ACOUSTIC DETECTION OF THE COLLAPSE OF A SODIUM VAPOR BUBBLE
IN AN INFINITE SEA OF SODIUM

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NOMENCLATURE

A	Surface area (m^2 , cm^2)
A_G	Amplifier gain response
B_e	Effective bandwidth (Hz)
C	Sonic velocity
C_l	Sonic velocity of liquid
C_M	Microphone calibration factor (Pico coulombs/bar)
D_T	Detection threshold
d	Detection index
$H(\omega)$	Filter response function
I_a	Acoustic intensity
K	Acoustic wave number
N_L	Noise level
Pa	Pascal
P	Pressure
P'	Peak acoustic pressure at bubble surface
$\bar{P}$	rms acoustic pressure
$\bar{P}_N$	rms acoustic noise pressure
$\bar{P}_{S+N}$	rms acoustic signal and noise pressure
P_a	Acoustic pressure
P_b	Internal bubble pressure
P_d	Probability of detection
P_{fa}	Probability of false alarm
P_∞	Static ambient pressure = distant pressure of the liquid
PSD	Power spectral density

NOMENCLATURE (cont'd)

Q	Charge
R	Radial distance, initial radius of bubble
R_o	Mechanical equilibrium radius of bubble
R_{min}	Minimum bubble radius
χ	Radial coordinate
S_L	Sound source level
S	$= (\overline{S + N} - \overline{N}) \overline{N}$, where $\overline{S + N}$ is the mean signal and noise power and $\overline{N}$ is the mean noise power
S_o	Source sound pressure level
S_E	Signal excess
T_1, T_2	Predetermined detection threshold
T_L	Acoustic transmission loss from source to the microphone detector
t	Time variable
V	Volume
V_o	Initial voltage, volume
V_{min}	Minimum volumes
W_e	External mechanical work
W_a	Acoustic power
β	Coefficient of compressibility
σ	Standard deviation
σ_s	Surface tension
ΔT_s	Degree of superheat
η_c	Conversion efficiency
ρ	Density

NOMENCLATURE (cont'd)

τ	Time; pulse width; system integration time constant
ρ_v	Vapor density
ρ_l	Liquid density
ω	Angular frequency
ϕ	Phase
ω_c	Center frequency of band with B_e

ABSTRACT

A discussion of the problem of sodium vapor bubble collapse is presented in this report. The physics of vapor collapse is presented in light of the work by Peppler et al. Theoretical estimates of the sound source level based on the work by Rayleigh and Judd are compared to an approximate pressure-volume work approach and recent experimental observations. Reactor ambient noise and transmission loss considerations are presented in regard to their impact on this detection problem. A methodology is proposed which considers the importance of the sound source level, ambient noise, transmission loss and a detection threshold and provides a means by which the feasibility of sodium vapor bubble collapse detection in an operating LMFBR may be assessed. The interrelationships between the detection threshold and the probability of detection and false alarm are discussed and applied to a standard acoustic square law detection system. This analysis clearly illustrates that the feasibility of such a detection system is strongly dependent on the knowledge of expected sound source levels, ambient noise levels and the transmission loss between the source and receiver. Furthermore, requirements of a high degree of probability of detection and a low probability of false alarm were found to require a high signal to noise ratio for a single sensor system but that the probability of false alarm requirement could be relaxed for systems multiple independent sensors. Finally, the need for additional experimental and theoretical information is presented in terms of sound source levels, ambient noise and a means for determining transmission loss.

1.0 INTRODUCTION

The objectives of this report are to discuss the detection, classification, and, possibly, the localization of boiling in a liquid metal fast breeder reactor by means of a few acoustic sensors positioned in the outlet plenum of the reactor. When boiling occurs, sounds are produced during the nucleation of a bubble, its subsequent growth, and its eventual collapse. As it is anticipated that the primary source of acoustic signal is the bubble collapse, this report discusses the phenomenon of a bubble collapse in an infinite sea of sodium with a random acoustic noise background. This report is based on a literature search conducted by the author and represents the preliminary results of a brief investigation. As this report deals primarily with an introductory overview of the detection process, much of the material must be considered tutorial rather than an original contribution.

Section 2 summarizes the physical processes of spherical bubble growth and collapse. Data presented by Peppler¹² et al. were chosen as representative of the collapse phenomenon. A plausible physical model is proposed which (1) distinguishes between initial bubble oscillation and the pressure pulses associated with bubble collapse; and (2) delineates the role of subcooling and superheating in the process of bubble growth and collapse. This model was not extended to more complex geometries and is presented to provide a qualitative description of the process.

Section 3 presents a bounding analysis on the peak acoustic pressures that may be generated as the result of bubble collapse. Three calculations are summarized, using the Rayleigh¹⁴ approach, the Judd^{9,10} approach, and the PV* approach, respectively. While none of these models is exact, each is

*Pressure-volume

included to bracket the range of peak acoustic pressures that may be encountered. Since the bounding analysis entails transient broad-band events, parameters of interest in characterizing these phenomena are peak acoustic pressure, pulse bandwidth, and effective pulse duration.

Some considerations of reactor ambient noise and transmission loss are discussed briefly in Section 4. The brevity of this discussion is due to the lack of experimental and theoretical data available on liquid metal fast breeder reactors.

In most acoustic systems, the detection process requires the determination of a detection threshold. Accordingly, in Section 5, a concept of the latter is discussed, including its interrelationship with the detection index, probability of detection, and probability of false alarm. These considerations are then applied to the problem of sodium vapor bubble collapse detection in a liquid metal fast breeder reactor.

In Section 6, a methodology is presented for determining the detectability of sodium vapor bubble collapse. Finally, this methodology is discussed, with emphasis on major areas of interest to a program concerned with research and development of a sodium boiling detection system.

2.0 PHYSICAL INTERPRETATION OF BUBBLE COLLAPSE

A detailed mathematical description of a sodium bubble collapse is complicated and beyond the scope of this current effort. Therefore, this section is confined to a summary of the physical processes involved in bubble growth and collapse. Although many experimental observations are available in the literature, this summary is based on observations by Peppler et al.,¹² and theoretical considerations by Rayleigh,¹⁴ Hickling and Plesset,⁸ and Judd.^{9,10}

Consider a spherical, superheated sodium vapor bubble released in a subcooled liquid. The initial radius of the bubble is R , and, initially, the vapor is superheated by a ΔT_s . The initial bubble is not constrained to be either in thermal or mechanical equilibrium with its surroundings. At time $t = 0$, the bubble is released in the subcooled liquid; this corresponds to the initiation of boiling in an infinite sea of sodium. For the purposes of this discussion, the vapor is assumed to obey the ideal gas law.

Due to the superheat, the bubble begins to expand which in this particular case can be considered adiabatic. Expansion of the bubble is rapid (the expansion rate exceeds the heat transfer processes governed by temperature gradient), until it reaches some mechanical equilibrium radius R_0 , which is determined by

$$P_{\infty} = P_b + \frac{2\sigma_s}{R_0},$$

whereupon the vapor pressure and temperature correspond to a saturated condition. At this time, the process is no longer adiabatic, and the

temperature potential due to the liquid subcooling initiates condensation of the bubble.

Condensation of the vapor produces a thermal boundary layer which effectively dampens the natural bubble oscillations and reduces the bubble radius, as shown in Fig. 1. As this process continues, the rate of decrease (compression) of the bubble radius increases and is controlled by both the inertial force of the liquid and the condensation rate; this causes a rarefaction around the bubbles, as shown in Fig. 1. At some time, the rate of collapse becomes very rapid and is controlled solely by the inertial force of the liquid. At this point, heat transfer by condensation may be neglected and the process again may be considered adiabatic. This rapid compression rate (which exceeds the acoustic velocity) causes very high pressures and temperatures in the bubble. These high pressures retard bubble collapse, and eventually the bubble reaches some radius R_{min} . If the radius at which the inertial forces become the governing consideration is R_0 , then $P_\infty \frac{4}{3} \pi (R_0^3 - R_{min}^3)$ is the external work that has been performed* and represents the potential energy available in the bubble for the rebound expansion.

After the bubble has reached R_{min} , a rapid expansion occurs, which causes the bubble to expand to some radius $< R_0$, and produces an acoustic wave which propagates out from the bubble (see pressure peak No. 1 in Fig. 1). This rapid expansion is halted by the inertial force of the sodium, and additional energy is dissipated by condensation of vapor. This process repeats itself,

*All effects due to surface tension, and energy losses due to heat transfer mechanisms are not considered.

Fig. 1. Pressure difference ($\Delta P = P_{\text{abs}} - P_{\text{system}}$) vs. time of a sodium boiling and recondensation experiment. (Data reproduced from Ref.12)

producing a lower pressure peak which is labeled No. 2 in Fig. 1. The difference in the peaks, of course, correspond to energy losses to the sodium by heat transfer; and by the time the second rebound has occurred, the bubble has effectively disappeared.

The foregoing description of the physical processes of bubble growth and collapse places in perspective the role of the various parameters such as superheat, subcooling, and bubble radius. Superheating of the vapor in the bubble may influence the determination of an initial bubble pressure and radius. The initial bubble radius and degree of subcooling of the surrounding liquid were observed to influence the bubble collapse and peak acoustic pressure. It is important to stress that the peak acoustic pressure observed from the collapse of a bubble can be related solely to the initial bubble radius and the subcooling of the liquid. The degree of superheating of the initial bubble is important only insofar as it determines the equilibrium radius of the bubble.

3.0 SOUND SOURCE LEVELS OF BUBBLES WITH DIFFERENT RADII

Intrigued by Reynolds' description of sounds emitted by water boiling in a kettle and his interpretation of the sounds as originating from the partial or complete collapse of the bubbles as they rose through the liquid, Lord Rayleigh¹⁴ in 1917 undertook to describe this phenomenon mathematically. Utilizing the continuity equation, and an expression for the PV work, he calculated the kinetic energy available, and the radial velocity of the bubble wall. This calculation was done initially for a cavity since Rayleigh reasoned that the ratio of the water vapor density to the liquid density was small ($\rho_v/\rho_l = 6.238 \times 10^{-4}$ for water; $\rho_v/\rho_l = 3.658 \times 10^{-4}$ for sodium) for a saturated vapor under one atmosphere of pressure. He determined that the time to collapse for an empty cavity

$$t = 0.91468 R_0 \sqrt{\rho_l / P_\infty}$$

where P_∞ is the distant pressure of the liquid. For a sodium vapor bubble of 1-cm radius, this time is calculated to be 0.83 milliseconds ($P_\infty = 1$ atm). Lord Rayleigh further compensated for the presence of a vapor which obeyed Boyle's law, by deriving an expression which quantitatively describes the peak acoustic pressure at the bubble surface:

$$P'^2 = (2/3)P/\beta(R_0^3/R^3 - 1) \rightarrow P_a = 236 \text{ dB re } \mu\text{Pa at 1 meter},^*$$

where β is the coefficient of compressibility of the sodium.

Since that time, many investigators have attempted to develop theoretical solutions which would describe, in more detail, the physical processes involved in the bubble collapse phenomenon. Although several of these developments are interesting, it is convenient to concentrate only on those developments which have a direct bearing on our specific problem. Judd⁹

*Note 1 $P_a = 1 \text{ N/m}^2$, 1 Atmosphere = 1 bu = $1.01325 \times 10^5 \text{ Pa}$ or 20 dB re atm is equivalent to 120 dB re μbar which is equivalent to 220 dB re μPa .

described the collapse of a single vapor bubble in liquid sodium by coupling the Hickling and Plesset⁸ treatment of the collapse of bubbles with a small quantity of noncondensable gas and the requirement of conservation of energy to describe the effects of heat transfer. This numerical calculation was performed on sodium vapor bubbles of various radii. For a 1-cm-radius bubble, the time to the collapse was estimated to be 1.2 milliseconds to a radius of 0.017 cm, and peak internal pressure of 20,000 atm, with an energy of 0.09 joules in the acoustic pulse. This represents a conversion efficiency $\eta_c = \text{Acoustic Energy}/\text{Initial Potential Energy}$ of 4%, and corresponds to a peak acoustic pressure of 231 dB re μPa at 1 meter.

The Judd and Rayleigh treatments yield close results as to the peak acoustic pressure to be observed in the collapse of a 1 cm radius sodium bubble; however, the collapse may be qualitatively outlined in less complicated terms. For example, the problem may be stated as follows:

Given a bubble of radius R_0 surrounded by a liquid with density ρ_l and sonic velocity C_l under a steady state distant pressure (P_∞), what is the upper limit on acoustic energy that may be released upon collapse of the bubble?

Again, neglecting surface-tension effects, the external mechanical work performed during compression of the bubble from some radius R_0 to some radius $R_{\min}$ is given by:

$$W_e = P_\infty (V_0 - V_{\min}) = 4/3 P_\infty \pi (R_0^3 - R_{\min}^3).$$

If $R_{\min} \ll R_0$, an approximate estimate of this potential is

$$W_e = 4/3 \pi P_\infty R_0^3.$$

For several values of R_0 with $P_\infty = 1$ atm:

R_0	$=$	W_e
2 cm		3.4 joules
1 cm		0.42 joules
0.1 cm		4.24×10^{-3} joules

If one assumes that this external work is converted into raising the temperature and pressure inside the bubble, and that the processes inside the bubble are adiabatic reversible, then this external work represents the potential energy available to be converted into kinetic and thermal energies during the rapid expansion of the bubble.

Two additional parameters are required to continue this problem.

The first is the time that it takes the bubble at $R_{\min}$ to expand back to its original radius. Experimental evidence indicates that this occurs in a very short time. Collapse times (neglecting the decrease due to condensation) appear to be on the order of a millisecond, and expansion times on the order of a microsecond. (Expansion of a very small bubble to a radius of 1 cm at the rate of the sonic velocity in sodium corresponds to an expansion time of 3.85 μ sec.)

The second parameter is a conversion efficiency which describes the fraction of the total energy which is converted into acoustic energy. Il'in and Morozov¹¹ reported that this fraction is of the order of 0.1% (10^{-3}); Dale⁶ and Judd⁹ reported a value of 0.2%. Thus the acoustic power available is

$$W_a = W_e \eta_c / \tau$$

Although observations indicate that rapid expansion of the bubble produces a steep acoustic pulse which cannot be described by the linearized acoustic wave equation, for simplicity, use shall be made of the relationships derived from the wave equation. Accordingly, the acoustic power $W_a = P_a^2 A / \rho C$, where A is the area over which the energy is spread (e.g., $4\pi R_o^2$), P_a represents the peak acoustic pressure, ρ the density of the sodium, and C the sonic velocity. Hence it follows that $P_a^2 A / \rho C = W_e \eta_c / \tau$, or that $P_a^2 = (\rho C / A) (W_e \eta_c / \tau)$. Now, substituting

$$\rho \sim 0.83 \times 10^3 \text{ kg/m}^3$$

$$C \sim 2.6 \times 10^3 \text{ m/sec}$$

$$A = 5.02 \times 10^{-3}, 1.26 \times 10^{-3}, 1.26 \times 10^{-5} \text{ m}^2$$

and performing the calculation yields the following

R_o , cm	W_e , joules	W_a , joules/sec	S_L , dB re μPa at 1m
2	3.40	0.425×10^3	199
1	0.42	0.106×10^3	193
0.1	0.004	0.106×10^3	173

Thus a 1-cm-radius bubble has a source level (S_L) of 193 dB re μPa at 1 meter. The reference to one meter is arbitrary. The conclusion from

this cursory analysis is that for bubbles with radii from 0.1 cm to 2 cm, one would expect sound source levels in the range 173-199 dB re μ Pa at 1 meter.

Anderson et al.¹ reported bubble and void collapse in the HTSL experiment with an equivalent sound source level of 148.8 dB re μ Pa at 1 meter (determined by correcting for geometrical considerations). Farahat et al.⁵ observed nucleate boiling with moderate subcoolings equivalent to 198.79 dB re μ Pa at 1 meter. Peppler et al.¹² determined that a 1-mm-radius bubble will produce sound source level of 180 dB re μ Pa at 1 meter. More recently, Anderson et al.⁴ have reported sound levels regarding bubble collapse with intervening structures to be of the order of 177 dB re μ Pa at 1/2 meter. The foregoing observations are summarized in Table 1.

TABLE 1. Theoretical and Experimentally Determined Sodium Vapor Bubble Collapse Sound Source Levels

	<u>Theoretical ($R_o = 1$ cm)</u>
•Rayleigh ¹⁴	234 dB re μ Pa at 1 m (134 dB re μ bar)
•Judd ^{9,10}	215-231 dB re μ Pa at 1 m (115-131 dB re μ bar)
•PV	193 dB re μ Pa at 1 m (93 dB re μ bar)
	<u>Experimental</u>
•Anderson ¹	$S_L = 148.8$ dB re μ Pa at 1 m (48.8 dB re μ bar)
•Farahat ⁵	$S_L = 199$ dB re μ Pa at 1 m (99 dB re μ bar)
•Peppler ¹²	$S_L = 180$ dB re μ Pa at 1 m ($R_o = 1$ mm); (80 dB re μ bar)
•Anderson ⁴	$S_L = 177$ dB re μ Pa at 1/2 m + attenuation (77 dB re μ bar)

4.0 REACTOR AMBIENT NOISE AND TRANSMISSION LOSS CONSIDERATIONS

A review of available literature on expected acoustic noise levels in the plenum of a reactor has revealed a paucity of experimental data.

The reactor noise field is probably due to:

Steady State Operations	{	Pump-induced cavitation
		Pump-induced vibrations
		Coolant flow noise
		Coolant flow-induced vibrations
Additional Transient Noise Sources	{	Control rod motions
		Valve openings and closings
		Thermal expansions and contractions of support structures

Experimental data have been obtained by Gavin⁷ on EBR-II in the YY05 experiment. During this experiment, which was designed to determine the radiation resistance of an ANL-developed high-temperature microphone, six microphones were placed at core level or higher in the EBR-II radial blanket. While the observed noise levels probably are not representative, Gavin reported an rms noise pressure level of 5.0×10^{-5} bars (134 dB re μPa) in a 150-Hz band width between 1 and 30 kHz; this corresponds to a band width-corrected spectrum level of 112 dB re $\mu\text{Pa}/\text{Hz}^{1/2}$.*

Recently, Anderson et al.⁴ reported loop noise levels at the FFM test loop to be of the order of 169 dB re μPa , broad band. Anderson et al.² also monitored the bulk sodium in the EBR-II Primary Tank, and reported background noise levels of 112 dB re $\mu\text{Pa}/\text{Hz}^{1/2}$. Thus for the lack of better information, one may use a noise level of 112 dB when normalized to 1 Hz, and of 169 dB when considering the broad-band noise. The data

*NOTE: Acoustic spectrum levels are quoted as acoustic energy in a 1 Hz band. As acoustic energy is proportional to acoustic pressure squared when referencing the spectrum level to pressure, one obtains a $\text{Hz}^{1/2}$ factor.

obtained by Anderson et al.² show (see Fig. 2) a dependency on primary sodium flow rate, and a resonance effect, between 20 and 30 kHz, of possibly a hydraulic nature. Based on the results of this experiment, Anderson was able to show a strong broad-band and flow-rate dependence.

Although transmission loss data for liquid metal fast breeder reactors are virtually not available at this time, it is possible to examine a given geometry and analytically determine an expected transmission loss. For the purpose of this report, only geometrical effects on acoustic intensity have been considered, with the following results:

<u>Geometry</u>	<u>Intensity Variation</u>	<u>Transmission Loss</u>
Tube	$I_a \propto R^0$	T_L due to attenuation only
Cylindrical	$I_a \propto R^{-1}$	$T_L = 10 \log(R) + \text{attenuation}$
Spherical	$I_a \propto R^{-2}$	$T_L = 20 \log(R) + \text{attenuation}$

For the case of the spherical geometry (i.e., a bubble in an infinite sea of sodium), the transmission loss is taken to be $20 \log(R)$, where R is the distance from the bubble center referenced to one meter.

Fig. 2. Sound pressure spectra (0-50 kHz range, 150-Hz band width) vs. primary sodium flow rate (per cent of rated at 62.5 MW) for the G1 microphone during an EBR-II shutdown.

(Reproduced from Ref. 2)

5.0 THE DETECTION THRESHOLD CONCEPT

The primary objective of a boiling detection system is to correctly identify the presence of a bubble-collapse signal in a random noise background. Thus a criterion is necessary on which a decision can be made that the signal is indeed identified correctly. Accordingly, the concept of a preassigned level requirement is introduced that ensures a certain degree of confidence in that decision. This preassigned level is defined as the detection threshold.

However, the determination of a preassigned threshold is strongly influenced by the desired degree of confidence. More specifically, once a threshold is determined, four possibilities arise:

- (1) One may observe that the signal at the detector input occurs with a certain frequency of correctness; i.e., with a certain probability of detection (P_d).
- (2) On the other hand, one may miss the occurrence of the signal with a certain frequency ($1-P_d$).
- (3) When no signal is present at the detector input and one is merely observing the presence of noise, there is the distinct possibility of determining falsely that the signal is present. This possibility is due to the fact that fluctuations in the noise background may cross the threshold and be misinterpreted as a signal. The frequency of these events is determined by the false alarm probability (P_{fa}).
- (4) No signal is present at the detector input and the decision is made accordingly and correctly. The frequency of occurrence of these events is determined by ($1-P_{fa}$).

These possibilities and their interactions are illustrated in Fig. 3.

One approach in the design of a detection system is to specify the detection threshold and then determine the system's probability of detection

<div> <div>DETECTOR OUTPUT</div> <div>DETECTOR INPUT</div> </div>	SIGNAL PRESENT	SIGNAL ABSENT
SIGNAL PRESENT	CORRECT DETECTION P_d	INCORRECT "MISS" $1 - P_d$
SIGNAL ABSENT	FALSE ALARM P_{fa}	NULL DECISION $1 - P_{fa}$

Fig. 3. Binary decision matrix

and probability of false alarm. Figure 4 illustrates such an approach. Figure 4a depicts acoustic signals with no noise, and Fig. 4b the random noise background; both are added and shown in Fig. 4c. Also shown in Fig. 4c are two predetermined thresholds. The first threshold (T_1) is set high enough to ensure that few false alarms occur; however, the probability of detection is low since only one of the signal plus noise levels passes over the threshold. The second threshold (T_2) is set at a lower level and all signals are correctly detected; however a few spurious noise pulses are also detected. In this instance, there is a high probability of detection, but also a high probability of false alarm.

It is apparent from this example that a relationship exists between the detection threshold and the probabilities of detection and false alarm. Accordingly, the designer of a detection system must either determine a threshold or specify a probability of detection and a probability of false alarm. Assuming that the system under consideration is a boiling detector which is to be a part of, or to augment, a reactor safety system, he must (1) specify a low false alarm probability and a high probability of detection, and (2) for a given noise background, determine a detection threshold. This specification poses the question: Is sufficient signal present to have a feasible detection system? In other words, given a detection probability, a false alarm probability, and a noise background, a certain signal-to-noise level is required.

The relationship between the detection threshold and the probabilities of detection and false alarm may be determined by considering a signal in a Gaussian noise background. This is illustrated in Fig. 5, which shows the rms output of an acoustic detection system, and is basically a histogram of observations of a noise background that has an rms value of $\bar{P}_N$ and a

Fig. 4. The problem of threshold setting

Fig. 5. A signal in a Gaussian noise background.

standard deviation of σ . Also shown is the distribution of the signal plus noise, and this is represented as a Gaussian distribution with a standard deviation of σ . The reason for this assumption is that, but the central limit theorem, the distribution of a large number of samples is Gaussian regardless of the distribution of the population from which the sample is chosen. In the case of an acoustic detection system, the number of samples is determined by the effective bandwidth (B_e), integration time constant (τ), and is statistically acceptable when this $B_e \tau$ product is greater than one. As evidenced by Fig. 5, an arbitrary threshold specifies both a false alarm probability and a probability of detection for a given separation of the means of the distributions. If one defines a detection index (d) equal to S^2/σ^2 , and specifies either a probability of detection or a probability of false alarm, then the detection threshold is determined. On the other hand, if one specifies both a probability of detection and a probability of false alarm, then one has specified a detection index and a threshold. This is the problem being considered here.

Considerable analytical effort has been focused on the relationship between probability of detection, probability of false alarm, and detection index. Recently these studies have been extended by Urick and Gaunard¹⁵ to cases of fluctuating signals in a random noise background. These cases are shown in Fig. 6, and the results are depicted in Fig. 7. Thus, specification of a required detection probability and a required false alarm probability determines the required detection index for this process.

For example, with a probability of detection (P_d) of 95%, Fig. 7 yields:

P_{fa}	$d = S^2/\sigma^2$
2%	36
5%	26
25%	16

Fig. 6. Waveform and amplitude distribution of fluctuating signals and noise . (Reproduced from Ref. 15)

Fig. 7. Plots of P_d vs. P_{fa} for various values of d , with and without fluctuations present. (Reproduced, in part, from Ref. 15)

Having specified the false alarm probability and the probability of detection, a detection index is determined. The problem at hand is: How does one relate this detection index to a detection threshold? For the case of a completely unknown signal in a background of Gaussian noise, Peterson and Birdsall¹³ have shown that, for a standard acoustic detector, the detection threshold (D_T) - defined as the ratio (in decibels) of the signal power (mean squared value) in the detector receiver bandwidth to the noise power (mean squared value) in a 1-Hz band measured at the receiver terminals - required to satisfy a specified probability of detection and false alarm may be expressed as follows:

$$D_T \approx 5 \text{ Log}(d B_e / \tau),$$

where B_e is the receiver bandwidth, τ is the system integration time, and d is the detection index, $d = S^2/\sigma^2$. Thus the detection threshold for a 95% probability of detection, a 5% probability of false alarm, and a detection index of 26 corresponds to 7.07 dB for a $B_e \tau$ product equal to one. (Note that decibel notation is used in referring to the detection threshold. The reason will be discussed in the next section.)

In summary, specification of a probability of detection (P_d) and a probability of false alarm (P_{fa}) specify a detection index d . This detection index in conjunction with the bandwidth B_e of the system and the system integration time (τ) determine the detection threshold. This determination of a detection threshold is basically a requirement on the signal-to-noise ratio to meet the detection criteria.

6.0 A METHOD FOR CALCULATING THE DETECTABILITY OF SODIUM VAPOR BUBBLE COLLAPSE

Collapse of a sodium vapor bubble in an operating liquid metal fast breeder reactor is indeed a complicated problem. It is not the intent of this report to solve this problem, rather to develop a method which may be applied to render an eventual solution.

Consider the problem of a single bubble in an infinite sea of sodium with a random distribution of noise sources. Although the actual collapse of the bubble will produce a broad band acoustic wave, for the purpose of this discussion consider the bubble as a monochromatic sound source. The linearized acoustic wave equation is assumed to be valid, whereupon the acoustic pressure at a radial distance r from the bubble can be expressed by:

$$P_a(r, \omega, t) = \frac{S_0}{r} \exp(iKr - i\omega t).$$

Furthermore, the distributed random noise sources throughout the volume produce a noise pressure field at r , which is given by:

$$P_n(r, \omega, t) = \iiint_V P_1(r_1, r, t, \omega_1, \phi_1) dV.$$

Thus addition of these two pressure fields yields the resultant noise at r . If this noise is sensed by an acoustic detection system, such as shown in Fig. 8, the following relations prevail, assuming that the microphone response and amplifier response is very broad band when compared to the band-pass filter.

Fig. 8. A typical acoustic detection system.

Let

$$P(t) = P_s(\chi, \omega, t) + P_n(\chi, \omega, t),$$

Then

$$Q(t) = C_M P(t) \text{ Coulombs,}$$

where C_M is the microphone response.

The voltage output of the amplifier is simply the amplifier transfer function (volts/coulomb)(A_G) multiplied by the charge output:

$$V(t) = C_M A_G P(t) .$$

This voltage is then passed through the band-pass filter, which, for the purposes of this discussion, has a step-function response such as

$$H(\omega) = 0, \text{ when } \omega < \omega_c - B_e/2 \text{ or } \omega > \omega_c + B_e/2;$$

$$H(\omega) = 1, \text{ when } \omega_c - B_e/2 < \omega < \omega_c + B_e/2 ,$$

where ω_c is the center frequency of the bandwidth B_e . Thus the filter function is the Fourier transform of $H(\omega)$:

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} H(\omega) [\exp(i2\pi\omega\tau)] d\omega \\ &= \frac{2\sin(\pi B_e \tau)}{\pi\tau} \cos(2\pi f\tau). \end{aligned}$$

The filter output then may be written as

$$V(t, \omega_c, B_e) = \int_{-\infty}^{\infty} H(\tau) (\omega, t-\tau) d\tau .$$

The square law detector circuit simply squares this voltage, which is integrated at the receiver output to yield:

$$V_{\text{out}} = \int_0^T V^2(t, \omega_c, B_e) dt.$$

It is interesting to note that when this integral is normalized by the product $B_e T$, an estimate of the power spectral density function for this band is realized:

$$\text{PSD} \simeq \frac{1}{B_e T} \int_0^T V^2(t, \omega_c, B_e) dt.$$

Thus the output of this detection scheme is proportional to the acoustic energy. Upon substitution of the previous variables

$$\text{PSD} \simeq \frac{1}{B_e T} \int_0^T dt \left[\int_{-\infty}^{\infty} d\tau H(\tau) (P_s + P_n) C_m A_G \right]^2,$$

or

$$\begin{aligned} \text{PSD} \simeq \frac{C_m^2 A_G^2}{B_e T} \int_0^T dt \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\epsilon H(\tau) H(\epsilon) [P_s(t-\tau) P_s(t-\epsilon) + P_s(t-\tau) P_n(t-\epsilon) \\ + P_s(t-\epsilon) P_n(t-\tau) + P_n(t-\tau) P_n(t-\epsilon)] \end{aligned}$$

On interchanging the t , ϵ and τ integrals, and if $P_n(t)$ and $P_s(t)$ are incoherent, the cross product terms vanish and

$$\begin{aligned} \text{PSD} \simeq \frac{C_m^2 A_G^2}{B_e T} \left[\int_0^T dt \int_{-\infty}^{\infty} d\epsilon \int_{-\infty}^{\infty} d\tau H(\tau) H(\epsilon) P_s(t-\tau) P_s(t-\epsilon) \right] \\ + \frac{C_m^2 A_G^2}{B_e T} \left[\int_0^T dt \int_{-\infty}^{\infty} d\epsilon \int_{-\infty}^{\infty} d\tau H(\tau) H(\epsilon) P_n(t-\tau) P_n(t-\epsilon) \right] \end{aligned}$$

The second term in this equation is the mean square noise power, designated by P_n^2 . The first term represents the mean square source value, with the $|x|^2$ removed from the integral;

$$\text{PSD} = \frac{C_M^2 A_G^2}{|x|^2} \cdot S^2 + (C_M^2 A_G^2) P_n^2$$

Using the assumptions employed in this example (which surprisingly hold in many instances), one may determine the signal power-to-noise power ratio at our standard detector output as

$$\frac{\text{Signal Power}}{\text{Noise Power}} = \frac{S^2}{|x|^2} \frac{1}{P_n^2} = \frac{\text{Mean Square Source Strength}}{(\text{Transmission Loss})(\text{Mean Square Noise Strength})},$$

where r^2 has been recognized as the geometrical transmission loss. The point of this laborious derivation is that the signal-to-noise ratio may be expressed in terms of multiplicative quantities or in decibel notation:

$$\begin{aligned} 10 \log((S/N)^2) &= 20 \log(S) - 20 \log(r) - 20 \log(P_n) \\ &= S_L - T_L - N_L \end{aligned}$$

Thus the output of a standard acoustic detection system may be expressed in a linear addition of logarithmic terms, and the signal excess at the receiver output may be defined by:

$$S_E = S_L - T_L - N_L.$$

The detection process requires that the S_E be greater than or equal to zero, with detection occurring with a preassigned level of confidence

when $S_E = 0$. It can be shown that the detection threshold discussed in the preceding equation may be added to this equation to yield

$$S_E = S_L - T_L - N_L - D_T .$$

When D_T is positive, a greater source level is required for detection to satisfy the point at which $S_E = 0$. As the D_T is determined by the P_d and P_{fa} , the value of DT can be raised or lowered by changing the specification on the required P_d and P_{fa} .

In summary, detection of a sodium bubble collapse in an infinite sea of sodium may be accomplished with a system such as illustrated in Fig. 8. The output of this system can be represented by an equation of the form

$$S_E = S_L - T_L - N_L - D_T$$

When $S_E = 0$, one has detectability with a degree of confidence equal to the probability of detection P_d and probability of false alarm P_{fa} , as specified by the detection threshold D_T . The term S_L is termed the source level at a reference distance from the bubble.

Thus to determine the detectability of a sodium vapor collapse in an infinite sea of sodium:

- (1) Specify P_d and P_{fa}
- (2) Determine detection index d
- (3) Calculate $D_T = 5 \log(dB_e/\tau)$
- (4) Determine the source level S_L
- (5) Determine the transmission loss T_L
- (6) Determine the expected noise level N_L

If $S_E = S_L - T_L - N_L - D_T \geq 0$, detection is possible with a probability of detection $\geq P_d$, and a false alarm probability $\leq P_{fa}$.

As an example of this method, on substituting values discussed earlier for a 1-cm diameter bubble with r^2 spreading in an infinite sea of sodium:

$$S_L \sim 200 \text{ dB re } \mu\text{Pa at 1 meter (Section 3.0),}$$

$$N_L \sim 169 \text{ dB re } \mu\text{Pa (Section 4.0),}$$

$$T_L \sim 0 \text{ at 1 meter (Section 4.0),}$$

and

$$D_T = 7.07 \text{ dB (Section 5),}$$

The signal excess at one meter is $S_E = 22.93 \text{ dB}$ for a $P_d \geq 95\%$ and $P_{fa} \leq 5\%$.

Another interesting concept is the figure of merit (FOM) of the system; this is defined for $S_E = 0$, and is equal to the transmission loss for this condition to arise:

$$\text{FOM} = S_L - N_L - D_T = 22.93 \text{ dB} ,$$

or detection with a confidence of $P_d = 95\%$ and $P_{fa} = 5\%$ up to 14.012 meters.

The degree of accuracy depends strongly on knowledge of each of the terms in this reactor sonar equation.

7.0 CONCLUSIONS AND RECOMMENDATIONS

This report is based on a survey of literature published up to 1975 on the subject of the detection of sodium boiling in a liquid metal fast breeder reactor.

Bounding calculations performed for a 1-cm-radius bubble indicate that reasonable amounts of energy are available close to the collapse of the bubble. When compared to experimental values, these bounding calculations yielded values higher than observed levels, but both theoretical and empirical values were found to be reasonable. The calculations for the 1-cm-radius bubble were performed because current hypothetical core accidents postulate flow blockage of more than one wire-wrapped fuel pin channel. Thus the implication is that subcooled nucleate boiling may occur near the center of the core, rapidly proceed into bulk nucleate boiling as the flow continues up the channel, and, finally, culminate in a void leaving more than one of these channels. This void probably will not be spherical and its collapse may be preceded by fragmentation. It is recognized that the overall situation is extremely complex and perhaps requires a more detailed mathematical model; nonetheless, these calculations provide a qualitative indication of the consequences.

The literature survey has revealed little information upon which one may realistically assign noise level values and transmission loss values. Noise level values may be obtained on prototypical and near prototypical liquid metal reactors. Transmission loss may be calculated with transmission line (lumped parameter) models, or empirically determined from testing prototypical structural members and fuel assemblies.

In this report, a 95% probability of detection and a 5% probability of false alarm were chosen. The question of accumulative probabilities with time should be addressed as well as coincident techniques from independent sensors. For example, the requirements on the probability of detection and false alarm may be relaxed if false alarms by each detector occur at a fairly reasonable random rate, since a coincident output from several detectors would eliminate many false alarms. On the other hand, the coincident output for non-random causal events is high.

A method is proposed in this report for determining feasibility of boiling detection in fast breeder reactors. Moreover, application of this methodology which requires solution of the equation:

$$S_E = S_L - N_L - T_L - D_T \quad ,$$

indicates that acoustic detection of boiling is feasible, but that the degree of feasibility is difficult to assess due to limited knowledge of fast breeder reactor system parameters.

The source level term(s) must be studied and characterized for various classes of hypothetical accidents; e.g., frequency bandwidth, center frequency, and time of duration for various bubble sizes characterized along with peak acoustic pressure. These source level studies may be accomplished by active participation in the SLSF, TREAT, and FFM series of experiments, which are designed to produce sodium boiling and voiding under a variety of conditions. However, participation generates still other considerations such as heat-transfer portions of these experiments.

The basic question that needs to be answered is: For a given flow coastdown, linear heat flux, and assembly geometry, what is the size of the bubble that is formed, and what is its resulting peak acoustic pressure, duration, bandwidth, and center frequency?

The noise level term (N_L) may be bracketed by performing reactor ambient noise studies on liquid metal reactors such as the EBR-II Upper Plenum Acoustic Experiment. These studies should produce power spectral density values for a variety of reactor operational conditions. Respective sources of noise should be identified and characterized in order that they may be distinguished from bubble collapse sounds; cavitation, flow noise, control rod motions, and pump-induced vibrations are sources of particular interest. Studies on prototypical LMFBRs are especially recommended from the standpoint of accumulating a library of reference sources and levels of noise.

The transmission loss term (T_L) is perhaps equally the most important and most difficult term to assess, since it can be composed of geometrical factors, energy loss by attenuation, scattering, and reflection considerations. A certain amount of analytical work may be done in this area, using lumped-parameter models to assess the ability of various paths to transmit acoustic energy to the detector. Nevertheless, more meaningful results can be obtained by introducing sound sources and measuring the transmission loss on actual fuel assemblies.

Finally, the detection threshold term (D_T). This term can be affected by the signal processing of the system. For the standard system discussed in this report, the detection threshold was shown to be dependent on the

detection index, system bandwidth, and system integration time. The spectral characterization of a given sound source should enable the design of an optimum filter and integration time constant to optimize this term. The information needed for these design considerations will be made available from the source level and noise level studies.

APPENDIX A: Units, Dimensions and Conventions

This report utilizes the recently adapted international system of units. Several of the more important of these units are tabulized as follows:

Quantity	Name	Symbol	Expression	Dimensions
Frequency	Hertz	Hz		s^{-1}
Force	Neutron	N		$m \cdot Kg \cdot s^{-2}$
Pressure	Pascal	Pa	N/m^2	$m^{-1} \cdot Kg \cdot s^{-2}$
Energy	Joule	J	$N \cdot m$	$m^2 \cdot Kg \cdot s^{-2}$
Power	Watt	W	J/s	$m^2 \cdot Kg \cdot s^{-3}$

Thus the standard reference acoustic pressure used this report is the μPa (10^{-6} Pa). The pascal is equal to 10^{-5} atmospheres or 10^{-5} bars. Secondly, pressures in this report are referred to as source levels referenced to 1 μPa . This dB notation may be explained as follows:

The acoustic intensity level of a sound of intensity I referenced to an intensity I_0 is

$$\text{Intensity level (dB)} = 10 \log(I/I_0)$$

As for plane waves, this intensity is proportional to the acoustic pressure (P) squared one has a sound pressure level or sound level as follows:

$$\text{Sound pressure level} = 10 \log(I/I_0) = 20 \log(P/P_0)$$

If one chooses the μPa as the reference pressure level, then a peak pressure of 1 ATM = 10^5 Pa results in a sound pressure level of

$$\text{SPL} = 20 \log(10^5 \text{ Pa} / 1 \mu \text{Pa}) = 220 \text{ dB re } \mu \text{Pa}.$$

To aid in the interpretation of these quantities referenced to 1 μPa Table A1 is presented.

TABLE A-1: dB Conversion Table

P(ATM)	dB re Psi	dB re μb	dB re dyne/cm^2	dB re μPa
10	43.35	140	100	240
1	23.35	120	80	220
10^{-1}	3.35	100	60	200
10^{-2}	-16.65	80	40	180
10^{-3}	-36.65	60	20	160
10^{-4}	-56.65	40	0	140
10^{-5}	-76.65	20	-20	120
10^{-6}	-96.65	0	-40	100

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