INELASTIC ELECTRON-PROTON SCATTERING
AND VECTOR MESON DOMINANCE⁺

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ABSTRACT

A simple phenomenological picture of scaling is proposed based on generalized vector meson dominance. The model suggests that a very weak spectral function over the large photon mass is sufficient to give the magnitude and the shape of scaling limit functions. Thus the measurement of νW_2 simultaneously explains why the rho-meson dominance works in high energy photon-reactions.

Experimental implications of the model are also discussed.

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The existence of scaling in inelastic electron-proton scattering predicted by Bjorken¹ has been verified by SLAC experiments² for some range of the scaling parameter ω . Various models which explain scaling laws have been proposed.³ One of the mysteries of scaling is, however, the failure of vector meson dominance (VMD) proposed by Sakurai^{4,3} although it predicted the correct order of magnitude of νW_2 . The purpose of the present note is to show that a rather weak spectral weight of dispersion integral in the large photon mass is sufficient to explain the scaling behaviour; thus the measurement of scaling functions simultaneously explains why the rho-meson dominance works in photo-reactions. The basic ingredient in our model is to approximate the dispersion integral over the photon mass by an infinite number of vector meson poles. This approximation is expected to be good for space like photons. By introducing heavy mesons we show that two of the basic properties of the parton model can be realized in the phenomenological level in a quite different manner from that of Bjorken and Paschos.⁵ These basic properties of the parton model are:

- (a) The mass of the particles which couple to massive photons (i.e., mass of the Parton) does not matter in the scaling limit.
- (b) The parton couple to massive photons as a point-like particle.

We can incorporate these two properties into the propagator and the coupling constant of heavy mesons.

The basic equation of VMD reads⁴

$$\langle A | j^\mu | B \rangle = \sum_V \frac{C_V}{m_V^2 - q^2} \langle A | J_V^\mu | B \rangle \quad (1)$$

where j^μ and J_V^μ are the electromagnetic and vector meson currents, respectively. C_V is a constant, and q^2 and m_V^2 the square of the mass

of the photon and the vector meson, respectively. Let us assume that heavy mesons with a mass which satisfies the relation⁶

$$m_V^2 \sim -q^2 \quad (2)$$

give a dominant contribution to the deep inelastic process. We call this "parton condition" for simplicity in the following although it has no direct connection with the parton model. Under the present assumption we can replace m_V^2 by $-q^2$ everywhere except in the propagator in Eq. (1). Then we get the property (a) above. Eq. (1) also shows that the "form factor" of the heavy meson does not fall off for $|q^2| \leq m_V^2$; this gives the property corresponding to (b). By this way we can transfer the mechanism of scaling to the strength of the coupling constant C_V and the property of the hadronic process, $V + p \rightarrow \text{anything}$. In the parton model^{3,5} this has been achieved by assuming quasi-free partons. In the present model the strongness of hadron interactions expressed by the Regge behaviour of the forward heavy meson -proton scattering amplitude combined with the "parton condition" determine the basic structure and the magnitude of scaling functions.⁷ It is our assumption that the mechanism of scaling based on the "parton condition" is working in the region where a pole dominance model like VMD is applicable (see Eq. (5) below). The "parton condition" is also the assumption that we can measure the "semi-local" structure of the spectral weight by varying q^2 .

Thus the measurement of νW_2 in some region of the scaling parameter is the measurement of the spectral weight for larger masses. By this way we can estimate the correction to the rho-meson dominance in photo-reactions due to the heavy mass states; this correction turns out to be small.⁸

In the following we briefly describe the basic idea of the model. Our starting equations are⁴

$$\sigma_Y^{(T)}(s, q^2) = \sum_V \frac{C_V^2}{(m_V^2 - q^2)^2} \cdot \sigma_V^{(T)}(s) + \text{Interference Terms} \quad (3)$$

$$\sigma_Y^{(L)}(s, q^2) = \sum_V \frac{C_V^2}{(m_V^2 - q^2)^2} \cdot \frac{(-q^2)}{m_V^2} \cdot \sigma_V^{(L)}(s) + \text{Interference Terms} \quad (4)$$

for

$$s \gg -q^2, m_V^2 \quad (5)$$

where $\sigma_Y^{(T)}(s, q^2)$ and $\sigma_Y^{(L)}(s, q^2)$ correspond to the transversal and longitudinal total cross sections of the process

$$"\gamma" + P \rightarrow \text{anything} \quad (6)$$

$\sigma_V^{(T)}(s)$ and $\sigma_V^{(L)}(s)$ are the corresponding cross sections for the process

$$V + P \rightarrow \text{anything} \quad (7)$$

Interference terms in Eqs. (3) and (4) describe the contribution due to the interference of different mesons. We neglect them in the following; qualitative features of the final result are little affected by this simplification. The condition in Eq. (5) is necessary to avoid the possible contribution from the anomalous complex singularities in the q^2 -plane for fixed s ⁹ and also to ensure the smooth photon mass extrapolation. Combined with the "parton condition" it also justifies to use the same Regge parameter for different mesons in Eqs. (8) and (9) below. Therefore our model is not valid for small scaling parameter $\omega = 1 + s/-q^2$. We next assume that the forward heavy meson-proton scattering amplitude has the Regge behaviour. Thus we get¹⁰

$$\sigma_V^{(T)}(s) = A \cdot \gamma(m_V)^2 \cdot \frac{1}{m_V^2} \cdot \left(\frac{s}{m_V^2}\right)^{\alpha-1} \quad (8)$$

$$\sigma_V^{(L)}(s) = B \cdot \gamma(m_V)^2 \cdot \frac{1}{m_V^2} \cdot \left(\frac{s}{m_V^2}\right)^{\alpha-1} \quad (9)$$

where A and B are constants and α the intercept of the leading Regge trajectory, $\alpha \leq 1$. $\gamma(m_V)$ is a function of m_V . The "parton condition" demands the following relation

$$(m_V)^0 < C_V \cdot \gamma(m_V) < m_V^2 \quad (10)$$

for $m_V^2 \gg m_\rho^2$. Thus we take the simplest choice

$$C_V \cdot \gamma(m_V) = C \cdot m_V \quad (11)$$

where C is a constant or a slowly varying function of m_V . This choice automatically ensures the condition in Eq. (5) as well as scaling. However note that the "parton condition" does not necessarily produce scaling. This means the main part of our conclusion is still valid even if scaling laws are weakly violated at large ω . With these assumptions we finally get

$$\sigma_\gamma^{(T)}(s, q^2) \approx f(\alpha) \cdot \left(\frac{m_\rho^2}{m^2}\right) \cdot \left(\frac{m_\rho^2}{-q^2}\right) \cdot \left(\frac{s}{-q^2}\right)^{\alpha-1} \cdot \sigma_\gamma^{(T)}(\infty, q^2=0) \quad (12)$$

$$\sigma_\gamma^{(S)}(s, q^2) \approx R \cdot \sigma_\gamma^{(T)}(s, q^2) \quad (13)$$

where

$$f(\alpha) \equiv \int_0^\infty \frac{dx}{(1+x)^2 x^{\alpha-1}} \quad \text{with } x = m_V^2/(-q^2) \quad (14)$$

$$R \equiv \int_{m_\rho^2/(-q^2)}^\infty \frac{dx}{x^\alpha (1+x)^2} / f(\alpha), \quad \int = \sigma_\rho^{(L)}(\infty) / \sigma_\rho^{(T)}(\infty) \quad (15)$$

We normalized the constants A, B and C at the rho-meson values; this will give the correct order of magnitude because of the experimental fact² that scaling sets in at fairly small q^2 . We also replaced the

summation over m_V^2 by an integration in Eqs. (3) and (4). The interval of two adjacent vector mesons, m^2 , is about $1(\text{BeV})^2$. Integrals in Eqs. (14) and (15) receive a major contribution from the region $x \sim 1$, i.e., $m_V^2 \sim -q^2$; this is exactly what we required. The values of $f(\alpha)$ and R are shown in the following:

	$f(\alpha)$	R
$\alpha = 1$	1	$\xi \cdot [\log(1-q^2/m^2) - 1/(1-m^2/q^2)]$
$\alpha = 1/2$	$\pi/2$	ξ

where $\xi = \sigma_\rho^{(s)}(\infty)/\sigma_\rho^{(T)}(\infty)$. R for $\alpha = 1$ depends on q^2 , and $R(q^2 = 4(\text{BeV})^2) \approx 2\xi$. We show νW_2 which is defined by⁴

$$\nu W_2 = \frac{\nu \cdot (S-H^2)}{\pi \cdot e^2 \cdot 2M} \cdot \left[\frac{(-q^2)}{(\nu^2 - q^2)} \right] \cdot [1+R] \cdot \sigma_\gamma^{(T)}(q^2, S) \quad (16)$$

for $\alpha = 1$ and $1/2$ in Fig. 1. We assumed that $R(\sim 2\xi) = 0.5$, $\sigma_\gamma^{(T)}(\infty, q^2=0) = 125 \mu\text{b}$ and $m^2 = 2m_\rho^2$. It is amusing to see that the value of νW_2 in our model is about same as predicted by Sakurai⁴ and it is not far from the experimental result.²

We next estimate the correction to the rho-meson dominance in photo-reactions due to the heavy mass states.

$$\begin{aligned} \sigma_\gamma^{(T)}(s, q^2=0) &\approx \sigma_{\gamma, \rho}^{(T)}(s) \cdot \left[1 + \left[\frac{m_\rho^2}{(m^2 + m_\rho^2)} \right]^2 + \left[\frac{m_\rho^2}{(2m^2 + m_\rho^2)} \right]^2 + \dots \right] \\ &\approx 1.2 \sigma_{\gamma, \rho}^{(T)}(s) \quad \text{for } \alpha \approx 1 \end{aligned} \quad (17)$$

where $\sigma_{\gamma, \rho}^{(T)}(s)$ is the value obtained by the rho-meson dominance. This shows that the rho-meson dominance is modified by about 20%.¹¹ This small modification of the rho-meson dominance explains why it works well in (inclusive) high energy photo-reactions.¹²

The present model suggests that the basic idea of VMD is still working in the deep inelastic process although the simplicity and usefulness of VMD is lost. It is the weak spectral integral that determines the magnitude and the shape of scaling limit functions. The real test of VMD would therefore be:

- (i) Measurement of R at small $q^2 (0 < -q^2 < m_\rho^2)$ and at large s .
SLAC data² suggests that R is also small in this region, namely
 $R \sim \xi = \sigma_\rho^{(L)}(\infty) / \sigma_\rho^{(T)}(\infty) < 0.5$.
- (ii) At large s and ω , some fraction (depending on ξ) of the final state particles will correspond to the longitudinal rho-meson-proton scattering almost independently of $q^2 (> m_\rho^2)$. This is the test of the gauge condition implemented by Sakurai.⁴

As for the test of our model.

- (iii) If the Regge-like behaviour of νW_2 observed at large ω ^{2,3} is confirmed to be a genuine one, it is a partial support for the "parton condition." If small mass mesons dominate the process independently of q^2 , it would be difficult to get a well-defined Regge behaviour with $\alpha < 1$.
- (iv) It is expected that we observe an increasingly heavier fire ball corresponding to projectile fragments when one increases $|q^2|$ at large s and large ω , or a heavier missing mass (up to $\sim |q^2|$) for larger q^2 at large fixed s ¹⁴ if only the final state proton is observed. The transversal momentum distribution of final state particles with respect to the incident "photon" could also be distinct from those observed in purely hadronic processes.

In the present note we studied the possible mechanism of scaling

by utilizing heavy particles instead of neglecting the mass of the particles involved.

What we found is a consistent picture about the Regge-like behaviour, scaling at fairly small q^2 , measured values of νW_2 and the success of the rho-meson dominance in (inclusive) photo-reactions.

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Figure Caption

Fig. (1); νW_2 given by Eq. (16) with $R=0.5$, $\sigma_Y(s=\infty)=125 \text{ } \mu\text{b}$, $m^2 = 2m_p^2$ and $\alpha = 1$ or $1/2$. Several experimental values taken from Ref. (2) are also shown with error flags to give a rough idea about the observed values of νW_2 .

References and Footnotes

1. J.D. Bjorken, Phys. Rev. 179, 1545 (1969).
2. E.D. Bloom et al., Report presented to the XV International Conference on High Energy Physics, Kiev, 1970 (SLAC-PUB-796).
3. For a list of earlier references and the present status of theoretical understanding of the deep inelastic process, see J.D. Bjorken, Lectures at the Scottish Universities Summer School.
4. J.J. Sakurai, Phys. Rev. Letters 22, 981 (1969). See also C.F. Cho, G.J. Gounaris, and J.J. Sakurai, Phys. Rev. 186, 1734 (1969). We follow the notation in this reference except the difference in the metric; $q^2 < 0$ corresponds to a space-like vector in our metric.
5. J.D. Bjorken and E.A. Paschos, Phys. Rev. 185, 1975 (1969).
6. Experiments also suggest the heavier mesons may be more important for larger momentum transfers. See L. Mo in High Energy Collisions (Proc. International Conference at Stony Brook, 1969, Gordon and Breach, New York). Eq. (2) is the naive identification, small distances $\sim$ heavier mesons.
7. This model was motivated by the observation made by Rittenberg and Rubinstein based on the dual resonance model. They suggest that the forward heavy meson-proton amplitude may scale by itself at high energies; V. Rittenberg and H.R. Rubinstein, "Scaling and Duality in Strong Interactions", Weizman Institute Preprint, 1970.
8. Our model should be regarded as a correction to the rho-meson dominance due to the heavy mass states.
9. A. Suri, Phys. Rev. Letters 26, 208 (1971).

References and Footnotes (continued)

10. We assume that $\sigma_V^{(L)}(s)/\sigma_V^{(T)}(s) = \text{constant}$. This is the result suggested by Rittenberg and Rubinstein.⁷ They also show that $\gamma(m_V) = \text{constant}$, and $\alpha < 1$ if one uses the dual resonance model.
11. This correction mainly depends on the measured values of νW_2 and the "parton condition" combined with scaling even at fairly small q^2 . If one includes interference terms the theoretical value for νW_2 would be increased, and the constants in Eqs. (8), (9) and (11) are forced to be smaller; the result in Eq. (17) is thus not so sensitive to the specific assumption we made about the interference terms.
12. D.O. Caldwell et al., Phys. Rev. Letters 25 609 and 613 (1970).
13. The violation of scaling by a logarithmic term in R for $\alpha = 1$ in our model is the result of the assumption $\sigma_V^{(L)}(s)/\sigma_V^{(T)}(s) = \text{constant}$.
14. If one uses the gluon model (vector meson-quark-quark coupling) and calculates the process " γ " + $q \rightarrow q + q + \bar{q}$, the invariant mass of the quark-antiquark pair, which could be converted into a multipion system via final state interactions, has these characteristics.
See also, J.D. Bjorken, SLAC-PUB-905, April, 1971. Experimental data does not contradict with this behaviour, see D.E. Andrews et al., Cornell preprint, CLNS-136, January, 1971

Fig.1