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ENGINEERING APPLICATIONS OF ANALOG COMPUTERS

by

Lawrence T. Bryant, Marion J. Janicke, Louis C. Just, and Alan L. Winiecki

February 1961

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</tr>
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</tr>
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<td>Temperature Distribution for an Infinite Slab - Case III</td>
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<td>Temperature Distribution for an Infinite Slab - Case IV</td>
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</tbody>
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ENGINEERING APPLICATIONS OF
ANALOG COMPUTERS

by

Lawrence T. Bryant, Marion J. Janicke,
Louis C. Just, and Alan L. Winiecki

INTRODUCTION

This publication is an extension of Bryant, L. T., Just, L. C., and Pawlicki, G. S., Introduction to Electronic Analog Computing, ANL-6187 (July 1960). Six experiments are presented from the fields of reactor engineering, heat transfer, and dynamics.

The mathematical representation for most of these experiments is in the form of nonlinear differential equations. In usual practice simplifying assumptions are introduced to linearize the equations. This linearization may alter the mathematical model sufficiently to cast doubt upon its applicability. If an analog computer is available, the nonlinear equation may be solved directly.

The presentation of these experiments has been designed to provide insight into physical phenomena and their mathematical representation. The steps required for producing the analog solution will be shown, as well as complete information for duplicating the solution. Graphical results are provided.

The format of each experiment will be:

1. Description of the problem

2. Mathematical statement of the problem including:
   a. Constants
   b. Initial Conditions

3. Preparation of machine equations
   a. Machine Variables
   b. Scale Factors

4. Analog circuit diagram
   a. Flow Sheet
   b. Potentiometer setting sheet
   c. Static Check sheet

5. Graphical representation of the solution.

6. Bibliography
I. DECELERATION OF A REACTOR CONTROL ROD

1. Problem Description

When a control rod is suddenly inserted or rejected from the core of a reactor, the rapid motion is quickly dampened by a dashpot or buffer mechanism, usually consisting of a hydraulic system which prevents sudden shock of the control drive mechanisms.

Constant deceleration-type dashpots give the most favorable characteristics for protection against shock loads. Essentially, a piston moves through oil, and the oil is squeezed into small clearances; this process in turn develops large amounts of frictional resistance. This friction, which is proportional to the speed of the moving piston, instigates the retarding force which slowly stops the motion of the control drive.

This hydraulic drag and the ensuing kinetic energy dissipation are frequently described by differential equations. Elias' equation of buffer motion(I-1) is given by

\[
\frac{dV_p}{dX} = - \frac{2 \mu \pi D_p^2 L_d^2 X}{W(L_d C - CX)^2}
\]

Various plots of the buffer characteristics are shown on Figs. 2, 3, 4, and 5.

Many parameters may be investigated before the design conditions for a particular problem are satisfied.(I-2,I-3)

2. Mathematical Statement of the Problem

a. Equations:

\[
\frac{dV_p}{dX} = - \frac{2 \mu \pi D_p^2 L_d^2 X}{W(L_d C - CX)^2}
\]

b. Constants and Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_p)</td>
<td>velocity into the dashpot</td>
<td>Variable</td>
<td>ft/sec</td>
</tr>
<tr>
<td>(D_p)</td>
<td>diameter of the dashpot</td>
<td>2</td>
<td>inches</td>
</tr>
<tr>
<td>(C)</td>
<td>dashpot clearance</td>
<td>0.03</td>
<td>inch</td>
</tr>
<tr>
<td>(L_d)</td>
<td>dashpot length</td>
<td>6</td>
<td>inches</td>
</tr>
<tr>
<td>(\mu)</td>
<td>viscosity of the dashpot fluid</td>
<td>Variable</td>
<td>lb/(ft)(sec)</td>
</tr>
<tr>
<td>(W)</td>
<td>weight of the control rod</td>
<td>200</td>
<td>lb</td>
</tr>
<tr>
<td>(X)</td>
<td>distance into the dashpot</td>
<td>6</td>
<td>inches</td>
</tr>
</tbody>
</table>

*References in each section are given at the end of each section.*
c. Initial Conditions

At \( t = 0 \):

\[ X = 0 \]

\[ \frac{dX}{dt} = 70 \text{ ft/sec} \]

\[ \frac{dV_p}{dt} = 0 \]

d. Analysis of Equations

Since

\[
\frac{d^2V_p}{dx} = \frac{dV_p}{dt} = \frac{d^2X/dt^2}{dx/dt},
\]

the original equation may be restated as

\[
\frac{d^2X}{dt^2} = \left( \frac{2 \mu \pi D_p^2 L_d^2}{WC^2} \right) \left( \frac{X}{dx/dt} \right) \left( \frac{1}{L_d - X} \right)^2.
\]

3. Preparation of Machine Equations

a. Machine Variables and Scale Factors

\[ X' = bX \quad ; \quad a = 10^3 \]

\[ t' = at \quad ; \quad b = 10^2 \]

b. Scaled Equations

\[
\frac{d^2X'}{dt'^2} = - \frac{b}{a^2} \left( \frac{2 \mu \pi D_p^2 L_d^2}{WC^2} \right) \left( \frac{X'}{b} \right) \left( \frac{dX'}{dt'} \right) \left( \frac{a}{b} \right) \left( \frac{b}{L_d b - X'} \right)^2
\]

\[ = - \frac{b}{a} \left( \frac{2 \mu \pi D_p^2 L_d^2}{WC^2} \right) \left( X' \frac{dX'}{dt'} \right) \left( \frac{1}{L_d b - X'} \right)^2. \] ... (5)

c. Machine Equation

When the values of constants and scale factors are introduced into Eq. (5), the machine equation results:

\[
\frac{d^2X'}{dt'^2} = -(5.07\mu) X' \left( \frac{dX'}{dt'} \right) \left( \frac{1}{50 - X'} \right)^2.
\]... (6)
The initial conditions (in terms of voltages) are:

\[ X' = bX(0) = 0 \]

\[ \frac{dX'}{dt} = \frac{b}{a} \frac{dX}{dt} = 7.0 \text{ volts} \]

\[ \frac{d^2X'}{dt'^2} = \frac{b}{a^2} \frac{d^2X}{dt^2} = 0 \]

4. Analog Circuit Diagram

a. Flow Sheet

Fig. 1. Circuit Diagram for Solution of Elias' Equation of Buffer Motion
### ANALOG COMPUTER

#### POTENTIOMETER SETTINGS

<table>
<thead>
<tr>
<th>POTENTIOMETER NO.</th>
<th>DRAWING NO.</th>
<th>MACHINE</th>
<th>V_p(0) (volts)</th>
<th>7.00</th>
<th>0700</th>
<th>0700</th>
<th>Correction</th>
<th>Setting</th>
<th>SET</th>
<th>PARAMETERS</th>
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<td>1</td>
<td>1</td>
<td></td>
<td>V_p(0) (volts)</td>
<td>7.00</td>
<td>0700</td>
<td>0700</td>
<td></td>
<td></td>
<td></td>
<td>a = 10^3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>X (volts)</td>
<td>-50.00</td>
<td>5000</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
<td>b = 10^2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td>0.2</td>
<td>0.2000</td>
<td>2000</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td>V_p(0) = 70 ft/sec</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td>5 / 100 (5.07 μ)</td>
<td>0.0125</td>
<td>0125</td>
<td>0125</td>
<td></td>
<td></td>
<td></td>
<td>D_p = 2 in</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>μ_1 = 0.0494</td>
<td>0.0125</td>
<td>0125</td>
<td>0125</td>
<td></td>
<td></td>
<td></td>
<td>C = 0.03 in</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>μ_2 = 0.0795</td>
<td>0.0202</td>
<td>0202</td>
<td>0202</td>
<td></td>
<td></td>
<td></td>
<td>L_d = 6 in</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>μ_3 = 0.102</td>
<td>0.0258</td>
<td>0258</td>
<td>0258</td>
<td></td>
<td></td>
<td></td>
<td>W = 200 lb</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X = 6 in</td>
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For Figs. 2 & 5

For Figs. 3 & 4
### Reactor Control Rod Deceleration

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<tr>
<th>UNIT</th>
<th>UNIT NUMBER</th>
<th>OUTPUT (VOLTS)</th>
<th>REMARKS</th>
<th>INTEGRATOR</th>
<th>INITIAL CONDITION</th>
<th>SET</th>
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<tr>
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<td></td>
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<td></td>
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<tr>
<td></td>
<td>2</td>
<td>-70.00</td>
<td></td>
<td></td>
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<td></td>
<td>3</td>
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<td></td>
<td></td>
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<td>FOR STATIC CHECK</td>
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<tr>
<td></td>
<td>4</td>
<td>-1.40</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>5</td>
<td>+0.02</td>
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<td></td>
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<td>AMP</td>
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<td>-0.02</td>
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<td>MULT</td>
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<td>3</td>
<td>+0.88</td>
<td></td>
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\[ \mu_z = 0.0795 \]
5. Graphical Results

**Fig. 2**
Velocity Versus Distance for Various Viscosities

**Fig. 3**
Velocity Versus Distance for Various Initial Velocities. Viscosity is 0.0795 Pound Force per Foot-Second

**Fig. 4**
Distance Versus Time for Various Initial Velocities
6. Bibliography


II. PRESSURE VARIATIONS THROUGH A PACKED BED

1. Problem Description

The future applications of nuclear power sources will depend on whether reactor technology can match the demand for higher power densities and higher operating temperatures. A popular concept for advanced application is the packed bed reactor. \(^{(II-1)}\) \(^{(II-2)}\) Equations of fluid flow and heat transfer for this concept are dependent upon the particular packed-bed system, particle shape, and the fluid for which they are developed.

The solutions to problems for this type of reactor design are usually obtained through use of empirical corrections. \(^{(II-3)}\) An equation derived by MacFarlane \(^{(II-4)}\) from the basic Bernoulli equation illustrates a fundamental method for calculating the performance of packed bed arrangements. This relationship

\[
\frac{dP}{dx} = \frac{(K + Hx)P}{E + Dx - P^2}
\]

expresses in differential form the variation of pressure and distance of a packed bed one square foot in cross section. MacFarlane also indicates four other general methods used for calculating laminar fluid flow in packed beds and describes their derivation.

2. Mathematical Statement of the Problem

a. Equations and Constants

\[
\frac{dP}{dx} = \frac{(K + Hx)P}{E + Dx - P^2}
\]

\(D = \frac{Q \rho_0 G}{g_c \rho_0 T_0 C_p} = 1.73 \times 10^4 \frac{lb^2}{ft^5}\)

\(G = \frac{fG^2 P_0}{2g_c D_p \rho_0} = 3.439 \times 10^8 \frac{lb^2}{ft^5}\)

\(H = \frac{fGQ P_0}{2g_c D_p \rho_0 T_0 C_p} = 5.22 \times 10^9 \frac{lb^2}{ft^6}\)

\(E = G^2 P_0 / g_c \rho_0 = 1130 \frac{lb^2}{ft^6}\)

\(K = D + G = 3.44 \times 10^8 \frac{lb^2}{ft^5}\)
b. Initial Conditions

\[ P_0 = 2.12 \times 10^4 \text{ lb/ft}^2 \]  
\[ x_0 = 0 \]  
\[ L = 0.2 \text{ ft.} \]

3. Preparation of the Machine Equations

a. Machine Variables and Scale Factors

\[ x = t \quad x(\text{final}) = t(\text{final}) \]
\[ t' = at \quad dt' = adt \quad a = 10^2 \]
\[ P' = bP \quad dP' = bdP \quad b = 10^{-3} \]
\[ P'_0 = bP_0 = (10^{-3})(2.12 \times 10^4) = 21.2 \text{ volts,} \]
\[ t'(\text{final}) = at(\text{final}) = 10^2 (0.2) = 20.0 \text{ volts} \]

b. The Scaled Equation

Substituting equations (2) into equation (1), the scaled equation is obtained:

\[ \frac{dP'}{dt'} = \frac{1}{a} \left\{ \frac{[K + (H/a) t'] P'}{E + (Dt'/a) - (P'^2/b^2)} \right\} \quad (3) \]
c. The Machine Equation

When numerical values are placed into equation (3), the machine equation (4) results:

\[
\frac{dP'}{dt'} = \frac{(3.44 + 0.522 t')P'}{0.0002 t' - P'^2}
\]  

(4)

4. Analog Circuit Diagram

a. Flow Sheet

![Circuit Diagram](image)

Fig. 6. Circuit Diagram for the Solution of MacFarlane's Equation
### b. POTENTIOMETER SETTINGS

<table>
<thead>
<tr>
<th>POTENTIOMETER NO.</th>
<th>DRAWING</th>
<th>MACHINE</th>
<th>MATHEMATICAL VALUE</th>
<th>VALUE</th>
<th>CORRECTION</th>
<th>SETTING</th>
<th>SET</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01</td>
<td>1</td>
<td>0.01a volt</td>
<td>-1.00</td>
<td></td>
<td>0100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>2</td>
<td>P₀b volt</td>
<td>+21.2</td>
<td></td>
<td>2120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>00</td>
<td>3</td>
<td>b²D/a</td>
<td>0.0002</td>
<td></td>
<td>0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>52</td>
<td>4</td>
<td>b²H/a²</td>
<td>0.5220</td>
<td></td>
<td>5220</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>03</td>
<td>5</td>
<td>b²K/a</td>
<td>3.44</td>
<td></td>
<td>0344</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>31</td>
<td>6</td>
<td>√10</td>
<td>3.16a</td>
<td></td>
<td>316a (10)</td>
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<tr>
<td>7</td>
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<td>7</td>
<td>aL</td>
<td>-20.00</td>
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### c. STATIC CHECK

<table>
<thead>
<tr>
<th>UNIT</th>
<th>UNIT NUMBER</th>
<th>OUTPUT (VOLTS)</th>
<th>REMARKS</th>
<th>INTEGRATOR</th>
<th>INITIAL CONDITION</th>
<th>SET</th>
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5. Graphical Results

![Graphical Results](image)

Fig. 7. Pressure Versus Distance

6. Bibliography


III. REACTOR KINETICS OVER MANY DECADES WITH THERMAL FEEDBACK (SIMULATION OF A TREAT TRANSIENT)

1. Problem Description

The TREAT reactor was designed to generate a very large, transient, thermal flux field of short duration. The maximum integrated flux is greater than $10^{15}$ neutrons/cm$^2$.

The core is a dispersion of highly enriched uranium (as the oxide or carbide) in a graphite matrix. The graphite serves as a moderator, a heat sink, and a generator of a sizeable negative temperature coefficient. The latter effect is due to the fact that the energy of the thermal neutrons increases with graphite temperature thus causing an increase in the leakage probability.

The purpose of this experiment is to simulate a TREAT transient initiated by control rod withdrawal and terminated by the negative temperature coefficient. Since a large excursion is expected, the reactor kinetics equations will be transformed by a substitution.

$$\eta = \ln \frac{n(t)}{n(0)}$$

The equations describing the neutron kinetics (with 6 delayed groups) will be solved on the analog computer. They will be forced by changes in $K_{ex}$.

2. Mathematical Statement of the Problem

a. Equations:

$$\frac{d\eta}{dt} = \frac{\beta}{\xi} \left[ (1 - \beta)K_{1ex} + \sum_{i=1}^{6} \frac{\beta_i}{\beta} \psi_i \right]$$

$$\frac{d\psi_i}{dt} = \lambda_i \beta K_{1ex} - \frac{d\eta}{dt} - \psi_i \left( \lambda_i + \frac{d\eta}{dt} \right) \quad i = 1, \ldots, 6,$$

where

$$\eta = \ln \frac{n(t)}{n(0)}$$

$$\epsilon_i = e^{-\eta} \frac{C_i(t)}{C_i(0)}$$

$$\psi_i = \epsilon_i - 1$$

$$K_{1ex} = K_{ex}/\beta$$
b. **Constants**

\[
\begin{align*}
\varphi &= 0.00755 \\
\xi &= 8.6 \times 10^{-4}
\end{align*}
\]

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<td>6</td>
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c. **Initial Conditions**

\[
\begin{align*}
\eta(0) &= 0 \\
\psi_i &= 0 \\
K_{lex} &= 0
\end{align*}
\]

3. **Preparation of Machine Equations**

a. **Machine Variables**

\[
\begin{align*}
t' &= at \\
\eta' &= b\eta \\
K_{lex}' &= cK_{lex} \\
\psi_i' &= d_i\psi_i
\end{align*}
\]

b. **Scale Factors**

\[
\begin{align*}
a &= 10 \\
b &= 2 \\
c &= 25 \\
d_1 = d_2 = \ldots = d_6 &= 2
\end{align*}
\]
c. Scaled Equations

\[
\frac{d\eta_i}{dt'} = \beta \frac{b(1 - \beta)}{a} K_{\text{lex}} + \sum_{i=1}^{6} \frac{b\beta_i}{d_\beta} \psi_i
\]

\[
\frac{d\psi_i}{dt'} = \frac{d_i \lambda_i \beta}{ac} K_{i\text{ex}}' - \frac{di}{b} \frac{d\eta_i}{dt'} - \psi_i \left(\frac{\lambda_i}{a} + \frac{1}{b} \frac{d\eta_i}{dt'}\right) \quad i = 1, \ldots, 6.
\]

d. The Generation of \( K_{i\text{ex}} \)

The expression for \( K_{\text{ex}} \) is made up of two parts: the contribution of the control rod and the contribution of the negative temperature coefficient, that is,

\[ K_{\text{ex}} = K(t) - K(n,t) \]

where

\[ K(t) = \begin{cases} 0.04t & \text{for } 0 \leq t \leq 0.5 \text{ sec} \\ 0.02 & \text{for } t > 0.5 \text{ sec} \end{cases} \]

and

\[ K(n,t) = 10^{-10} \int n \, dt \]

Since

\[ K_{i\text{ex}}' = \frac{c}{\beta} K_{\text{ex}} \]

then

\[ K_{i\text{ex}}' = \frac{c}{\beta} K(t) - \frac{c}{\beta} K(n,t) \]

or, more simply,

\[ K_{i\text{ex}}' = K_i'(t) - K_i'(n,t) \]

\[ K_i'(t') = \begin{cases} 0.04 \frac{c}{\beta} \frac{t'}{a} & \text{for } 0 \leq t' \leq 0.5 \text{a sec} \\ 0.02 \frac{c}{\beta} & \text{for } t' > 0.5 \text{a sec} \end{cases} \]

\( K_i'(t') \) can be generated by means of an integrator and a relay.
The generation of $K_i(n,t)$ is more complicated: a function generator is needed. If the machine variables and scale factors are substituted into

$$K(n,t) = 10^{-10} \int ndt,$$

the result is

$$K_i(n,t) = \frac{c}{\beta a} 10^{-10} \int n dt'.$$

But

$$n = n_1 n(0);$$

therefore

$$K_i(n,t) = \frac{c n(0)}{\beta a} 10^{-10} \int n_1 dt.$$

The analog computer will supply $\eta = ln n_1$ (due to a change in variable) and since $e^{ln n_1} = n_1$,

$$K_i(n,t) = \frac{c n(0)}{\beta a} 10^{-10} \int e^{\eta} dt'.$$

After the terms are collected,

$$K_i(n,t) = c \int e^{\alpha} dt',$$

where

$$\alpha = \eta + ln n(0) - ln 10^{10} - ln a \beta.$$

For the values given the constants, and for $n(0) = 10^2$,

$$\alpha = \eta - 15.836.$$

Then $25 e^\alpha$ will be generated with a diode-function generator (DFG). In order to decrease the slope of the function, the DFG will be driven by $10\eta - 100$. 
In the actual experiment, the input to integrator 3 is removed (by means of a relay) until \(25e^\alpha = 0.01\) volt.

### DFG DATA

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<th>(\alpha)</th>
<th>(25e^\alpha)</th>
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<td>-100</td>
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### e. Machine Equations

\[
\frac{d\eta'}{dt'} = 0.0697 K_{\text{lex}} + 0.3512 (0.0331\psi_1' + 0.2199\psi_2')
\]

\[
+ 0.2821\psi_3' + 0.3192\psi_4' + 0.1126\psi_5' + 0.0331\psi_6')
\]

\[
\frac{d\psi_1'}{dt'} = - \frac{d\eta'}{dt'} - 0.0013\psi_1' - 0.5\psi_1' \frac{d\eta'}{dt'}
\]

\[
\frac{d\psi_2'}{dt'} = - \frac{d\eta'}{dt'} - 0.0032\psi_2' - 0.5\psi_2' \frac{d\eta'}{dt'}
\]

\[
\frac{d\psi_3'}{dt'} = - \frac{d\eta'}{dt'} - 0.0154\psi_3' - 0.5\psi_3' \frac{d\eta'}{dt'}
\]

\[
\frac{d\psi_4'}{dt'} = - \frac{d\eta'}{dt'} - 0.0456\psi_4' - 0.5\psi_4' \frac{d\eta'}{dt'}
\]
\[ \frac{d\psi_5}{dt'} = 0.0001 K_{ex}^i - \frac{d\eta_1'}{dt'} - 0.1612 \psi_5' - 0.5\psi_5' \frac{d\eta_1'}{dt'} \]

\[ \frac{d\psi_6}{dt'} = 0.0009 K_{ex}^i - \frac{d\eta_1'}{dt'} - 1.43\psi_6' - 0.5\psi_6' \frac{d\eta_1'}{dt'} \]

\[ K_{ex}^i = K_i^i(t) - K_i^i(n,t) \]

\[ K_i^i(t) = \begin{cases} 13.25 & t' \text{ volts for } 0 \leq t' \leq 5 \text{ sec} \\ 66.23 & t' > 5 \text{ sec} \end{cases} \]

\[ K_i^i(n,t) = 25 \exp (\eta - 15.836) \]

4. **Analog Circuit Diagram**

   a. **Flow Sheet**

   ![Circuit Diagram](Image)

   **Fig. 8. Circuit Diagram for Duplication of TREAT Transient**
### Problem No. 4

#### The Reactor

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<td>( \beta = 8.6 \times 10^{-4} )</td>
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**ALL MULTIPLIER CHANNELS = +5 volts**
5. **Graphical Results**

![Graphical Results](image)

Fig. 9

\[ K_{ex} \text{ and } \eta = \ln n, \]

*Versus Time*

6. **Bibliography**


IV. A VIBRATING SYSTEM WITH TWO DEGREES OF FREEDOM

1. Problem Description

Problems of vibration must be considered in the design of power plants using fissionable fuel. Fuel elements, control rods, and structural supporting members are capable of vibrating; their characteristics must be analyzed, for vibration problems prove to be of importance to eliminate concern for the safe operation of the power plant. Good representations of the true situation usually involve systems with several degrees of freedom. (IV-1)

A typical vibration problem which will serve as an introduction to multi-degree-of-freedom systems is shown in Fig. 10. The two masses $m_1$ and $m_2$ are suspended vertically by springs $k_1$ and $k_2$. The masses are constrained such that they only move vertically. The displacements $x_1$ and $x_2$, taken positive for a downward motion, are measured using static equilibrium as reference. The elongation of the upper spring is $x_1$ and the elongation of the lower spring is $(x_2 - x_1)$. The restoring force acting on $m_1$ is $[-k_1x_1 + k_2(x_2 - x_1)]$, and on $m_2$ the restoring force is $-k_2(x_2 - x_1)$, where $k_1$ and $k_2$ are the spring constants of the respective springs.

Effects due to energy dissipation in the elastic spring, wind friction, and springs that have appreciable mass have been neglected in the equations of motion given below.

\[
m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1) \tag{1}
\]


a. Equations
\[ m_2 \frac{d^2x_2}{dt^2} = -k_2(x_2 - x_1) \]  

(2)

b. **Equation Constants**

\( m_i \): mass (lb) \( (i = 1, 2) \)

\( k_i \): spring constant (lb force/ft) \( (i = 1, 2) \)

\( A \): Initial displacement of the springs (feet)

c. **Initial Conditions**

It is obvious that with the springs displaced a certain distance, \( A \), the initial conditions will have the following values

\[ x_1(0) = x_2(0) = A \]

\[ \frac{dx_1(0)}{dt} = \frac{dx_2(0)}{dt} = 0 \]  

(3)

\[ \frac{d^2x_1(0)}{dt^2} = \frac{d^2x_2(0)}{dt^2} = 0 \]

3. **Preparation of Machine Equations:**

In transforming to the machine equations, the following relationships are made. Let

\[ x'_i = b x_i \quad (i = 1, 2) \]

and

\[ t' = a t \]

Substitution of equations (4) into equations (1) and (2) yields

\[ \frac{d^2x'_1}{dt'^2} = - \frac{k_1}{a^2 m_1} x'_1 + \frac{k_2}{a^2 m_1} (x'_2 - x'_1) \]  

(5)

\[ \frac{d^2x'_2}{dt'^2} = \frac{k_2}{a^2 m_2} (x'_2 - x'_1) \]  

(6)
where

\[ x_1(0) = bx_1(0) = bA \]

and

\[ x_2(0) = bx_2(0) = bA \]

The solution to the equations will vary with \( m_1, m_2, k_1, k_2, \) and \( A \). In the solution given here, we consider the following physical constants:

\[ k_1 = k_2 = 0.2 \text{ lb force/ft} \]

\[ m_1 = m_2 = 1 \text{ lb mass} \]

\[ A = 1 \text{ ft} \]

4. Analog Circuit Diagram

a. Flow Sheet

![Circuit Diagram](image)

Fig. 11. Circuit Diagram for the Solution of the Equations Describing a Vibrating System with Two Degrees of Freedom
### b. POTENTIOMETER SETTINGS

A VIBRATION SYSTEM WITH TWO DEGREES OF FREEDOM

<table>
<thead>
<tr>
<th>POTENTIOMETER NO.</th>
<th>MATHEMATICAL VALUE</th>
<th>VALUE</th>
<th>CORRECTION</th>
<th>SETTING</th>
<th>SET</th>
<th>PARAMETERS</th>
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<tr>
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### c. STATIC CHECK

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<th>REMARKS</th>
<th>INTEGRATOR</th>
<th>INITIAL CONDITION</th>
<th>SET</th>
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</table>
5. Graphical Results

![Graphical Results](image)

**Fig. 12**
Distance Versus Time for a System with Two Degrees of Freedom

6. Bibliography

V. TEMPERATURE DISTRIBUTION IN A RADIATING FIN

1. Problem Description

The only economical method for rejecting heat from an outer-space power plant is by thermal radiation. If the working fluid of the power plant passes through tubes, the additions of extended surfaces to the tubes in the form of fins reduces the number of tubes required. This reduction decreases the probability that a meteor will puncture a vital coolant-carrying passageway. (The puncture of a fin is of lesser concern for the continued operation of the power plant.) An analysis of the temperature distribution of these extended surfaces is very important in calculating the effectiveness (and, indirectly, the safety of the plant) of various fin geometries.

2. Mathematical Statement of the Problem

In the development of a differential equation for conduction,

\[ dq = d/dx \left( 2kW Y_x \frac{dT}{dx} \right) \]  

(1)

A general heat balance requires this differential equation (1) to be equal to

\[ dq = 2\sigma \varepsilon \left( T^4 - T_s^4 \right) dA \]  

(2)

the heat rejected by radiation.

By assuming that the arc length (ds) on the arbitrary surface is equivalent to dx on the abscissa and assigning \( Y_x \) equal to a constant thickness for a straight fin geometry, the differential equation for temperature is

\[ \frac{d^2T}{dx^2} = \frac{\sigma \varepsilon}{Hk} \left( T^4 - T_s^4 \right) \]  

(3)

A constant heat source will be assumed at one end of the fin and

\[ \frac{dT}{dx} \bigg|_{x=L} = 0 \]  

at the other end. This will correspond to the fin in Fig. 13.

![Fig. 13](image)

Geometry of Radiation Fin and Coolant Tubes
a. **Constants and Variables**

- \( T \) = Absolute temperature along the fin
- \( T_s \) = Absolute temperature of the sink
- \( \sigma \) = Stefan-Boltzmann constant
- \( \varepsilon \) = Emissivity
- \( W \) = Width of the fin in the z-direction
- \( L \) = Total length of the fin in the x-direction
- \( q \) = Heat dissipated
- \( H \) = Half-thickness of the fin
- \( k \) = Thermal conductivity of the fin material

Typical values are:

- \( T_s = 0^\circ R \)
- \( \sigma = 0.173 \times 10^{-8} \text{ BTU/(hr)(ft}^2)(^\circ R^4) \)
- \( \varepsilon = 0.9 \)
- \( W = 1.0 \text{ ft} \)
- \( L = 0.25 \text{ ft} \)
- \( H = 1.250 \times 10^{-3} \text{ ft} \)
- \( k = 25.0 \text{ BTU/(hr)(ft)(^\circ R)} \)

b. **Initial Conditions**

\( T(0) = 2000^\circ R \)

The most efficient use of radiator material weight dictates the arrangement of the finned tubes in a straight bank. The general temperature distribution, of this arrangement, along the fin is given in Fig. 15.

3. **Preparation of Machine Equations**

a. **Machine Variables and Scale Factors.**

\[
\begin{align*}
x' &= ax \\
T' &= bT \\
x^2 &= a^2 dx^2 \\
dT' &= bdT \\
d^2T' &= bd^2T \\
a &= 10^2 \\
b &= 5 \times 10^{-2}
\end{align*}
\]
b. Scaled Equation

Substituting Equations (5) into Equation (3) we get,

\[
\frac{d^2 T'}{dt'^2} = \frac{\sigma \varepsilon T'^4}{a^2 b^3 kH} \quad \text{(Note } T_S = 0.)
\]

c. Machine Equation

\[
\frac{d^2 T'}{dt'^2} = 0.03986 \left( \frac{T'^4}{10^5} \right)
\]

d. Initial Conditions

\[ T' = bT = 100 \text{ volts} \]

\[ \frac{dT'}{dt'} = Y \text{ volts} \]

so that

\[ \left. \frac{dT'}{dt'} \right|_{t=aL} = 0 \]

4. Analog Circuit Diagram

a. Flow Sheet

![Flow Sheet](image)

Fig. 14. Circuit Diagram for Solution of Second-order, Fourth-degree Differential Equation
### b. POTENTIOMETER SETTINGS

<table>
<thead>
<tr>
<th>POTENTIOMETER NO.</th>
<th>MATHEMATICAL VALUE</th>
<th>VALUE</th>
<th>CORRECTION</th>
<th>SETTING</th>
<th>SET</th>
<th>PARAMETERS</th>
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<td>(-\frac{dT}{dt}) volts</td>
<td>*</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(10^6 \sigma c/\alpha^2 b^3 kH)</td>
<td>0.03986</td>
<td></td>
<td>0399</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Variable to produce \(\frac{dT}{dx}\) = 0 at \(x = L\)

### c. STATIC CHECK

<table>
<thead>
<tr>
<th>UNIT</th>
<th>UNIT NUMBER</th>
<th>OUTPUT (VOLTS)</th>
<th>REMARKS</th>
<th>INTEGRATOR</th>
<th>INITIAL CONDITION</th>
<th>SET</th>
<th>PARAMETERS</th>
</tr>
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<td>-100v</td>
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<td></td>
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<tr>
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<td>+3.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Graphical Results

Fig. 15. Temperature Versus Length and $dT/dx$ Versus Length for a 0.25-ft Fin ($K = 25.0$)

6. Bibliography


VI. TEMPERATURE DISTRIBUTION IN AN INFINITE SLAB CONSIDERING VARIABLE THERMAL PROPERTIES

1. Problem Description

When the thermal properties of various materials are studied, thermal conductivity, specific heat and density are usually considered as constants; they are, however, dependent upon temperature.\(^{(VI-1)}\) In this experiment, an insulated zirconium slab is studied. Four cases are considered:

1. Diffusivity \((\kappa = k/\rho c)\) is constant;

2. \(\kappa = F\left(\frac{1}{5} \sum_{i=1}^{5} T_i\right)\);

3. \(\kappa = F\) (Temperature of the region described by the heat balance);

4. \(\kappa = F\) (Average temperature across an interface).

2. Mathematical Statement of the Problem\(^{(VI-2)}\)

a. Equations

Fig. 16. Model of the Infinite Slab Showing Regions Used for Analysis
\[
\frac{dT_1}{dt} = \frac{S}{\rho c\Delta x} - \frac{k}{\Delta x^2} (T_1 - T_2)
\]
\[
\frac{dT_2}{dt} = \frac{k}{\Delta x^2} (T_1 - T_2) - \frac{k}{\Delta x^2} (T_2 - T_3)
\]
\[
\frac{dT_3}{dt} = \frac{k}{\Delta x^2} (T_2 - T_3) - \frac{k}{\Delta x^2} (T_3 - T_4)
\]
\[
\frac{dT_4}{dt} = \frac{k}{\Delta x^2} (T_3 - T_4) - \frac{k}{\Delta x^2} (T_4 - T_5)
\]
\[
\frac{dT_5}{dt} = \frac{k}{\Delta x^2} (T_4 - T_5) - \frac{\varepsilon \sigma}{\rho c \Delta x} (T_5^4 - T_0^4).*
\]

b. Constants

(1) Constant case

- \(k\) = thermal conductivity = 11 BTU/(hr)(ft)(°F)
- \(c\) = specific heat = 0.066 BTU/(lb)(°F)
- \(\rho\) = density = 0.397 lb/ft³
- \(\kappa = k/c\rho\) = diffusivity = 0.4198 ft²/hr
- \(S\) = heat source = 18.3 BTU/(ft²)(sec)
- \(\varepsilon\) = emissivity
- \(\sigma\) = Stephan-Boltzmann constant

\(\Delta x = 1/60\) ft

(2) As a function of temperature

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<th>(T, \circ F)</th>
<th>(\kappa(T))</th>
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<tr>
<td>200</td>
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<tr>
<td>700</td>
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<tr>
<td>800</td>
<td>0.309</td>
</tr>
<tr>
<td>900</td>
<td>0.298</td>
</tr>
</tbody>
</table>

*Radiation heat loss will be neglected.
c. **Initial Conditions**

\[ T_1 = T_2 = T_3 = T_4 = T_5 = 100^\circ F \]

3. **Preparation of Machine Equations**

a. **Machine Variables**

\[ t' = aT \]
\[ T' = bT \]
\[ S'' = \Delta xSb/k \]

b. **Scale Factors**

\[ a = 1.0 \]
\[ b = 0.1 \]

c. **Machine Equations**

\[
\frac{dT_1'}{dt'} = \frac{\kappa S''}{\Delta x^2a} - \frac{\kappa}{\Delta x^2a} (T_1' - T_2')
\]

\[
\frac{dT_2'}{dt'} = \frac{\kappa}{\Delta x^2a} (T_1' - T_2') - \frac{\kappa}{\Delta x^2a} (T_2' - T_3')
\]

\[
\frac{dT_3'}{dt'} = \frac{\kappa}{\Delta x^2a} (T_2' - T_3') - \frac{\kappa}{\Delta x^2a} (T_3' - T_4')
\]

\[
\frac{dT_4'}{dt'} = \frac{\kappa}{\Delta x^2a} (T_3' - T_4') - \frac{\kappa}{\Delta x^2a} (T_4' - T_5')
\]

\[
\frac{dT_5'}{dt'} = \frac{\kappa}{\Delta x^2a} (T_4' - T_5')
\]

d. **Initial Conditions**

\[ T_1' = T_2' = \ldots T_5' = 10 \text{ volts} \]

\[ S'' = 10 \text{ volts} \] [\( S = 18.3 \text{ BTU/(ft}^2)(\text{sec}) \)] for 50 sec
4. Analog Circuit Diagrams

a. Flow Sheets

Case I - (κ = constant)

Fig. 17. Circuit Diagram for an Infinite Slab with Thermal Conductivity κ = Constant

Fig. 18. Relay Circuit for the Heat Pulse Used in Experiment VI
Case II - [$\kappa = F(T_{av})$]

Fig. 19. Circuit Diagram for an Infinite Slab with Thermal Conductivity $\kappa = F(T_{average})$
Case III - \( \kappa = F(\text{Temperature of the Region Described by the Heat Balance}) \)

Fig. 20. Circuit Diagram for an Infinite Slab with Thermal Conductivity \( \kappa = F(\text{Temperature of Region Described by the Heat Balance}) \)

Case IV - \( \kappa = F(\text{Average Temperature Across an Interface}) \)

Fig. 21. Circuit Diagram for an Infinite Slab with Thermal Conductivity \( \kappa = F(\text{Average Temperature Across an Interface}) \)
## Temperature Variation in a One Face Insulated Slab Considering Variable Thermal Properties

### Case I

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<th>SETTING</th>
<th>SET</th>
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<td></td>
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<td>S = 18.3</td>
</tr>
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<td></td>
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<td>S'' = 10 volts</td>
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<td></td>
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<td>b = 0.1</td>
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<td></td>
<td>( \kappa = \frac{0.4198}{3600} )</td>
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<tr>
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**Slrgonne National Laboratory**  
**APPLIED MATHEMATICS DIVISION**  
**ANALOG COMPUTER**

**b. POTENTIOMETER SETTINGS**

TEMPERATURE VARIATION IN A ONE FACE INSULATED SLAB  
CONSIDERING VARIABLE THERMAL PROPERTIES  
CASE III - IV

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### Temperature Variation in a One Face Insulated Slab Considering Variable Thermal Properties

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5. **Graphical Results**

**Case I - ($\kappa = \text{constant}$)**

Heat Input = 18.3 BTU/ft$^2$sec for 50 sec

![Graph of Temperature Distribution for an Infinite Slab - Case I](image)

**Case II - [$\kappa = F(T_{av})$]**

![Graph of Temperature Distribution for an Infinite Slab - Case II](image)

**Fig. 22. Temperature Distribution for an Infinite Slab - Case I**

**Fig. 23. Temperature Distribution for an Infinite Slab - Case II**
Case III - \[ \kappa = F(T \text{ of region described by the heat balance}) \]

![Graph Case III](image)

Fig. 24. Temperature Distribution for an Infinite Slab - Case III

Case IV - \[ \kappa = F(\text{Average T across an interface}) \]

![Graph Case IV](image)

Fig. 25. Temperature Distribution for an Infinite Slab - Case IV

6. Bibliography
