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SERVOMECHANISMS
WITH FORCE FEEDBACK

by

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SERVOMECHANISMS WITH FORCE FEEDBACK

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ABSTRACT

The effect of positive force feedback on a linear, proportional, positional servomechanism is analyzed, using a second-order model. This type of feedback is shown to cancel the effect of a load disturbance on the behavior of the servomechanism. A vacuum-tube analog for the servomechanism is developed, with voltage and current analogous to velocity and force, respectively; then, nullifying the effect of load forces on the behavior of a servomechanism becomes simply a matter of lowering the output impedance of the analog. Bode's results on the effect of feedback on vacuum-tube circuit impedances are readily applied to the analog, and current (force) feedback is shown to offer a promising means for lowering output impedance and thus making the response insensitive to external loads. Experimental evidence is presented in support of the analysis.

The effect of deflection in the force transducer is also analyzed, when an inertial load is assumed. It is shown that the spring-mass combination of the transducer-load undergoes sustained oscillation at its natural frequency when force feedback is applied to the servomechanism. However, the amplitude of the oscillation can be made insignificantly small by making the force transducer stiff enough so that the period of oscillation of the transducer-load combination is much shorter than the rise time of the servomechanism.

When the nature of the load is purely inertial, force feedback becomes acceleration feedback, and a chapter is devoted to exploration of the effects of positive acceleration feedback on the stability and speed of response of a positional servomechanism. For a second-order model, analytical and

experimental results are presented which demonstrate that positive acceleration feedback reduces the effective inertia of the servomotor and thus improves both damping and speed of response.

The mathematical model is extended to include third-order systems as well, and when the extra, third-order time delay is significant the use of positive acceleration feedback is shown to produce a great reduction in stability. Here, analog computer results are presented which demonstrate the oscillatory nature of a system with positive acceleration feedback when significant higher-order effects are present.

Two types of force-reflecting servomechanisms are also analyzed. These are systems which produce, at the input, a sense of feel of the load forces. Criteria for evaluating the systems mathematically are established and the application of these criteria shows the adverse effects on performance produced by the friction and inertia of the components. The use of velocity and acceleration feedback to compensate for friction and inertia is analyzed, as well as the use of negative force feedback. Just as positive force feedback is shown to lower the output impedance of a positional servomechanism, so also is negative force feedback shown to increase output impedance. When negative force feedback is applied around the input end of a force-reflecting servomechanism, the resulting increase in input impedance is manifested by an apparent reduction in inertia and friction, and thus an improvement in the sense of feel.

Laboratory models of both types of force-reflecting servomechanisms have been constructed, and the effect of force feedback in improving the sense of feel is described. Some quantitative measurements are given which verify the analytical results and offer evidence that the mathematical model (second-order) is a good representation of the physical model actually constructed.

CHAPTER 1

INTRODUCTION

It is the purpose of this dissertation to examine a class of linear proportional servomechanisms in which an electrical signal proportional to output force is used to improve performance. The effect of this "force feedback" on a positional servomechanism is analyzed as well as the effect on a special type of servomechanism which reflects load forces back to the input. This latter type of servomechanism is called "force reflecting."

Laboratory models of these servomechanisms have been designed and constructed, and experimental data are presented in support of the analysis.

1.1 The Mathematics of Servomechanism Theory

Many servomechanisms are made up of electrical and mechanical components for which familiar circuit laws and Newton's laws of motion give, for the relation between input and output quantities, a linear integro-differential equation with constant coefficients. This dissertation is concerned with such servomechanisms, customarily termed linear and proportional; i.e., they can be represented by linear integro-differential equations with constant coefficients. In order to place its feedback nature in evidence, a servomechanism is described mathematically by considering first its component parts, called control elements, and then adding the effect of its feedback arrangement.

A control element (linear, proportional) is a device which produces a known response (output) when excited by a known stimulus (input). In terms of the mathematical model shown schematically in Fig. 1.1, the stimulus, r , and response, c , are functions of time and are related by a linear integro-differential equation. As a simple example, the control element may be a mass whose response is a positional movement, $c(t)$,

when its stimulus is a force, $r(t)$. In this example, the integro-differential equation relating stimulus to response is

$$r = m \frac{d^2c}{dt^2} \quad . \quad (1.1)$$

A slightly more complicated example is the response (voltage) of a simple resistance-capacitance low-pass filter when the stimulus is a voltage.

The equation is

$$r = c + RC \frac{dc}{dt} \quad . \quad (1.2)$$

Finally, the system shown in Fig. 1.2 has an integro-differential equation relating input force (stimulus) and output force (response) that can be derived by d'Alembert's Principle:

$$r = M \frac{d^2x_1}{dt^2} + K_1x_1 + B \frac{dx_1}{dt} - B \frac{dx_2}{dt} \quad (1.3)$$

$$0 = K_2x_2 + B \frac{dx_2}{dt} - B \frac{dx_1}{dt} \quad (1.4)$$

$$c = K_2x_2 \quad . \quad (1.5)$$

From Eq. (1.4), x_1 can be written as follows:

$$x_1 = \frac{K_2}{B} \int x_2 dt + x_2 \quad . \quad (1.6)$$

Inserting x_1 and its derivatives into Eq. (1.3) gives

$$r = \frac{MK_2}{B} \frac{dx_2}{dt} + M \frac{d^2x_2}{dt^2} + \frac{K_1K_2}{B} \int x_2 dt + K_1x_2 + K_2x_2 \quad . \quad (1.7)$$

Since, from Eq. (1.5), $c = K_2x_2$, it follows that

$$r = \frac{M}{K_2} \frac{d^2c}{dt^2} + \frac{M}{B} \frac{dc}{dt} + \frac{K_2 + K_1}{K_2} c + \frac{K_1}{B} \int c dt \quad . \quad (1.8)$$

By the use of operational methods, the integro-differential equation of a control element yields a relation between the Laplace transform

Fig. 1.1 General block diagram of a control element

Fig. 1.2 Schematic of a mechanical system

of the input and that of the output. This relation is of the form

$$C(s) = R(s)G(s) \quad , \quad (1.9)$$

where $C(s)$ is the transform of the output, $R(s)$ is the transform of the input, and $G(s)$ is a rational algebraic fraction which characterizes the control element completely. $G(s)$ is called the transfer function of the control element. In the example of Fig. 1.2, Eq. (1.8) can be transformed into

$$R(s) = \frac{M}{K_2} s^2 C(s) + \frac{M}{B} s C(s) + \frac{K_2 + K_1}{K_2} C(s) + \frac{K_1}{Bs} C(s) \quad . \quad (1.10)$$

Thus, the transform of the output is

$$C(s) = \frac{BK_2 s R(s)}{BMs^3 + K_2 Ms^2 + B(K_1 + K_2)s + K_1 K_2} \quad . \quad (1.11)$$

In Eq. (1.11), the input is $R(s)$, the output is $C(s)$, and the transfer function of the system is

$$G(s) = \frac{BK_2 s}{BMs^3 + K_2 Ms^2 + B(K_1 + K_2)s + K_1 K_2} \quad . \quad (1.12)$$

A control system may consist of several control elements. For example, Fig. 1.3 shows a system in which the response of control element #1 is the stimulus for control element #2. It is evident that the Laplace transform of the output of the entire system, in terms of its input, is given by

$$C(s) = G_2(s)G_1(s)R(s) \quad (1.13)$$

or

$$C(s) = G(s)R(s) \quad , \quad (1.14)$$

where G_1 is the transfer function of control element #1, G_2 is the transfer function of control element #2, and $G = G_1 G_2$ is the transfer function of the combination.

Now consider the system shown in Fig. 1.4. This system consists of linear proportional control element #1 with transfer function $G(s)$ and linear proportional control element #2 with transfer function $H(s)$ in a "feedback loop." The stimulus for control element #2 is the response of the system, $c(t)$, while the stimulus for control element #1 is not the input to the system, $r(t)$, but the difference between the input to the system and the response of control element #2. The Laplace transform of the equation of this system is

$$C(s) = G(s)R(s) - G(s)C(s)H(s) \quad , \quad (1.15)$$

from which is obtained

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \quad . \quad (1.16)$$

The feedback control system of Fig. 1.4 is called a positional servomechanism if the output quantity $c(t)$ is a mechanical position. Equation (1.16) is the equation which characterizes any linear, proportional, single-input servomechanism. It is with such a servomechanism that this dissertation will be concerned.

In terms of Eq. (1.16), the problem of servomechanism analysis may be stated as follows: given the transfer function of the entire system, $C/R = G/(1 + HG)$, what is the response, $c(t)$, of the servomechanism to a specified input $r(t)$? The problem of servomechanism synthesis may be stated as follows: given the desired response $c(t)$ to a specified input $r(t)$, what must $G/(1 + HG)$ be, and what physical components will yield the required $G(s)$ and $H(s)$? Since the engineering approach to a servomechanism problem always involves both analysis and synthesis, the essentials of elementary servomechanism theory are:

1. determining $G(s)$ and $H(s)$ for a particular configuration of physical components and, conversely, determining a suitable configuration of components to obtain a particular $G(s)$ and/or $H(s)$;

Fig. 1.3 Block diagram of a combination of control elements

Fig. 1.4 Block diagram of a feedback control system

2. discovering the relationship between $G/(1 + HG)$ and the actual physical response of the system to particular inputs.

The first of these two essentials involves the writing of integro-differential equations describing each of the components in a system. The second classically involves the often tedious procedure of determining inverse Laplace transforms. The servomechanism designer avoids the latter procedure for all but the simplest cases and settles for methods which yield useful engineering information about the relation between the component values and the system response in a more efficient manner. One such method is the root-locus^{(4)*} procedure, and this is used throughout Chapter 3 of this dissertation. Furthermore, analog computer results are presented in order to eliminate entirely the necessity for hand calculation of inverse transformations.

1.2 Scope and Division of Problem

Chapter 2 of this dissertation discusses the use of force feedback in a positional servomechanism. The motivation for incorporating force feedback is to compensate automatically for external loads. The reasoning, based on physical intuition, is described below.

1. If a servomechanism is designed to respond in a particular way when it is unloaded, its behavior under load will usually depart from the desired operation.
2. There are many requirements for a servomechanism which is expected to operate under a wide variety of load conditions.
3. Measuring the external load forces on the output member and supplying a control signal to produce automatically a counterforce appear to be a logical method of nullifying the effect of an external load on the response of a servomechanism.

The instrument used to perform the actual measurement of load force is essentially a Hooke's Law device; deflection is produced and this

* Superior numbers, when they occur in the text proper, refer to entries in the Bibliography.

deflection is the measured quantity. Although the instrument itself is not described in detail, the effect of its deflection on the servomechanism performance is analyzed in Chapter 2.

As a logical consequence to the study of force feedback, the analysis of a system with acceleration feedback is undertaken in Chapter 3. Information regarding the effect of additional system time delays is obtained, and this information can be made to apply to force feedback as well as to acceleration feedback.

The final chapter of this dissertation, Chapter 4, treats of force-reflecting servomechanisms - a class of control systems in which an ordinary servomechanism controls the output position from an input or command position, and in addition, the external loads at the output are applied remotely to the input. Such a system has a bilateral nature, and conventional servomechanism theory plays only a small part in its analysis; rather, two-terminal pair network ideas are used.⁽³⁾ Control elements, as defined in Sec. 1.1, are present in the system, but the relation between output and input is a relation between two input variables and two output variables (force and position) rather than between a single input $r(t)$ and an output $c(t)$.

Force-reflecting servomechanisms play an important role in control system applications whenever a sense of feel must accompany the remote positioning of an output member.^(6,7) Examples are: power steering of a land, sea or air vehicle; remote performing of manual operations, as with a robot. Operations of this latter type have become increasingly important with the increase in the use of radioisotopes and the handling of radioactive materials, and manipulators, devices which act as remote extensions of the arms of their operators, are required to perform a variety of delicate tasks, including operating precision machines and running all kinds of chemical analyses. The proper performance of this sort of work obviously requires a sense of feel, particularly if, as is generally

the case, the very nature of the remote operation impairs direct vision or precludes it altogether. Chapter 4 is concerned with the design of force-reflecting manipulators which employ servomechanisms as their principal parts.

CHAPTER 2

POSITIONAL SERVOMECHANISM WITH FORCE FEEDBACK

This chapter discusses the use of force feedback in a positional servomechanism. Of chief interest is the fact that the response of such a servomechanism can be made independent of external loads or disturbances.

2.1 The Mathematical Model

The servomechanism to be used as a reference model in this discussion is shown schematically in Fig. 2.1. It consists of a synchro pair which generates a voltage proportional to the error between the input position, r , and the output position, c ; an amplifier; a two-phase induction motor and a mechanical load geared to the motor shaft. The gear ratio is 1:1.

It is assumed that the speed-torque curves of the motor, for rated voltage on the fixed phase and several control phase voltages, are as shown in Fig. 2.2, with stalled (zero-speed) torque being proportional to control phase voltage. The load is assumed to be purely inertial, with moment of inertia J_1 lb-ft-sec², the moment of inertia of the motor rotor is J_m lb-ft-sec²; the stalled torque is K_m lb-ft per control phase volt; the gain of the amplifier is K_a , the synchro pair yields one volt per radian of difference between r and c ; the slope of the torque-speed curves is $-K_b$ lb-ft per radian per second. From the speed-torque curves of Fig. 2.2,

$$T_{\text{motor}} = -K_b \frac{dc}{dt} + K_a K_m V_a \quad , \quad (2.1)$$

where V_a is the amplifier input voltage. The motor torque is also given by

$$T_{\text{motor}} = (J_m + J_1) \frac{d^2c}{dt^2} \quad . \quad (2.2)$$

Equations (2.1) and (2.2) produce

$$(J_m + J_1) \frac{d^2c}{dt^2} + K_b \frac{dc}{dt} = K_a K_m V_a \quad . \quad (2.3)$$

Fig. 2.1 Schematic of a servomechanism

Fig. 2.2 Speed-torque curves of a servomotor. Rated voltage is applied to the fixed phase and curves 1, 2, 3 and 4 have .25 rated, .5 rated, .75 rated and 1.0 rated voltage respectively on the control phase.

Fig. 2.3 Block diagram of a servomechanism.

The transfer function of the amplifier, motor and load is obtained by taking the Laplace transform of Eq. (2.3) and factoring:

$$(J_m + J_l)s^2C + K_b sC = K_a K_m V_a \quad ; \quad (2.4)$$

$$\frac{C(s)}{V_a(s)} = G_m(s) = \frac{K_a K_m}{(J_m + J_l)s^2 + K_b s} \quad . \quad (2.5)$$

The block diagram of the system is shown in Fig. 2.3. The transfer function of the entire servomechanism is

$$\frac{C(s)}{R(s)} = \frac{G_m(s)}{1 + H(s)G_m(s)} \quad . \quad (2.6)$$

In this case, $H(s)$ is unity and

$$\frac{C(s)}{R(s)} = \frac{G_m(s)}{1 + G_m(s)} \quad . \quad (2.7)$$

Substitution of the value of $G_m(s)$ from Eq. (2.5) gives

$$\frac{C(s)}{R(s)} = \frac{\frac{K_a K_m / s}{(J_m + J_l)s + K_b}}{1 + \frac{K_a K_m / s}{(J_m + J_l)s + K_b}} \quad . \quad (2.8)$$

The equation of the system is

$$\frac{C(s)}{R(s)} = \frac{K_a K_m}{(J_m + J_l)s^2 + K_b s + K_a K_m} \quad . \quad (2.9)$$

2.2 Response of the Servomechanism

Equation (2.9) contains all the essential information about the performance of the servomechanism of Fig. 2.1. Given an input $r(t)$, it is possible to compute $R(s)$, the Laplace transform of $r(t)$. Insertion of this transformed expression into Eq. (2.9) produces the transformed output $C(s)$. The actual response, $c(t)$, is obtained by computing the inverse Laplace transform of $C(s)$.

A common method of analyzing the servomechanism of Fig. 2.1 is to consider its response to a unit step function input. This is the response when the input member is suddenly moved from a reference position to a position one radian away, i.e., $r(t)$ is given by

$$r(t) = \begin{cases} 0, & t \leq 0 \\ 1, & t > 0 \end{cases} \quad (2.10)$$

The Laplace transform of $r(t)$ is $1/s$ and insertion of this expression into Eq. (2.9) gives

$$C(s) = \frac{\frac{K_a K_m}{(J_m + J_l)s}}{s^2 + \frac{K_b s}{J_m + J_l} + \frac{K_a K_m}{J_m + J_l}} \quad (2.11)$$

Because of the quadratic nature of the expression in the denominator of Eq. (2.11), there are three possible forms for the inverse transform $c(t)$. If the discriminant, $[K_b/(J_m + J_l)]^2 - 4[K_a K_m/(J_m + J_l)]$, is negative, the system is oscillatory, i.e., underdamped; if zero, the system is nonoscillatory and critically damped; if positive, the system is nonoscillatory and over-damped. The designer must determine from the system specifications what degree of damping is required, consistent with other requirements (e.g., speed of response). In arriving at the final design, he selects a motor, fixing K_b , J_m and K_m , and an amplifier, fixing K_a , in order to realize suitable behavior of the servomechanism. The only factor over which he has no control is the load inertia, and it is apparent that if the load inertia J_l varies, then the behavior of the servomechanism of Fig. 2.1 will also vary. For those applications where the response of a servomechanism must not depend on its load, force feedback - or, in the particular case considered here, torque feedback - provides a direct way of making a servomechanism insensitive to its load.

2.3 Effect of Force (Torque) Feedback

Figure 2.4 is a schematic of the servomechanism of Fig. 2.1 with an additional feedback loop which measures torque on the output shaft and provides a torque-proportional signal to the amplifier. In addition to the constants defined previously, the torque transducer produces K_t volts per lb-ft of load torque. Since the load in this example is purely inertial, the transfer function of the torque transducer is

$$\frac{V(s)}{C(s)} = K_t J_1 s^2 \quad . \quad (2.12)$$

Figure 2.5 is a block diagram of the torque feedback loop alone without the positional feedback. The transfer function of this loop is

$$\frac{C}{R'} = \frac{\frac{K_m K_a}{(J_m + J_1)s^2 + K_b s}}{1 - \frac{J_1 K_m K_a K_t s^2}{(J_m + J_1)s^2 + K_b s}} \quad (2.13)$$

or

$$\frac{C}{R'} = \frac{K_m K_a}{(J_m + J_1 - K_m K_a K_t J_1)s^2 + K_b s} \quad . \quad (2.14)$$

When the positional feedback is considered, the block diagram of Fig. 2.6 applies and the overall system transfer function is

$$\frac{C}{R} = \frac{\frac{C}{R'}}{1 + \frac{C}{R'}} = \frac{K_m K_a}{(J_m + J_1 - K_m K_a K_t J_1)s^2 + K_b s + K_m K_a} \quad . \quad (2.15)$$

It is evident that if $K_t = 1/K_m K_a$,

$$\frac{C}{R} = \frac{K_m K_a}{J_m s^2 + K_b s + K_m K_a} \quad , \quad (2.16)$$

and the system response no longer depends on the load J_1 .

Fig. 2.4 Schematic of a servomechanism with torque feedback

Fig. 2.5 Block diagram of a torque feedback loop

Fig. 2.6 Block diagram of a servomechanism with torque feedback

Although the above analysis assumes an inertial load, the use of torque feedback nullifies the effect of any type of load - active or passive - on the response of the servomechanism of Fig. 2.4. Equation (2.2) can be rewritten for the case of a general load, $w(t)$, in the form

$$T_{\text{motor}} = J_m \frac{d^2c}{dt^2} + w(t) \quad , \quad (2.17)$$

and Eq. (2.1) can be rewritten, when position and torque feedback are both present, in the form

$$T_{\text{motor}} = -K_b \frac{dc}{dt} + K_a K_m (r - c + K_t w) \quad . \quad (2.18)$$

Transformed and combined, Eqs. (2.17) and (2.18) become

$$K_a K_m R = (J_m s^2 + K_b s + K_a K_m) C + W(1 - K_m K_a K_t) \quad . \quad (2.19)$$

Again, for $K_t = 1/K_m K_a$, the effect of W disappears and the system equation is

$$\frac{C}{R} = \frac{K_a K_m}{J_m s^2 + K_b s + K_a K_m} \quad . \quad (2.20)$$

Equation (2.20) shows that the servomechanism response is independent of load.

2.4 Experimental Verification of the Mathematical Model

The servomechanism of Fig. 2.1 has been assembled in the laboratory, in an attempt to verify the results of the previous section. To obtain reliable quantitative verification it is necessary that the experimental model of the servomechanism fit the mathematical model as closely as possible. Essentially, it must be demonstrated that the motor and its servo-amplifier are accurately described by the transfer function of Eq. (2.5). For this demonstration, five experiments have been performed and a detailed description follows.

Experiment #1 was a measurement of the factor $K_a K_m$ of Eq. (2.5). Since, from Eq. (2.1), the motor torque is given by

$$T_{\text{motor}} = -K_b sC + K_m K_a V_a \quad , \quad (2.21)$$

a measurement of stalled torque ($sC = 0$) as a function of input voltage to the amplifier gives

$$\left. \frac{T_{\text{motor}}}{V_a} \right|_{sC = 0} = K_m K_a \quad . \quad (2.22)$$

Experiment #2 was a measurement of the term K_b of Eq. (2.5). In this experiment the servomechanism of Fig. 2.1, unloaded, was operated with positive tachometric (velocity) feedback, as shown in the block diagram of Fig. 2.7. The mathematical model now has the transfer function

$$\frac{C}{R} = \frac{\frac{K_a K_m}{s(J_m s + K_b)}}{1 - \frac{K_a K_m K_v s}{s(J_m s + K_b)}} \cdot \frac{1}{1 + \frac{\frac{K_a K_m}{s(J_m s + K_b)}}{1 - \frac{K_a K_m K_v s}{s(J_m s + K_b)}}} \quad , \quad (2.23)$$

which reduces to

$$\frac{C}{R} = \frac{K_a K_m}{J_m s^2 + (K_b - K_a K_m K_v)s + K_a K_m} \quad . \quad (2.24)$$

It is evident from Eq. (2.24) that, if $K_v = K_b / K_a K_m$, the servomechanism will undergo sustained oscillation. For experiment #2, the transfer function of the tachometer, K_v volt-sec per radian, was measured and K_b was obtained from

$$K_b = K_v K_a K_m \quad (2.25)$$

when K_v was just large enough to produce sustained oscillations.

Fig. 2.7 Block diagram of a servomechanism with positive tachometric feedback.

EXPERIMENT	$K_a K_m \frac{\text{lb-ft}}{\text{volt}}$	$K_b \frac{\text{lb-ft-sec}}{\text{rad}}$	$J_m \frac{\text{lb-ft-sec}^2}{\text{rad}}$
1	4.0×10^{-4}		
2		2.21×10^{-5}	
3			4.5×10^{-6}

Fig. 2.8 Results of experiments #1, #2, and #3.

Experiment #3 was designed to measure J_m and was performed simultaneously with Experiment #2. When the value of K_v was just sufficient to produce sustained oscillation, the frequency of oscillation, ω_n , was measured. Equation (2.24) with $K_v = K_b/K_a K_m$ is

$$\frac{C}{R} = \frac{\frac{K_a K_m}{J_m}}{s^2 + \frac{K_a K_m}{J_m}}, \quad (2.26)$$

and the frequency at which the servomechanism oscillates is clearly

$$\omega_n = (K_a K_m / J_m)^{\frac{1}{2}}, \quad (2.27)$$

from which is obtained

$$J_m = \frac{K_a K_m}{\omega_n^2}. \quad (2.28)$$

Experiments #1, #2 and #3 produce numerical values for the quantities $K_a K_m$, K_b and J_m , under the assumption that the mathematical model of Sec. 2.1 is an accurate representation of the servomechanism. Actually, the system can be expected to be linear only over a restricted range. Thus, $K_a K_m$ is measured at values of voltage which do not cause amplifier or motor saturation; K_b and J_m are measured at low speeds, where the slope of the actual torque-speed curves is relatively constant.

Experiment #4 was a frequency-response test of the motor-amplifier combination. The purpose of this test was to provide a check on the validity of the mathematical model and the measured values of K_b and J_m . Under steady-state sinusoidal conditions, the response of the unloaded motor bears a certain relationship in amplitude and phase to the amplifier input voltage. The rms values of velocity and voltage and the phase between them were the measured data, and, as is customary, two curves were plotted with these data; first, a curve of the ratio (in decibels) of rms velocity to rms voltage versus the logarithm of frequency; second, a curve of the phase angle between velocity and voltage versus the logarithm of frequency.

In the mathematical model the sinusoidal steady-state relation between velocity and input voltage is obtained from Eq. (2.5):

$$\frac{sC(s)}{V_a(s)} = \frac{K_a K_m}{J_m s + K_b} \quad (2.29)$$

Under sinusoidal steady-state conditions,

$$\text{Gain} = \left| \frac{j\omega C(j\omega)}{V_a(j\omega)} \right| = \frac{\text{rms velocity}}{\text{rms voltage}} = \frac{\frac{K_a K_m}{J_m}}{[(K_b/J_m)^2 + \omega^2]^{\frac{1}{2}}} \quad (2.30)$$

and

$$\theta = \text{phase angle between velocity and voltage} = -\tan^{-1} \frac{\omega J_m}{K_b} \quad (2.31)$$

From Eqs. (2.30) and (2.31) curves were plotted and compared with the data of Experiment #4, and the check on the model and measurements was obtained.

The results of Experiments #1, #2 and #3 are tabulated in Fig. 2.8. With these values, Eq. (2.30) becomes

$$\left| \frac{j\omega C}{V_a} \right| = \frac{88.8}{(24.2 + \omega^2)^{\frac{1}{2}}} \quad (2.32)$$

and Eq. (2.31) becomes

$$\theta = -\tan^{-1} 0.204 \omega \quad (2.33)$$

Figure 2.9 contains the graph of Eq. (2.32), the mathematical model, and of the corresponding physical data of Experiment #4. Figure 2.10 contains the graph of Eq. (2.33), the mathematical model, and of the corresponding physical data of Experiment #4. The agreement is sufficiently close and the mathematical model represents the physical system adequately. In the center of the graph of Fig. 2.9, the experimental data fall below the calculated curve and the maximum discrepancy is about $2\frac{1}{2}$ db. The experimental values here are thus $2\frac{1}{2}$ db lower (about 25% less) than for the calculated curve. Of much greater importance, however, is the fact that

Fig. 2.9 Frequency response of a servomotor - amplitude

Fig. 2.10 Frequency response of a servomotor - phase

the data coincide with the curve at high frequencies, demonstrating that the system does actually possess a single significant time delay (equal to J_m/K_b) and that this delay has been accounted for in the analysis.

As a final check on the validity of the mathematical model, the response of the servomechanism of Fig. 2.1 to a step function, as obtained in the laboratory with a recorder, is plotted in Fig. 2.11 along with the calculated response of the model of Eq. (2.16) with the values of $K_a K_m, K_b$ and J_m obtained in Experiments #1, #2 and #3. Again, the agreement is good. The position-measuring device and recorder are limited in accuracy to within about 5% of the total range used and the error between the experimental and calculated curves is seen to be within this tolerance.

2.5 Experimental Verification of the Effect of Torque Feedback

Having established the validity of the mathematical model of Sec. 2.1, an experiment to verify the conclusion of Sec. 2.3 is described here. First, sufficient tachometric feedback (negative) is employed to make the servomechanism critically damped at zero load. Then graphs of the actual response of the laboratory servomechanism with various loads are obtained on a recorder. These graphs are contained in Fig. 2.12. Curve A is the response of the servomechanism with zero load; curves B and C are the responses with two different inertial loads and no torque feedback; curve D (indistinguishable from A) is the response to the loads of curves B and/or C when torque feedback is added and set so that $K_t = 1/K_a K_m$. The experiment demonstrated that torque feedback, when applied to a servomechanism described by Fig. 2.4 and the mathematical model of Sec. 2.3, produces a response which is independent of load.

2.6 Impedance Analysis

The problem of this chapter, that of making the response of a servomechanism independent of its load, can be approached from quite a different viewpoint. An electrical-mechanical analogy makes this new approach clear.

Fig. 2.11 Step-function response of a servomechanism with no load.

Consider the analogy suggested by the following pairs:

voltage V - angular velocity $s\theta$

current I - torque T

conductance G - coefficient of viscous friction B

capacitance C - moment of inertia J

inverse inductance $\Gamma = 1/L$ - stiffness K .

In terms of this analogy, a possible definition of mechanical impedance is given by the following equation:

$$Z(s) = \frac{s\theta(s)}{T(s)} \quad (2.34)$$

In order to make the response of a servomechanism insensitive to external load torques, it is necessary to lower the output impedance of the servomechanism. One means for accomplishing this reduction in output impedance is by the use of additional feedback in the servomechanism, and this section of Chapter 2 discusses the effect of feedback on impedance. A vacuum tube amplifier serves as the model for the discussion.

Consider the general vacuum tube circuit of Fig. 2.13, where the tube is operated under small-signal, linear conditions. If the grid connection between P and P' is broken and a unit electrical signal (1 volt) is impressed between P' and K , the voltage measured at P (with respect to K) is the return voltage that appears by transmission around the circuit. This voltage is called the return ratio for the tube and is designated by the symbol T . It is also convenient to refer to a quantity F , called the return difference for the tube, which is given by

$$F = 1 - T \quad (2.35)$$

This latter quantity is a generalization of the familiar $(1 + \mu B)$ of conventional feedback amplifier theory.

The effect of feedback on impedance levels is readily examined on the basis of the return difference F . Bode⁽¹⁾ has shown that the impedance viewed at any two terminals of a vacuum-tube network is given by

$$Z = Z_0 \frac{F(0)}{F(\infty)} \quad (2.36)$$

Fig. 2.12 Step-function response of a servomechanism with inertial load and torque feedback. Curves A, B, and C are for no-load, half-load and full-load respectively. Curve D (indistinguishable from A) is the response when full torque feedback is added for all loads.

Fig. 2.13 Vacuum-tube circuit for defining return difference.

Fig. 2.14 Schematic of a cathode-follower

Here, Z_0 is the impedance when a tube is replaced by its internal impedance and the μ or g_m is zero. The quantity $F(0)$ is the return difference for the tube when the two terminals in question are short-circuited; $F(\infty)$ is the return difference for the tube when the two terminals in question are open-circuited.

As an example, consider the output impedance of the cathode follower of Fig. 2.14. By conventional analysis, one writes equations as follows:

$$E_2 = (I_2 + I_p)R_k \quad (2.37)$$

$$E_g = -\mu E_2 = I_p(r_p + R_k) + I_2 R_k \quad (2.38)$$

Elimination of I_p gives

$$-\mu E_g = \frac{E_2 - I_2 R_k}{R_k} (r_p + R_k) + I_2 R_k \quad (2.39)$$

and

$$E_2 \frac{r_p + R_k + \mu R_k}{R_k} = I_2 r_p \quad (2.40)$$

The output impedance is

$$\frac{E_2}{I_2} = Z = \frac{r_p R_k}{r_p + R_k (1 + \mu)} \quad (2.41)$$

The output impedance for the cathode follower of Fig. 2.14 could also have been obtained using the concept of return difference. With the tube replaced by its internal impedance, the output impedance is

$$Z_0 = \frac{r_p R_k}{r_p + R_k} \quad (2.42)$$

The return difference for the tube when the output is short-circuited is

$$F(0) = 1 \quad (2.43)$$

and when the output is open-circuited,

$$F(\infty) = 1 + \frac{R_k \mu}{r_p + R_k} \quad (2.44)$$

Substitution of Eqs. (2.42), (2.43) and (2.44) into Eq.(2.36) gives

$$Z = \frac{r_p R_k (r_p + R_k)}{(r_p + R_k) (r_p + R_k + \mu R_k)} = \frac{r_p R_k}{r_p + R_k (1 + \mu)} \quad (2.45)$$

for the output impedance of the active network.

To apply this theory to a servomechanism, consider again the servomechanism of Sec. 2.1 with no load. The equation of this system, in transform notation, is

$$K_a K_m (R - C) = (J_m s + K_b) s C \quad (2.46)$$

When Eq. (2.46) is written in matrix form, as

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{-K_a K_m}{s} & J_m s + K_b + \frac{K_a K_m}{s} \end{bmatrix} \begin{bmatrix} sR \\ sC \end{bmatrix}, \quad (2.47)$$

it becomes apparent that an analogous electrical network exists whose node-basis matrix is

$$N = \begin{bmatrix} 0 & 0 \\ \frac{-\mu}{Ls} & (Cs + \frac{1}{R} + \frac{1}{Ls}) \end{bmatrix}. \quad (2.48)$$

Figure 2.15 is the required analog when

$$\left. \begin{aligned} R &= 1/K_b \\ C &= J_m \\ L &= 1/K_a K_m \\ V_1 &= sR \\ V_2 &= sC \\ \mu &= 1 \\ r_p &= 0 \end{aligned} \right\}. \quad (2.49)$$

The transformer is ideal and used for phase reversal.

Fig. 2.15 Vacuum-tube analog of a servomechanism

Fig. 2.16 A second analog of a servomechanism

Another analog exists in the circuit of Fig. 2.16, whose node-basis matrix is

$$N = \begin{bmatrix} 0 & 0 \\ -\frac{g_m}{s} & \left(Cs + \frac{1}{R} + \frac{g_m}{s} \right) \end{bmatrix}. \quad (2.50)$$

For $g_m = K_a K_m$, the latter circuit is topologically a closer analog of the servomechanism itself. The motor-amplifier combination is represented in the analog by the pentode, whose grid represents the amplifier input. The plate current, $g_m E_g$, is the torque $K_a K_m (R - C)$, developed by the motor at stall, and the reduction in torque with increasing velocity is exactly represented by the shunt resistance R . Any load on the motor can be represented in the analog by a load across the output terminals. Finally, the integrators are required since the synchros generate positional signals rather than velocity signals.

For the purposes of this analysis, it is desirable to separate the usual position feedback, shown so clearly in Fig. 2.16, from any other feedback that may be used to lower output impedance. Therefore, a compromise analog is shown in Fig. 2.17. The pentode produces only that part of the output current (torque) which is due to feedback signals other than the basic positional one. Once again, the transformers are ideal and used for phase reversal, and the terminals $Y - Y'$ represent an input to the servo-amplifier intended for additional feedback signals.

The output impedance (Thevenin's equivalent impedance) of the circuit of Fig. 2.17 with the pentode input shorted (no auxiliary feedback) is seen to be

$$Z = \frac{1}{sC + \frac{1}{R} + \frac{1}{Ls}}. \quad (2.51)$$

Fig. 2.17 A final analog of a servomechanism

Fig. 2.18 Analog with voltage feedback

Fig. 2.19 Analog with current feedback

Two possible feedback schemes, sometimes called voltage and current feedback, respectively, are shown in Figs. 2.18 and 2.19. In the circuit of Fig. 2.18, the return differences for the pentode g_m are seen from inspection to be

$$F(0) = 1 + 0 \quad (2.52)$$

and

$$F(\infty) = 1 + \frac{kg_m}{sC + \frac{1}{R} + \frac{1}{Ls}} \quad (2.53)$$

In the circuit of Fig. 2.19 with $R_1 \ll R$ the return differences for the pentode g_m are seen from inspection to be

$$F(0) = 1 - kR_1g_m \quad (2.54)$$

and

$$F(\infty) = 1 - 0 \quad (2.55)$$

For the feedback scheme of Fig. 2.18, the output impedance is, from Eq. (2.36),

$$Z = \frac{1}{sC + \frac{1}{R} + \frac{1}{Ls}} \cdot \frac{1}{1 + \frac{kg_m}{sC + \frac{1}{R} + \frac{1}{Ls}}} \quad (2.56)$$

or

$$Z = \frac{1}{sC + \frac{1}{R} + kg_m + \frac{1}{Ls}} \quad (2.57)$$

In terms of the analogous servomechanism parameters, the impedance is

$$Z = \frac{1}{J_ms + K_b + kK_aK_m + \frac{K_aK_m}{s}} \quad (2.58)$$

It is evident that the feedback of Fig. 2.18 is an analog of negative velocity feedback and that k is analogous to K_v , the transfer function of the tachometer.

For the feedback scheme of Fig. 2.19, the output impedance is

$$Z = \left[\frac{1}{sC + \frac{1}{R} + \frac{1}{Ls}} \right] \left[\frac{1 - kR_1 g_m}{1} \right] \quad (2.59)$$

$$Z = \frac{1 - kR_1 g_m}{sC + \frac{1}{R} + \frac{1}{Ls}} \quad (2.60)$$

In terms of the analogous servomechanism parameters, the impedance is

$$Z = \frac{1 - kR_1 K_a K_m}{J_m s + K_b + \frac{K_a K_m}{s}} \quad (2.61)$$

It is evident that the feedback of Fig. 2.19 is an analog of torque feedback, and that kR_1 is analogous to K_t , the transfer function of the torque transducer.

Equation (2.58) and Equation (2.61) indicate that both of these feedback schemes lower the output impedance of the servomechanism. It is not the purpose of this section to discuss the relative advantages of one or the other, but only to provide an added insight into the effects of torque - and incidentally, velocity - feedback. The method of this section, drawing heavily on the well-established analysis of impedance in feedback amplifiers, is an often-overlooked tool for the servomechanism designer.

2.7 Effect of Deflection in Force Transducer

Present-day devices for measuring torque on a shaft generally measure deflection of the shaft, and considerations of transducer sensitivity make it desirable to have as much deflection per unit torque as possible. Thus it is important to discover the effect of torque feedback on systems with considerable deflection in their output members. The servomechanism of Sec. 2.1, Fig. 2.1, with the added consideration of stiffness K lb-ft per radian in the output shaft, is the model for discussion. An inertial load is assumed.

The synchro generator can be geared to the motor side or the load side of the output shaft of stiffness K . Thus, two cases are analyzed, corresponding to the two synchro connections.

With the synchro geared to the load end of the flexible shaft, the system equation is, in transform notation,

$$K_a K_m (R - C + K_t J_1 s^2 C) = J_m s^2 \theta_m + J_1 s^2 C + K_b s \theta_m, \quad (2.62)$$

where θ_m is the motor position (different from the output, C , by the amount of deflection in the output shaft) and all other parameters are as previously defined. The output shaft is assumed to behave according to Hooke's law, and thus

$$K(\theta_m - C) = J_1 s^2 C, \quad (2.63)$$

from which is obtained

$$\theta_m = [1 + (J_1 s^2 / K)] C. \quad (2.64)$$

Insertion of Eq. (2.64) into Eq. (2.62) gives

$$K_a K_m (R - C + K_t J_1 s^2 C) = (J_m s^2 + K_b s) [1 + (J_1 s^2 / K)] C + J_1 s^2 C, \quad (2.65)$$

from which is obtained

$$\frac{C}{R} = \frac{K_m K_a}{\frac{J_m J_1}{K} s^4 + \frac{J_1 K_b}{K} s^3 + (J_m + J_1 - K_m K_a K_t J_1) s^2 + K_b s + K_a K_m}. \quad (2.66)$$

There is good reason to suppose, from physical intuition alone, that the dynamic behavior of this servomechanism may be troublesome. First, the placement of the synchro on the load side of the output shaft places a mechanically resonant structure (the spring-mass combination of the load and compliant shaft) within the positional feedback loop. If the natural frequency of this structure is within the bandwidth of the entire system, one might expect a dynamic behavior which evidences dominant oscillations at this natural frequency. Second, if the term $K_a K_m$, which represents the stiffness of the entire servomechanism, is too large, the

servomechanism will be stiffer than one of its members, the output shaft. This would imply that the servomechanism has added a "negative spring" onto the end of the actual spring (the compliant shaft) in order to reduce total compliance. The use of such a negative spring, a device which produces decreasing deflection with increasing force, would seem to warrant particular attention.

A mathematical basis for these suspicions is provided by a Routh-Hurwitz test of the denominator of Eq. (2.66). This test determines whether the roots of the denominator of Eq. (2.66) have positive real parts. If so, the response of the servomechanism will contain a component which increases indefinitely with time, indicating instability. The Routh-Hurwitz evaluation follows. For a discussion of the use of this test, the reader is referred to the bibliography.⁽¹⁰⁾

$$\begin{array}{c}
 \frac{J_m s}{K_b} \\
 \hline
 \frac{J_1 K_b}{K} s^2 + K_b s \left[\frac{\frac{J_m J_1}{K} s^4 + (J_m + J_1 - J_1 K_a K_m K_t) s^2 + K_a K_m}{\frac{J_m J_1}{K} s^4 + J_m s^2} \right] \\
 \hline
 J_1 s^2 (1 - K_a K_m K_t) + K_a K_m
 \end{array}$$

$$\begin{array}{c}
 \frac{K_b s}{K(1 - K_a K_m K_t)} \\
 \hline
 J_1 s^2 (1 - K_a K_m K_t) + K_a K_m \left[\frac{\frac{J_1 K_b}{K} s^3 + K_b s}{\frac{J_1 K_b}{K} s^3 + \frac{K_a K_m K_b s}{K(1 - K_a K_m K_t)}} \right] \\
 \hline
 K_b s - \frac{K_a K_m K_b s}{K(1 - K_a K_m K_t)}
 \end{array}$$

The remainder after two steps in the test is

$$R = K_b s - \frac{K_a K_m K_b s}{K(1 - K_a K_m K_t)} \quad (2.67)$$

For the servomechanism to remain stable, it is necessary that

$$K_b - \frac{K_a K_m K_b}{K - K K_a K_m K_t} \geq 0 \quad (2.68)$$

and

$$K - K K_a K_m K_t - K_a K_m \geq 0 \quad (2.69)$$

For $K_t = 1/K_a K_m$ (full torque feedback), Eq. (2.69) becomes

$$K_a K_m \leq 0 \quad (2.70)$$

which is impossible, and the servomechanism is always unstable. For $K_t = 0$ (no torque feedback), Eq. (2.69) becomes

$$K \geq K_m K_a \quad (2.71)$$

which demonstrates that the servomechanism stiffness cannot exceed that of the compliant output shaft. For stability with intermediate values of torque feedback, Eq. (2.69) requires that

$$K_t \leq \frac{1}{K_a K_m} - \frac{1}{K} \quad (2.72)$$

A more usual synchro connection, that on the motor end of the flexible shaft, is described by the equation

$$K_m K_a (R - \theta_m + K_t J_1 s^2 C) = J_m s^2 \theta_m + J_1 s^2 C + K_b s \theta_m \quad (2.73)$$

Since $\theta_m = [1 + (J_1 s^2 / K)] C$, Eq. (2.73) can be written as

$$\frac{C}{R} = \frac{K_m K_a}{\frac{J_m J_1}{K} s^4 + \frac{J_1 K_b}{K} s^3 + \left[J_m + J_1 - J_1 K_a K_m K_t + \frac{K_a K_m J_1}{K} \right] s^2 + K_b s + K_a K_m} \quad (2.74)$$

For the condition $K_t = 1/K_m K_a$, Eq. (2.74) becomes

$$\frac{C}{R} = \frac{K_m K_a}{J_m s^2 + K_b s + K_m K_a} \cdot \frac{1}{\frac{J_1}{K} s^2 + 1} \quad (2.75)$$

Equation (2.75) shows that the effect of the compliant shaft is isolated from the servomechanism proper. No value of servomechanism $K_m K_a$ will make the system unstable. However, the step function response will always have a component which exhibits sustained oscillation with a frequency

$$\omega_n = (K/J_1)^{\frac{1}{2}} \quad (2.76)$$

The magnitude of this oscillation will depend on the relative magnitudes of the residues of the function C/R of Eq. (2.75) at its various poles.

In physical terms, it is apparent that the above servomechanism and its resonant load are completely isolated (due to the zero output impedance of the servomechanism with torque feedback). Thus, the question of the oscillation of the output shaft and inertial load is not made more difficult because of the presence of the servomechanism. Any spring-mass combination oscillates when moved, and the degree of oscillation depends on how fast it is moved in comparison to the period of its own natural mode of oscillation. Thus, if the servomechanism response (unloaded) is slow, the output shaft can be quite flexible; if the response is fast, the output shaft must be relatively stiff.

The step-function response of the servomechanism of Fig. 2.1, when a compliant output shaft is used and the synchro generator is connected to the motor end of the shaft, has been obtained with an analog computer. For these results, the assumed servomechanism parameters are those of Fig. 2.8 with some modifications. A 40:1 gear ratio was employed, raising $K_a K_m$ and J_m by a factor of 1600. Then K_b was adjusted to obtain critical damping with no load. Thus, the modified parameters used in the computation were

$$\begin{aligned} K_a K_m &= (1600) (4 \times 10^{-4}) = 0.64 \text{ lb-ft/volt} \\ J_m &= (1600) (4.5 \times 10^{-6}) = 0.0072 \text{ lb-ft-sec}^2/\text{rad} \\ K_b &= (4 J_m K_a K_m)^{\frac{1}{2}} = 0.135 \text{ lb-ft-sec/rad} \end{aligned} \quad (2.77)$$

In the computation, two values of load inertia, J_1 lb-ft-sec² per radian, and three values of shaft stiffness, K lb-ft per radian, were assumed; in all,

twelve curves were obtained with an x-y plotter, and they corresponded to the responses with and without torque feedback. These are shown in Figs. 2.20, 2.21 and 2.22

Figure 2.20 illustrates again the effect of inertial loads on the response of a second-order servomechanism and the usefulness of torque feedback in nullifying this effect. The stiffness of the output shaft in this case is infinite.

Figure 2.21 is the graph of the analog computation for the system when finite shaft stiffness is assumed. When no torque feedback is present, the response does not differ appreciably from that of the infinitely stiff case. However, in the presence of torque feedback, the resonant structure (spring-mass) on the output is isolated from any damping afforded by the output impedance of the servomechanism, and sustained oscillation is evident. The frequency of oscillation can be measured from the graphs to be 22 rad/sec and 15.7 rad/sec for $J_1 = 0.052$ and 0.104 respectively. The predicted frequencies (since $K = 26$ lb-ft/rad) were $(26/0.052)^{\frac{1}{2}}$ or 22.3 rad/sec and $(26/0.104)^{\frac{1}{2}}$ or 15.8 rad/sec, which are in very good agreement with the results read from Fig. 2.21.

Figure 2.22 is the analog computation for an even more compliant system. When no torque feedback is present, the response again does not differ appreciably from that of the infinitely stiff case. However, in the presence of torque feedback sustained oscillations are evident. (In Fig. 2.22 these oscillations even diverge, but this can be attributed to the limited accuracy of the analog computer.) The measured frequencies were 14.2 and 9.7 rad/sec for $J_1 = 0.052$ and 0.104, respectively. These corresponded well with the predicted values of $(10.4/0.052)^{\frac{1}{2}} = 14.1$ and $(10.4/0.104)^{\frac{1}{2}} = 10$, respectively.

Finally, comparison of Figs. 2.21 and 2.22 shows that the magnitude of the oscillation increases as the resonant frequency decreases. This supports the general remarks made above on the dependence of the degree of oscillation on the speed of movement relative to the natural period.

Fig. 2.20 Step-function response of a servomechanism as obtained on an analog computer. Two graphs are presented corresponding to two different loads. The stiffness of the output shaft is infinite. Curves A and B are for no torque feedback and full torque feedback respectively.

Fig. 2.21 Step-function response of a servomechanism as obtained on an analog computer. Two graphs are presented corresponding to two different loads. The stiffness of the output shaft is 26 lb-ft/rad. Curves A and B are for no torque feedback and full torque feedback respectively.

Fig. 2.22 Step-function response of a servomechanism as obtained on an analog computer. Two graphs are presented corresponding to two different loads. The stiffness of the output shaft is 10.4 lb-ft/rad. Curves A and B are for no torque feedback and full torque feedback respectively.

2.8 Higher-order Effects

When a servomechanism departs significantly from the mathematical model of Sec. 2.1, the effect of torque feedback changes drastically. Many practical servomechanisms are represented more accurately by the block diagram of Fig. 2.23 where the additional control element may represent a motor field, electrical filter, or other energy-storing device which produces additional time delay in the servomechanism response to some input. The usefulness of torque feedback depends on the significance of such an additional element, and thus, an examination of higher-order effects is necessary whenever the use of torque feedback is contemplated. However, Chapter 3 of this dissertation is a more convenient place for such an examination, and so further discussion of this point is reserved for Chapter 3.

Fig. 2.23 Block diagram of a system with an additional time delay

CHAPTER 3

POSITIONAL SERVOMECHANISM WITH ACCELERATION FEEDBACK

In Chapter 2 it was demonstrated that torque feedback applied to a linear proportional servomechanism can nullify the effect of a load on the behavior of the servomechanism. The specific example considered was that of an inertial load, and Eq. (2.15), which applies to this case, is reprinted for convenience:

$$\frac{C}{R} = \frac{K_m K_a}{(J_m + J_l - K_a K_m K_t J_l) s^2 + K_b s + K_a K_m} \quad (2.15)$$

In addition to demonstrating that the effect of load inertia can be cancelled, Eq. (2.15) suggests that the effect of the motor inertia can also be reduced, merely by setting $K_t > 1/K_a K_m$. When the load inertia is fixed, torque feedback becomes simply acceleration feedback, and the fixed inertial load and torque transducer comprise an accelerometer,

This chapter discusses the effect of acceleration feedback in lowering the effective inertia of a servomotor and extends the mathematical model of Chapter 2 to include the analysis of systems with an additional time delay.

3.1 Second-order Servomechanism

The servomechanism of Chapter 2 is often termed a second-order system (since its differential equation is of order two), and the nature of its response to a typical input depends on the roots of a quadratic equation. Such a servomechanism has a transfer function of the general form.

$$\frac{C}{R} = \frac{K}{J s^2 + F s + K} \quad (3.1)$$

where J is the inertia of the load and motor, F is the effect of the viscous friction of the load and motor in addition to the effect of the back emf of the motor, and K is the stiffness of the servomechanism; that is, K is the

amount of static torque required to deflect the output member by one radian from the reference position. Within the framework of this second-order model, the designer can study the transient behavior of a servomechanism as K , J and F are varied by examining the expression

$$m_1, m_2 = -\frac{F}{2J} \pm \left[\frac{F^2}{4J^2} - \frac{K}{J} \right]^{\frac{1}{2}}, \quad (3.2)$$

which gives the poles of the function C/R of Eq. (3.1) and, ultimately, the formula for the transient response of the system.

From purely physical considerations, certain generalizations can be made concerning the performance of the second-order servomechanism. First, increasing stiffness results in faster response to a step-function input, but poorer relative stability, i.e., greater overshoot and hunting. Second, increasing friction results in slower response, but better stability. Thus it is natural in designing a system to effect a compromise between speed of response and relative stability, two of the more usual criteria by which a servomechanism is evaluated. As long as the motor operates in the linear region, stiffness is easily increased by increasing amplifier gain, and friction (viscous) is increased either with dampers or synthetically by velocity (tachometric) feedback of such a polarity that increased velocity produces increased motor torque in the opposite direction. By incorporating sufficient friction and stiffness into the system, the designer may be able to arrive at a point where the compromise between speed of response and relative stability results in a servomechanism that meets specified requirements.

It is apparent from the results in Chapter 2 that the compromise is not necessary if the motor and load inertia can be reduced, since this appears to result in faster response and greater relative stability. For this reason servomotors are constructed with as low an inertia as possible. When physical restrictions put a lower limit on motor inertia, it seems natural to look to a synthetic means of reducing its effect, for

example, by the use of acceleration feedback of such a polarity that increased acceleration produces increased motor torque in the same direction, thus automatically aiding the motor in overcoming its inertia.

The possibilities suggested by the above physical considerations warrant a mathematical analysis of the system, and so the response of the servomechanism to a unit step-function input is obtained by finding the inverse Laplace transform of

$$C = \frac{K}{s(Js^2 + Fs + K)} \quad (3.3)$$

By conventional means, the inverse Laplace transform is found to be

$$c = 1 - \frac{e^{-\frac{F}{2J}t}}{\left[1 - \frac{F^2}{4KJ}\right]^{\frac{1}{2}}} \sin \left\{ \left[\frac{K}{J} - \frac{F^2}{4J^2} \right]^{\frac{1}{2}} t + \tan^{-1} \frac{\left[1 - \frac{F^2}{4KJ}\right]^{\frac{1}{2}}}{\frac{F}{2(KJ)^{\frac{1}{2}}}} \right\} \quad (3.4)$$

and the substitutions $\omega_n^2 = K/J$, $\zeta = F/2(KJ)^{\frac{1}{2}}$, and $\tau = \omega_n t$ produce

$$c = 1 - \frac{e^{-\zeta\omega_n t}}{(1 - \zeta^2)^{\frac{1}{2}}} \sin \left[(1 - \zeta^2)^{\frac{1}{2}} \omega_n t + \tan^{-1} \frac{(1 - \zeta^2)^{\frac{1}{2}}}{\zeta} \right] \quad (3.5)$$

or

$$c = 1 - \frac{e^{-\zeta\tau}}{(1 - \zeta^2)^{\frac{1}{2}}} \sin \left[(1 - \zeta^2)^{\frac{1}{2}} \tau + \tan^{-1} \frac{(1 - \zeta^2)^{\frac{1}{2}}}{\zeta} \right] \quad (3.6)$$

The response is plotted as a function of normalized time, $\tau = \omega_n t$, for various values of ζ in Fig. 3.1. A clear mathematical basis for the physical observations of the two preceding paragraphs is established; since $\omega_n = (K/J)^{\frac{1}{2}}$ and $\zeta = F/2(KJ)^{\frac{1}{2}}$, the following results are obtained by inspection of the curves of Fig. 3.1: increased K results in increased ω_n but decreased ζ (faster response but poorer stability); increased F results in increased ζ (greater stability but slower response); decreased J results

Fig. 3.1 Normalized step-function response of a second-order servomechanism for various values of damping factor.

in increased ω_n and increased ζ (greater stability and, since the effect on response of ω_n is greater than that of ζ , faster response).

That acceleration feedback provides a reduction in the inertia term of Eq. (3.1) is easily demonstrated. Consider the servomechanism of Chapter 2 whose motor-amplifier has the transfer function, from Eq. (2.5),

$$G_m = \frac{K_a K_m}{J_m s^2 + K_b s} \quad (3.7)$$

when the servomechanism is unloaded. If an accelerometer with transfer function $K_\alpha s^2$ is attached to the output and its signal is fed back to the input, the transfer function (exclusive of the positional feedback) is

$$G_m' = \frac{\frac{K_a K_m}{J_m s^2 + K_b s}}{1 - \frac{K_a K_m K_\alpha s^2}{J_m s^2 + K_b s}} \quad (3.8)$$

or

$$G_m' = \frac{K_a K_m}{(J_m - K_a K_m K_\alpha) s^2 + K_b s} \quad (3.9)$$

When the positional loop is added, the transfer function of the entire servomechanism is

$$\frac{C}{R} = \frac{K_a K_m}{(J_m - K_a K_m K_\alpha) s^2 + K_b s + K_a K_m} \quad (3.10)$$

which demonstrates that acceleration feedback reduces the effective inertia and thus improves both speed of response and relative stability of the second-order servomechanism.

The same experimental servomechanism whose mathematical model has been verified experimentally (Chapter 2) has been modified to include an accelerometer, consisting of a stiff torque-measuring shaft with inertia on the end. The curves of Fig. 3.2 contain the experimentally determined response to a step-function input. Curve A is the response of

the servomechanism with the parameters as defined in the table of Fig. 2.8, Chapter 2. Thus, it is the same curve that appears on Fig. 2.11, Chapter 2. Curve B is the response when sufficient tachometric feedback was added to make the system critically damped, i.e.,

$$(K_b + K_a K_m K_v)^2 = 4 J_m K_a K_m \quad (3.11)$$

$$K_v = \frac{2(J_m K_a K_m)^{\frac{1}{2}} - K_b}{K_a K_m} \quad (3.12)$$

$$K_v = \frac{2(4.5 \times 10^{-6} \times 4.0 \times 10^{-4})^{\frac{1}{2}} - 2.21 \times 10^{-5}}{4.0 \times 10^{-4}} \quad (3.13)$$

$$K_v = 0.157 \frac{\text{volt-sec}}{\text{rad}} \quad (3.14)$$

Curve C is the response when sufficient acceleration feedback was added to make the servomechanism critically damped, i.e.,

$$(J'_m - K_a K_m K_\alpha) = K_b^2 / 4 K_a K_m \quad (3.15)$$

$$K_\alpha = \frac{J'_m - \frac{K_b^2}{4 K_a K_m}}{K_a K_m} \quad (3.16)$$

$$K_\alpha = \frac{9.0 \times 10^{-6} - \frac{(2.21 \times 10^{-5})^2}{4 \times 4 \times 10^{-4}}}{4 \times 10^{-4}} \quad (3.17)$$

$$K_\alpha = 2.175 \times 10^{-2} \frac{\text{volt-sec}^2}{\text{rad}} \quad (3.18)$$

Here, J'_m includes the inertia of the accelerometer.

In addition to the experimentally determined curves (A, B, C), Fig. 3.2 also contains the corresponding calculated responses (A', B', C') for the three experiments. Curves A and B agree, within the limits of accuracy of the experiment, with Curves A' and B'. However, Curve C, the response with acceleration feedback, is substantially different from the

Fig. 3.2 Step-function response of a servomechanism, as obtained by experiment. Curves A, B, and C represent the response with no feedback, tachometric feedback and acceleration feedback respectively. The solid curves are experimental; the dashed curves are calculated.

calculated curve, Curve C', and the reason for this difference lies in a consideration of system linearity.

Following the application of the unit step input, the combination of positional error voltage and accelerometer voltage produced considerable saturation in the amplifier-motor combination. Visual observation of metered motor voltages, during the course of the experiment, indicated that the system was operating in a nonlinear region during the first half of its excursion toward the final position. Thus, the servomechanism was called upon to respond faster than was physically possible. An exact experimental verification of the calculated effects of acceleration feedback would require a different experiment altogether. A larger amplifier and motor and a more sensitive position-measuring device (to allow smaller step-function inputs and still present a readable recording of output position) would be necessary.

In defense of the actual experiment, it should be noted that, with the available laboratory components, the stabilizing effect of acceleration feedback has been adequately demonstrated by the experimental curves of Fig. 3.2. To a lesser degree (less, by reason of saturation), the speed of response is also seen to be improved by the use of acceleration feedback.

3.2 Third-order System

When an additional time delay is present in the system, acceleration feedback may no longer be desirable. As an example of a third-order system, consider the dc servomechanism of Fig. 3.3 (this is termed a Ward-Leonard system). It consists of an amplifier, a generator, a motor, and position pickoffs. The equations of the various components are easily derived, under the assumptions that there are no armature inductance and no armature reaction. These assumptions are quite reasonable; compensating windings are customarily used to eliminate armature reaction and motor-generator sets intended for servomechanism applications are designed with small armature inductance-to-resistance ratios in order that an additional time delay will not be present.

Fig. 3.3 Schematic of a third-order servomechanism.

In the following equations, K_a is the amplifier gain; V_{in} is the amplifier input voltage; I_f is the field current; L_f and R_f are the field inductance and resistance, respectively; K_{mg} is the generator voltage constant; K_{mm} is the motor torque constant; R_a is the combined armature resistance; J_m is the motor inertia; and K_b is the motor back emf constant.

For the amplifier and generator field the equation is

$$K_a V_{in} = I_f R_f + s I_f L_f \quad , \quad (3.19)$$

and thus the transfer function of the amplifier-field is

$$G_1 = \frac{I_f}{V_{in}} = \frac{K_a}{s L_f + R_f} \quad . \quad (3.20)$$

For the generator armature and motor, with no load on the motor, the equations are

$$K_{mg} I_f = V_a = I_a R_a + K_b s C \quad (3.21)$$

and

$$K_{mm} I_a = J_m s^2 C \quad . \quad (3.22)$$

Combination of Eqs. (3.21) and (3.22) produces

$$K_{mg} I_f = \frac{R_a J_m}{K_{mm}} s^2 C + K_b s C \quad . \quad (3.23)$$

Thus, the armature-motor transfer function is

$$G_2 = \frac{C}{I_f} = \frac{K_{mg}}{\frac{K_a}{K_{mm}} J_m s^2 + K_b s} \quad . \quad (3.24)$$

The transfer function of the complete motor-generator-amplifier set (exclusive of positional feedback) is

$$G = G_1 G_2 = \frac{K_a K_{mg}}{(s L_f + R_f) \left[\frac{R_a}{K_{mm}} J_m s^2 + K_b s \right]} \quad (3.25)$$

or

$$G = \frac{K}{s(\tau_1 s + 1)(\tau_2 s + 1)} \quad , \quad (3.26)$$

where

$$\begin{aligned} K &= \frac{K_a K_{mg}}{R_f K_b} \\ \tau_1 &= \frac{L_f}{R_f} \\ \tau_2 &= \frac{R_a J_m}{K_{mm} K_b} \end{aligned} \quad (3.27)$$

When the positional feedback is added, the transfer function of the entire third-order servomechanism is

$$\frac{C}{R} = \frac{G}{1 + G} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2)s^2 + s + K} \quad (3.28)$$

For any input $R(s)$, Eq. (3.28) can be solved for $C(s)$; then $c(t)$, the actual response, can be computed. Unlike the second-order system, however, general statements about the relation between speed of response or degree of stability and the values of τ_1 , τ_2 and K cannot be made. Even a general expression for the poles of the function C/R of Eq. (3.28) is unwieldy and the general formula for the actual response, corresponding to that of Eq. (3.5) for the second-order case, is much too complicated to allow one to see the effect of changes in τ_1 , τ_2 and K . Therefore in this chapter a solution will be found for the poles of the function C/R of Eq. (3.28) by the graphical procedure of root-locus diagrams, which point up well the effect of the system parameters on the poles. The importance of these poles in determining the actual response of the servomechanism to a step-function input depends on the residues of the function

$$C = \frac{\frac{K}{s}}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2)s^2 + s + K} \quad (3.29)$$

at these poles. Some idea of the sizes of the residues is obtained by inspection of the root-locus diagrams, and trends are established which relate the servomechanism parameters, τ_1 , τ_2 and K , to the actual response.

Finally, in this chapter will be included analog computer results which give the actual responses of the third-order servomechanism as its parameters assume several values.

The reader has already been referred to the Bibliography for readings on root-locus procedures, and no attempt is made here to explain or justify the techniques used in obtaining the graphs. It suffices to say that a root-locus is the graph of the locus, in the $s = \sigma + j\omega$ plane, of the poles of a function $G(s)/[1 + G(s)]$ of a complex variable $s = \sigma + j\omega$ as some servomechanism parameter changes. An understanding of the method of obtaining the graph is not essential to an understanding of its use in this dissertation. What is essential, however, is a rather general understanding of the relation between the poles of a function together with its residues at these poles and the actual dynamic response of the servomechanism described by the function. For a thorough understanding of this relation the reader is again referred to the Bibliography,⁽⁹⁾ but the present purpose is sufficiently served by the general discussion in the following paragraphs.

The inverse transformation of the rational algebraic fraction $C/R(s)$ gives an expression of the form

$$c(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + \dots + a_n e^{s_n t} \quad (3.30)$$

for the response of an n^{th} -order servomechanism in terms of the poles s_i , and residues, a_i , of its transfer function when the input is a unit impulse function ($R = 1$). (The case of two identical poles, s_i , gives rise to a term of the form $a_i t e^{s_i t}$ and is avoided here for simplicity.) Integration of Eq. (3.30) gives the step-function response, and, by reason of the exponential nature of the terms, the form of the resulting expression is similar to Eq. (3.30). Thus, the presence of oscillatory components in the response of a servomechanism and their relative importance is known when the poles and residues of the transfer function are known. The poles are, in general, complex; the real part gives the rate of exponential decay

of each term, and the imaginary part gives the frequency of oscillation. When a graph in the $s = \sigma + j\omega$ plane (such as the root-locus) is available, the value of each pole is read directly, and the residues are obtained indirectly by the method outlined below.

Given a rational fraction in factored form,

$$G(s) = A \frac{(s + z_1)(s + z_2) \dots (s + z_m)}{(s + p_1)(s + p_2) \dots (s + p_n)}, \quad (3.31)$$

where $-z_1, -z_2, \dots, -z_m$ are the m zeros of the function, $-p_1, -p_2, \dots, -p_n$ are its n poles, and A is a constant factor, the residue of the function at a particular pole is easily obtained graphically. On the plot of the poles and zeros in the s -plane, vectors are constructed from the particular pole in question to each of the zeros and other poles. Since the residue of $G(s)$ at the pole at $-p_i$ is

$$k_i = A \left. \frac{(s + z_1)(s + z_2) \dots (s + z_m)(s + p_i)}{(s + p_1)(s + p_2) \dots (s + p_n)} \right|_{s = -p_i}, \quad (3.32)$$

it follows that

$$k_i = A \frac{r_1 e^{j\theta_1} r_2 e^{j\theta_2} \dots r_m e^{j\theta_m}}{R_1 e^{j\phi_1} R_2 e^{j\phi_2} \dots R_n e^{j\phi_n}} \quad (3.33)$$

and

$$k_i = A \frac{r_1 r_2 \dots r_m}{R_1 R_2 \dots R_n} e^{j(\theta_1 + \theta_2 \dots \theta_m - \phi_1 - \phi_2 \dots - \phi_n)}, \quad (3.34)$$

where r_1, r_2 , etc., are the lengths of the vectors in the s -plane from the pole at $-p_i$ to each zero; R_1, R_2 , etc., are the lengths of the vectors to each other pole; θ_1, θ_2 , etc., and ϕ_1, ϕ_2 , etc., are the angles of these vectors. It should be understood that the factor $R_i e^{j\phi_i}$ is missing in Eqs. (3.33) and (3.34).

A root-locus plot thus contains the loci of the poles of a transfer function as a servomechanism parameter of interest is varied. From this plot, the designer reads the poles directly and either estimates or measures the residue at each pole by Eq. (3.34). A good general understanding is thereby obtained of the effect of a parameter on pole and zero locations in the s -plane, and from these locations the behavior of the system can be quickly estimated.

3.3 Root-locus Plot for a Third-order Servomechanism with K_a as a Parameter

The stiffness of the servomechanism of Fig. 3.3 is given by

$$K_s = \frac{K_a K_{mg} K_{mm}}{R_f R_a} \quad , \quad (3.35)$$

and K_a , the amplifier gain, is a convenient parameter to vary. Thus, attention here is focused upon the effect of amplifier gain (and, ultimately, stiffness) on the response of a servomechanism. The equation of interest is Eq. (3.28), with K_a expressed explicitly. This equation is

$$\frac{C}{R} = \frac{K' K_a}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2) s^2 + s + K' K_a} \quad (3.36)$$

where $K' = K/K_a$, and τ_1 , and τ_2 are as previously defined. Figures 3.4a and 3.4b contain plots of the loci of the three poles of the transfer function in Eq. (3.36) as K_a varies from zero to infinity. Four graphs are shown, corresponding to different values of τ_1 and τ_2 . The loci begin at the poles of the open-loop transfer function, $-1/\tau_1$, $-1/\tau_2$ and 0, and move in the direction of the arrows as K_a increases. Figure 3.4a is particularly useful in illustrating how an experienced designer can learn the effect of a change in a parameter such as K_a on the relative stability of a system; if a line is drawn from the origin to a pole on this locus (see Fig. 3.4a), the angle of this line with the negative real axis is given by

$$A = \cos^{-1} \left[\frac{\sigma_1}{(\sigma_1^2 + \omega_1^2)^{1/2}} \right] \quad , \quad (3.37)$$

where σ_1 and ω_1 are the real and imaginary parts of the pole in question. Since this locus (the locus in Fig. 3.4a) is for a second-order system, with $\tau_1 = 0$,

$$A = \cos^{-1} \frac{\frac{F}{2J}}{\left\{ \frac{F^2}{4J^2} + \frac{K}{J} - \frac{F^2}{4J^2} \right\}^{\frac{1}{2}}} = \cos^{-1} \left[\frac{F}{2(KJ)^{\frac{1}{2}}} \right] = \cos^{-1} \zeta. \quad (3.38)$$

Furthermore, the length of this line from the origin to the pole is given by the expression $[(F^2/4J^2) + (K/J) - (F^2/4J^2)]^{\frac{1}{2}}$, which is simply ω_n . From Fig. 3.1 of Sec. 3.1, the response of the servomechanism as a function of ζ and ω_n is clearly established. Thus, the root-locus shows the movement of the servomechanism poles as a parameter is varied and the response is obtained by inspection of the length and angle of the radius vector to the locus.

The above interpretation of a root-locus applies strictly to a second-order system. However, in the third-order system shown in Fig. 3.4b, the extra locus on the left end of the negative real axis is far enough out that its real part is large, and thus the rate of exponential decay of the term corresponding to this pole is large, and the pole has no significant effect on response.⁽⁵⁾ Furthermore, a graphical computation of the residue at a pole on this negative real part of the locus would show that the residue is small compared to the residues at the poles near the origin. Thus the locus which breaks away from the real axis controls the behavior of the servomechanism, and the interpretation of ζ and ω_n in terms of the radius vector to this dominant locus is a valid one. The effect of the extra, third-order locus is simply to "bend" the dominant locus toward the right-half plane. This effect is important, but once it has been taken into account, the response of the servomechanism may be estimated by considering only the dominant locus and ignoring the locus of the real root.

It is often desirable in designing a servomechanism to vary some parameter other than K_a , which may be rigidly specified by considerations

Fig. 3.4a Root-locus plots for two second-order servomechanisms with K_a varying. Numerals along the loci are the values of K_a at certain roots.

Fig. 3.4b Root-locus plots for two third-order servomechanisms with K_a varying. Numerals along the loci are the values of K_a at certain roots.

of stiffness, noise problems, etc. If this is the case, it is clear from the graphs of Figs. 3.4a and 3.4b that, for a constant $K'K_a$, the stability of the servomechanism is improved when either τ_1 or τ_2 is made smaller. The proper selection of system components may accomplish this reduction, but for the purposes of this dissertation, attention is directed instead to the use of auxiliary feedback arrangements. In the second-order servomechanism of Sec. 3.4, $\tau_1 = 0$ and $\tau_2 = J/F$. Tachometric feedback increases F and acceleration feedback (of proper polarity) decreases J ; both effects result in a reduction in τ_2 . In a third-order system, however, the situation is not quite so simple because of the additional time delay that the feedback signals encounter. In the analysis of the next section this is demonstrated and there are introduced root-loci and analog computer results to illustrate the effect of tachometer and acceleration feedback on a third-order system of fixed stiffness.

3.4 Tachometric and Acceleration Feedback in a Third-order System

Figures 3.5 and 3.6 are block diagrams of the third-order system of Fig. 3.3 with tachometric (velocity) and acceleration feedback, respectively. The transfer functions of these two systems are given below. For the servomechanism with velocity feedback, the equation is

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2)s^2 + (1 + KK_V)s + K} \quad , \quad (3.39)$$

and with acceleration feedback the equation is

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2 - KK_A)s^2 + s + K} \quad . \quad (3.40)$$

Parenthetically, it is to be noted that the tachometric and acceleration feedback could be modified by an additional transfer function of the form

$$G = (\tau_1 s + 1) \quad (3.41)$$

Fig. 3.5 Block diagram of a third-order servomechanism with tachometric feedback.

Fig. 3.6 Block diagram of a third-order servomechanism with acceleration feedback.

in order to produce ideal effects (i.e., the same effects as are evident in a second-order system). Such a modification would cancel the effect of the additional time delay and produce

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2) s^2 + (1 + KK_V + KK_V \tau_1 s) s + K} \quad (3.42)$$

and

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2 - KK_\alpha - KK_\alpha \tau_1 s) s^2 + s + K} \quad (3.43)$$

These equations can be written as

$$\frac{C}{R} = \frac{\frac{K}{1 + KK_V}}{\tau_1 \left[\frac{\tau_2}{1 + KK_V} \right] s^3 + \left[\tau_1 + \frac{\tau_2}{1 + KK_V} \right] s^2 + s + \frac{K}{1 + KK_V}} \quad (3.44)$$

and

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 (1 - KK_\alpha) s^3 + [\tau_1 + \tau_2 (1 - KK_\alpha)] s^2 + s + K} \quad (3.45)$$

respectively, and it is evident that the suggested modification is effective in controlling the size of τ_2 directly. As a practical matter, it is difficult to obtain the "compensating" transfer function of Eq. (3.41) physically, although it can be approximated by an ordinary lead network. No more will be said here about the modification.

Consider again Eq. (3.39) and (3.40). The remainder of this chapter is devoted to a study of the behavior of the servomechanism as K_V and K_α are varied. Root-locus plots are given first and then analog computer results. At the outset, however, in order to provide a general description of the stability of the system when tachometric and/or acceleration feedback are present, a Routh-Hurwitz evaluation of the denominator polynomials of Eqs. (3.39) and (3.40) shows that for the system to be stable,

$$(\tau_1 + \tau_2)(1 + KK_V) \geq \tau_1 \tau_2 K \quad (3.46)$$

for tachometric feedback, and

$$(\tau_1 + \tau_2 - KK_\alpha) \geq \tau_1 \tau_2 K \quad (3.47)$$

for acceleration feedback.

It appears from Eq. (3.46) that a third-order servomechanism becomes more stable with increasing tachometric feedback. At least, the value of K at which the servomechanism becomes unstable is raised. On the other hand, it appears from Eq. (3.47) that increasing acceleration feedback results in poorer stability. This latter observation is disturbing since the relative stability of the second-order system of Sec. 3.1 is increased by the addition of acceleration feedback, and one expects a similar effect when a system departs slightly from second-orderedness. A further analysis of third-order systems must therefore discuss not only the response of the system as K_α is increased, but also the effect of the size of the additional time constant, τ_1 .

To accomplish this analysis, root-locus plots with K_α varying, and for different values of τ_1 , are necessary. Tachometric feedback is also of interest and the system with both types of feedback is characterized by the equation

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2 - KK_\alpha)s^2 + (1 + KK_V)s + K} \quad (3.48)$$

The root-locus plots of Eq. (3.48) for various τ_1 and K_V , with K_α as the continuously varying parameter, are contained in Figs. 3.7-3.10. These graphs are drawn assuming the following parameters: $\tau_2 = 0.0642$ and $K = 45.5$. The numbers corresponding to K_V and K_α are more easily handled with the following substitutions: $132K_V = K_V'$; $132K_\alpha = K_\alpha'$, and only the top half of the loci are shown. The numerals that appear along the loci are values of K_α' and the arrows indicate the direction of movement of the system poles as K_α' is increased. The following paragraphs contain comments on the curves of Figs. 3.7-3.10.

Fig. 3.7 Root-locus plots for a servomechanism with K'_α varying. Numerals along the loci are values of K'_α . The servomechanism is second-order and the three graphs correspond to three values of tachometric feedback.

Fig. 3.8 Root-locus plots for a servomechanism with K'_α varying. Same as Fig. 3.7, but for a third-order system with $\tau_1 = .001$.

Fig. 3.9 Root-locus plots for a servomechanism with K'_α varying. Same as Fig. 3.7, but with $\tau_1 = .005$.

Fig. 3.10 Root-locus plots for a servomechanism with K_V^1 varying. Same as Fig. 3.7, but with $\tau_1 = .0154$, and only two values of tachometric feedback.

Figure 3.7 demonstrates the results of Sec. 3.1; acceleration feedback increases the relative stability and speed of response of a second-order servomechanism, since ζ and ω_n become larger as K'_α is increased.

Figure 3.8 demonstrates that a system which departs very little from second-orderedness ($\tau_1 = \tau_2/64$) is also improved, particularly when tachometric feedback is also added. In this graph, as in the ones to follow, the parts of the loci which are far out on the negative real axis are not considered in arriving at general conclusions about stability, for the reasons given earlier. The extent to which this omission produces errors can best be judged on the basis of the analog computer results presented later.

Figure 3.9 shows that a system with significant third-order effects ($\tau_1 = \tau_2/12.8$) behaves badly with acceleration feedback. The dominant locus moves in the direction of reduced ζ and soon enters the right-half plane, the region of instability. This system requires a considerable amount of tachometric feedback to realize an improvement in stability over a system with no auxiliary feedback at all.

Figure 3.10 shows that a system which is decidedly third-order ($\tau_1 = \tau_2/4.17$) is made less stable when acceleration feedback is added; so much so that the addition of tachometric feedback does not prevent the locus from moving directly toward the right-half plane.

The above comments refer to implications that can be drawn from the root-locus plots. For precise information attention is now directed to the analog computer results of Figs. 3.11, 3.12, 3.13 and 3.14. These are graphs of the step-function response of the systems whose root-loci appear on Figs. 3.7, 3.8, and 3.9. Figure 3.11 proves the result of Sec. 3.1; both stability and speed of response of a second-order system are improved by the addition of acceleration feedback, and that for constant stiffness, tachometric feedback improves stability but reduces speed of response - a well-known result.

Fig. 3.11 Step-function response of a servomechanism as obtained on an analog computer. The servomechanism is second-order and has the root-locus plots shown in Fig. 3.7.

Fig. 3.12 Step-function response of a servomechanism as obtained on an analog computer. Same as Fig. 3.11 but for a third-order system with $\tau_1 = .001$, corresponding to the root-locus plots shown in Fig. 3.8.

Fig. 3.13 Step-function response of a servomechanism as obtained on an analog computer. Same as Fig. 3.11 but with $\tau_1 = .005$, corresponding to the root-locus plots shown in Fig. 3.9

Fig. 3.14 Step-function response of a servomechanism as obtained on an analog computer. Same as Fig. 3.13, but with a different selection of parameters from the root-locus plots of Fig. 3.9

Figure 3.12 shows that a system with small departure from second-orderedness ($\tau_1 = 0.001$) is affected by acceleration and tachometric feedback in the same way as a second-order system.

Figures 3.13 and 3.14 show that a system with appreciable third-order effects ($\tau_1 = 0.005$) has increased speed of response with acceleration feedback, but its relative stability is greatly reduced. Figures 3.13 and 3.14 also show that the use of tachometric feedback with this higher-order system continues to improve relative stability.

3.5 Conclusion

In conclusion, in this chapter there has been discussed the advantages that accrue to the use of acceleration feedback in a second-order servomechanism, and there has been illustrated that, when significant third-order effects are present, simple acceleration feedback causes poor relative stability. From this latter result, the use of force feedback as described in Chapter 2, with inertial loads, is also suspect whenever higher-order effects such as additional time delays are present. For a third-order servomechanism with inertial load and torque feedback, the transfer function is of the form

$$\frac{C}{R} = \frac{K}{\tau_1 \tau_2 s^3 + (\tau_1 + \tau_2 - KK_t J_1) s^2 + s + K} \quad , \quad (3.49)$$

where τ_2 now includes the load inertia, J_1 , in addition to the motor inertia, J_m . The similarity of Eqs. (3.49) and (3.40) shows that the use of torque feedback modifies the same denominator term in the same direction as the use of acceleration feedback. Thus, the behavior of the poles of the function of Eq. (3.49) as K_t is increased will be similar to the behavior when acceleration feedback is used, and third-order effects will produce the same difficulties. These arguments are self-evident when one considers that torque feedback with an inertial load is identical to acceleration feedback.

CHAPTER 4

FORCE-REFLECTING SERVOMECHANISMS

Control systems which incorporate a sense of feel at the input are the subject of this chapter. Such systems are not properly termed servomechanisms in the restricted sense of Chapters 1, 2 and 3; however, the systems to be described here have ordinary positional servomechanisms as important parts, and this dissertation conforms with the literature on the subject^(6,7) in terming these systems "force-reflecting servomechanisms."

In physical terms, a force-reflecting servomechanism is a control system which provides for remote positioning of an output member according to the position of an input or command member, and produces forces at the input member which are equal to the output member forces or load. Stated differently, a force-reflecting servomechanism is a control system in which the stimulus produces a response at the input as well as at the output. Thus, the operator moves the input handle (stimulus) and this motion produces a force (inclination to move) against the load at the output as well as a force (resistance to movement) at the input. Alternatively, the operator could be regarded as applying a force to the input, and this force causes force and motion at the output as well as some resultant motion back at the input. The situation is analogous to that obtaining for a two-terminal-pair electric network. Application of a voltage at the input terminals produces an output voltage and current as well as an input current. Thus, the control systems to be described in this chapter are analyzed following the methods of two-terminal-pair network analysis. The transformed integro-differential equations are written and manipulated into the form

$$T_1(s) = f_1(s)\theta_1(s) + f_2(s)\theta_2(s) \quad (4.1)$$

and

$$T_2(s) = g_1(s)\theta_1(s) + g_2(s)\theta_2(s) \quad (4.2)$$

Here T_1 and T_2 are the input and output torques, θ_1 and θ_2 are the input and

output positions, and the functions f_1 , f_2 , g_1 , and g_2 are rational algebraic fractions. The general model is that of Fig. 4.1.

Fig. 4.1 General model for a force-reflecting servomechanism.
Note the reference polarities for torque and velocity.

For convenience, the following matrix equation is written from Eqs. (4.1) and (4.2) with the substitutions $y_{11} = f_1/s$, $y_{12} = f_2/s$, $y_{21} = g_1/s$, and $y_{22} = g_2/s$:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} s\theta_1 \\ s\theta_2 \end{bmatrix} \quad (4.3)$$

In terms of the electrical-mechanical analogy of Sec. 2.6, y_{11} is the short-circuit input admittance, y_{22} is the short-circuit output admittance, and y_{12} and y_{21} are the short-circuit transfer admittances, i.e.,

$$y_{11} = \left. \frac{T_1}{s\theta_1} \right|_{s\theta_2 = 0} \quad (4.4)$$

$$y_{22} = \left. \frac{T_2}{s\theta_2} \right|_{s\theta_1 = 0} \quad (4.5)$$

$$y_{21} = \left. \frac{T_2}{s\theta_1} \right|_{s\theta_2 = 0} \quad (4.6)$$

$$y_{12} = \left. \frac{T_1}{s\theta_2} \right|_{s\theta_1 = 0} \quad (4.7)$$

4.1 Establishment of Performance Criteria

It is now possible to establish certain performance criteria for remote manipulation with a force-reflecting servomechanism in terms of the y -matrix [the matrix of Eq. (4.3)] of the system.

1. For ease in manipulation it is essential that a force-reflecting manipulator require little torque to move the input handle from one place to another when the output handle has no load. Otherwise the operator will become tired and also the feel of an actual load will be masked by the effort required to move the input member alone. Thus the open-circuit (unloaded) input admittance must be small.

From Eq. (4.3), under the condition $T_2 = 0$, Cramer's rule gives

$$s\theta_1 = \frac{T_1 Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}, \quad (4.8)$$

and the open-circuit input admittance is

$$\left. \frac{T_1}{s\theta_1} \right|_{T_2 = 0} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}}. \quad (4.9)$$

Since $(Y_{11}Y_{22} - Y_{12}Y_{21})$ is the determinant, Δ , of the y -matrix of Eq. (4.3), the open-circuit input admittance can be written as

$$\left. \frac{T_1}{s\theta_1} \right|_{T_2 = 0} = \frac{\Delta}{Y_{22}}. \quad (4.10)$$

Thus, for a good sense of feel, the force-reflecting servomechanism must have a small value of (Δ/Y_{22}) .

2. Load forces must be reflected accurately, so that the operator derives a good sense of feel of the size, weight, hardness or other property of the manipulated object. Under static conditions this requires that the stiffness of the system be high and that a steady output torque, T_2 , produce an equal and opposite input torque, $-T_1$. Thus, if the output member encounters a rigid load, the deflection of the input member in producing a force on this load should be small. Also, the input torque should equal this load torque, with appropriate algebraic sign.

In terms of the admittance parameters, Eq. (4.4) ($s\theta_2 = 0$ characterizes a rigid load) gives

$$T_1 = y_{11}s\theta_1 \quad . \quad (4.11)$$

When the displacement, $\theta_1(t)$, is a constant, its transform is θ_1/s and Eq. (4.11) becomes

$$T_1(s) = y_{11}\theta_1 \quad . \quad (4.12)$$

The final value theorem of the Laplace transformation gives the static torque, when steady-state conditions are reached:

$$\lim_{t \rightarrow \infty} T_1(t) = \lim_{s \rightarrow 0} sT_1(s) = \lim_{s \rightarrow 0} sy_{11}\theta_1 \quad . \quad (4.13)$$

Thus, the ratio of static torque to deflection (the stiffness of the system) is

$$\text{Stiffness} = \lim_{s \rightarrow 0} sy_{11} \quad . \quad (4.14)$$

For static load torques [$T_2(s) = T_2/s$] to be reflected accurately it is necessary that

$$\lim_{t \rightarrow \infty} T_1(t) = \lim_{s \rightarrow 0} sT_1(s) = -T_2 \quad . \quad (4.15)$$

From Eqs. (4.4) and (4.6) ($s\theta_2 = 0$ characterizes the static case),

$$\frac{T_1(s)}{T_2(s)} = \frac{y_{11}}{y_{21}} \quad (4.16)$$

and

$$T_1(s) = \frac{y_{11}}{y_{21}} T_2(s) \quad . \quad (4.17)$$

Since $T_2(s) = T_2/s$, Eq. (4.17) becomes

$$T_1(s) = \frac{y_{11}}{sy_{21}} T_2 \quad (4.18)$$

and

$$\lim_{t \rightarrow \infty} T_1(t) = \lim_{s \rightarrow 0} sT_1(s) = \lim_{s \rightarrow 0} \frac{y_{11}}{y_{22}} T_2 \quad . \quad (4.19)$$

Comparison of Eqs. (4.19) and (4.15) shows that, for proper static force-reflection,

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = -1 \quad . \quad (4.20)$$

Recapitulating, Eqs. (4.14) and (4.20) indicate that, for good static feel, (sy_{11}) must be large in the limit as s approaches zero, and (y_{11}/y_{21}) must equal -1 in the limit as s approaches zero. This pair of requirements constitutes criterion 2.

3. It is imperative that the system exhibit a high degree of stability under normal operating conditions. Since the system is intended to operate with a wide variety of loads, stability must be examined with the output short-circuited (infinite mechanical load or perfectly rigid structure) and open-circuited (no load). Furthermore, the source impedance - that of the human operator - varies from zero (tight grip on the handle) to infinity (loose grip on the handle). In fine, it is necessary that the system exhibit short-circuit and open-circuit stability at each end.

For the case of an open-circuited input and output ($T_1 = T_2 = 0$), Eq. (4.3) gives

$$y_{11}s\theta_1 + y_{12}s\theta_2 = 0 \quad (4.21)$$

and

$$y_{21}s\theta_1 + y_{22}s\theta_2 = 0 \quad . \quad (4.22)$$

These equations have a nontrivial solution when

$$y_{11}y_{22} - y_{12}y_{21} = \Delta = 0 \quad , \quad (4.23)$$

and the transient response of the system is determined by the zeros of Δ . Stated differently, Eqs. (4.21) and (4.22) may be solved for $s\theta_1$ and $s\theta_2$ as follows:

$$s\theta_1 = -\frac{y_{12}}{y_{11}}s\theta_2 \quad (4.24)$$

$$-\frac{y_{21}y_{12}}{y_{11}}s\theta_2 + y_{22}s\theta_2 = 0 \quad (4.25)$$

$$s\theta_2(y_{11}y_{22} - y_{21}y_{12}) = 0 \quad . \quad (4.26)$$

Similarly, one can obtain

$$s\theta_1(y_{11}y_{22} - y_{12}y_{21}) = 0 \quad (4.27)$$

Equations (4.26) and (4.27) are the homogeneous differential equations (in transformed notation) of the system when the input and output are open-circuited. Their solutions (the particular integrals, transformed) give the transient response of the system. Thus, the transient behavior, under the open-circuit condition, is determined by the zeros of $(y_{11}y_{22} - y_{12}y_{21})$, which are the zeros of Δ .

When the input is open-circuited and the output is short-circuited ($T_1 = s\theta_2 = 0$), Eq. (4.4) gives

$$y_{11}s\theta_1 = 0 \quad (4.28)$$

and the zeros of y_{11} control the response of the system.

When the input is short-circuited and the output is open-circuited ($T_2 = s\theta_1 = 0$), Eq. (4.5) gives

$$y_{22}s\theta_2 = 0 \quad (4.29)$$

and the zeros of y_{22} control the response of the system.

The case of short-circuited input and output is treated by recognizing it as the case where the system equations are derived on the basis of velocity (short-circuit) drivers. Thus a z-matrix is written by applying Cramer's rule to the y-matrix of Eq. (4.3):

$$\begin{bmatrix} s\theta_1 \\ s\theta_2 \end{bmatrix} = \begin{bmatrix} \frac{y_{22}}{\Delta} & -\frac{y_{12}}{\Delta} \\ -\frac{y_{21}}{\Delta} & \frac{y_{11}}{\Delta} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad (4.30)$$

Of course, the elements in the z-matrix of Eq. (4.30) are analogous to the open-circuit impedance parameters of circuit theory and may be written as follows:

$$\frac{y_{22}}{\Delta} = \frac{s\theta_1}{T_1} \bigg|_{s\theta_2 = 0} = z_{11} \quad (4.31)$$

$$-\frac{y_{12}}{\Delta} = \left. \frac{s\theta_1}{T_2} \right|_{s\theta_2 = 0} = z_{12} \quad (4.32)$$

$$-\frac{y_{21}}{\Delta} = \left. \frac{s\theta_2}{T_1} \right|_{s\theta_1 = 0} = z_{21} \quad (4.33)$$

$$\frac{y_{11}}{\Delta} = \left. \frac{s\theta_2}{T_2} \right|_{s\theta_1 = 0} = z_{22} \quad (4.34)$$

For the case where the input and output are both short-circuited ($s\theta_1 = s\theta_2 = 0$), Eqs. (4.30) to (4.34) give

$$z_{11}T_1 + z_{12}T_2 = 0 \quad (4.35)$$

and

$$z_{21}T_1 + z_{22}T_2 = 0 \quad (4.36)$$

Again, a nontrivial solution exists when

$$z_{11}z_{22} - z_{12}z_{21} = 0 \quad (4.37)$$

and thus the zeros of the z -matrix determinant govern the transient behavior of the system. In general, the analyses to follow will not produce a z -matrix directly, but the determinant of the z -matrix is the inverse of the determinant of the y -matrix. The following equation proves this:

$$z_{11}z_{22} - z_{12}z_{21} = \frac{y_{22}y_{11} - y_{12}y_{21}}{\Delta^2} = \frac{\Delta}{\Delta^2} = \frac{1}{\Delta} \quad (4.38)$$

Therefore, the poles of the determinant for a force-reflecting servomechanism determine the transient behavior of the system when input and output are both short-circuited.

In summary, the degree of stability of a force-reflecting servomechanism is determined by investigating the zeros of y_{11} , y_{22} and Δ and the poles of Δ . This determination constitutes Criterion 3.

The table of Fig. 4.2 relates the transient response under each load and source condition to the zeros and poles of the appropriate admittance function.

Condition	Input	Output	Relative Stability Controlled by
1	Short-circuit	Short-circuit	poles of Δ
2	Short-circuit	Open-circuit	zeros of y_{22}
3	Open-circuit	Short-circuit	zeros of y_{11}
4	Open-circuit	Open-circuit	zeros of Δ

Fig. 4.2 List of the functions whose poles and zeros determine stability.

The following section of this chapter (Sec. 4.2) presents two different schemes for a force-reflecting servomechanism and applies the three criteria outlined in the above paragraphs to evaluate each. In Secs. 4.3 to 4.6 are discussed improvements on each of the two schemes, and finally, some experiments to support the analysis are discussed in Sec. 4.7.

4.2 Two Force-reflecting Servomechanisms

One method of obtaining force-reflection is to connect two positional servomechanisms in such a way that the overall system is bilateral. A block diagram of such a scheme is shown in Fig. 4.3 and the detailed schematic is shown in Fig. 4.4. The analysis of this system proceeds under the following assumptions: the servomechanisms are identical, the speed-torque curves are linear, and the parameters of Chapter 2, Sec. 2.1 apply. From the speed-torque curves, the motor torque at the input handle is

$$T_m = K_b s \theta_1 + K_a K_m (\theta_1 - \theta_2) \quad (4.39)$$

Summation of torques at the input gives

$$T_1 = J_m s^2 \theta_1 + T_m = (J_m s^2 + K_b s + K_a K_m) \theta_1 - K_a K_m \theta_2 \quad (4.40)$$

Similarly, for the output,

$$T_2 = J_m s^2 \theta_2 + T_m = (J_m s^2 + K_b s + K_a K_m) \theta_2 - K_a K_m \theta_1 \quad (4.41)$$

Fig. 4.3 Block diagram of a positional-positional force-reflecting servomechanism.

Fig. 4.4 Schematic of a positional-positional force-reflecting servomechanism.

In matrix notation,

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} J_m s + K_b + \frac{K_a K_m}{s} & -\frac{K_a K_m}{s} \\ -\frac{K_a K_m}{s} & J_m s + K_b + \frac{K_a K_m}{s} \end{bmatrix} \begin{bmatrix} s\theta_1 \\ s\theta_2 \end{bmatrix} \quad (4.42)$$

It is now possible to make some statements about the performance of this force-reflecting servomechanism, according to the criteria established in Sec. 4.1. The open-circuit input admittance is

$$Y_1 = \frac{\Delta}{y_{22}} = J_m s + K_b + \frac{K_a K_m}{s} - \frac{(K_a K_m/s)^2}{J_m s + K_b + K_a K_m/s} \quad (4.43)$$

$$Y_1 = J_m s + K_b + \frac{(J_m s + K_b)K_a K_m/s}{J_m s + K_b + (K_a K_m/s)} \quad (4.44)$$

Interpretation of Eq. (4.44) in terms of sinusoidal steady-state admittance is accomplished by substituting $j\omega$ for s . Then, the low-frequency behavior of the system is obtained by letting ω approach zero, in which case Eq. (4.44) becomes

$$Y_1(j\omega) \approx 2[J_m(j\omega) + K_b] \quad (4.45)$$

Thus, for low frequencies (that is, steady movement, as is recommended in delicate remote manipulation) the operator feels the effect of twice the inertia and friction of the input member. It is reasonable to assume (and will become obvious later) that the inertia and friction of each member, output and input, contribute to produce this effect.

Application of criterion 1 to Eq. (4.45) indicates that, for a good sense of feel, the inertia and friction of the members should be made as small as possible.

Criterion 2 of Sec. 4.1 requires that stiffness be large and that

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{12}} = -1 \quad (4.46)$$

Application of Eqs. (4.14) and (4.20) to the matrix of Eq. (4.42) gives

$$\lim_{s \rightarrow 0} sy_{11} = K_a K_m \quad (4.47)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{12}} = -1 \quad (4.48)$$

Equation (4.48) indicates that the force-reflection requirement is automatically satisfied, and Eq. (4.47) shows that greater stiffness is obtainable by the use of higher amplifier gain.

Finally, the relative stability of the system is determined by investigating the zeros of Δ , y_{11} , and y_{22} (there are no poles of Δ other than the one at $s = 0$). The zeros of y_{11} and y_{22} are

$$s_{1,2} = -\frac{K_b}{2J_m} \pm \left[\frac{K_b^2}{4J_m^2} - \frac{K_a K_m}{J_m} \right]^{1/2}, \quad (4.49)$$

and the damping factor, ζ , is

$$\zeta = \frac{K_b}{2(K_a K_m J_m)^{1/2}} \quad (4.50)$$

The zeros of Δ are the roots of

$$(J_m s + K_b)(J_m s + K_b + 2K_a K_m) = 0, \quad (4.51)$$

and the damping factor for the dominant zeros is

$$\zeta = \frac{K_b}{2(2J_m K_a K_m)^{1/2}} \quad (4.52)$$

From Eqs. (4.50) and (4.52) it is evident that reducing J_m increases relative stability, but reducing K_b decreases relative stability.

As a sidelight to this mathematical analysis, it should be noted that an exact electrical analog of this system exists in the network of Fig. 4.5, and an excellent physical picture of the behavior of the system and its feel is obtained by consideration of the geometry of this network. The shunt capacitance and conductance are the input and output inertia and friction,

respectively, and the series inductance is the compliance of the system. The ideal system, of course, is that of Fig. 4.6.

Application of the criteria of Sec. 4.1 has shown that reducing friction and inertia (and, of course, increasing stiffness) improves the sense of feel at the input handle, but that reducing friction and increasing stiffness decreases relative stability. Discussion of methods for producing these changes and reconciling their effects is deferred until the next section.

A second, and more direct, method of obtaining force-reflection is to measure the output torque and drive the input handle to produce equal and opposite input torque. A detailed schematic for accomplishing this is shown in Fig. 4.7. In this system, the output is positioned by an ordinary positional servomechanism driven from the input. The torque at the output is measured and the input handle is driven against the operator's hand by the torque signal, resulting in a sense of feel at the input handle.

If the same assumptions are used as for the previous system, the input and output torques are, respectively,

$$T_1 = J_m s^2 \theta_1 + K_b s \theta_1 - K_a K_m K_t T_2 \quad , \quad (4.53)$$

where K_t is the gain of the torque transducer, and

$$T_2 = J_m s^2 \theta_2 + K_b s \theta_2 + K_m K_a \theta_2 - K_m K_a \theta_1 \quad . \quad (4.54)$$

When T_2 from Eq. (4.54) is inserted into Eq. (4.53), the result, in matrix notation, is as follows:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} J_m s + K_b + \frac{K_t K_a^2 K_m^2}{s} & -K_t K_a K_m (J_m s + K_b + K_m K_a / s) \\ -\frac{K_m K_a}{s} & J_m s + K_b + K_m K_a / s \end{bmatrix} \begin{bmatrix} s \theta_1 \\ s \theta_2 \end{bmatrix} \quad (4.55)$$

The y-matrix of Eq. (4.55) is used to determine how this system meets the three performance criteria of Sec. 4.1. First, the feel of the system when unloaded is indicated by

Fig. 4.5 Electrical analog of a positional-positional force-reflecting servomechanism.

Fig. 4.6 Electrical analog of an ideal force-reflecting servomechanism.

$$Y_1 = \frac{\Delta}{y_{22}} = \frac{[J_m s + K_b + (K_a K_m / s)](J_m s + K_b)}{J_m s + K_b + (K_a K_m / s)} \quad (4.56)$$

or

$$Y_1 = J_m s + K_b \quad (4.57)$$

Equation (4.57) shows that the operator feels the inertia and friction of the input handle only when he attempts to move the force-reflecting system without a load. As in the previous system, reduction of friction and inertia lowers the input admittance and improves feel.

The application of Criterion 2 gives

$$\lim_{s \rightarrow 0} sy_{11} = K_t K_a^2 K_m^2 \quad (4.58)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{12}} = -K_t K_a K_m \quad (4.59)$$

When

$$K_t = \frac{1}{K_a K_m} \quad (4.60)$$

the stiffness of this system is the same as that of the previous system and static force-reflection is also proper.

Finally, the question of stability leads to the examination by Criterion 3 of the zeros of

$$\Delta = [J_m s + K_b + (K_m K_a / s)](J_m s + K_b) \quad (4.61)$$

$$y_{22} = J_m s + K_b + K_m K_a / s \quad (4.62)$$

and

$$y_{11} = J_m s + K_b + \frac{K_t K_a^2 K_m^2}{s} \quad (4.63)$$

For $K_t = 1/K_a K_m$, y_{11} , y_{22} and Δ have the same zeros as y_{11} and y_{22} in the previous system. Again, an effort to reduce friction and increase stiffness, although resulting in better feel, will produce poorer stability.

4.3 Improving Stability

Both of the systems of the last section display a reduction in stability when either a reduction in friction or an increase in stiffness is

effected in order to improve feel. This section considers a way to increase stability without an adverse effect on feel, and therefore opens the door to further considerations directed at improving feel. The method considered is the inclusion of "mutual" friction in place of "self" friction at each end. By the use of tachometric feedback, wherein both sides are driven by a signal proportional to the difference in velocity between the handles, this "mutual" friction can be inserted synthetically. The changes that this modification produces in Eqs. (4.40) and (4.41) are given below. The added "mutual" friction term is $K_a K_m K_v (s\theta_1 - s\theta_2)$, where K_v is the tachometer gain.

$$T_1 = (J_m s + K_b) s\theta_1 + K_a K_m K_v (s\theta_1 - s\theta_2) + \frac{K_a K_m}{s} (s\theta_1 - s\theta_2) \quad (4.64)$$

and

$$T_2 = (J_m s + K_b) s\theta_2 + K_a K_m K_v (s\theta_2 - s\theta_1) + \frac{K_a K_m}{s} (s\theta_2 - s\theta_1) \quad (4.65)$$

Equations (4.64) and (4.65) refer to the modification, by differential tachometric feedback, of the first force-reflecting servomechanism in Sec. 4.2. A new electrical analog for this system is contained in Fig. 4.8.

Fig. 4.8 Analog of a positional-positional force-reflecting servomechanism with differential tachometric feedback.

Comparison of this schematic with that of Fig. 4.5 shows that damping (friction) has been included between the two handles.

The important property of this mutual damping, in physical terms, is that the velocity signals subtract to zero and provide no feeling of viscous drag when the input and output members are moving in synchronism, as is the case during steady operation. Yet, damping is provided whenever a transient condition (change of force or motion) is present, and, thus, hunting is prevented. Mathematically, the effect of this mutual damping is seen from the application of the criteria of Sec. 4.1 to the new system matrix written below.

$$[y] = \begin{bmatrix} J_m s + K_b + K_a K_m K_v + \frac{K_a K_m}{s} & -K_a K_m K_v - \frac{K_a K_m}{s} \\ -K_a K_m K_v - \frac{K_a K_m}{s} & J_m s + K_b + K_a K_m K_v + \frac{K_a K_m}{s} \end{bmatrix} \quad (4.66)$$

Criterion 1 produces

$$Y_1 = J_m s + K_b + K_a K_m K_v + (K_a K_m / s) - \frac{[K_a K_m K_v + (K_a K_m / s)]^2}{J_m s + K_b + K_a K_m K_v + (K_a K_m / s)} \quad (4.67)$$

$$Y_1 = J_m s + K_b + \frac{[J_m s + K_b + K_a K_m K_v + (K_a K_m / s)][K_a K_m K_v + (K_a K_m / s)]}{J_m s + K_b + K_a K_m K_v + (K_a K_m / s)} - \frac{[K_a K_m K_v + (K_a K_m / s)]^2}{J_m s + K_b + K_a K_m K_v + (K_a K_m / s)} \quad (4.68)$$

$$Y_1 = J_m s + K_b + \frac{(J_m s + K_b)[K_a K_m K_v + (K_a K_m / s)]}{J_m s + K_b + K_a K_m K_v + (K_a K_m / s)} \quad (4.69)$$

For large $K_a K_m K_v$ and steady movement of the input member, the input admittance becomes

$$Y_1 \approx J_m s + K_b + J_m s + K_b = 2(J_m s + K_b) \quad (4.70)$$

Comparison of Eq. (4.70) with Eq. (4.45) shows that no change in no-load input admittance has resulted from the addition of differential tachometric feedback.

The application of Criterion 2 gives

$$\lim_{s \rightarrow 0} sy_{11} = K_a K_m \quad (4.71)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{12}} = -1 \quad (4.72)$$

Comparison with Eqs. (4.47) and (4.48) shows that no change in stiffness or force ratio has resulted.

Finally, application of Criterion 3 demonstrates that a significant increase in relative stability has been accomplished by the addition of differential tachometric feedback.

$$\Delta = (J_m s + K_b) [J_m s + K_b + 2K_a K_m K_v + (2K_a K_m / s)] \quad (4.73)$$

$$y_{22} = J_m s + K_b + K_a K_m K_v + (K_a K_m / s) \quad (4.74)$$

and

$$y_{11} = J_m s + K_b + K_a K_m K_v + (K_a K_m / s) \quad (4.75)$$

The functions y_{11} and y_{22} now have a damping factor,

$$\zeta = \frac{K_b + K_a K_m K_v}{2(J_m K_a K_m)^{1/2}} \quad (4.76)$$

which indicates a direct improvement in stability by increasing K_v . Furthermore, the damping factor for the dominant zeros of Δ is

$$\zeta = \frac{K_b + 2K_a K_m K_v}{2(2J_m K_a K_m)^{1/2}} \quad (4.77)$$

which is also increased by increasing K_v .

The stability of the second force-reflecting servomechanism of Sec. 4.2 is also increased by the use of differential tachometric feedback without adversely affecting the feel of the system. With differential tachometric feedback, Eqs. (4.53) and (4.54) become

$$T_1 = J_m s^2 \theta_1 + K_b s \theta_1 + K_a K_m K_v (s \theta_1 - s \theta_2) - K_a K_m K_t T_2 \quad (4.78)$$

and

$$T_2 = J_m s^2 \theta_2 + K_b s \theta_2 + K_a K_m K_v (s \theta_2 - s \theta_1) + K_a K_m (\theta_2 - \theta_1) . \quad (4.79)$$

The new y -matrix is, for $K_t = 1/K_a K_m$,

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} J_m s + K_b + 2K_a K_m K_v + \frac{K_a K_m}{s} & -2K_a K_m K_v - (J_m s + K_b) - \frac{K_a K_m}{s} \\ -K_a K_m K_v - \frac{K_a K_m}{s} & J_m s + K_b + K_a K_m K_v + \frac{K_a K_m}{s} \end{bmatrix} \quad (4.80)$$

Criterion 1 gives

$$Y_1 = \frac{\Delta}{y_{22}} = J_m s + K_b + K_a K_m [K_v + K_v + (1/s)] - \frac{[K_a K_m K_v + (K_a K_m/s)][J_m s + K_b + 2K_a K_m K_v + (K_a K_m/s)]}{J_m s + K_b + K_a K_m K_v + (K_a K_m/s)} \quad (4.81)$$

$$Y_1 = J_m s + K_b + \frac{(J_m s + K_b)(K_a K_m K_v)}{J_m s + K_b + K_a K_m K_v + (K_a K_m/s)} \quad (4.82)$$

For $K_a K_m K_v$ very small, the input admittance is

$$Y_1 \approx J_m s + K_b \quad (4.83)$$

For $K_a K_m K_v$ very large (the practical case) and for steady movement of the input member, the input admittance is

$$Y_1 \approx 2(J_m s + K_b) \quad (4.84)$$

which is just twice the value of the case when no differential tachometric feedback is employed, and the operator again feels the inertia and friction of both members.

Application of Criterion 2 gives

$$\lim_{s \rightarrow 0} sy_{11} = K_a^2 K_m^2 K_t \quad (4.85)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = -K_a K_m K_t \quad (4.86)$$

which reduce to $K_a K_m$ and -1 , respectively, when $K_t = 1/K_a K_m$. Thus, no change in reflection of static forces has resulted from the modification.

Finally, stability is shown to be improved by the application of Criterion 3. After some algebraic manipulation,

$$\Delta = (J_m s + K_b)[J_m s + K_b + 2K_a K_m K_v + (K_a K_m/s)] \quad (4.87)$$

$$y_{22} = [J_m s + K_b + K_a K_m K_v + (K_a K_m/s)] \quad (4.88)$$

and

$$y_{11} = [J_m s + K_b + 2K_a K_m K_v + (K_a K_m/s)] \quad (4.89)$$

The above results are obtained subject to $K_t = 1/K_a K_m$.

The damping factor for the zeros of y_{22} is now

$$\zeta = \frac{K_b + K_a K_m K_v}{2(J_m K_a K_m)^{\frac{1}{2}}} \quad (4.90)$$

For the zeros of y_{11} and the dominant zeros of Δ , the damping factor is

$$\zeta = \frac{K_b + 2K_a K_m K_v}{2(J_m K_a K_m)^{\frac{1}{2}}} \quad (4.91)$$

Both Eqs. (4.90) and (4.91) indicate a direct improvement in damping with increasing K_v .

The conclusion to be drawn from this section is that differential tachometric feedback increases stability in both force-reflecting systems without adversely affecting feel. The next two sections discuss two

methods for improving feel by reducing friction and inertia. Independently applied, both these methods have an adverse effect on stability and thus this section has provided a means of readjusting stability after friction and inertia have been reduced.

4.4 Improving Feel by Positive Tachometric and Acceleration Feedback

It is evident from Sec. 4.2 that stiffness (and thus, feel) is improved by increasing amplifier gain, K_a . The deleterious effect this has on stability can be counteracted by differential tachometric feedback, and in this section no more will be said about the use of increased stiffness to improve feel. Instead, attention is directed toward reducing friction and inertia, since such reduction lowers the input admittance, and thus improves feel, for both systems.

The first method to be considered for reducing friction and inertia is suggested by the results of Chapter 3, and uses positive tachometric and acceleration feedback around each end of the force-reflecting servomechanism, as proposed by Schmidt.⁽⁸⁾ Application of such feedback to the first system of Sec. 4.2 (the positional-positional system) changes Eqs. (4.40) and (4.41) into

$$T_1 = (J_m - K_a K_m K_\alpha) s^2 \theta_1 + (K_b - K_a K_m K_v) s \theta_1 + \frac{K_a K_m}{s} (s \theta_1 - s \theta_2) \quad (4.92)$$

and

$$T_2 = (J_m - K_a K_m K_\alpha) s^2 \theta_2 + (K_b - K_a K_m K_v) s \theta_2 + \frac{K_a K_m}{s} (s \theta_2 - s \theta_1) \quad (4.93)$$

The electrical analog for this system is now the diagram of Fig. 4.9, and the y -matrix follows.

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} (J_m - K_a K_m K_\alpha) s + K_b - K_a K_m K_v & -K_a K_m / s \\ + K_a K_m / s & (J_m - K_a K_m K_\alpha) s + K_b - K_a K_m K_v \\ -K_a K_m / s & + K_a K_m / s \end{bmatrix} \quad (4.94)$$

It is apparent from Eq. (4.94) that inertia and friction are directly controlled by the use of acceleration and tachometric feedback.

Fig. 4.9 Analog of a positional-positional force-reflecting servo-mechanism with positive tachometric and acceleration feedback.

Application of Criterion 1 gives

$$Y_1 \approx 2(J_m - K_a K_m K_\alpha)s + 2(K_b - K_a K_m K_v) \quad (4.95)$$

for steady movement of the input member. Thus input admittance can be effectively reduced.

Application of Criterion 2 gives

$$\lim_{s \rightarrow 0} s y_{11} = K_a K_m \quad (4.96)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = -1, \quad (4.97)$$

demonstrating that static force-reflection is unaffected.

Finally, Criterion 3 relates to the zeros of

$$\Delta = (J's + K_b')(J's + K_b' + (2K_a K_m/s)) \quad (4.98)$$

$$y_{11} = J's + K_b' + (K_a K_m/s) \quad (4.99)$$

and

$$y_{22} = J's + K_b' + (K_a K_m / s) \quad , \quad (4.100)$$

where $J' = (J_m - K_a K_m K_\alpha)$ and $K_b' = (K_b - K_a K_m K_v)$. The damping factor for the dominant zeros of Δ is

$$\zeta = \frac{K_b'}{2(2J'K_aK_m)^{\frac{1}{2}}} \quad , \quad (4.101)$$

and for the zeros of both y_{11} and y_{22} is

$$\zeta = \frac{K_b'}{2(J'K_aK_m)^{\frac{1}{2}}} \quad . \quad (4.102)$$

Although the introduction of positive acceleration and tachometric feedback clearly improves the no-load feel by reducing the apparent inertia and friction, the effect on stability is not so favorable. The damping factor for the zeros of y_{11} and y_{22} is related to that for the system with no auxiliary feedback by the ratio

$$\frac{\zeta_{\text{new}}}{\zeta_{\text{old}}} = \frac{1 - (K_a K_m K_v / K_b)}{[1 - (K_a K_m K_\alpha / J_m)]^{\frac{1}{2}}} \quad . \quad (4.103)$$

The reduction of inertia has a favorable effect, the reduction of friction an unfavorable one. It is, of course, impossible to say what the overall combined effect is, since this is a function of the original inertia and friction and of the degree to which each is reduced. If an attempt is made to reduce each by the same factor, then

$$1 - (K_a K_m K_v / K_b) = 1 - (K_a K_m K_\alpha / J_m) = k \quad , \quad (4.104)$$

and damping is reduced by $k^{\frac{1}{2}}$.

Application of positive tachometric and acceleration feedback to the second system of Sec. 4.2 (the positional-force system) produces the y -matrix of Eq. (4.105):

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} J's + K_b' + (K_a K_m/s) & -[J's + K_b' + (K_a K_m/s)] \\ -K_a K_m/s & J's + K_b' + (K_a K_m/s) \end{bmatrix} \quad (4.105)$$

Again, the inertia and friction are directly controlled and the first two performance criteria of Sec. 4.1 yield

$$Y_1 = J's + K_b' \quad (4.106)$$

$$\lim_{s \rightarrow 0} s y_{11} = K_a K_m \quad (4.107)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = -1 \quad (4.108)$$

For the application of Criterion 3, the following functions are written from the matrix of Eq. (4.105):

$$\Delta = [J's + K_b' + (K_a K_m/s)](J's + K_b') \quad ; \quad (4.109)$$

$$y_{22} = y_{11} = J's + K_b' + (K_a K_m/s) \quad (4.110)$$

Comparison of Eqs. (4.106) to (4.110) with Eqs. (4.95) to (4.100) shows that this system experiences the same improvement in feel and the same reduction in stability as the positional-positional system when positive acceleration and tachometric feedback are added.

4.5 Improving Feel by Torque Feedback

The last section showed that positive tachometric and acceleration feedback could be used to lower friction and inertia and thereby to improve the feel of either of the two force-reflecting servomechanisms under consideration. In this section another method which uses torque feedback for improving feel, is discussed following the suggestion of Burnett and Schiewe.⁽²⁾

From Sec. 2.6 of Chapter 2 it is evident that positive torque feedback lowers the output impedance of a servomechanism by the factor $(1 - K_a K_m K_t)$. For a force-reflecting servomechanism it seems reasonable to assume that negative torque feedback can raise the output impedance by the factor $(1 + K_a K_m K_t)$. The following analysis justifies this assumption. Consider the application of torque feedback around each end of the positional-positional force-reflecting servomechanism. The schematic of Fig. 4.10 applies.

At the input,

$$T_1 = (J_m s + K_b) s \theta_1 + K_a K_m (\theta_1 - \theta_2) - K_a K_m K_t T_1 \quad , \quad (4.111)$$

and at the output,

$$T_2 = (J_m s + K_b) s \theta_2 + K_a K_m (\theta_2 - \theta_1) - K_a K_m K_t T_2 \quad . \quad (4.112)$$

The matrix equation is

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{J_m s + K_b + (K_a K_m / s)}{1 + K_t K_a K_m} & - \frac{K_a K_m / s}{1 + K_t K_a K_m} \\ - \frac{K_a K_m / s}{1 + K_t K_a K_m} & \frac{J_m s + K_b + (K_a K_m / s)}{1 + K_t K_a K_m} \end{bmatrix} \begin{bmatrix} s \theta_1 \\ s \theta_2 \end{bmatrix} \quad . \quad (4.113)$$

The matrix of Eq. (4.113) is symmetrical and an analogous electrical network exists in the schematic diagram of Fig. 4.11. The no-load feel of the system is given by

$$Y_1 = \frac{J_m s + K_b + (K_a K_m / s)}{1 + K_t K_a K_m} - \frac{(K_a K_m / s)^2}{(1 + K_t K_a K_m)[J_m s + K_b + (K_a K_m / s)]} \quad (4.114)$$

or

$$Y_1 = \frac{J_m s + K_b}{1 + K_t K_a K_m} + \frac{(J_m s + K_b) K_a K_m / s}{(1 + K_t K_a K_m)[J_m s + K_b + (K_a K_m / s)]} \quad . \quad (4.115)$$

Fig. 4.10 Schematic of a positional-positional force-reflecting servomechanism with torque feedback.

Fig. 4.11 Analog of a positional-positional force-reflecting servo-mechanism with torque feedback.

For steady movement of the input handle, the input admittance is

$$Y_1 \approx \frac{2(J_ms + K_b)}{1 + K_t K_a K_m} \quad (4.116)$$

Comparison of Eq. (4.116) with Eq. (4.45) shows that the input admittance (and also the output, since the system is symmetrical) has been reduced by the factor $(1 + K_t K_a K_m)$ over the system without torque feedback loops.

Application of Criterion 2 to the matrix of Eq. (4.113) shows that

$$\lim_{s \rightarrow 0} sy_{11} = \frac{K_a K_m}{1 + K_t K_a K_m} \quad (4.117)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = -1 \quad (4.118)$$

Although the force ratio is suitable, the stiffness has been reduced over that of the first system of Sec. 4.2. Thus, the amplifier gain K_a and torque transducer gain K_t must be adjusted separately to produce both a specified $(1 + K_t K_a K_m)$ for input admittance reduction and a specified $K_a K_m / (1 + K_t K_a K_m)$ for stiffness control. This can easily be accomplished in practice. From Eq. (4.117), the amplifier gain for this new

system (redesignated K_a') must bear the following relationship to that of the first system [see Eq. (4.47) of Sec. 4.2]:

$$\frac{K_a' K_m}{1 + K_t K_a' K_m} = K_a K_m \quad (4.119)$$

$$K_a' K_m = K_a K_m + K_a' K_m (K_t K_a K_m) \quad (4.120)$$

and

$$K_a' = \frac{K_a K_m}{1 - K_t K_a K_m} \quad (4.121)$$

This, of course, leads directly to a stability problem since the zeros of

$$\Delta = \frac{(J_m s + K_b)(J_m s + K_b + 2K_a' K_m)}{1 + K_t K_a' K_m} \quad (4.122)$$

$$y_{22} = \frac{J_m s^2 + K_b s + K_a' K_m}{1 + K_t K_a' K_m} \quad (4.123)$$

and

$$y_{11} = \frac{J_m s^2 + K_b s + K_a' K_m}{1 + K_t K_a' K_m} \quad (4.124)$$

are affected adversely by the large K_a' . To show this, consider the following argument: just as with the use of positive tachometric and acceleration feedback, the effect is to operate on a function of the form

$$y_{ij} = J_m s^2 + K_b s + K_a K_m \quad (4.125)$$

and produce

$$y_{ij} = \frac{J_m s^2 + K_b s}{1 + K_t K_a' K_m} + K_a K_m \quad (4.126)$$

The damping factor for the zeros of y_{ij} in Eq. (4.125) is

$$\zeta = \frac{K_b}{2(J_m K_a K_m)^{1/2}} \quad (4.127)$$

For the zeros of y_{ij} in Eq. (4.126) the damping factor is

$$\zeta = \frac{K_b}{2[J_m K_a K_m (1 + K_t K_a' K_m)]^{\frac{1}{2}}} \quad (4.128)$$

It is evident from Eqs. (4.127) and (4.128) that damping is reduced by the use of torque feedback when stiffness is kept fixed.

The difference between the use of torque feedback and positive tachometric and acceleration feedback is seen by comparing Eq. (4.126) with Eqs. (4.101) and (4.102). Torque feedback reduces both friction and inertia by the same factor $(1 + K_t K_a' K_m)$ and therefore reduces damping by the factor $(1 + K_t K_a' K_m)^{\frac{1}{2}}$. In order to restabilize the system the use of differential tachometric feedback (Sec. 4.3) may be required.

The use of torque feedback to reduce input admittance may also be applied to the force-reflecting servomechanism of Sec. 4.2, (the positional-force system), according to the schematic diagram of Fig. 4.12. In this modification the input torque is added to the output torque to produce the required signal to drive the input member. In physical terms, the behavior of the system is the following: when a torque is applied to the input handle, the input torque transducer delivers a signal to the input motor to cause it to move the handle in the direction of the applied torque; now the positional servomechanism moves the output handle to follow the input; if there is some load on the output, the torque required to handle this load is delivered back to the input handle, against the operator's hand. With this system, the operator feels only this true load torque, since the torque required to move the input member alone is supplied by the input motor.

The analysis of this system proceeds from the same assumptions used in Sec. 4.2. Torques are assumed at the input and output, with reference polarities as indicated in Fig. 4.1.

At the input, the equation of torques gives

$$T_1 = J_m s^2 \theta_1 + K_b s \theta_1 - K_t K_a K_m T_1 - K_t K_a K_m T_2 \quad (4.129)$$

Fig. 4.12 Schematic of a force-positional force-reflecting servomechanism with torque feedback.

The choice of signs in Eq. (4.129) is made on the basis of the following arguments. The operator's torque, T_1 , must move the input handle against its friction and inertia, and so the first two terms on the right are positive. The third term is negative since the input torque transducer delivers a signal to aid in moving the input handle. The last term is negative because of the assumed reference polarity of T_2 ; load torques have the opposite reference polarity (consistent with conventional two-terminal-pair theory) and would appear in Eq. (4.129) with a positive sign, opposing the operator's torque, T_1 .

At the output, the equation of torques gives

$$T_2 = J_m s^2 \theta_2 + K_b s \theta_2 + K_a K_m (\theta_2 - \theta_1) \quad (4.130)$$

Equations (4.129) and (4.130) can be written in a more suitable manner as follows:

$$T_1 = \frac{[J_m s + K_b + (K_t K_a^2 K_m^2 / s)] s \theta_1 - K_t K_a K_m [J_m s + K_b + (K_a K_m / s)] s \theta_2}{1 + K_t K_a K_m} \quad (4.131)$$

and

$$T_2 = J_m s^2 \theta_2 + K_b s \theta_2 + K_a K_m (\theta_2 - \theta_1) \quad (4.132)$$

In matrix notation, Eqs. (4.131) and (4.132) become

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{J_m s + K_b + (K_t K_a^2 K_m^2 / s)}{1 + K_t K_a K_m} & - \frac{K_t K_a K_m [J_m s + K_b + (K_a K_m / s)]}{1 + K_t K_a K_m} \\ - K_a K_m / s & J_m s + K_b + (K_a K_m / s) \end{bmatrix} \begin{bmatrix} s \theta_1 \\ s \theta_2 \end{bmatrix} \quad (4.133)$$

Because the matrix of Eq. (4.133) is not symmetrical, no passive electrical equivalent exists to give a simple physical picture of this system's

performance and feel. However, the performance criteria of Sec. 4.1 can easily be applied. For an indication of no-load feel,

$$Y_1 = \frac{J_m s + K_b}{1 + K_t K_a K_m} \quad (4.134)$$

Equation (4.134) shows that the input admittance is effectively decreased. Criterion 2 relates to

$$\lim_{s \rightarrow 0} s y_{11} = \frac{K_t K_a^2 K_m^2}{1 + K_t K_a K_m} \quad (4.135)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} = - \frac{K_t K_a K_m}{1 + K_t K_a K_m} \quad (4.136)$$

Equation (4.136) shows that, for proper force-reflection, $K_t K_a K_m$ must be much greater than unity. For this condition,

$$\lim_{s \rightarrow 0} s y_{11} \approx K_a K_m \quad (4.137)$$

and

$$\lim_{s \rightarrow 0} \frac{y_{11}}{y_{21}} \approx -1 \quad (4.138)$$

Thus, for $K_t K_a K_m \gg 1$, the stiffness of the system is unchanged over that of this system without torque feedback, and the force-reflection ratio is proper.

Finally, the relative stability of this system is examined on the basis of the zeros of

$$\Delta = \frac{[J_m s + K_b + (K_a K_m / s)](J_m s + K_b)}{1 + K_t K_a K_m} \quad (4.139)$$

$$y_{22} = J_m s + K_b + (K_a K_m / s) \quad (4.140)$$

and

$$y_{11} = \frac{J_m s + K_b + K_t K_a^2 K_m^2}{1 + K_t K_a K_m} \quad (4.141)$$

The damping factors for the roots of Eq. (4.139), (4.140) and (4.141) are

$$\xi = \frac{K_b}{2(J_m K_a K_m)^{\frac{1}{2}}} \quad (4.142)$$

for the dominant zeros of Δ ,

$$\xi = \frac{K_b}{2(J_m K_a K_m)^{\frac{1}{2}}} \quad (4.143)$$

for the zeros of y_{22} , and

$$\xi = \frac{K_b}{2(J_m K_t K_a^2 K_m^2)^{\frac{1}{2}}} \quad (4.144)$$

for the zeros of y_{11} .

Comparison of Eq. (4.139) to Eq. (4.141) with Eq. (4.61) to Eq. (4.63) indicates that this system has the same relative stability as that of the system without input admittance reduction, except for the short-circuit output condition. The quantity y_{11} of Eq. (4.141), whose roots control the stability of the system under this condition, takes the same form as y_{ij} of Eq. (4.126) when an effort is made to reduce inertia and friction, i.e., when $K_t K_a K_m \gg 1$:

$$y_{11} = \frac{J_m s + K_b}{1 + K_t K_a K_m} + K_a K_m \quad (4.145)$$

and

$$\xi = \frac{K_b}{2(J_m K_t K_a^2 K_m^2)^{\frac{1}{2}}} \quad (4.146)$$

Again, mutual damping can be used to restabilize the system without affecting Criteria 1 and 2.

4.6 General Comments on the Two Force-reflecting Systems and the Two Methods of Improving Feel

The two basic configurations of Figs. 4.4 and 4.7 have been shown to have approximately the same feel, stiffness and stability. Comparison of Eqs. (4.45) with (4.57) would indicate that the positional-force system of Fig. 4.7 would have half the input admittance of the positional-positional system of Fig. 4.4, and thus would have a better sense of feel. However, with differential tachometric feedback (whose presence would be almost a certainty in a high-performance force-reflecting system), the systems have the same input admittance. [Compare Eq. (4.70) with Eq. (4.84).] Availability of components (a good torque transducer may be difficult to obtain) might cause the system of Fig. 4.4, the positional-positional system, to be preferred over the positional-force system of Fig. 4.7, but analysis alone has indicated little to recommend one over the other.

Two separate schemes have been discussed for modifying these two basic systems so as to produce lower no-load input admittance: the use of positive tachometric and acceleration feedback, and the use of negative torque feedback. In both cases the reduction of input friction and inertia by the same factor produces an adverse effect on stability. Thus, it would at first appear that the method of positive tachometric and acceleration feedback is to be preferred, since separate control over inertia and friction is possible and a compromise can be obtained which might produce suitable stability. However, the same compromise can also be effected for the torque feedback method by adding negative tachometric feedback deliberately to replace part of the friction that is removed by the $(1 + K_t K_a K_m)$ factor of torque feedback. In any case, the use of differential tachometric feedback would be added to restabilize a system in which friction and inertia have been reduced. Thus, the choice between the two methods for improving feel, based on the results of this chapter, remains quite arbitrary. However, two practical considerations do exist which argue strongly for the method of torque feedback, and these considerations are discussed in the next paragraphs.

Up until now, this thesis has considered only linear proportional systems, characterized by linear integro-differential equations with constant coefficients. Specifically, the forces of friction have been assumed to be of the viscous (velocity-proportional) variety such as result from the back-emf of electric motors (K_b) and/or the movement of well-lubricated bearings. In any mechanical system a different kind of friction may also be encountered, in which the force required to move a member is large until the member begins moving and then becomes proportional to velocity. This type of "break-away" or static friction is difficult to analyze mathematically, but its effects are obviously troublesome in remote manipulation. Since torque feedback reduces all the forces necessary to move the input member in a force-reflecting system, it naturally reduces the feel of static friction as effectively as the feel of inertia or viscous friction. Individual compensation for inertia and viscous friction (such as by the method of positive acceleration and tachometric feedback) cannot cope with the problem of static friction, and suffers in this important way by comparison with torque feedback.

Secondly, considerations of sensitivity favor the use of torque feedback to improve feel. In essence, torque feedback modifies inertia (or friction), as shown in Eq. (4.147):

$$J' = \frac{J}{1 + K_t K_a K_m} \quad (4.147)$$

On the other hand, positive acceleration feedback modifies inertia as shown in Eq. (4.148):

$$J' = J[1 - (K_a K_m K_\alpha / J)] \quad (4.148)$$

Since parameters such as amplifier gain, K_a , are subject to change during normal operation of the system, the effective inertia, J' , is subject to change and, thus, changes will occur in feel, stiffness and stability. The sensitivity of J' to changes in K_a may be expressed mathematically by relating the percent change in J' caused by a given percent change⁽¹⁾ in K_a .

For the system with torque feedback, Eq. (4.147) gives, for this percent-change relation,

$$\frac{dJ'/J'}{dK_a/K_a} = \frac{K_a dJ'}{J' dK_a} = - \frac{K_t K_a K_m J}{J' (1 + K_t K_a K_m)^2} = - \frac{K_t K_a K_m}{1 + K_t K_a K_m} \quad (4.149)$$

For $1 + K_t K_a K_m \gg 1$, Eq. (4.149) becomes

$$- \frac{dJ'/J'}{dK_a/K_a} \approx 1, \quad (4.150)$$

and a given percent change in K_a produces an equal and opposite change in J' .

For the system with positive acceleration feedback, Eq. (4.148) gives

$$\frac{dJ'/J'}{dK_a/K_a} = \frac{K_a dJ'}{J' dK_a} = - \frac{K_a K_m K_\alpha}{J'} = - \frac{K_a K_m K_\alpha}{J - K_a K_m K_\alpha} \quad (4.151)$$

For the methods to be compared on an equal basis, it is necessary that

$$1 + K_t K_a K_m = \frac{1}{1 - (K_a K_m K_\alpha/J)} \quad (4.152)$$

The inequality $1 + K_t K_a K_m \gg 1$ implies

$$\frac{1}{1 - (K_a K_m K_\alpha/J)} \gg 1 \quad (4.153)$$

and

$$K_a K_m K_\alpha/J \approx 1 \quad (4.154)$$

Insertion of Eqs. (4.153) and (4.154) into Eq. (4.151) gives

$$- \frac{dJ'/J'}{dK_a/K_a} = \frac{K_a K_m K_\alpha/J}{1 - (K_a K_m K_\alpha/J)} \gg 1, \quad (4.155)$$

and a given percent change in K_a produces a much larger change in J' .

Comparison of Eqs. (4.150) and (4.155) shows that, from the viewpoint of

sensitivity to changes in system parameters, the use of torque feedback is to be preferred to the use of positive acceleration and/or tachometric feedback.

4.7 Experimental Results

Models of both of the basic force-reflecting configurations (Figs. 4.4 and 4.7) have been constructed in the laboratory and appear to be quite capable of performing remote manipulation, i.e., the position of the output member follows the input, load forces are reflected back to the input, the input admittance is reasonably small, and the relative stability is such as to provide near-critical damping. As discussed in the analysis (Sec. 4.3), differential tachometric feedback provides the required stability without a discernable effect on input admittance.

In an effort to reduce the no-load input admittance and thus improve feel, the more promising of the two feedback arrangements, torque feedback, has been applied to both systems. Its effect is immediately evident in improving feel, particularly when static friction is deliberately incorporated into the system. As predicted in the analysis, when the stiffness remains fixed, torque feedback produces a reduction in relative stability, and differential tachometric feedback is again effective in re-stabilizing the system.

In order to provide quantitative results, the first force-reflecting servomechanism of Sec. 4.2 (positional-positional) has been tested using frequency-response techniques. The method consists in the measurement of the short-circuit admittance parameter, y_{11} , which by reason of the physical symmetry of the system (Figs. 4.3 and 4.4) is the same as y_{22} . Specifically, sinusoidal steady-state measurements of the magnitude of admittance - the ratio of rms torque to rms velocity - were made as a function of frequency when one handle was driven by a mechanical device producing sinusoidal velocity and the other handle was locked down. Figure 4.13 shows a schematic of the test arrangement. The mechanical

Fig. 4.13 Schematic of the experiment for measuring y_{11} . Except for their output pinions, servo-mechanisms A and B are not shown.

linkage attached to an eccentric provides sinusoidal motion of a controlled frequency to the input handle. Frequency and torque were the measured quantities. Velocity was calculated from frequency and length of stroke.

Figure 4.14 contains a plot of

$$\left| y_{11}(j\omega) \right| = \frac{\text{rms torque at input}}{\text{rms velocity at input}} \quad (4.156)$$

when the input was driven sinusoidally and the output was locked down.

Two curves are presented: curve A corresponds to the system with no torque feedback, and curve B corresponds to the system with torque feedback adjusted so that $(1 + K_t K_a K_m) = 1.6$. When an attempt was made to use much greater amounts of torque feedback, the input admittance became too small to be measured with the laboratory equipment available. As will be pointed out in Chapter 5, future experimental work on this subject should be based on a bigger system and a better torque transducer. Then the inertia and friction will be greater (of course, the power output of the servoamplifier has to be higher) and large amounts of torque feedback can be applied.

No attempt is made here to use the experimental data of Fig. 4.14 for an exact verification of a mathematical model. The experiment was conducted with a large amount of tachometric feedback (to add friction, synthetically, and make admittance measurable) and high amplifier gain. As a result, the system moved into and out of a nonlinear region of operation, due to saturation in the motor and amplifier. However, under the assumption that the short-circuited force-reflecting servomechanism behaves roughly as an R-L-C parallel circuit, the values of K_b (analogous to $1/R$), $K_a K_m$ (analogous to $1/L$), and J_m (analogous to C) can be inferred from Fig. 4.14. Then, the calculated admittance of a system with these values of K_b , $K_a K_m$, and J_m can be plotted in an attempt to fit the experimental curve. This is done in the paragraphs below.

Fig. 4.14 Experimentally determined curves of short-circuit input admittance of a force-reflecting servomechanism. Curves A and B are without torque feedback and with torque feedback, respectively.

Fig. 4.15 Calculated curves of short circuit input admittance of a force-reflecting servomechanism. Curves A and B are without torque feedback and with torque feedback, respectively.

The same servomotor is used as in the experiment of Chapter 2, Sec. 2.4, but with a 40:1 gear reduction (see Fig. 4.13). Thus J_m is taken to be

$$J_m = (40)^2(4.5 \times 10^{-6}) = 0.0072 \text{ lb-ft-sec}^2 \quad . \quad (4.157)$$

The value of K_b (including the effect of tachometric feedback) is taken to be the minimum value of admittance shown on the experimental Curve A of Fig. 4.14. At the frequency corresponding to this minimum, J_m and $K_a K_m$ are assumed to cancel (antiresonance). Thus,

$$K_b = 0.25 \text{ lb-ft-sec} \quad . \quad (4.158)$$

Finally, $K_a K_m$ is taken to be the value that produces antiresonance with the known value of J_m at the null frequency (2.5 cycles per second) observed from Curve A. Thus,

$$K_a K_m = (6.28f)^2 J_m = (246)(0.0072) = 1.77 \text{ lb-ft/rad} \quad . \quad (4.159)$$

The input admittance of an R-L-C parallel circuit (in terms of the appropriate mechanical quantities) is

$$Y = \left[K_b^2 + [6.28fJ_m - (K_a K_m / 6.28f)]^2 \right]^{-\frac{1}{2}} \quad . \quad (4.160)$$

When torque feedback is added ($1 + K_t K_a K_m = 1.6$), the input admittance is

$$Y = \left[\frac{1}{1.6} K_b^2 + [6.28fJ_m - (K_a K_m / 6.28f)]^2 \right]^{-\frac{1}{2}} \quad . \quad (4.161)$$

Equations (4.160) and (4.161), with values of J_m , K_b , $K_a K_m$ from Eqs. (4.157) to (4.159), are plotted in Fig. 4.15.

Although the graphs of Fig. 4.14 (experimental) are by no means identical with those of Fig. 4.15 (calculated), they possess the same general shape and support the analysis in two important ways. First, the short-circuit input admittances of the physical system are, at least approximately, of the same form as those of the mathematical model, i.e., the admittance of an R-L-C parallel circuit. Second, the effect of torque feedback in reducing input admittance is evident.

CHAPTER 5

CONCLUSIONS

This dissertation presents a study of force feedback in linear, proportional servomechanisms of both the positional and force-reflecting types.

In a positional servomechanism, positive force feedback has been shown to reduce the effect of load disturbances (both active and passive) on the behavior of the servomechanism. A second-order analysis was presented and an experiment which verifies the analytical conclusions has been described.

The effect of positive acceleration feedback on a positional servomechanism was also studied. Both analytical (second-order) and experimental results showed that positive acceleration feedback reduces the apparent inertia of the servomotor and thus improves damping and speed of response. A third-order analysis, however, indicated that positive acceleration feedback greatly reduces damping when additional time delays become significant.

Two basic force-reflecting servomechanisms were described and criteria were established by which these mechanisms can be evaluated in terms of their suitability in performing remote manipulations. Methods for improving the characteristics of both systems were analyzed, and negative force feedback was shown to be particularly effective in improving the sense of feel of a force-reflecting remote manipulator. An experiment to demonstrate this improvement has been performed and data which support the analysis were presented.

For further study in the application of force feedback to a force-reflecting servomechanism, it is recommended that a larger model be constructed. Then, the degree to which force feedback reduces the apparent inertia and friction can be more easily determined, and further physical measurements can be made. Of prime importance is the design of a

suitable force transducer. Finally, a higher-order analysis, accounting for additional time delays, should be applied to the force-reflecting system, and compensation networks for counteracting higher-order effects should be investigated.

BIBLIOGRAPHY

1. Bode, H. W., Network Analysis and Feedback Amplifier Design, Van Nostrand, New York, 1945, p. 47, p. 67.
2. Burnett, J. R., and Schiewe, A. J., Fifth Quarterly Progress Report on Bilateral Force-reflecting Servomechanism Research, Purdue University Engineering Experiment Station, 1955, p. 8.
3. Chalmers, W. J., Servomechanism Synthesis through Network Parameters, Ph.D. Thesis, Purdue University, 1955.
4. Evans, W. R., Graphical Analysis of Control Systems, Trans. AIEE, 67, 547-51 (1948).
5. Fett, G. H., Feedback Control Systems, Prentice-Hall, New York, 1954, p. 221.
6. Goertz, R. C., and Bevilacqua, F., A Force-reflecting Positional Servomechanism, Nucleonics, 10 (11), 33-45 (1952).
7. Goertz, R. C., Burnett, J. R., and Bevilacqua, F., Servos for Remote Manipulation, IRE Convention Records, Part 9, 1953.
8. Schmidt, E. M., Effective Reduction of Inertia and Friction in a Positional Servomechanism, M. S. Thesis, Purdue University, 1957.
9. Truxal, J. G., Automatic Feedback Control System Synthesis, McGraw-Hill Book Co., New York, 1955, p. 221-277.
10. Van Valkenburg, M. E., Network Analysis, Prentice-Hall, New York, 1955, p. 418.