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TWO METHODS OF SPACE-TIME ENERGY DENSIFICATION

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TWO METHODS OF SPACE-TIME ENERGY DENSIFICATION*

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ABSTRACT

With a view to the goal of net energy production from a DT microexplosion, we study two ideas (methods) through which (separately or in combination) energy may be "concentrated" into a small volume and short period of time--the so-called space-time energy densification or compression. We first discuss the advantages and disadvantages of lasers and relativistic electron-beam (E-beam) machines as the sources of such energy and identify the amplification of laser pulses as a key factor in energy compression. The pulse length of present relativistic E-beam machines is the most serious limitation of this pulsed-power source. The first energy-compression idea we discuss is the reasonably efficient production of short-duration, high-current relativistic electron pulses by the self interruption and restrike of a current in a plasma pinch due to the rapid onset of strong turbulence. A 1-MW plasma focus based on this method is nearing completion at this laboratory. The second energy-compression idea is based on laser-pulse production through the parametric amplification of a self-similar or solitary wave pulse, for which analogs can be found in other wave processes. Specifically, the second energy-compression idea is a proposal for parametric amplification of a solitary, transverse magnetic pulse in a coaxial cavity with a Bennett dielectric rod as an inner coax. Amplifiers of this type can be driven by the pulsed power from a relativistic E-beam machine. If the end of the inner dielectric coax is made of LDT or another fusible material, the amplified pulse can directly drive a fusion reaction--there would be no need to switch the pulse out of the system toward a remote target.

INTRODUCTION

DT MICROEXPLOSIONS

It is useful to consider the conditions required to attain net energy release from a DT-fusion microexplosion, because these define particularly demanding space-time energy compression goals. For purposes of discussion we will assume as the primary energy source, a 1-MW capacitor bank with a 1- to 10-nsec quarter-cycle time when driving an appropriate load.

The theory of high-density fusion has been expertly reviewed by Linhart.¹ To attain the $\rho r = 1$ g/cm² burn condition for uncompressed solid-density fuel requires input energy of about 10^9 J in time of 2×10^{-9} sec to a pellet with an approximately 1-cm radius, i.e., a power of 5×10^{17} W and power flux of 5×10^{16} W/cm². An initial value of $\rho r < 1$ can be increased by convergent compression to $\rho r = 1$ because ρ increases as the compression ratio n , while for spherical compression r varies as $1/n^{1/3}$, and thus ρr is proportional to $n^{2/3}$. For $n = 1000$, the uncompressed ρr of the solid-density target can be $\rho r = 0.01$. Consequently, the total energy requirement can be reduced by

a factor of 10^6 to about 1000 J, the energy delivery time decreases to 2×10^{-11} sec and the power requirement to 2×10^{14} W, and the power-flux remains unchanged.

If compression is achieved by the Nuckolls ablation-driven rocket,² then the efficiency of energy coupling to the compressed-pellet core is only about 5%, making the energy and power requirements given above too low by a factor of 20. More efficient compression is possible as noted by Hora,³ and in Section 5 we will discuss the possibility of 100% efficient ablation-free compression.

The $\rho r = 1$ criteria arises from two independent factors: the requirement that inertial confinement times be equal to the burn time, and the demand that the particles from the DT burn are deposited in the fuel. It is possible to independently satisfy these two criteria, and thus obtain exothermic burn conditions for ρr of the fuel < 1 . The inertial confinement time can be increased by means of a tamper, or by transient containment of the hot plasma in a strong field such that the microinstability time of the contained plasma is equal to its burn time. On the other hand, a magnetic field with $B r = 1$ MG-cm may couple burn α 's to the system

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even though $\rho r \ll 1$.

In a tamped system with $\rho r < 1$, a field is required to reduce electron conduction losses to the tamper. It may also be possible to catch the low-density ablated fuel used to compress the pellet core in a strong field for a Bohm diffusion time and achieve yield from this ablation product. In fact it may be easier to obtain net yield from the contained corona than to burn the dense core produced by the ablation process. The idea that it is easier to reach exothermic burns of the rocket fuel than to achieve breakeven energy release from the rocket is suggested by study of the plasma focus.

It may also be possible to decrease the input energy to obtain a net DT yield by relatively unexplored direct conversion of fusion α to field energy in a field-contained explosive with $\rho r < 1$. For example, a pellet that is relatively transparent to its own α particles and surrounded by a strong magnetic field will permit the energetic α 's to diffuse across the field more easily than the electrons. As a result a net charge will develop on this quasi-contained pellet to the point where the α loss time is increased and the crossfield electron diffusion time is decreased so that a quasi-neutral plasma wind becomes possible. This results in the radius of positive charge becoming slightly larger than that of the negative charge, and as a result causes electrostatic implosion of the pellet. In addition, the DT-burn-driven anomalous diffusion of the target electron across the magnetic field might appear as a negative resistance and amplify the confining field.

At least one of the above means, or an as yet undiscovered equivalent trick, is mandatory to reduce the energy and power requirements well below the 10^9 J, 5×10^{17} W values for an inertially confined, uncompressed solid pellet.

ENERGY SOURCES

The pulsed power sources that most nearly approach the energy and power requirements for driving a microexplosion with a potential yield $Y > 1$ MJ are the laser and the relativistic E-beam machine. Existing E-beam machines have greater pulse energy and higher efficiency than high-power pulsed lasers, but also have a lower power level due to their 30- to 200-

pulse length and smaller power flux density--there is greater difficulty in focusing large relativistic currents, compared to the relative focusing ease of the laser. It is often argued that an advantage of the laser relative to the relativistic electron-beam pulsed power source is that the range for deposition of the laser light is shorter than that for relativistic electrons.

It is presently possible to convert 5% of a relativistic E-beam energy to collectively accelerated ions and 15% of the energy to few-cm microwaves.⁴

The efficiencies of these processes are likely to increase soon, and efficient conversion to microwaves can probably be extended to wavelengths shorter than 1 cm. Thus microwave pulses and energetic ion bursts must be considered as pulsed power sources that are competitive with relativistic E-beam and pulsed lasers for fusion-microexplosion applications.

Short-wavelength microwaves have only recently been explored seriously⁵ for laser applications because of the widely accepted but erroneous belief that far shorter wavelengths are necessary, and that a submicron laser is ideal for microexplosion applications.⁶ We will suggest that the "brand x" laser should actually be in the 0.01- to 1-cm wavelength range, and thus modern extension of well established microwave technology could be a more effective source of the coherent radiation in this wavelength range than a long-wavelength laser (maser). Combined use, by preexcitation of a mode of a microwave generator cavity with a maser followed by amplification of this field energy of the mode with a relativistic electron burst, might provide an effective hybrid pulsed power source.

The present low efficiency of the pulsed laser relative to that for the production of relativistic electron bursts is a transitory advantage of electron beams. The efficiencies of lasers will increase, and no doubt approach the values of present E-beam machines. On the other hand only small improvements in E-beam efficiency are possible, and efforts to reduce the multnanosecond pulses of these machines into the subnanosecond range will certainly result in some reduction in source to E-beam energy conversion efficiency. The ability to focus laser light is also not a very fundamental

advantage, since present lasers can be focused to spot sizes substantially smaller than the pellet diameter, while experimental efforts to focus relativistic electron bursts⁷ have achieved results approaching the focusings required to reach the diameter of a microexplosive having a potential yield at 20% burn efficiency of 1 MJ. The electron bursts obtainable from a 1 MJ plasma focus will have a diameter in the range $0.1 \text{ cm} > d \geq 100 \text{ }\mu\text{m}$, and thus may even exceed the required E-beam focusing.

The argument that deposition of relativistic E-beam energy is a problem relative to the range of laser light in a target cannot be maintained. The reason for this statement is the fact that E-beam, pulsed power sources that approach breakeven requirements will need a v/γ of > 100 , and thus the energy is primarily in the field rather than in the kinetic energy of the electrons. When a plasma is subjected to an electromagnetic field the E field is screened out in a Debye length $\lambda_D = v/\omega_p$, where ω_p is the electron plasma frequency, while the B field is screened in a collisionless skin depth $\delta = c/\omega_p$, a distance larger than λ_D by a factor c/v . Thus, from the point of view of the plasma, the electromagnetic field is an oscillatory B field, i.e., in effect one is dealing with an ac pinch.

Coupling of the field energy is not governed by a corona with a density corresponding $\omega = \omega_p$, where ω is the field frequency, but by the corona density that can be held by the radiation pressure of the field at a temperature where the plasma becomes collisionless. The resistivity is then governed by strong turbulence. We have estimated this critical field intensity to be $B = 2\pi \times 10^{17.8}$ sufficient to contain a corona with density $n = 5 \times 10^{22}$ at a temperature of 1 kV; at this temperature and density, the plasma becomes sufficiently collisionless to become strongly turbulent.

The greatly increased resistivity of the skin layer in the turbulent state results in rapid heating of the skin layer, and the skin's driven Bohm-like diffusion across the field is an ablation process that multiplies the field's radiation pressure by a factor of order v_A/v , where v_A is the Alfvén velocity for inward transport of field energy in the low-density plasma outside the corona, and v is the ion thermal velocity of the

material ablated from the dense corona surface. Since the Alfvén velocity v_A approaches c , the speed of light, the ablation amplification of the field pressure becomes a factor of order 100, identical to the results for laser driven ablation.

The anomalous resistivity leading to the ablation is due to Langmuir turbulence, i.e., to the current driven amplification of the skin-layer thermal electron plasma oscillation, and thus the frequency associated with energy deposition is not the electromagnetic driving field frequency ω , but the plasma frequency ω_p of the driven corona. ω_p is proportional to $\sqrt{n_e}$, where n_e is the corona density and is determined by the radiation pressure of the driving field, not by the frequencies of this field.

Thus, one realizes that large-energy-density fields result in skin currents that can drive strong turbulence in the corona whose density depends on the field's radiation pressure, and the energy transport to the pellet is due to field diffusion governed by the turbulent resistivity. These skin currents are in effect dc phenomena on the time scale of ω_p of the corona, and the Langmuir oscillation can become large in the time required for a few plasma oscillations. Thus for $\omega < \omega_p$, in first approximation, effective coupling of field energy to the pellet is governed by the driving field's intensity, not by its frequency. This frequency-independent equivalence of fields of the same energy density is valid provided $\omega < \omega_p$, and the field power flux density approaches 10^{15} to 10^{16} W/cm^2 , the value required to drive a fusion micro-explosion.

It is an accidental coincidence that the critical density for $1\text{-}\mu\text{m}$ Nd radiation of $n = 10^{21}$ is high enough that radiation pressure effects at current power flux densities make a difference of only about a factor of 2 in corona density, and thus energy coupling occurs at approximately the place where $\omega = \omega_p$. It is apparently this accidental coincidence that has given rise to the widespread belief that short-wavelength radiation is required for effective energy coupling without super-thermal electrons because energy is deposited where $\omega = \omega_p$.

This accidental coincidence is not scalable to other field frequencies,

since a decrease in frequency at constant Poynting vector power flux density will still result in a dense corona (where the radiation pressure now plays a dominant rather than an insignificant role), and since the Langmuir turbulent-field diffusion rate (which governs the rate of energy transport from the field into skin layer heating) is governed by ω_p of the corona; effective energy deposition at fixed field intensity will continue as the driving field frequency is decreased.

The most extreme form of this argument is achieved by reducing the field frequency to the point where the burn time is a quarter cycle time of the field, and the "ac pinch" becomes an ordinary dc pinch. For dc pinch it is well known that the field plasma interface occurs at the point of pressure balance $B^2/8\pi = 2nkT$, provided the resistivity is not strongly anomalous, and thus the Alfvén velocity (snowplow velocity) exceeds the field diffusion velocity.

If we consider as an example a 10-kV plasma, then a field of $2\pi \times 10^7$ G will be in pressure balance with the plasma for $n = 5 \times 10^{21}$, for which the burn time is 2×10^{-8} sec, corresponding to a quarter-cycle time of a field with frequency $\omega = 2.5\pi \times 10^7$. If we assume energy deposition occurs at $\omega = \omega_p$, the corresponding electron density for the plasma target would be $n = 2 \times 10^{26}$, a factor of 2×10^{15} less than the actual density of the skin layer, the location of the real resistive heating. This most extreme case clearly indicates that in general the idea that energy is deposited in a corona at or near $\omega = \omega_p$ is fallacious.

If the use of very-short-wavelength radiation is not fundamental, then we are led to ask what, if anything, does determine the optimum field frequency? If one goes beyond the question of energy deposition and considers stability, then it is possible to arrive at an optimum frequency range. Ensley² has shown that the interface between the field and the pellet is stabilized if the wavelength of the field is equal to or greater than the pellet radius. Evidence for instabilities for $\lambda \ll R$ has been provided both by actual laboratory experiment and numerical simulation computer experiments.⁹ On the other hand, the dynamic stabilization of MHD instabilities¹⁰ such as the $m = 0$ instability demand a field frequency ν such that $1/\nu < R/v$, where R

is the pellet radius and v is the ion thermal velocity, which for 10-kV burn condition becomes $v \geq 10^8$ cm/sec. Thus we can conclude that the wavelength λ of the driving field should be in the range $R \leq \lambda \leq 100 R$. We conclude that λ should be in the range: 250 μ m to 1 cm.

Thus the pulse power problem becomes that of producing pulses in this frequency range with total energy $E \leq 10^6$ J, at a power of $E/T > 10^{14}$ W, and flux $E/TA \geq 10^{16}$ W/cm², where A is the surface area of the uncompressed solid-density DT pellet.

OUR PROPOSED ENERGY SOURCE

As a result of the above discussion we are left with only the short pulse length as a unique advantage of the laser as a pulsed power source for driving fusion microexplosion. This advantage is fundamental and cannot be dismissed. The object of this paper is to present two methods that separately or in combination can approach the short pulse requirements with a nonlaser source. The production of small-diameter electron bursts at a pulse length less than that of existing E-beam machines by self-interruption and relativistic restrike of the current of a pinch due to the sudden onset of a strongly turbulent anomalous resistance is the basis for the first energy densification idea.

The short-pulse production in a laser amplifier is the particularly unique feature that results in the attractiveness of the laser for pulsed fusion application. In essence this process is the amplification of a self-similar or solitary wave in an active medium and results in an effective pulse width $\tau \ll L/c$, where L is the length of the energy storage device, in this case the active medium. The pulse width resulting from this parametric amplification is much shorter than the maximum passive delivery time τ of the energy stored in a reservoir of length L , which is $\tau \geq L/c$.

Parametric amplification of a solitary wave is not unique to the laser but can be formulated for a much broader class of wave processes. The most important idea contained in the present paper is a specific means of amplification of a transverse magnetic solitary wave in a coaxial cavity with a relativistic electron beam. If the idea presented

here proves viable in practice then presently unique laser-pulse amplification can be extended to microwave generation technique in the optimum wavelength range 250 μ m to 1 cm.

Section 1 of the paper discusses briefly the concepts of transit time amplification, the production of strong turbulence in a plasma, the resulting collapse of Langmuir solitons, and the idea of Alfvén and Lawson limiting currents. These concepts are central to the idea developed in the remainder of the paper. Section 2 presents the idea for short-duration intense relativistic-electron burst production in a plasma by self interruption of a pinch current. Section 3 discusses a specific target for the short electron burst produced by a 1-MJ plasma focus. In Section 4 analogs between fundamental atomic and molecular process and their macroscopic counterparts, high-Q cavities, are developed. The results of Sections 1 and 4 are then used to develop the second energy-compression idea, parametric amplification of a solitary wave pulse. The mathematical details of this parametric solitary-wave amplification are given in the Appendix, and Section 6 summarizes the paper.

1. SOME BASIC CONCEPTS

In this section we will discuss briefly several concepts that are fundamental to developments in later sections of this paper. Specifically we will discuss the idea of a transit-time oscillator, and, with the aid of the result, introduce the concept of strong turbulence and the Zakharov collapse of plasmons, and also introduce the various limiting currents for electron beams.

TRANSIT-TIME OSCILLATOR

The idea of a transit-time oscillator is an important cornerstone for the study of parametric amplification. In simplest form one can imagine a beam of randomly distributed electrons passing through small holes in a pair of capacitor plates so that the electron current path is perpendicular to those plates.¹¹ The capacitor is assumed tied to an inductor to form a microscopic harmonic oscillator in the form of an L-C resonant circuit with frequency $\omega = 1/\sqrt{LC}$, where L and C are respectively the inductance and capacitance of the system. An oscillating E field is present between

the plates and directed perpendicular to them, parallel to the electron beam threading the plates through the small hole drilled in each plate.

Thus the separate electrons of the beam experience a force along their line of motion pointing either in the direction of motion or against the direction of motion, depending on the phase of the L-C circuit during the transit of the electrons between the plates. For simplicity we consider nonrelativistic electrons. Those electrons that pass through the plates when the E field is pointed in a direction opposed to that of their motion have work done on them, being accelerated at the expense of the energy of the oscillator. On the other hand, electrons that transit the active region between the plates when the E field points in their direction of motion do work on the field increasing the energy of the L-C circuit at the expense of their kinetic energy. Electrons that do work on the field are slowed down and have an increased transit time relative to that of the electrons when the oscillator is unexcited. However, an electron encountering the oscillator in the opposite phase will accelerate, thus decreasing the transit time relative to that for the case of an unexcited oscillator.

If the transit time of an electron is in the vicinity of an odd number of half cycles, then about half the electrons will undergo varying degrees of acceleration, while the other half undergo varying amounts of deceleration. Because of the larger transit time of the decelerated electrons than the accelerated electrons, the net effect of the electrons in the beam, even for a completely random distribution, is to increase the excitation level of the L-C circuit.

A string of such L-C circuits with variable capacitor plate spacing to compensate for changing transit time provides a simplified model of a microwave generator, where, in such a generator, the analog of our L-C circuit becomes a mode of a resonant cavity. Nonlinear effects of such a string of oscillators can act to bunch the initially randomly distributed electrons in the beam. The bunching greatly increases the effectiveness of coupling between the beam and the oscillator. If the bunches encounter the oscillator with phase such that the

bunched electrons decelerate, putting energy in the field, then the oscillator string behaves like a microwave generator, and if the beam becomes bunched with opposite phase then the bunches are accelerated at the expense of energy stored in the oscillators, and the system becomes a model of a linear accelerator.

In the case of the relativistic electron the spacing of the plates does not have to be changed because the transit time of relativistic electrons is not changed by small gains or losses of electron energy. In this case, if the beam consists of a set of bunches that do work on the oscillators and an alternate set of bunches that receive energy from the oscillators, one has a system analogous to that involved in autoacceleration of portions of a relativistic electron beam at the expense of energy supplied by other portions of the same beam.¹²

The oscillators in these examples can represent specific modes of macroscopic accelerator cavities, or one can think of them as the equivalent harmonic oscillator corresponding to transition between two energy levels of "microscopic cavities," i.e., atoms or molecules. If the L-C circuit in the above example is initially unexcited it will nonetheless possess a zero-point oscillation and may also have shot-noise-like thermal fluctuations. The small random field will become transit-time amplified by the electron beam, and thus it is possible to amplify the thermal or zero-point quantum noise to obtain macroscopic coherent excitation of the oscillators. Of course, the thermal or zero-point excitation will also be present when the oscillator is macroscopically excited.

ZAKHAROV COLLAPSE OF PLASMONS

If one considers as a model of a plasma a space-filling set of nonoverlapping Debye spheres, then in first approximation the ions in a sphere constitute a fixed "uniform" neutralizing background. Each thermally excited electron will oscillate with random phase and direction in the field of the ions and the remaining electrons. The motion, like that of a rock dropped in a hole drilled through the earth, will be harmonic. It will have an amplitude proportional to $\sqrt{T}$, where T is the

temperature. The electric field due to the similar independent thermal motion of all the electrons in the Debye sphere will have an average value of zero and an rms value proportional to $\sqrt{N}$ where N is the number of electrons in a Debye sphere.

Now if a string of randomly distributed electrons constituting a current with drift velocity equal to the thermal velocity of the Debye-sphere electrons passes through the Debye sphere, the electrons' transit time is $1/v_0$ and it can cause transit-time amplification of initially random thermal-plasma oscillation. Since the population of electrons in a Debye sphere is changed in a plasma period, the amplified oscillation will convect to a neighboring Debye sphere, where amplification can continue if the diameter of the electron current is greater than the Debye length. Nonetheless it is clear that the electron-ion collisions, and random thermal motion will act to damp coherent oscillation produced by transit-time amplification. Thus to obtain a significant nonthermal excitation of the plasma oscillation, the density of electrons in the driving current must not be too much smaller than the electron density of the medium.

Numerical simulations show that for a beam density greater than 1% of the background plasma density the plasma oscillation can be driven to a large amplitude in less than 100 plasma periods, provided the plasma temperature is high enough to make the electron-ion large-angle collision frequency less than the electron plasma-frequency. This transit-time amplification of plasma oscillation is in effect the Buneman¹³ or two-stream instability. For large-amplitude oscillation the nonlinear effect can act to bunch the electrons of the driving current.

The amplified plasma oscillations correspond to a large coherent electric field in the Debye sphere, and this field can accelerate electrons out of the sphere, resulting in a space charge that will also cause ions to be expelled. If the plasma oscillations are strongly driven, the plasmon pressure that remains after damping of the wave by particle expulsion from the Debye sphere is large enough to more than compensate for the thermal pressure of the expelled particles. For this condition the system is unstable towards developing a local cavity of decreased density that is held open

against the particle pressure of the surrounding medium by radiation pressure of the coherent Langmuir oscillation of the cavity.

Virtually any low-entropy, intense energy source can drive the plasma oscillation, and thus the collapse of Langmuir oscillation first predicted by Zakharov¹⁴ is a relatively universal phenomenon in strongly driven plasmas. Nonlinearities in frequency space provide the means by which energy is coupled across the density and hence the plasma-frequency gradient as a cavity develops.

Both laboratory experiment and computer simulation have now firmly established the reality of the collapse phenomenon, and local density depressions approaching a factor of 2 have been observed experimentally. The cavity development, which tends to have a diameter of about $10 \lambda_D$, or roughly the order of the collisionless skin depth, will stop decreasing in density despite the decreased damping in the density-depressed region because the nonlinearities may not be strong enough to couple energy across too great a change in local plasma frequency.

It is nonetheless instructive to consider the extreme state of collapse where the cavity density goes to zero and the longitudinal plasma excitation is replaced by the transverse lowest-standing-wave-mode excitation of the vacuum cavity in the plasma, and the radiation pressure of the excitation holds open the cavity against the particle pressure of the surrounding medium. In this limiting case we can speak of the local cavities or "plasma-contained microwaves."

For a strongly driven medium the number of initially randomly distributed collapse regions will become large, and this can lead to conglomeration of cavities into a larger collapse region. The location of the bubbles that arise as the separate cavities fall into each other should be determined by the state of minimum dissipation, i.e., the condition of highest cavity Q. If the cavity is in the center of the surrounding experimental cavity we have the plasma-contained microwave state. If on the other hand the resonator has a very high Q, e.g., because it is a superconductor, then it may be energetically favorable for the bubble to develop between the supercon-

ducting wall and the plasma, thus turning the plasma-contained microwave state inside out to produce a microwave-contained plasma.

We suspect that the state of minimum dissipation may correspond to confining the plasma in a shell between a centered and a surrounding electromagnetic field standing-wave bubble. It is of considerable interest to study this system to see if the state of minimum dissipation is in fact the one postulated here because we are dealing with a phenomenon that is unstable towards forming in a strongly driven plasma.

There is a problem associated with this conceptual cavity limit where one approaches vanishingly small particle density. The longitudinal plasma oscillations are scalar waves and can be a node-free excitation in a simply connected region. On the other hand the electromagnetic-wave excitation of a particle-free cavity satisfies a vector equation, and must have an axial nodal line in a simply connected region such as a sphere. Thus as the collapse of the plasma density goes to completion a nodal line must develop, and plasma will tend to accumulate in this nodal region, rather than be expelled.

For this reason the natural limiting case for complete collapse may not be a spherical-Bessel-function spherical-harmonic product that is an eigenstate of a simple spherical cavity and has an axial node. Instead the mode may be a spherical-Newmann-function spherical-harmonic product that can exist only if there is a donut-plasma axial stem along a diameter of the cavity, which in fact provides a collapse that is multiply connected, i.e., a torus or doughnut with a small hole, where the doughnut is a plasma-free region of strong standing-wave excitation and the plasma fills the space not occupied by the doughnut-shaped standing wave.

In the case where the collapse is driven by a relativistic electron beam, the return current that flows along the electron beam path when the relativistic beam is injected into the plasma will be expelled from the relativistic beam for current densities high enough to drive strong turbulence. The return-current expulsion will occur in the form of non-relativistic filaments, and this return-

current expulsion is known as the Weible instability.¹⁰ If the path of the initial relativistic beam's return-current composite has a region of slightly lower density than the surrounding plasma, then the return-current expulsion will occur in the local region to form a pseudo-spherical region with a relativistic current along a diameter of the depressed density sphere, and the return-current filaments will flow in the spherical surface of the region of depressed density.

The return currents will drive the Langmuir turbulence, which will cascade into the depressed-density spherical region, and the resulting field will expel electrons and ions to further depress the density. In the zero-density limit one obtains a spherical cavity with nonrelativistic return currents in the form of filaments flowing on the vacuum-plasma interface with the relativistic beam forming a axial current along the diameter of the sphere, and the sphere will contain a standing-wave excitation whose radiation pressure will hold open the spherical cavity and at the same time confine the axial relativistic beam to a tightly pinched small-diameter path. It is this type of spherical collapse that we postulate as the form the Langmuir collapse in three dimensions must approach. Such a cavity is similar to a toroidal vortex¹¹ with the added feature that the doughnut-shaped region of vanishingly small particle density is held open by a standing-wave electromagnetic excitation.

The confinement of the relativistic beam to a small-diameter thread passing through the center of the standing-wave-excited vacuum bead also facilitates the conceptual complete collapse. It may lead to an actual complete collapse, because this axial relativistic current can now directly drive the cavity excitation, rather than indirectly cause collapse by driving plasma oscillations in the surrounding dense plasma that must then cascade into the lower density collapsing region due to nonlinearity of the intense plasma oscillation. The indirectly driven collapse will not go to completion because the nonlinearities will be ineffective in driving plasma energy across too steep a density gradient. The cavity directly driven by a radiation-pressure-confined, axial relativistic electron beam does not depend on the nonlinearity to supply the

energy to drive the collapse.

At a high level of excitation it will become energetically favorable for random local collapse regions to become a regular array of collapse beads threaded by the axial relativistic-electron-beam string. Such a threaded string of collapse beads is a natural linear acceleration structure for relativistic electrons, and if formed in a plasma with an appropriate density gradient becomes a string of variable-diameter, relativistic-beam-threaded beads that could provide coherent linear acceleration of ions.

Collapse in the form of a string of cavities has been suggested recently by several theoretical studies and also has been found in an experimental study.¹² The new point contained in the present discussion is the argument that in the case of collapse driven by a relativistic beam, the beam will become an electromagnetically pinched axial current along the cavity diameter. Again, such structure is one of particular interest precisely because such a collapse phenomena appears to be mandatory for strongly driven plasmas.

LIMITING CURRENTS FOR ELECTRON BEAMS

In the remainder of this section we turn our attention to the topic of limiting currents for relativistic electron beams. A beam of charged particles possesses kinetic energy per unit length proportional to number of particles per unit length. When the beam exists in a conducting plasma, both the electric and magnetic fields are screened out of existence. On the other hand, such a beam in a plasma-free region will have electric and magnetic field energy proportional to the square of the number of particles per unit length. In the absence of space-charge neutralization, the current can be space-charge or Langmuir-Chiles limited, and in the case of space-charge neutralization the current can become magnetic-field or Alfvén-Lawson limited.

Relativistic electron beams are particularly interesting because the magnetic field is less than the electric field by a factor v/c and an ion density $n_i = (1/\gamma^2)n_e$ is all that is necessary in the lab frame to balance the electric and magnetic fields of the beam. In the above expression, γ is the relativistic

contraction factor $\gamma = 1/\sqrt{1 - \beta^2}$, where $\beta = v/c$; v is the electron drift velocity, c is the speed of light, and n_0 is the density of the electron beam as viewed in the lab frame.

One must consider the limiting currents in two different contexts. First the question arises as to whether a particular current configuration can exist without being cut off by its self-field, independent of the question of how the field and particle energy were initially established. A second question arises when one asks how the current was produced in the first place.

If the only source of field energy is the kinetic energy of particles emitted from a field-screened region, e.g., from the cathode of a diode on the relativistic E-beam machine, then the electric and magnetic fields of the resulting particles cannot exceed the initial kinetic energy of the particle that produced this field, and as a consequence the total field energy and the total particle energy in an impedance-matched diode are roughly equal, i.e., the system has a "global" v/γ of 1 even though the local v/γ may be much higher than 1. The factor v/γ is the ratio of the field energy to the kinetic energy of a beam of charged particles.

Ordinary currents have very high values of v/γ , e.g., $v/\gamma > 10^4$, while highly relativistic beams have $v/\gamma \leq 1$. The value of v/γ is given by Mr_0/γ where r_0 is the classical electron radius, and $M = 2 \times 10^{10}$ is the number of electrons per unit length, where I is the beam current in amperes. The factor γ should actually be written as $\gamma - 1$, which subtracts the electron rest energy from its "relativistic kinetic energy."

We will consider first the limiting currents in the case of space-charge neutralization so that one need consider only the magnetic field. For a solid beam of radius a with concentric return-current path of radius R the Alfvén or Lawson limiting current is $I = 1.7 \times 10^4 \sqrt{B}$ amperes. Physically this is the current whose self-field becomes strong enough for the Larmor radius of a single electron moving in the self-consistent beam field to become equal to $\frac{1}{2}a$, so that the beam field is able to prevent further progress of the electron, i.e., the beam is stopped by "self magnetic

resistance."

For the concept of this and related limiting currents to be applicable, it is necessary for the beam to be sufficiently collisionless that the electron-cyclotron frequency ω_c exceeds the electron-ion large-angle collision frequency ω_{ci} . For beams with $\omega_{ci} > \omega_c$ the system is collision dominated, the resistance is due to particle collision rather than to the beam's self field, and the concept of the Alfvén or Lawson limit does not apply.

If one considers a hollow cylindrical shell beam of radius R and thickness Δ surrounded by a concentric cylindrical return path where, e.g., Δ could be the collisionless skin depth $\Delta = c/\omega_c$, then the limiting current becomes $I = 1.7 \times 10^4 \sqrt{B} R/\Delta$, a factor R/Δ larger than that of the solid beam. In this case the limiting current corresponds to the Larmor radius of a beam electron in the self-field of the beam becoming equal to $\Delta/2$, i.e., the beam is limited by its own "self magnetic resistance." A beam with both B_0 and B_z fields corresponding to a force-free configuration has essentially no Alfvén or Lawson limit.¹⁹

If we consider the emission of a relativistic beam from a cathode into a vacuum region, then the particle kinetic energy provides the source for the field energy, and the diode current can be both space-charge limited and magnetic-field limited. We will assume that a sufficient number of ions are evolved from the anode, the cathode, and the residual background gas (which typically has a pressure $p \leq 10^{-4}$ Torr) to provide space-charge neutralization so that we need consider only the magnetic field. If the cathode of such a relativistic-beam machine has a radius R , then the impedance-matched anode-cathode spacing for the diode corresponds to a distance d such that the outermost electrons of the beam will be moving tangential to the anode. The diode current corresponding to this condition is $I = 8500 \sqrt{B} R/d$ amperes, the impedance Z of the diode is $Z = 30 (v/\gamma)$, and the ratio $R/d = v/\gamma$.

A plasma exists at both the cathode and the anode, and the electrons are field emitted from the cathode plasma. When the beam enters the anode plasma the remaining E-field will be screened out in a Debye length and the magnetic

field will be screened out in a collisionless skin depth, i.e., a return current is generated back along the relativistic beam as it penetrates into the anode plasma and this return current flows radially outward in the skin layer of the growing anode plasma. A self-consistent anode plasma will result that is contained by the magnetic field in the diode, prevented from penetrating deeply into the anode plasma because of the presence of a return current. The temperature of the anode plasma will increase, eventually reaching a value of the order of 1 kV so that it becomes collisionless, and then the return current is able to drive the Langmuir oscillation resulting in an anomalous resistance that diffusively rips the return current out of the relativistic current path, permitting the field to penetrate the anode in a small-diameter channel surrounding the relativistic current.

This complex sequence of events requires a careful and extensive two-dimensional simulation to properly follow the initial development of the return current--the self-consistent evolution of the anode plasma against the excluded magnetic field, the onset of strong turbulence as the anode plasma reaches a temperature such that it is collisionless, and the resulting axial field penetration. A complete self-consistent simulation of this process has not been made to date. We believe that the self-consistent evolution of the anode plasma will result in the relativistic beam pinching to the density of the anode, with field penetration in a concentric channel along the pinched beam, where the channel diameter is of the order of a collisionless skin depth of the anode. Such a channel will be unstable toward development of a set of Langmuir collapse beads strung along the pinched relativistic beam. One interpretation of recent microscopic study of anode targets is that such a string of cavities forms along current filaments that have pinched to solid or near-solid density.²⁰

2. PRODUCTION OF RELATIVISTIC ELECTRON BURSTS VIA A PLASMA FOCUS

ENERGY STORAGE

To provide space-time concentration of energy by passive switching of the energy content of a reservoir, it is

necessary both to have a high energy density, and a large effective sound speed of the energy storage medium. The highest sound speeds attainable approach the speed of light and occur when the energy storage medium is a vacuum electromagnetic field in a cavity.

The desirability of high energy density favors energy storage in magnetic fields rather than electric fields, since magnetic fields do not have magnetic pole sources while the sources of the electric fields are charged particles. Thus large electric fields tend to be limited by electrical breakdowns, while magnetic fields experience this problem only in second orders of v/c due to induced electric fields resulting from any rate of change of the magnetic field. Thus, in efforts to produce short-duration, high-energy-density, pulsed power sources, one is naturally driven to consider inductive storage, and the desirability of a large effective sound speed dictates that the magnetic field exist in either a low-density plasma or ideally in vacuum.

Despite the limitation on energy density, one advantage of energy storage in capacitors is the fact that switching energy out of a capacitor bank involves the relatively straightforward problem of closing a switch to establish current flow to a load. Switching an inductive storage device requires solution of the more difficult problem of opening a switch, i.e., interrupting the current flow on one path, followed by restrike of the current at higher voltage along a different path that carries the stored energy to the load. This asymmetry in case of switching is a direct consequence of the asymmetry in field source, i.e., the fact that electrons exist but magnetic poles apparently do not exist.²¹

ION AND ELECTRON ACCELERATION

The earliest efforts on Project Sherwood concentrated on Bennett Z-pinches experiments,²² and resulted in unexpectedly large neutron yields that proved to be due to fast-ion acceleration, which was often correlated with $m=0$ instabilities. The fast ions were usually accompanied by hard x rays due to bremsstrahlung from accelerated electrons, and also by high-frequency microwave bursts whose origin is more difficult to explain. Considerable effort to associate the ion acceleration with IL voltages due to MHD

instabilities, such as the $m = 0$ sausage instability, have not been fully successful, and it is now clear that microinstabilities, strong turbulence, and the associated large anomalous resistance play a central role, and that $m = 0$ instabilities tend to be the natural region for nucleation of Langmuir and other turbulent phenomena.

Although a large variety of mechanisms for ion and electron acceleration in plasma, beam plasma, and beam neutral-gas systems are known,²³ two specific types of acceleration occur in plasma that are of primary importance in general, as well as to the specific objectives of this paper. Type I acceleration is associated with the turbulent-resistance interruption of an existing current, often in a pinch, followed by restrike of the current with more energetic electrons. Type II acceleration occurs in processes related to establishing a current in an initially field-free region²⁴ for example, injection of a relativistic beam into a target plasma. In the Type II process the initially induced return current is later expelled from the relativistic-beam path by the turbulent resistivity associated with amplification of Langmuir oscillations.

Both Type I and Type II acceleration can be associated with the Langmuir collapse of plasma due to a very high level of driven plasma oscillations.²⁴ In both the Type I and Type II acceleration schemes, the ions can be given an energy 10 to 25 times the internally applied voltage. The distinguishing feature of Type I acceleration relative to Type II is that the Type I ion acceleration is in the direction of the externally applied field, while the Type II ion acceleration is in the direction of relativistic current flow and opposite to that of the internally applied potential. This latter type of ion acceleration is often termed collective acceleration.

We wish to suggest that the Type I and Type II acceleration are actually basically the same process because in each case the nonrelativistic current is instrumental in driving Langmuir turbulence. The apparent differences arise because the nonrelativistic current in Type I acceleration is driven by the externally applied potential, while in the case of Type II the nonrelativistic current is driven by the induced electric

field due to the changing magnetic field of the advancing primary relativistic electron burst, and flows in a direction opposite that of the relativistic beam, and thus against the electron flow caused by the externally applied potential.

PLASMA PINCH: PLASMA FOCUSING

The most extreme form of the classic pinch is provided by the $r=Z$ pinch of a plasma focus, which in a sense can be thought of as a giant $m = 0$ instability. For detailed discussion of the plasma focus, consult the review article by Mather.²⁵ The detailed structure of the energy of the focus and its release into heat at maximum pinching is intimately associated with the focus-free vortex filaments studied by Bostick²⁶ and co-workers.

A fully correct, detailed theory of electron or ion burst production in a plasma focus must involve these filaments and their creation and annihilation in a fundamental way. In the present discussion we will emphasize the importance of strong turbulence in the gas-particle production in the focus. The sudden onset of strong turbulence near maximum pinching and a resultant 2-3 order-of-magnitude jump in resistivity is now firmly established experimentally.²¹ The onset of the turbulent resistance must be intimately associated with the annihilation of Bostick filament pairs. In the present note our description will ignore this filamentary structure. We nonetheless believe that the present description treats the particle acceleration phenomena, at least in gross detail, in a correct way. We hope to present a detailed theory that fully incorporates the filaments and associated filament-annihilation process in a fundamental way in a future publication.

The current-sheet snowplow of the plasma focus driven supersonically, relative to the low-density field gas, by JxB forces, results in half of the bank energy going into inductively stored energy behind the snowplow, while half of the energy goes into particles. This particle energy occurs half in the form of directed kinetic energy of the snowplow, and half goes into direct heating of the snowplow ions and indirect heating of the electrons through electron-ion collision. The thermal energy is carried away both by hydrodynamic flow of the

snowplowed plasma parallel to the snowplow density step and perpendicular to its directed motion, and also by radiation processes: bremsstrahlung plus bound-bound and bound-free radiation of high-Z partially ionized impurity ions in the current sheet. The net result of these processes is to produce at maximum pinch a dense plasma column with density of 10^{19} to 10^{20} with an ion temperature of about a kilovolt and an electron temperature that is less than the ion temperature by a factor of 2 to 10. The pinch current in a megajoule system driven by a 40-kV capacitor bank will be several megamps.

Two modes of operation, a high density mode and a low density mode, result from different initial fill-gas pressures.²⁸ The basic distinction between the two modes is due to the fact that in the high density mode the diffusion of the skin currents into the back side of the snowplow is slower than the snowplow growth, and thus at maximum pinch the high-density shock front leads the magnetic field, and at maximum pinch a relatively field-free, dense plasma "wire" is formed on axis. By decreasing the fill-gas pressure, the relative ratio of field diffusion into the snowplow is increased, and the magnetic-field front and the shock front tend to coincide, so that at maximum pinch one has a hollow plasma shell with density of about 10^{19} with a low-density center of ionized fill gas, which typically will have a density $n < 7 \times 10^{17}$ corresponding to an initial fill-gas pressure of ≤ 1 Torr.

As maximum pinching is approached, the energy demand exceeds the supply rate from the capacitor bank and the system begins to draw on the inductive energy behind the snowplow. This is accomplished by the addition to this applied bank voltage of an IL voltage with the same direction as the external field. The energy supply is provided by IL, where I is not localized but occurs throughout the system.

The production of fast particles, both electron and ions, in a plasma focus, particularly when operated in the low-density mode, is now well documented experimentally.²⁹ A number of theoretical explanations of this and similar phenomena in vacuum sparks³⁰ have been published. The complex behavior that follows the development of maximum pinching has been

clarified due to the early work of Bostick et al.,³¹ and recently by double-exposure holography in a number of laboratories. The work of Bernard et al. deserves special mention.³²

These measurements show that there is sudden onset of Langmuir oscillations, and simultaneously the dense plasma stem at maximum pinch disappears, starting at the anode face and proceeding away from the anode. The result of this sequence of events is a larger diameter plasma, with a diameter a factor of 10 greater than the diameter at maximum pinch, and with density of 10^{18} to 5×10^{17} , or two orders of magnitude smaller than that at maximum pinching. The energy per particle in this lower density plasma approaches 10 to 20 keV, and in the high-density mode the plasma may not differ too greatly from a Maxwellian. The primary contribution to the neutron yield of the focus is a "quasi-thermonuclear" reaction in this low-density hot plasma. In the low density mode, substantial electron and ion acceleration occurs, the accelerated ions are concentrated along the axis, and the neutron yield is substantially of a non-thermonuclear beam-target nature. Experimentally the largest neutron yields are produced in the high density mode and the yield falls off as the fill-gas pressure is decreased. It is normally concluded that the high density mode favors neutron production. The conclusion must be carefully reexamined in view of the fact that sufficiently energetic ions $E \geq 1$ MeV are produced so that the yield may fall off simply because there is not enough target gas to stop the fast ions.

Another factor that reduces the yield at low pressures is ion starvation, i.e., the supply of ions available to be accelerated becomes limited and the ion-starved condition should favor electron acceleration. Low-density-mode operation of a 1-MV system should result in about 10% of the bank energy going into a small-diameter, 10-nsec relativistic electron burst, and 1% of the bank energy may go into an oppositely directed axial MeV ion burst. Careful efforts directed toward deeper understanding and optimization of the electron and ion burst production could result in increasing the coupling efficiency to relativistic electrons and to energetic ions by about a factor of 3.

A further increase in efficiency requires the elimination of the snowplow assembly, which squanders half of the bank energy in the form of particle kinetic energy, and then only 50% of the bank energy exists as inductive energy storage behind the snowplow. It is the inductive energy that drives the pseudo-thermonuclear yield in the high-density mode, where up to 60% of the inductive energy is converted into relatively random 10-to-20-kV ions; in the low-density mode at least 20% of the inductive energy goes into fast-ion and electron production during a period of 10 nsec or less.

The rise times of the electron and ion bursts in the low density mode are very short and may be in the 100-psec range. The actual initial diameters of the electron and ion bursts are unknown. The diameter must be at least as small as the focus, i.e., a diameter of the order of 1 mm. Work by Postick et al.³³ suggests that the diameter of these energetic particle beams could be as small as 100 μ m.

In order to understand the focus phenomena we first consider the current sheet well before maximum pinching. As discussed in Part 1, the Alfvén or Lawson limiting current for a hollow electron beam is $I = 1.7 \times 10^4 \text{ kA} \sqrt{R/\Delta}$, where R is the beam radius and Δ the collisionless skin depth c/ω_p ; c is the speed of light and ω_p is the electron plasma frequency. The current that leads to self pinching in a vacuum diode of radius R and anode-cathode spacing d , in the case of space-charge neutralization, is $I = 8500 \text{ kA} \sqrt{R/d}$. These concepts apply for a drifting beam in the case where the electron-cyclotron frequency for electrons in the self-field of the beam is greater than the electron-large-angle collision frequency, and then the "resistance" is field rather than collision dominated.

In the case of the collisionless limiting current for the diode the electrons are first accelerated by the external field at the surface of the cathode plasma. These fast electrons then are decelerated to provide the field energy, and consequently the field energy of the diode must be of the same order as the initial total kinetic energy of electrons undergoing transit between the cathode and anode.

In either the case of self-field-dominated resistance or in the case of collision-dominated resistance, the presence of an axial E-field can drive the particles against the resistance, continuously resupplying the energy to the particles, or they give their energy to the field or to plasma heating, and in this way the field energy can greatly exceed the initial kinetic energy of the drifting electrons. The resulting high-v/y beam can continue to exist in the collisionless case in the absence of an applied voltage if the current field configuration is less than the appropriate Alfvén or Lawson limiting current, i.e., if the configuration is relatively force free so the self magnetic resistance of the system is small.

The current is collision dominated during most of the collapse phase of the plasma focus. As maximum pinch condition is approached the sum of the bank voltage and the IL contribution to the voltage accelerates current-carrying electrons, between large-angle collisions with ions, to approach a drift velocity comparable to the random thermal velocity of the electrons, and one reaches the condition for resonant amplification of the longitudinal plasma oscillation. The amplified plasma oscillation rapidly reaches a significant amplitude, and causes turn-on of a turbulent resistivity in at most a few dozen plasma periods. The resistance increases due to the observed plasma oscillations by a factor of 100 to 1000, and this results in a rate of diffusion of the field into the snowplow plasma that approaches $5 \times 10^8 \text{ cm/sec}$, or about 10 times the rate of snowplow motion. The sudden increase in resistivity greatly reduces the particle current, resulting in a large displacement current, and the field diffusion through the plasma produces rapid heating.

EXPLODING PLASMA

Thus the pinched plasma in the high density mode is in effect transformed into an exploding plasma wire, and in the low density mode into an exploding plasma shell, i.e., the rapid particle heating drives the particle pressure well above the local field pressure, and the plasma undergoes driven Bohm-like diffusion across the field, further increasing its temperature. The density

of the plasma drops by a factor of 100, the radius increases by a factor of 10, and the energy per particle (i.e. the effective temperature) increases by a factor of 10 or more to a value of 10-20 keV. More than half of the inductive energy stored behind the snowplow is transformed into heat, and the hot, expanded low-density plasma achieves near pressure balance with the remaining field, where it can be contained for the order of a Bohm diffusion time, during which the primary yield of the focus is produced.

The explosion of the plasma wire or plasma shell must be accompanied by a momentum-conserving recoil, i.e., a convergent implosion of some of the plasma toward the axis. Thus one can look upon the explosion of the plasma wire as an ablation process that causes implosion. It is not clear if the creation of a dense plasma on axis by the implosion recoil to the outward-directed plasma has been observed experimentally, although it is possible that observation of repinching reported in some experimental studies is due to this process.

The σ of the imploded portion of the current sheet in the plasma focus is far too small to produce significant yield. The surprising fact is that the exploding portion of the plasma is caught by the remaining magnetic field and the yield actually comes from the rocket fuel rather than from the convergently compressed rocket recoil with larger σ . The high-density-mode yield from a 50-50 DT plasma in a 1-MJ focus will approach 0.5% of the bank energy, 1% of the inductively stored energy, or about 2% of the field energy that is destroyed to produce plasma heating. This leads to the conclusion that it is easier to produce net yield from the exploded rocket fuel than it is to obtain yield from the imploded rocket.

If this exploding-plasma-wire phenomenon can be applied to the same number of particles but at an initial density of about 10 times solid density, then the initial density is increased by 10^4 , the burn rate is increased by 10^4 , the disassembly time is decreased by a factor of 10^2 , and the net yield from the ablated rocket fuel becomes 100 times larger, thus approaching breakeven relative to the 1-MJ bank energy. By directing the electron burst from the low density focus mode

over a small diameter ($d = 0.01$ cm), solid-density DT wire,⁶⁶ it may be possible to achieve scaling approaching that of the above example.

The recoil compression of the central portion of the wire even in this case will have a σ too small to give significant yield in the absence of an exotic effect such as direct conversion to field energy of the particles produced by fusion. However if the exploding-wire-surface, imploding-wire-core phenomenon is extended to the condition where the field energy initially possesses 10 to 100 MJ, the wire core can reach about 100-fold compression at $\sigma = 1$ burn conditions and one may obtain breakeven yield from both the imploded core and the exploded corona. In addition, the detonation of the core will drive a shock outward through the field-contained corona and can produce additional yield by "refrying" this corona.

The high-density-mode, exploding-plasma-wire phenomenon is analogous in a number of respects to ordinary or relativistic-E-beam-driven exploding-wire work³⁴ and the exploding plasma shell in the low density mode has similarity to exploding cylindrical foils.³⁵

In the low density mode the field diffusion precedes the inward imploding half of the plasma shell and in the low density core the restrike of the hollow cylindrical current is initially impossible because the current would exceed the Alfvén limit $I = 1.7 \times 10^4$ By R/A until the induced axial E field (in other words, the displacement current) is able to increase the value of B_y . As the plasma-foil explosion proceeds outward from the anode, the unexploded portion of the plasma cylinder will act as a plasma cathode that will emit a relativistic electron beam that has $v/\gamma = 100$, and this beam will self pinch because of space-charge neutralization in a distance d about $1/100$ of the plasma cathode radius, producing a virtual cathode. Because the electrons are temporarily prevented from propagating due to the beam's self-field, the normally more sluggish ions have a chance to respond to the axial field, and ions are accelerated until the ion-containing region between the anode and the unexploded end of the plasma shell are exhausted; the system becomes

ion-starved and the space-charge neutralization is destroyed, and then the current is reestablished between the virtual cathode and the anode as a small-diameter relativistic electron burst.

The triggering of the complex sequence of events that leads to the ion and electron burst is provided by the acceleration of electrons of the current shell to a drift velocity equal to the electron thermal velocity. Since an applied voltage will distribute itself proportional to the resistance, both the material and shape of the anode center can produce significant variation in the voltage drop across the skin layer of the metal anode, and this anode-skin-layer potential drop subtracts from the voltage applied across the pinch, and consequently can alter ("fine tune") the point during final pinching at which amplification of Langmuir oscillation begins, and causes the sudden increase in resistivity of the current sheet.

The different hydrogen-adsorption properties of different anode-center materials could alter the availability of ions, and thus change the ratio of energy that goes into the electron burst and the oppositely directed ion burst. A hollow anode should favor ion acceleration over electron-burst production in low-density-mode operation. The presence of impurity ions in the current sheet changes the radiation rate, and this affects the extent to which the electron temperature lags behind the ion temperature, and this in turn can change the time of onset of strong Langmuir turbulence. Turbulent enhancement of bound-bound and bound-free impurity radiation could also be important.

As pointed out by Gribkov,³⁶ one can enter the region of strong turbulence either by accelerating the current-carrying electrons to a drift velocity equal to the thermal velocity, or by reducing the thermal velocity to a value equal to or less than the drift velocity. The rapid cooling that will occur when the current sheet reaches the rod, if a small-diameter rod is placed on axis extending from the anode, may help to trigger the strong turbulence by quenching the electron thermal energy.

Careful experimental study of the

variables could lead to increased efficiency of electron and ion burst production, and probably also permit increase of the size of the ion burst relative to that of the electron burst, and vice versa.

The rate of increase of turbulent resistivity could be very large, the change by a factor of 100 or more taking place in as few as 10 electron plasma periods, i.e., in a time $\tau = 100$ psec. However in most cases the strong turbulence will probably start at one point on the current sheet, and then spread through a full 2π angle. Thus the cutoff of the current will usually require several nanoseconds instead of only 100 psec. In occasional high-symmetry shots, the 100-psec cutoff time may be approached, and this could explain the occasional very large, energetic, very-short-duration electron burst that is observed experimentally. Effort to achieve high-symmetry with respect to angle of onset of strong turbulence and current cutoff could help in increase the intensity, and decrease the rise time of the electron burst. A rod extending from the anode center with a disk mounted perpendicular to the rod end, where the disk has a razorsharp edge, might achieve the desired result for highly symmetrical collapsing current sheets.

Since the current sheet actually consists of many small filaments, the unexploded plasma shell end, which acts as a cathode, should actually emit a large number of separate electron beams, one from each filament end. It is tempting to visualize as an orderly phenomenon the self pinching of these separate beams to form a virtual cathode, so that each current stream winds up in a coil equal to a Larmor radius, to produce a toroidal vortex-like structure with a circulating current greater than the net pinch current. The decay of toroidal vortices of that type have been observed by Bostick and co-workers.³⁷ The model for the process suggested here is presently under study.

3. A TARGET FOR THE PLASMA-FOCUS ELECTRON BURST

In this section we consider some key features of one type of target to be utilized for the focus-produced relativistic-electron burst. An extended

treatment of this and other targets will be presented elsewhere. The purpose of the present discussion is to provide an answer to the question, "Why produce relativistic electrons?" The self interruption and relativistic restrike of the plasma-focus current discussed in the previous section is, in effect the transformation of a "super-high"- v/v ordinary nonrelativistic current into a high- v/v relativistic current with $v/v = 100$. What is the fundamental use of such a v/v reduction?

The answer to this question is best provided by discussing the electron-burst target shown along with the plasma-focus anode in Fig. 1. The target is a hollow glass shell with an inside diameter of roughly 1/2 cm, with a dielectric axial rod of DT, LiDT, or perhaps CDT with diameter $d \approx 0.02$ cm. The space inside the sphere is filled with DT gas at a pressure of 1 atm or less. The collapsing current sheet of the focus will wrap around the sphere as shown, and produce a pinch of axis beyond the end of the axial target rod just outside the glass shell. For low-density-mode operation this will result in restrike of the current as a hollow, relativistic electron beam that will travel along the surface of the dielectric rod, by virtue of the phenomenon discovered by Bennett et al.³⁰ A nonrelativistic return current will be generated and it will flow back along the relativistic current path. Thus, the initial consequence of the transduction of the pinched nonrelativistic current into a relativistic beam is that

the relativistic beam is able, by virtue of its greater electron inertia and energy, to penetrate into an initially current-free region, and because of the induced return current the new charged-particle path remains current free.

What in effect has been created is a zero-inductance current loop folded back upon itself. By appropriate choice of the gas pressure in the sphere, it is possible to achieve plasma conditions along the relativistic-nonrelativistic current path for the return current to drive longitudinal Langmuir oscillations, once the plasma becomes hot enough along the current path to prevent strong collisional damping of plasma oscillation, i.e., when the temperature reaches roughly 1 keV.

The onset of strong turbulence results in the Weibel instability expelling the return current as a set of filaments from the return relativistic path until they flow along the inner surface of the plasma shell. The shape of this target has been chosen to have the doughnut-shaped limit of spherical Zakharov-Langmuir collapse discussed in Section 1. The net effect of driving the return current filaments from the relativistic current path on the surface of the dielectric rod is to rapidly fill the sphere with 10^9 J of field energy that was initially stored outside the sphere.

It is this switching of field energy, and only this switching of field energy, that is the real benefit of restrike of the super-high- v/v nonrelativistic pinch current as a high- v/v relativistic current. The v/v of the field energy inside the sphere will be of order 100, and thus the effect this energy has on the central rod will be due to turbulent diffusion of the field into the rod, not the deposition of single relativistic electrons. We believe that this example contains the real essence of the usefulness of high- v/v relativistic-electron bursts to pulsed fusion.

The pressure of the field switched into the cavity will cause the dielectric rod to undergo an insulator-to-metal transition, become an exploding-implosioning wire and, as indicated in the previous section, there is a chance at the 10^9 J level of obtaining non-trivial yield from

Fig. 1 Plasma-focus anode and electron-burst target.

the low-density-ablated corona even though the ρr of the compressed rod core may well be much less than the burn value $\rho r = 1$. It is presently unclear if the Langmuir collapse will drive the fill-gas plasma density to zero, faster than the rod explodes. If this turns out to be the case, then the explosion-implosion of the rod will be driven by standing-wave electromagnetic energy of one or more modes of this multiply connected rod-sphere cavity.

It is amusing to note that the target discussed here is in effect a miniature version of the imploded-belt-pinch concept of Velekov,³⁹ except that we are interested in destroying the central rod and he wants the rod to remain intact long enough to burn the compressed belt pinch. By changing the objective of his implosion scheme to destruction of a central fissionable dielectric rod, a means of scaling the present target to the multi-megajoule driving energy range might result. Such an approach must be preceded by study of the Bennett rod phenomenon over periods of time of 10^{-3} sec, and also at high fields, to learn where the ability of the rod to carry high surface currents is destroyed and the rod becomes an exploding-implosioning wire. For the scheme to work the dielectric rod phenomenon must persist to field pressure of at least 1 to 10 Mbar, where the insulator-to-metal transition will take place, in order that sufficient field energy is created by the slow implosion process in the Velekov scheme prior to onset of rod destruction, so that enough energy is available in the field for the explosion-implosion phase to reach burn conditions both in the imploded core and in the field-contained exploded corona.

4. ATOMS AND GIANT ATOMS

In this section we wish to present a picture that will facilitate our discussion in the following section of parametric amplification of solitary waves. We wish to emphasize the parallel between the fundamental processes in atoms and molecules and analogous processes in macroscopic analogs that are most naturally provided by superconducting cavities such as those developed for linear accelerators at Stanford.⁴⁰

To work toward optimum energy compression concepts it is useful to begin by considering fundamental atomic

processes and the results obtained by combining these processes in various ways. The discrete energy levels of an atomic system may be excited from the ground state by a photon of the appropriate energy, by an inelastic collision with an electron or other charged particle, or by a combination of radiation and charged particles acting together. It is even possible to excite energy levels with an uncharged particle such as a neutron. Level excitation and deexcitation are subject to selection rules that are a consequence of symmetry considerations, which determine the allowed processes connecting various energy levels. The excitation of atomic energy levels by inelastic scattering was first seen in the famous Frank-Hertz experiment.⁴¹

An excited state of an atom has a finite lifetime because of its coupling to zero-point field fluctuations and charge-density fluctuations of the vacuum that are the cause of spontaneous decay. An excited level can decay by photon emission, an Auger process in which energy is transferred to electron, by an electron capture by the nucleus, or in numerous other more complex, less probably ways. In the present discussion we will concentrate on level excitation and deexcitation by single photon emission or absorption, or by inelastic electron scattering.

It is nonetheless instructive to keep in mind the full range of excitation-deexcitation mechanisms, not only for bound-bound transitions, but also for bound-free and free-free transitions of atoms, as well as the even richer electronic and nuclear transitions of diatomic and polyatomic molecules and the optical and acoustical branch transitions of giant molecules, such as single crystals, because each of these fundamental processes can have a macroscopic analog in our giant-atom superconducting cavities, and the microscopic-atom, macroscopic-atom parallel is a useful tool for thinking on either the microscopic or the macroscopic level.

In addition to the spontaneous decay of an energy level, the level decay can be induced by a photon of appropriate energy corresponding to a times the difference frequency of the two levels ω_1, ω_2 . The stimulated emission is the inverse of the photon excitation. A particularly

important aspect of the stimulated emission process is that the Bose nature of photons favors growth of the final state. This also applies to deexcitation by plasma emission, or photon or other excitation-emission processes that are possible for an atom in a plasma, solid, or other dielectric medium.

Energy-level decay by the inverse of the inelastic Frank-Hertz excitation mechanism exists, and is often referred to as a collision of the second kind or a superelastic collision. In this case, a free electron is accelerated at the expense of the excitation energy of the atomic level. Processes involving electrons do not favor growth of the final state because electrons obey Fermi statistics, and it is this fundamental difference that makes lasers possible, while the analogous amplification phenomenon for single-electron wave functions is not presently known to exist.

It is possible to consider deexcitation of a level with a two-electron state, with the electrons in either a singlet or a triplet state. The wave function of these quasi-Bose particles can be amplified, and a near perfect realization of this process is provided by certain Josephson-junction phenomena in the gap between two superconductors. The growth of charged Bose-particle final states also requires appropriate charge neutralization to prevent the amplification process from being space-charge limited.

Near perfect analogs of the fundamental process are provided by considering superconducting cavities as giant atoms, whose longitudinal standing-wave excitations provide the analog of atomic energy levels. The analogy approaches perfection because the macroscopic quantum states of the quasi-Boson electron pair in a superconductor find their most appropriate description in terms of states, known as coherent states, that are eigen functions of the annihilation operator. The eigen functions in this case have the forms of single-particle wave functions, known as the complex order parameters, and apart from a self-consistent potential term obey a single-particle Schrödinger-like equation known as the Ginzburg-Landau equation. The details of this picture are provided in the Appendix, in conjunction with the mathematical formulation of the solitary wave amplification that is presented in Section 5.

The $\bar{Q}$ of our macroscopic-atom superconducting cavity is a natural analog of the spontaneous decay of an atom. The excitation of a standing-wave cavity mode by means of a traveling wave, produced, for example, by a microwave generator, provides the analog of photon excitation of an energy level, and the use of the stored energy in the cavity to feed a traveling wave provides an analog of stimulated emission. Electron acceleration and deceleration by the field of a transverse magnetic standing-wave mode, depending on the field phase during the electron transit time of the cavity, provides the parallel to Frank-Hertz level excitation and superelastic level excitation collision, or collision of the second kind.

With laser light it is possible to excite a coherent superposition of nearly degenerate atomic energy levels, and the decay of such a state has considerable structure due to the interference terms between different states of the coherent superposition. The mode-locking of nearly degenerate cavity levels provides a macroscopic analog of this process.

A string of giant-atom cavities may be excited by a random beam of electrons so that no particular phase relation exists between cavities, a process analogous to creating a population inversion in a lasing medium with an incoherent electron beam. The deexcitation could in both cases be by a stochastic electron acceleration process. An appropriately phased, bunched electron beam can coherently excite a cavity string, and the coherent deexcitation of the cavities with bunches of the opposite phase is a linear accelerator. An analog of bunching for a single electron is probably possible for the microscopic analog, provided the density of electrons occupying the electron wave function is less than two particles per phase-space volume $\bar{h}^3$, at which point Fermi statistics play a decisive role. The more natural analog of a bunched beam requires the use of quasi-Boson electron pairs. The cavity string can be coherently excited by means of a traveling wave, and a string of atomic resonators can be similarly excited by use of coherent laser light. The linear acceleration of an electron or of a relativistic ion by such a coherently excited atom string via a phased string of superelastic collisions has been considered in an unpublished Laboratory report by Hofstadter.

and some closely related work has appeared in the Russian literature.⁴²

An initially randomly distributed electron beam passing through a string of coherently excited cavities can become bunched by nonlinear effects. An incoherent electron beam scattering from a crystal of atoms in the ground state will produce a diffraction pattern that is the Fourier transform or reciprocal lattice of the crystal. This is possible with incoherent electrons (the Davison-Germer experiment) or with incoherent x rays because the periodic array acts like a passive filter, selecting the accidentally coherent electrons or photons of the initial beam to produce the diffraction pattern. It is of considerable interest to study incoherent electron scattering from a periodic crystal array of coherently excited atoms, because in this case the crystal can act as an active filter and one should obtain an analog of bunching. An initially incoherent electron beam interacting with an active generalized crystal, produced by interference in an active medium of a reference wave and a wave scattered from some object, is of particular interest. The active generalized crystal in this case is a structured population inversion or active three-dimensional hologram whose reciprocal lattice is a three-dimensional image of the object.

One process is significantly absent in present practice from our analogy. The most important means of deexcitation of excited microscopic cavities, i.e., atoms, is provided by stimulated emission. The amplification of a traveling wave by the standing-wave energy stored in a string of linear acceleration cavities is a perfectly viable process that has not to our knowledge been considered previously. Such a macroscopic stimulated-emission process provides the ultimate in series/parallel switching of energy storage reservoirs, and forms the basis for the discussion in the next section.

5. PARAMETRIC AMPLIFICATION OF SOLITARY WAVES

In this section we develop the second and most fundamental of our energy-conversion ideas. In section 1 we identified laser pulse amplification as the feature primarily responsible for the unique advantage of lasers for pulsed fusion applications.

PULSE AMPLIFICATION IN LASERS

The pulse amplification is obtained by first Q-switching and then mode-locking an oscillator, i.e., a laser with mirrors, to produce a string of very short temporal width pulses. One of these pulses is switched out of the oscillator and sent to an amplifier that is a pre-inverted volume of atoms of the same species as those of the oscillator. The population inversion is provided by means of flashlamp, electrons, etc., driven by a primary energy source, such as the 1-kV capacitor bank discussed previously. The amplifier is a laser without mirrors. The initial pulse, if it is intense enough, can extract a significant fraction of the energy stored in the active medium, and in a single pass become amplified in a self-similar fashion to very high intensity. A self-similar nonlinear solution of a wave equation is known as a solitary wave⁴³ or soliton.

One type of pulse with self-similar behavior arises in the study of the concept of self-induced transparency. If an intense pulse passes through an initially unexcited medium, energy can be extracted from the front of the pulse to excite the medium oscillators, i.e., atoms, molecules, etc., and the back half of the pulse can then stimulate emission back into the pulse from the coherently excited atoms. In this way a self-similar pulse shape evolves that can pass through the normally opaque medium in an almost lossless fashion known as self-induced transparency.

For the losses to be small the medium must be pure, and the transient energy storage in excited state of the medium oscillator must be short relative to the spontaneous-decay time of the excited energy levels. The spontaneous decay will result in gradual damping or decreases in amplitude of the solitary wave, assuming that other loss mechanisms are negligible. If a separate energy source is employed to re-excite levels that decay spontaneously, then compensation for the spontaneous-decay damping is provided and the pulse will in principle undergo lossless transmission through the medium.

If the separate energy source excites energy levels so that more energy is provided than is lost by the spontaneous decay or other loss mechanisms, then

the self-similar pulses can experience inverse damping, i.e., become parametrically amplified. Thus the process of pulse amplification in the laser amplifier is in effect self-similar parametric amplification of a solitary wave by externally driving the medium in which the pulse propagates. The concept of parametric amplification of a solitary wave is a phenomenon more general than the specific example provided by the laser pulse amplification, and we now turn our attention to finding a concrete realization of the process for microwave generation.

UBITRON MICROWAVE GENERATOR

One of the most successful microwave generators for short-wavelength radiation was invented by Phillips and was known as the Ubitron.⁴⁵ The Ubitron is a coaxial system where an electron beam collimated by a B_z field forms the inner coax, and the outer coax is a smooth-walled cylinder fitted with concentric rings of periodically spaced magnets. The purpose of the periodic magnetic field is to dynamically modulate the electron-beam inner coax and thus avoid the process of careful machining of a periodic cavity structure on the outer coax, the procedure normally employed in microwave tubes and which becomes increasingly difficult at shorter and shorter wavelengths. The resulting slow-wave structure proved to be an efficient source of short-wavelength radiation. The B_z field in the system served only to prevent space-charge blowup of the electron-beam inner coax, and played no fundamental role in the microwave generation.

Recent work on the microwave generation with relativistic electron bursts in cavity structures similar to the Ubitron have successfully produced efficient conversion of energy to 3-cm microwaves, and prospects for high efficiency in the millimeter and sub-millimeter range appear promising.⁴⁶ The radiation produced in the relativistic analog of the Ubitron, in some cases, does show B_z field dependence, perhaps because the cavitron mode that plays a central role in the Drummond-Silman autoresonant acceleration concept is involved.⁴⁷ A beam in the absence of a B_z field can have both longitudinal and transverse oscillations, while the transverse plasma modes tend to be suppressed by the stiffness provided by the B_z field.

OUR VERSION OF THE UBITRON

We wish to consider a version of the Ubitron in which the B_z field is replaced by a Bennett cathode.⁴⁸ Such a dielectric rod is known to develop a plasma layer and electrons embedded in the dielectric rod float the electron current off the surface of the rod. Such a "plasma silvered" rod can carry a large current without damaging the dielectric.

When the rod is employed as a cathode for a relativistic E-beam machine, the plasma surface rapidly drops to the cathode potential, and the current flow along the rod is nonrelativistic. The relativistic electrons are produced at the growing plasma tip, off the end of the rod. If no anode/cathode gap is provided at the rod end, then the rod would constitute a short circuit.

The large magnetic field of the small-diameter-rod surface current provides self magnetic insulation, inhibiting the formation of a radial emission or shank fire. For this reason a concentric return path can be present with a spacing of a few rod diameters without shorting problems, a fact established by unpublished work of Smirnov et al.⁴⁹ It is interesting to make the outer coax out of a dielectric also, so that it acts as a return current path only after it evolves a similar plasma layer. By providing this plasma-silvered dielectric coax with the periodic Ubitron magnets, one should obtain a tube that will work as well as the original Ubitron, the basic change being merely replacing the collimating B_z field with the collimating Bennett rod.

At sufficiently high excitation levels, the radiation pressure of the amplified slow wave begins to become significant, and will be able to help cause the periodic beam modulation, and for strong self-excitation it should be possible to remove the periodic magnetic field and still have a working system.⁵⁰ One might use a raster to produce the initial level of slow-wave excitation sufficient for the system to become self driving, and thus avoid the periodic magnetic structure altogether. For strong electromagnetic excitation it should also be possible to remove the dielectric rod. This would result in a system that is essentially identical to the pinched-relativistic-beam-threaded string of Langmuir collapse cavities discussed in

the first section of this paper.

The slow-wave structure for a flared outer coax provides an increased propagation velocity with distance, and for transverse magnetic excitation one obtains a method of coherent ion acceleration with the E field of this transverse magnetic mode, rather than the cyclotron mode of the Drummond-Sloan autoresonant accelerator. The present structure is an ion accelerator that should be as viable as the Drummond-Sloan approach, and has the virtue that the ion acceleration from a single Langmuir collapse region has been demonstrated by computer simulation,^{4,5} the results of which agree with those from collective-acceleration experiments.⁶ Further it has a structure known from experiment and from computer simulation to be unstable towards forming.

Once this Ubitron-like tube becomes loaded, the potential difference will become distributed proportional to the load, and for a dielectric rod of appropriate length, the potential drop of the driving E-beam power source should occur along the rod. Then the rod can be extended to touch the anode without resulting in a short circuit, i.e., the loaded rod/cavity system itself becomes an impedance-matched load for the relativistic E-beam machine.

MATHEMATICAL TREATMENT

Now we wish to consider a cylindrical-plasma surfaced, coaxial dielectric cavity of inner radius a , outer radius R , length L , and with vacuum spacing between the coaxial electrodes of $\Delta = (R - a)$. We will initially assume that the cavity also has end plates so that it is a completely closed resonator.

The lowest cylindrically symmetric, transverse magnetic mode of the cavity has the form

$$B_0(r, z) = B_0 \sin(k_z z)$$

$$x \frac{\sin[k_r(r-a)]}{(r-a)} e^{-iat}$$

where $k_z = \pi/L$, and $k_r = \pi/\Delta$. The substitution of this form into the wave equation yields the dispersion relation

$$\omega^2 = \omega_c^2 + (k_z c)^2,$$

where ω_c , the cutoff frequency, is given by $\omega_c = \pi c/\Delta$. Since $\Delta \ll L$, it follows that $\omega_c^2 = \omega^2 \gg (k_z c)^2$, and thus the dispersion relation for the "nonrelativistic" photon (i.e., $v_g = \partial\omega/\partial k \ll c$, where v_g is the group velocity) is approximately

$$\omega = \omega_c + \frac{k_z^2 c^2}{2\omega_c} + \dots$$

If we multiply this expression by $\hbar$, we obtain $E = E_0 + p_z^2/2m_0$, where $\hbar\omega_c = E_0$, and $m_0 = E_0/c^2$.

Now assume that near the coaxial guide center ($z_0 = L/2$), an ($m = 0$)-like local pinched region is slowly introduced so that the waveguide mode adapts adiabatically to the perturbation. When this local constriction increases beyond the point where $\int p_z dz$ over the constriction exceeds $\hbar$, then: (1) the original mode, which was a "bound state" of the entire guide, becomes a bound state of the constriction, (2) the electromagnetic energy originally distributed proportional to the unperturbed $B_0^2 + B_z^2$ in the entire guide becomes localized near the constriction, and (3) the perturbed mode decays exponentially outside the constriction, becoming vanishingly small at the guide ends. The end plates of the coax, having served their conceptional purpose, can now be imagined away.

In reality we are interested in the case where this localized, solitary electromagnetic excitation is the cause, via its radiation pressure, of the constriction in which it is self-trapped. The radiation pressure causes a local depression in the electron density of the plasma silvering layer on the dielectric rod, and this depression becomes the well in which the electromagnetic field is localized.

A fully correct treatment of this self-trapping requires a self-consistent formulation that is provided in the Appendix. The Appendix also provides a coherent-state formulation of the waveguide problem for the case of superconducting guides. A remarkable similarity between the superconducting

treatment and the self-consistent collisionless-plasma treatment is noted. This similarity arises because for both cases, Ohm's law is replaced by a London-like relation where the current $\vec{j}$ is proportional to the vector potential $\vec{A}$. In this section we will employ a simplified treatment of the self-trapped wave which is shown in the Appendix to be the lowest order term in the more nearly correct self-consistent formulation.

The ($m = 0$)-like adiabatic local pinch in the inner coax causes Δ to become a function of z , i.e., $\Delta = \Delta(z)$, and $E_0 = E_0(z)$ now plays the role of a potential. For convenience we will place the origin of the coordinate system at the minimum of the constrictor, and then Taylor expand $\omega_c(z) = \pi c/\Delta(z)$ about the minimum at $z = 0$:

$$\begin{aligned} \omega_c(z) &= \omega_c(0) + \frac{\partial \omega_c}{\partial z} \bigg|_{z=0} z \\ &+ \frac{1}{2} \left[\frac{\partial^2 \omega_c}{\partial z^2} \bigg|_{z=0} \right] z^2 + \dots \\ &+ \left[\frac{\partial \omega_c}{\partial z} \frac{\partial^2 \Delta}{\partial z^2} \bigg|_{z=0} \right] z^2 + \dots \end{aligned}$$

Since $\frac{\partial \Delta(0)}{\partial z} = 0$ we obtain

$$\omega_c(z) = \omega_c(0) + \frac{1}{2} \omega_c(0) \frac{1}{\Delta(0)} \left[\frac{\partial^2 \Delta(0)}{\partial z^2} \right] z^2 + \dots$$

and thus $\epsilon = p^2/2m_0 + 1/2 m_0 \omega_c^2 z^2$, the Hamiltonian of a harmonic oscillator, where $\epsilon = E - E_0(0)$, $E = \hbar \omega'$, $E_0 = \hbar \omega_0(0)$, $m_0 = \hbar \omega_c(0)/c^2$, and

$$\omega_0'^2 = \frac{E_0(0)}{\Delta(0)} \left[\frac{\partial^2 \Delta(0)}{\partial z^2} \right],$$

and we have neglected a higher order term proportional to $p^2 z^2$. Thus, in this

approximation the $\sin(k_z z)$ portion of the wave evolves into $\exp(-\alpha z^2/2)$, the ground state of a harmonic oscillator, and the localized excitation becomes

$$\psi_0(r, z) = B_0 \frac{\sin \left[\frac{\pi}{\Delta} (r - a) \right]}{(r - a)}$$

$$\exp \left[-\frac{1}{2} (\alpha z^2 + i \hbar \omega' t) \right],$$

where the eigenvalue ϵ is now equal to $\hbar \omega'/2$, and this in turn implies $\hbar^2 \alpha^2 / 2m_0 = \hbar \omega'/2$, or $\alpha = \omega' \omega_c(0)/c^2$, and in addition

$$\frac{\hbar^2 \alpha^2}{2m_0} = \frac{1}{2} \frac{\hbar \omega_c(0)}{\Delta(0)} \left[\frac{\partial^2 \Delta}{\partial z^2} \right]_{z=0},$$

or

$$\alpha^2 = \frac{\omega_c^2(0)}{\Delta(0)} \left[\frac{\partial^2 \Delta(0)}{\partial z^2} \right].$$

Thus we obtain two expressions relating α to ω' , Δ , and $\partial^2 \Delta(0)/\partial z^2$.

SOLITARY-WAVE AMPLIFICATION

A current in the surface plasma can result in transit-time amplification of the localized field excitation because it has an axial electric field, and in this way we may obtain parametric amplification of the standing solitary-wave pulse. If the localized well and its cause, the self-trapped wave, are moving with velocity $U \ll c$ in the lab system, then the pulse will be viewed as a propagating solitary wave that is undergoing transit-time amplification by the electron current in the plasma silvering layer.

The initial pulse to be amplified in this way could be produced by a microwave analog of the laser oscillator, which could be a closed cavity (like that of our initial discussion in this section) whose nearly degenerate modes are mode locked to produce a finite number of new states of equal spacing ΔE . The coherent superposition of such a set of states will result in a string of equally spaced

*Note: ω' , the oscillator frequency, should not be confused with ω_c , the wave frequency.

narrow-width pulses. By rendering the coaxial-oscillator end wall temporarily transparent, one pulse in the string could be sent into the amplifier section of the coax. Alternatively, one might use a mode-locked Q-switched maser to provide the initial pulse.

The viability in practice of this parametric amplification must be determined by experiment. We have made an effort to present the idea in a form that we believe may actually be workable experimentally. The earliest form of this idea is contained in an unpublished note on "An Electromagnetic Brake."⁵¹

One potential application of such large-amplitude solitary-wave pulses would be for coherent ion acceleration, where one would employ a flared outer coax to permit gradual acceleration of the solitary wave and thus of the ions riding the wave. As the amplitude of the solitary becomes very large, the radiation pressure of the pulse will reach the point where it can begin to cause an insulator-metal transition and thus transform the dielectric rod into a metal rod, followed rapidly by it becoming an exploding-implosioning wire similar to the focus plasma of section 2 or the electron-burst target rods of section 3.

Typically, insulator-metal transition occurs in the pressure range of 1 to 10 Mbar. Because of the short transit time of the solitary pulse, the transition pressure in this case may exceed the "static" pressure for such a transition by as much as a factor of 10. It is clear that an interesting means of employing the amplified solitary wave for pulsed fusion is to make the high-amplitude end of the dielectric inner coax from DT, LiDt, or perhaps CDT, or from a hollow glass tube containing DT gas at high pressure.

By appropriate choice of parameters it is possible to set the solitary-wave propagation velocity U at a desired value. This opens the possibility of obtaining highly efficient ablation-free compression of the rod. Compression that is subsonic with respect to the electron Fermi velocity, and supersonic relative to the ion thermal-vibrational velocities of the solid, presents an interesting and genuinely complex problem. The compression of the electrons of the solid in this case will be adiabatic. Space-charge effects

will force the ions to also be compressed, and at the same time will drive them to high vibrational levels which ultimately would melt the solid, but not necessarily destroy the electron degeneracy.

The ionic vibrations could relax by emitting radiation or, because large molecules tend to readily couple energy between electronic and nuclear motion, the highly excited nuclear vibrations might relax by Auger emission, resulting in heating of electrons at the Fermi surface, thus tending to destroy the degeneracy. However, polyatomic molecules tend not to have an electronic spectrum but to relax by nonadiabatic effects into excitation of nuclear vibration and rotation states.⁵² Such a decay mechanism for excitation at the Fermi surface would result in cooling, tending to preserve the electron degeneracy. Thus nonradiative cooling of the Fermi surface is analogous to the role played by bremsstrahlung in cooling electrons in the plasma-focus snowplow plasma, so that the electron temperature lags by as much as a factor of 10 behind the ion temperature.

The effect of the time-dependent radiation pressure in this case would be to drive coherent standing waves into the Fermi gas, and the ions would take on the same structure due to space-charge effects, so that the electron density would vary as $1/r^2$ and have a periodic modulation of the standing wave in the Fermi gas "resonator;" the electrostatic waves in the degenerate medium are the extension of the electromagnetic-wave excitation in the vacuum cavity region. A self-consistent interface between the field and the plasma should develop that could provide impedance matching, so that the time-dependent radiation pressure could feed energy at the speed of light from the vacuum continuum as it evolves at the radiation-plasma interface into the Fermi-gas continuum. The pressure amplification of the ablation-driven compression is replaced in this case by the presence of many more waves in the degenerate gas than would exist for the electromagnetic field in the cavity in the absence of the degenerate plasma. As a result the outer-wall radiation pressure is amplified by a factor equal to the number of standing waves as one passes radially inward.

Analogous possibilities exist for

microwave confinement of hot plasmas, and have been considered by Ensley⁵³ and by the present author. The idea of Mora⁵⁴ on increased compression efficiency are also apparently of a similar nature. From the point of view of the field-free plasma region electromagnetic (i.e., radiation pressure) confinement of a plasma is similar to the electrostatic confinement invented by Farnworth and studied experimentally by Hirsch and others.⁵⁵ The reason for this fact is that the electrons directly follow the time-dependent radiation pressure while the ions indirectly follow the field due to space-charge effects.

The preceding discussion is a suggestion that a new type of highly efficient compression may be possible that is in essence an isentropic radiation-pressure-driven-electrostatic confinement of ions by standing charge-density waves in a degenerate Fermi gas. For this type of compression, burn would be initiated when the decreased ion spacing and increased vibrational amplitude reached the level at the rod center where barrier penetration becomes sufficiently probable. Since this penetration probability varies exponentially with nuclear separation, the onset of burn in the rod center could be very rapid.

5. SUMMARY

The pulsed power sources that most nearly approach fusion-microexplosion requirements (pulse energy $>10^3$ J, power $>10^{14}$ W, and power flux density $>10^{16}$ W/cm²) are relativistic E-beam machines and high-power pulsed lasers. The ability to convert 5% of existing E-beam energy to accelerated ions and 15% to short-wavelength microwaves makes collective ion bursts and microwave pulses competitive alternatives for microexplosion applications.

The greater efficiency of relativistic E-beam, the greater ability to focus laser light relative to electron bursts, and the apparent greater ease of energy deposition of laser light in a target than electron beam energy are not decisive differences. For fields in excess of

2×10^7 G, and for power-density fluxes of $>10^{15}$ W/cm² the deposition of high- v/γ energy is in first approximation a function of energy density, not of field frequency, because the radiation pressure determines the corona density and above 1 kV the skin currents drive Langmuir oscillations that result in large anomalous resistivity, and enhanced field diffusion, rapid energy deposition and thus causes ablation-driven compression.

When stability is considered, the optimum wavelength range for pulsed power sources intended for pulsed fusion application is $100 R > \lambda > R$, where R is the pellet radius. This singles out the wavelength range $1 \text{ cm} > \lambda > 100 \text{ } \mu\text{m}$ as particularly interesting. This is a wavelength range accessible to advanced microwave generation techniques utilizing both relativistic E-beam machines, or to long-wavelength laser and masers, or to a combination of these sources.

The short-pulse capability of the lasers is a decisive advantage. The laser-pulse amplification is a special case of a more widely applicable phenomenon of parametric amplification of a solitary wave.

One means under development at LLL for production of short-duration, small diameter, high- v/γ ($v/\gamma = 100$) relativistic E-beam pulses is by the strong-turbulence-triggered self interruption of the current of the pinch produced by low-density-mode operation of a 1-MW plasma focus. This current interruption is followed by restrike of the initial "super high"- v/γ ordinary current as a lower v/γ , relativistic electron burst. In this way, one can produce small-diameter, 10^3 -J, 10-nsec multi-megamp electron bursts and oppositely directed 10^4 -J-MeV ion bursts with diameter d of 1 mm and perhaps as small as 100 μm . The rise time of these pulses under the most ideal conditions can be as short as 100 psec.

Study of the neutron-yield-producing mechanisms of the focus suggests that a significant increase in the focus yield could be obtained by scaling the focus phenomena to the case of exploding-imploding DT wire at initial density 10 times solid density by employing the relativistic electron burst produced by the focus. The energy delivery to the fusible rod occurs by the relativistic

electron flow over the rod being initially cancelled out by a return current. This phase is followed by expulsion of the return current from the relativistic beam path by the Weibel instability due to Langmuir turbulence enhanced resistivity.

The return-current expulsion results in rapid transfer of 10^5 J of magnetic field energy to the region around the rod. The primary yield will be due to the turbulent, heated, lower density plasma ablated from the rod, and temporarily contained in the field that remains after a significant portion of the field energy has been transferred into particle energy, to produce a 10-kV plasma at $1/10$ solid density. These conclusions are based on scaling of the experimentally relatively well-studied plasma-focus deflagration burn mode, and are presently untested experimentally. Experiment as always must provide the acid test for these theoretical scaling arguments.

An alternative approach, utilizing the focus-produced electron burst or the longer duration bursts of a relativistic E-beam machine, is provided by the parametric amplification of a solitary transverse magnetic wave in a plasma-coated-dielectric coaxial cavity. This idea may permit the fundamentally important pulse amplification of laser light to be extended to microwave generation techniques. The inner coax of the proposed parametric amplifier is a Bennett dielectric rod.

The important applications of this solitary-wave parametric amplification are coherent ion acceleration and pulsed fusion. For ion acceleration the outer coax is gradually increased in diameter to permit acceleration of the solitary wave and thus of any ions riding the wave. The application to pulsed fusion is provided in the simplest way by making the high-energy end of the dielectric central coax of a fusible dielectric material or of a glass tube containing high pressure DT gas. The amplified soliton radiation pressure results in an insulator-metal transition in the dielectric rod, and this will be followed by rapid explosion-implosion self destruct of the rod end, which for pulse fields of about 2×10^7 G could result in significant DT yield. This latter idea has no experimental antecedents, and its value in practice must await appropriate experimental tests. We believe the approach could be tested

experimentally in the specific form discussed in this paper.

APPENDIX

NONLINEAR ELECTROMAGNETIC WAVES

In this appendix we develop the mathematical details of the treatment of nonlinear electromagnetic waves both for the case of interaction with a superconducting medium and for the case of a collisionless plasma, and apply the nearly identical results of the two cases to a more complete treatment of a solitary wave, to provide support for the oversimplified discussions in Section 5 of the main body of this paper.

We will simplify the treatment by assuming only nonrelativistic electrons; we also assume the ions merely serve as a neutralizing background, i.e., we use a quasi-neutral formulation. The central problem is to determine the fields from Maxwellian equations self-consistently from the current and charge-density sources whose distribution depends on the fields caused by these sources.

We begin with the Maxwell equation

$$\nabla \times \vec{H} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \vec{J}, \quad (1)$$

where $\vec{H}$ is the magnetic field, $\vec{E}$ the electric field, c the speed of light, and $\vec{J}$ the current density. Now introduce the vector and scalar potential $\vec{A}$ and ϕ . In terms of these potentials, the field becomes

$$\vec{H} = \nabla \times \vec{A}, \text{ and } \vec{E} = -(\nabla\phi + \frac{1}{c} \frac{\partial \vec{A}}{\partial t})$$

which, substituted into (1) with the use of the identity $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, gives

$$\nabla(\nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t}) = \nabla^2 \vec{A} - \frac{1}{c} \frac{\partial^2 \vec{A}}{\partial t^2} + \frac{4\pi}{c} \vec{J}. \quad (2)$$

The left side of (2) vanishes by virtue of the Lorentz condition. Recall that the potentials $\vec{A}$ and ϕ may be changed to the potentials $\vec{A} + \vec{\nabla}\chi$ and $\phi - (1/c)(\partial\chi/\partial t)$, and become $\nabla \times \vec{\nabla}\chi = 0$. This gauge transformation causes the new potentials to produce the same fields as the old potentials.

To solve Eq. (2) in specific cases we must obtain an expression for $\vec{J}$:

$$\vec{J} = \frac{e}{m} \langle m\vec{v} \rangle = \frac{e}{m} \langle \vec{p} - \frac{e}{c} \vec{A} \rangle$$

where $m\vec{v}$ is the particle momentum and $\vec{p}$ is the canonical momentum. The equation of motion for $\vec{p}$ is

$$\frac{d\vec{p}}{dt} = -\vec{\nabla}(\phi + \frac{e}{c} \vec{V} \cdot \vec{A})$$

and thus for force-free motion $d\vec{p}/dt = 0$, $\vec{p}$ is a constant, and $\vec{\nabla} \times \vec{p} = 0$. For $\vec{\nabla} \times \vec{p} = 0$, $\vec{p}$ is the gradient of some scalar λ , i.e., $\vec{p} = \vec{\nabla} \times \lambda$, and $\vec{p} = (e/c) \vec{A}$ may be looked upon as a new vector potential $-(e/2) \vec{A}'$, i.e., $\vec{p}$ provides a gauge transformation, and consequently Eq. (2) becomes

$$\nabla^2 \vec{A} - \frac{1}{c} \frac{\partial^2 \vec{A}}{\partial t^2} - \frac{e^2}{c^2} \vec{A} = 0, \quad (3)$$

where $\frac{e^2}{c^2} = 4\pi n e^2 / m$; $n = \int f(\vec{x}, \vec{p}, t) d\vec{p}$ and the electron distribution function satisfies the self-consistent truncation of the many-particle Boltzmann equation, i.e., the Vlasov equation. We will obtain an identical equation for the superconducting case.

Anticipating the results we note that the similarity between the collisionless plasma and the superconductor arises because in both cases (for $\vec{p} = (e/c) \vec{A}$), a London-like relation^{5,6} that replaces Ohm's law. The difference between the problem for the superconductor treated as a coherent charge boson fluid, and the collisionless Vlasov plasma is contained in the definition of the density n . The parallel between the collisionless plasma and the quantum fluid can be made particularly close by introducing the Wigner distribution function $f_w(\vec{x}, \vec{p}, t)$ given by

$$f_w(\vec{x}, \vec{p}, t) = f_w^*(\vec{x} + \frac{\vec{p}}{2}, t) e^{i\vec{p} \cdot \vec{y}} \psi(\vec{x} - \frac{\vec{p}}{2}, t) \psi^*,$$

It is the most nearly analogous function for a quantum system of the classical phase-space distribution function, and in the limit $\hbar \rightarrow 0$, $f_w \rightarrow f$. The Wigner distribution function has the peculiarity in general that unlike a classical distribution function it can sometimes be negative. The expectation value of a quantum operation $\langle \hat{O}(\vec{x}, \vec{p}) \rangle$ expressed in terms of the Wigner distribution has the form $\langle \hat{O}(\vec{x}, \vec{p}) \rangle = \int f_w(\vec{x}, \vec{p}, t) \hat{O}(\vec{x}, \vec{p}) d\vec{x} d\vec{p}$, which is the same form as the classical phase-space average in statistical mechanics. Formulation of a quantum problem in terms of the Wigner distribution function is particularly similar to the classical problem in the special case of a single harmonic oscillator in a coherent state.

We will summarize briefly the results for a single oscillator at the end of the appendix. The coherent-state picture we will employ for the second quantized field is equal to a superposition of countable infinity of oscillators in coherent states, because of the well known fact that a second quantized Bose field and an infinite set of noninteracting harmonic oscillators are isomorphic.

We assume that the charged Bose fluid is a model of the quasi-Bose electron pairs of a superconductor. The second quantized field for such a system has annihilation and creation operators, $\hat{\psi}(\vec{x}, t)$ and $\hat{\psi}^*(\vec{x}, t)$, that satisfy the equal-time Bose commutation relations

$$[\hat{\psi}(\vec{x}', t), \hat{\psi}^*(\vec{x}, t)] = \delta(\vec{x}' - \vec{x}).$$

The Heisenberg equation of motion of the annihilation operation is

$$i\hbar \frac{\partial \hat{\psi}}{\partial t} + [\hat{H}, \hat{\psi}] = 0,$$

where $\hat{H} = \hat{H}_0 + \hat{H}_1$ and

$$\hat{H}_0 = \frac{1}{2m} \int \hat{\psi}^* (\vec{p} - \frac{e}{c} \vec{A})^2 \hat{\psi} d\vec{x} + e \int \hat{\psi}^* \phi(\vec{x}) \hat{\psi} d\vec{x}$$

and

$$\hat{H}_1 = \frac{1}{2} \iint \hat{\psi}^*(\vec{x}) \hat{\psi}^*(\vec{y}) V(\vec{x} - \vec{y}) \hat{\psi}(\vec{y}) \hat{\psi}(\vec{x}) d\vec{x} d\vec{y}$$

where V is a pair potential and $e\phi$ is an externally applied potential. Substitution of these forms of $\hat{H}$ into the equation of motion for $\hat{\psi}$ yields a Hartree-like operator equation

$$i\hbar \frac{\partial \hat{\psi}}{\partial t}(\vec{x}, t) = \frac{(\vec{p} - \frac{e}{c} \vec{A})^2}{2m} \hat{\psi}(\vec{x}, t) + U\hat{\psi}(\vec{x}, t) + \frac{1}{2} \int \hat{\psi}^*(\vec{y}) V(\vec{x} - \vec{y}) \hat{\psi}(\vec{y}) d\vec{y} \hat{\psi}(\vec{x}, t).$$

Now we make the basic assumption that superfluidity is the condition corresponding to all the Bosons occupying the same wave function, and thus the system is in a coherent state or Glauber state.^{5,6} Such a state $|\psi\rangle$ is an eigenstate of the annihilation operation $\hat{\psi}$, i.e., $\hat{\psi}|\psi\rangle = \psi(\vec{x}, t)|\psi\rangle$; the wave-function-like quantity ψ is the eigenvalue, and is known in the jargon of superfluidity and superconductivity as the complex order parameter. Projecting the equation for $\partial\hat{\psi}/\partial t$ onto the coherent state $|\psi\rangle$, we obtain the

Hartree-like equation for the complex order parameter known as the Ginsberg-Landau equation:

$$\text{if } \frac{\partial \psi}{\partial t} = \frac{(\bar{\psi} - \frac{e}{c} \bar{A})^2}{2m} \psi + e\psi\psi \\ + \frac{1}{2} \int |\psi|^2 V(\bar{x} - \bar{y}) d\bar{y} \psi(\bar{y}). \quad (4)$$

In obtaining Eq. (4), higher terms that arise because the overcomplete set of states $|\psi(\bar{x}, t)\rangle$ are not orthogonal.

We should also treat the electromagnetic field as a second quantized field and introduce the operator $\hat{E}(\bar{x}, t)$, which is a superposition of positive frequency field components and annihilates a photon at a point. Then for a coherent (superfluid-like) photon field we would introduce the coherent state $|E(\bar{x}, t)\rangle$, where

$$\hat{E}(\bar{x}, t)|E(\bar{x}, t)\rangle = E(\bar{x}, t)|E(\bar{x}, t)\rangle;$$

the eigenvalue order parameter $E(\bar{x}, t)$ looks like and satisfies the same equation as a classical electric field.

We omit the details of the procedure outlined above and simply state that we arrive at the same mathematical form as if we had used an unquantized electromagnetic (EM) field in the first place with the slightly subtle distinction that $E(\bar{x}, t)$ is an order parameter for the positive frequency part of the field, not a classical field. The only apparent consequence of this fact is that complex exponential solutions for waves are used as a mathematical device in the classical case, and only the real part of the answer has meaning, whereas $\hat{E}(\bar{x}, t)$, the order parameter, must have complex solutions corresponding to directed energy and momentum. The distinction is usually conveniently overlooked in classical EM books when they cover the topic of an evanescent wave, e.g., one at a throat below cutoff between two cavities. The book's author, having carefully pointed out in some earlier part of the book that complex fields are a mathematical device, will then go on to deduce the wave "inside the quantum barrier" ignoring this fact.

Returning to the complex order parameter, Eq. (4), we note that $V(\bar{x} - \bar{y})$ will be a screened Coulomb potential and by oversimplifying this potential into a delta function of strength a we may obtain the nonlinear Schrödinger equation.

$$\text{if } \frac{\partial \psi}{\partial t} = \frac{(\bar{\psi} - \frac{e}{c} \bar{A})^2}{2m} \psi + e\psi\psi + a|\psi|^2 \psi = 0.$$

We will obtain an equation of precisely this form for the nonlinear wave equation Eq. (3) presently.

The complex order parameter, or macroscopically occupied wave function, has the property that $|\psi|^2$ and

$$\bar{J} = \frac{1}{2m} \{ \psi [(\bar{\psi} - \frac{e}{c} \bar{A}) \psi]^* - \psi^* [\bar{\psi} - \frac{e}{c} \bar{A}] \psi \}$$

are "honest to God" particle and current densities rather than probability densities as is the case for the mathematically identical Hartree equation for a single particle. For this reason the Hartree-like equation for the order parameter is "exact" given the validity of the assumption of a coherent state as an exact description of superfluidity.

If we take ψ of the form $\psi = \sqrt{n} \exp(-i/\hbar S)$, then $|\psi|^2 = n$, and $\bar{J} = a n (\nabla S - (e/c) \bar{A})$, and from the real and imaginary parts of the complex order parameter equation for this form of ψ we obtain

$$\frac{1}{2m} (\nabla S - \frac{e}{c} \bar{A})^2 + e\psi + an^2 + \frac{\partial \psi}{\partial t} = \frac{1}{2m} \frac{\nabla^2 n}{\sqrt{n}},$$

which is the usual Hamilton-Jacobi equation with quantum potential $(\hbar^2/2m) \nabla^2 n / \sqrt{n}$. As noted by Feynman⁵⁶ the requirement of space-charge neutralization makes the quantum term small almost everywhere. We also obtain the continuity equation

$$\frac{\partial n}{\partial t} + \frac{\nabla}{m} \cdot (\nabla S - \frac{e}{c} \bar{A}) = 0.$$

If we interpret the role of ∇S and $\partial S/\partial t$ as providing a gauge transformation, then these two equations become

$$\frac{e^2}{2mc^2} k^2 + e\psi + an^2 = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}},$$

and

$$\partial n/\partial t = \nabla \cdot [n(e/mc) \bar{A}],$$

and the quantum-mechanical order-parameter current reduces to $(\hbar m/c) \bar{J} = (\hbar^2/c^2) \nabla \bar{A}$, which, as promised, leads to Eq. (3). Thus for either the collisionless plasma or the superconductor the problem is to solve

$$\nabla^2 \bar{A} - \frac{1}{c^2} \frac{\partial^2 \bar{A}}{\partial t^2} - \frac{\omega_p^2}{c^2} \bar{A} = 0.$$

We will assume that the problem of interest will involve only a single component of $\bar{A}$ and a single field frequency ω so that we may take A in the form $A = a(\bar{r}) \exp(i\omega t)$, for which the wave equation becomes

$$\left[\nabla^2 + \left(\frac{\omega}{c} \right)^2 - \left(\frac{\omega_p}{c} \right)^2 \right] a(\bar{r}) = 0$$

We must now find an expression for n the density in ω_p in order to have a completely defined problem. In the case of the Vlasov plasma we assume that $f(\bar{x}, \bar{p}, t)$ is an equilibrium distribution function

$$f = \exp \left[-\beta \left(\bar{p} - \frac{e}{c} \bar{A} \right)^2 + e\phi \right]$$

and then interpret $\bar{p}$ as a gauge

$$\left(\bar{p} - \frac{e}{c} \bar{A} \right)^2 + \frac{e^2}{2mc^2} A^2 = \frac{e^2}{2mc^2} A^2$$

The quasi-neutrality condition demands that $e_0 = -\frac{1}{2} (e^2/2mc^2) A^2$, and approximating A^2 by its time average a^2 we obtain $\omega_p^2 = (\omega_0^2)^2 \exp(-\beta e^2 a^2/4mc^2)$, where $(\omega_0^2)^2$ is $4\pi n_0 e^2/m$, n_0 being the density in the asymptotic region where the fields are small. Thus we obtain the equation

$$\nabla^2 a + \left[\left(\frac{\omega}{c} \right)^2 - \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right) \right] a = 0$$

If we further specialize this to a one dimensional problem then we obtain

$$\frac{\partial^2 a}{\partial z^2} + \left[\left(\frac{\omega}{c} \right)^2 - \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right) \right] a = 0$$

If this equation is multiplied by $\partial a/\partial z$, it becomes

$$\frac{\partial}{\partial z} \left(\frac{\partial a}{\partial z} \right)^2 + \left[\left(\frac{\omega}{c} \right)^2 - \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right) \right] a \left(\frac{\partial a}{\partial z} \right) = 0$$

or

$$\frac{\partial}{\partial z} \left[\left(\frac{\partial a}{\partial z} \right)^2 + \left(\frac{\omega}{c} \right)^2 a^2 + \frac{2mc^2}{\beta e^2} \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right) \right] = 0$$

The integral of this equation is

$$\left(\frac{\partial a}{\partial z} \right)^2 + \left(\frac{\omega}{c} \right)^2 a^2 + \frac{4mc^2}{\beta e^2} \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right) = E_0$$

where E is the constant of integration. The solution of this equation for z is given by

$$z = z_0 + \int_{a(z_0)}^a \frac{da}{\sqrt{E - \left(\frac{\omega}{c} \right)^2 a^2 - \frac{2mc^2}{\beta e^2} \left(\frac{\omega_p}{c} \right)^2 \exp \left(-\frac{\beta e^2}{4mc^2} a^2 \right)}}$$

It is not possible to obtain an analytic solution to this integral. However, in the approximation where we may linearize the exponential, the integral becomes

$$z = z_0 + \int_{a(z_0)}^a \frac{da}{\sqrt{E - \left(\frac{\omega}{c} \right)^2 a^2 - \frac{4mc^2}{\beta e^2} \left(\frac{\omega_p}{c} \right)^2 + \frac{4mc^2}{\beta e^2} \left(\frac{\omega_p}{c} \right)^2 a^2}}$$

If we now consider the problem corresponding to this equation as that of a one-dimensional Langmuir collapse, where the particle-free field region is at the origin and the field-free particle region is at $\pm z$ for large z , i.e., $z \rightarrow \pm \infty$, $n \rightarrow n_0$, then the first integration can be taken as f_0 and then E is the integrand at $z = 0$, where $\partial a/\partial z = 0$ and $\omega_p = 0$, and consequently $E = (\omega/c)^2$; and for this value of E the second integral becomes

$$z = \alpha \int_{a(0)}^a \frac{a}{a^2 - 1} da, \text{ where } \alpha = \frac{1}{4\sqrt{4\pi n_0 T}}$$

$$z = \alpha [\cosh^{-1} a(z) - \cosh^{-1} a(0)]$$

or

$$a(z) = a(0) + \cosh(z/\alpha)$$

We note that this is the solution of the linearized equation

$$\frac{\partial^2 a}{\partial z^2} + \left[\left(\frac{\omega}{c} \right)^2 - \left(\frac{\omega_p}{c} \right)^2 \left(1 - \frac{\beta e^2}{4mc^2} a^2 \right) \right] a = 0$$

an equation of exactly the same form as the nonlinear Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\bar{p} - \frac{e}{c} \bar{A} \right)^2 \psi + a|\psi|^2 \psi + e\phi\psi$$

if $\psi = ae^{-i\omega t}$.

The nonlinear equation for a is also of the form employed by Shiff⁵⁸ at the basis of a model of an elementary particle, and the treatment in this section is also similar in a number of respects to the work of de Broglie.⁵⁹ The

linearization is actually not valid near $z = 0$ in the case treated above, which corresponds to the case where the electromagnetic field via its radiation pressure holds open its own wave guide.

In the case where the vacuum well exists even in absence of the radiation pressure, such as for a superconducting cavity or the plasma-coated dielectric-wall cavity for the parametric amplifier in Section 5 we may introduce an effective potential u that simulates the field-free guide, i.e.,

$$\frac{\omega_p^2}{c^2} = \frac{\omega_0^2}{c^2} \exp(-\beta u) \exp\left(-\frac{e^2}{4\pi mc^2} n^2\right),$$

and for this case the major burden of producing the vacuum region is due to the "known" potential u , the radiation pressure provides a perturbation.

In this case the linearization is valid since $(\omega_p/c)^2$ vanishes at small r values because of u , i.e., in the region where n vanishes in the absence of the linearization would be invalid. For this case one may write ω_p^2/c^2 as

$$\left(\frac{\omega_p}{c}\right)^2 = \frac{\omega_0^2}{c^2} \left\{ \exp(-\beta u) \left(1 - \frac{e^2}{4\pi mc^2} n^2\right) \right\}$$

where u can be taken as a strong function of r , so that ω_p drops rapidly to zero at the guide surface in a physically realistic manner.

Note that if we assume n is of the form $\exp(-\lambda x^2/2)$, e.g., as a trial function for a variational approach to the non-linear Schrödinger-like equation for (n) , and in addition if we approximate the non-linear term as $(\omega_p/c)^2(1 - \lambda x^2/2)$, then the equation for λ becomes

$$\frac{\partial^2 a}{\partial z^2} + \left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_0}{c}\right)^2 \left(1 - \frac{\lambda x^2}{2}\right) \right] a;$$

this equation has the form of a harmonic oscillator equation from which λ can be determined in terms of ω and ω_0 . Apart from the present problem being a one-dimensional linear formulation instead of a one-dimensional radial problem, the problem and the result are identical to the simplified treatment of a standing, solitary transverse magnetic wave and this establishes the fact that the simplified treatment is the lowest-order self-consistent solution.

Finally we turn to the determination of the form of n for the superconduction problem. The equations relating $\tilde{A}$ and n in this case are

$$\frac{e^2}{4\pi mc^2} \tilde{A}^2 + an = \frac{\tilde{A}^2}{2m} \frac{\partial^2 \sqrt{n}}{\partial z^2} \sqrt{n}, \text{ or}$$

$$\frac{e^2}{4\pi mc^2} \tilde{A}^2 + an + \frac{\tilde{A}^2}{2m} \left[\left(\frac{1}{n} \frac{\partial n}{\partial z} \right)^2 - \frac{2}{n} \frac{\partial^2 n}{\partial z^2} \right],$$

$$\frac{e^2}{4\pi mc^2} \tilde{A}^2 + an = \frac{\tilde{A}^2}{4m} \left[\frac{1}{n} \frac{\partial^2 n}{\partial z^2} - \frac{1}{2} \left(\frac{1}{n} \frac{\partial n}{\partial z} \right)^2 \right],$$

If we neglect the Hartree term on and use the fact that for quasi-neutrality $e\phi = -(e^2/4\pi mc^2) \tilde{A}^2$, then we obtain

$$\left(\frac{e}{\hbar c} \right)^2 \tilde{A}^2 = \left[\frac{1}{4m} \frac{\partial^2 n}{\partial z^2} - \frac{1}{2} \left(\frac{1}{n} \frac{\partial n}{\partial z} \right)^2 \right].$$

Now let $n = n_0 e^{-f}$ to obtain

$$\left(\frac{e}{\hbar c} \right)^2 \tilde{A}^2 = \left[\left(\frac{df}{dz} \right)^2 \frac{1}{2} - \frac{\partial^2 f}{\partial z^2} \right].$$

To obtain a first approximation we assume $\partial^2 f / \partial z^2$ is negligible, and thus

$$f = \frac{\sqrt{2} e}{\hbar c} \int_{z_0}^z \tilde{A} dz' \quad \text{which implies}$$

$$n = n_0 \exp - \frac{\sqrt{2} e}{\hbar c} \int_{z_0}^z \tilde{A} dz'.$$

OSCILLATOR COHERENT STATES

Finally we outline the coherent-state picture in the case of a single harmonic oscillator. The Hamiltonian H of a one-dimensional oscillator in dimensionless form is $H = \frac{1}{2}(p^2 + x^2)$, or, in terms of creation and annihilation, operator $H = \tilde{n} + \frac{1}{2}$, where $\tilde{n} = \tilde{a}^+ \tilde{a}$, $[\tilde{a}, \tilde{a}^+] = 1$, and $|n\rangle$ are the number eigen states; $\tilde{n}|n\rangle = n|n\rangle$, $\tilde{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle$, $\tilde{a}|n\rangle = \sqrt{n}|n-1\rangle$, and the Schrödinger wave functions are the position representatives of $|n\rangle$, i.e., $\langle x|n\rangle = \phi_n(x) \exp(-x^2/2) \psi_n(x)$, and the time-dependent wave functions are

$$\psi_n(x, t) = A \exp \left[-\left(n + \frac{1}{2}\right) t \right] \exp \left(-\frac{x^2}{2} \right) \psi_n(x)$$

where A is a normalization constant and ψ_n a Hermite polynomial.

The coherent state of the oscillator $|\alpha\rangle$ is an eigenstate of annihilation operator $\hat{a}$, $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. The position representative of $|\alpha(t)\rangle$ is given by

$$\langle x|\alpha(t)\rangle = \exp\left[-\frac{1}{2}(x - x(t))^2\right] \exp\left[-\frac{i}{\hbar}x(t)p(t)\right] \exp\left[-\frac{it}{2}\right]$$

where $x(t)$ and $p(t)$ are the classical oscillator position and momentum coordinates; $x(t) = x_0 \cos(\omega t - \phi)$ and $p(t) = x_0 \sin(\omega t - \phi)$. Thus we see that the coherent-state wave function $\psi(\alpha(t), x) = \langle x|\alpha(t)\rangle$ is a Gaussian, nonspreading wave packet that travels on the classical path with classical momentum, and, if expressed in dimensional rather than dimensionless form, shows that as $\hbar \rightarrow 0$ the wave packet becomes more sharply peaked and packet approaches a delta-function. The probability density superimposes a classical particle undergoing a harmonic oscillation. By Fourier transform of $\psi(\alpha(t), x)$, we obtain the momentum representation $\psi(p, \alpha(t))$, where

$$\psi(p, \alpha(t)) = B \exp\left[-\frac{1}{2}(p - p(t))^2\right] \exp\left[-\frac{i}{\hbar}p(t)x(t)\right] \exp\left[-\frac{it}{2}\right]$$

The Wigner distribution function for this state is $f_W(x, p, t) = |\psi(\alpha(t), x)|^2 |\psi(p, \alpha(t))|^2$, a genuine probability that satisfies Liouville's equation. $f(x, p, t) = C \exp[-(x - x(t))^2 - (p - p(t))^2]$, where C is a normalization constant. We may also write $f(x, p, t)$ in the form $f(x, p, t) = \chi^*(x, p, t) \chi(x, p, t)$, where $\chi = C \exp\left[-\frac{i}{2}(x - x(t))^2 - \frac{i}{2}(p - p(t))^2\right] \exp\left[-\frac{i}{\hbar}(p(t)x(t) - px(t))\right] e^{-it/2}$, a "phase-space wave function" that satisfies Liouville's equation and represents a wavepacket traveling with directed motion on the classical phase-space path with the classical energy and momentum. These packets and their excited states are of the type studied by Della Rocca and Wiener⁶¹ in Wiener's last published work. Excited coherent states or Kennard packets also exist and are of the form

$$\psi_n(x, t) = C_n \exp\left[-\frac{1}{2}(x - x(t))^2\right] \exp\left[-\frac{i}{\hbar}x(t)p(t)\right] \exp\left[-\frac{it}{2}\right] \times H_n(x - x(t)) \exp\left[-i\left(n + \frac{1}{2}\right)t\right]$$

with Fourier transform

$$\phi_n(x, t) = C_n \exp\left[-\frac{1}{2}(p - p(t))^2\right] \exp\left[-\frac{i}{\hbar}p(t)x(t)\right] H_n(p - p(t))$$

and Wigner distribution function

$$f_n(x, p, t) = \chi_n^*(x, p, t) \chi_n(x, p, t) = |C_n|^2 |H_n|^2$$

where

$$\chi_n(x, p, t) = \exp\left[-\frac{1}{2}(n - n(t))^2\right] \exp\left[-\frac{i}{2}(x - x(t))^2\right] \exp\left[-\frac{i}{\hbar}(xp(t) - px(t))\right] \exp\left[-i\left(n + \frac{1}{2}\right)t\right]$$

the f_n and χ_n satisfy Liouville's equation.

The boson annihilation operation $\psi(x, t)$ in the discussion of the coherent-state picture of a superconductor is a superposition of harmonic-oscillator creation operators $a_n^\dagger$, i.e.,

$$\psi(x, t) = \sum_n a_n^\dagger \exp(i\omega_n t) \phi_n(x)$$

The $\phi_n(x)$ is a complete set of states, and the coherent state $|\psi(x, t)\rangle$ is a product $\prod_n |\phi_n\rangle$ of the separate oscillator states. Thus, the position representation of $|\psi\rangle$ is a product of the separate oscillator position representations. The Fourier transform of the product states is a product of the simple-oscillator Fourier transforms, and the Wigner distribution function is a product of the separate-oscillator Wigner distribution functions. The product of coherent states, and the sum or coherent superposition of such states are equivalent — a particularly unique property of the coherent state.⁶²

The form of the eigenvalue $\psi(x, t)$ of annihilation operator $\hat{\psi}$ is the complex order parameter

$$\psi(x, t) = \sum_n a_n(t) \phi_n(x)$$

where $a_n(t)$ is the eigenfunction of the separate oscillator state; $a_n^\dagger |a_n(t)\rangle = a_n(t) |a_n(t)\rangle$; and $\phi_n(x)$ is any complete set of states.

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