MASTER

MATHEMATICAL TECHNIQUES FOR VIBRATION ANALYSIS
OF REACTOR SYSTEMS

by

C. Noble
Idaho Nuclear Corporation
Idaho Falls, Idaho

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MATHEMATICAL TECHNIQUES FOR VIBRATION ANALYSIS OF REACTOR SYSTEMS

I. INTRODUCTION

The vibration analysis of a nuclear reactor by the finite element method can be divided into three major steps: generating the stiffness and mass matrices, determining the natural frequencies (eigenvalues) and mode shapes (eigenvectors), and solving the uncoupled equations of motion for displacements in normal coordinates. The emphasis in this paper is on the mathematical techniques used to solve the eigenvalue problem

$$K\vec{x} = \lambda M\vec{x} \quad (1)$$

where K and M are the stiffness and mass matrices, respectively, and on techniques to solve the differential equations

$$\ddot{\eta}_i + 2\zeta\lambda\dot{\eta}_i + \lambda\eta_i = h(t), \quad i = 1, 2, \dots, n \quad (2)$$

for displacements in normal coordinates.

The eigenvalue problem, Equation (1), is solved by the STRAP-D code⁽¹⁾, which reduces the dimensions of the stiffness matrix by condensing, and by the STRAP-MS code which reduces the dimensions by modal synthesis⁽²⁾. The solution of the equations of motion, Equation (2), is accomplished by Laplace transform techniques.

II. SOLUTION OF THE GENERALIZED PROBLEM

If K and M are such that no reduction of dimensions is required, then any one of many methods may be used to determine eigenvalue and eigenvector solutions of Equation (1). The threshold Jacobi and QR methods⁽³⁾ of solving eigenvalue problems that require no reduction are described briefly.

First, the threshold Jacobi method is used to determine the solutions of Equation (1). A matrix, S , is assumed to exist such that

$$S^T M S = \text{diag}(m_1, m_2, \dots, m_n) \quad (3)$$

and

$$S^T K S = I. \quad (4)$$

Then Equation (1) is transformed to an equivalent eigenvalue problem

$$\det(S^T K S - \lambda \text{diag}(m_1, m_2, \dots, m_n)) = 0. \quad (5)$$

Since the mass matrix, M , is positive definite, Equation (5) can be transformed to

$$\det(N^{-1/2} S^T K S N^{-1/2} - \lambda I) = 0 \quad (6)$$

where

$$N^{-1/2} = \text{diag}(m_1^{-1/2}, m_2^{-1/2}, \dots, m_n^{-1/2}). \quad (7)$$

By defining

$$Q = N^{-1/2} S^T K S N^{-1/2} \quad (8)$$

the original eigenvalue problem, Equation (1), is transformed to

$$\det(Q - \lambda I) = 0 \quad (9)$$

where Q is a real symmetric matrix. The threshold Jacobi technique can be used to determine solutions of the eigenvalue problems

$$\det(Q - \lambda I) = 0 \quad (10)$$

and

$$\det(M - \lambda I) = 0. \quad (11)$$

Given a real symmetric matrix A , the diagonalization of A by the threshold Jacobi method is as follows: A threshold value, t , is set and plane rotations are applied repeatedly to A until every off-diagonal element of A has magnitude equal to or less than t . An off-diagonal element of A induces a rotation if and only if it is greater in magnitude than t . Now t is reduced in magnitude and the process repeated until the threshold value, t , is smaller than some specified convergence criterion. The eigenvalues are the diagonal elements of the diagonalized matrix, A , and the eigenvectors are the accumulated products of the plane rotation matrices. Convergence for any real symmetric matrix is guaranteed by this process. Thus, the threshold Jacobi method can be used to determine orthogonal matrices S and T such that

$$S^T M S = \text{diag}(m_1, m_2, \dots, m_n) \quad (12)$$

and

$$T^T Q T = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n). \quad (13)$$

Determination of a particular subcollection of the eigenvalues and corresponding eigenvectors by the threshold Jacobi method, requires that a complete set of eigenvalues and eigenvectors be determined. However, for large matrices, determination of a complete set may not be practical from a computer storage and computation time standpoint.

The eigenvalue problem, Equation (1), can also be solved by the QR method. In this case the matrix, M , is factored into

$$S^T S = M \quad (14)$$

by using the Cholesky algorithm, where S is a lower triangular matrix. Since M is a symmetric, positive definite matrix, S is real. Now Equation (1) can be written as

$$\det(I - \lambda S^T S) = 0 \quad (15)$$

which is equivalent to

$$\det((S^T)^{-1} K S^{-1} - \lambda I) = 0. \quad (16)$$

The symmetric matrix, Q , is defined to be

$$Q = (S^T)^{-1} K S^{-1}. \quad (17)$$

The eigenvalues of Q can be found by a QR algorithm that performs the following operations: (1) transformation of the matrix, Q , to tridiagonal form by Householder transformations, (2) iterations by using plane rotations, and (3) origin shifts to accelerate convergence. The eigenvectors are not determined by this process. Given an eigenvalue, λ , the eigenvector, $\bar{x}$, corresponding to λ can be determined by inverse iteration⁽³⁾. Only the desired eigenvectors (mode shapes) must be computed. Difficulties are encountered if S is ill-conditioned with respect to inversion or if the dimensions of S are too large for computer manipulation. Some minor problems also occur when using inverse iteration if Q has multiple eigenvalues.

III. EIGENVALUES AND EIGENVECTORS OF LARGE MATRICES

A structural model that has many degrees of freedom, or equivalently, a structure that induces large stiffness and mass matrices, is considered. The analyst has the problem of reducing the dimensions of the stiffness and mass matrices to smaller matrices that are amenable to solution. This reduction of dimensions can be done by condensing or by modal synthesis.

Condensing is accomplished as follows: The stiffness matrix

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \quad (18)$$

is partitioned where K_{11} represents the degrees of freedom that are to be considered, and K_{22} represents the degrees of freedom that are to be condensed. An effective stiffness matrix

$$\bar{K} = K_{11} - K_{12} K_{22}^{-1} K_{21} \quad (19)$$

is computed. The mass matrix, $\bar{M}$, corresponding to the considered degrees of freedom is computed. Then the reduced eigenvalue problem

$$\det(\bar{K} - \lambda \bar{M}) = 0 \quad (20)$$

is solved.

An alternate technique of reducing the dimensions of the eigenvalue problem is modal synthesis. When modal synthesis is used a structure, S , is partitioned into substructures $S_1, S_2, \dots, S_p$ each of which can be analyzed individually. Then approximate stiffness and mass matrices, K' and M' , for the structure are computed using the mode shapes of the substructures. Since K' and M' depend on the mode shapes of the substructures, p matrices are determined that satisfy:

$$\phi_j^T K_j \phi_j = \text{diag} (v_{1,j}, v_{2,j}, \dots, v_{m(j),j}) \quad (21)$$

and

$$\phi_j^T M_j \phi_j = I \quad (22)$$

where K_j and M_j are the stiffness and mass matrices of substructure, S_j .

The ϕ_j matrices are computed by assuming the nodes at the junctions of subsystems are fixed. The modal synthesis technique also requires some means of describing interactions among the subsystems. The interactions are determined

by relaxing the nodes at the junctions and computing relaxation⁽⁴⁾ or constraint modes⁽²⁾. By utilizing the normal modes of the substructures and the relaxation modes a transformation matrix:

$$T_0 = (\phi_1^{(j_1)}, \phi_2^{(j_2)}, \dots, \phi_p^{(j_p)}, C) \quad (23)$$

is constructed, where $\phi_i^{(j_i)}$ is a subcollection, with j_i members of the modes of subsystem, S_i , and C is the matrix of relaxation modes. T_0 is a rectangular matrix with column dimension equal to the sum of the j_i plus the number of relaxation modes, and row dimension equal to the row dimension of K and M . Approximate stiffness and mass matrices are computed as follows:

$$K' = T_0^T K T_0 \quad (24)$$

and

$$M' = T_0^T M T_0. \quad (25)$$

The reduced eigenvalue problem

$$\det(K' - \lambda M') = 0 \quad (26)$$

is now solved. This solution yields a collection of eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_m$ and the corresponding eigenvectors $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m$. These eigenvectors are transformed to eigenvectors of the system by

$$\vec{z}_k = T_0 \vec{x}_k. \quad (27)$$

If $\|K \vec{z}_i - \lambda_i M \vec{z}_i\|$ is not sufficiently close to zero, a procedure given by Bajan and Feng⁽²⁾ is used for constructing a new transformation T_1 , using the $\vec{x}_i$ vectors and the modes of the substructures to refine the eigenvectors until the desired accuracy is obtained. Experience to date with matrices representative of structural problems has shown the solution of Equation (26) to be accurate initially and not requiring subsequent refinement.

IV. SOLUTION OF THE EQUATIONS OF MOTION

The eigenvalue problem Equation (1) arises from the need to uncouple the equation

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = f(t). \quad (28)$$

Assuming that ϕ is a solution of Equation (1) and by changing of coordinates as

$$\vec{\eta} = \phi \vec{x} \quad (29)$$

the uncoupled equations of motion

$$\ddot{\eta}_i + 2\zeta \lambda_i \dot{\eta}_i + \lambda_i^2 \eta_i = \phi^T f(t) \quad i = 1, 2, \dots, n \quad (30)$$

follow. Viscous damping is assumed with ζ equal to the percent of critical damping. The problem of determining the structural response has been reduced to solving n one-degree of freedom systems of the form

$$\ddot{y} + \xi \beta \dot{y} + \beta^2 y = \frac{\beta^2}{k} q(t, y, \dot{y}) \quad (31)$$

where $\beta > 0$ and $0 \leq \xi < 1$.

Some of the widely used methods of solving the differential equations, Equation (30), assume the solution to be locally well behaved and compute an approximate solution using the truncated Taylor series. A typical representative of these methods is the Runge-Kutta⁽⁵⁾ method which stepwise solves

$$\dot{y} = h(t, y, \dot{y}), \quad y(t_0) = y_0, \quad \dot{y}(t_0) = \dot{y}_0 \quad (32)$$

assuming

$$\begin{aligned} y_{n+1} &= y_n + (\Delta t) \dot{y}_n + \frac{1}{6} (\Delta t)^2 (k_1 + k_2 + k_3) \\ \dot{y}_{n+1} &= \dot{y}_n + \frac{1}{6} (\Delta t) (k_1 + 2k_2 + 2k_3 + k_4) \end{aligned} \quad (33)$$

where

$$\begin{aligned} k_1 &= h(t_n, y_n, \dot{y}_n) \\ k_2 &= h\left(t_n + \frac{(\Delta t)}{2}, y_n + \frac{(\Delta t)}{2} \dot{y}_n, \dot{y}_n + \frac{(\Delta t)}{2} k_1\right) \\ k_3 &= h\left(t_n + \frac{(\Delta t)}{2}, y_n + \frac{(\Delta t)}{2} \dot{y}_n + \frac{(\Delta t)^2}{4} k_1, \dot{y}_n + \frac{(\Delta t)}{2} k_2\right) \\ k_4 &= h\left(t_n + \Delta t, y_n + (\Delta t) \dot{y}_n + \frac{(\Delta t)^2}{2} k_2, \dot{y}_n + (\Delta t) k_3\right). \end{aligned} \quad (34)$$

As evidenced by Equation (32), the forcing function can be dependent upon the spatial coordinates as well as their velocities. Also methods that use a truncated Taylor series can usually be adapted to solving the equation

$$M\ddot{x} + C\dot{x} + Kx = f(t) \quad (35)$$

without having to solve the eigenvalue problem, Equation (1). One of the methods involving a truncated Taylor series would be necessary if the stiffness matrix, K , were time dependent.

A common problem is the case in which $q(t, y, \dot{y})$ is a discretely defined function that is dependent upon time only. This case is compatible with blowdown analysis and many seismic analyses. In this case $\phi(s)$ is defined as:

$$\phi(s) = L(y(t)) = \int_0^\infty y(t) e^{-st} dt \quad (36)$$

where $\phi(s)$ is the Laplace transform of $y(t)$. Then

$$\phi(s) = \frac{L\left(\frac{\beta^2}{k} g(t)\right)}{s^2 + 2\xi\beta s + \beta^2} + \frac{(s^2 + 2\xi\beta)y_0 + \dot{y}_0}{s^2 + 2\xi\beta s + \beta^2} \quad (37)$$

where y_0 and $\dot{y}_0$ are the initial displacement and velocity, respectively. Equation (37) implies that

$$y(t) = L^{-1}(F(s)G(s)) + L^{-1}(H(s)) \quad (38)$$

where

$$F(s) = L\left(\frac{\beta^2}{k} q(t)\right) \quad (39)$$

$$G(s) = (s^2 + 2\xi\beta s + \beta^2)^{-1} \quad (40)$$

and

$$H(s) = ((s^2 + 2\xi\beta)y_0 + \dot{y}_0) G(s). \quad (41)$$

Use of the convolution integral produces*

$$L^{-1}(F(s)G(s)) = \frac{\beta^2}{\xi\beta(1-\beta^2)^{\frac{1}{2}}} \int_0^t g(\lambda) e^{-\xi\beta(t-\lambda)} \sin(\beta(1-\xi^2)^{\frac{1}{2}}(t-\lambda)) d\lambda \quad (42)$$

So far no assumptions have been made regarding the character of $g(t)$. Now $g(t)$ may be defined as

$$q(t) = a_i + b_i t, \quad t_i \leq t \leq t_{i+1} \quad (43)$$

where a_i , b_i and t_i are known for $i = 0, 1, 2, \dots, p$.

The value of $y(T)$ is determined, where $t_k \leq T \leq t_{k+1}$ for an arbitrary fixed $1 \leq k \leq p$. From Equation (42)

$$\begin{aligned} y(t) = & \sum_{i=0}^{k-1} \frac{\beta^2}{k} \int_{t_i}^{t_{i+1}} (a_i + b_i \lambda) e^{-\xi\beta(T-\lambda)} \sin(\beta(1-\xi^2)^{\frac{1}{2}}(T-\lambda)) d\lambda \\ & + \frac{\beta^2}{k} \int_{t_k}^T (a_k + b_k \lambda) e^{-\xi\beta(T-\lambda)} \sin(\beta(1-\xi^2)^{\frac{1}{2}}(T-\lambda)) d\lambda \\ & + z(T) \end{aligned} \quad (44)$$

where

$$z(t) = L^{-1}(H(s))$$

Thus, integrals of the type

$$\int_{T_1}^{T_2} (a + b\lambda) e^{-c(t-\lambda)} \sin d(t-\lambda) d\lambda, \quad (45)$$

where a, b, c , and d are scalars, must be solved. Equation (45), through integration by parts, can be reduced to

$$\begin{aligned} & \left[\frac{c^{c(t-\lambda)} \sin d\lambda}{c^2 + d^2} (-c \cos d\lambda + d \sin d\lambda) (a + b\lambda) \right. \\ & - \frac{c^{c(t-\lambda)} \cos d\lambda}{c^2 + d^2} (-c \sin d\lambda - d \cos d\lambda) (a + b\lambda) \\ & + \frac{be^{c(t-\lambda)}}{(c^2 + d^2)^2} \sin d\lambda ((d^2 - c^2) \cos d\lambda + 2cd \sin d\lambda) \\ & \left. + \frac{be^{-c(t-\lambda)}}{(c^2 + d^2)^2} \cos d\lambda (d^2 - c^2) \sin d\lambda - 2cd \cos d\lambda \right] \Bigg|_{\lambda=T_1}^{\lambda=T_2} \end{aligned} \quad (46)$$

The advantage of using this method is that the time step used is not a function of any local conditions as in Runge-Kutta or other methods that involve the truncated Taylor series. Laplace transforms are particularly advantageous when β is large; then the time step required to maintain stability of solution for the local numerical schemes must necessarily be small compared to the period of $\cos(\beta(1-\xi^2)^{\frac{1}{2}}t)$.

If the problem to be solved has a forcing function that is dependent on the spatial coordinates, then the Laplace transform techniques discussed do not apply; whereas, the techniques that are dependent on local behavior usually can be adapted quite easily to solve such problems.

V. STRAP (STRUCTURAL ANALYSIS PACKAGE)

The STRAP⁽¹⁾ package was developed to analyze the response of structures to static and dynamic loads. The present STRAP package consists of two computer codes, STRAP-S and STRAP-D, for static and dynamic analyses, respectively. A third computer code STRAP-MS is being written that utilizes the modal synthesis technique to find the frequencies and mode shapes, and Laplace transforms or Runge-Kutta techniques to solve the differential equations of motion.

To use the STRAP-MS code, the structure is partitioned into substructures. STRAP-MS computes the stiffness matrices of the individual substructures and determines the relaxation modes by solving a series of static problems. The mass matrices are defined as lumped masses or a consistent mass is generated according to McCalley's⁽⁶⁾ technique.

Matrices satisfying Equations (21) and (22) are computed using the threshold Jacobi method. The matrix, Equation (23), is constructed by STRAP-MS using the desired substructure modes and the relaxation modes. The code performs the necessary matrix manipulation until solutions of the eigenvalue problem

$$\det(K' - \lambda M') = 0 \quad (47)$$

are determined where K' and M' are defined by Equations (24) and (25).

The STRAP-MS code has the capability of combining the two reduction techniques, condensing and modal synthesis, to solve large systems. After the structure has been partitioned into substructures, desired degrees of freedom are condensed from the substructures. Then according to Equation (18) a stiffness matrix of the form

$$K^{(j)} = \begin{pmatrix} K_{11}^{(j)} & K_{12}^{(j)} \\ K_{21}^{(j)} & K_{22}^{(j)} \end{pmatrix} \quad (48)$$

is generated for substructure, S_j , and the condensed stiffness matrix is

$$\bar{K}^{(j)} = K_{11}^{(j)} - K_{12}^{(j)} (K_{22}^{(j)})^{-1} K_{21}^{(j)} \quad (49)$$

The mass matrix is rearranged as

$$K^{(j)} = \begin{pmatrix} M_{11}^{(j)} & M_{12}^{(j)} \\ M_{21}^{(j)} & M_{22}^{(j)} \end{pmatrix} \quad (50)$$

for compatibility with Equation (48), and

$$\bar{M}^{(j)} = M^{(j)} \quad (51)$$

is defined.

An eigenvector, $\bar{x}_k^{(j)}$, satisfying

$$\bar{K}^{(j)} \bar{x}_k^{(j)} = v_k^{(j)} \bar{M}^{(j)} \bar{x}_k^{(j)} \quad (52)$$

generally has fewer components than the number of columns of K . With the eigenvector, $\bar{x}_k^{(j)}$, considered as a deflected shape of the substructure, S_j , a compatible displacement of the condensed degrees of freedom is computed by

$$\vec{x}_{ck}^{(j)} = - (K_{22}^{(j)})^{-1} K_{21} \vec{x}_k^{(j)} \quad (53)$$

Now

$$\vec{x}^{(jk)} = \begin{pmatrix} \vec{x}_k^{(j)} \\ \vec{x}_{ck}^{(j)} \end{pmatrix} \quad (54)$$

where $\vec{x}^{(jk)}$ satisfies

$$(\vec{x}^{(jk)})^T K^{(j)} \vec{x}^{(jk)} = v_k^{(j)} \quad (55)$$

If Ψ_j is defined to be

$$\Psi_j = (\vec{x}^{(j1)}, \vec{x}^{(j2)}, \dots, \vec{x}^{(jm)}) \quad (56)$$

then

$$(\Psi_j)^T K^{(j)} \Psi_j = \text{diag}(v_1^{(j)}, v_2^{(j)}, \dots, v_m^{(j)}) \quad (57)$$

In general $(\Psi_j)^T M^{(j)} \Psi_j$ is not a diagonal matrix. It will however, have more off-diagonal zeros than the consistent mass matrix. The transformation matrix for the modal synthesis process, Equation (23), is constructed using mode shapes computed according to Equation (54).

VI. NUMERICAL RESULTS

Two problems were run to compare the methods of determining frequencies and mode shapes discussed: (1) an analysis of the Semiscale Blowdown and Emergency Core Cooling loop shown in Figure 1, and (2) an analysis of one loop of a typical pressurized water reactor (PWR) shown in Figure 2.

Fig. 1 Semiscale Blowdown System

Fig. 2 Typical Large PWR Piping

1. ANALYSIS OF SEMISCALE BLOWDOWN AND EMERGENCY CORE COOLING PROJECT

The semiscale piping system was modeled as a 3-dimensional frame having 21 members and 22 nodal points as shown in Figure 3. This model with supports considered, had 114 unrestrained degrees of freedom. The dimensions of the stiffness

matrix were reduced to 63 by condensing most of the rotational degrees of freedom. This condensed stiffness matrix and an assumption that the mass was lumped at the nodal points determined the eigenvalue problem

$$\det(\bar{K} - \lambda \bar{M}) = 0$$

where $\bar{M}$ was a diagonal matrix. The resulting eigenvalue problem was solved for the fundamental frequencies of the system using STRAP-D.

For the STRAP-MS analysis, the semiscale loop model was partitioned into two substructures. The particular nodes and members defining each of the substructures is illustrated by Figure 4. The substructures, S_1 , and S_2 , had 52 and 56 unrestrained degrees of freedom, respectively. Since the substructures were connected at only one junction (Node 18), only 6 relaxation modes exist.

Fig. 3 Semiscale Blowdown System Model

Fig. 4 Subsystems of Semiscale Blowdown System

Using 20 modes from each subsystem and the 6 relaxation modes, the dimensions of K and M were reduced according to Equations (24) and (55). The resulting eigenvalue problem

$$\det(K' - \lambda M') = 0$$

with K' and M' of dimensions 46 by 46, was solved for the fundamental frequencies of the system using STRAP-MS.

The frequencies determined by STRAP-D and STRAP-MS are compared in Table I.

TABLE I

Comparison of First Fifteen Frequencies
Computed by STRAP-D and STRAP-MS for
Semiscale Blowdown System

Frequency (Hz)		
	STRAP-D	STRAP-MS
1	4.70	4.68
2	6.97	6.75
3	8.73	8.59
4	9.76	9.76
5	13.25	13.15
6	13.86	13.95
7	20.24	20.22
8	30.24	30.34
9	31.86	30.43
10	34.46	33.51
11	37.58	37.85
12	45.98	45.87
13	53.85	54.10
14	55.97	55.88
15	61.42	61.55

This problem was also solved using STRAP-MS without any reduction of dimension. The first 40 mode shapes and frequencies agreed with the results of STRAP-MS with subsystems within three percent. For this particular problem condensing is seen to be the most economical method of solution, since it involves finding the eigenvalues and eigenvectors of a 63 by 63 matrix and inverting a 51 by 51 matrix, whereas modal synthesis involves finding the eigenvalues and eigenvectors of a 52 by 52 matrix, a 56 by 56 matrix, and a 46 by 46 matrix.

2. FOUR LOOP PWR ANALYSIS

For the four-loop PWR, one loop of the piping system was modeled with 30 nodal points and 177 unrestrained degrees of freedom. The mass matrix was a non-diagonal consistent mass matrix generated according to McCalley's⁽⁶⁾ technique. Again the eigenvalue problem was solved by using three different models: (1) STRAP-D

with degrees of freedom condensed as shown in Figure 5; (2) STRAP-MS with two substructures as shown in Figure 6; and (3) STRAP-MS with substructures having condensed degrees of freedom. A degree of freedom was condensed in the STRAP-MS model if and only if that degree of freedom was condensed in the STRAP-D model.

INC-A-16230

Fig. 5 Model of PWR

INC-A-17520

Fig. 6 Subsystems of PWR

The frequencies resulting from these models are compared in Table II.

The STRAP-MS run with condensing and STRAP-MS without condensing gave slightly different answers for the lower frequencies, with a maximum difference of 20 percent at the eighteenth frequency. The different results given by STRAP-MS with condensing and STRAP-D with the same degrees of freedom condensed are realistic; they are for different models of the same structure. The vectors computed by Equation (54) imply that the reduced stiffness and mass matrices, K' and M' , used in the modal synthesis depend upon the masses at the nodes of the condensed degrees of freedom, whereas the frequencies computed by the STRAP-D code are independent of the inertial effects of these degrees of freedom.

The inertial dependence is illustrated by the frequencies of the condensed substructures and the frequencies of the complete substructures shown in Table III for the STRAP-MS models.

TABLE II

Comparison of First Twenty Frequencies
Computed by STRAP-MS, STRAP-MS with
Condensing, and STRAP-D for PWR

Frequency (Hz)

	STRAP-MS	STRAP-MS Condensed	STRAP-D Condensed
1	6.38	6.38	6.057
2	8.03	8.03	8.12
3	8.62	8.62	9.32
4	10.43	10.43	9.36
5	11.15	11.15	10.39
6	13.63	13.63	11.90
7	15.86	15.86	15.71
8	17.97	17.38	16.07
9	19.36	19.34	20.29
10	21.42	21.41	21.86
11	21.66	21.66	28.04
12	22.96	22.96	24.93
13	23.17	23.16	27.68
14	27.81	27.80	28.41
15	28.76	28.70	28.88
16	29.71	29.69	31.54
17	37.42	37.40	32.34
18	40.04	39.99	47.34
19	59.29	59.25	59.99
20	66.13	65.99	60.24

TABLE III

Comparison of Frequencies For
Condensed and Complete Substructure 1 of PWR

Frequency (Hz)		
	Condensed	Complete
1	10.54	10.54
2	16.37	16.28
3	29.45	28.75
4	40.50	39.96
5	59.60	59.29
6	64.45	63.66
7	103.54	93.96
8	125.93	122.49
9	143.75	136.28
10	223.77	141.50

The difference at the tenth frequency in Table III is strictly due to condensing.

VII. CONCLUSIONS

The comparisons indicate that the frequencies and mode shapes of a structure can be determined adequately either by the condensing technique or by the modal synthesis technique. The advantage of the modal synthesis technique is that more dynamic degrees of freedom can be considered as illustrated by the PWR analysis, in which 177 dynamic degrees of freedom were considered. The present STRAP-D code is restricted to at most 100 dynamic degrees of freedom. No significant differences in computation time are required to find the frequencies and mode shapes by either of the two methods.

The combining of the modal synthesis and condensing techniques is an important feature of the STRAP-MS code, in that it allows the structure to be defined with fewer substructures. Correspondingly fewer eigenvalue problems must be solved and the number of relaxation modes required is minimized. When the two techniques are combined in the manner described, empirical results indicate the vectors defined by Equation (54) should be included in the analysis.

For equations involving high frequency components, the solution of the equations of motion by the Laplace transform method for time-dependent forcing functions has proven to be considerably faster than the Runge-Kutta method. Both types of differential equation routines are needed - one to solve nonlinear problems and one to solve time-dependent forcing functions for high frequency equations.

REFERENCES

1. J. A. Dearien et al., STRAP--A Computer Code for Static and Dynamic Analysis with Studies Made Using the Code, IN-1362 (June 1970).
2. R. L. Bajan and C. C. Feng, Free Vibration Analysis by the Modal Substitution Method, Engineering Research Center, University of Colorado, Boulder, Colorado (June 1968).
3. J. H. Wilkinson, The Algebraic Eigenvalue Problem, Clarendon Press, Oxford (1965).
4. M. Pakstys, Application of Matrix Techniques to Analysis of Complex Structures, Proceedings for Short Course on Normal Modes, Shock and Vibrations, Pennsylvania State University (July 1966).
5. J. Todd, Survey of Numerical Analysis, McGraw Hill, New York (1962).
6. R. B. McCalley, Jr., Mass Matrix for a Prismatic Beam Segment, KAPL-M-6913, Knolls Atomic Power Lab., General Electric Co., Schenectady, New York (March 1968).