SEP 9 1953

UCRL 7478-7

MASTER

University of California

Ernest O. Lawrence
Radiation Laboratory

Livermore, California

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

Facsimile Price \$ 1.60
 Microfilm Price \$.80

Available from the
 Office of Technical Services
 Department of Commerce
 Washington 25, D. C.

This paper was submitted for publication in the open literature at least 6 months prior to the issuance date of this Microcard. Since the U.S.A.E.C. has no evidence that it has been published, the paper is being distributed in Microcard form as a preprint.

UNIVERSITY OF CALIFORNIA
 Lawrence Radiation Laboratory
 Livermore, California

CONTRACT No. W-7405-eng-48

ON THE CONNECTION BETWEEN THE DISTORTED WAVE AND DISPERSION
THEORIES OF DIRECT NUCLEAR REACTIONS*

Arnold Tubis

August 21, 1963

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

ON THE CONNECTION BETWEEN THE DISTORTED WAVE AND DISPERSION
THEORIES OF DIRECT NUCLEAR REACTIONS^{*}

Arnold Tubis[†]

Lawrence Radiation Laboratory, University of California
Livermore, California

In a recent series of papers, Shapiro and his collaborators,^{1,4} and Saperstein⁵ have outlined a program for treating direct nuclear reactions which is based on dispersion theory techniques^{6,7}. This program, which superficially appears to be much different from the widely used method of distorted waves^{8,9}, was motivated by an observation of Amado¹⁰ that the amplitude for a stripping reaction contains a pole in the momentum transfer variable quite close to the physical region.

It was originally hoped that the residue at the stripping pole, which is proportional to the reduced width of the captured particle state, could be determined by extrapolating experimental data¹⁰. Attempts to determine reduced widths by extrapolation techniques, however, tend to be frustrated by distortions due to initial and final state interactions and the form factors for the deuteron and captured particle vertex functions^{9,11,12}.

The program of Shapiro et al and Saperstein is not so much concerned with the determination of reduced widths as it is with constructing the direct reaction amplitude starting with known (or assumed) vertex form factors and the (complex) phase shifts associated with initial and final state scattering. It is claimed^{3,5} that the dispersion theory method is superior to the method of distorted waves since it uses only the asymptotic behavior (phaseshifts) of the scattering wave functions. The

distorted wave method makes use of the entire scattering wave functions and thus would yield ambiguous results if several different optical potentials account for the same set of scattering phase shifts. The dispersion theory method ^{1,5}, as formulated up to now, does not seem to suffer from such ambiguities.

The purpose of this note is to show that the distorted wave and dispersion methods are not as dissimilar as has been suggested and in fact probably give essentially the same results. The ambiguity mentioned above in connection with the distorted wave approach is also present in the dispersion theory method when the latter is properly formulated.

Let us consider the non-relativistic stripping process $A(d,p)B$ corresponding to zero angular momentum for the captured neutron state. For simplicity, we ignore Coulomb effects and the spin and statistics of the particles and assume that there are only two possible channels, $(A + d)$ and $(B + p)$. In the following discussion, these channels will be referred to as α and β respectively.

The center of mass energy of the system is

$$E = \frac{\hbar^2 k_d^2}{2\mu_\alpha} + \epsilon_\alpha = \frac{\hbar^2 k_p^2}{2\mu_\beta} + \epsilon_\beta \quad (1)$$

$$\mu_\alpha = \frac{M_d M_A}{M_d + M_A} \quad \mu_\beta = \frac{M_p M_B}{M_p + M_B}$$

where M_d , M_A , M_p and M_B are respectively the masses of the deuteron, nucleus A, the proton and nucleus B. ϵ_α and ϵ_β are the internal energies

of the deuteron and nucleus A, and nucleus B respectively. k_d and k_p are respectively the relative motion wave numbers in the initial and final states.

The reduced two by two scattering matrix for center of mass angular momentum l satisfies the unitarity condition

$$t_l - t_l^\dagger = 2i t_l \rho_l t_l^\dagger = 2i t_l^\dagger \rho_l t_l \quad (2)$$

$$(\rho_l)_{ij} = k_i^{2l+1} \Theta(E - \epsilon_i) \delta_{ij} \quad i, j = \alpha, \beta \quad (3)$$

$$(t_l^\dagger)_{ij} = (t_l)_{ji}^*$$

where $\Theta(x)$ is the unit step function which vanishes for $x < 0$. Time reversal invariance implies that t_l is symmetric. If we write the diagonal elements of t_l as

$$(t_l)_{jj} = \frac{e^{i\delta_j} \sin \delta_j}{k_j^{2l+1}} \quad (4)$$

where the δ_j are complex,* it follows from (2) and the symmetry of t_l that

$$\begin{aligned} \text{Im}(t_l)_{\beta\alpha} &= (t_l)_{\beta\alpha} (e^{i\delta_\alpha} \sin \delta_\alpha)^* \Theta(E - \epsilon_\alpha) \\ &+ (t_l)_{\beta\alpha}^* (e^{i\delta_\beta} \sin \delta_\beta) \Theta(E - \epsilon_\beta) \end{aligned} \quad (5)$$

since the right hand side of (5) is real, we see that the phase of $(t_l)_{\beta\alpha}$ is given by the phase of

$$e^{i\delta_\alpha} \sin \delta_\alpha \Theta(E - \epsilon_\alpha) - e^{i\delta_\beta} \sin \delta_\beta \Theta(E - \epsilon_\beta) \quad (6)$$

* They become real if the stripping amplitude vanishes.

The above result, which is a generalization of the Fermi-Watson final-state interaction theorem, was derived by Saperstein in reference 5).

His result, however, contains an error in sign [see his equations (17) and (18) and the discussion following them.]

Now $(t_\ell(E))_{\beta\alpha}$ has branch cuts on the physical sheet of the energy plane which start below the threshold for physical scattering^{6,7}. The cut nearest the physical region is determined by partial wave contributions of the Butler stripping diagram (Figure 1.), i.e. the ordinary plane wave Born approximation. If we assume constant form factors (neglect the internal structure of the deuteron and nuclei A and B). The Butler pole contribution to $(t_\ell(E))_{\beta\alpha}$ is, aside from constant factors,^{8,9,10}

$$\frac{1}{(k_p k_d)^\ell} \int_0^\infty r^2 dr e^{-k_\beta r} j_\ell(k_d r) j_\ell\left(k_p \frac{M_A}{M_A + M_n}\right) = \frac{1}{2(k_p k_d)^{\ell+1}} Q_\ell\left(\frac{k_n^2 + k_d^2 + \left[\frac{M_A}{M_A + M_n}\right]^2 k_p^2}{2 \left[\frac{M_A}{M_A + M_n}\right] k_p k_d}\right) \quad (7)$$

$$k_n = \sqrt{\frac{2 M_n M_A}{M_n + M_A} \frac{|\epsilon_\beta - \epsilon_\alpha + \epsilon_d|}{n^2}} \quad (8)$$

where ϵ_d is the internal energy of the deuteron, M_n is the neutron mass. j_ℓ is the spherical Bessel function¹³ and Q_ℓ is the Legendre function of the second kind.¹⁴

The branch points of (7) on the physical energy sheet are determined by the equations

$$\left(k_p + \frac{M_A}{M_A + M_n} k_p\right)^2 = -k_n^2 \quad (9)$$

$$\text{Im } k_p, \quad \text{Im } k_d \geq 0 \quad (10)$$

(7) is real on the real energy axis to the right of $E = E_0$ where E_0 is the largest value of E satisfying (9) and (10).

The form factors may be calculated on the basis of models for the deuteron and neutron - A bound state wave functions^{2,3,5,8,9} or alternatively in terms of the effective (optical) potentials for n - p and n - A scattering^{2,3}. Such form factors lead to additional terms in (7) which have branch cuts starting to the left of E_0 . The positions of these cuts have been determined by Shapiro^{2,3} on the basis of assumed optical potentials and by Schnitzer using the Feynman diagram techniques of Landau. Since the dispersion theory calculation of form factors must rely, at least for the present, on the wave function-potential picture of the distorted wave theory, the consideration of form factors sheds no light on the difference between the two methods.

The dispersion theory calculation^{1,3,5} of the stripping amplitude is based on the Omnes-Muskhelishvili^{6,7,15,16} representation of the scattering amplitude in terms of its phase for energies above the physical scattering threshold [i.e. the phase of the expression (6)] and discontinuities across various branch cuts in the unphysical energy region. Now the only branch cuts in the unphysical region considered in the references cited above are those associated with diagrams of the Butler type with possible non-constant form factors. However, other branch cuts further removed from the physical region may also be important. In fact we shall see below that the branch cuts associated with interactions in the initial and final states, when properly accounted for, bring the dispersion theory into essential congruence with the method of distorted waves.

According to the method of distorted waves ^{8,9}, the stripping amplitude (t_{ℓ}) _{$\beta\alpha$} is, aside from constant factors,

$$\frac{1}{(k_p k_d)^{\ell}} \int_0^{\infty} r^2 dr \frac{e^{-k_n r}}{r} \mu^{(-)*}(k_p, r) \mu^{(+)}(k_d, r) \quad (11)$$

$$\left[-\frac{\hbar^2}{2\mu_{\alpha}} \frac{d^2}{dr^2} + V_{\alpha}(r) \right] \left\{ r \mu^{(+)}(k_d, r) \right\} = \frac{\hbar^2 k_d^2}{2\mu_{\alpha}} \left\{ r \mu^{(+)}(k_d, r) \right\} \quad (12)$$

$$\left[-\frac{\hbar^2}{2\mu_{\beta}} \frac{d^2}{dr^2} + V_{\beta}(r) \right] \left\{ r \mu^{(-)}(k_p, r) \right\} = \frac{\hbar^2 k_p^2}{2\mu_{\beta}} \left\{ r \mu^{(-)}(k_p, r) \right\} \quad (12a)$$

$$\mu^{(+)}(k_d, r) \xrightarrow{r \rightarrow \infty} e^{i\delta_{\alpha}(k_d)} \sin(k_d r - \frac{\ell\pi}{2} + \delta_{\alpha}(k_d)) / k_d r \quad (13)$$

$$\mu^{(-)}(k_p, r) \xrightarrow{r \rightarrow \infty} e^{-i\delta_{\beta}(k_p)} \sin(k_p r - \frac{\ell\pi}{2} + \delta_{\beta}(k_p)) / k_p r \quad (13a)$$

where V_{α} and V_{β} are respectively the optical potentials for $d - A$ and $p - B$ scattering. We have employed the zero range approximation ^{8,9} and neglected internal structure effects in writing (11).^{*} We have also assumed that $\frac{M_A}{M_A + M_n} \approx 1$ which is usually the case in practice and helps simplify the discussion.

Now if the stripping amplitude is negligible in comparison with that of $d - A$ and $n - B$ elastic scattering, the potentials V_{α} and V_{β}

* This procedure is practically equivalent to assuming constant form factors.

are Hermitian and the phase of (11) is just $\delta_\alpha + \delta_\beta$. Thus (5) is exactly satisfied by (11) and the phase of the distorted wave stripping amplitude in the physical region is the same as that used in the dispersion theory calculation.

Furthermore, it can be shown, using the method of A. Martin¹⁷ and de Alfaro and Rossetti,¹⁸ that the nearest cut of (11) to the left of the physical scattering threshold is correctly described by the Butler pole contribution (7).

If V_α and V_β can be written as superpositions of Yukawa wells, i.e.

$$V_{\alpha,\beta} = \int_{A_{\alpha,\beta}}^{\infty} C_{\alpha,\beta}(\sigma) e^{-\sigma r} d\sigma \quad (14)$$

the next nearest cuts will occur for

$$(k_p + k_d)^2 \leq -(k_n + A_{\alpha,\beta})^2 \quad (15)$$

$$\text{Im } k_p, \text{Im } k_d \geq 0 \quad (16)$$

and correspond to the Feynman-Landau diagrams in Figures 2a and 2b.^{**} There will also be poles associated with bound states of d-A and n-B systems.⁵

* Nucleon and deuteron optical potentials are usually described by the Saxon-Woods well which, as is well known, cannot be represented by (14). If, however, the potentials can be represented by (14) at large distances and have the general Saxon-Woods shape at small distances, there will still be branch points at the locations given by (15) and (16)^{2,3}. In addition, there will be an essential singularity at infinity which can easily be accounted for in a dispersion theory calculation.^{2,3}

** (21) and (22), for the case when only initial or only final state scattering is important may be immediately deduced from equations (12), (13), (33), (34) and (35) of reference 18. Note that the discontinuity across the Butler branch cut is not modified by initial or final state scattering except where the cuts described by (15) and (16) overlap with it.

If these additional cuts and poles are to be accounted for in a dispersion theory calculation, they will at present have to be treated via the potential-wave function picture and it should be now fairly obvious that both the distorted wave and dispersion methods in the present case are practically equivalent. It is also clear that the consideration of branch cuts due to initial and final state interactions adds to the dispersion method the ambiguity, mentioned in connection with the distorted wave method at the beginning of this note. Potentials which yield practically the same scattering results over a limited energy region may, in general, lead to different singularities in the unphysical region. If the position of singularities described by (15) and (16) are sufficiently far from the physical region, for the class of potentials fitting the initial and final state scattering data, then both methods should yield fairly unambiguous results.

The above discussion has been based on the assumption of negligible inelastic scattering in the initial and final states. A small amount of inelastic scattering should not change the general conclusions drastically. When there is a large amount of inelastic scattering the relationship between the two approaches is not particularly clear. The importance of short range interactions means that singularities considerably removed from the physical region must be carefully considered in the dispersion approach. Also one must probably take all important channels into account explicitly. All of the points mentioned in this note will be dealt with at greater length in a forthcoming paper to be published elsewhere.

The author wishes to thank Dr. Sidney Fernbach and other members of the Theoretical Division at the Lawrence Radiation Laboratory,

Livermore, California for the hospitality extended him during his summer visit. He also wishes to acknowledge useful conversations with Dr. Harry Lutz and Dr. Ervin Schwarcz.

References

- 1) I.S. Shapiro, JETP 41, 1148 (1962); Nuclear Physics 28, 244 (1961)
- 2) I.S. Shapiro, JETP 43, 755 (1962); Nuclear Physics 38, 327 (1962)
- 3) I.S. Shapiro in Selected Topics in Nuclear Theory, edited by F. Janovch, International Atomic Energy Agency, Vienna (1963)
- 4) V.A. Kaminskii and Y.V. Orlov, JETP 43, 1517 (1963)
- 5) A.M. Saperstein, Phys. Rev. 130, 2054 (1963)
- 6) M.L. Goldberger and G.F. Chew in Dispersion Relations and Energy Particles, edited by C. DeWitt and R. Omnes, Hermann, Paris (1960)
- 7) J.D. Jackson in Dispersion Relations, edited by G.R. Screaton, Oliver and Boyd, London (1961)
- 8) W. Tobocman, Phys. Rev. 115, 98 (1959); Theory of Direct Nuclear Reactions, Oxford University Press (1961)
- 9) Lectures by N. Austern in reference 3.
- 10) R.D. Amado, Phys. Rev. Lett. 2, 399 (1959)
- 11) H.J. Schnitzer, Nuclear Physics 36, 505 (1962)
- 12) C.D. Dullemond and H.J. Schnitzer, Phys. Rev. 129, 821 (1963)
- 13) P.M. Morse and H. Feshbach, Methods of Theoretical Physics, Part II, McGraw Hill, New York (1953), p. 1573
- 14) P.M. Morse and H. Feshbach, op. cit., p. 1325
- 15) R. Omnes, Nuovo Cimento 8, 316 (1958)
- 16) M. Sugawara and A. Tubis, Phys. Rev. 130, 2127 (1963)
- 17) A. Martin, Nuovo Cimento, 14, 403 (1959); 15, 99 (1960)
- 18) V. DeAlfaro and C. Rossetti, Nuovo Cimento 18, 783 (1960)

* Work supported in part by funds from the U.S. Atomic Energy Commission, the U.S. Air Force and the National Science Foundation.

+ Permanent address: Department of Physics, Purdue University
Lafayette, Indiana.

Fig. 1

The Butler stripping pole diagram

(a)

(b)

Fig 2

Lowest order diagrams representing interactions in the initial and final states. The dotted lines represent the exchange of a particle of mass

$$\frac{\hbar A_a}{c} \quad \text{and} \quad \frac{\hbar A_b}{c}$$

respectively [see (14)]. Note that the exchanged particle cannot be a pion in (a) since the deuteron has an isotopic spin of zero.