OCT 30 1967

MASTER

SC-R-67-1185

September 1967

Monograph

A TABLE OF OUTPUT PROBABILITY DISTRIBUTIONS FOR A QUANTIZING ENVELOPE DETECTOR WITH A SINUSOIDAL SIGNAL IN GAUSSIAN NOISE AT THE INPUT

T. S. Edrington

SANDIA LABORATORIES

OPERATED FOR THE U. S. ATOMIC ENERGY COMMISSION BY SANDIA CORPORATION

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ABSTRACT

The quantizing envelope detector is a memoryless device with binary output -- 1 if the input voltage exceeds a threshold; 0 otherwise. This document provides probability distributions of the detector output when the input is a sinusoidal signal in Gaussian noise.

ACKNOWLEDGMENT

The programming described in the Appendix is the work of John Norton, Division 9422.

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Introduction

Introduction

The quantizing envelope detector is a memoryless device with the input-output characteristic shown below.

The object of the present effort is the derivation of probability distributions of the output when the input is a sinusoid plus Gaussian noise. The results obtained here extend the tables in SCR-409.¹

¹ Williams, C. S., Output Mean and Deviation Versus Input S/N for Linear, Quadratic, and Quantizing Envelope Detectors with Saturation, SCR-409, May 1961. Williams deals with Gaussian signal plus Gaussian noise and sinusoidal signal plus sinusoidal noise (and with linear and quadratic, as well as quantizing, detectors). A comparison of our data with SCR-409 requires only the relationships $\bar{I} = AP_I(A)$ and $A = 1/P_I(A) \Big|_{S=0}$.

Analysis

Let S , N , and R denote the power of a stationary sinusoid, power of a stationary Gaussian process, and envelope voltage of the sinusoid plus noise, respectively. The probability density function of R

$$P_R(x) = (x/N) \exp(-x^2/2N - S/N) I_0(x \sqrt{2S/N})$$

is taken from Rice.² The probability mass function of I , the detector output voltage, is clearly

$$P_I(y) = \begin{cases} \int_{R_0}^{\infty} P_R(x) dx, & y = A \\ 1 - \int_{R_0}^{\infty} P_R(x) dx, & y = 0. \end{cases}^*$$

The problem, then, is the evaluation of

$$\int_{R_0}^{\infty} P_R(x) dx$$

with various values of R_0 and S/N . Machine solutions were obtained by the method described in the Appendix.

Results

Table I lists $P_I(A)$, the probability of nonzero output, as a function of S/N and $P_I(A) \Big|_{S=0}$.

²Rice, S. O., "Mathematical Analysis of Random Noise," BSTJ 23, 24, p. 106, 1944, 45.

*The notation $P_I(y)$ is equivalent to $\text{Prob}\{I = y\}$.

TABLE I
 $P_I(A)$ as a Function of S/N and $P_I(A)$ $\Big|_{S=0}$

$S/N \backslash P_I(A)$ $\Big _{S=0}$	0.0100	0.0500	0.1000	0.2000	0.2500	0.3333	0.5000
0.000	0.0100	0.0500	0.1000	0.2000	0.2500	0.3333	0.5000
0.001	0.0100	0.0501	0.1002	0.2003	0.2503	0.3337	0.5003
0.010	0.0105	0.0515	0.1023	0.2032	0.2535	0.3370	0.5035
0.100	0.0149	0.0653	0.1232	0.2319	0.2841	0.3691	0.5335
0.126	0.0163	0.0694	0.1293	0.2400	0.2928	0.3782	0.5419
0.158	0.0180	0.0745	0.1368	0.2500	0.3034	0.3892	0.5520
0.200	0.0204	0.0814	0.1466	0.2631	0.3172	0.4033	0.5650
0.251	0.0234	0.0898	0.1586	0.2787	0.3336	0.4202	0.5802
0.316	0.0275	0.1007	0.1740	0.2983	0.3541	0.4410	0.5989
0.398	0.0330	0.1148	0.1935	0.3226	0.3793	0.4664	0.6213
0.501	0.0405	0.1329	0.2179	0.3524	0.4100	0.4970	0.6479
0.631	0.0507	0.1564	0.2486	0.3888	0.4470	0.5333	0.6788
0.794	0.0648	0.1866	0.2869	0.4323	0.4909	0.5756	0.7140
1.000	0.0845	0.2255	0.3344	0.4842	0.5422	0.6243	0.7531
1.259	0.1120	0.2752	0.3922	0.5443	0.6007	0.6784	0.7951
1.585	0.1503	0.3378	0.4614	0.6120	0.6653	0.7364	0.8383
1.995	0.2033	0.4145	0.5414	0.6853	0.7336	0.7958	0.8803
2.512	0.2751	0.5057	0.6303	0.7608	0.8020	0.8530	0.9184
3.162	0.3687	0.6082	0.7229	0.8328	0.8652	0.9037	0.9499
3.981	0.4845	0.7154	0.8118	0.8955	0.9184	0.9443	0.9731
5.012	0.6162	0.8164	0.8880	0.9436	0.9576	0.9725	0.9878
6.310	0.7493	0.8992	0.9441	0.9749	0.9819	0.9890	0.9956
7.943	0.8635	0.9554	0.9779	0.9913	0.9940	0.9966	0.9988
10.000	0.9423	0.9852	0.9936	0.9978	0.9986	0.9993	0.9998
19.953	0.9966	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31.623	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

APPENDIX

Evaluation of $\int_{R_0}^{\infty} p_R(x) dx$

We let $N = 1$ and write

$$\int_{R_0}^{\infty} p_R(x) dx = e^{-S} \int_{R_0}^{\infty} x e^{-x^2/2} I_0(x \sqrt{2S}) dx \quad (1)$$

where

$$I_0(x \sqrt{2S}) = \sum_{k=0}^{\infty} \frac{(x \sqrt{2S})^{2k}}{2^{2k} (k!)^2} \quad (2)$$

Substitution of Equation (2) into (1) gives

$$\int_{R_0}^{\infty} p_R(x) dx = e^{-S} \int_{R_0}^{\infty} x e^{-x^2/2} \sum_{k=0}^{\infty} \frac{x^{2k} S^k}{2^k (k!)^2} dx$$

$$= e^{-S} \sum_{k=0}^{\infty} \frac{S^k}{2^k (k!)^2} \int_{R_0}^{\infty} x^{2k+1} e^{-x^2/2} dx$$

$$= e^{-S} \sum_{k=0}^{\infty} \frac{S^k}{(k!)^2} \int_{R_0^2/2}^{\infty} t^k e^{-t} dt$$

where

$$t = x^2/2.$$

But

$$\int_{R_o^2/2}^{\infty} t^k e^{-t} dt = e^{-R_o^2/2} \sum_{j=0}^k \frac{k!}{(k-j)!} \left(\frac{R_o^2}{2}\right)^{k-j},$$

so

$$\int_{R_o}^{\infty} p_R(x) dx = e^{-S-R_o^2/2} \sum_{k=0}^{\infty} \frac{S^k}{k!} \sum_{j=0}^k \frac{1}{(k-j)!} \left(\frac{R_o^2}{2}\right)^{k-j} \quad (3)$$

If $S = 0$,

$$\begin{aligned} P_I(A) \Big|_{S=0} &= \int_{R_o}^{\infty} x e^{-x^2/2} dx \\ &= e^{-R_o^2/2}. \end{aligned}$$

We pick values of $P_I(A) \Big|_{S=0}$; find

$$\frac{R_o^2}{2} = -\ln P_I(A) \Big|_{S=0};$$

substitute values of $R_o^2/2$ and $S = S/N$ into Equation (3); and evaluate Equation (3) by machine. Table II is a listing of the program.

TABLE II
Program Listing

08/01/67

```

PROGRAM BESMAG
DIMENSION P(50),S(50)
EPS=1,E-4
CHECK=EPS*,001
READ 26,N,M
26 FORMAT(16I5)
READ 27,(P(I),I=1;N)
27 FORMAT(8F10,0)
READ 27,(S(I),I=1;M)
DO 50 L = 1,N
DO 51 LL = 2,M
3 ARG=-LOGF(P(L))
PA=EXPF(-S(LL))
PB=P(L)
SUM=0.
TERMKA=PA*PB
TERMK=1.
TERMJO=1.
SUM=TERMK
JS=0
DO 10 K=1,100
KS=K
XK = K
TERMK=TERMK*S(LL)/XK
TERMJO=TERMJO*ARG/XK
TERMJ=TERMJO
TERM=TERMK*TERMJO
IF (TERM-0.) 46,45,46
46 TMAX=0.
SUM=SUM+TERM
DO 20 J=1,K
SAVE=TERM
AK = K-J+1
TERMJ=TERMJ*AK/ARG
TERM=TERMK*TERMJ
IF (TERM-TMAX) 47,47,48
48 TMAX = TERM
47 JS=J
SUM=SUM+TERM
IF (ABSF (SAVE/TERM)- 1.) 20,20,25
25 IF (ABSF (TERM) - EPS) 35,20,20
20 CONTINUE
GO TO-10
35 IF (TMAX - CHECK) 45,10,10
10 CONTINUE
45 SUM=SUM+TERMKA
6 PRINT 220,P(L), S(LL), SUM, KS, JS
220 FORMAT(1H0,4HP = E14.7,5X,4HS = E14.7,5X,11HINTEGRAL = E14.7,5X,
1K = 13,5X,4HJ = 13)
51 CONTINUE
50 CONTINUE
END

```