 1. Gamma-ray transition energies and intensities in the ^{168}Ho and ^{169}Ho decays

Gamma-ray number	Energy (keV)	Intensity
^{168}Ho transitions		
4	79.82 ± 0.02^a	-
9	184.30 ± 0.02^a	14.8 ± 2.3
-	(557.1)	-
12	629.3 ± 0.5	weak
18	729.1 ± 0.6	3
19	741.32 ± 0.03^a	100.0 ± 6.0
24	815.95 ± 0.03^a	59.7 ± 5.9
25	821.11 ± 0.03^a	98.2 ± 4.3
29	914.7 ± 0.7	weak
^{169}Ho transitions		
1	64.5 ± 0.2	-
2	67.3 ± 0.3	-
3	73.6 ± 0.5	-
3	74.5 ± 0.5	-
5	84.6 ± 0.2	-
6	149.6 ± 0.3	14.4 ± 3.2
6	151.9 ± 0.3	54.3 ± 6.1

^aCalibration energies based on data from reference (6).

Table 1. (Continued)

Gamma-ray number	Energy (keV)	Intensity
¹⁶⁹ Ho transitions (cont)		
7	159.6 ± 0.1	13.1 ± 3.7
8	167.5 ± 0.2	7.4 ± 2.1
10	623.6 ± 0.3	24.5 ± 3.1
11	628.9 ± 0.3	27.0 ± 3.6
13	676.5 ± 0.2	40.5 ± 4.0
14	695.0 ± 0.2	12.7 ± 3.8
15	(698.0) ± 0.3	13.9 ± 2.8
16	705.0 ± 0.1	12.8 ± 3.1
17	717.0 ± 0.2	30.5 ± 2.1
20	761.0 ± 0.2	100 ± 5
22	778.4 ± 0.1	98 ± 6
23	788.4 ± 0.1	206 ± 21
26	852.9 ± 0.2	109 ± 13
27	866.4 ± 0.2	42.8 ± 5.6
28	876.4 ± 0.3	20.2 ± 3.9
¹⁶⁸ Ho or ¹⁶⁹ Ho transitions		
21	773.0 ± 0.5	weak
30	1373 ± 2	weak
31	1442 ± 2	weak

Table 1. (Continued)

Gamma-ray number	Energy (keV)	Intensity
^{168}Ho or ^{169}Ho transitions (cont)		
32	1517 $\pm$ 2	weak
33	1678 $\pm$ 2	weak
34	1769 $\pm$ 2	weak
35	1850 $\pm$ 2	weak

schemes described below. The doublet at 149.6-151.9 keV (Figure 8) was resolved by the computer to yield separate peak centroids and areas for the two components. The width of the 74 keV peak indicated the probable presence of another doublet, but poor statistics and insufficient resolution prevented separation of the two lines.

Absorption of the low energy gamma rays in the source prevented intensity measurements below 100 keV. The non-uniform distribution of activity throughout the irregularly shaped, high-Z source made analytic correction of self-absorption impossible. Metal samples thin enough to reduce this absorption would have had insufficient activity to be counted.

A search for high energy (> 900 keV) gamma rays following the ^{168}Ho or ^{169}Ho half-lives was made using a 55 cc Ge(Li) detector and a target of irradiated erbium metal. A gamma-ray spectrum of the source accumulated with this detector is shown in Figure 9. These high energy transitions are also listed in Table 1.

Figure 8. Computer resolution of the 149.6-151.9 keV doublet gamma-ray peak

Figure 9. Gamma-ray spectrum of the erbium metal target accumulated with the 55 cc detector

Figure 9. (Continued)

THE DECAY SCHEMES

The transitions assigned to the decay of ^{168}Ho indicated that virtually all of the beta-ray feeding of ^{168}Er levels was occurring to previously characterized excited states of that nucleus (6). In Figure 10 a decay scheme for ^{168}Ho is proposed. The 557.1 keV transition connecting the 821.1 and 264.1 keV levels was not observed in this work, but this transition has been reported (6) as a very low intensity depopulation mode of the 821.1 keV state relative to the two other strong gamma rays from that state (821.1 and 741.3 keV). Weak gamma rays observed with short half-lives and energies corresponding to the decay of the 994.7 keV state of ^{168}Er to the 79.8 and 264.1 keV states are dotted in the proposed decay scheme. These transitions have been seen in the ^{168}Tm decay (6). The intensities of the ^{168}Ho gamma rays relative to the 741.3 keV transition are also shown in Figure 10, along with the spin and parity assignments for the low lying levels of ^{168}Er proposed by Keller et al. (6). Direct beta feeding of the levels at 895.8, 821.1, and 264.1 keV must occur based on the intensity of gamma-ray transitions to and from these levels. The difficulties described earlier in obtaining accurate relative intensities for the 79.8 keV gamma ray made it impossible to calculate the beta-ray feeding to the first excited state of ^{168}Er . Additional beta-ray feeding to the 994.7 keV state probably

Figure 10. Proposed decay scheme for ^{168}Ho

occurs. Since this state is the 4^+ rotational member of the $K = 2$ γ -vibrational band in ^{168}Er , it is a little surprising that it is not fed as strongly by the ^{168}Ho beta decay, as are the 2^+ and 3^+ members of this band, which have the same intrinsic nuclear structure. Although beta-ray branching intensities and $\log ft$ values could not be calculated for the decay of ^{168}Ho , an estimate of the spin and parity of the ^{168}Ho ground state was made. Since allowed beta decay connects states of the same parity and differing in spin by zero or one unit of $\hbar$, and since the states in ^{168}Er that are strongly fed by the ^{168}Ho decay have spins and parities of 2^+ , 3^+ , and 4^+ , the most likely spin and parity for the ^{168}Ho ground state is 3^+ . In addition predictions of the ground state spin and parity for odd-odd nuclei can be made by the method of Gallagher and Moskowski (18). If the Nilsson states occupied by the odd proton and odd neutron in ^{168}Ho are the $7/2^-(523)$ and $1/2^-(521)$ orbitals respectively (based on the ground state spins and parities of ^{169}Ho and ^{169}Er) then the Gallagher-Moskowski rules predict $I^\pi = 3^+$ for ^{168}Ho . This probable assignment for ^{168}Ho has been included in Figure 10.

Construction of the decay scheme for ^{169}Ho (Figure 11) was greatly facilitated by the recent report on the ^{169}Er levels by Mulligan et al. (11). As was correctly deduced by the earlier workers (7, 8), the beta decay of ^{169}Ho feeds

Figure 11. Proposed decay scheme for ^{169}Ho

predominately the two lowest members of the $5/2^-(523)$ band in ^{169}Er . This decay pattern is easily understood with the help of Figure 2. The asymptotic selection rules for allowed beta decay (1) are $\Delta N = 0$, $\Delta n_3 = 0$, and $\Delta \Lambda = 0$. Of the filled neutron orbitals in ^{169}Ho which can participate energetically in beta decay, only the $5/2^-(523)$ state satisfies these rules. This leaves the residual ^{169}Er nucleus with an unpaired $5/2^-(523)$ neutron. The much more intense beta feeding of the $5/2^-$ member of this band over the feeding of the $7/2^-$ member is consistent with the picture of the most favored beta decay occurring between states with the same even-even core excitation.

The ground state transition of 852.9 keV fixes the band-head energy of this $5/2^-(523)$ band. Eleven transitions to low lying states of ^{169}Er were observed from this level and the $7/2^-$ rotational state built upon it. The 852.9 keV state is observed to decay to the first four members of the ground state band by transitions thought to be predominately of E2 character based on the $\Delta K \leq L$ selection rule (1). This state also decays to the first two members of the $5/2^-(512)$ band by either M1 or E2 transitions. The ΔK -selection rule allows E1 transitions between the $5/2^-(523)$ and $7/2^+(633)$ states. This decay however was not observed. A theoretical explanation for this behavior is given in the next section. The $7/2^-$ member of the $5/2^-(523)$ band also

decays to rotational states of the ground state band, but not to members of the $5/2^-(512)$ band. This $7/2^-$ level however does decay by an E1 transition to the first rotational state built on the $7/2^+(633)$ Nilsson state.

The decay of the $7/2^+(633)$ state to the two lowest levels of the $5/2^-(512)$ band is as expected. The ΔK -selection rule requires E3 transitions between the $7/2^+(633)$ band and the $1/2^-(521)$ ground state band, whereas E1 transitions can occur to the $5/2^-(512)$ band. These hindered transitions have been previously reported by Bonitz (12). The decay of the $9/2^+$ member to the band-head state of $7/2^+(633)$ has not been previously reported. Direct evidence for the decay of the 91.8 keV, $5/2^-(512)$ state was not observed. Bonitz (12) has observed the hindered decay of this state to the $3/2^-$ and $5/2^-$ members of the ground state band (but not to the ground state itself as one might expect by purely energetic considerations) by very low energy transitions, and has commented on the evidence for band-mixing between the $1/2^-(521)$ and the $5/2^-(512)$ bands. A similar situation occurs in the level structure of ^{171}Yb (19), the nucleus with one more pair of protons than ^{169}Er .

Mulligan et al. (11) have pointed out that inter-band and vibrational mixing occur in the ^{169}Er levels. This situation causes admixture of gamma-ray multipolarities

for transitions connecting two levels within a band or between bands. One is therefore unable to assign unique gamma-ray multipolarities to a large number of ^{169}Ho gamma rays. Consequently internal conversion coefficients for these transitions are difficult to estimate. The large number of low energy transitions between the ^{169}Er levels below 400 keV, which will be considerably internally converted, prevent one from deducing beta feeding to these levels, even if accurate gamma-ray intensities could have been measured. The previously reported (7) 1.95 MeV beta branch to ^{169}Er levels around 100 keV probably occurs to members of the $5/2^-(512)$ band based on the $\Delta I = 0, 1; \Delta \pi =$ No allowed beta decay selection rule.

Although pairs of high energy gamma rays having energy differences approximately corresponding to the energy differences of a number of ^{168}Er or ^{169}Er states, no conclusive assignment of these transitions to either decay scheme could be made.

THEORETICAL DISCUSSION

The principal contribution of this work to the knowledge concerning the energy level structure of ^{169}Er is the data on the gamma-ray decay of the 852.9 keV $5/2^-$ (523) band-head. It is of interest to compare the observed decay pattern of this state with predictions of the Nilsson model. This state strongly decays to the first four members of the ground state band and to the first two members of the $5/2^-$ (512) band. No decay however is observed to the $7/2^+$ state at 244 keV, which could be populated by E1 transitions from the $5/2^-$ (523) band-head. The following theoretical arguments attempt to explain this seemingly unusual behavior.

In Table 2 the linear expansion coefficients and their squares, given by Davidson (1), are shown for relating the Nilsson state wave functions to the ordinary infinite spherical harmonic oscillator functions with $l \cdot A$ coupling. These coefficients for the four band-head states are based on the values of the deformation parameter ($\beta = 0.2$) and the spin-orbit coupling parameters ($\mu_5 = 0.450$, $\mu_6 = 0.448$) which give the correct ordering of Nilsson state energies observed in ^{169}Er . Each coefficient is characterized by the principal oscillator number N , the orbital angular momentum l , and the total angular momentum j of the harmonic oscillator function used to expand the Nilsson state.

Table 2. Expansion coefficients for the Nilsson state wave functions^a

Nilsson state	ℓ	j	Coefficient	Coefficient squared
$1/2^-(521)$	1	$1/2$	- .508434	.2585
	1	$3/2$	.304393	.0926
	3	$5/2$	- .495173	.2451
	3	$7/2$	- .434983	.1892
	5	$9/2$	.445203	.1982
	5	$11/2$	.127389	.0162
$5/2^-(512)$	3	$5/2$	- .056945	.0032
	3	$7/2$	.880763	.7757
	5	$9/2$	.434443	.1887
	5	$11/2$	- .179644	.0323
$7/2^+(633)$	4	$7/2$	- .017510	.0003
	4	$9/2$	.197435	.0389
	6	$11/2$	- .080460	.0065
	6	$13/2$	.976851	.9542
$5/2^-(523)$	3	$5/2$	.235259	.0553
	3	$7/2$	- .378521	.1433
	5	$9/2$	.875482	.7664
	5	$11/2$	.186831	.0349

^aTabulated in reference (1).

Using the relationship $N = 2 (n_r - 1) + \ell$ between the principal oscillator number, the orbital angular momentum, and the radial node number n_r of the harmonic oscillator functions, one can express these functions in their usual shell model form of $|n_r \ell j \rangle$. As one can see from the squares of the coefficients in Table 2, the $5/2^-(523)$ state is predominately a sum of the $|2f7/2 \rangle$ and $|1h13/2 \rangle$ functions, while the $7/2^+(633)$ state is almost completely made up of the $|1g13/2 \rangle$ function.

Since the E1 operator can connect states differing in spin by no more than one unit of angular momentum, the decay of the $5/2^-(523)$ state to the $7/2^+(633)$ Nilsson state must occur through transitions connecting those components of the initial and final state wave functions having $\Delta J \leq 1$. These cases involve the portions of the wave functions having small amplitude. In contrast, M1 and E2 operators connect several of the major components of the $1/2^-(521)$ and $5/2^-(512)$ band-head states with the $5/2^-(523)$ state. The same argument explains the hindered E1 decay of the $7/2^+(633)$ state to the $5/2^-(512)$ band reported by Bonitz (12) in terms of the wave function expansions of Table 2.

CONCLUSIONS

The radioactive decays of ^{168}Ho and ^{169}Ho have been investigated using high resolution gamma-ray spectrometers. Many new transitions have been observed in both decays. A decay scheme for ^{168}Ho is proposed for the first time. The decay scheme of ^{169}Ho is shown to be consistent with the recently determined energy level structure for ^{169}Er to which ^{169}Ho decays. The gamma-ray decay pattern deduced for the $5/2^-(523)$ band-head state in ^{169}Er is in excellent theoretical agreement with the predictions of the Nilsson model.

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