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NEUTRON WALL LOAD DISTRIBUTIONS
IN A CIRCULAR CROSS SECTION
TOKAMAK

BY

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Neutron Wall Load Distributions in a Circular Cross Section Tokamak

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ABSTRACT

The distributions of the angular and scalar flux and current around the wall of a circular cross section tokamak are calculated by numerically solving the integral form of the neutron transport equation. The effect of the toroidal geometry is taken into account while investigating three different isotropic plasma source distributions - uniform, peaked, and shifted. The results of the calculations for a typical large scale tokamak design indicate a strong dependence of the scalar flux and the current (of DT neutrons) on the wall position, resulting in "hot spots" on the wall. The calculations also indicate a marked variation of the angular flux of DT neutrons with the wall position, which may be an important consideration in wall sputtering and tritium breeding evaluations. The angular distribution is used as a source condition for the neutron transport code ANISN, to investigate the effect on the flux and tritium breeding ratio in a fusion reactor blanket.

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I. INTRODUCTION

The angular distribution and the magnitude of the DT neutron flux incident on the first wall of a tokamak power reactor will be an important consideration in the reactor design for a number of reasons. It could be important in the divertor design, e.g. will a large number of neutrons enter the divertor region and thus be lost from the system, in which case special shielding may be required near the divertor. Also, the neutron flux may have an effect on the wall sputtering and thus affect the production of impurity neutrals which could react with the hot plasma. The angle at which the neutrons strike the wall may determine the angular distribution of the sputtered neutrals, e.g., whether they emerge normal to the wall (and thus perhaps interact with the plasma) or whether they emerge nearly tangent to the wall and are transported into the divertor region. The neutron wall load is, of course, a basic parameter in the neutronics design of a fusion reactor, e.g., in considerations of the tritium breeding ratio and the nuclear heating rates in the blanket.

In this report we calculate the neutron wall flux and current for a circular cross section tokamak due to an isotropic source of neutrons assumed to be produced in a DT plasma. The three dimensional characteristics of the toroidal geometry are accounted for using a ray tracing process to numerically solve the integral neutron transport equation. In

section II the theory used and the numerical method of solution are discussed. In section III a comparison with previous work is presented. In sections IV and V the results of a calculation of the wall load distribution of a typical tokamak reactor design are presented and discussed. In section VI the calculated angular distributions are used as source conditions for the discrete ordinates transport code ANISN in order to study the effect on the breeding ratio for a typical blanket design. A summary and conclusions drawn from the analyses are given in section VII.

II. METHOD OF SOLUTION

A. Equations and Geometry

The integral solution at any particular point in space of the neutron transport equation may be written as¹

$$F(\underline{\Omega}) = \int_0^{\infty} s(\rho) \exp(-\Sigma\rho) d\rho \quad (1)$$

where $F(\underline{\Omega})$ is the neutron angular flux in the direction $\underline{\Omega}$, ρ is the path length, $s(\rho)$ is the volumetric source, and Σ is the macroscopic absorption cross section. For our case, we are interested only in the streaming of the neutrons from the plasma to the wall, so that there are no interactions and $\Sigma = 0$. Thus, equation (1) reduces to

$$F(\underline{\Omega}) = \int_0^{\rho_w} s(\rho) d\rho \quad (2)$$

where ρ_w indicates where the ray hits the wall. The scalar flux F is then found by integrating $F(\underline{\Omega})$ over all solid angles $d\underline{\Omega}$, e.g.,

$$F = \int F(\underline{\Omega}) d\underline{\Omega} \quad (3)$$

Also, the neutron current J across a surface perpendicular to a particular direction $\underline{\Omega}'$ is defined by

$$J(\underline{\Omega}') = \int F(\underline{\Omega}) \underline{\Omega} \cdot \underline{\Omega}' d\underline{\Omega} \quad (4)$$

The differential solid angle $d\underline{\Omega}$ may be written as $d\underline{\Omega} = \sin\theta d\theta d\phi = -d\phi d\mu$, where θ and ϕ are the angles in a spherical coordinate system and $\mu = \cos\theta$. Choosing the $\theta = 0$ axis of this system parallel to $\underline{\Omega}'$, four basic quantities may be calculated:

$$F(\underline{\Omega}) = F(\mu, \phi) = \int s(\rho) d\rho \quad (5a)$$

$$F(\mu) = \int F(\mu, \phi) d\phi = \iint s(\rho) d\rho d\phi \quad (5b)$$

$$F = \int F(\mu) d\mu = \iiint s(\rho) d\rho d\phi d\mu \quad (5c)$$

$$J = \int \mu F(\mu) d\mu = \iiint s(\rho) \mu d\rho d\phi d\mu \quad (5d)$$

We have written a code to numerically calculate, at any point on the wall of a torus, the two angular fluxes, $F(\mu, \phi)$ and $F(\mu)$, the scalar flux F , and the current J .

The geometry and coordinate systems involved in the solution of equations (5) are shown in Figure 1. To account for the toroidal geometry, it is convenient to define three coordinate systems, one at the wall point, one at the center of the torus, and one at the plasma centerline. The primary system is placed at the center of the torus and is composed of cylindrical coordinates (R, λ, Z) . A second system consists of the polar coordinates (r, χ) measured from the plasma center in any (R, Z) plane. At any wall point P , designated by the angle χ up from the outside of the torus, a spherical coordinate system (ρ, θ, ϕ) , or equivalently (ρ, μ, ϕ) , is used.

Three lengths of interest in the tokamak are also shown in Figure 1: the major radius R_t , the plasma radius r_p , and the wall radius r_w . We can define two ratios from these lengths which are useful in comparing different torus sizes. The first is the aspect ratio A , defined as

$$A \equiv \frac{R_t}{r_p} \quad (6)$$

The second, which we denote by Y , is a measure of the size of the plasma in relation to the torus cross section:

$$Y = \frac{r_w}{r_p} \quad (7)$$

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Fig. 1. Torus geometry showing the three coordinate systems used in the ray-tracing process.

B. Sources

We have used three different source distributions - uniform, peaked, and shifted - in our investigation of the wall load. We assume all are isotropic sources, so that the source strength is only a function of (r, χ) , the location in the plasma cross section. For the uniform source, we assume $s(r)$ is simply constant within the plasma for $r < r_p$.

In the peaked source approximation, we assume the source is a quadratic in r , e.g.,

$$s(r, \chi) \sim 1 - \left(\frac{r}{r_p}\right)^2, \quad r < r_p \quad (8)$$

With the shifted source, we allow for the first-order effect of the toroidal geometry on the plasma, by which the magnetic flux surfaces, on which the plasma density is constant, are shifted outward.² That is, the source strength is a function of circles in the (R, Z) plane, which are shifted to progressively greater R , rather than being concentric at R_t . If the radius of the shifted flux surface is denoted by ψ , and α is used as the polar angle, the ψ -circles are centered at $R_t + f(\psi)$, and the values of (r, χ) corresponding to (ψ, α) can be found from

$$r^2 = (\psi \sin \alpha)^2 + (f + \psi \cos \alpha)^2 \quad (9a)$$

$$\cos \chi = (f + \psi \cos \alpha)/r \quad (9b)$$

We approximate the plasma shift $f(\psi)$ by the simple expression

$$f(\psi) = \epsilon \frac{(r_p^2 - \psi^2)}{r_p} \quad (10)$$

where ϵ is a constant (less than 1) which measures the maximum shift, ϵr_p .

The shifted source strength is written as

$$s(r, \chi) \sim 1 - \frac{\psi^2}{r_p^2} \quad (11a)$$

where the value of ψ must be found by solution of shift equations 9a and 9b. Without reproducing the derivation, we indicate that

$$\frac{\psi^2}{r_p^2} = \frac{2\epsilon^2 - 4\epsilon \frac{r}{r_p} \cos \chi + 2 \frac{r^2}{r_p^2}}{1 + 2\epsilon^2 - 2\epsilon \frac{r}{r_p} \cos \chi + \sqrt{1 + 4\epsilon^2 - 4\epsilon \frac{r}{r_p} \cos \chi - 4\epsilon^2 \frac{r^2}{r_p^2} \sin^2 \chi}} \quad (11b)$$

Note that for $\epsilon = 0$, this form for the shifted source strength reduces to that of the peaked source, as required. The shifted source is thus represented by a quadratic whose peak occurs at a radius ϵr_p . In Section IV the effect of these three different source representations on the neutron wall flux is investigated.

In order to compare the fluxes calculated from the different sources it is necessary to normalize the different

volumetric source distributions $s(r)$ such that S is the same for each, where

$$S \equiv \iiint s(r, \chi) \, dr d\chi d\lambda \quad (12)$$

and the integral is over the entire plasma volume. This is achieved by integrating each $s(r, \chi)$ in equation (12) and then rescaling by a constant. The normalized distributions $s(r, \chi)$ for each source across the plasma cross section are shown in Figure 2. From equation (12), it was found that the maximum value of the peaked source is twice the value of the uniform source, while for $\epsilon = 1/2$, the maximum value of the shifted source is approximately 95% that of the peaked source.

C. Discretization

It would be relatively straightforward to analytically solve equation (5a) for $F(\mu, \phi)$ with a particular source $s(\rho)$, except for the difficulty in determining when the path crosses the torus wall r_w . To avoid this problem, it was decided to solve for $F(\mu, \phi)$ using numerical methods based on a ray tracing scheme. The angular mesh is first discretized by dividing μ from 0 to 1 into L parts and ϕ from 0 to 2π into $6L$ parts. The ray tracing process then entails choosing a particular direction (μ_i, ϕ_i) and following the path out from the wall point P in discrete steps $\Delta\rho$, sampling the source s_k

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Uniform Source

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Peaked Source

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Shifted Source
 $\epsilon=0.5$

Fig. 2. Normalized plasma source strength as a function of position in the circular plasma cross section for the three source distributions.

whenever the step is within the plasma, until the path comes to the torus wall (denoted by point P'), at which point it stops. The sum of all the sources encountered along this ray is then the angular flux $F(\mu_i, \phi_i)$ for the particular direction (μ_i, ϕ_i) . A different direction is then chosen and the process is repeated, and in total there are $L \times 3L = 3L^2$ rays traced. (Symmetry in ϕ eliminates half of the $6L \phi_i$.) In terms of this discretization process, equations (5) are written as

$$F(\mu_i, \phi_i) = \Delta\rho \sum_{\rho} s_k \quad (13a)$$

$$F(\mu_i) = 2 \Delta\phi \Delta\rho \sum_{\phi} \sum_{\rho} s_k \quad (13b)$$

$$F = 2 \Delta\mu \Delta\phi \Delta\rho \sum_{\mu} \sum_{\phi} \sum_{\rho} s_k \quad (13c)$$

$$J = 2 \Delta\mu \Delta\phi \Delta\rho \sum_{\mu} \mu \sum_{\phi} \sum_{\rho} s_k \quad (13d)$$

where $\Delta\mu = 1/L$ and $\Delta\phi = \frac{\pi}{3L}$. The factor of 2 arises from the symmetry in the angle ϕ , as shown in Figure 1. We have chosen the step size to be

$$\Delta\rho = \eta r_w/L \quad (14)$$

where η is a constant on the order of 1. The particular direction cosines μ_i were chosen at the midpoints of the $\Delta\mu$'s, so that $\mu_1 = \frac{2L-1}{2L}$ and $\mu_L = \frac{1}{2L}$, and thus the angular flux is

not calculated for the end points of $\mu = 1$ and $\mu = 0$. Similarly the ϕ_i are chosen as the midpoints of the $\Delta\phi$, and the midpoints of the $\Delta\rho$ are used to evaluate the s_k . Whenever a point along the ray is found to be inside the plasma, the value of the source at that point is calculated and added to the angular flux, so that the ray tracing process is independent of the source distribution. Thus it is possible to calculate the angular flux for all three of the source distributions during one ray trace. This results in a significant reduction in computing time, since it is not necessary to repeat the ray process for each source.

It is of interest to compare the accuracy of the discretized solution as L and η are varied. Clearly fewer steps will be needed (and thus less computer time) for larger values of η and smaller values of L . In Table 1 the results of varying η and L on the current and scalar flux are shown for a number of different cases. In Tables 1a and 1b L is varied from 10 to 20 for a torus with $A = 2.5$ and $Y = 1.25$ for a uniform source with $\eta = 1$. It is seen that the accuracy is very good for both the wall points, $\chi = 0^\circ$ and $\chi = 135^\circ$, ranging from 2% for $L = 10$ to less than 0.1%. For the variation of η from 0.5 to 3 shown in Table 1c with $A = 2.5$, $Y = 1.25$, and $L = 12$ at $\chi = 135^\circ$, the agreement is again found to be within less than 1% for the uniform source. The large decrease in the number of steps necessary in the ray tracing process is also evident in Table 1, with the decrease going as

Table 1. Variation of the Flux and Current with L and η

Table 1a: $R_t=4$, $r_w=2$, $r_p=1.6$, uniform source, $\chi=0^\circ$, $\eta=1.0$

L	Flux F_L	Current J_L	Ray Steps	CPU Time (sec)	$\frac{F_L - F_{20}}{F_{20}}$	$\frac{J_L - J_{20}}{J_{20}}$
10	10.849	8.2984	4834	1.594	-2.04%	-2.40%
12	10.884	8.3225	8434	2.777	-1.72%	-2.12%
14	11.037	8.4504	13465	4.401	-0.34%	-0.61%
16	11.088	8.5181	20160	6.388	0.12%	-0.08%
18	11.057	8.4880	28729	9.203	-0.16%	-0.17%
20	11.075	8.5026	39498	12.931	0.0	0.0

Table 1b: $R_t=4$, $r_w=2$, $r_p=1.6$, uniform source, $\chi=135^\circ$, $\eta=1.0$

L	Flux F_L	Current J_L	Ray Steps	CPU Time (sec)	$\frac{F_L - F_{20}}{F_{20}}$	$\frac{J_L - J_{20}}{J_{20}}$
10	12.127	7.3685	5039	1.782	-0.09%	0.17%
12	12.132	7.3421	8743	2.952	-0.05%	-0.18%
14	12.160	7.3711	13839	4.660	0.18%	0.21%
16	12.144	7.3694	20649	6.699	0.05%	0.19%
18	12.146	7.3578	29443	9.785	0.07%	0.03%
20	12.138	7.3557	40348	13.352	0.0	0.0

Table 1c: $R_t=4$, $r_w=2$, $r_p=1.6$, uniform source, $\chi=135^\circ$, L=12

η	Flux F_η	Current J_η	Ray Steps	CPU Time (sec)	$\frac{F_\eta - F_{.5}}{F_{.5}}$	$\frac{J_\eta - J_{.5}}{J_{.5}}$
0.5	12.158	7.3604	17498	5.412	0.0	0.0
1.0	12.132	7.3421	8743	2.952	-0.21%	-0.25%
1.5	12.213	7.4049	5825	1.972	0.45%	0.60%
2.0	12.217	7.4127	4391	1.529	0.49%	0.71%
2.5	12.119	7.2720	3498	1.268	-0.32%	-1.20%
3.0	12.223	7.4402	2915	1.086	0.53%	1.08%

the ratio of the step size η or as the cube of the ratio of the discretization L . In Figure 3 the variation of the angular flux $F(\mu, \phi)$ with L is shown for the case of Table 1b. From these plots it is evident that the angular flux is again nearly the same for different L , but it is also seen that a larger L tends to smooth out the $F(\mu, \phi)$. For the finer mesh the flux is more detailed and the rough edges are smoothed somewhat, as expected, since for the larger L more angles are analyzed. From these tests with different L and η we were able to conclude that an accurate calculation of the flux and current was possible using coarse angular meshes and large step sizes, thus reducing the computer time needed for the solution.

D. Computer Program

A computer program named WALLOAD was written to solve equations (13) for the flux and current. Input to the code consists of r_p , r_w , R_t , χ , η , L , an integer designating the source to be used, and the value of ϵ for the shifted source when necessary. WALLOAD then calculates the angular mesh points and begins the ray tracing process. This procedure entails converting from the wall coordinate system (ρ, θ, ϕ) to the torus system (R, λ, Z) as the steps are taken along the path. At each step the source value is calculated if the point is within the plasma, until the ray reaches the torus

L=10

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L=16

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L=20

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Fig. 3. Angular Flux $F(\mu, \phi)$ for different values of L for $R_t=4.0$, $r_w=2.0$, $r_p=1.6$, uniform source, $\chi=135^\circ$, $\eta=1.0$.

wall (as determined by the (R,Z) coordinate of the path point). The sum of all the source values encountered along the particular ray is then multiplied by $\Delta\rho$ to give the value of $F(\mu,\phi)$ for that ray. Then the values of $F(\mu)$, F , and J are calculated by summing over all the angles and multiplying by the appropriate mesh size, as given in equation (13).

III. COMPARISON WITH PREVIOUS RESULTS

In order to test our method of solution, we compared the results of WALLOAD to two previously studied cases. The first test case was to calculate the scalar flux and current for an infinite cylinder filled with a uniform (constant) isotropic source ($A = \infty$, $Y = 1$). It can be shown that for this case the scalar flux is constant around the cylinder wall and the current is equal to one-half of the flux.

To simulate an infinite cylinder we used a very large R_t , so that the aspect ratio is extremely large and $R_t \gg r_w$. Running WALLOAD for a number of points around the wall for this cylindrical approximation with a uniform source, we found that the calculated flux was within 1% of the theoretical value. Furthermore, the current was found to be one-half of the flux to a similar accuracy. Hence we concluded that WALLOAD was able to accurately simulate the flux in an infinite cylinder.

The second test case was a previous study of neutron flux asymmetry in toroidal geometries by W. Dänner.³ He

solved for the angular flux from a homogeneous (uniform) isotropic source in a circular cross-section torus by solving a 4th order equation whose roots correspond to points where a ray from the wall crosses or just touches the plasma source. This exact method of solution is limited to the uniform source case, but it still provided a check on the results of WALLOAD.

A different coordinate system was used in Ref. 3 than the one employed by WALLOAD (as in Figure 1). Dänner uses the angles γ and δ , which are related to (θ, ϕ) by

$$\tan \phi = \frac{\sin \gamma}{\tan \delta} \quad \text{and} \quad \sin \theta = \frac{\sin \delta}{\cos \phi} \quad (15)$$

To check the results of our code, we compared our analysis to the results in Ref. 3 for a circular cross-section torus with $A = 4$ and $Y = 1.25$ with a uniform source. The comparison of Dänner's angular flux $F(\delta, \gamma)$ for both methods of solution is shown in Figure 4. Two different wall points, $\chi = 0^\circ$ and 135° , are compared. It is clear from Figure 4 that the angular flux calculated by WALLOAD is in excellent agreement with the results of the exact solution of Ref. 3. The agreement was found to be equally as good at other wall points. By integrating first over δ and then over γ , we were able to also compare the calculations of $F(\gamma)$ and F (the scalar flux) for both methods. Again good agreement was found between both calculations, indicating that our numerical ray tracing scheme predicted the same fluxes as the exact solution

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Fig. 4. Comparison of the angular flux $F(\delta, \gamma)$ calculated by WALLOAD and ref. 3 for a torus with $A=4.0$ and $Y=1.25$ for a uniform source distribution.

of Ref. 3 for a uniform source.

From these two comparisons of WALLOAD, we were able to conclude that our code was capable of solving the neutron streaming problem in a circular cross-section tokamak.

IV. WALL LOAD DISTRIBUTION

There have been a number of recent, detailed design studies of tokamak type fusion power reactors.⁴⁻⁶ Most of these are for non-circular cross-section toruses, with aspect ratios A ranging from 2.6 to 3.7 and effective wall to plasma radius ratios Y of 1.1 to 1.2. From these designs, we concluded that to study the neutron wall load distribution in a "typical" tokamak, we could run WALLOAD for a torus with $A = 3.0$ and $Y = 1.1$, ignoring the asymmetry of the torus cross section of these typical designs since our code is presently suitable only for a circular cross section torus. From the results of the discretization and step fraction tests shown in Table 1 and Figure 3, we decided to use $L = 14$ and $\eta = 1.0$. The three alternate sources - uniform, peaked, and shifted - were used in the analysis, with $S = 1.0$ in each case. In the shifted source case, a shift ϵ of 0.5 was chosen. For each of these sources we calculated the current J , scalar flux F , and the angular fluxes $F(\mu)$ and $F(\mu, \phi)$ for different wall points with angles $\chi = 0^\circ$ to $\chi = 180^\circ$.

The results of our calculation of the angular neutron flux $F(\mu, \phi)$ are shown as contour plots in Figures 5-11 for the

three sources at seven wall points - $\chi = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, \text{ and } 180^\circ$. The scale of the curves on each plot is the same: each contour line represents an increment of 1.0 in the value of the flux. Hence it can be seen that the maximum value of $F(\mu, \phi)$ at $\chi = 0^\circ$ (the outer edge of the torus) is approximately twice that at $\chi = 180^\circ$ (inner edge of the torus).

The second angular flux $F(\mu)$, found by integrating $F(\mu, \phi)$ over ϕ , is shown in Figures 12-14 as a function of wall angle χ for each of the three sources. For ease of illustration, both perspective and contour plots of $F(\mu)$ vs. χ are given for each source distribution. On the contour plots, the scale is again the same for each source, but each line represents an increment of 2.0 in the value of $F(\mu)$. Hence it can be seen that the maximum value of $F(\mu)$ is nearly the same for each source, and occurs at $\chi = 0^\circ$.

In Figure 15 the scalar flux F and current J are shown as functions of the wall angle χ for each of the three sources. These curves show that for a circular cross-section tokamak with $A = 3$ and $Y = 1.1$, the flux and current are definite functions of wall position χ , with different shapes and magnitudes depending on the source distribution.

V. DISCUSSION

The plots of the angular flux in Figures 5-11 show a very interesting behavior in $F(\mu, \phi)$ for different wall

positions. At the outside wall point $\chi = 0^\circ$, it is seen that $F(\mu, \phi)$ is symmetric about the angle ϕ , as expected since there is obviously symmetry about the torus at this point. The effect of each source distribution on the angular flux is also evident from Figure 5, with a maximum value of 30 for the peaked source, 28 for the shifted source and 21 for the uniform source. Hence the peaked source angular flux maximum is about 43% greater than the uniform source maximum at $\chi = 0^\circ$. The angle $\theta_{\max}$ at which the maximum occurs is also different for each source - for the uniform source, $\theta_{\max} = 35^\circ$; for the peaked source, $\theta_{\max} = 39^\circ$; and for the shifted source $\theta_{\max} = 45^\circ$. One other effect of the different sources is the range over which there are neutrons incident on the wall point. For example, at $\chi = 0^\circ$ there is a flux up to $\theta = 78^\circ$ for the uniform source but only up to 73° for the peaked source.

At the wall point $\chi = 30^\circ$, shown in Figure 6, we now see that the angular flux is no longer symmetric about ϕ . The maximum flux, which is about 5% less than the maximum at $\chi = 0^\circ$, is now shifted to an angle ϕ of about 105° for all the sources. Also, the angle of maximum flux $\theta_{\max}$ now ranges from 38° for the uniform source to 48° for the shifted source. The range of the flux has also increased slightly, so that now there is a flux up to $\theta = 80^\circ$ for the uniform source.

By examining Figures 7-11 for other values of χ up to 180° , one can establish certain trends in the angular flux vs.

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Fig. 5. Angular flux $F(\mu, \phi)$ at $\chi=0^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 6. Angular flux $\bar{F}(\mu, \phi)$ at $\chi=30^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 7. Angular flux $F(\mu, \phi)$ at $\chi=60^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 8. Angular flux $F(\mu, \phi)$ at $\chi=90^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 9. Angular flux $F(\mu, \phi)$ at $\chi=120^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 10. Angular flux $F(\mu, \phi)$ at $\chi=150^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

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Fig. 11. Angular flux $F(\mu, \phi)$ at $\chi=180^\circ$ for a "typical" tokamak with $A=3.0$ and $Y=1.1$ for three sources. Each line represents an increment of one in the value of the flux. The semi-circles are lines of constant $\cos \theta$; while the radial lines are at constant ϕ .

wall position. The first is that the maximum flux tends to decrease with increasing χ , so that by 180° the maximum is less than one-half that at 0° . Also, the angle $\theta_{\max}$ at which this maximum occurs increases with χ , so that at the inside of the torus ($\chi = 180^\circ$) the maximum is nearly tangent to the wall ($\theta_{\max} \approx 90^\circ$). However, note that the total flux for any θ (i.e., $F(\mu)$) is still larger at smaller values of θ , since for those angles the flux ranges over all values of ϕ while for $\theta \approx 90^\circ$ it does not. The range of θ over which the angular flux is nonzero is also seen to increase with χ , so that by $\chi = 120^\circ$ the entire range of $\theta(0^\circ - 90^\circ)$ is covered. The behavior of $F(\mu, \phi)$ with the angle ϕ shows an interesting behavior, as evident in Figures 5-11. As noted earlier, at $\chi = 0^\circ$ it is symmetric about ϕ . However, the flux then becomes shifted to higher values of ϕ up to $\chi = 90^\circ$ (top of the torus) where the maximum flux occurs for $\phi = 120^\circ - 135^\circ$, depending on the source (Figure 8). For χ greater than 90° , ϕ at the maximum then decreases until the flux is again symmetric about ϕ at $\chi = 180^\circ$, as expected. Thus at the inner and outer points of the torus ($\chi = 180^\circ$ and 0°) the angular flux is symmetric about ϕ , but the maximum flux is nearly tangent to the wall at $\chi = 180^\circ$ and is much more normal to the wall at $\chi = 0^\circ$.

The effects of the different source distributions on the angular flux are evident by examining Figures 5-11. In general, it can be seen that the peaked source has the highest

value of $F(\mu, \phi)$ at each wall point. The angle of maximum flux $\theta_{\max}$ tends to be largest for the shifted source, while the uniform source generally results in the largest range of θ over which there is an angular flux (for χ less than 120°).

The second angular flux, $F(\mu)$, found by integrating $F(\mu, \phi)$ over all angles ϕ , is shown in Figures 12-14 as a function of wall angle χ for each of the three sources, for the "typical" tokamak with $A = 3$ and $Y = 1.1$. Both a perspective plot and a contour plot are given for each source in order to emphasize the variation of $F(\mu)$ with μ (i.e., θ) and χ . From these figures it can be seen that there is no flux tangent to the wall ($\theta = 90^\circ$) until χ is greater than about 90° , at the top of the torus. For smaller values of χ , there is a sharp decrease in $F(\mu)$ as θ increases. Hence most of the flux at these points is directed fairly perpendicular to the wall. Note also that the θ of maximum $F(\mu)$ increases as χ increases. That is, $F(\mu)$ at $\chi = 0^\circ$ has its maximum at $\theta \approx 35^\circ$, while at $\chi = 45^\circ$ the maximum is at $\theta \approx 47^\circ$. However, for points toward the inside of the torus ($\chi > 90^\circ$) this shifting of the maximum $F(\mu)$ has ceased. At those wall points the maximum is more normal to the wall ($\theta \approx 0^\circ - 15^\circ$), although the magnitude has decreased and instead the flux is fairly evenly distributed over all θ . This behavior was found to occur for all three sources, although the flux is highest for the peaked and shifted sources. In conclusion we see that the angular flux as a function of θ is very highly peaked for wall

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Fig. 12. Angular flux $F(\mu)$ as a function of $\cos \theta$ (i.e. μ) and wall angle χ for a uniform source distribution. The lines in the right hand figure represent an increment of two in the flux.

Fig. 13. Angular flux $F(\mu)$ as a function of $\cos \theta$ (i.e. μ) and wall angle χ for a peaked source distribution. The lines in the right hand figure represent an increment of two in the flux.

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Fig. 14. Angular flux $F(\mu)$ as a function of $\cos \theta$ (i.e. μ) and wall angle χ for a shifted source distribution. The lines in the right hand figure represent an increment of two in the flux.

points on the outside of the torus, but for inner wall points the flux is much smoother and extends over all values of θ . In section VI we have taken these results for $F(\mu)$ at certain wall points and formed a shell source for the flux for use in the discrete ordinates code ANISN, to study the effect of different $F(\mu)$ on the tritium breeding ratio in a blanket.

We next integrated the angular flux over all μ to calculate the scalar flux F and the current J at each wall point, which are shown in Figure 15. From this plot it is clear that there is a much different behavior of F and J for each source distribution. Both the uniform and peaked source were found to have a maximum flux slightly inside the torus, at $\chi \approx 105^\circ$, with the uniform source producing a larger flux. The maximum flux for the uniform source is 7.7% greater than the minimum (at $\chi = 0^\circ$), while for the peaked source the maximum is 10.4% larger. The shifted source flux is much different. For that case, the maximum occurs at $\chi = 0^\circ$ and is 42.7% larger than the minimum at $\chi = 180^\circ$. Since the source intensity for the shifted source is strongest at the outer edge of the plasma, the outer wall points feel a stronger flux than those on the inside. This effect is also evident for the shifted source current, where there is a large difference of 61.3% between maximum and minimum. The current for the uniform and peaked source are nearly the same, with maxima occurring at $\chi \approx 25^\circ$ which are $\sim 17\%$ larger than the minima at $\chi = 180^\circ$. The reason for this behavior in the current vs. χ

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Fig. 15. Scalar flux F and current J as a function of wall angle χ for the three sources. The units are arbitrary. The percent values are the difference between the minimum and maximum flux or current for each source.

can be understood from Figures 12-14, in which we found that $F(\mu)$ was more normal to the wall point for smaller values of the wall angle χ .

Hence we see that the neutron wall load is not constant about the wall for the tokamak analyzed here, but that a "hot spot" occurs whose position and magnitude depend on the source. It is interesting to note that a uniform wall load might be obtained for a different shift ϵ in a shifted source. This is seen by comparing the flux vs. wall angle for a shift of 0.5 and for the peaked source, which can be considered a shifted source with shift of zero.

VI. EFFECT OF THE ANGULAR FLUX ON TRITIUM BREEDING RATIO

In addition to the gross variation in flux and current with χ , there will be second-order effects due to the variation in the angular distribution. In this section we use the results of the angular flux calculation to generate a source condition for the discrete ordinates transport code ANISN.⁷ For the four wall points, $\chi = 0^\circ, 60^\circ, 120^\circ$, and 180° , we used $F(\mu)$ due to a peaked source (shown in Figure 13) to form four different shell source angular distributions of 14 MeV neutrons suitable for input to ANISN. Slab geometry was used with an S_8 angular quadrature, and the $F(\mu)$ for each χ was split up into appropriate ranges of $\Delta\mu$ to form each shell source. The composition of the blanket was that of the standard or "benchmark" blanket⁸ of lithium, carbon, and niobium, shown in Figure 16. For comparison, the shell source

Distances in cm	0	150	200	200.5	203.5	204		264	294	300
Origin →	Plasma	Vacuum	Nb	94 % Li	Nb		94% Li	C	94 % Li	
				6% Nb			6% Nb		6% Nb	
Zone Number	1	2	3	4	5	20 cm	20 cm	20 cm	7	8
Region Number	1	2	3	4	5	6	7	8	9	10
Material	A	B	C	D	C		D	E	D	
Number of Intervals Per Zone	1	1	3	6	3		30	15	3	
Thickness (cm)	150	50	0.5	3	0.5		60	30	6	

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Comment: The intervals in each zone are equal step length. There are 62 intervals all together.

Fig. 16. Configuration of the Bench-Mark Blanket Model (from Ref. 8).

formed from $F(\mu)$ for an infinite cylinder approximation with very large A and $Y = 1.1$ was also used with ANISN.

The effect of each of these sources on the tritium breeding ratio T is shown in Table 2, broken down into two contributions to T : T_6 for the reactions ${}^6\text{Li}(n, \alpha)t$ and T_7 for the reaction ${}^7\text{Li}(n, n'\alpha)t$. The breeding ratios for two other shell sources - isotropic and beam - are also given. The total T is found to be nearly the same for all sources, with agreement to less than 1%.

However, there is a larger difference in the individual contributions T_6 and T_7 for the different sources. The breeding from the ${}^6\text{Li}$ reaction varies by 2%, while for the ${}^7\text{Li}$ reaction there is a 5% variation. T_6 is largest for the sources at the wall points $\chi = 120^\circ$ and 180° , while the T_7 is highest for the $\chi = 0^\circ$ point. The reason for this difference is due to the variation of $F(\mu)$ with χ , as shown in Figure 13 for the peaked source, and also because of the nature of the different Li reactions. The ${}^6\text{Li}$ reaction has a large cross section at thermal neutron energies, while the ${}^7\text{Li}$ is a threshold reaction requiring neutron energies larger than 2 MeV. Hence for a source of 14 MeV neutrons more perpendicular to the wall, as in the $\chi = 0^\circ$ case and the beam case, the T_7 is large because more of the high energy neutrons are able to penetrate through the niobium into the lithium. Conversely, for a more uniform neutron source (such as at $\chi = 120^\circ$ or 180° or the isotropic source) there is more

Table 2

Breeding Ratios for Different Source
Distributions in the Benchmark Blanket
Design.

Source	T6	T7	T
Isotropic (P-O)	.933659	.500787	1.434446
Beam	.900072	.534875	1.434948
$F(\mu)$, $\chi=0^\circ$	.917460	.525490	1.442950
$F(\mu)$, $\chi=60^\circ$	.920567	.521373	1.441940
$F(\mu)$, $\chi=120^\circ$	.923944	.514143	1.438087
$F(\mu)$, $\chi=180^\circ$	.923557	.514533	1.438090
$F(\mu)$, ∞ cylinder	.925663	.511695	1.437358

thermalization of the high energy neutrons as they enter the blanket (at oblique angles and are scattered), and thus there is more tritium production from the ^6Li . This difference in T6 and T7 could be an important factor in optimization of the blanket.

VII. SUMMARY AND CONCLUSIONS

We have investigated the neutron wall load in a circular cross-section tokamak using a numerical ray tracing process to solve the integral neutron transport equation. Three different plasma source distributions were analyzed. It was found that the angular neutron flux, as well as the scalar flux and the current, were highly dependent on the wall position. The calculations indicate that significant "hot spots" may occur on the first wall, affecting the wall design and also the design of the blanket. In addition to this gross normalization, the variations in the angular flux distribution were found to affect the tritium breeding in a typical blanket.

Variation of the wall load in tokamaks with non-circular cross sections is presently under investigation; this will require a more sophisticated treatment of the ray-wall-crossing test.

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