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Engineering and
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ERROR DUE TO DEFLECTION IN A TWO-AXIS
MEASURING MACHINE

MASTER

J. R. Johnson, Jr.

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

operated for the ATOMIC ENERGY COMMISSION under U. S. GOVERNMENT Contract W-7405 eng 26

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ABSTRACT

Error due to the deflection caused by probe-spring forces in a two-axis measuring machine has been derived. Tests were run to obtain data for the determination of this error, and the results of a computer program were used to calculate errors for different amounts of probe displacement with part contact at various angles.

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SUMMARY

Error due to deflection caused by probe-spring forces in a two-axis measuring machine is dependent upon the part contact angle, amount of probe displacement, probe spring constant, and compliances of the measuring machine. By derivation, this error can be expressed as a function of the measuring machine deflection rates, zero-position components of the probe displacement, actual probe displacement, and part contact angle. Knowing these deflection rates and zero-position components, the error can then be calculated for any probe displacement at any part contact angle.

Results indicate that errors up to 72 microinches can be introduced in the inspection of variable-surface-angle parts without any part surface deviation at all, and that errors of ± 45 microinches can be introduced with part surface deviations of ± 2 mils and only rectilinear surfaces.

INTRODUCTION

With the inspection technology developed to date, it becomes increasingly difficult to ignore sources of error which were neglected in the past. For measuring machines where physical contact is made with the part, static forces between the part and measuring-head probe are developed which cause a deflection of the measuring head, resulting in reading errors which can no longer be neglected. These forces are usually caused by the displacement of a spring-loaded probe, and will depend on the probe spring constant, amount of probe displacement, and part contact angle. The resulting measuring-head deflection depends on the forces and the corresponding compliances, which are unique for a given machine design. If all readings were taken at the same part contact angle and amount of probe displacement, there would be no problem. However, if the setup procedure specifies zeroing the measuring probe on horizontal and vertical surfaces, errors will be introduced when measuring surfaces at other angles, and they will be magnified for surfaces that deviate from the zero position of the measuring probe.

ERROR DUE TO DEFLECTION

MEASURING MACHINE DESCRIPTION

Of particular interest in this report is a two-axis measuring machine with a constant-angle probe. The measuring head is mechanically positioned along two axes of motion in a vertical plane, such that contact with a perfect part produces a zero reading of the measuring probe. The probe is displaced only in a constant direction which, in this case, is at an angle of 45 degrees from the vertical. Knowing the part surface angle, deviation from zero along this constant direction is then adjusted to provide the deviation along the direction normal to the part surface.

DERIVATION OF THE READING ERROR

Assuming a probe spring constant of K , the measuring probe displacement, D_g , due to a force, F_g , in the probe direction of motion is:

$$D_g = K F_g. \quad (1)$$

From Figure 1 it can be seen that if friction is neglected, a force, F , applied to the probe at any angle is related to F_g by:

$$F = \frac{F_g}{\cos (45^\circ - \theta)}, \quad (2)$$

and the component of probe displacement, D , in the direction of the applied force, F , is related to D_g by:

$$D = D_g \cos (45^\circ - \theta). \quad (3)$$

This force can be resolved into the horizontal and vertical components shown in Figure 1 by using:

$$F_h = F \sin \theta = \frac{D}{K} \frac{\sin \theta}{\cos^2 (45^\circ - \theta)}, \text{ and} \quad (4a)$$

$$F_v = F \cos \theta = \frac{D}{K} \frac{\cos \theta}{\cos^2 (45^\circ - \theta)}. \quad (4b)$$

Figure 1. RELATION AND RESOLUTION OF FORCE APPLIED TO A MEASURING PROBE.

From elastic theory it can be shown that the components of the measuring-head deflection are related to the force components and corresponding compliances, C_{ij} , by:

$$Y = C_{yv} F_v + C_{yh} F_h = \frac{D}{\cos^2 (45^\circ - \theta)} \left[\frac{C_{yv}}{K} \cos \theta + \frac{C_{yh}}{K} \sin \theta \right], \text{ and } (5a)$$

$$Z = C_{zv} F_v + C_{zh} F_h = \frac{D}{\cos^2 (45^\circ - \theta)} \left[\frac{C_{zv}}{K} \cos \theta + \frac{C_{zh}}{K} \sin \theta \right]. \quad (5b)$$

At $\theta = 0^\circ$, using linear elasticity, let:

$$Y_v = \left(\frac{dY}{dD} \right)_{\theta=0^\circ} = \left(\frac{Y-0}{D-0} \right)_{\theta=0^\circ} = \left(\frac{Y}{D} \right)_{\theta=0^\circ}, \text{ and } (6a)$$

$$Z_v = \left(\frac{dZ}{dD} \right)_{\theta=0^\circ} = \left(\frac{Z-0}{D-0} \right)_{\theta=0^\circ} = \left(\frac{Z}{D} \right)_{\theta=0^\circ}. \quad (6b)$$

At $\theta = 90^\circ$, using linear elasticity, let:

$$Y_h = \left(\frac{dY}{dD} \right)_{\theta = 90^\circ} = \left(\frac{Y - 0}{D - 0} \right)_{\theta = 90^\circ} = \left(\frac{Y}{D} \right)_{\theta = 90^\circ}, \text{ and} \quad (6c)$$

$$Z_h = \left(\frac{dZ}{dD} \right)_{\theta = 90^\circ} = \left(\frac{Z - 0}{D - 0} \right)_{\theta = 90^\circ} = \left(\frac{Z}{D} \right)_{\theta = 90^\circ}. \quad (6d)$$

Then, substituting into Equation 5a and 5b with the appropriate θ values:

$$\frac{C_{yv}}{K} = Y_v \cos^2 45^\circ, \quad (7a)$$

$$\frac{C_{zv}}{K} = Z_v \cos^2 45^\circ, \quad (7b)$$

$$\frac{C_{yh}}{K} = Y_h \cos^2 (-45^\circ) = Y_h \cos^2 45^\circ, \text{ and} \quad (7c)$$

$$\frac{C_{zh}}{K} = Z_h \cos^2 (-45^\circ) = Z_h \cos^2 45^\circ. \quad (7d)$$

From trigonometry:

$$\frac{\cos^2 45^\circ}{\cos^2 (45^\circ - \theta)} = \left[\frac{\cos 45^\circ}{\cos 45^\circ \cos \theta + \sin 45^\circ \sin \theta} \right]^2 = \frac{1}{(\cos \theta + \sin \theta)^2}. \quad (8)$$

Using Equations 5, 7, and 8:

$$Y = D \frac{Y_v \cos \theta + Y_h \sin \theta}{(\cos \theta + \sin \theta)^2}, \text{ and} \quad (9a)$$

$$Z = D \frac{Z_v \cos \theta + Z_h \sin \theta}{(\cos \theta + \sin \theta)^2}. \quad (9b)$$

Assuming that the setup procedure specifies zeroing the measuring probe on the horizontal and vertical surfaces; then, at $\theta = 0^\circ$ and a zero-position component of probe displacement, D'_v , deflection in the vertical direction is:

$$Z' = D'_v Z_v; \quad (10a)$$

at $\theta = 90^\circ$ and a zero-position component of probe displacement, D'_h , deflection in the horizontal direction is:

$$Y' = D'_h Y_h. \quad (10b)$$

Then, from Figure 2 it is apparent that the reading error is given by:

$$E = (Y' - Y) \sin \theta + (Z' - Z) \cos \theta. \quad (11)$$

A list of the symbols used in these derivations is contained in Appendix A.

MEASUREMENT OF DEFLECTION RATES

From the derivation of the reading error, it is seen that the measurements of the four deflection rates (Y_v , Y_h , Z_v , and Z_h) and the zero-position components

Figure 2. READING ERROR AS A FUNCTION OF THE DEFLECTION COMPONENTS. (Z' and Y' are the Amounts the Machine is Adjusted to Compensate for Deflection at Setup)

of probe displacement (D_h^i and D_v^i) are all that are necessary to determine the reading error for any probe displacement at any part contact angle. These measurements were made for a typical measuring machine by mounting two Dyna-Gage transducers, as seen in Figure 3. Deflection of the measuring head was detected by measuring the capacitance change between the Dyna-Gage transducer tips and plates mounted on a clamp attached to the measuring head; capacitance was measured and amplified by using DG-400 Dyna-Gages connected to the transducers; deflections were read from Varian recorders calibrated with a full-scale travel equivalent to 500 microinches; measuring-probe displacement was measured by the LVDT integral with the machine. Tests were run by inserting different combinations of shims between the measuring probe and a supporting block. Although no special effort was made to eliminate the effect of probe friction on deflection in these tests, it is felt that the shims were moved around enough to remove any significant residual friction forces.

Results for these tests are summarized by the graphs of Figure 4 and reveal that the deflection rates are linear. The following values were determined from the data:

$$Y_v = -9.3 \text{ microinches per mil,}$$

$$Y_h = 22.5 \text{ microinches per mil,}$$

$$Z_v = 2.3 \text{ microinches per mil,}$$

$$Z_h = -2.6 \text{ microinches per mil, and}$$

$$D_h^i = D_v^i = 5.0 \text{ mils.}$$

It is observed that for forces applied in both the horizontal and vertical directions, the horizontal deflection of the measuring head is greater than the vertical. If it is assumed that most of the deflection is in the arm supporting the measuring head, this indicates that the arm is more rigid in the vertical direction than in the horizontal.

ERROR COMPUTATION

Using the deflection rates and zero-position components of probe displacement, a computer program was prepared to calculate reading errors every two degrees for part surface deviations of zero, plus and minus one, and plus and minus two mils. This program and the results are contained in Appendix B. The data

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Figure 3. DYNA-GAGE TRANSDUCERS MOUNTED ON A TWO-AXIS MEASURING MACHINE.

(a) Horizontal Applied Force

(b) Vertical Applied Force

Figure 4. RESULTS OF DEFLECTION-RATE TESTS.

have been plotted with polar coordinates, as shown in Figure 5. It is interesting to observe that angles close to 90 degrees produce the greatest errors, indicative again of greater arm rigidity in the vertical direction.

Figure 5. READING-ERROR CURVES FOR VARIOUS PART-SURFACE DEVIATIONS.

CONCLUSIONS

From the results of these tests and computation it is evident that errors up to 72 microinches can be introduced during the inspection of variable-surface-angle parts on this type of measuring machine, even without any part-surface deviation at all. Furthermore, errors of ± 45 microinches can be introduced when the part surface deviates by ± 2 mils, even if all surfaces are rectilinear.

APPENDIX A

LIST OF SYMBOLS

- C_{ij} - measuring head compliance in the "i" direction due to the component of force in the "j" direction (microinches per pound).
- D - component of probe displacement in the direction of applied force (mils).
- D_j - component of probe displacement in the "j" direction (mils).
- E - inspection error (microinches; positive for readings showing more stock than actual, negative for less than actual).
- F - applied force (pounds).
- F_j - component of force in the "j" direction (pounds).
- g - subscript to denote the measuring probe direction of motion.
- h - subscript to denote the horizontal direction.
- K - probe spring constant (mils per pound).
- v - subscript to denote the vertical direction.
- Y - deflection of the measuring head in the horizontal direction (microinches).
- Y_j - rate of change of "Y" with respect to "D" due to component of force in the "j" direction (microinches per mil).
- y - subscript to denote the horizontal direction of deflection.
- Z - deflection of the measuring head in the vertical direction (microinches).
- Z_j - rate of change of "Z" with respect to "D" due to component of force in the "j" direction (microinches per mil).
- z - subscript to denote the vertical direction of deflection.
- θ - part contact angle (angle, in degrees, between the vertical line and the normal to the part surface).

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APPENDIX B

ERROR COMPUTATION PROGRAM (FORTRAN IV)

```

      DIMENSION E(5)
      READ (5,10) YV,YH,ZV,ZH,DQ
10  FORMAT (5F5.1)
      PI # 3.1416
      WRITE (6,15)
15  FORMAT (50H1 DISPLACEMENT      0      +1      +2      -1      -2)
      WRITE (6,20)
20  FORMAT (11H      THETA)
      DO 45 I # 1,46
      AI # (2*I) - 2
      T # (AI*PI)/180.
      C # COS(T)
      S # SIN(T)
      FT # (C + S)**2
      FY # (YV*C + YH*S)/FT
      FZ # (ZV*C + ZH*S)/FT
      DO 30 J # 1,3
      D # FLOAT(J) - 1. + DQ
30  E(J) # (DQ*YH - D*FY)*S + (DQ*ZV - D*FZ)*C
      DO 40 J # 4,5
      D # 3. - FLOAT(J) + DQ
40  E(J) # (DQ*YH - D*FY)*S + (DQ*ZV - D*FZ)*C
45  WRITE (6,50) AI,(E(J),J#1,5)
50  FORMAT (F10.0,1X,5F8.1)
      STOP
      END

```

ERROR COMPUTATION RESULTS (IN MICROINCHES)

THETA (DEGREES)	Deviation (mils)				
	0	+1	+2	-1	-2
0.	0.	-2.3	-4.6	2.3	4.6
2.	6.5	4.7	2.9	8.3	10.1
4.	12.4	11.0	9.7	13.8	15.2
6.	17.9	16.8	15.8	18.9	20.0
8.	22.9	22.1	21.3	23.7	24.6
10.	27.6	27.0	26.3	28.3	28.9
12.	32.0	31.4	30.9	32.5	33.0
14.	36.0	35.6	35.1	36.5	37.0
16.	39.8	39.4	38.9	40.3	40.7
18.	43.4	42.9	42.5	43.9	44.3
20.	46.7	46.2	45.7	47.2	47.8
22.	49.9	49.3	48.7	50.4	51.0
24.	52.8	52.1	51.4	53.5	54.2
26.	55.5	54.6	53.8	56.3	57.1
28.	58.0	57.0	56.0	59.0	60.0
30.	60.3	59.1	58.0	61.5	62.7
32.	62.5	61.1	59.7	63.8	65.2
34.	64.4	62.8	61.2	66.0	67.6
36.	66.2	64.3	62.4	68.0	69.9
38.	67.7	65.6	63.5	69.8	72.0
40.	69.1	66.7	64.2	71.5	73.9
42.	70.2	67.5	64.8	72.9	75.7
44.	71.2	68.1	65.1	74.2	77.3
46.	71.9	68.5	65.1	75.3	78.7
48.	72.4	68.6	64.8	76.2	80.0
50.	72.7	68.5	64.3	76.9	81.0
52.	72.7	68.1	63.5	77.3	81.9
54.	72.5	67.4	62.4	77.6	82.6
56.	72.0	66.5	61.0	77.6	83.1
58.	71.3	65.2	59.2	77.3	83.4
60.	70.2	63.6	57.0	76.8	83.4
62.	68.9	61.7	54.5	76.0	83.2
64.	67.2	59.4	51.6	75.0	82.8
66.	65.2	56.7	48.3	73.6	82.1
68.	62.8	53.6	44.5	72.0	81.1
70.	60.0	50.1	40.1	69.9	79.9
72.	56.8	46.0	35.3	67.5	78.3
74.	53.1	41.5	29.8	64.7	76.4
76.	48.9	36.3	23.7	61.5	74.1
78.	44.2	30.5	16.9	57.8	71.5
80.	38.8	24.0	9.2	53.6	68.4
82.	32.8	16.7	0.7	48.8	64.9
84.	26.0	8.6	-8.9	43.4	60.8
86.	18.3	-0.6	-19.5	37.3	56.2
88.	9.7	-10.9	-31.5	30.3	51.0
90.	-0.0	-22.5	-45.0	22.5	45.0