FIXED CUTS AND πN BACKWARD SCATTERING*

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ABSTRACT

The present status of fermion Regge pole theory in backward $\pi N \rightarrow N\pi$ scattering is discussed. Fixed cuts are introduced to remove parity partners and a fit is attempted. However the dip in $\pi^+ p \rightarrow p\pi^+$ scattering, a natural consequence of a nonsense wrong signature zero without the fixed cut, is now impossible to explain with a reasonable N_α residue.

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I Introduction

Since the beginnings of Regge pole theory it has been realized that a fermion Regge pole, in general, consists of two towers of resonances of opposite parity.¹ In particular, with Regge trajectories that are linear in u , these resonances will occur in pairs with same mass, but opposite parity. This prediction has been an experimental embarrassment to the assumption of linear trajectories. Previous fits² within this assumption have had to constrain the parametrization to remove the lowest parity partners and then ignored their Regge recurrences (which often emerge from the fit with an anomalously large partial decay width.) Only the experimental and theoretical successes³ of linear trajectories have made this unaesthetic approach tolerable.

Recently it has been shown that if resonances of only one parity on a linear trajectory are inserted in a Van Hove model the result is a fermion Regge pole and a fixed Regge cut.⁴ The parity partners are then present only on the unphysical sheet with respect to this cut. Thus, the elegance of linear trajectories and the absence of parity partners suggest that fermion Regge poles are accompanied by a fixed cut. In Section III we parametrize the resultant fermion Regge pole - fixed cut ensemble and test it in πN scattering. Before this, in Section II we review the successes and failures of the conventional theory.

II Conventional Regge Theory

The successes of the conventional application of fermion Regge poles to backward $\pi N \rightarrow N\pi$ scattering are:

1^o) Linear trajectories with a universal slope.

Here the Δ trajectory, with five resonances on a linear trajectory, is very convincing.

2^o) The s dependence of the differential cross-section.

We can calculate from experimental data at different s and the same u , the effective trajectory:

$$\alpha_{eff}(u, s_1, s_2) = 1 + \ln \left[\frac{\frac{d\sigma}{du}(s_1, u)}{\frac{d\sigma}{du}(s_2, u)} \right] \cdot \left[2 \ln(s_1/s_2) \right]^{-1}$$

In Figure I we graph the predictions for $\alpha_{\Delta, eff}$ of various models for $\pi^- p \rightarrow p \pi^-$ scattering and experimental results.⁵ The results are not accurate enough to discriminate between the different models but are consistent with the linear trajectory that is successful in the resonance region.

3^o) There is a dip in the $\pi^+ p \rightarrow p \pi^+$ cross-section at $u \sim -.15 \text{ (BeV/c)}^2$, a nonsense wrong-signature point for the N_α trajectory. This dip is well fit by an $N_\alpha + \Delta$ Regge pole model.

4^o) The N_α residue function that emerges from cross-section fits is consistent with the πNN coupling constant and the $N^*(1688)$'s partial decay width.

The failures are:

1^o) The parity partners discussed above.

2^o) The Δ dip, predicted at $u \sim -1.6 \text{ (BeV/c)}^2$, is missing.

This can be remedied by the addition of s channel absorption effects to the model.⁵

3⁰) The Δ residue predicts resonance widths too small by a factor of 20. This difficulty is worse than has usually been stated. Those fits that come closest to a reasonable Δ width lead to huge partial decay widths for its recurrences. The widths along the trajectory are in fact smooth and well fit⁷ by a Veneziano model. The residue appears to be smooth in both the resonance and cross-section regions but discontinuous in between. This is in marked contrast to the N_α trajectory.

4⁰) The recent $\pi^-p \rightarrow n\pi^0$ backward data⁸ are hard to understand, even when the N_α trajectory is added. We can calculate from experiment the phase φ between the u-channel $I = 1/2$ and $I = 3/2$ amplitudes by:

$$\begin{aligned}\cos \varphi &= \frac{1}{4x} \left(1 + 3 \frac{w_2 - 2w_3}{w_1} \right) \\ x^2 &= \frac{3}{2} \frac{(w_3 + w_2)}{w_1} - \frac{1}{2} \\ w_{1,2} &= \frac{d\sigma}{du} (\pi^+ p \rightarrow p \pi^+) , w_3 = \frac{d\sigma}{du} (\pi^- p \rightarrow n \pi^0)\end{aligned}$$

In Figure II we graph $\varphi(u)$ extracted from the Orear data and compare it with Regge fits. In a Δ - N_α fit:

$$\begin{aligned}\varphi &= \frac{\pi}{2} (\alpha_\Delta - \alpha_{N_\alpha} \pm 1) \\ &\approx 45^\circ \text{ or } 135^\circ\end{aligned}$$

depending upon the relative residue sign. In either case, as the N_α passes through its dip the phase should jump, although near the dip the N_α should be important. Experimentally^{5,8}, (see Fit. II) φ remains close to 45° over a large range of u . The one extant fit⁹ to this data, Halzen et. al., utilizes an N_α residue proportional to $(\alpha_{N_\alpha} + 1/2)$ so the dip is a double zero. This would be inconsistent with the dip's shape in a pure Regge pole model.

III Fixed Cuts

It is convenient to work with the πN partial wave amplitudes $f_{J\pm}$. Here f_{J+} will contain only resonances of normal parity, and f_{J-} only resonances of abnormal parity. We can generalize these to complex J : $f_\pm(u, J)$. To remove one set of resonances we must satisfy the MacDowell constraint:¹⁰

$$f_+(\sqrt{u}, J) = -f_-(-\sqrt{u}, J)$$

and the Regge pole eqns.:

$$f_+^{-1}(\sqrt{u}, \alpha(u)) = 0$$

$$f_-^{-1}(\sqrt{u}, \alpha(u)) \neq 0$$

A general form that fulfills these conditions is

$$f_\pm(\sqrt{u}, J) = - \frac{E \mp M}{2\sqrt{u}} \frac{\sqrt{bu} \pm \sqrt{J-a}}{J - (a + bu)} \zeta(J)$$

where we have added a fixed cut to the Regge structure. The Regge pole of $f_-(\sqrt{u}, J)$ has now moved through this cut onto the unphysical J sheet. The $\sqrt{u}$ cut will not

be present in the invariant amplitudes A' and B .

Here $G(J)$ is a general function of J and need not be analytic. It will determine the resonance partial decay widths and the Regge pole residue. To analyze the data we choose the following parametrization:

$$G(J) = f(J) \left\{ a_1 + \sqrt{\frac{b m^2}{J-a}} \left(a_2 + a_3 \frac{J-a}{b m^2} \right) \right\}$$

$$f(J) = \Gamma(J + \frac{1}{2})$$

This choice of $f(J)$ is suggested by the Veneziano model and is successful in the resonance region.⁷ The rest of the parametrization is sufficiently general to allow the Regge cut to occur in either the A' or B invariant amplitude or both. At the same time it will not substantially change the successful behavior in the resonance region guaranteed by $f(J)$. This form does involve difficulties with the unitarity condition which we ignore here.

The cut contributions can be evaluated numerically. Formally the needed integral diverges, but the Regge limit ($s \rightarrow \infty, |u/s| \ll 1$) justifies the insertion of a cut-off.

This form allows a reasonable Δ trajectory-plus-cut fit to $\pi^- p \rightarrow p \pi^-$ scattering. The Δ dip is now nicely filled in by the cut and the width extrapolation is no worse. However the N_α dip is also filled in by the cut and we cannot fit the

$\pi^+ p \rightarrow p \pi^+$ data. We can see the problem here by considering a typical cut integral:

$$\begin{aligned}
 I &= \int_0^\infty \frac{f(-J) \sqrt{J} (s/s_0)^{-J}}{(J+\alpha)} \\
 &= \int_0^\infty \frac{(s/s_0)^{-J} \sqrt{J}}{J+\alpha} \left\{ f(\alpha) + f'(\alpha)(J+\alpha) \dots \right\} \\
 &= f(\alpha) \left[-\pi \sqrt{\alpha} (s/s_0)^\alpha \operatorname{erfc}(\sqrt{\alpha \ln s/s_0}) + \sqrt{\pi/\ln(s/s_0)} \right] \\
 &\quad + f'(\alpha) \sqrt{\frac{\pi}{\ln^3(s/s_0)}} + O(f''(\alpha) \ln^{-5/2}(s/s_0))
 \end{aligned}$$

Here the first term will have the pole's dip but the second term will not. And it is smaller by a factor of $(\ln(s/s_0))^{-1}$. Although s_0 can be chosen small to make the first term appear dominant, the resultant f will then have a large derivative especially if we are to demand consistency in the resonance region. Halzen et. al.⁹ choose $s_0 = .1$ and a similar f . Thus they are able to fit the dip but extrapolate badly to the πNN coupling constant where they err by a factor of 60. A better statement of their and our result is that present information on the N_α trajectory indicates that the fixed cut model (without parity partners) is not reasonable from a phenomenological point of view.

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Figure CaptionsCaption for Figure I:

Model predictions for α_{eff} compared to experimental information.

Caption for Figure II:

The experimentally determined relative phase φ of the $I^u = 1/2$ to $I^u = 3/2$ amplitudes is graphed. The dotted line represents an $N_\alpha + \Delta$ Regge pole model. The data used are all at $p_L \approx 6$ (BeV/c).

Fig.1

Fig.2