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LOGARITHMIC SPIRAL LIGHT PIPE FOR SCINTILLATORS

by

William C. Kaiser, Anthony J. MacKay and William W. Managan

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The use of light pipes between scintillators and photomultipliers is sometimes necessary when the scintillator must be placed in a strong magnetic field. The use of a logarithmic spiral to efficiently couple the light from the scintillator into a long cylindrical light pipe has been described previously.⁽¹⁻⁶⁾ The purpose of this report is to describe a procedure which offers a straightforward means of designing and making the logarithmic spirals. The method will be outlined and an example described. A brief review of the theory and tabulation of several useful cases is given in the Appendix.

In essence, the method is to produce a full-size cross-section template which can be used in a machine, such as a lathe, equipped with an automatic curve-follower attachment, thus converting the cross section into a body of revolution. First, the ratio of the diameters of scintillator to photomultiplier must be determined. Then the desired portion of the logarithmic spiral must be calculated. If one of the diameter ratios given in Tables II-VII can be used, the calculation is unnecessary. The calculated points are transferred to an oversized drawing, the scale being chosen to permit precision reduction to actual size. For the master drawing, it is necessary to use a material having good dimensional stability, such as Mylar film. The reduction can be accomplished by the usual photographic methods for making slides. If equipment for making printed circuit boards is available, this may also be used successfully. In the Electronics Division, a reduction ratio of 2 to 1 from master drawing to printed circuit board (Figure 1) for each dimension has been standardized. Hence, it is convenient to scale the masters larger by factors of 2, 4, 8, etc.

After the master has been reduced to actual size, an ultraviolet light contact print onto the printed circuit board (precoated with Kodak Photoresist) is made, developed in trichloroethylene, and the copper is etched off in a ferric chloride bath.

The printed circuit boards used had an epoxy-fiberglas reinforced base, 0.0625 inch thick, which could be machined or filed to follow the contour of the copper outlining the desired curve. It also provided sufficient mechanical rigidity and strength for use directly with a "follower attachment" on a lathe or similar machine.

Figure 1

Steps in Construction of Logarithmic Spiral Pattern

Since the light pipe must be highly polished after machining, it is desirable to make the template slightly larger than the desired finished

dimensions. This in turn means that it would be unrealistic to demand too high a degree of precision in the master drawing. For this reason a scale factor of 2 has seemed to be sufficient.

In connection with studies of the Mössbauer effect using the 14.4-kev gamma ray from Fe^{57} , it was necessary to have the scintillator in a high magnetic field. The low energy of the gamma ray and the nature of experiment made it necessary to use the most efficient light-collection system possible. It was decided to mount a NaI(Tl) crystal 1 in. in diameter by 0.040 in. thick onto a light pipe 8 in. long, using the logarithmic spiral. The diameter of the cylindrical portion of the light pipe was chosen to be 2 in. The calculations for this case are given in Table III. The master drawing and template are shown in Figure 1. The detector layout is shown in Figure 2. The material chosen for the light pipe was Lucite.*

Figure 2 Scintillation Counter Head with Logarithmic Spiral on Light Pipe

For comparison of light-collecting efficiency, the pulse height and line width (full width at half maximum) for the 14.4-kev line were to be observed for four cases: (a) crystal directly on photomultiplier tube, (b) crystal on a cylindrical light pipe of same length and diameter but without the spiral section, (c) the light pipe with the logarithmic spiral, (d) the same as (c) but with a 0.002-in. aluminum foil filter between source and crystal. The line was not sufficiently resolved to permit measuring the line width. Therefore, the ratio of line peak height to valley was recorded. The valley was between the signal and the noise with the tube at room temperature. The test conditions and results are summarized in Table I.

*Obtained under the trade name Cadco Cast Acrylic Resin Rods from Cadillac Plastic Co., 727 W. Lake Street, Chicago 6, Illinois

TABLE I

Condition	Light Pipe	Al ₂ O ₃ Reflector	Peak Position Channel Number	Ratio of Peak to Valley
(a)	None	No	62	1.05
(b)	Cylinder, 2 in. O.D. x 8 in. long	No	50	1.05
		Yes	58	1.05
(c)	Cylinder with log spiral 2 in. O.D. x 8 in. long	No	69	1.47
		Yes	70	1.50
(d) Same as (c) plus 0.002 in. Al foil filter between source and crystal	Cylinder with log spiral 2 in. O.D. x 8 in. long	Yes	70	2.57

Window: 0.0008-in. aluminum foil

Crystal: NaI(Tl), 1 in. in diameter by 0.040 in. thick

Photomultiplier: Selected Type 6363, Du Mont

Voltage = 1,140 v. Electronic gain = 1,500

Source: 14.4-kev gamma rays from Fe⁵⁷

The two cases (b) and (c) were tried with and without a powder reflector* between the polished Lucite surface and an aluminum housing. The presence of the reflecting powder increased the observed pulse height in case (b) and made no difference in case (c), as would be expected from the theory. The inference is that in case (b) the reflector was effective only near the scintillator, where the conditions for total internal reflection are not satisfied. The nonuniform character of the photocathode prevented a clear-cut interpretation of the results. Relative to case (a), case (b) showed a lower pulse height and a greater line width. Case (c) showed a slightly greater pulse height, much better resolution, and indicated that the logarithmic spiral is a very efficient means of introducing light from a scintillator into a long cylindrical light pipe. The improvement effected by the X-ray filter was significant. The filter removed the 6-kev K X ray while transmitting the 14.4-kev gamma ray from iron.

*Al₂O₃, A-5175 alumina, Linde Air Products, Division of Union Carbide and Carbon, 30 East 42nd Street, New York 17, New York.

APPENDIX

The derivation of the equations of the logarithmic spiral adapter below is based on the arrangement of the scintillator and light pipe shown in Figure 3. It is desired that rays of light emerging from the scintillator at any angle ϕ shall strike the curved surface so as to insure total internal reflection. This condition is met if the angle θ between the incident ray and the normal to the surface is greater than the critical angle, i.e., $\theta \geq \arcsin(1/n)$, where n is the index of refraction of the medium. For Lucite, $n = 1.5$. As the angle ϕ increases, a limiting condition is reached when $\phi = \theta$. At this point the tangent of the curve is parallel to the cylinder. It is only necessary to evaluate points on the logarithmic spiral for $0 < \phi < \theta$.

Figure 3 Arrangement of Scintillator and Logarithmic Spiral Light Pipe Adapter

Consider the radius vector r from the origin to point P (see Figure 3). The angle $(90 - \theta)$ between the radius vector and the tangent to the spiral at P is given by

$$\tan(90 - \theta) = r \frac{d\phi}{dr} \quad (1)$$

and is a constant, i.e.,

$$\tan(90 - \theta) = 1/a \text{ or } \tan \theta = a \quad (2)$$

Figure 4

Graph of Minimum D/d Versus the Critical Angle θ Showing Values for Several Light Pipe Materials

Material	Index	Wavelength, $\text{\AA}$	Reference
Fused Quartz	1.468	4200	7
Lucite (Polymerized Methyl Methacrylate)	1.502	4358	8
Glass			
a. Ordinary Crown	1.528	4340	9
b. Dense Flint	1.681	4340	9
Synthetic Sapphire	1.781	4358	10
Sodium Iodide*	1.847	4250	

*This value obtained by experiment using a NaI prism inside our dry room facility.

Separation of the variables in (1) gives

$$\frac{dr}{r} = (\tan \theta) d\phi \quad . \quad (3)$$

Integration of (3) leads to

$$\ln r = (\tan \theta) \phi + \text{constant} \quad (4)$$

or

$$r = r_0 e^{(\tan \theta) \phi} \quad , \quad (5)$$

where r_0 equals the diameter $\underline{d}$ of the crystal

In evaluating the curve, one may assign a constant value to θ in fitting the spiral to arbitrarily chosen values of D/d , where D is the diameter of the photomultiplier. Figure 4 shows the ratio D/d vs. θ and indicates the limiting condition for Lucite. For any particular material, the limiting values of θ may be found from its index of refraction. For Lucite, the limiting value is seen to be about 41.8 degrees. The corresponding limiting ratio D/d is about 1.86. It should be emphasized that each material on Figure 4 is suitable for use in all cases where the ratio D/d is greater than the value given on the graph. Tables II and VII have been prepared for limiting conditions for Lucite and synthetic sapphire, respectively. Tables III, IV, V, and VI have been prepared for values of θ corresponding to D/d values of 2, 2.5, 3 and 4, respectively.

From Figure 3 the relationship

$$\frac{D}{d} = \frac{2r}{d} \cos \theta - 1 \quad (6)$$

is found for $\phi = 0$.

Given d for the scintillator and D for the photomultiplier, a check may be made to see whether Tables II - VII can be used. If not, substitute D and d into equation (6) and solve for $(r/d) \cos \theta$, where $r = d e^{\theta \tan \theta}$. Rewriting equation (6), one gets

$$(r/d) \cos \theta = 1/2[(D/d) + 1]$$

Then, using limiting values of θ corresponding to available materials, one tries to find cases where $e^{\theta \tan \theta} \cos \theta$ is less than $1/2[(D/d) + 1]$. These cases will give total internal reflection for all light rays emitted from the crystal which strike the sides of the logarithmic spiral light pipe.

TABLE II

$\theta = 42^\circ$ for $D = 1.876d$
(Minimum D/d for Lucite)

ϕ , degrees	r	$r \sin \phi$	$r \cos \phi$
0	1.0000	0.0000	1.0000
5	1.0817	0.0943	1.0776
10	1.1702	0.2032	1.1524
15	1.2658	0.3276	1.2227
20	1.3693	0.4683	1.2867
25	1.4812	0.6260	1.3425
30	1.6023	0.8012	1.3877
35	1.7333	0.9942	1.4198
40	1.8750	1.2052	1.4363
42	1.9349	1.2947	1.4379

TABLE III

$\theta = 43^\circ 44'$ for $D = 2.000d$

ϕ , degrees	r	$r \sin \phi$	$r \cos \phi$
0	1.0000	0.0000	1.0000
5	1.0912	0.0951	1.0870
10	1.1817	0.2052	1.1638
15	1.2846	0.3325	1.2408
20	1.3965	0.4776	1.3122
25	1.5181	0.6416	1.3758
30	1.6502	0.8251	1.4291
35	1.7939	1.0290	1.4695
40	1.9501	1.2535	1.4939
$43^\circ 44'$	2.0754	1.4347	1.4997

TABLE IV

$\theta = 48^\circ 55'$ for $D = 2.5d$

ϕ , degrees	r	$r \sin \phi$	$r \cos \phi$
0	1.0000	0.0000	1.0000
5	1.1053	0.0963	1.1011
10	1.2217	0.2121	1.2031
15	1.3503	0.3495	1.3043
20	1.4925	0.5105	1.4025
25	1.6496	0.6972	1.4951
30	1.8233	0.9117	1.5790
35	2.0153	1.1559	1.6508
40	2.2275	1.4318	1.7064
45	2.4620	1.7409	1.7409
$48^\circ 55'$	2.6634	2.0076	1.7501

TABLE V

 $\theta = 52^\circ 23'$ for $D = 3.00d$

ϕ , degrees	r	r sin ϕ	r cos ϕ
0	1.0000	0.0000	1.0000
5	1.1199	0.0976	1.1156
10	1.2542	0.2178	1.2351
15	1.4045	0.3635	1.3567
20	1.5729	0.5380	1.4780
25	1.7615	0.7444	1.5964
30	1.9726	0.9863	1.7084
35	2.2091	1.2671	1.8096
40	2.4740	1.5902	1.8952
45	2.7706	1.9591	1.9591
50	3.1027	2.3768	1.9944
52° 23'	3.2745	2.5937	1.9989

TABLE VI

 $\theta = 56^\circ 51'$ for $D = 4.00d$

ϕ , degrees	r	r sin ϕ	r cos ϕ
0	1.0000	0.0000	1.0000
5	1.1430	0.0996	1.1386
10	1.3064	0.2269	1.2865
15	1.4932	0.3865	1.4423
20	1.7066	0.5837	1.6037
25	1.9506	0.8244	1.7679
30	2.2295	1.1148	1.9308
35	2.5483	1.4616	2.0874
40	2.9126	1.8722	2.2312
45	3.3290	2.3540	2.3540
50	3.8050	2.9148	2.4458
55	4.3490	3.5625	2.4945
56° 51'	4.5700	3.8263	2.4987

TABLE VII

$\theta = 34.2^\circ$ for $D = 1.482d$
 (Minimum D/d for Synthetic Sapphire)

ϕ , degrees	r	r sin ϕ	r cos ϕ
0	1.0000	0.0000	1.0000
5	1.0611	0.0925	1.0571
10	1.1259	0.1955	1.1088
15	1.1947	0.3092	1.1540
20	1.2677	0.4336	1.1913
25	1.3452	0.5685	1.2192
30	1.4274	0.7137	1.2362
34.2	1.5003	0.8433	1.2409

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