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**A STUDY OF ADIABATIC,
FULLY-DEVELOPED, ANNULAR-DISPERSED
TWO-PHASE FLOW
IN A VERTICAL ROUND TUBE**

R.L. SIMPSON

U.S. ATOMIC ENERGY COMMISSION
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ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
SAN JOSE, CALIFORNIA

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TWO-PHASE FLOW IN A VERTICAL ROUND TUBE

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SUMMARY

The detailed time-averaged flow structure of upward, annular-mist, two-phase flow is used to predict pressure drop, film liquid thickness, and average void fraction, which are compared with experimental results. The several flow phenomena incorporated in this computational scheme are expressed by four phenomenological models. Many sets of experimental data are rationalized to form a consensus upon which to test these models.

● **FILM LIQUID FLOW RATE – LIQUID FILM THICKNESS RELATIONSHIP IN ADIABATIC TWO-PHASE FLOW**

The film liquid flow rate–liquid film thickness data of Alia, et al., and Quandt are presented along with the data of Schraub, Simpson, and Janssen. These data, taken with different fluids and different geometries, are in good agreement and indicate a universal G_f^+ versus δ^+ relationship for pressures in the range $1 \text{ atm} < P < 22 \text{ atm}$, $0.05 < X < 0.74$, and mass flow rates between 0.1×10^6 and $2 \times 10^6 \text{ lb/h-ft}^2$. An analysis, using mixing length theory with allowance for voids in the liquid film by $a = CU^+$, is in good agreement with these experimental data.

● **ENTRAINMENT FRACTION IN FULLY-DEVELOPED, ANNULAR-DISPERSED TWO-PHASE FLOW**

Much entrainment fraction data from adiabatic annular dispersed two-phase flows have been examined. Using the entrainment fraction independent variable of Paleev and Fillipovich, one is able to account for area, density, surface tension, and flow rate effects for fully-developed flows. The data of Steen, taken with several fluids covering an extensive range of liquid viscosities, indicate that the Paleev and Fillipovich variable does not account correctly for the liquid viscosity. Based on Steen's silicone data, a modified Paleev and Fillipovich variable that correctly accounts for the liquid viscosity is

presented. Also presented is the resulting correlation of the data of Steen, Minh and Huyghe, and Alia, et al., with this variable.

● **CORE LINEAR GAS VELOCITY PROFILES**

The core gas linear velocity profiles from the Schraub, Simpson, and Janssen annular-dispersed round-tube data are found to agree with the results of Alia, et al., in terms of $(U_{cg} - U_{rg})/U_{rg}$ versus y/R . These profiles, for flow qualities greater than 11%, lie between single-phase pipe and flat-plate correlations.

● **LIQUID VOLUMETRIC DISTRIBUTION MODEL FOR THE GAS CORE**

The liquid volumetric concentration in the gas core is not uniform at the asymptotic condition. Since droplet concentration gradient diffusion must not be the only transport mechanism, the idea of preferential turbulent drift of droplets toward the wall is briefly investigated. However, it is concluded that the unity core local slip ratio assumption is a simple and reasonable assumption since it produces a rather flat liquid volumetric distribution and eliminates the need for the local slip ratio to vary over the core.

The results from the computational scheme indicate that the entrainment fraction and pressure drop are intimately related. When a known entrainment is input, the pressure drop is predicted within 25% of the experimental value for $X > 25\%$. In many cases, the agreement was better than 25%, and improves as the mean quality is increased. Since it is apparent that the entrainment fraction is rig dependent to some degree, the pressure drop will be affected accordingly; i.e., an increase in entrainment fraction will produce a decrease in pressure drop. It was also found that when the pressure drop was closely predicted, the film thickness and average void fraction were also closely predicted.

1. INTRODUCTION

This study concerns the upward flow of an adiabatic two-phase mixture in a round tube for the annular-mist flow regime. The flow is not simple. Liquid flows in a thin film along the tube wall. Large waves exist on the liquid film surface. Other liquid is entrained in the gaseous core. Eventually, an equilibrium between core droplet deposition and liquid film entrainment rates is attained, and the entrainment fraction becomes essentially constant. The flow is *fully developed* at this asymptotic condition.

This study has a two-fold objective:

- To rationalize existing experimental results to form a consensus upon which to test semi-empirical modeling, and
- To model the several flow phenomena and demonstrate

the inter-model consistency of the results.

To meet the first objective, four phenomenological models, or correlations, are developed and compared with experimental data:

- a. The film liquid flow rate—liquid film thickness relationship.
- b. The fraction of liquid entrained in the gas core model,
- c. The core gas linear velocity profile relationship, and
- d. A liquid volumetric distribution model for the gas core.

These four models are considered together to produce average void fraction, liquid film thickness, and pressure drop predictions that are compared with experimental results. The effect of perturbations of the individual models on the final results are also considered.

2. THE FILM LIQUID FLOW RATE—LIQUID FILM THICKNESS RELATIONSHIP IN ADIABATIC TWO-PHASE FLOW

2.1 EXPERIMENTAL RESULTS

In an annular two-phase flow there is apparently a distinct liquid film covering the conduit walls, and a gas flow with entrained liquid droplets in the core. Several investigators^{1 2 3 4} report large amplitude roll waves on the surface of the liquid film so that, in general, the interface is not smooth. To describe this type of flow, it is useful to describe the relationship between the mean liquid film flow rate and the mean liquid film thickness even though these waves are not explicitly considered. Only upward co-current flow is considered here.

Several investigators have measured either the mean liquid film flow rate⁵ or the mean liquid film thickness^{1 6 7 8}. The investigations of Quandt⁹ and Alia, et al.,¹⁰ and the present program are known to report direct measurements of both quantities for adiabatic flow. Quandt used a 1/4 × 3 inch rectangular plexiglass channel for visual observations in a vertical upward co-current air-water flow at near-atmospheric conditions. He used a dye concentration technique (which he described in detail) to determine the liquid film flow rate and liquid film thickness.

Alia, et al., studied argon-water and argon-ethyl alcohol flows in a 1-inch round tube at pressures between 11 and 22 atmospheres, flow rates between 0.2×10^6 and 2×10^6 lb/h-ft², and qualities between 5 and 79%. An isokinetic sampling scoop, forming a segment of an annulus, was traversed through the liquid film to obtain the total liquid flowing between the wall and some position y in the film. The thickness of the liquid film was determined by the film

conductance method, described in the Tenth Quarterly Progress Report¹¹.

In the P.A. 44 program, air-water flows in a 1-inch round tube at about 85 psia were run with an approximately constant air flow rate of 0.11 lb/sec for qualities between 11 and 62%. The liquid film was sucked off through a porous sinter, as described in Quarterly Progress Reports 4 and 5¹², while the liquid film thickness was determined by the needle contact method described in Quarterly Report Numbers 7 and 10^{13 11}.

All results reported from these investigations are shown in Figure 1 as G_f^+ versus δ^+ , where

$$G_f^+ \equiv \frac{\delta G_f}{\mu_l}, \quad \delta^+ \equiv \frac{\delta}{\nu_l} \sqrt{\frac{\tau_{wg}}{\rho_l}},$$

and δ = film thickness.

(2-1)

Quandt and the P.A. 44 investigators determined τ_{wg} from pressure drop and "quick closing valve" void fraction data, while Alia, et al., measured τ_{wg} directly with a floating sleeve hydraulic method described in Reference 14. Note that there is substantial agreement among these data, with the best agreement being between the data of Quandt and of Alia, et al.

The uncertainties associated with these data should be borne in mind. The liquid film flow rate obtained by the porous sinter method cannot be distinctly determined as

shown in Figure A-16 of Reference 12. This problem is more acute at low quality flows (large δ^+). In the needle contact method, the needle can disturb the liquid film by contact hysteresis¹⁵ and by deflecting gas into the liquid film. Alia, et al., point out that air in the liquid film, as indicated by the scoop data, causes some uncertainty in the relationship between the electrical conductance film thickness and the actual extension of the region where the liquid is the continuous phase. Quandt did not estimate the uncertainty of his method, except to state that there was less than $\pm 25\%$ scatter in his data.

Alia, et al., presented the empirical equation, $G_f^+ = 5.3 (\delta^+)^{1.1}$, fitted through their data which agrees well with the other data as a whole. It is interesting to note that Alia, et al., found that the G^+ between the wall and a given y^+ in the film also followed this relation until $y^+ \geq \delta^+$. For $y^+ > \delta^+$, the data departed markedly from this relation.

The good agreement of these data taken with different fluids and different geometries indicates a universal G^+ versus δ^+ relationship for pressures in the range $1 \text{ atm} < P < 22 \text{ atm}$, $0.05 < X < 0.74$, and mass flow rates between 0.1×10^6 and $2 \times 10^6 \text{ lb/h-ft}^2$.

2.2 ANALYSIS

As a first approximation, several investigators^{9 16 17} have assumed that the single-phase "law of the wall" of the mean velocity profile applies to the film liquid and that

$$\frac{G_f \delta}{\mu_L} = G_f^+ = \int_0^{\delta^+} U_L^+ dy_L^+ \quad (2-2)$$

Levy¹⁶ used a three-layer Von Karman U_L^+ versus y_L^+ model, while Hewitt¹⁷ used a two-layer model. The results of these analyses for the range of presented data are shown in Figure 1 to overestimate G_f^+ about 30 to 60% at a given δ^+ . Note that the Hewitt model does not predict G_f^+ as a unique function of y_L^+ .

This general approach seems reasonable, except for the fact that the possibility of gas flow in the film has not been considered. Alia, et al.,¹⁰ report void fractions of the order of 0.5 at the outer edge of the film from their film isokinetic sampling data. Hanratty and Hershman³ report large roll waves on the film surface that could conceivably entrain gas bubbles into the film. Hence, allowance for gas will be made in the present analysis.

In the present analysis, a two-layer model will be used to derive the film velocity profile: (1) a viscous sublayer near the wall ($y^+ \leq 11$) and (2) a fully turbulent region described by the Prandtl mixing length theory. The void fraction is considered a function of U^+ (or y^+), to be consistent with the idea that G^+ is a universal function of

y^+ . Two different liquid film void fraction distributions, each a unique function of U^+ , will be considered here. The first,

$$a = CU^+, \quad (2-3a)$$

assumes that entrained air is distributed throughout the film. The motivation is that

$$a \rightarrow 0 \text{ as } U \rightarrow 0, \text{ while } a \rightarrow 1 \text{ as } U_g/U_g, G_L \rightarrow 1.$$

If Equation (2-3a) is approximately

$$a \sim U_g/U_g, G_L, \quad (2-3b)$$

then the constant

$$C \approx U_r/U_g, G_L$$

can be evaluated by the isokinetic sampling data of Reference 19, which produces $C \approx 0(10^{-2})$. Furthermore, with α of the order of 0.5 at the outer film edge, it is necessary that $C \approx 0.03$ for U_L^+ of the order of 10 to 20 for $10 < y^+ < 200$.

The second distribution approximates the void fraction distribution obtained from the CISE film isokinetic sampling data^{10 14} using a local slip ratio of 1 and presented in Reference 10. For $y^+ \leq 11$, $a = 0$, while $a \sim \ln|y^+|$ out to the film edge, where $a \approx 0.5$. Since $U^+ \sim \ln|y^+|$, then

$$a = AU^+ + B \quad (2-4)$$

is an approximate relation fitting the data with $A = 0.082$ and $B = -0.9$.

A force balance on the film for a given y position produces

$$\tau - \tau_W + \rho_L \int_0^y (1-a) g dy + \frac{dP}{dz} y$$

or

$$\frac{\tau}{\tau_W} = 1 + \frac{\nu_{Lg}}{U_{\tau L}^3} \left[\int_0^{y^+} (1-a) dy^+ + \frac{y_L^+}{\rho_{Lg}} \frac{dP}{dz} \right] \quad (2-5)$$

for small y/R values. The coefficient of the bracketed term on the right side of Equation (2-5) is of the order of 10^{-3} while the bracketed quantities tend to cancel. Hence, in the present analysis

$$\tau \approx \tau_W^*$$

In the viscous sublayer,

$$\tau_W = \mu \frac{\partial U}{\partial y}, \quad (2-6)$$

with

$$\mu = \mu_\ell (1 - a) + \mu_g a.$$

For the first void distribution,

$$\mu \approx \mu_\ell (1 - CU^+) \text{ since } \mu_g/\mu_\ell \ll 1.$$

Thus,

$$U_\ell^+ - \frac{CU_\ell^{+2}}{2} = y_\ell^+ \quad (2-7)$$

for the sublayer. With $a = 0$ in the sublayer, the second void distribution produces the classical single phase relation for $U_\ell^+ \leq 11$,

$$U_\ell^+ = y_\ell^+ \quad (2-8)$$

For the fully-turbulent region, the mixing length relation,

$$\tau = \rho \ell^2 \left[\frac{\partial U}{\partial y} \right]^2 \quad (2-9)$$

will be used with

$$\rho = \rho_\ell \left[1 - (1 - \rho_g/\rho_\ell) a \right], \quad (2-10)$$

and $\ell = 0.4 y$.

For $a = 0.03U^+$ and $\rho_g/\rho_\ell \ll 1$, one obtains

$$\frac{dy_\ell^+}{0.4y_\ell^+} = \sqrt{1 - 0.03U_\ell^+} dU_\ell^+ \quad (2-11)$$

from Equations (2-9) and (2-10). Integrating, using the single-phase constants,¹⁹ $U^+ = 11$ at $y^+ = 11$, produces

$$\frac{1}{0.4} \ln \left| \frac{y_\ell^+}{11} \right| = \frac{-2/3}{0.03} \left\{ (1 - 0.03 U_\ell^+)^{3/2} - (0.67)^{3/2} \right\} \quad (2-12)$$

Equation (2-12) applies outside the point of intersection of Equations (2-7) and (2-12), while Equation (2-7) applies inside that point. The justification for using the single-phase constants is somewhat intuitive since these constants produce agreement of the mixing length theory with single-phase velocity profile data for mass transfer and moderate pressure gradient cases^{19 20}.

Likewise, using Equations (2-9) and (2-10) and $\rho_g/\rho_\ell \ll 1$ with the second void distribution, $a = 0.082 U^+ - 0.9$, produces

$$\frac{dy_\ell^+}{0.4y_\ell^+} = \sqrt{1 - (0.082 U_\ell^+ - 0.9)} dU_\ell^+ \quad (2-13)$$

Integrating and using the single-phase constants produces

$$\frac{1}{0.4} \ln \left| \frac{y_\ell^+}{11} \right| = \frac{-2/3}{0.082} \left\{ (1.9 - 0.082 U_\ell^+)^{3/2} - 1 \right\} \quad (2-14)$$

To obtain G_f^+ , one finds that

$$G_f^+ = \int_0^\delta \frac{\rho U dy}{\mu_\ell} = \int_0^\delta (1 - a) U_\ell^+ dy_\ell^+ \quad (2-15)$$

The results of performing the integration indicated in Equation (2-15) for the two void distributions are shown in Figure 1 and labeled Analysis A and Analysis B, respectively. In summary, the Analysis A results used Equations (2-3a), (2-7) and (2-12) in Equation (2-15), while Analysis B used Equations (2-4), (2-8), and (2-14).

Although Analysis B used the data input of the CISE film void distribution, Analysis A fits the trend and position of the experimental G_f^+ -versus δ^+ results much better (within 10% of mean values). Both analyses predict the film liquid flow rate better than the zero void analyses for the thicker films.

3. ENTRAINMENT FRACTION IN ADIABATIC, ANNULAR-DISPERSED TWO-PHASE FLOW

There is a need to predict liquid entrainment rates along a developing two-phase, annular-dispersed flow. However,

the phenomena of liquid entrainment and deposition are complicated. As a beginning, a description of the entrain-

Figure 1. Dimensionless Film Liquid Flow Rate Versus Film Thickness

ment fraction in the gas core for a fully-developed, adiabatic two-phase flow would be a stride forward. (By definition, an asymptotic or fully-developed, adiabatic two-phase flow is one where $d\rho_g/dz$ is negligible and the entrainment fraction and dP/dz remain essentially constant.)

Several investigators have suggested that large roll wave motions on the gas core-liquid film interface produce entrainment. For very thin films, these waves cease to exist and no further entrainment occurs. For fully-developed adiabatic conditions, the rate of entraining liquid into the air core equals the rate of depositing drops on the interface. Any characteristic length associated with the interface area must have little relevance in determining the asymptotic fraction of the core occupied by the entrained droplets.

Using the idea that the characteristic length is nondetermining, Paleev and Fillipovich²¹ proposed that

$$\frac{\rho_g}{\rho_l} \left(\frac{\mu_l G_g}{\rho_g \sigma} \right)^2 = \left(\frac{\mu_l^2 l}{\rho_l \sigma L} \right) \left(\frac{G_g^2 L}{\rho_g \sigma} \right) \quad (3-1)$$

be the form for an entrainment fraction independent variable. Paleev and Fillipovich claim that the two dimensionless groups on the right side of Equation (3-1) are natural choices since they successfully correlate data from the interaction between a gas and liquid during nozzle spraying. (Though they do not reference any nozzle spraying work, the work of Hinze²² supports their statement.) Taking into account the entrained liquid in calculating the core density, they modify Equation (3-1) by substituting $(\rho_g/\rho_l)(1 + E G_l/G_g)$ for ρ_g/ρ_l to obtain

$$\left(\frac{\rho_g}{\rho_l} \right) \left(1 + E \frac{G_l}{G_g} \right) \left(\frac{G_g \mu_l}{\rho_g \sigma} \right)^2 = \Pi_1 \quad (3-2)$$

Using data from several sources (21 and 23-26), they obtained

$$1 - E = 0.985 - 0.44 \log_{10} \left[\Pi_1 \times 10^4 \right] \quad (3-3)$$

as a fit through the mean of the data. For a given Π_1 , most of the data points used by them fell within ± 0.1 around this line, which is presented in Figure 2. The asymptotic (fully-developed) entrainment datum of the Gill, et al.,⁶ developing flow is in excellent agreement with Equation (3-3), as shown in Figure 3.

The results of Minh and Huyghe²⁷ for air-water and air-alcohol flows in round tubes are shown in Figure 3. These data alone examine the effects of cross-sectional area

and different fluid properties on the entrainment fraction. These data were taken over a range of pressures from 1 to 8 atmospheres. The liquid film was sucked off through a uniform width slit around the tube perimeter. All the film liquid was assumed to be removed when an increase in slit width produced no increase in the flow rate of removed liquid. The entrainment fraction between $L/D = 80$ and $L/D = 300$ was reported unchanged, indicating a fully-developed condition. Apparently independent of Paleev and Fillipovich, they deduced a kinetic energy per unit volume parameter*

$$\frac{G_g^2}{\rho_g} \left(1 + E \frac{G_l}{G_g} \right) \quad (3-4)$$

that correlates the area and density effects in their air-water data. As shown in Figure 3, their air-water and air-alcohol data are nearly superimposed when plotted in terms of Π_1 .

Alia, et al.,¹⁰ report film liquid flow rate data at $L/D = 140$ for argon-alcohol and argon-water flows. An isokinetic sampling scoop, forming a segment of an annulus, was traversed through the liquid film to obtain the total liquid flowing between the wall and some position y in the film. The thickness of the liquid film was determined by a film conductance method. These results, shown in Figure 3 in terms of Π_1 , are in basic agreement with the results of Minh and Huyghe, although the argon-alcohol results appear low. (Alia, et al., also present E versus flow quality curves (Figure 32 of Reference 10) smoothed through their air-water data. Evaluating these curves in terms of Π_1 , the results surround the Minh-Huyghe data.)

The air-water data of Cousins, et al.,²⁸ and Cousins and Hewitt²⁹ for a 3/8-inch-diameter acrylic resin tube were obtained by removing the wall liquid film through a porous section. The data²⁹ were found to agree with the air-water data of Minh and Huyghe for $Re_f > 2000$. For $Re_f < 2000$, Cousins and Hewitt indicate that the data of Cousins, et al., fall above the Minh and Huyghe results and are Reynolds number dependent in the formulation. They found no length dependency on the data taken at $L/D > 80$.

The air-water data of Cousins and Hewitt²⁹ for a 1-1/4-inch-diameter copper tube, obtained by suction through a porous sinter, were found to agree with the Paleev and Fillipovich Equation (3-3) for $Re_f > 2000$ and $L/D > 80$, as did the data of Collier and Hewitt²⁵ for the same diameter pipe. They propose that the discrepancy between Equation (3-3) and the Minh and Huyghe data is due to a diameter effect (1-1/4 : 3/8). However, the data of Alia, et al.,¹⁰ taken for a 2.5-cm-diameter tube agree with the Minh and Huyghe data and contradict the proposed diameter effect.

* Minh and Huyghe suggest that expression (3-4) be divided by $\bar{a}^2$. At high flow qualities $\bar{a} \rightarrow 1$, and the expression is then essentially unchanged.

Figure 2. Data Presented by Paleev and Fillipovich

MINH AND HUYGHE⁽²⁷⁾

- × WATER ϕ 12 mm diameter
- WATER ϕ 6 mm diameter
- ALCOHOL ϕ 12 mm diameter

CISE DATA⁽¹⁰⁾, 2.5-cm DIAMETER

- ◻ ARGON - ALCOHOL
 - ◆ ARGON - WATER
- } FILM SAMPLING

SINGH DATA⁽³⁰⁾

- 1/2-inch SINTER
- 1-inch SINTER
- ◊ 2-inch SINTER

GE FILM DATA (AIR - WATER)⁽³¹⁾

- △ ROUND TUBE, 1-inch diameter
 - ◻ CONCENTRIC ANNULUS
 - ◊ ECCENTRIC ANNULUS
- } GF ROD AND TUBE,
1-inch-diameter TUBE,
0.5-inch-diameter ROD.

GIL, HEWITT, AND HITCHON⁽²⁷⁾

⊙

Figure 3. Further Entrainment Data

The 1000-psia adiabatic steam-water round tube entrainment results of Singh, et al.,³¹ are shown in Figure 3 in terms of Π_1 . These data, obtained by suction through a porous sinter at $L/D = 200$ from the entrance, fall below the correlation of Paleev and Fillipovich. Longer sinter lengths were used for higher total mass flow rates in an effort to catch all liquid in the film.

The air-water entrainment data of the P.A. 44 investigation³¹ are presented in Figure 3 in terms of Π_1 for the round tube and the concentric and eccentric annulus. These data were taken under constant air flow rate—variable water flow rate conditions at essentially the same pressure and temperature by sucking off the film through 1-inch-long porous sinters 12 feet from the entrance. The flow is presumed to be fully developed, since isokinetic sampling profile measurements at 9 and 10.5 feet from the entrance were identical (within the uncertainty of the isokinetic sampling technique). There appear to be three general groupings from these data:

- The data that are fairly well correlated by Equation (3-3). These data were taken by core liquid flow rate sampling techniques, with the exception of the Dowtherm and carbon dioxide data of Mondin and Huyghe who used the peripheral slit technique used by Minh and Huyghe, and some of the porous sinter method data of Cousins and Hewitt.
- The slit and scoop data of Minh and Huyghe, and Alia, et al., and the 3/8-inch-diameter porous sinter data of Cousins, et al., and Cousins and Hewitt, that fall above Equation (3-3).
- The porous sinter data of Singh, et al., and the present investigation, that fall below Equation (3-3).

At first glance, it would appear that E is either not a unique function of Π_1 , or improper properties are contained in Π_1 . However, each set of these data taken on a given apparatus with a given measurement technique, is self-consistent with the E versus Π_1 formulation. The data of Minh and Huyghe show the effect of changing all variables in Π_1 , although μ_l and ρ_l for water and alcohol are nearly the same. The data of Alia, et al., show mainly the effect of the surface tension for argon-water and argon-alcohol flows. With the exception of the Mondin and Huyghe work, the remaining investigations used water as the working liquid.

The data of Steen³² are plotted in Figure 4. These data, taken at $L/D > 90$ for air-silicone, air-ethylene glycol, and air-water flows, examine the effect of different fluid properties with substantial variations in μ_l . The air-water data agree with the data of Minh and Huyghe, but the remaining data fail to correlate with Π_1 . Only in Steen's experiments was μ_l varied substantially so that the liquid viscosity dependence could be examined. Because these data indicate a higher liquid viscosity results in higher Π_1 but lower E , an empirical viscosity ratio factor to correct

Π_1 was derived from the silicone data ($9 < Re_f < 1100$). (The silicones have essentially the same ρ_l and σ , but μ_l varies from 5 to 100 centipoises.) This viscosity factor is

$$\left(\frac{\mu_g}{\mu_l} \right)^{11/5}$$

normalizing on the gas viscosity, which varies little in all these data. Hence,

$$\begin{aligned} \Pi_3 &= \Pi_1 \left(\frac{\mu_g}{\mu_l} \right)^{11/5} \\ &= \left(\frac{\rho_g}{\rho_l} \right) \left(1 + E \frac{G_l}{G_g} \right) \left(\frac{G_g \mu_g}{\rho_g \sigma} \right)^2 \left(\frac{\mu_g}{\mu_l} \right)^{1/5} \end{aligned}$$

should correlate the effect of liquid viscosity as well as the effect of properties correlated by Π_1 . The parameter Π_3 is in substantial agreement with Steen's parameter Π_2 , since

$$\Pi_3 = \Pi_2^2 \left(1 + E \frac{G_l}{G_g} \right) \left(\frac{\mu_g}{\mu_l} \right)^{1/5}$$

where

$$\Pi_2 = \frac{G_g \mu_g}{\rho_g \sigma} \left(\frac{\rho_g}{\rho_l} \right)^{1/2}$$

and the liquid viscosity plays a lesser role than the gas viscosity.

As shown in Figure 5, Steen's air-water and air-ethylene glycol data are nearly superimposed on the air-silicone data when plotted in terms of Π_3 . These data correlate with the Minh and Huyghe, and Alia, et al., data which, in turn, are better correlated by Π_3 than Π_1 , as shown in Figure 6. The equations

$$1 - E = 0.819 - 0.112 \log_{10} |\Pi_3 \times 10^7| \quad (3-5a)$$

$$\text{for } \Pi_3 < 4.5 \times 10^7, \text{ and}$$

$$1 - E = 0.966 - 0.210 \log_{10} |\Pi_3 \times 10^7| \quad (3-5b)$$

for $\Pi_3 > 4.5 \times 10^7$ fit these data, as shown in Figures 5 and 6.

The data of Singh, et al., and the present investigation do not correlate with the other data in terms of Π_3 . Likewise, the ambient temperature air-water data that correlate with Equation (3-3) are merely shifted by nearly the same viscosity ratio factor and will not correlate with the Steen, Minh and Huyghe, and Alia, et al., data.

In conclusion, there is considerable evidence that the E versus Π_3 formulation contains proper parameters for fully developed adiabatic two-phase annular dispersed flows. Although all the results from these several investigations do not agree, the results from a given apparatus with a given

Figure 4. Data of Steen (See Ref. 32)

Figure 5. Steen's Entrainment Data in Terms of π_3

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Figure 6. Entrainment in Terms of π_3

measurement technique are self-consistent with the E versus Π_3 formulation when parameters contained in Π_3 are varied.

It is difficult to assess which measurement technique is best since no investigation has covered obtaining the same entrainment data by more than one method. Steen³² has shown that the one radial position core liquid sampling technique he used has shortcomings for obtaining high entrainment data, the results being probe position and size dependent. However, because Steen used the same size probe at the same radial location for the air-silicone data used here, it is felt that the qualitative property dependence shown by his data is valid. The spacer results of Alia, et al.,³³ indicate that small protrusions from the walls can produce further entrainments. This could be the case for the data of the present investigation, since slight misalignment of the void fraction quick-closing valves upstream of the porous sinters could have produced a low film flow

rate, as the data indicate. As pointed out in Reference 31, there is some uncertainty in interpreting the flow rate results from the porous sinter method. The isokinetic scoop and peripheral slit techniques data are consistent, and probably disturb the flow the least. Hence, it is concluded that some of the data discussed here are rig and measurement technique dependent. In light of this discussion, it is also concluded that the data of Minh and Huyghe and Alia, et al., are likely the best quantitatively.

More careful fully-developed flow entrainment experiments with careful measurement of all flow structure quantities (dP/dx , α , film thickness, wave structure, film liquid flow rate, and isokinetic sampling to obtain air-water velocity profiles) at several axial positions would indicate whether the flow was fully developed and would help resolve the proper E versus Π_3 relationship, as well as provide more information about the entrainment phenomena.

4. CORE GAS LINEAR VELOCITY PROFILES

In fully-developed, turbulent, single-phase flow in a round tube, a definite core ($y/R \gtrsim 0.05$) velocity profile similarity has been established; namely,

$$\frac{U_{Cg} - U_g}{U_{rg}} = f(y/R) \quad (4-1)$$

where

$$f(y/R) \sim \ln |y/R|$$

as the wall is approached³⁵. One would expect that for a low entrained liquid annular two-phase flow, the formulation of Equation (4-1) may well hold for the gas linear velocity

$$U_g = G_g / \rho_g \alpha \quad (4-2)$$

The gas linear velocity results of the present investigation are shown in Figure 7 to be substantially independent of mean flow quality within the scatter of the results. The experimental gas linear velocity results of Alia, et al.,¹⁰ are shown in Figure 8 for fully-developed annular dispersed flows. These sets of data are in close agreement for $y/R > 0.1$. (All these data were obtained by isokinetic sampling and impact pressures. Near the wall, the isokinetic probe interferes with the liquid film so that the isokinetic sampling results from both investigations for $y/R < 0.1$ are not completely reliable.) Apparently, the constant local slip ratio assumption of Alia, et al., in reducing their data produces nearly the same results as obtained in the present investigation using a variable local slip ratio assumption.³⁶

Also shown in Figures 7 and 8 are the single-phase turbulent flow results of Laufer³⁷ for the pipe, and Wieghardt³⁸ for the flat plate. These single-phase results have been theoretically described by³⁸

$$f(y/R) = \frac{1}{0.4} \ln |y/R| + g(y/R) \quad (4-3)$$

for large Reynolds numbers.³⁴ Note that Equation (4-3) is composed of a logarithmic term and a large eddy correction term, the latter being slightly different for the pipe and the flat plate³⁴. The gas linear velocity results appear to fall between the pipe and flat plate results. This is not surprising since the single-phase pipe flow is constrained by the walls, while the flat-plate flow can propagate into the freestream. The two-phase annular flow is constrained within the pipe, but the interface between the liquid film and the core remains free. Hence, the large eddy structure of the two-phase annular core flow is likely to be a combination of pipe and flat-plate characteristics, as indicated by the data.

Alia, et al., fitted

$$f(y/R) = \frac{1}{K} \ln |y/R| \quad (4-4)$$

to their data for $y/R > 0.1$, obtaining $K = 0.3$. They contended that the liquid droplets decreased the turbulence level in the core since single-phase flows were known to produce $K = 0.4$. It should be noted that in single-phase flow, the region $y/R > 0.1$ is also described by an additive large eddy turbulence function such as the one contained in Equation (4-3). Thus, this interpretation of the effect of droplets on turbulence level need not hold to explain the

Figure 7. Core Gas Linear Velocity Data of Schraub, Simpson, and Janssen (See Ref. 31)

Figure 8. Data of Alia, Et al. (See Ref. 10)

gas linear velocity profile. Equation (4-3), with a hybrid large eddy term from the pipe and flat-plate single-phase flows, appears to be applicable to the gas linear velocity

profiles for annular-dispersed flows with qualities greater than 11%.

5. LIQUID VOLUMETRIC DISTRIBUTION IN THE GAS CORE

In a fully-developed, annular-dispersed two-phase flow, one would expect the droplet volumetric concentration in the gas core to be uniform, providing droplet gradient diffusion is the only mechanism of transport. Some sampling data support this view,³⁸ while other data^{10 31 38} show concave upward $1-a$ profiles as a function of r . The latter group of data indicates that another mechanism, in addition to gradient diffusion, plays an important transport role.

Turbulent drift of droplets toward the tube walls has been observed to be a deposition mechanism in aerosol research³⁹. Sternling, as cited by Cousins and Hewitt³⁸, also suggests turbulent drift as a possible mechanism of droplet deposition. Sternling contends that droplets near the centerline are likely to encounter a large eddy and be preferentially thrown toward the wall. The gradient diffusion mechanism would tend to even out the concentration.

Thus, Cousins and Hewitt³⁸ wrote:

$$\ddot{Q}_D = V_r (1-a) - \epsilon_D \frac{d(1-a)}{dr} \quad (5-1)$$

where V_r is the turbulent drift velocity, $\dot{Q}_D$ is net radial liquid volumetric flux, and ϵ_D is the turbulent gradient diffusion coefficient.

In a fully-developed flow, $\ddot{Q}_D$ is zero, and

$$\frac{V_r}{\epsilon_D} dr = \frac{1}{(1-a)} d(1-a) \quad (5-2)$$

Integrating from the centerline to radius r yields

$$\int_{\zeta}^r \frac{V_r}{\epsilon_D} dr = \ln \left| \frac{(1-a)}{(1-a)_\zeta} \right| \quad (5-3)$$

If one sets the eddy diffusion and momentum coefficients equal and uses Laufer's³⁵ pipe results, then

$$\epsilon_M = \epsilon_D = 0.07 U_{\tau g} R$$

for $r/R < 0.8$. Schmel³⁹ suggests that $V_r \sim U_\tau$. Requiring a fit of the form $V_r \sim (r/R)^n U_\tau$, one finds (see Figure 9)

$$V_r = \frac{1}{2} (r/R)^2 U_\tau \quad (5-4)$$

and the resulting equation

$$\frac{1}{0.42} (r/R)^3 = \ln \left| \frac{(1-a)}{(1-a)_\zeta} \right| \quad (5-5)$$

to fit roughly the isokinetic sampling results of Reference 31. The data of Reference 38 show (see Figure 10) less curvature, the higher the quality. These data indicate that the droplet volumetric profile is not a unique function of quality.

The 0.4784 quality run results shown in Figure 9 indicate the dependence of sampling probe results on the data reduction scheme. Data reduced by the British homogeneous method⁶ (unity local slip ratio, void weighted density) show a rather flat profile, while the same data reduced by the isokinetic sampling equations of Schraub¹⁸ show more profile curvature. (As pointed out in Reference 31, and in Figure 11 there is little difference in the local air linear velocity and local quality results obtained by these two reduction schemes. There is more difference in the $(1-a)$ results.

Thus, while the rate Equation (5-1) may be applicable, there is no a priori basis for Equation (5-4). To predict the profile shape, one must have V_r and ϵ_D expressed as a function of quality as well as position. Alia, et al.,¹⁰ indicate that the void profile flattens with increasing flow rates, increasing gas density at constant volume flow rate and decreasing surface tension. Hence, V_r and ϵ_D must also be a function of fluid properties. In addition, the sampling data void results are dependent upon the data reduction scheme, making the deduction of ϵ_D and V_r from sampling data uncertain.

The results of Figure 9 indicate that when a unity local slip ratio is used in reducing the data, the assumption of a constant core droplet volumetric concentration is not bad. This assumption is not only useful, in view of the uncertainties mentioned above, by eliminating the core a variation, but it also eliminates the need to know the local slip ratio variation.

Figure 9. Radial Distribution of Liquid, Data of Schraub, Simpson, and Janssen (See Ref. 31)

Figure 10. Radial Distribution of Liquid, Data of Cousins and Hewitt (See Ref. 38)

Figure 11. Comparison of Data Reduced by the Isokinetic and British Homogeneous Methods

6. THE PREDICTION OF PRESSURE DROP, MEAN VOID FRACTION, AND FILM THICKNESS

The second objective of demonstrating the inter-model consistency of these several models was attained by comparing calculated pressure drop, mean void fraction, and film thickness results with experimental results. A schematic diagram of the calculation steps is shown in Figure 12.

In the present approach, one must first start with an entrainment fraction model such as one derived in Section 3, and the given input information, $\dot{W}_L$, $\dot{W}_g$, R , and fluid properties. With this information one uses Equation (6-1) to determine G^+ :

$$G^+ = \frac{\dot{W}_{Lf}}{2\pi R \mu_L}, \quad (6-1)$$

where $\dot{W}_{Lf}$ is the film liquid flow rate in the film model of Section 2. The film gas flow rate, $\dot{W}_{gf}$, the dimensionless film thickness δ^+ or y_{gi}^+ , and the dimensionless film-core mean interface velocity U_{gi}^+ are determined from the film model.

One now can calculate the mean core void fraction $\bar{a}_c$. The liquid flow rate in the core $\dot{W}_{Lc}$ is determined from $\dot{W}_L$ and the entrainment fraction E . $\dot{W}_{gc}$, the gas flow rate in the core, is determined by subtracting $\dot{W}_{gf}$ from $\dot{W}_g$. As noted in Section 5, a constant core void with a local slip ratio of 1 is not in bad agreement with data reduced, assuming a local slip ratio of 1. Hence, with local slip ratio of 1, one obtains

$$\bar{a}_c = \frac{1}{\left(1 + \frac{\rho_g \dot{W}_{Lc}}{\rho_L \dot{W}_{gc}}\right)} \quad (6-2)$$

Also, from the film model one can calculate y_{gi}^+ and U_{gi}^+ of the core-film interface from U_{gi}^+ and y_{gi}^+ ; i.e.,

$$\left. \begin{aligned} y_{gi}^+ &= y_{gi}^+ \left(\frac{\nu_L}{\nu_g} \right) \left(\frac{\rho_L}{\rho_g} \right)^{1/2} \\ U_{gi}^+ &= U_{gi}^+ \left(\frac{\rho_g}{\rho_L} \right)^{1/2} \end{aligned} \right\} \quad (6-3)$$

which satisfies the kinematic condition that the mean quantities $U = U_i$ and $y = y_i$ at the mean core-film interface. Using the logarithmic gas linear velocity profile ($g(y/R) \rightarrow 0$ as $y/R \rightarrow 0$),

$$U_g^+ = \frac{1}{K} \ln \left(\frac{y_g^+}{y_{gi}^+} \right) + U_{gi}^+ - g(y/R). \quad (6-4)$$

One can now obtain a value for U_{rg} , the shear velocity based on gas properties, by a gas mass balance for the core:

$$U_{rg} = \frac{2\pi \rho_g \nu_g^2 \bar{a}_c}{\dot{W}_g} \int_{y_{gi}^+}^{R^+} U_g^+ (R_g^+ - y_g^+) dy_g^+, \quad (6-5)$$

where

$$\begin{aligned} & - \int_{y_{gi}^+}^{R^+} U_g^+ (R_g^+ - y_g^+) dy_g^+ \\ &= \frac{1}{K} \left\{ R_g^{+2} \left[\frac{-\ln}{2} \left(\frac{R_g^+}{y_{gi}^+} \right) + \frac{3}{4} \right] + \frac{y_{gi}^{+2}}{4} - R_g^+ y_{gi}^+ \right\} \\ &+ \frac{y_{gi}^{+2} U_{gi}^+}{2} \left[\left(\frac{R_g^+}{y_{gi}^+} \right)^2 - 1 \right] - R_g^+ y_{gi}^+ U_{gi}^+ \left[\frac{R_g^+}{y_{gi}^+} - 1 \right] \\ &- \int_{y_{gi}^+}^{R^+} g(y_g^+/R_g^+) y_g^+ dy_g^+ + R_g^+ \int_{y_{gi}^+}^{R^+} |g(y_g^+/R_g^+)| dy_g^+. \end{aligned} \quad (6-6)$$

Figure 12 Schematic Diagram of Pressure Drop and Void Fraction Computation Method

If we use Hinze's³⁴ $g(y/R)$ function for a pipe (p. 518, Figure 7-30) approximated by

$$g(y/R) = \frac{-1}{2} \left[1 - \cos\left(\frac{5\pi y}{4R}\right) \right], \quad (6-7)$$

we obtain

$$\begin{aligned} & - \int_{y_{gi}^+}^{R^+} U_g^+ (R_g^+ - y_g^+) dy_g^+ \\ &= \frac{R_g^{+2}}{K} \left\{ \frac{3}{4} + \frac{1}{2} \ln \left| \frac{y_i}{R} \right| + \frac{1}{4} \left(\frac{y_i}{R} \right)^2 - \frac{y_i}{R} \right\} \\ & - \frac{R_g^{+2} U_{gi}^+}{2} \left\{ \frac{y_i}{R} - 1 \right\}^2 - 0.204 R_g^{+2} \end{aligned} \quad (6-8)$$

from Equation (6-6) to order $(y_i/R)^2$ since $(y_i/R) \ll 1$. Equation (6-5) with Equation (6-8) can be solved for U_{rg} and y_i by iteration.

As shown in Figure 12, one can now calculate the average void fraction over the entire cross section, summing the film and core contributions. Finally, one can calculate the pressure drop by the force balance relation:

$$\frac{dP}{dz} = - \left[\frac{2 U_{rg}^2 \rho_g}{R} + (1 - \bar{\alpha}) \rho_l g \right]. \quad (6-9)$$

Tables 1, 2, and 3 contain calculated and available experimental results for dP/dz , $\bar{\alpha}$, and δ for the same input mean data. The effects of different input entrainment values and different values of K , the constant in the gas core velocity profile, are examined.

Schraub, Simpson, and Janssen³¹ reported the experimental values given in Table 1 for E , dP/dz , $\bar{\alpha}$, and δ . It is apparent, from Table 1, that for $\bar{X} = 0.38$ the best agreement between predicted and experimental results is obtained with $K = 0.3$ and the experimentally determined value for E . These results show that the dP/dz , $\bar{\alpha}$, and δ are strongly dependent upon the entrainment fraction. Results for other quality runs verify that $K = 0.3$ produces the best overall agreement between calculated and experimental results for $X > 30\%$. For $X < 30\%$, it appears, from Figures 13 and 14, that $K = 0.4$ produces better agreement. At first, one might attempt to specify K as an empirical function of X , varying from 0.4 (say) at low qualities to some lower value at higher qualities. This function would be merely an

artifice, clearly not in line with the suggestion of Alia, et al.,¹⁰ that more core droplets (lower mean quality) should produce a lower value for K . Based on the data of Schraub, Simpson, and Janssen,³¹ one could simply state that this computational scheme does not apply for $X < 30\%$. However, there is sufficient uncertainty in the measured value of entrainment to question this trend with quality, and to ask how the trend would be affected if the entrainment fraction at $X < 30\%$ were less than measured. For the $X = 22\%$ run, for example, the calculated values of δ and $\left| \frac{dP}{dz} \right|$ would be correspondingly raised and the calculated value of $\bar{\alpha}$ would be lowered, all three thereby coming into better agreement with the data.

Calculations from the data of Alia, et al.,¹⁰ are shown in Table 2. Unfortunately, only experimental dP/dx was reported by Alia, et al., for these flows. However, these results confirm that $K = 0.3$ predicts dP/dx in closer agreement with experimental values than $K = 0.4$. Also, the Π_1 entrainment correlation produces a much higher entrainment than Π_3 which causes much lower pressure drops, in better agreement with the reported experimental values.

In Table 3, with the data of Gill, et al.,⁶ it is found that the Π_1 entrainment correlation (with which the experimental entrainment data of Gill, et al., agrees) and $K = 0.3$ produce a dP/dx value in best agreement with experiment. Also, predicted δ is within 20% of the experimental value.

It is noted that Gill and Hewitt⁷ obtained asymptotic dP/dz data on the same apparatus used by Gill, et al.,⁶. Gill and Hewitt introduced the liquid at the tube center, while Gill, et al., introduced the liquid around the periphery. Consequently, E was higher and dP/dz was lower for Gill and Hewitt than for Gill, et al. Again, $K = 0.3$ produces predicted dP/dz in better agreement with experiment than $K = 0.4$.

One must conclude from this discussion that E and dP/dz are intimately related and that with $K = 0.3$ and the known E , dP/dz can be closely predicted (within 25% of experimental value). (It is realized that the change from $K=0.4$ in Section 4 to $K=0.3$ in the present computations amounts to changing K so as to absorb the inaccuracies of the phenomenological models in this computational method.)

On the other hand, one generally does not know E except from a correlation. As discussed in Section 3, it is apparent that some entrainment data are rig and measurement technique dependent, making the Π_1 and Π_3 correlations uncertain. Thus, the conclusion is reached that an E versus dP/dz correlation is more universal. Of course, Π_1 and Π_3 entrainment correlations may be determined experimentally for a given rig in order that dP/dz can be predicted more accurately from that rig.

Figure 13. Comparison of Calculated and Experimental Pressure Drop and Void Fraction for the Data of Schraub, Simpson, and Janssen (Ref. 31)

Figure 14. Comparison of Calculated and Experimental Liquid Film Thickness for the Data of Schraub, Simpson, and Janssen (Ref. 31).

Table 1
CALCULATED AND EXPERIMENTAL RESULTS
FOR THE ROUND TUBE DATA OF SCHRAUB,
SIMPSON, AND JANSSEN
($G_g = 7.2 \times 10^4$ lb/ft²-sec)

$\bar{X}$	Entrainment Correlation	K	E	δ (in.)	Calculated			Experimental		
					U_T (ft/sec)	$\bar{\alpha}$	dP/dz (lbf/ft ³)	dP/dz (lbf/ft ³)	$\bar{\alpha}$	δ (in.)
0.38	Π_1	0.4	0.24	0.020	7.8	0.920	-45	-21.9	0.945	0.004 to 0.010
0.38	Experi- mental Value	0.4	0.74	0.009	6.2	0.956	-28	-21.9	0.945	0.004 to 0.010
0.38	Π_1	0.3	0.24	0.023	6.7	0.907	-35	-21.9	0.945	0.004 to 0.010
0.38	Experi- mental Value	0.3	0.74	0.011	5.2	0.949	-21	-21.9	0.945	0.004 to 0.010
0.29	Experi- mental Value	0.4	0.80	0.010	6.2	0.947	-28	-25.9 to -24.9	0.925	0.009 to 0.014
0.29	Experi- mental Value	0.3	0.80	0.012	5.2	0.939	-21	-25.9 to -24.9	0.925	0.009 to 0.014
0.22	Experi- mental Value	0.4	0.84	0.011	6.4	0.934	-31	-35.2 to -32.7	0.900 to 0.885	0.015 to 0.020
0.22	Experi- mental Value	0.3	0.84	0.014	5.4	0.926	-24	-35.2 to -32.7	0.900 to 0.885	0.015 to 0.020
0.48	Experi- mental Value	0.4	0.71	0.008	5.9	0.965	-25	-20.2 to -17.8	0.962 to 0.954	0.005
0.48	Experi- mental Value	0.3	0.71	0.009	4.9	0.959	-18	-20.2 to -17.8	0.962 to 0.954	0.005
0.62	Experi- mental Value	0.4	0.67	0.005	5.5	0.975	-21	-21.8 to -13.9	0.971 to 0.975	0.005
0.62	Experi- mental Value	0.3	0.67	0.007	4.5	0.970	-15	-21.8 to -13.9	0.971 to 0.975	0.005

Note: See Reference 31.

Table 2
CALCULATED AND EXPERIMENTAL RESULTS
FOR THE ROUND TUBE DATA OF ALIA, et al.

Gas-Liquid Flow	$\bar{X}$	Entrainment Correlation	K	E	δ (in.)	U_{τ} (ft/sec)	$\bar{\alpha}$	Calculated dP/dz (lbf/ft ³)	Experimental dP/dz (lbf/ft ³)
Argon-water	0.351	Π_1	0.3	0.404	0.045	3.6	0.807	-56	-40.3
Argon-water	0.351	Π_3	0.3	0.228	0.053	3.9	0.788	-65	-40.3
Argon-water	0.351	Π_3	0.4	0.228	0.049	4.2	0.801	-73	-40.3
Argon-water	0.786	Π_1	0.3	0.312	0.013	2.4	0.947	-23	-17.7
Argon-water	0.786	Π_3	0.3	0.213	0.013	2.5	0.944	-25	-17.7
Argon-water	0.786	Π_3	0.4	0.213	0.012	2.9	0.951	-32	-17.7
Argon-alcohol	0.294	Π_1	0.3	0.999	0.001	1.8	0.896	-16	-31.8
Argon-alcohol	0.294	Π_3	0.3	0.383	0.062	4.0	0.741	-67	-31.8
Argon-alcohol	0.294	Π_3	0.4	0.383	0.057	4.3	0.754	-75	-31.8
Argon-alcohol	0.786	Π_1	0.3	0.882	0.051	2.0	0.969	-15	-13.7
Argon-alcohol	0.786	Π_3	0.3	0.330	0.016	2.5	0.942	-24	-13.7
Argon-alcohol	0.786	Π_3	0.4	0.330	0.012	2.9	0.950	-31	-13.7

Note: See Reference 10.

Table 3
 CALCULATED AND EXPERIMENTAL RESULTS
 FOR THE ROUND TABLE DATA OF GILL, et al.,
 AND GILL AND HEWITT
 (1-1/4-inch Inside Diameter, $\bar{X} = 0.33$,
 $\bar{W} = 1500 \text{ lb/h}$)

Water Introduced on Wall (Ref. 6)
 Experimental $E \approx 0.5$

Experimental

Entrainment Correlation	K	E	δ (in.)	U_τ (ft/sec)	$\bar{a}$	dP/dz (lb/ft ³)	dP/dz (lb/ft ³)	δ (in.)
Π_1	0.4	0.488	0.012	20	0.958	-58	-33	0.013
Π_3	0.4	0.192	0.017	22	0.945	-73	-33	0.013
Π_1	0.3	0.488	0.015	16	0.950	-41	-33	0.013
Π_3	0.3	0.192	0.021	18	0.934	-53	-33	0.013

Water Introduced on Centerline (Ref. 7)

Experimental	0.4	0.75	0.008	17	0.973	-42	-23	0.008
Value	0.3	0.75	0.009	14	0.968	-29	-23	0.008

Note: See References 6 and 7.

7. CONCLUDING COMMENTS

During the course of this study, a low-pressure, horizontal, air-water channel was constructed at APED, as reported in Reference 40. Liquid film thickness and flow rate measurements indicated that when no void was observed in the liquid film, the zero void Von Kármán theory as applied by Levy¹⁶ described the film thickness-flow rate relation. At the same time, no liquid entrainment or roll waves were observed. This is in agreement with the idea that entrainment and roll waves are related. It also suggests that when there is no entrainment, one must use the zero void film model, while the results of Section 2 require the consideration of void when there are roll waves and entrainment.

The apparently universal G_f^+ versus δ^+ film flow rate-thickness relationships described by the experimental data in Section 2 imply that there is a universal nondimensional velocity profile in the film. Hence, this is necessary proof that the Preston tube method of relating wall shear to impact pressure is valid for the film. Furthermore, this strengthens the confidence for using the Preston tube calibration presented by Schraub, Simpson, and Janssen³¹ as long as the probes are small enough to remain within the wall-film region.

The results from the computational scheme indicate that the entrainment fraction and pressure drop are closely related. When a known entrainment is input, the pressure

drop is predicted within 25% of the experimental value for $X > 25\%$. In many cases, the agreement is better than 25%, and improves as the mean quality is increased. Since it is apparent that the entrainment fraction is rig-dependent to some degree, it is evident that the pressure drop will be affected accordingly; i.e., an increase in entrainment fraction will produce a decrease in pressure drop. Thus, if one wishes to minimize pressure drop in a round tube, one can design the flow inlet configuration to promote liquid entrainment, such as introducing the liquid along the centerline, as done by Gill and Hewitt⁷. It was further found that when the pressure drop was closely predicted, the film thickness and average void fraction were also closely predicted.

Certainly this prediction scheme is more complicated than a simple pressure drop correlation such as the Lockhart-Martinelli⁴¹. However, more physical phenomena are considered so that the pressure drop prediction is sensitive to the entrainment fraction while also predicting the film thickness. Also, the present study of the entrainment fraction indicates that the liquid surface tension is important in predicting the entrainment fraction from a given rig. This implies that the surface tension should influence the pressure drop—in contrast to the Lockhart-Martinelli correlation which includes no surface tension dependency.

NOMENCLATURE

D	=	Tube diameter [L]
E	=	Entrainment fraction
G	=	Mass flux [M/L ² T]
G ⁺	=	G _f δ/μ _g , dimensionless
g	=	Gravitational constant [L/T ²]
L	=	Characteristic length; also axial length
ℓ	=	Mixing length [L]
P	=	Pressure [F/L ²]
R	=	Tube radius [L]
r	=	Local radius [L]
U	=	Local velocity in flow direction [L/T]
U _g ⁺	=	U/U _{τg} , dimensionless
U _ℓ ⁺	≡	U/U _{τℓ} , dimensionless
U _{τg}	≡	$\sqrt{\tau_w/\rho_g}$, [L/T]
U _{τℓ}	≡	$\sqrt{\tau_w/\rho_\ell}$, [L/T]
V _r	=	Turbulent radial drift velocity [L/T]
X	=	Quality
y	=	Distance measured from wall [L]
y _g ⁺	≡	y U _{τg} /ν _g , dimensionless
y _ℓ ⁺	≡	y U _{τℓ} /ν _ℓ , dimensionless
z	=	Axial length [L]
α	=	Local void fraction
ᾱ	=	Average void fraction
δ	=	Time-averaged liquid film thickness [L]
ε	=	Diffusion coefficient
μ	=	Dynamic viscosity [F · T/L ²]
ν	=	Kinematic viscosity [L ² /T]
ρ	=	Density [M/L ³]
σ	=	Surface tension [F/L]
τ	=	Shear stress [F/L ²]

Subscripts

c	=	Core
CL	=	Tube centerline value
f	=	Film
g	=	Gas phase
i	=	Interface condition
ℓ	=	Liquid phase
max	=	Maximum value
w	=	Wall value

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