CONF

CMELIN REFERENCE NUMBER

AED-Conf- 63-202-9

(1, d)

AN RF TECHNIQUE FOR PLASMA NUMBER DENSITY MEASUREMENT

by

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CONF-363-12

International Symposium on
Plasma Phenomena and Measurement
San Diego, California
October 29-November 1, 1963

Internal Memorandum

Contract AT(04-3)-326-1

M. L. Report No. 1087

October 1963

MASTER

(To be presented at the IEEE International
Symposium on Plasma Phenomena and Measure-
ment at San Diego, October 1963.)

Proc. published by IEEE Transactions on
Nuclear Science

Microwave Laboratory
W. W. Hansen Laboratories of Physics
Stanford University
Stanford, California

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ABSTRACT

It is well-known that a cylindrical plasma column in a transverse rf electric field is subject to a dipole resonance. The frequency at which this occurs can be used to determine the average electron density across the plasma section. This paper describes the experimental application of the technique to plasmas with and without a confining axial static magnetic field, and gives theoretical curves to assist in rapid reduction of the data. Higher order multipole resonances can also be excited. The feasibility of applying these to determination of the number density profile is discussed.

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I. INTRODUCTION

Several rf techniques are available for plasma electron density measurements, their ranges of application depending generally on the system dimensions and the electron density to be measured. Some involve measurements of wave propagation along or across the column, while others depend on perturbation by the plasma of the resonant frequency of a probing structure within it, or of a microwave cavity surrounding the column. In this paper we shall deal with a technique based on a dipole resonance of the plasma column. The way in which this occurs can be understood from consideration of the idealized system shown in Fig. 1. If the rod is of uniform, isotropic, dielectric material of permittivity ϵ' , relative to free space, then the electric field inside, E_1 , is related to that outside at infinity, E_0 , by the expression,¹

$$E_1 = \frac{2E_0}{1 + \epsilon'} \quad (1)$$

If the dielectric of interest is the positive column of a gas discharge, we may substitute the value of ϵ' appropriate to a plasma. Neglecting the damping effect of collisions, and assuming a cold plasma, this is

$$\epsilon' = 1 - \frac{\omega_p^2}{\omega^2} \quad (2)$$

where ω is the applied frequency and ω_p is the electron plasma frequency given by

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m} \quad (3)$$

Here n is the electron number density, e is the electronic charge, m is the electronic mass and ϵ_0 is the permittivity of free space.

FIG. 1--Dielectric rod perturbing a uniform electric field.

After substitution of Eq. (2) in Eq. (1), we note that there will be a resonance when

$$\omega_1^2 = \frac{\omega_p^2}{2}, \quad (4)$$

where ω_1 is the dipole resonant frequency. This is the basic dipole resonance effect first described by Tonks,² and by Herlofson.³ Physically, the resonance occurs between the free-space capacitance and the inductive component of plasma impedance due to inertia of the electrons.

Strictly, the analysis leading to Eq. (4) is incorrect since the quasistatic approximation cannot apply at nonzero frequency in an infinite system. Nevertheless, we might expect the analysis to predict substantially the correct resonant frequency if the free-space wavelength is long compared to the dimensions of the region strongly perturbed by the presence of the column. Another assumption which is not physically correct is that the column is suspended alone in free space; the effect of the glass tube surrounding it should be taken into account. In Section II we shall deal with the modifications required to the simplified analysis to take into account such factors as tube and electrode geometry, nonuniform electron density, nonzero electron temperature, electron-neutral collisions, and the presence of a static axial magnetic field.

The use of the dipole resonance effect as a diagnostic for electron density measurement is attractive in the range where propagation and cavity perturbation techniques are usually applied. The resonance can be excited by a circular strip-line system, which is more easily constructed than a microwave cavity, and which is used at much lower frequencies. The resolution of density variations along the tube axis is considerably greater than that of propagation measurements, which usually require a length of substantially uniform column considerably longer than the width of a strip-line exciting system. It will be noted, also, that the dipole resonance occurs only in the presence of the plasma, so that if number density fluctuations are occurring, the proportionate changes in the resonant frequency are much greater than in that of a resonant cavity. This means that fluctuating discharges may be more easily studied with the dipole resonance method.

It is the purpose of this paper to provide a key to the analyses bearing on the dipole resonance phenomena and to the related experimental work, and in doing so, define the conditions under which it may be most readily applied.

II. THEORY OF THE RESONANCE

The modifications to Eq. (4) demanded by a practical set-up can be separated into two parts; those dependent on the region external to the plasma, and those associated with the plasma itself. If the exciting electrode dimensions are small compared to a free-space wavelength of the applied frequency, the quasistatic assumption is accurate^{4,5} and will allow us to reduce the problem in the region external to the plasma to the solution of Laplace's equation. In this work, we shall be interested in the cylindrical system shown in Fig. 2a. For this case, Eq. (4) becomes^{6,7}

$$\omega_1^2 = \frac{\omega_p^2}{k_1} \quad (5)$$

The geometrical constant k_1 is obtained by putting $m = 1$ in the expression

$$k_m = 1 + \epsilon_1 \frac{\left[1 - \left(\frac{1-K}{1+K} \right) \left(\frac{a}{b} \right)^{2m} \right]}{\left[1 + \left(\frac{1-K}{1+K} \right) \left(\frac{a}{b} \right)^{2m} \right]}, \quad (6)$$

where K is given by

$$K = \frac{1}{\epsilon_1} \left(\frac{1 + \left(\frac{b}{c} \right)^{2m}}{1 - \left(\frac{b}{c} \right)^{2m}} \right) \quad (7)$$

In these expressions, ϵ_1 is the relative permittivity of the glass, and a , b , and c are, respectively, the internal and external radii of the glass tube, and the radius of the exciting electrodes. Most commercial glasses and quartz have ϵ_1 in the range 4-5. Computed values of k_1 are shown in Fig. 3 for this range.

(a) GEOMETRY

(b) ELECTRIC FIELD DISTRIBUTION

FIG. 2.--Ideal cylindrical resonating system.

FIG. 38--Variation of k_1 with strip-line dimensions. $\epsilon_1 = 4.0$.

FIG. 3b--Variation of k_1 with strip-line dimensions $\epsilon_1 = 4.5$.

Fig. 3. Variation of k_1 with stub-line dimensions $\epsilon_1 = 5.0$.

It is worth noting at this point that the field configuration of the dipole mode shown in Fig. 2b is not the only one possible. Higher-order multipole modes can be excited. In the infinite system of Fig. 1 these are degenerate, but the presence of a surrounding glass tube, or of nonuniform electron density, removes this degeneracy. We shall return to these modes later in our discussion of electron density profile measurements. Figures 4, 5, and 6 give computed values of k_m for these modes.

The analysis of the resonance has been extended to hollow discharges, and shows that splitting of the resonance should occur.^{8,9} This has been checked experimentally.⁸ A similar effect has been predicted and observed for solid elliptical plasma columns.^{10,11}

We now turn to consideration of the assumptions concerning the plasma column, and the necessary modifications to Eq. (5) if they are relaxed.

A. THE EFFECT OF ELECTRON-NEUTRAL COLLISIONS.

It has been shown that the Q of the resonance is given by (ω/ν) where ν is the electron-neutral collision frequency.⁷ For the technique to be of value, we will normally work under conditions of electron density and neutral gas pressure such that Q is at least 10. The dipole resonant frequency will then be shifted by about 0.3% so that Eq. (5) will still apply to a very close approximation.

B. THE EFFECT OF A STATIC AXIAL MAGNETIC FIELD

If a magnetic field, B , is applied to the column, the relative permittivity of the plasma is no longer isotropic, but becomes a tensor quantity. In the analysis, this difficulty can be overcome; in the case of an axial magnetic field, by resolving the electric field into two equal counter-rotating components. The right-hand polarized component rotates in the same sense as the electrons in the magnetic field and has a relative permittivity given by

$$\epsilon_R = 1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \quad (8)$$

FIG. 1a--Variation of k_2 with strip-line dimensions $\epsilon_1 = 4.0$.

FIG. 4b--Variation of k_2 with strip-line dimension $\epsilon_1 = 5.0$.

FIG. 4c--Variation of k_2 with strip-line dimensions $\epsilon_1 = 5.0$.

FIG. 5a--Variation of k_3 with strip-line dimensions $\epsilon_1 = 4.0$.

FIG. 50--Variation of k_3 with (a/b) for different values of (c/b) .

FIG. 5c--Variation of k_3 with strip-line dimensions $\epsilon_1 = 5.0$.

FIG. 6a--Variation of k_4 with strip-line dimensions $\epsilon_1 = 4.0$.

FIG. 6b--Variation of k_4 with strip-line dimensions $\epsilon_1 = 4.5$.

FIG. 6.c--Variation of k_4 with stripline dimensions $\epsilon_1 = 5.0$.

where ω_c is the electron cyclotron frequency given by

$$\omega_c = \frac{eB}{m} \quad (9)$$

The left-hand polarized component has relative permittivity

$$\epsilon_L' = 1 - \frac{\omega_p^2}{\omega(\omega + \omega_c)} \quad (10)$$

There are now two dipole resonant frequencies ω_R and ω_L in the rotating coordinate systems given by the modified forms of Eq. (5):

$$\omega_R(\omega_R - \omega_c) = \frac{\omega_p^2}{k_1} \quad (11)$$

and

$$\omega_L(\omega_L + \omega_c) = \frac{\omega_p^2}{k_1} \quad (12)$$

These are equivalent to

$$\omega_R - \omega_L = \omega_c \quad (13)$$

and

$$\omega_R \omega_L = \frac{\omega_p^2}{k_1} \quad (14)$$

This implies that the splitting increases directly with ω_c , and that the density measurement depends on the product of the two resonant frequencies.

This splitting was first observed by Tonks² and has been the subject of several more recent studies.¹²⁻¹⁵

C. EFFECT OF NONUNIFORM DENSITY

If the electron density is nonuniform, simple solutions of Laplace's equation do not satisfy the conditions within the plasma. We must solve the equation

$$\nabla \cdot \left(1 - \frac{\omega_p^2}{\omega^2} \right) \nabla \phi_1 = 0 \quad , \quad (15)$$

where ϕ_1 is the rf component of potential and ω_p^2 is now a function of the space coordinates. In the situations of interest to us, ω_p^2 will be a function of the radius, r , only, so that we have

$$\omega_p^2 = \omega_{p0}^2 f \quad , \quad (16)$$

where ω_{p0} is the local plasma frequency on the axis, and f is a function of r .

It has been shown elsewhere^{6,7} by a variational method, that if the solution to Laplace's equation appropriate to the uniform density case is used as a trial function in the determination of a multipole resonant frequency, the resulting expression is

$$\omega_m^2 = \frac{1}{k_m} \int_0^1 \omega_p^2 \frac{d}{dx} \left(x^{2m} \right) dx \quad , \quad (17)$$

where x has been written for (r/a) . In the dipole resonance case this simplifies to

$$\omega_1^2 = \frac{\overline{\omega_p^2}}{k_1} \quad , \quad (18)$$

where $\overline{\omega_p^2}$ is dependent on the average electron density across the section.

The use of the solution of Laplace's equation appropriate to uniform density as a trial function can be expected to be satisfactory provided that $[1 - (\omega_p^2/\omega^2)]$ does not change sign anywhere in the column. If this occurs, poles are introduced into the exact solution for the potential and a radically different trial function is needed. Physically, such poles will not exist, of course, due to effects such as nonzero electron temperature. To assess the error involved in using Eq. (18), these aspects must be examined further.

D. EFFECTS OF NONZERO ELECTRON TEMPERATURE

If a hydrodynamic description of the plasma is used, the modified form of Eq. (15) can be shown to be^{16,17}

$$\nabla \cdot \left\{ \nabla^3 \phi_1 - \frac{\nabla^2 \phi_1}{\gamma} \left(\frac{\nabla f}{f} \right) + \left(\frac{\omega^2}{\omega_{p0}^2} - f \right) \frac{\nabla \phi_1}{\gamma \lambda_{D0}^2} \right\} = 0 \quad (19)$$

where λ_{D0} is the local Debye length at the tube axis and γ is the compression constant for the electron gas.

This equation can be solved numerically. Such a calculation has been carried out recently by Mickel, Parker and Gould.¹⁶ To do this, it was first necessary to determine the form of the electron density profile, f . This was done numerically by Parker¹⁸ for the Tonks-Langmuir "free-fall" conditions applicable to low-pressure discharges when mean free paths for ion-neutral collisions exceed the tube dimensions.

By using the computed solutions for f , it was possible to obtain numerical solutions to Eq. (19) subject to the boundary conditions of zero radial electron velocity, and continuity of the logarithmic derivative of potential at the tube wall. These were found to predict the experimental resonant frequency to within about 10%, comparison measurements of density being made by cavity perturbation methods.¹⁶ Similar accuracy has been obtained in comparisons with probe measurements.¹⁷ From the computed results we may deduce a modified form of Eq. (18):

$$\omega_1^2 = \frac{\omega_p^2}{k_{eff}^2} \quad (20)$$

The value of (k_{eff}/k_1) derived from the results of Nickel, Parker and Gould, is shown in Fig. 7 plotted against the dimensionless parameter (d/λ_D) , where d is the column diameter and λ_D is the electronic Debye length based on the mean electron density across the section. It is clear that the variational solution of Eq. (19) is good for (d/λ_D) of the order of 200 or above. For lower values, a correction should be made, based on a curve such as that of Fig. 7, or an improved trial function should be used in the variational expression for the frequency. It should be noted that Fig. 7 is not a universal curve, but has been given for one, typical, experimental value of k_1 only.

FIG. 7.--Variation of (k_{eff}/k_1) with (d/λ_D) .

III. THE EXPERIMENTAL METHOD

The theories discussed so far cover the case of an infinitely long discharge tube, with no axial variations of electron density, and contained within a concentric, infinitely long, metallic cylinder. A practical set-up is shown in Fig. 8a in a form suitable for measuring the axial variation of electron density along a discharge column.

A. OBSERVATIONS ON RESONANCE SHAPE

If the proportions of the exciting electrodes are such that their width is greater than their diameter, a relatively clean dipole resonance peak is usually observed with an experimental arrangement such as that shown in Fig. 8b. However, the finite nature of the system allows longitudinally-propagating modes to be excited which may show up as indicated in Fig. 9a. These can be effectively suppressed by the use of the short-circuiting rings shown in Fig. 8a. The separation of these is adjusted so as to minimize the subsidiary peaks. By this method, strip-lines with their widths equal to half their diameter, or less, may be used with less than 5% error in the resonance frequency measurements.¹⁵

When the spurious longitudinal propagation effects have been minimized, there may remain weak resonances such as that shown in Fig. 9b, due to excitation of multipole modes. These may result from asymmetry, or eccentricity of the tube, plasma, or electrodes, and are normally of no importance.

A further important series of resonances occurring at higher frequencies than that of the dipole mode is that first studied by Tonks in 1931,² and which has only recently been explained.^{16,17,19-21} Excitation of these resonances depends on the existence of nonuniform electron density and nonzero electron temperature. Their frequencies may be predicted from Eq. (19). Since the dipole resonance is normally stronger and well separated from the first resonance in this subsidiary series, there is no difficulty in recognizing it.

The Q of the dipole resonance has been predicted as (ω/v) for an infinitely long system. In practice, somewhat lower values will be obtained due to loading by the external circuit, and losses from the ends. The latter takes two forms; an

(a) STRIP - LINE GEOMETRY

(b) SCHEMATIC CIRCUIT

FIG. 8.--Typical strip-line exciting system, and experimental circuit.

(a) WITHOUT SHORT-CIRCUITING RINGS

(b) WITH SHORT-CIRCUITING RINGS

FIG. 9.--Typical dipole resonance shapes.

energy loss due to longitudinal propagation, and a further loss due to thermal and drift motions of the electrons. For the conditions under which most of the work on dipole resonance has been carried out, i.e., mercury-vapor discharges at pressures of about 1 μ Hg, these effects combine to give a Q of between 10 and 100. Power absorption at resonance is typically 40%.

B. MEASUREMENTS WITH STATIC MAGNETIC FIELDS

The experimental evidence suggests that for $\omega < (\omega_1/2)$ theory and experiment give agreement to within 10-15% with Eq. (13).^{14,15c} For higher cyclotron frequencies, however, one of the resonances disappears rapidly. A possible explanation for this is that the magnetic field has a powerful effect on the electron density profile. At higher magnetic field strengths, the droop from the value on the axis increases and for the reasons discussed earlier, the analysis may become invalid. The implication is, then, that the technique is applicable to measurements in relatively weak magnetic fields.

C. INDEPENDENT MEASUREMENTS OF ELECTRON DENSITY

Equation (18) was checked by Crawford, et al.,^{7,14,15} by cavity perturbation, longitudinal propagation, and Langmuir probe methods. Agreement between computed and measured dipole resonant frequencies was to within about 20% for measurements made with and without an axial magnetic field.

Equation (20) has been checked to within 10% by Gould, et al.,¹⁶ by reference to cavity perturbation measurements, and by Crawford¹⁷ with reference to Langmuir probe measurements, with no axial magnetic field in either case.

D. MULTIPOLE RESONANCES

These are of interest from at least two points of view. First, they allow additional comparisons with theory to be made, and second, it seems at first sight that it might be feasible to use them to determine the electron density profile in the column.⁷ Figure 10 shows electrodes that have been used to excite resonances up to the octopole mode. In using these, care must be taken not to excite additional modes. Careful balancing of the rf source is required to avoid this.

a) DIPOLE

b) QUADRUPOLE

c) HEXAPOLE

d) OCTOPOLE

FIG. 10.--Electrodes for exciting multipole resonances.

It will be realized from study of Figs. 3-6 that the multipole resonances will not be widely separated unless the electrodes fit closely round the discharge tube. Some work on density profile determination has been carried out by the author and his colleagues under these conditions, but without success. The difficulties derive from the fact that an expression of the form of Eq. (17) is an approximation only. When these frequencies are manipulated algebraically to determine the electron density profile, however, very high accuracy is demanded if a reasonable result is to be obtained. At low values of (d/λ_D) , where the discrepancy between k_m and $(k_m)_{\text{eff}}$ is high, the difficulties are aggravated, of course, and $(k_m)_{\text{eff}}$ cannot be determined without prior knowledge of the density profile! It seems then that accurate profile determination by use of several modes will not be feasible.

E. MEASUREMENTS ON FLUCTUATING DISCHARGES

The dipole resonance technique has been applied to the study of fluctuating discharges having electron density fluctuation amplitudes of the order of a few per cent.²² The method is illustrated in Fig. 11. The system is tuned to a point off resonance. Any fluctuations in number density will then appear as fluctuations in the output signal. If the resonance curve is approximately linear over the working range, a comparison calibration can be obtained by injecting a measured sinusoidal component of current, at a frequency below about 1 kc/s, and noting the corresponding output signal. The calibration is then given by

$$\frac{n_{ac}}{n_{dc}} = \frac{I_{ac}}{I_{dc}} \quad (21)$$

where n_{dc} is calculated from the dipole resonant frequency, and n_{ac} corresponds to the measured output signal.

Although the frequency sensitivity of the dipole resonance is about two orders of magnitude higher than that of a similar cavity perturbation measurement,²³ the output sensitivity is not. This factor depends on the Q of the resonance which is an order of magnitude lower for the dipole resonance system. There is still, however, a substantially higher output from the dipole resonance system.

FIG. 11.--Measurement of fluctuating density components.

IV. DISCUSSION

It will be seen from the foregoing sections that the technique described provides a simple, easily-realizable method of measurement for both dc and ac components of plasma number density. The strip-line can be used conveniently at low frequencies, where a microwave cavity might become impractically large. It has the advantage over propagation measurements that its spatial resolution is high, particularly when short-circuiting rings are used, and it does not perturb the plasma in the way that rf or Langmuir probes may.

The low-frequency limit to the method is set by the Q-value. At pressures of about 1 μ Hg, this implies a lower limit of about $10^8/\text{cm}^3$ for the electron density measurable. At the high density end, the diameter of the exciting electrodes must remain small compared to a free-space wavelength at the working frequency. This probably sets an electron density limit of about $10^{12}/\text{cm}^3$ in a 5 mm diameter tube.

A number of possible developments still remain to be examined. For example, the Q of the resonance depends on collisions, and on end losses from the exciting system. By using strip-lines of different widths and observing the variation in Q, it might be possible to measure the collision frequency, and to estimate the electron temperature in the column. An alternative method of obtaining the electron temperature might be to measure several of the additional resonances, ω_p , due to nonzero temperature, and to plot curves of (ω_p/ω_1) . If the electron density profiles are given by the "frec-fall" theory, the results of Gould, et al., could be compared with those to give an estimate of the electron temperature.

ACKNOWLEDGEMENTS

The technique described has been developed over a period of several years. During this time, major contributions have been made by Dr. G. S. Kino, and S. A. Self, and considerable assistance has been given by Dr. J. Spalter, A. B. Cannara and G. F. Haggquist. The work was supported by the U. S. Atomic Energy Commission.

LIST OF REFERENCES

1. W. R. Smythe, Static and Dynamic Electricity (McGraw-Hill Book Co., New York, 1950), 2nd ed., pp. 65-68.
2. L. Tonks, Phys. Rev. 37, 1458 (1931).
3. H. Herlofson, Arkiv för Fysik 3, 247 (1951).
4. A. Dattner, Ericsson Technics 8, 1 (1963).
5. C. E. Bryant and W. R. Franklin, Proc. Phys. Soc. 81, 531 and 790 (1963).
6. F. W. Crawford and G. S. Kino, Comptes Rendus 256, 1939 and 2798 (1963).
7. F. W. Crawford, G. S. Kino, S. A. Self, and J. Spalter, J. Appl. Phys. 34, 2186 (1963).
8. A. M. Messiaen and P. E. Vandenplas, Jour. Nucl. En. Pt. C, 4, 267 (1962).
9. F. W. Crawford, J. Nucl. En. Pt. C, 5, 69 (1963).
10. L. Tonks, Phys. Rev. 38, 1219 (1931).
11. E. Åström, Arkiv för Fysik 19, 163 (1961).
12. A. Dattner, Ericsson Technics 2, 309 (1957).
13. A. M. Messiaen and P. E. Vandenplas, Physica 28, 537 (1962); Phys. Letters 2, 193 (1962).
14. F. W. Crawford and G. S. Kino, Phys. Letters 4, 240 (1963).
15. F. W. Crawford, G. S. Kino and A. B. Cannara, J. Appl. Phys. 34 (to appear November 1963).
16. J. C. Nickel, J. V. Parker and R. W. Gould, Phys. Rev. Letters 11, 183 (1963).
17. F. W. Crawford, Microwave Laboratory Report No. 1086, Stanford University (September 1963).
18. J. V. Parker, California Institute of Technology, Electron Tube and Microwave Laboratory Technical Report No. 19, Contract Nonr 220(13), (December 1963).
19. P. Weissglas, Phys. Rev. Letters 10, 206 (1963).
20. F. W. Crawford, Phys. Letters 5, 244 (1963); Microwave Laboratory Report No. 1045, Stanford University (June 1963).
21. F. W. Crawford and G. S. Kino, Proc. Vith International Conference on Ionization Phenomena in Gases, Paris (July 1963) (in press).
22. F. W. Crawford, G. S. Kino, S. A. Self and J. Spalter, J. Appl. Phys. 34, 2196 (1963).
23. F. W. Crawford and J. D. Lawson, J. Nucl. En. Pt. C, 3, 179 (1961).