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UNDERWATER ACOUSTIC IMAGING:
PRELIMINARY FEASIBILITY STUDY
FINAL REPORT

S. Frankel
Sidney Frankel and Associates
Menlo Park, California
For
Sandia Laboratories, Albuquerque

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FA-136

Underwater Acoustic Imaging: Effects of Some
Parameter Variations on Automatic
Power-Level Control Signals

S. Frankel

July 9, 1968

MEMORANDUM FOR FILE: FA-136

S. Frankel
July 9, 1968

Subject: Underwater Acoustic Imaging: Effect of Some Parameter Variations
on Automatic Power-Level Control Signals

1. Introduction

In previous memoranda we have discussed an imaging system which uses automatic power-level control (APC) to ameliorate the extreme diffraction effects from specularly-reflecting regions of acoustically smooth objects [1, 2]. The proper operation of such a system assumes, among other things, reciprocity for the transmission path in the propagating medium. Such reciprocity exists if the propagation system is isotropic* and, of course, time-invariant.**

The first condition, - isotropy, - is a normal property of the medium. This memorandum is concerned with deterioration in APC resulting from violation of the assumption of time-invariance. We consider the effects of two sources of variation: (1) relative motion between imager and object (although, strictly, variation of the condition of the medium is not involved); (2) variation in propagation-velocity gradient with time. It turns out that while the first of these can impose a significant limitation on the permitted relative lateral motion, the effect of the second appears, under ordinary circumstances, to be negligible.

* but not necessarily homogenous

** In that case, the transmission path is the same in both directions between imaging system and target.

2. Effect of Relative Motion

The method of operation of the imager is to send out a pulse of acoustic energy to be partially reflected back and detected by a mosaic of individual receiving elements located in the image plane of the system. The transmitted pulse is the combined effect of pulses transmitted simultaneously from each of the image-plane elements; the level of power transmitted from each element is controlled by the amount of signal received by that same element, in such a way as to compress the dynamic range of signals reflected from various regions of the target. If the imager moves laterally to the line-of-sight-to-target during the transmission round-trip time, sufficiently far to cause the reflected energy to fall into a channel other than the one of its origin, the APC control is lost or at least diminished, and the system operation deteriorates correspondingly.

Let

R = range to target, meters

c = velocity of propagation, meters/second

t_r = time for signal round trip, seconds

$$t_r = \frac{2R}{c} \quad (1)$$

Let

v_s = lateral component of speed of imager, meters/sec.

l_s = lateral motion (meters) of imager in time t_r

$$l_s = v_s t_r = \frac{2R v_s}{c} \quad (2)$$

The signal received by an image element will appear to come from a region displaced from the region to which that element transmitted energy, by an amount, l_s . Let D_o be the side of a square representing the projection of one imaging element on the object (or target). Let η_s be the fraction of lateral slip permitted:

$$\eta_s = \frac{l_s}{D_o} = 2 \left(\frac{R}{D_o} \right) \left(\frac{v_s}{c} \right) \quad (3)$$

whence

$$v_s = \frac{\eta_s D_o c}{2R} \quad (4)$$

For example, suppose $R = 300$ m, $D_0 = 1$ m, $c = 1500$ m/sec., and $\eta_s = 1/2$. Then

$$v_s = \frac{\frac{1}{2} \times 1 \times 1500}{2 \times 300} = 1.25 \text{ m/sec.}$$

or about 2.4 kt.

This simple argument ignores the fact that as lateral motion causes the sonar echo to lose register with its own channel, the same effect introduces energy from an adjacent channel. Thus the relative change in excitation of a channel with lateral motion is proportional, not only to the lateral displacement in the round-trip interval, but also to the gradient of the field in the direction of displacement. Unfortunately, the only regions where this gradient is large enough to validate the foregoing analysis is in those yielding specular reflection. Thus it appears advisable to stay with that analysis, which evidently represents a worst case.

3. Effect of Varying Speed Gradient

Consider a propagation medium in which the speed of propagation varies in the vertical direction (say, y -) only. Consider a non-horizontal element of a ray trajectory, and choose the x -coordinate so that the element lies in the xy -plane (Fig. 1). Let θ be the angle the element makes with the x -axis, - i.e., with the normal to velocity-gradient vector, and c , $\dot{x}$, $\dot{y}$ the speed of propagation and its x - and y - components respectively. Then we have [Ref. 3, Section 1.6.1; also Ref. 4, Section 5.6].

$$\frac{c}{\cos \theta} = \frac{c}{\sqrt{1 - \sin^2 \theta}} = \text{const.} = K, \text{ say} \quad (5)$$

From Fig. 1,

$$\frac{1}{\cos \theta} = \sec \theta = \frac{ds}{dx} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} \quad (6)$$

while we can write

$$\cos \theta = \frac{\dot{x}}{c} \quad (7)$$

$$= \sqrt{1 - \sin^2 \theta} = \left[1 - \left(\frac{\dot{y}}{c} \right)^2 \right]^{\frac{1}{2}} \quad (8)$$

Figure 1. Trajectory Coordinate System

(6) in (5) gives the form

$$c \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} = K \quad (9)$$

while (8) in (5) gives

$$\frac{c}{\left[1 - \left(\frac{\dot{y}}{c} \right)^2 \right]^{\frac{1}{2}}} = K ; \quad (9a)$$

that is,

$$\frac{c^2}{K^2} = 1 - \left(\frac{\dot{y}}{c} \right)^2 \quad (10)$$

The nature of the problems of interest are such that we can understand the quantitative effects sufficiently well for our purposes by assuming that the speed, c , varies linearly with y , and the gradient, g , varies linearly with t . Thus, write

$$c = c_0 + g_0 (1 + \beta t)y \quad (11)$$

where c_0 is the speed at the level $y = 0$ [i.e., at any point $(x,0)$], g_0 is the speed gradient at $t = 0$, and β is the relative time-rate of change of the gradient.

3.1 Trajectory Equation for Linear, Time-Invariant Gradient

A linear, time-invariant gradient yields trajectories which are arcs of circles [4, loc. cit]. For this case, $\beta = 0$, and (11) becomes

$$c = c_0 + g_0 y \quad (12)$$

From (9) we have

$$\frac{dy}{dx} = \pm \frac{\sqrt{K^2 - c^2}}{c} = \pm \tan \theta, \quad (13)$$

using (5). Clearly, (Fig. 1), only the upper sign is valid. Now

$$\frac{dc}{dx} = \frac{dc}{dy} \frac{dy}{dx} = g_0 \frac{dy}{dx} \quad (14)$$

by (12). Then (14) in (13) gives

$$\frac{dc}{dx} = g_0 \frac{\sqrt{K^2 - c^2}}{c}$$

whence

$$g_0 x + c_1 = \int \frac{cdc}{\sqrt{K^2 - c^2}} = - (K^2 - c^2)^{\frac{1}{2}} \quad (15)$$

where c_1 is an integration constant. Then, using (12) and re-arranging,

$$\left(y + \frac{c_0}{g_0}\right)^2 + \left(x + \frac{c_1}{g_0}\right)^2 = \left(\frac{K}{g_0}\right)^2 \quad (15)$$

which is the equation of a circle centered at

$$\left(-\frac{c_1}{g_o}, -\frac{c_o}{g_o} \right), \text{ with radius, } \frac{K}{g_o}.$$

Let the origin of coordinates, (0,0) be a point on the trajectory. Then (15) gives

$$\left(\frac{c_o}{g_o} \right)^2 + \left(\frac{c_1}{g_o} \right)^2 = \left(\frac{K}{g_o} \right)^2$$

i.e.,

$$c_1 = \sqrt{K^2 - c_o^2} \quad (16)$$

Write

$$\theta_o = \theta \Big|_{\substack{x=0 \\ y=0}}$$

Then (12) and (5) imply

$$\frac{c_o}{\cos \theta_o} = K \quad (17)$$

and (17) in (16) gives

$$c_1 = -c_o \tan \theta_o \quad (18)$$

Substituting (18) in (15) and writing

$$\alpha_o = \frac{c_o}{g_o} \quad (19)$$

yields

$$(y + \alpha_o)^2 + (x - \alpha_o \tan \theta_o)^2 = (\alpha_o \sec \theta_o)^2 \quad (20)$$

3.1.1 Parametric Trajectory Equations for Linear, Time-Invariant Speed Gradient

Parametric form for the trajectory function for the time-invariant case is of interest for comparison with the time-variant case. To emphasize that we are dealing with the case for $\beta = 0$, write (x_o, y_o) in place of (x, y) . From Eq. (9a)

$$\frac{\dot{y}_o}{c} = \frac{\sqrt{K^2 - c^2}}{K} \quad (21)$$

Furthermore,

$$\dot{c} = g_o \dot{y}_o$$

so that (21) may be re-written

$$\frac{\dot{c}}{c \sqrt{K^2 - c^2}} = \frac{g_o}{K}$$

whence elementary integration and manipulation yields

$$c \cosh (c_2 - g_o t) = K \quad (22)$$

Next, by (5) and (7)

$$\begin{aligned} \dot{x}_o &= c \cos \theta = \frac{c^2}{K} \\ &= K \operatorname{sech}^2 (c_2 - g_o t) \end{aligned}$$

by (22). Therefore

$$x_o = K \int \operatorname{sech}^2 (c_2 - g_o t) dt + c_3$$

whence

$$x_o - c_3 = -\frac{K}{g_o} \tanh (c_2 - g_o t) \quad (23)$$

and from (22), with the help of (11) with $\beta = 0$,

$$y_0 + \frac{c_0}{g_0} = \frac{K}{g_0} \operatorname{sech} (c_2 - g_0 t) \quad (24)$$

Eqs. (23) and (24) are parametric equations of the trajectory. The value of c_2 depends on the time origin while c_3 depends on the position origin. Squaring and adding (23) and (24) yields

$$(x_0 - c_3)^2 + \left(y_0 + \frac{c_0}{g_0}\right)^2 = \left(\frac{K}{g_0}\right)^2 \quad (25)$$

which eventually identifies with (20) if the trajectory passes through the origin.

It is of interest to re-write (24) in an approximate form for certain initial conditions. Assume the trajectory is at the origin at $t = 0$:

$$x_0 = y_0 = 0 \text{ for } t = 0;$$

then (24) implies

$$\frac{c_0}{g_0} = \frac{K}{g_0} \operatorname{sech} c_2 = \frac{c_0}{g_0} \sec \theta_0 \operatorname{sech} c_2$$

whence

$$\left. \begin{aligned} \operatorname{sech} c_2 &= \cos \theta_0 \\ \sinh c_2 &= \sqrt{\cosh^2 c_2 - 1} = \sqrt{\sec^2 \theta_0 - 1} = \tan \theta_0 \\ \tanh c_2 &= \sinh c_2 \operatorname{sech} c_2 = \sin \theta_0 \end{aligned} \right\}$$

Then (24) becomes

$$\begin{aligned} y_0 + \frac{c_0}{g_0} &= \frac{c_0 \sec \theta_0}{g_0} \left[\cosh (c_2 - g_0 t) \right]^{-1} \\ &= \frac{c_0 \sec \theta_0}{g_0} \left[\cosh c_2 \cosh g_0 t - \sinh c_2 \sinh g_0 t \right]^{-1} \\ &= \frac{c_0 \sec \theta_0}{g_0} \left[\sec \theta_0 \cosh g_0 t - \tan \theta_0 \sinh g_0 t \right]^{-1} \\ &= \frac{c_0}{g_0} \operatorname{sech} g_0 t \left[1 - \sin \theta_0 \tanh g_0 t \right]^{-1} \end{aligned} \quad (26)$$

For small $\sin \theta_0 \cdot \tanh g_0 t$, (26) becomes

$$y_0 + \frac{c_0}{g_0} \approx \frac{c_0}{g_0} \operatorname{sech} g_0 t \left[1 + \sin \theta_0 \tanh g_0 t \right] \quad (27)$$

If, more specifically, $|g_0 t| \ll 1$,

$$y_0 \approx \frac{c_0}{g_0} \left\{ -\frac{1}{2} (g_0 t)^2 + \sin \theta_0 \cdot (g_0 t) \right\} \quad (28)$$

Since $y_0 = 0$ at $t = 0$, (28) also gives the change in height, Δy , for time, t .

For the special case where the trajectory is horizontal at $x_0 = y_0 = t = 0$, we have also $\theta_0 = 0$, and

$$\Delta y \Big|_{\theta_0 = 0} \approx -\frac{1}{2} c_0 g_0 t^2 \quad (29)$$

If the gradient is not constant, but varies linearly with time, as in Eq. (11), it might reasonably be expected, provided $|\beta t|$ is sufficiently small, that Δy could be approximated by using an average $\bar{g}$:

$$\bar{g} = g_0 \left(1 + \frac{1}{2} \beta t \right)$$

in (29), in place of g_0 , to get

$$\Delta y \Big|_{\theta_0 = 0} \approx -\frac{1}{2} c_0 g_0 \left(1 + \frac{1}{2} \beta t \right) t^2 \quad (30)$$

That this, in fact, turns out to be the case, is demonstrated in the next section.

3.2 Trajectory Equation for Linear, Time-Varying Speed Gradient

Start with Eq. (10):

$$\frac{c^2}{K^2} = 1 - \left(\frac{\dot{y}}{c} \right)^2 \quad (10)$$

For the subsequent analysis it is convenient to choose the origin of coordinates at the center of curvature for $t = 0$. We will also assume that the trajectory is

horizontal at $t = 0$. Writing $u = \beta t$, the appropriate expression for the propagation speed is

$$c = g_0 (1 + u) y - c_0 u \quad (31)$$

To see this, note first that the vertical gradient is

$$\frac{\partial c}{\partial y} = g_0 (1 + u) \quad (32)$$

as in Eq. (11)

Next, since the radius of curvature is $\frac{c_0}{g_0}$ at $t = 0$ [Eqs. (19) and (20)], we

have $y = \frac{c_0}{g_0}$ at $t = 0$, so that (31) is

$$c = (1 + u) c_0 - c_0 u = c_0$$

as required at $t = 0$. Furthermore,

$$K = \frac{c}{\cos \theta} = \frac{c_0}{\cos 0} = c_0$$

Thus (10) becomes

$$\frac{c^2}{c_0^2} = 1 - \frac{y^2}{c^2}$$

whence

$$\dot{y} = \pm c \left[1 - \frac{c^2}{c_0^2} \right]^{\frac{1}{2}} \quad (33)$$

Substitute

$$\left. \begin{aligned} y &= \frac{c_0}{g_0} \eta \\ v &= g_0 t \end{aligned} \right\} \quad (34)$$

whence

$$\left. \begin{aligned} u &= \beta t = \frac{\beta}{g_0} v = \gamma v \\ \dot{\eta} &= \dot{v} \frac{d\eta}{dv} \\ &= g_0 \frac{d\eta}{dv} \end{aligned} \right\} \quad (35)$$

In (31) this gives

$$\frac{c}{c_0} = \eta + \gamma (\eta - 1) v = h, \text{ say} \quad (36)$$

while (33) becomes

$$\frac{d\eta}{dv} = \pm h (1 - h^2)^{\frac{1}{2}} \quad (37)$$

In case $\gamma = 0$, write $\eta = \eta_0$, and $h = \eta_0$, so that (37) is

$$\frac{d\eta_0}{dv} = \pm \eta_0 (1 - \eta_0^2)^{\frac{1}{2}} \quad (38)$$

with solution

$$\eta_0 = \pm \operatorname{sech} (v + \text{const}) \quad (39)$$

The assumption that the trajectory is horizontal at $t = 0$ (i.e., at $v = 0$) implies from (38) that either $\eta_0 = 0$ or $\eta_0 = \pm 1$ at $v = 0$. For γ and g_0 to have the same sign, and for $|g_0| < \infty$, the only appropriate value is $\eta_0 = +1$, whence the constant in (39) is zero, and the lower sign must be used:

$$\eta_0 = \operatorname{sech} v \quad (40)$$

Thus (37) becomes restricted to

$$\frac{d\eta}{dv} = - h (1 - h^2)^{\frac{1}{2}} \quad (41)$$

Eq. (40) shows that η_0 is analytic at $v = 0$. Assuming that η is also analytic at $v = 0$ for γ sufficiently small, we can write, provisionally,

$$\eta = \sum_{k=0}^{\infty} a_k v^k \quad (42)$$

Typical values of g_0 and t of interest in this study suggest that v is small enough to justify approximating η_0 by

$$\eta_0 \approx 1 - \frac{1}{2} v^2 \quad (43)$$

Therefore, for the perturbed function η , we assume provisionally, expansion out to the v^3 term:

$$\eta = a_0 + a_1 v + a_2 v^2 + a_3 v^3 \quad (44)$$

where the a_k ($k = 0, \dots, 3$) are to be determined. Immediately, at $v = 0$, we have $\eta = \eta_0 = 1 = a_0$, so that

$$\eta = 1 + a_1 v + a_2 v^2 + a_3 v^3 \quad (45)$$

From (36),

$$\begin{aligned} h &= 1 + a_1 v + (a_2 + \gamma a_1) v^2 + (a_3 + \gamma a_2) v^3 \\ h^2 &= 1 + 2 a_1 v + \left[a_1^2 + 2 (a_2 + \gamma a_1) \right] v^2 \\ &\quad + 2 \left[(a_3 + \gamma a_2) + a_1 (a_2 + \gamma a_1) \right] v^3 + O(v^4) \\ 1 - h^2 &= -2a_1 v - \left[a_1^2 + 2a_2 + 2\gamma a_1 \right] v^2 - 2 \left[a_3 + \gamma a_2 + a_1 a_2 + \gamma a_1^2 \right] v^3 + O(v^4) \\ (1 - h^2)^{\frac{1}{2}} &= (-2a_1 v - \dots)^{\frac{1}{2}} \end{aligned} \quad (46)$$

The right member is clearly non-analytic as $v \rightarrow 0$, unless $a_1 = 0$, while $\frac{d\eta}{dv}$ must be analytic at least for $\gamma = 0$, by (40). A necessary condition that $(1 - h^2)^{\frac{1}{2}}$ be analytic near $v = 0$ is, by (46),

$$a_1 = 0 \quad (47)$$

which implies

$$\eta = 1 + a_2 v^2 + a_3 v^3 \quad (48)$$

$$h = 1 + a_2 v^2 + (a_3 + \gamma a_2) v^3 \quad (49)$$

$$h^2 = 1 + 2a_2 v^2 + 2(a_3 + \gamma a_2) v^3 + 0(v^4)$$

$$(1-h^2)^{\frac{1}{2}} = (-2a_2)^{\frac{1}{2}} v \left[1 + \frac{a_3 + \gamma a_2}{a_2} v + 0(v^2) \right]^{\frac{1}{2}}$$

which implies $a_2 < 0$. Furthermore, as $v \rightarrow 0$,

$$(1-h^2)^{\frac{1}{2}} = (-2a_2)^{\frac{1}{2}} v \left[1 + \frac{a_3 + \gamma a_2}{2a_2} v + 0(v^2) \right] \quad (50)$$

Also, from (48),

$$\frac{d\eta}{dv} = 2a_2 v + 3a_3 v^2 \quad (51)$$

Using (51), (49), and (50) in (41)

$$\begin{aligned} 2a_2 v + 3a_3 v^2 &= -(-2a_2)^{\frac{1}{2}} v \left[1 + a_2 v^2 + (a_3 + \gamma a_2) v^3 \right] \left[1 + \frac{a_3 + \gamma a_2}{2a_2} v + 0(v^2) \right] \\ &= -(-2a_2)^{\frac{1}{2}} v \left[1 + \frac{a_3 + \gamma a_2}{2a_2} v + 0(v^2) \right] \end{aligned} \quad (52)$$

Equating coefficients of like powers of v in (52) yields

$$2a_2 = -(-2a_2)^{\frac{1}{2}} \quad (53)$$

$$3a_3 = -(-2a_2)^{\frac{1}{2}} \left(\frac{a_3 + \gamma a_2}{2a_2} \right) \quad (54)$$

The non-trivial solution of (53) is

$$a_2 = -1/2 \quad (55)$$

Then (55) in (54) gives

$$3a_3 = - \frac{a_3 - \frac{1}{2}\gamma}{-1} = a_3 - \frac{1}{2}\gamma$$

whence

$$a_3 = - \frac{1}{4}\gamma \quad (56)$$

Then (55) and (56) in (48) yields

$$\eta = 1 - \frac{1}{2}v^2 - \frac{1}{4}\gamma v^3$$

Since $\eta(0) = 1$, we have, using (34),

$$\begin{aligned} \Delta y \Big|_{\theta_0 = 0} &= \frac{c_0}{g_0} [\eta - \eta(0)] \\ &= - \frac{1}{2} \frac{c_0}{g_0} v^2 (1 + \frac{1}{2}\gamma v) \end{aligned}$$

which checks Eq. (30) when appropriate substitutions are made for v and γ [Eqs. (34) and (35)].

3.2.1 Discussion

The error in register, by Eq. (30), is

$$\epsilon = - \frac{1}{4} c_0 g_0 \beta t^3 \text{ meters} \quad (57)$$

where t is the round-trip time to the target.

$$t \approx \frac{2R}{c_0} \quad (58)$$

where R is the range to the target. (58) in (57) implies

$$\epsilon = -2 \frac{g_0 \beta R^3}{c_0^2} \text{ meters} \quad (59)$$

If we take for typical values, $g_0 = 0.02 \text{ sec.}^{-1}$, $c_0 = 1500 \text{ meters/sec.}$, we get

$$e = -1.78 \times 10^{-8} \beta R^3 \text{ meters} \quad (60)$$

Typical ranges of interest are $R = 30, 100, 300 \text{ meters.}$ For these we have, respectively

$$\left. \begin{aligned} e_{30} &= -4.8 \times 10^{-4} \beta \\ e_{100} &= -1.78 \times 10^{-2} \beta \\ e_{300} &= -0.48 \beta \end{aligned} \right\} \quad (\text{meters})$$

Although I do not have information at present on likely values of β , it does not seem probable that seriously bad values can occur often, or that they can persist very long if they do. For example, if we want to restrict $|E_{100}|$ to 0.1 meter, then $\beta \leq 5.6/\text{sec.}$ If a value for β of 5/sec. occurs it obviously cannot last very long. At 300 meters a more reasonable permissible error is one meter, which implies a maximum β of 2/sec., still (one would guess!) very unlikely for any length of time.

REFERENCES

1. Memorandum to Sandia, "Summary Point of View: System Proposed for Underwater Acoustic Imaging", Frankel Associates Report FA-131, October, 1967.
2. Underwater Acoustic Imaging: Preliminary Feasibility Study. Frankel Associates Report FA-133, January, 1968.
3. Born, M., and Wolf, E., Principles of Optics, Pergamon, London et. al., 1959.
4. Urick, R. J., Principles of Underwater Sound, McGraw-Hill, New York, 1967.

FA-137

"Overshoot" Effects in the Smoothing of
Two-Dimensional Quantized Signals

S. Frankel

June 11, 1968

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"Overshoot" Effects in the Smoothing of Two-Dimensional Quantized Signals

In Reference 1 we observed that when a quantized two-dimensional transient function is transmitted through a two-dimensional low-pass filter an oscillatory response is observed, reminiscent of that obtained in the one-dimensional case. [2]. *

If one considers the case where one moves from a region of zero signal, abruptly to one of non-zero signal, then certain differences are observable in the two cases:

1. The peak amplitude of oscillation appears to be greater in the two-D case. For the one-D case, the limiting example of unit step shows an overshoot of 9%.** For the two-D case of Reference 1 the maximum overshoot was about 37%.
2. For the one-D case, the amplitudes of oscillations around the zero level of the unit step are the same as the amplitudes around the unity value, - in fact, the function is skew symmetrical around the $\frac{1}{2}$ -unit amplitude. In the two-D case, the amplitude of oscillation around the non-zero level is much larger than that around the zero level.

* See Reference List

** Fig. 201 of Reference 2 appears to be in error

3. In the one-D case, the amplitude swings positive and negative around the zero level. In the two-D case it swings only positive.

These effects may be explained in part by comparing the responses of low-pass filters to certain step functions in the one-D and two-D cases respectively.

First, reviewing the one-D case [2], consider the unit pulse of width a:

$$\left. \begin{aligned} g_1(x) &= 1, \quad 0 < x < a \\ &= 0 \text{ otherwise} \end{aligned} \right\} \quad (1)$$

Its Fourier transform is

$$\begin{aligned} G_1(f_x) &= \int_{-\infty}^{\infty} g_1(x) e^{-j\omega_x x} dx, \quad \omega_x = 2\pi f_x \\ &= \int_0^a e^{-j\omega_x x} dx \\ &= \frac{1 - e^{-j\omega_x a}}{j\omega_x} \end{aligned} \quad (2)$$

Let this signal be applied to a l.p. filter with the following characteristic:

$$\left. \begin{aligned} Y_1(f_x) &= 1, \quad |f_x| < f_c \\ &= 0 \text{ otherwise} \end{aligned} \right\} \quad (3)$$

Then the response to (1) is

$$\begin{aligned} h_1(x) &= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{1 - e^{-j\omega_x a}}{j\omega_x} e^{j\omega_x x} d\omega_x \\ &= \frac{1}{j2\pi} \left\{ \int_{-\omega_c}^{\omega_c} \frac{e^{jx\omega_x} d\omega_x}{\omega_x} - \int_{-\omega_c}^{\omega_c} \frac{e^{j(x-a)\omega_x} d\omega_x}{\omega_x} \right\} \\ &= \frac{1}{2\pi} \left\{ \int_{-\omega_c}^{\omega_c} \frac{\sin(x\omega_x) d\omega_x}{\omega_x} - \int_{-\omega_c}^{\omega_c} \frac{\sin[(x-a)\omega_x] d\omega_x}{\omega_x} \right\} \end{aligned} \quad (4)$$

since

$$\int_{-\omega_c}^{\omega_c} \frac{\cos b\omega_x}{\omega_x} d\omega_x = 0 \text{ for real } b.$$

Again, for any real b ,

$$\begin{aligned} \int_{-\omega_c}^{\omega_c} \frac{\sin b\omega_x}{\omega_x} d\omega_x &= \int_{-b\omega_c}^{b\omega_c} \frac{\sin t}{t} dt \\ &= 2 \int_0^{b\omega_c} \frac{\sin t}{t} dt = 2 \text{Si}(b\omega_c) \end{aligned} \quad (5)$$

where $\text{Si}(u)$ is the sine-integral function [3,4]. Then (4) becomes

$$h_1(x) = \frac{1}{\pi} \left\{ \text{Si}(\omega_c x) - \text{Si}[\omega_c(x-a)] \right\} \quad (6)$$

If now, the original pulse (1) is stretched out into a unit step ($a \rightarrow \infty$), then for any finite x ,

$$\text{Si}[\omega_c(x-a)] \rightarrow \text{Si}(-\infty) = -\text{Si}(\infty) = -\frac{\pi}{2} \quad [3]$$

Therefore, for the unit step, (6) becomes

$$h_1(x) = \frac{1}{2} \left[1 + \frac{2}{\pi} \text{Si}(\omega_c x) \right] \quad (7)$$

This function is plotted in Figure 1. Further discussion is available in Reference 2.

Now turning to the two-D case, consider the two-D pulse defined by

$$\left. \begin{aligned} g_2(x,y) &= 1, & 0 < x < a \\ & & 0 < y < b \\ &= 0 & \text{otherwise} \end{aligned} \right\} \quad (8)$$

Its Fourier transform is

$$G_2(f_x, f_y) = \left. \begin{aligned} &\int_0^a \int_0^b e^{-j(\omega_x x + \omega_y y)} dx dy, \\ &\omega_x = 2\pi f_x \\ &\omega_y = 2\pi f_y \end{aligned} \right\} \quad (9)$$

which integrates to

$$G_2(f_x, f_y) = \left(\frac{1 - e^{-j\omega_x a}}{j\omega_x} \right) \left(\frac{1 - e^{-j\omega_y b}}{j\omega_y} \right) \quad (10)$$

Apply this signal to a filter with the characteristic

$$Y_2(f_x, f_y) = 1, \quad |f_x|, |f_y| < f_c \quad (11)$$

= 0 otherwise

Then the response to (8) is

$$\begin{aligned} h_2(x, y) &= \frac{1}{4\pi^2} \int_{-\omega_c}^{\omega_c} \int_{-\omega_c}^{\omega_c} \left(\frac{1 - e^{-j\omega_x a}}{j\omega_x} \right) \left(\frac{1 - e^{-j\omega_y b}}{j\omega_y} \right) e^{j(\omega_x x + \omega_y y)} d\omega_x d\omega_y \\ &= \frac{1}{4\pi^2} \left\{ \int_{-\omega_c}^{\omega_c} \frac{(1 - e^{-j\omega_x a})}{j\omega_x} e^{j\omega_x x} d\omega_x \right\} \left\{ \int_{-\omega_c}^{\omega_c} \frac{(1 - e^{-j\omega_y b})}{j\omega_y} e^{j\omega_y y} d\omega_y \right\} \\ &= \frac{1}{\pi^2} \left\{ \text{Si}(\omega_c x) - \text{Si}[\omega_c(x-a)] \right\} \left\{ \text{Si}(\omega_c y) - \text{Si}[\omega_c(y-b)] \right\} \quad (12) \end{aligned}$$

Again, allowing a and b to become infinite as in the preceding case, we get the result for a two-D unit step:

$$h_2(x, y) = \frac{1}{4} \left[1 + \frac{2}{\pi} \text{Si}(\omega_c x) \right] \left[1 + \frac{2}{\pi} \text{Si}(\omega_c y) \right] \quad (13)$$

If we consider the response along the line $y = x$, for instance, we should obtain a result somewhat similar to Figure 6 of [1]. Thus,

$$\begin{aligned} h_2(x, x) &= \frac{1}{4} \left[1 + \frac{2}{\pi} \text{Si}(w_c x) \right]^2 \\ &= h_1^2(x) \end{aligned} \quad (14)$$

That is, the response, along the diagonal, $y = x$, in the two-D case, is the square of the response in the one-D case. Thus, the oscillation peak of 9% in the one-D case yields a peak of 0.09 in the "zero" region and a peak of 1.09 in the "one" region. The corresponding squares are 0.008 and 1.19 which compare with 0.03 and 1.37 in Fig. 6 of Reference 1. This explains, at least qualitatively, the extreme difference in swing around "zero" and "one" respectively. The squaring operation [Eq. (14)] explains why the function never goes negative in contrast to the one-D case (Fig. 1).

Although there is considerable quantitative difference between the present result and that of Reference 6, this should not be surprising in view of the difference in assumed configurations. On the other hand, it is interesting to see whether the function-squaring idea still gives the same relation between the peak deviation in the "one" region, and the peak deviation in the "zero" region. Thus if the peak deviation for the un-squared function is δ , we have, for the peaks in the "one" region and "zero" region respectively [c.f. Eq. (14)] :

$$\begin{aligned} u_1 &= \frac{1}{4} \left\{ 1 + \left[(1 + \delta) \right] \right\}^2 = 1.37^* \\ u_0 &= \frac{1}{4} \left[1 - (1 + \delta) \right]^2 = \frac{\delta^2}{4} \end{aligned}$$

The first of these yields $\delta = 0.34$, whence the second checks the value of 0.03.*

* Fig. 6 of Ref. 1

REFERENCES

1. Frankel, S., Sampling Theorems and the Smoothing of Low-Resolution, Quantized, Acoustic-Image Data. Report Number FA-134, Frankel Associates, Menlo Park, California, April 1968.
2. Guillemin, E.A., **Communication Networks, Vol. II; Wiley, New York, 1935.**
3. **Jahnke, E. and Emde, F., Tables of Functions; Dover, New York, 1945.**
4. Handbook of Mathematical Functions, U.S. Dept. of Commerce, Bureau of Standards, Applied Math. Series 55; June 1964.

FA-138

Effect of Target Curvature on the Strength
of Acoustic Image Signals

S. Frankel

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INTRODUCTION:

In a previous report, in deriving the range equation for the specular return from acoustic targets, we assumed that the target surface is a nearly-smooth plane [1] . Curvature of the target modifies this result. For most targets of interest, where the surfaces are not nearly plane, they tend to be convex.* In that case, the analogy with geometrical optics suggests that the reflected angular beam width will tend to be greater than the incident width, with consequent reduction in the received specular intensity and improvement in the specular/diffuse return ratio.

Since the surfaces in question have dimensions much larger than a wavelength, simple geometric optics methods are adequate for estimating the effects of simple curved surfaces, although more **rigorous** methods are sometimes employed. [2,3] .

In radar technology, the quantity that measures a target's capacity for scattering incident energy back to a receiver is the radar cross-section, σ [4] . In sonar technology, the corresponding (but not **equivalent**) quantity is the **target strength, t.** [2,3,5.] If a σ , analogous to that of radar, were defined for sonar, we would have

$$\sigma = 4\pi t \quad (1)$$

If a target is located at range, R_T , from a source, the acoustic intensity incident at the target is

$$I_{R_T} = \frac{I_1}{R_T^2} \quad (2)$$

* That is, the target is situated between the acoustic viewing system and the target's centers of curvature.

where I_1 is a proportionality constant which may be interpreted as the value of the radiated intensity, I_R , in the direction of the target at a range, R_T , of 1 meter.

If we consider the target as an equivalent source, then it is clear that the strength of this equivalent source must be proportional to the incident intensity, I_{R_T} , and the echo intensity at a large distance, R_R , must be proportional to this source strength, and inversely proportional to R_R^2 :

$$I_{R_R} \propto \frac{I_{R_T}}{R_R^2}$$

The proportionality constant is the target strength, t :

$$I_{R_R} = t \frac{I_{R_T}}{R_R^2} = \frac{I_{R_T} \sigma}{4\pi R_R^2} \quad (3)$$

Eq. (3) defines t and a procedure for determining its value. Thus

$$t = \frac{I_{R_R}}{I_{R_T}} R_R^2 \quad (4)$$

$$= R_T^2 R_R^2 \frac{I_{R_R}}{I_1} \quad (5)$$

by Eq. (2)

Some of the simple surfaces of interest include the sphere, the cylinder, and, for comparison, the plane. These are discussed in References 2 and 3. Reference 3 uses the unnecessarily sophisticated Kirchoff-Fresnel approach. Urick in Reference 2 calculates the target strength of a sphere by geometric optics, but assumes the incident wave is a plane wave of finite dimensions. Aside from the fact that this is physically impossible in the (assumed) Fraunhofer region, the the assumption leads to incorrect results whenever one or both of the principal radii of curvature of the surface are not much less than the source/target distance. Thus, the assumption of a plane wave gives erroneous results for the plane and the cylinder.

In the following analysis the insonified surfaces are assumed to be in the Fraunhofer region, large compared to a wavelength, and with curvature radii also large compared to a wavelength, thus justifying the approximations of geometric optics. In addition, the effects of power absorption in the medium are ignored (to compensate for the fact that they were ignored in the derivation of Eq. (5) .

Target Strengths of Simple Surfaces: Plane (Reference)

Consult Fig. 1. The total path length from source to receiver is $(R_T + R_R)$. Thus the intensity at the receiver is, by Eq. (2),

$$I_{R_R} = \frac{I_1}{(R_T + R_R)^2} \quad (6)$$

Then (6) in (5) gives

$$t_{\text{plane}} = \frac{R_T^2 R_R^2}{(R_T + R_R)^2} \quad (7)$$

If source and receiver are located at the same point, $R_R = R_T$, and (7) becomes

$$t_{\text{plane}} = \frac{R_T^2}{4} \quad * \quad (8)$$

Sphere

Consult Fig. 2: The transmitter is located at T; the surface, S, is a sphere of radius, a. The incident beam is a right circular cone of angle, θ . The receiver is on the same line as the transmitter/target axis, but, generally, at a different range, R_R , to the target. ψ is the angle of incidence of the extreme ray of the incident beam.

We assume, for acoustic imaging purposes, that the incident beam covers an area including no more than one-half of the sphere diameter; then we can write the following approximate relationships

$$\begin{aligned} \frac{\sin \phi}{\sin \theta} &\approx \frac{\phi}{\theta} \approx \frac{R_T}{a} \\ \psi = \theta + \phi &= \left(1 + \frac{R_T}{a}\right) \theta \approx \frac{R_T}{a} \theta \text{ for } \frac{R_T}{a} \gg 1 \\ \alpha = 2\psi - \theta &= \left(1 + 2\frac{R_T}{a}\right) \theta \approx 2\frac{R_T}{a} \theta \text{ for } \frac{R_T}{a} \gg 1 \end{aligned} \quad \left. \vphantom{\begin{aligned} \frac{\sin \phi}{\sin \theta} &\approx \frac{\phi}{\theta} \approx \frac{R_T}{a} \\ \psi = \theta + \phi &= \left(1 + \frac{R_T}{a}\right) \theta \approx \frac{R_T}{a} \theta \text{ for } \frac{R_T}{a} \gg 1 \\ \alpha = 2\psi - \theta &= \left(1 + 2\frac{R_T}{a}\right) \theta \approx 2\frac{R_T}{a} \theta \text{ for } \frac{R_T}{a} \gg 1 \end{aligned}} \right\} \quad (9)$$

* cf. Table 9.1 of Reference 2

Assuming total reflection from the target, we can equate incident and reflected powers:

$$I_{R_T} \left(\pi R_T^2 \theta^2 \right) \approx I_{R_R} \left(\pi R_R^2 \alpha^2 \right)$$

whence

$$\frac{I_{R_R}}{I_{R_T}} = \frac{R_T^2 \theta^2}{R_R^2 \alpha^2} = \left(\frac{R_T}{R_R} \right)^2 \left(\frac{a}{2R_T} \right)^2 \quad \text{by (9)}$$

Then by (4)

$$t_{\text{sphere}} = \frac{a^2}{4} \quad * \quad (10)$$

Cylinder

Again we assume the cylinder radius large enough so that the incident beam covers no more than 1/2 the surface projection in a plane parallel to the axis. The length of the cylinder is assumed sufficient to intercept the whole beam in the axial direction.

Fig. 2 may be used again for determining angles, with the understanding that we are now looking at the trace of a cylinder normal to its axis, rather than that of a sphere through its center. The reflected beam, instead of being a circular cone of total angle, 2α , is now essentially an elliptical cone of major half-angles θ and α . Equating incident and reflected power now gives

$$I_{R_T} \left(\pi R_T^2 \theta^2 \right) = I_{R_R} \left(\pi R_R^2 \theta \alpha \right)$$

* See footnote, page 5

whence

$$\begin{aligned}
 t_{\text{cylinder}} &= \frac{I_{R_R}}{I_{R_T}} R_R^2 \\
 &= \left(\frac{R_T}{R_R} \right)^2 \left(\frac{\theta}{\gamma} \right) R_R^2 \\
 &= \frac{a R_T}{2} * \quad (11)
 \end{aligned}$$

Small Neighborhood of Arbitrary Surface

By this class of surfaces we intend surfaces whose two principal **radii of curvature** may be assumed constant over the portion of the surface of interest. The principal curvature circles lie in orthogonal planes. Each of these curvatures causes the reflected wave to diverge in the same way as the single curvature did in the case of the cylinder. Let a_1, a_2 , be the radii of the principal circles of curvature, and α_1, α_2 , the corresponding half beam widths of the reflected wave when the incident half beam width is θ . Then arguments analogous to the foregoing case yield

$$I_{R_T} R_T^2 \theta^2 = I_{R_R} R_R^2 \alpha_1 \alpha_2$$

and

$$\begin{aligned}
 t_{\text{any}} &= \left(\frac{R_T}{R_R} \right)^2 \left(\frac{\theta}{\alpha_1} \right) \left(\frac{\theta}{\alpha_2} \right) R_R^2 \\
 &= R_T^2 \left(\frac{a_1}{2R_T} \right) \left(\frac{a_2}{2R_T} \right) \\
 &= \frac{a_1 a_2}{4} * \quad (12)
 \end{aligned}$$

* See footnote, page 5

Note that (12) reduces to (10) when $a_1 = a_2 = a$; i.e., when the principal curvature radii are equal.

Comparisons With Plane Surface

The main comparisons of interest are for the cylinder and the sphere. We have

$$\begin{aligned}\rho_{\text{cylinder}} &= \frac{t_{\text{cylinder}}}{t_{\text{plane}}} = \left(\frac{aR_T}{2} \right) \left[\frac{(R_T + R_R)^2}{R_T^2 R_R^2} \right] \\ &= \frac{1}{2} \frac{a (R_T + R_R)^2}{R_T R_R^2} = \frac{a}{2R_T} \left[1 + \left(\frac{R_T}{R_R} \right)^2 \right]\end{aligned}\quad (13)$$

For $R_T = R_R$,

$$\rho_{\text{cylinder}} = \frac{2a}{R_T} \quad (14)$$

Similarly,

$$\rho_{\text{sphere}} = \frac{a^2}{4} \left[\frac{(R_T + R_R)^2}{R_T^2 R_R^2} \right] \quad (15)$$

and, if $R_T = R_R$

$$\rho_{\text{sphere}} = \frac{a^2}{R_T^2} \quad (16)$$

As examples, suppose $R_T = 30$ meters while $a = 0.2$ meter (for either the cylinder or the sphere). Then

$$\rho_{\text{cylinder}} = \frac{2 \times 0.2}{30} = \frac{1}{75}, \text{ or } -19 \text{ db.}$$

$$\rho_{\text{sphere}} = \left(\frac{0.2}{30}\right)^2 = \left(\frac{1}{150}\right)^2, \text{ or } -44 \text{ db.}$$

Concluding Remarks

1. The sample calculations show that curvature of the target surface can represent a very significant benefit in the battle to bring the specular/diffuse return ratio to manageable proportions.
2. The definition we have used for t [Eq. (4)] is essentially that given in Reference 3. Urick [2] defines t as "the ratio of the intensity of sound returned by the target, at a distance of 1 yd. from its acoustic center in some direction, to the incident intensity from a distant source". In our notation, the incident intensity from a distant source is I_{R_T} ; the received intensity at a distant (R_R) receiver (distant enough for the wave front to be sensibly spherical) is I_{R_R} . If a distance of 1 yd. is sufficiently large to cause the wave front to appear sensibly spherical, then the reflected intensity at 1 yd. is $R_R^2 I_{R_R}$. Urick's definition then gives

$$t = \frac{R_R^2 I_{R_R}}{I_{R_T}}$$

which is Eq. (4).

The conceptual difficulty with Urick's definition is that, except for a spherical (or in the limit, a plane) surface, there is no guarantee that the target possesses an "acoustic center", so that the "1 yd." distance cannot be uniquely determined in all cases. In fact, to play safe, Eq. (4) should be written

$$t = \lim_{R_R \rightarrow \infty} \left[\frac{R_R^2 I_{R_R}}{I_{R_T}} \right]$$

REFERENCES

1. S. Frankel, Underwater Acoustic Imaging: Preliminary Feasibility Study. Sandia Corporation Contract Report SC-CR-68-3545, March 1968.
2. R. J. Urick, Principles of Underwater Sound for Engineers. McGraw-Hill, New York, 1967.
3. Physics of Sound in the Sea, Part III, N.D.R.C. Division of Summary Technical Report. Volume 8, pp 358 - 362 (1946).
4. Skolnik, M. I., Introduction to Radar Systems, McGraw-Hill, New York, 1962.
5. V. M. Albers, Underwater Acoustics Handbook - II. Pennsylvania State University Press, University Park, Penna., 1965.

FIG. 1. PLANE TARGET

FIG. 2. SPHERICAL TARGET

FA-139

Some Thoughts Concerning Restrictions on
Insonifier Modulation for Underwater
Acoustic Imaging Systems

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July 15, 1968

SOME THOUGHTS CONCERNING RESTRICTIONS ON INSONIFIER MODULATION FOR
UNDERWATER ACOUSTIC IMAGING SYSTEMS

In previous discussions on the subject of Underwater Acoustic Imaging we have ignored the question of type of modulation to be used at the acoustic source [1]. In fact, implicit in the analysis was the assumption of an unmodulated carrier,* an assumption which is valid for bi-static systems, but generally impractical for monostatic systems, the class of systems to which our study has been confined.

In monostatic systems, in order to avoid having the receiver continuously in an overloaded, inoperable condition, it is practically necessary to have the transmitter turned off part of the time. Furthermore, it is clear that if the whole return of a target is to be received during a silent transmitter period, the transmitter must be turned off at least half the time; otherwise the duration of the returned pulse is greater than the time available to receive it. In fact, because the target is generally finite in depth, and also because the receiver does not recover its sensitivity instantaneously after the transmitter is turned off, it is desirable to make the transmitter duty-cycle considerably less than $1/2$.

The target to be inspected may be isolated, - that is, sufficiently distant in range and bearing from other possible targets - so that the other targets are not a factor in designing for optimum viewing; or it is non-isolated if other targets can impose design restrictions. Included in such extraneous targets must be both ocean bottom and ocean surface.

* Except that it is readily generalized to the "instantaneous" behavior of a modulated carrier, averaged over a large number of carrier cycles; a generalization which is valid in the pre-detection part of the system when the modulation bandwidth is much less than the carrier frequency.

Isolated Targets

Since ocean surface and bottom are **always** present, a target can be isolated only if it is sufficiently distant from these boundaries compared to its distance from the acoustic viewer.

We assume that the target to be viewed has already been located by another procedure (which may involve the same system operating in another mode), and that we are not too much interested in target position, but, rather in target appearance. In that case, the transmitted pulse repetition frequency is limited only by the requirement that the frequency be so related to the range that the received echo fall between transmitted pulses. To simplify the discussion we stipulate that it fall half-way between successive transmitted pulses. Thus, if T_p is the pulse period and n a positive integer, or zero, then

$$(n + \frac{1}{2})T_p = \frac{2R_T}{v} \quad (1)$$

where R_T is the range to the target, and v is the velocity of propagation. The pulse repetition frequency, f_p , is

$$f_p = \frac{1}{T_p} = \frac{(n + \frac{1}{2}) v}{2 R_T} = \frac{(2n + 1) v}{4 R_T} \quad (2)$$

If we take $v \approx 1500$ meters/second

$$f_p = 375 \frac{2n+1}{R_T} \quad (3)$$

Similarly, if for some reason we want the received pulse to coincide with a transmitted pulse,* then we need

$$f_p = \frac{n'v}{2R'_T} = 750 \frac{n'}{R'_T} \quad (4)$$

where n' is a positive integer.

* We might want to obliterate an undesired target by having its return coincide with a transmitted pulse.

With no other limitations imposed, the lowest permissible f_p is given by $n = 0$ in Eq. (3):

$$f_{p_{\min}} = \frac{375}{R_T} \quad (5)$$

For instance, at $R_T = 30$ meters, this yields $f_p = 12.5$ Hz, while at $R_T = 300$, the rate is down to 1.25 Hz. If the target appearance is changing rapidly, and one wishes to obtain the illusion of continuous motion, a picture frame rate of at least 20 per second is desirable, which in turn implies a sampling rate of at least the same amount. Thus, even in the case of a target at 30 meters, the minimum sampling rate is inadequate, and we require $n > 0$. In fact, from (3),

$$n > \frac{1}{2} \left(\frac{f_p R_T}{375} - 1 \right)$$

With $f_p > 20$ and $R_T = 30$,

$$n > \frac{1}{2} \left(\frac{20 \times 30}{375} - 1 \right) = 0.6$$

i.e., $n = 1$, at least

and $f_p = 37.5$ Hz, at least.

On the other hand, if the target is relatively static, the lower repetition frequency may be used, with the added minor advantage of unambiguous range information. Nevertheless, it may appear more desirable, from an implementation standpoint*, to use the higher rep. rate, and obtain the advantage of greater range, or better signal/noise ratio, accruing from the permitted low information rate, by the use of pulse integration. These are matters of detailed design which cannot be determined within the scope of this review.

Effect of Target Depth

Up to now we have been using R_T as "the" target range. In fact, the target is generally spread out in range. As a result the echoes to the various receiving elements are spread out in time.

* e.g., filter design and amplifier design

Let

τ_p = pulse duration

D_T = depth (in range) of target

Then the returns to all the elements covering the target are spread out over an interval

$$T_r = \tau_d + \frac{2D_T}{v} \quad (6)$$

If we allow a time guard band, τ_g , on either side of the return interval, the total transmitter silent period is

$$T_s = T_r + 2\tau_g = \tau_d + 2 \left(\frac{D_T}{v} + \tau_g \right) \quad (7)$$

Therefore the total pulse period is

$$T_p = T_s + \tau_d = 2 \left[\tau_d + \tau_g + \frac{D_T}{v} \right] \quad (8)$$

The pulse duty cycle is

$$\begin{aligned} \eta_p &= \frac{\tau_d}{T_p} = \frac{1}{2} \left(\frac{\tau_d}{\tau_d + \tau_g + \frac{D_T}{v}} \right) \\ &= \frac{1}{2} \left(\frac{1}{1 + \left(\frac{\tau_g + \frac{D_T}{v}}{\tau_d} \right)} \right) = f_p \tau_d \end{aligned} \quad (9)$$

The same arguments for choosing f_p apply here as in the case of the "flat" isolated target. For a given f_p , τ_d will generally be smaller than in the flat case; so, therefore, will also be the maximum permissible average power transmitted.* However, f_p must not be so large that ambiguity occurs over the depth of the target.

* Owing to peak power cavitation effects.

Inherent in the time-spread of the returns is information which is usable for third-dimension display. If the receiving system image plane is the XY-plane, displacement along the Z-axis is proportional to the relative time delay of the received signal. The matter of 3-D displays to take advantage of this information should be investigated with display manufacturers.

Non-Isolated Target

For the acoustic imaging problem it is reasonable to assume, in most cases, that if undesired reflectors are in the vicinity of a desired target, the desired target may be separated from the extraneous by maneuvering the viewing system so that the desired target (a) is separated angularly from the undesired or (b) the desired target shadows the undesired. If either of these situations holds, then high prf pulsing may be used without confusion in interpreting the image. If neither holds, the desired target may be enhanced vis-a-vis the extraneous ones by range-gating the desired target, while using a sufficiently low f_p to avoid range ambiguity. This approach should even be useful in separating a target from nearby bottom or surface.

In order to insure good separation of targets it may not be necessary to use a low f_p ; it may be that integers n and n' exist (within a specified error), satisfying Eqs. (3) and (4) for a common value of f_p , and for specified values of R_T , R'_T . The near integers would have to satisfy

$$375 \frac{2n+1}{R_T} = 750 \frac{n'}{R'_T}$$

which can be re-written

$$n' = (n + \frac{1}{2}) \frac{R'_T}{R_T} \quad (10)$$

Conclusion

We have tried to indicate some of the factors imposing limitations on the choice of modulation schemes for acoustic viewing sonar. The question of greater sophistication in signal design for optimum parameter estimation in the given environment has not been considered.

Reference

Underwater Acoustic Imaging: Preliminary Feasibility Study. Sandia Corporation Contract Report, SC-CR-68-3545, March 1968.

FA-145

Effects of Reverberation on Effectiveness of
Acoustic Imaging at High Frequencies:
Data Requirements

S. Frankel

October 23, 1968

SUMMARY

The purpose of this report is to display the parameters that control (volume reverberation/signal) ratio in acoustic imaging arrangements in the low MHz range, in order to determine the areas in which new information, if any, is needed. The result obtained [Eq. (17)] , unlike the conventional sonar result [Eq. (21)] , is independent of range and director gain. However, when the target is only slightly rough, the expression for (rev./sig.) ratio contains a factor - the fourth power of the wavelength - not present in the conventional case.

The cumulative reverberation effect of high pulse repetition frequency is also discussed.

It is concluded that additional data on the strength of volume scatterers in the MHz region is required before feasibility in the face of volume reverberation can be judged.

EFFECTS OF REVERBERATION ON EFFECTIVENESS OF ACOUSTIC IMAGING METHODS
AT HIGH FREQUENCIES: DATA REQUIREMENTS

1. Introduction

It has been suggested that we assess the requirements of an experimental investigation to determine what essential data, not otherwise available, is required to assist in evaluating the feasibility of high frequency acoustic-imaging arrangements.

In order to make such an assessment it is desirable to review the available theory and data to determine whether, in the presence of reverberation, such imaging is (1) obviously feasible by such a wide margin that new experiments represent an unnecessary expenditure of time and money; (2) obviously unfeasible by such a wide margin as to lead to the same conclusion; (3) questionably feasible - i.e., no conclusion with a high probability of validity can be drawn from the available data.

2. Reverberation Theory: General [1, 2, 3]

Conceptually, reverberation (i.e., back scattering or clutter from distributed targets, superposed upon, and tending to degrade the quality of the image from a desired target) falls into two classes; - volume reverberation, the back scattering from undesired targets distributed over a volume; and surface reverberation, the back scattering from undesired targets distributed over a surface. For practical technical reasons the latter category is further divided into two sub-categories, - sea-surface reverberation, and bottom reverberation, with obvious meanings. Reverberation from thin layers within a volume is sometimes treated as a surface phenomenon.

Because of the nature of the imaging missions, we can assume that conditions can be arranged so that sea-surface reverberation ordinarily does not enter as a complication. Targets may be suspended within the sea at some finite depth below the surface, or they may be partly or wholly embedded in the bottom. In

2. (Continued)

the latter case, experience indicates that, normally, reflections from sea-bottom and from man-made objects differ sufficiently to yield a good probability of location and identification [4]. For objects suspended within the sea volume, reverberation from the bottom need only be troublesome in the case of high p.r.f. systems with range ambiguity [5]. In that case, the available remedies are (1) to maneuver into a position that avoids bottom reflection*, (2) to reduce the p.r.f.

Whether or not the preceding speculations are wholly justified, it seems reasonable to concentrate for the moment on the question of volume reverberation. Here the problem is one of adequate contrast between reflecting elements from diffuse targets and reverberation scatter from adjacent targets at the same range.

2.1 Volume Reverberation Theory

The theory is analogous to radar range theory [7, 8, 2].

Let

P_t = transmitter power, watts

$G_t(0, \phi)$ = projector power gain as a function of the standard spherical coordinates θ, ϕ

r = range to scatterer, meters

α = pressure attenuation constant, ncpers/meter

* Not inconceivable, in view of the short ranges involved, except possibly for side lobe problems, a matter for separate consideration [6].

2.1 (Continued)

The incident signal intensity on a scatterer located at (r, θ, ϕ) is

$$I_i = \frac{P_t G_t (\theta, \phi)}{4\pi r^2} e^{-2\alpha r}$$

Let m_v = scattering cross-section per unit volume, (meter⁻¹) [7, 8, 2]. Then the power scattered back to the projector from a differential volume, dV , is

$$\begin{aligned} dI_R &= \frac{I_i (m_v dV)}{4\pi r^2} e^{-4\alpha r} \\ &= \frac{P_t G_t m_v dV}{(4\pi r^2)^2} e^{-4\alpha r} \end{aligned} \quad (1)$$

In sonar technology it is more common to use the concept of scattering strength in place of that of scattering cross-section [2]. If scattering strength per unit volume is represented by s_v , we have [9]

$$s_v = \frac{m_v}{4\pi} \quad (2)$$

Eq. (2) in (1) yields

$$dI_R = \frac{P_t G_t s_v}{4\pi r^4} e^{-4\alpha r} dV \quad (3)$$

Let A_r = the effective area of the receiving hydrophone. Then the received power from the element of volume, dV , is, by (3)

$$dP_R = \frac{P_t G_t A_r s_v}{4\pi r^4} e^{-4\alpha r} dV \quad (4)$$

2.1 (Continued)

and integration over the appropriate volume yields

$$P_r = \frac{P_t}{4\pi} \int_V \frac{G_T A_r s_v e^{-4\alpha r}}{r^4} dV \quad (5)$$

It is now appropriate to introduce some simplifying assumptions. First, it is assumed that the scatterers behave homogeneously, over the volume of interest, so that s_v may be removed from under the integral sign. Second, the receiving and transmitting antennas are considered to be the same antenna, and use is made of the identity [10, Section 2.14].

$$A_r = \frac{G_r \lambda^2}{4\pi} = \frac{G_t \lambda^2}{4\pi} \quad (5a)$$

where λ is the wavelength of the transmission, meters. Thus, Eq. (5) becomes

$$P_r = \frac{P_t s_v \lambda^2}{(4\pi)^2} \int_V \frac{G_t^2 (\theta, \phi) e^{-4\alpha r}}{r^4} dV \quad (5b)$$

Third, we assume that the emission is in the form of a brief, rectangular pulse, of duration τ , such that $(c\tau) \ll r$, where c is the velocity of propagation, meters/second. In that case, we can assume r constant over the volume of interest, and write, [2, Section 8.4]

$$dV = r^2 \left(\frac{c\tau}{2} \right) d\Omega \quad (6)$$

where $d\Omega$ is the differential solid angle.

Equate this constant, r , to the target range, R .

Then, from (5b) and (6), the ratio of received reverberation power to transmitted power is

$$\rho_v = \frac{P_R}{P_t} = \frac{c\tau s_v \lambda^2 e^{-4\alpha R}}{32\pi^2 R^2} \int_{\Omega} G_t^2 (\theta, \phi) d\Omega \quad (7)$$

We have the elementary transformation (Fig. 1)

$$d\Omega = \sin \theta \, d\phi \, d\theta$$

which changes (6) to

$$\rho_v = \frac{c \tau s_v \lambda^2 e^{-4\alpha R}}{32\pi^2 R^2} \int_0^{2\pi} \int_0^{\pi} G_t^2(\theta, \phi) \sin \theta \, d\theta \, d\phi \quad (7a)$$

Finally, for estimating purposes, we assume G_t to have a 3 db beamwidth, B , independent of ϕ , and we replace $G_t(\theta, \phi)$ by

$$\left. \begin{aligned} G_t(\theta, \phi) &= G_0 = \text{constant}, \quad 0 \leq \theta < \frac{B}{2} \\ &= 0 \text{ otherwise} \end{aligned} \right\} \quad (8)$$

Fig. 1. Differential Solid Angle in Spherical Coordinates

2.1 (Continued)

Eq. (8) in (7a) gives

$$\begin{aligned}\rho_v &= \frac{c\tau s_v \lambda^2 G_o^2 e^{-4\alpha R}}{32\pi^2 R^2} (2\pi) \left[1 - \cos \frac{B}{2} \right] \\ &= \frac{c\tau s_v \lambda^2 G_o^2 e^{-4\alpha R}}{16\pi R^2} \left[1 - \cos \frac{B}{2} \right]\end{aligned}\quad (9)$$

B is the element resolution beamwidth which we can safely take as very much less than unity $\left[B \text{ is the order of } 10^{-2} \text{ radians} \right]$ for typical applications, so that

$$1 - \cos \frac{B}{2} \approx \frac{B^2}{8}$$

Thus Eq. (9) is

$$\rho_v = \frac{c\tau s_v \lambda^2 G_o^2 B^2 e^{-4\alpha R}}{128\pi R^2} \quad (10)$$

The gain and beamwidth are not independent; in fact, in terms of the assumed approximations, we can take*

$$B^2 = \frac{16}{G_o} \quad (11)$$

* On a sphere at unit distance from the source, the beam illuminates an area approximately $\frac{\pi B^2}{4}$. Since the whole area of the unit sphere is 4π , the gain is

$$G_o = \frac{4\pi}{\left(\frac{\pi B^2}{4}\right)} = \frac{16}{B^2}$$

2.1 (Continued)

Eq. (11) in (10) yields, finally,

$$\rho_v = \frac{c \tau s_v \lambda^2 G_o e^{-4\alpha R}}{8\pi R^2} \quad (12)$$

On the other hand, we have, for the relative diffuse power reflected back from a small plane target element [11, Section 2.2, Eq. (2)]

$$\rho_{rd} = \frac{P_{rd}}{P_t} = \frac{|R|^2 \theta_\sigma^4 A_r}{\pi R^2 s^2} \exp \left\{ - \left[\left(\frac{\theta_\sigma}{s} \right)^2 + 4 \alpha R \right] \right\} \quad (13)$$

where

R = complex pressure reflection coefficient at the **insonified** surface

$$\theta_\sigma = \frac{2\pi\sigma}{\lambda} \quad (14)$$

$$s = \frac{\sigma}{T} \quad (15)$$

σ = r.m.s. deviation of the random rough surface from an imaginary "average" plane surface, meters.

T = surface correlation distance (a measure of the average distance between hills or valleys of the roughness).

Usually we will have $\left(\frac{\theta_\sigma}{s} \right)^2 \ll 4 \alpha R$. Using, also, Eq. (5a), with G_T replaced by G_o ,

2.1 (Continued)

$$\rho_{rd} = \frac{|R|^2 \theta_\sigma^4 G_o \lambda^2 e^{-4\alpha R}}{4\pi^2 R^2 s^2}$$

Using (14)

$$\rho_{rd} = \frac{4\pi^2 |R|^2 \sigma^4 G_o e^{-4\alpha R}}{R^2 s^2 \lambda^2} \quad (16)$$

The ratio of back-scattered volume reverberation to target signal is, then, from (12) and (16):

$$\mu_d = \frac{\rho_v}{\rho_{rd}} = \frac{c\tau s_v^2 \lambda^4}{32\pi^3 |R|^2 \sigma^4} \quad (17)$$

It is interesting to compare this result with the corresponding case of conventional detection and ranging sonar. For such systems the range equation is analogous to that for radar.

$$\rho_p = \frac{P_r}{P_t} = \frac{G_t A_r m_t}{(4\pi R^2)^2} e^{-4\alpha R} \quad (18)$$

where m_t is the target scattering cross-section. Substituting the target strength

$$s_t = \frac{m_t}{4\pi} \quad (19)$$

and replacing A_r via Eq. (5a), with $G_r = G_o$ results in

$$\rho_p = \frac{G_o^2 \lambda^2 s_t}{16\pi^2 R^4} e^{-4\alpha R} \quad (20)$$

2.1 (Continued)

Then, from (12) and (20),

$$\mu_p = \frac{\rho_v}{\rho_p} = \frac{2\pi\tau s_v R^2}{G_o s_t} \quad (21)$$

In the conventional sonar case, where the beam spreads over a much larger area than that occupied in the projection of the target, the reverberation-to-signal ratio worsens with the square of the range, but improves linearly with antenna gain. In the imaging case, neither of these factors is present. However, apart from variations in scatterer strength with wavelength, which are likely to be small at the high imaging frequencies, the reverberation/signal ratio in the imaging case has a strong fourth-power variation with wavelength. Thus, decreasing the wavelength by a factor of 3 improves the ratio by about 20 db.

2.1.1 Effect of High Pulse Repetition Frequency (p.r.f.)

If the p.r.f. is high* it may be necessary to consider the cumulative effect of volume scatter from additional ranges beyond the target range, R. Thus, if T_p is the pulse repetition frequency, reflections that coincide with that from the target range, R, will occur at $R + \frac{1}{2} cT_p$, $R + 2(\frac{1}{2} cT_p)$, ..., $R + k(\frac{1}{2} cT_p)$, ..., ($k = 1, 2, \dots$).

Thus Eq. (12) needs to be re-written

$$\rho_v = \frac{c\tau s_v \lambda^2 G_o e^{-4\alpha R}}{8\pi R^2} \sum_{k=0}^{\infty} \left\{ \frac{e^{-2k\alpha cT_p}}{\left[1 + \frac{kcT_p}{2R} \right]^2} \right\} \quad (12a)$$

* See Reference 5.

2.1.1 (Continued)

Typically, for an operating frequency of 1 MHz, $\alpha \approx 0.038$ Np/meter.

For $R = 30$ meters and $T_p = 0.02$ second, the correction for the summation is 0.7 db.

3. Discussion

Take c at approximately 1500 meters/second, so that

$$\frac{c}{32\pi^3} = 1.5$$

and, therefore, write Eq. (17)

$$\begin{aligned}\mu_d &= 1.5 \left(\frac{s_v s^2}{|R|^2 \sigma^4} \right) (\tau \lambda^4) \\ &= 1.5 \left(\frac{s_v}{|R|^2 \sigma^2 T^2} \right) (\tau \lambda^4)\end{aligned}$$

$$\mu_d = Qx^4 \tag{22}$$

where

$$\left. \begin{aligned}Q &= 1.5 \frac{s_v}{|R|^2 \sigma^2 T^2} \\ x &= \lambda \tau^{\frac{1}{4}}\end{aligned} \right\} \tag{23}$$

3. (Continued)

In Eqs. (23), reasonable estimates can be made for all quantities with the probable exception of s_v . Assuming s_v ranges from 10^{-6} to 10^{-9} [1, 2] and $|R|^2 \sigma^2 T^2$ ranges from 10^{-20} to 10^{-22} , we have Q ranging from about 10^{11} to 10^{16} .

In Fig. 2 is plotted μ_d (db) vs. x with Q as parameter. Suppose, for instance, we take $\lambda = 0.5$ mm = 5×10^{-3} meter, and $\tau = 0.02$ second, so that $x = (5 \times 10^{-3}) (0.376) = 1.88 \times 10^{-3}$. Then Fig. 2 yields, at best, a rev./signal ratio of zero db, a mark of non-feasibility. On the other hand, if the frequency is doubled, $\lambda = 0.25$ mm, and the rev./signal ratio is (at best) -12 db, which is potentially useable. Nevertheless, the optimistic use of $Q = 10^{11}$ is far from justified, with the greatest element of uncertainty, for the present, residing in s_v .

4. Conclusion

It is necessary to acquire more information about scattering strength of ocean volume scatterers in the millimeter and fractional millimeter wavelength range. The information should cover a variety of conditions, such as variation with frequency in the low MHz range, depth below surface, time of day and year, geographical location [2, 3]. Such data may already exist. We strongly suggest that as a start an interview be arranged with appropriate personnel at, say, Scripps's Institute of Oceanography in San Diego, for quick orientation regarding available information and problems in researching new information in this area.

REFERENCES

1. L. E. Kinsler and A. R. Frey, Fundamentals of Acoustics, Wiley, New York, 1962.
2. R. J. Urick, Principles of Underwater Sound for Engineers, McGraw-Hill, New York, 1967.
3. V. M. Albers, Underwater Acoustics Handbook, Pennsylvania State University Press, University Park, Pa.
4. EOGG International, Side Scan Sonar: Mark 1 Dual Channel System Application Notes (95 Brookline Avenue, Boston, Mass. 02215).
5. S. Frankel, Some Thoughts Concerning Restrictions on Insonifier Modulation, Etc.; Memorandum to Sandia Corporation, FA-139.
6. P. Jacquinot and B. Roizen-Dossier, Apodisation. In "Progress in Optics", Vol. III, ed. E. Wolf. Interscience Publishers, New York.
7. L. Ridenour, Radar Systems Engineering; Vol. 1, MIT Rad. Lab Series, McGraw-Hill, New York, 1947.
8. M.I. Skolnik, Introduction to Radar Systems, McGraw-Hill, New York, 1962.
9. S. Frankel, Effect of Target Curvature on the Strength of Acoustic Image Signals. Report to Sandia Corporation, FA-138, July 11, 1968.
10. S. Silver, Microwave Antenna Theory and Design; Vol. 12 of MIT Rad. Lab Series, McGraw-Hill, New York, 1949.
11. S. Frankel, Underwater Acoustic Imaging: Preliminary Feasibility Study. Sandia Labs Report, SC-CR-68-3545, March, 1968.

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An Assessment of Apodisation Requirements
for Underwater Acoustic Imaging Systems

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SUMMARY

This report presents results of computations of radiation patterns of circularly-symmetric sonic projectors employing a well-known class of pressure distributions, namely, $f(r) = (1-r^2)^m$, where r is radial distance from the center of the projector and m is a positive integer. Radiation maxima and radiation "averages", defined for limited regions off the main lobe, are given for a range of values of m . Special computation procedures required to extend available tabulated results are described.

GLOSSARY

a : Radius of circular aperture.

$B_{\frac{1}{2}}$: Value of u at 3 db point of radiation pattern; see p.10.

$f(r, \phi)$: Relative distribution of plane wave of pressure over radiator aperture; see p.3.

$F_m(u)$: See Eq. (34).

$g(u, \phi)$: Relative radiation pattern; see Eq. (1).

$G_m(u)$: See Eq. (37).

$\overline{G}_m(u)$: See Eq. (41).

$\tilde{G}_m(u)$: See Eqs. (F-5), Appendix F.

$h(u)$: Relative radiation pattern; see Eq. (13).

$h_o(u)$: Design radiation pattern; see Section 5.

H_m : See Eq. (46).

j : $\sqrt{-1}$

j_n, s : s^{th} zero of the n^{th} order Bessel function of the first kind.

$J_0(ur)$: Bessel function of first kind, zeroth order, argument (ur).

m : Positive integer.

M_m : See Eq. (F-8).

p_m : $\Gamma(u_m)$; see Eq. (8).

p_{av} : See Eq. (10).

P_n : See Eqs. (30b)

Q_n : See Eqs. (30b)

r : Radial distance from center to variable point in circular aperture.

$R_m(u)$: Eq. (E-6).

$T_p(u)$: Eq. (42).

$$u = \frac{2\pi a}{\lambda} \sin \theta = \alpha \sin \theta \approx \alpha \theta$$

u_{min} : Lower limit of u in a range of values; corresponds to θ_{min} ;
 $u_{min} = \alpha \sin \theta_{min}$, etc. See "u".

U : Value of u corresponding to total transmitted half beamwidth, $\mathcal{H}$;
 $U = \alpha \sin \mathcal{H}$.

$$\alpha = \frac{2\pi a}{\lambda}$$

$\gamma(u)$: Relative amplitude pattern normalized to unit main lobe. See Eq. (5).

$\Gamma(u)$: Relative power pattern. See Eq. (6).

$\overline{\delta h}(u)$: Deviation in radiation pattern resulting from deviation, $\delta(r)$,
in desired aperture distribution. See Section 5.

$\delta(r)$: Deviation from desired aperture distribution. See Section 5.

$$\eta(r) = \overline{\delta h} / h_0$$

θ : Angular displacement from direction of main lobe.

$\theta_{\frac{1}{2}}$: Angular displacement from direction of main lobe to direction of
3 db. reduction in radiation level of a single resolution element.

$\theta_{\min}$: Lower limit of θ in a range of values.

$\mathcal{H}$: Value of total transmitted half beamwidth.

λ : Acoustic wavelength.

σ : See Eqs. (F-3) - (F-5), Appendix F.

ϕ : Azimuth angle of a point in radiation space.

ϕ' : Azimuth angle of a point in aperture plane.

$$\Psi = \theta / \theta_{\frac{1}{2}} \quad [\text{Eq. (44)}]$$

$\Psi_{\min}$: Lower limit of Ψ in a range of values.

$$\Psi = \frac{\mathcal{H}}{\theta_{\frac{1}{2}}} \quad . \quad \text{See also Eqs. (48).}$$

1. INTRODUCTION

1.1 Statement of Typical System Problem

In Reference 1 we emphasized our discouragement with the problem of reducing diffraction crosstalk from specularly reflecting regions to an acceptable level.* In a later report [2] * we discussed the salutary effect of convex surface curvature in reducing specular reflected intensity.

Thus, for a typical machined plane surface, ratio of specular-to-diffuse reflection could be of the order of 120 db. [1] . For a typical cylindrical surface of interest we computed an improvement of 19 db, while a typical spherical surface showed an improvement of 44 db.

It might be argued that when looking at a plane surface, the probability of specular return (assuming all angles of tilt of the surface normal to the line-of-sight to be equally probable) is very small. It could further be claimed that such probability is still not great for a cylindrical surface. For a doubly-curved surface, however, (such as a mine or a missile nose), there is always at least one point of specular reflection; the probability of specular return is unity.**

* See Reference List

** Any physically-realizable, completely closed surface must have at least one point of specular reflection. For an ideal cylinder capped with flat plates at the ends, such reflection points would almost always be on the circle of intersection between cylinder and end plate. Since this edge has one principle curvature radius equal to zero, the received intensity is also zero [Ref. 2, Eq. (12), ff]

Therefore, as a minimal problem, it seems appropriate to be concerned with controlling the glare diffraction crosstalk from typical spherical surfaces. The numbers given in the opening remarks indicated that the offending specular/diffuse level is of the order of 75 db. Possibly, another 10 db or so can be gained by operating at higher than optimum frequency [1]. Thus, in order to yield equal (main beam/diffuse) and (side-lobe/specular) levels, side lobes must be down at least 65 db. With some margin to allow diffuse signal level to be the controlling level, add a factor of 10 db. Thus, as a suitable typical figure for assessing the dimensions of this problem, we arrive at a side-lobe design objective of -75 db.

1.2 Statement of Analytical Problem

The design objective may be made more quantitative in a variety of ways, with a corresponding variety in the resulting design specifications. One of the simplest design statements is a specification for the maximum value of the side-lobe level; for instance, we could specify that no side lobe maximum should be less than 75 db down from the main lobe. If this is too restrictive, we might modify the requirement to state that side-lobe maxima beyond a specified number of beam widths from the main lobe shall be below a specified value, - e.g., -75 db. This method of specification is generally excessively restrictive, since it states, in effect, that the interfering signal shall never exceed a certain level.

A second method consists of specifying an average, - or, expected, - value of side-lobe level. The limitation of this method is its failure to be concerned with the frequency with which the average is exceeded.

More significant than either of these is an analytical statement of the probability that the side-lobe level will exceed a specified value, - e.g., -75 db.

In this report, all three methods are formulated analytically, but only the first two are discussed in any detail for a specified class of aperture distributions.

2. ANALYTICAL FORMULATIONS

Before proceeding with the analytical formulations suggested in the preceding section, it is well to emphasize the fact that even a level of -75 db is a very small number. While the idealized analysis may demonstrate conceptual feasibility for some design objective, the physical realization may readily be impaired by limitations of mechanical design and manufacturing tolerances to a point where the design parameters become meaningless. We have no information with which to make a judgment in this regard. It seemed desirable to first glean a quantitative impression of the influence of pertinent parameters before investigating fabrication feasibility.

2.1 Radiation Pattern, Circular Aperture, Azimuth-Independent Distribution

As indicated previously, [1], we can apply, by analogy, the results of scalar electromagnetic theory. From Silver [3] start with the relative pattern

$$g(u, \phi) = a^2 \int_0^{2\pi} \int_0^1 f(r, \phi') e^{j u r \cos(\phi - \phi')} r dr d\phi' \quad (1)$$

where

a = radius of circular aperture

f = relative distribution of plane wave of pressure over the aperture

$$u = \frac{2\pi a}{\lambda} \sin \theta \quad (2)$$

λ = acoustic wavelength

θ = angular position of field point from acoustic axis (co-latitude) (Fraunhofer region)

r = radial distance of aperture point from aperture center (on acoustic axis)

ϕ = azimuth angular position of field point (Fraunhofer region)

ϕ' = azimuth angular position of aperture point

$$j = \sqrt{-1}$$

For azimuth-independent distribution, $f(r, \phi') = f(r)$:

$$\begin{aligned}
 g(u, \phi) &= a^2 \int_0^1 r f(r) \left\{ \int_0^{2\pi} e^{j u r \cos(\phi - \phi')} d\phi' \right\} dr \\
 &= 2\pi a^2 \int_0^1 r f(r) J_0(ur) dr \quad * \\
 &= g(u)
 \end{aligned} \tag{3}$$

where $J_0(ur)$ is the Bessel function of the first kind, zeroth order.

In the subsequent analysis we will limit discussion to distributions which yield the main lobe at $\theta = 0$. This implies $u = 0$, and the relative amplitude of this main lobe is then

$$g(0) = 2\pi a^2 \int_0^1 r f(r) dr \tag{4}$$

since $J_0(0) = 1$. The relative pattern normalized to unit main lobe is

$$\gamma(u) = \frac{g(u)}{g(0)} = \frac{\int_0^1 r f(r) J_0(ur) dr}{\int_0^1 r f(r) dr} \tag{5}$$

Assuming $f(r)$ is real, γ is real, but not necessarily positive. The relative power pattern is

$$\Gamma(u) = \gamma^2(u) \tag{6}$$

* See for instance, Appendix B of Reference 1

2.2 Quantitative Forms for Design Objectives

2.2.1 Side-Lobe Maxima

The maxima of $\Gamma(u)$ occur at the maxima of $|\gamma(u)|$, given by

$$\gamma'(u_m) = 0, \quad m = 1, 2, \dots \quad (7)$$

provided that

$$\gamma''(u_m) \neq 0$$

The side-lobe power maxima are then

$$p_m = \Gamma(u_m), \quad m = 1, 2, \dots \quad (8)$$

or, in db

$$N_{pm} = 10 \log \Gamma(u_m) = 20 \log |\gamma(u_m)| \quad (9)$$

2.2.2 Average Side-Lobe Power

In Appendix A we argue that a reasonable formulation for averaging the specular power received by all image elements beyond a specified angular displacement from the specular main lobe is

$$p_{av} = 2 \frac{\int_{\theta_{min}}^{\theta} \theta \Gamma(u) d\theta}{\theta^2 - \theta_{min}^2} \quad (10)$$

where θ is the half-width of the total transmitted beam, θ_{min} is the largest angle away from the specular lobe at which excessive average specular crosstalk is acceptable.

2.2.3 Probability of Exceeding Specified Power Level

The probability that Γ will exceed a specified level, Γ_b , is

$$\text{Prob } (\Gamma > \Gamma_b) = \int_{\Gamma_b}^{\Gamma_{\max}} \text{pr } (\Gamma) d\Gamma \quad (11)$$

where $\Gamma_{\max}$ is the maximum value Γ attains in the specified range of θ and $\text{pr}(\Gamma)$ is the probability density, or frequency function, of the received power level [4, Chapter 15].

2.3 Choice of Class of Aperture Distribution Functions

According to the main references used in this study [3, 5-8], a rational procedure for determining an optimum aperture to satisfy a specified set of conditions is not available as of the dates of those references. Nevertheless, the range of possibilities that need to be considered turns out to be fairly well limited by the usual types of requirements.

The first limitation on an acceptable form for $f(r)$ is that it be an even function. Justification for this limitation is that it is analytically convenient, without being seriously restrictive. Furthermore, given the value of a physically reasonable function at a finite number of points in the positive (or negative) range only, an even function can be found passing through those points. In addition, all aperture distributions, $f(r)$, of interest to us, have the properties

$$\left. \begin{array}{l} f(0) \neq 0 \\ f'(0) = 0 \end{array} \right\} , \quad (12)$$

a set of properties guaranteed by an even function, provided it, and its derivative, are defined at $r = 0$. Finally, it has been shown that, for one-dimensional distributions (function of x - only, or of y - only) the best use of

the aperture is obtained by employing an even function [5].

2.3.1 Asymptotic Behavior

Because we suspect that we must get some angular displacement from the main lobe before we can realize the small values of side lobe power required, it appears of interest to become familiar with the asymptotic behavior of the radiation pattern, $g(u)$, for large u . Jacquinet & Roizen-Dossier [5] give such an expansion for even functions, $f(r)$. Details appear in Appendix B of this report.

If, in Eq. (8), $f(r)$ is an even function of r , with all derivatives existing in $0 \leq r \leq 1$, write

$$\left. \begin{aligned} f(r) &= T(r^2) = T(s) \\ s &= r^2 \end{aligned} \right\} \quad (12)$$

Then, in Appendix B we show a derivation of Jacquinet's result:

$$\begin{aligned} h(u) &= \frac{g(u)}{2\pi u^2} \\ &= \frac{1}{u} \left\{ T(1)J_1(u) - \left(\frac{2}{u}\right) T'(1)J_2(u) + \left(\frac{2}{u}\right)^2 T''(1)J_3(u) - \dots \right. \\ &\quad \left. + (-1)^m \left(\frac{2}{u}\right)^m T^{(m)}(1) J_{m+1}(u) + \dots \right\} \end{aligned} \quad (13)$$

where $J_n(u)$ is the Bessel function of the first kind, n^{th} order and

$$T^{(n)}(1) = \left[T^{(n)}(s) \right]_{s=1} \quad (14)$$

Eq. (13) shows that the rate at which the side-lobe level decreases with large increasing u depends essentially on the lowest order non-vanishing derivative of the distribution function at $r = 1$. If $T(1) \neq 0$, the behavior is [9]

$$\begin{aligned}
h(u) \sim h_0(u) &= T(1) \frac{J_1(u)}{u} \\
&\sim T(1) \sqrt{\frac{\pi}{2}} \frac{\cos\left(u - \frac{3\pi}{4}\right)}{u^{3/2}}
\end{aligned} \tag{15}$$

If $T(1) = 0$ but $T'(1) \neq 0$

$$\begin{aligned}
h(u) \sim h_1(u) &= -\frac{2}{u^2} T'(1) J_2(u) \\
&\sim -2 T'(1) \sqrt{\frac{\pi}{2}} \frac{\cos\left(u - \frac{5\pi}{4}\right)}{u^{5/2}}
\end{aligned} \tag{16}$$

And, in general, if $T(1) = T'(1) = \dots = T^{(m-1)}(1) = 0$,

while $T^{(m)}(1) \neq 0$,

$$\begin{aligned}
h(u) \sim h_m(u) &= (-1)^m \frac{2^m}{u^{m+1}} T^{(m)}(1) J_{m+1}(u) \\
&\sim (-1)^m 2^m T^{(m)}(1) \sqrt{\frac{\pi}{2}} \frac{\cos\left[u - \left(m + \frac{3}{2}\right) \frac{\pi}{2}\right]}{u^{m+3/2}}
\end{aligned} \tag{17}$$

2.3.2 Class of Functions, $T(r^2)$ With Specified Behavior; Normalized Pattern

A function, $T(r^2)$, possessing all the necessary derivatives, with the behavior leading to (17), may be written

$$T(r^2) = (1 - r^2)^m \left[a_0 + a_1 (1-r^2) + a_2 (1-r^2)^2 + \dots \right] \tag{18}$$

where $a_0 \neq 0$. Most of our attention in this report is confined to the special case of (18) where $a_0 = 1$, $a_k = 0$, $k > 1$. Thus, our primary interest from this point is in the aperture distribution

$$f(r) = (1 - r^2)^m \tag{19}$$

We mention in passing that Taylor [6, 7] uses (18), with $-1 < m < \infty$, (m not restricted to integer values) as a basis for designing aperture distributions that give optimum aperture efficiency with nearly uniform side lobes. Silver [3] gives data based on (19) for $m = 0, 1, 2, 3, 4$.

Eq. (19) in (3) now yields

$$h(u) = \frac{g(u)}{2\pi a^2} = \int_0^1 r (1 - r^2)^m J_0(ur) dr \quad (20)$$

Evaluation of (20) results in *

$$h(u) = \frac{2^m (m!) J_{m+1}(u)}{u^{m+1}} \quad (21)$$

Following Jahnke and Emde [9] we define the lambda-functions.

$$\Lambda_p(u) = \left(\frac{u}{2}\right)^p (p!) J_p(u), \quad p = 0, 1, 2, \dots \quad (22)$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{p!}{(p+k)!} \left(\frac{u}{2}\right)^{2k} \quad (22a)$$

It is easy, using (22a), to check that

$$\left. \begin{aligned} \Lambda_p(0) &= 1 \\ \Lambda_p'(0) &= 0 \\ \Lambda_p''(0) &< 0 \end{aligned} \right\} \quad (23)$$

* See Appendix C

where, as usual, primes represent derivatives. Thus, Λ_p have maxima at $u = 0$, the maximum value being unity. $u = 0$ also represents the direction of the main lobe [3].

Using (22) in (21) yields

$$h(u) = \frac{\Lambda_{m+1}(u)}{2(m+1)} \quad (23a)$$

and

$$g(u) = 2\pi a^2 h(u) = \frac{\pi a^2}{m+1} \Lambda_{m+1}(u) \quad (24)$$

However, if the main-lobe amplitude is normalized to unity, the pattern is simply $\Lambda_{m+1}(u)$, and this is the function with which we will deal primarily.

3. Computation Procedures

After a brief discussion of a convenient approximate expression for beam width we consider (1) determination of the maximum side-lobe levels, (2) determination of the average side-lobe level as specified in Section 2.2.2 and Appendix A.

3.1 Approximate Expression for Beam Width

The half-beam width is the angular displacement from the center of the main lobe to the direction for 3db power reduction. Although these beamwidths can be determined exactly from available tables, some accuracy can be sacrificed for the sake of analytical convenience in a preliminary study. From (22a)

$$\Lambda_p(u) \approx 1 - \frac{1}{p+1} \left(\frac{u}{2} \right)^2 \quad (25)$$

for sufficiently small u . At the 3 db point, $\Lambda_p = \frac{1}{\sqrt{2}}$. Writing $B_{\frac{1}{2}}$ for u at the 3 db point, (25) yields, approximately

$$\frac{1}{\sqrt{2}} = 1 - \frac{B_{\frac{1}{2}}^2}{u(p+1)}$$

whence, for an antenna pattern of the form $\Lambda_p(u)$,

$$B_{\frac{1}{2}} = 2 \left[1 - \frac{1}{\sqrt{2}} \right]^{\frac{1}{2}} \sqrt{p+1} = 1.08 \sqrt{p+1} \quad (26)$$

Values of $B_{\frac{1}{2}}$ computed from (26) are compared with exact tabulated values [9] in the following table:

Table I: Beam Width Approximation Error

p	$1.08 \sqrt{p+1}$	Table Values	Error	%
0	1.08	1.12	-0.04	-3.6
1	1.53	1.62	-0.09	-5.6
2	1.87	2.00	-0.13	-6.5
3	2.16	2.32	-0.16	-6.9
4	2.42	2.60	-0.18	-6.9
5	2.64	2.85	0.21	-7.4
6	2.86	3.08	-0.22	-7.1
7	3.05	3.30	-0.25	-7.6
8	3.24	3.50	-0.26	-7.4

The actual half-power, half-beamwidth is

$$\theta_{\frac{1}{2}} = \sin^{-1} \left[\frac{\lambda B_{\frac{1}{2}}}{2\pi a} \right] \approx \frac{\lambda B_{\frac{1}{2}}}{2\pi a} \quad (27)$$

Thus, for an aperture distribution given by (19), the half-power half beamwidth is, approximately,

$$\theta_{\frac{1}{2}}(m) = \frac{1.08 \sqrt{m+2}}{\alpha} \quad (28)$$

where

$$\alpha = \frac{2\pi a}{\lambda} \quad (29)$$

3.2 Determination of Maximum Side-Lobe Levels: Maxima/Minima of $\Lambda_n(u)$

Calculations were made out to the fifth side lobe up to whatever value of m [Eq. (24)] was necessary (in each case*) to assure that larger values of m would yield levels below -75 db. Tables of $\Lambda_p(u)$ are in Jahnke v. ~~Ende~~ [9] for $p = 0, 1, \dots, 8$. Maxima/minima can be obtained by inspection of those tables for two values (i.e., side lobes) of Λ_1 ($m=0$), and one value each of Λ_2 to Λ_5 ($m = 1, \dots, 4$). These results were clearly inadequate. To go beyond them we derived a simple theorem (Appendix D), relating the maxima/minima of the Λ -functions to the zeros of the Bessel functions of the first kind. The theorem simply is that the maxima/minima of $\Lambda_p(u)$ coincide with the zeros of $J_{p+1}(u)$. The Bureau of Standards tables [10] give the first twenty zeros of $J_n(u)$ ($u \neq 0$) for $n=0, 1, \dots, 8$.

In order to attain the -75 db level for each of the first five side lobes, it was necessary to obtain $J_n(u)$ for the largest n and the largest u indicated in the following table:

<u>Side Lobe Number</u>	<u>Largest n Required in $J_n(u)$</u>	<u>Largest u Required</u>
1	14	20.00
2	9	18.47
3	7	19.55
4	6	21.64
5	5	23.59

The Bureau of Standards tables have values of $J_n(u)$ in the following ranges:

$$n = 0, 1, 2 \quad (0 \leq u \leq 17.5)$$

$$n = 3, \dots, 9 \quad (0 \leq u \leq 20)$$

$$n = 10, 11, 20, 21 \quad (0 \leq u \leq 20)$$

* Except for $m=1$, for which the side lobes were evidently decreasing much too slowly.

Thus, we needed data beyond the tables for both n and u .

To obtain $J_{12}(u)$, $J_{13}(u)$, and $J_{14}(u)$ we used the following recurrence relation as suggested in the Bureau of Standards tables:

$$\left. \begin{aligned} J_{12} &= \frac{22}{u} J_{11} - J_{10} \\ J_{13} &= \frac{24}{u} J_{12} - J_{11} \\ J_{14} &= \frac{26}{u} J_{13} - J_{12} \end{aligned} \right\}$$

When u exceeded the tabulated values for $J_n(u)$, we used the asymptotic expansion [9, 10]

$$J_n(u) = \sqrt{\frac{2}{\pi u}} \left[P_n(u) \cos \phi - Q_n(u) \sin \phi \right] \quad (30)$$

where

$$\phi = u - \left(n + \frac{1}{2}\right) \frac{\pi}{2} \quad (30a)$$

and

$$\left. \begin{aligned} P_n(u) &= 1 - \frac{(4n^2 - 1)(4n^2 - 9)}{2! (8u)^2} + \frac{(4n^2 - 1)(4n^2 - 9)(4n^2 - 25)(4n^2 - 49)}{4! (8u)^4} \\ Q_n(u) &= \frac{4n^2 - 1}{8u} - \frac{(4n^2 - 1)(4n^2 - 9)(4n^2 - 25)}{3! (8u)^3} + \dots \end{aligned} \right\} \quad (30b)$$

Usually, it was unnecessary to go beyond the first term for Q_n , and the first two terms for P_n .

The first zeros for $n = 9, \dots, 14$, and the second zeros for $n = 9, 10$ were needed, but were not available. Those were calculated using the following formulas from Jahnke u. Emde, p. 143, Section 3 for u_1 and u_2 , the first and second zeros of $J_n(u)$ respectively:

$$\left. \begin{aligned} u_1 &\approx n + 1.856 n^{1/3} + \frac{1.033}{n^{1/3}} - \frac{0.004}{n} - \frac{0.091}{n^{5/3}} + \dots \\ u_2 &\approx n + 3.245 n^{1/3} + \frac{3.158}{n^{1/3}} + \dots \end{aligned} \right\}$$

The formula for u_1 yields adequate accuracy, but that for u_2 is only an approximation. Newton's method was used to get improved approximations. For the present problem this method translates to the following statement: If u_q is an approximate solution to

$$J_m(u) = 0 ,$$

a better approximation, u_{q+1} , may be obtained from

$$\begin{aligned} u_{q+1} &= u_q - \frac{J_m(u_q)}{J'_m(u_q)} \\ &= u_q - \frac{J_m(u_q)}{\frac{m}{u_q} J_m(u_q) - J_{m+1}(u_q)} \end{aligned}$$

In this way we obtained the following positive second roots of J_9 and J_{10} :

$$\left. \begin{aligned} J_{9,2} &= 17.24 \\ J_{10,2} &= 18.47 \end{aligned} \right\} *$$

B. of S. [10]notation.

3.3 Average Side-Lobe Level

The formulation for average side-lobe level is given in Eq. (10). For the aperture distribution given by (19) we have

$$\begin{aligned} \gamma(u) &= \frac{g(u)}{g(0)} = \frac{h(u)}{h(0)} \quad [\text{Eq. (5)}] \\ &= \Lambda_{m+1}(u) \end{aligned} \quad (23b)$$

by Eq. (23a). Then (6) and (22) result in

$$\Gamma(u) = \left(\frac{2}{u}\right)^{2m+2} [(m+1)!]^2 J_{m+1}^2(u). \quad (31)$$

Using this result in (10), with $u \approx \alpha\theta$, yields

$$p_{av} = 2 \left[\left(\frac{2}{\alpha}\right)^{m+1} (m+1)! \right]^2 \frac{\int_{\theta_{min}}^{\theta} \frac{J_{m+1}^2(\alpha\theta) d\theta}{\theta^{2m+2}}}{\left(\theta^2 - \theta_{min}^2\right)} \quad (32)$$

The integral in (32) can be evaluated in closed form (see Appendix E). Evaluation and reduction results in

$$p_{av} = \frac{4}{(2m+1)} \left\{ \frac{F_m(u_{min}) - F_m(U)}{U^2 - u_{min}^2} \right\} \quad (33)$$

where

$$\left. \begin{aligned} u_{min} &= \alpha \theta_{min} \\ U &= \alpha \theta \end{aligned} \right\} \quad (33a)$$

$$F_m(u) = (m+1)^2 \Lambda_m^2(u) + \frac{u}{2}^2 \Lambda_{m+1}^2(u) \quad (34)$$

and, generally,

$$u = \alpha \theta \quad (33b)$$

Use (22) in (34) to get

$$F_m(u) = [(m+1)!]^2 \left(\frac{2}{u}\right)^{2m} \left[J_m^2(u) + J_{m+1}^2(u) \right] \quad (35)$$

With the help of Eq. (30) we get

$$F_m(u) = \frac{1}{\pi} \left[(m+1)! \right]^2 \left(\frac{2}{u}\right)^{2m+1} G_m(u) \quad (36)$$

where

$$G_m(u) = \left[P_m(u) \cos \phi_m - Q_m(u) \sin \phi_m \right]^2 + \left[P_{m+1}(u) \cos \phi_{m+1} - Q_{m+1}(u) \sin \phi_{m+1} \right]^2 \quad (37)$$

and

$$\left. \begin{aligned} \phi_m &= u - (m+\frac{1}{2})\frac{\pi}{2} \\ \phi_{m+1} &= u - (m+\frac{3}{2})\frac{\pi}{2} = \phi_m + \frac{\pi}{2} \end{aligned} \right\} \quad (38)$$

In Appendix F we show that (37) may be written

$$G_m(u) = \bar{G}_m(u) + \tilde{G}_m(u) \sin(2\phi_m + \sigma) \quad (39)$$

where $\bar{G}_m$, $\tilde{G}_m$, and σ are slowly varying functions of u .* For our averaging purposes, the term in $\tilde{G}_m$, which goes through a full sine-wave cycle in approximately one beam width, is not significant. Thus in (36), we replace G_m by $\bar{G}_m$:

$$F_m(u) = \frac{1}{\pi} \left[(m+1)! \right]^2 \left(\frac{2}{u}\right)^{2m+1} \bar{G}_m(u) \quad (40)$$

* i.e., they change relatively little in one cycle of ϕ , or in an interval $\Delta u = 2\pi$

where, according to Appendix F,

$$\bar{G}_m(u) = \frac{1}{2} \left(T_m^2 + T_{m+1}^2 \right) \quad (41)$$

and [10; Section 9.2.28] we have the asymptotic expansion

$$T_p^2(u) = \frac{\pi u}{2} M_p^2(u) = \left[1 + \frac{1}{2} \frac{q-1}{(2u)^2} + \frac{1.3}{2.4} \frac{(q-1)(q-9)}{(2u)^4} + \frac{1.3.5}{2.4.6} \frac{(q-1)(q-9)(q-25)}{(2u)^6} + \dots \right] \quad (42)$$

with $q = 4 p^2$

Thus (40) is written

$$F_m(u) = \frac{1}{2\pi} \left[(m+1)! \right]^2 \left(\frac{2}{u} \right)^{2m+1} \left[T_m^2 + T_{m+1}^2 \right] \quad (43)$$

In Appendix F we show that T_p^2 , Eq. (42), may be approximated by the first two terms of the series for values of u exceeding those in Table II. (Page 18)

In Table II, the quantity Ψ is θ measured in half-beamwidths.

Since $u \approx \alpha \theta$, Eqs. (26) and (29) imply

$$\Psi = \frac{\theta}{\theta_{\frac{1}{2}}} = \frac{u}{1.08 \sqrt{m+2}} \quad (44)$$

Table II. Minimum Values of u and Ψ For Approximating $T_p^2(u)$

<u>m</u>	<u>u</u>	<u>Ψ</u>
1	1.63	0.98
2	2.64	1.22
3	4.58	1.91
4	6.37	2.41
5	8.03	2.81

Within these limits we can then write

$$\begin{aligned}
 T_m^2 + T_{m+1}^2 &\approx 2 \left[1 + \frac{1}{2} \left(\frac{2m+1}{2u} \right)^2 \right] \\
 &\approx 2 \left[1 + \frac{(2m+1)^2}{9.36(m+2)\Psi^2} \right] \\
 &= 2 H_m(\Psi)
 \end{aligned} \tag{45}$$

where

$$H_m = 1 + \frac{(2m+1)^2}{9.36 (m+2)\Psi^2} \tag{46}$$

With the help of Eqs. (43) - (46), Eq. (33) now becomes

$$p_{av} = 1.09 \left\{ \frac{(1.852)^{2m+1} [(m+1)!]^2}{(2m+1)(m+2)^{m+3/2}} \right\} \left\{ \frac{\left[\frac{H_m(\Psi_{min})/\Psi_{min}^{2m+1}}{\Psi^2 - \Psi_{min}^2} \right] - \left[\frac{H_m(\Psi)/\Psi^{2m+1}}{\Psi^2 - \Psi_{min}^2} \right]}{\Psi^2 - \Psi_{min}^2} \right\} \tag{47}$$

where Ψ is the number of element beam widths contained in the total transmitted beam width; or it is, approximately, the square root of the number of resolution elements.

For a receiving antenna with $n \times n$ independent receiving elements in the image plane, $\Psi \approx n$. H_m is given by (46) for values of Ψ greater than those shown in Table II.

Note that, via Eq. (44)

$$\left. \begin{aligned} \Psi_{\min} &= \left[\Psi \right]_{u=u_{\min}} \\ \Psi &= \left[\Psi \right]_{u=U} \end{aligned} \right\} \quad (48)$$

4. Computation Results

The results of computations based on the foregoing discussion are summarized in the following table and curves.

Table III shows values of side lobe maxima (in db down from the main lobe) and their angular displacements, in half beamwidths, Ψ , from the main lobe, for values of m from 1 to 7, when the relative aperture distribution is

$$f(r) = (1-r^2)^m \quad (19)$$

Since it was not considered worthwhile to carry the computations beyond $m = 7$, the table shows no value of m for which the "design objective" of -75 db is reached at the first side lobe. Beyond the first side lobe the desired objective is reached for $m \leq 7$, with a trade-off existing between the required value of m , and the number of half beamwidths over which the design objective fails to be realized.

Figure 1 shows average side lobe level in the interval $\Psi_{\min} \leq \Psi \leq \Psi = 10$, where the meaning of "average" is defined in the preceding section, $\Psi_{\min}$ is the minimum angle (half beamwidths) beyond which the average is as shown, and Ψ is the number of element beamwidths contained in the total transmitted beamwidth (see Section 3.3 following Eq. (47).)

Table III. Side-Lobe Maxima vs. m

m	Side Lobe Number									
	1		2		3		4		5	
	Ψ	db	Ψ	db	Ψ	db	Ψ	db	Ψ	db
1	3.4	24.6	5.2	33.6	7.0	39.8	8.7	44.4	10.4	48.3
2	3.5	30.6	5.1	42.7	6.7	49.6	8.2	55.6	9.7	60.6
3	3.6	36.0	5.1	48.9	6.5	58.0	7.9	65.4	9.2	71.5
4	3.8	40.9	5.1	55.4	6.4	65.8	7.7	74.3	8.9	81.6
5	3.9	47.7	5.2	61.1	6.4	72.9	7.6			
6	4.0	50.1	5.3	66.8	6.4	79.6				
7	4.1	54.3	5.3	74.0						

Side Lobe Level
(db)

Fig. 1: AVERAGE SIDE-LOBE LEVEL IN
THE INTERVAL $\psi_{\min} \leq \psi \leq \psi$

5. DISCUSSION AND CONCLUSIONS

In order to meet the design objective stipulated at the beginning of this report, it would appear that a value of m of at least 4 is required in Eq. (19). In that case the maximum side-lobe level meets the design objective at angles greater than 8 half beamwidths from the main lobe. For the same case the average side-lobe level is within the objective beyond 6 half beamwidths.

The question that next presents itself is that of tolerances on $f(r)$ required to realize the predicted performance without excessive deviation. No serious attempt is made to answer this question in the present report. The answer may well be available in error studies in connection with microwave antennas; hence a next step in this regard is a literature search of the subject.

However, we can take a crude look at the problem by introducing a small variation in the assumed distribution, $f(r)$. Thus, if the actual distribution is

$$F(r) = f(r) + \delta(r) \quad (49)$$

then the relative pattern is

$$h(u) = \frac{A_{m+1}(u)}{2(m+1)} + \int_0^1 r \delta(r) J_0(ur) dr \quad (50)$$

[See Section 2.3.2]

Next, if we assume $\delta(r)$ can be expressed as an even function, then the leading term in the asymptotic expansion of the variation in h is

$$\overline{\delta h}(u) \sim \delta(1) \frac{J_1(u)}{u} \sim \sqrt{\frac{\pi}{2}} \delta(1) \frac{\cos\left(u - \frac{3\pi}{4}\right)}{u^{3/2}} \quad (51)$$

Similarly, the leading term in the asymptotic expansion of Λ_{m+1} is

$$\Lambda_{m+1} \sim \left(\frac{2}{u}\right)^{m+1} (m+1)! \cos \left[u - \left(m + \frac{3}{2}\right) \frac{\pi}{2} \right] \cdot \sqrt{\frac{\pi}{2u}} \quad (52)$$

In this crude estimate we are interested only in comparing the peaks of the variation with the side lobes produced by the undisturbed distribution. Therefore, the comparison is made with the trigonometric functions omitted. Write

$$\eta = \frac{\overline{\delta h}}{h_0} \quad (53)$$

$$\text{where } h_0 = \frac{\Lambda_{m+1}}{2^{m+1}} \quad (54)$$

We get

$$\begin{aligned} \eta &= \left[\frac{\sqrt{\frac{\pi}{2}} \delta(1)}{u^{3/2}} \right] \left[\frac{\sqrt{\frac{2}{\pi}} u^{m+3/2}}{2^m (m)!} \right] \\ &= \frac{\delta(1) u^m}{2^m (m)!} \end{aligned} \quad (55)$$

Using (44)

$$\eta = \delta(1) \frac{(m+2)^{\frac{m}{2}}}{m!} (0.54 \Psi)^m \quad (56)$$

For example, with $m = 4$ and $\Psi = 6.2$ (Fig. 1)

$$\begin{aligned} \eta &= \delta(1) \frac{6^3}{24} (3.35)^3 \\ &= 341 \delta(1) \end{aligned} \quad (57)$$

If $\eta = 2$, then $h_0 + \overline{\delta h} = 3$, representing an increase in side lobe level of 9.5 db over the normal design figure. It must be emphasized again that this approach represents the crudest of estimates. However, with this in mind we get, from (57), that the permitted variation is

$$\delta(1) = \frac{2}{341} \approx 0.006$$

or 0.6% of the distribution maximum.

Finally it should be noted that we have inspected the behavior of only one class of aperture pressure distributions. A number of other types are available for more refined consideration [5, 7], among them special cases of Eq. (18) other than Eq. (19).

APPENDIX A

Average Spurious Specular Power Received By Diffraction

Assume a continuous (one piece) circular piezoelectric transducer in the receiver image plane. Assume, also, a spherical target on the sonic axis of the receiver at such a distance that its (projected) image covers, approximately, the image-plane transducer.

The specular reflection from the target is, therefore, on the sonic axis, and should, therefore, properly be received by the central element of the circular matrix. However, because of the diffraction lobes possessed by all elements of the matrix, every other element will, in general, receive some diffracted energy. An element with its main lobe at an angle, θ , to the sonic axis of the system will see the specular reflection from the target with the side lobe at angle, θ , to its own axis.

In Fig. A-1, the image-plane points with main lobe at angle θ to the sonic axis lie on a circle of radius ρ , given, for small total beamwidths, by

$$\rho = f \theta \quad (A-1)$$

where f is the focal distance to the image plane.

Fig. A-1. Image-plane Geometry

The power received in the axial direction by elements in the annular ring ($\rho, \rho + d\rho$) is

$$\begin{aligned} dP_s &= \Gamma(u) (2\pi \rho \cdot d\rho) \\ &= 2\pi r^2 \Gamma(u) \cdot \theta d\theta \end{aligned}$$

Let $\theta =$ half width of total transmitted beam

$\theta_{\min}$ = largest angle away from the specular lobe at which excessive average specular crosstalk is acceptable.

Then the power received in an annular interval ($\theta_{\min}, \theta$) is

$$P_s = 2\pi r^2 \int_{\theta_{\min}}^{\theta} \theta \Gamma(u) d\theta \quad (A-2)$$

If p_{av} designates the average power received over the annular ring, then, also,

$$P_s = 2\pi r^2 p_{av} \int_{\theta_{\min}}^{\theta} \theta d\theta = \pi r^2 p_{av} [\theta^2 - \theta_{\min}^2] \quad (A-3)$$

Combining (A-2) and (A-3),

$$p_{av} = 2 \frac{\int_{\theta_{\min}}^{\theta} \theta \Gamma(u) d\theta}{\theta^2 - \theta_{\min}^2} \quad (A-4)$$

Furthermore, the geometry selected (target on sonic axis) appears to be a worst case. For, in order to see a target appreciably off axis, the target must be appreciably further away from the receiver. The first effect of this change is that the relative specular return to the receiver will be smaller [2]. The second effect is that the average angle, θ , at which elements of the transducer see the specular reflection will be greater, hence the diffraction energy received by these elements will, on the average, be smaller [See Section 2.3.1]

APPENDIX B

Asymptotic Expansion of $\int_0^1 r f(r) J_0(ur) dr$, when $f(r)$ is an Even Function

---

Let

$$f(r) = T(r^2) = T(s) \quad (B-1)$$

where

$$\left. \begin{aligned} s &= r^2 \\ \frac{ds}{dr} &= 2r \\ s \Big|_{r=1} &= 1 \end{aligned} \right\} \quad (B-2)$$

First we obtain

$$\int r^{a+1} J_a(ur) dr = \frac{1}{u} r^{a+1} J_{a+1}(ur), \quad a = 1, 2, 3, \dots \quad (B-3)$$

From [9], p.45 we have

$$\int t^{a+1} J_a(t) dt = t^{a+1} J_{a+1}(t) \quad (B-4)$$

Write $t = ur$; $dt = u dr$

Then (B-4) becomes

$$\int (ur)^{a+1} J_a(ur) (u dr) = (ur)^{a+1} J_{a+1}(ur);$$

that is,

$$u \int r^{a+1} J_a(ur) dr = r^{a+1} J_{a+1}(ur)$$

which is essentially (B-3).

Now write

$$\begin{aligned} h(u) &= \frac{g(u)}{2\pi a^2} = \int_0^1 r f(r) J_0(ur) dr \\ &= \int_0^1 r T(s) J_0(ur) dr \end{aligned} \quad (B-5)$$

Integrate by parts; let

$$x = T(s), \quad dy = r J_0(ur) dr$$

then

$$dx = T'(s)ds = 2rT'(s)dr$$

and, by (B-3), with $a = 0$,

$$y = \int_0^1 r J_0(ur) dr = \frac{r}{u} J_1(ur)$$

Therefore,

$$\begin{aligned} h(u) &= \left[T(s) \cdot \frac{r}{u} J_1(ur) \right]_0^1 - \int_0^1 \frac{r}{u} J_1(ur) [2rT'(s)] dr \\ &= \frac{1}{u} \left[T(1) J_1(u) \right] - \frac{2}{u} \int_0^1 T'(s) \cdot r^2 J_1(ur) dr \end{aligned} \quad (B-6)$$

Continuing integrating by parts, let

$$x = T'(s), \quad dy = r^2 J_1(ur) dr$$

$$dx = T''(s) \frac{ds}{dr} dr = 2r T''(s) dr$$

$$y = \int r^2 J_1(ur) dr = \frac{r^2}{u} J_2(ur)$$

by (B-3), with $a = 1$

Then

$$\begin{aligned} \int_0^1 T'(s) r^2 J_1(ur) dr &= \left[T'(s) \frac{r^2}{u} J_2(ur) \right]_0^1 \\ &\quad - \frac{2}{u} \int_0^1 T''(s) r^3 J_2(ur) dr \\ &= \frac{1}{u} T'(1) J_2(u) - \frac{2}{u} \int_0^1 T''(s) r^3 J_2(ur) dr \end{aligned}$$

and, by (B-6)

$$h(u) = \frac{1}{u} \left[T(1) J_1(u) \right] - \frac{2}{u^2} \left[T'(1) J_2(u) \right] + \frac{4}{u^2} \int_0^1 T''(s) r^3 J_2(ur) dr \quad (B-7)$$

The foregoing suggests, - and we will prove by induction, - that

$$h(u) = \frac{1}{u} \left\{ T(1) J_1(u) - \left(\frac{2}{u} \right) T'(1) J_2(u) + \left(\frac{2}{u} \right)^2 T''(1) J_3(u) - \dots + (-1)^k \left(\frac{2}{u} \right)^k T^{(k)}(1) J_{k+1}(u) \right\} + R_k \quad (B-8)$$

where

$$R_k = (-1)^{k+1} \left(\frac{2}{u} \right)^{k+1} \int_0^1 T^{(k+1)}(s) r^{k+2} J_{k+1}(ur) dr \quad (B-9)$$

For, in (B-9), let

$$x = T^{(k+1)}(s); \quad dy = r^{k+2} J_{k+1}(ur) dr$$

Then

$$dx = T^{(k+2)}(s) [2r dr]; \quad y = \frac{1}{u} r^{k+2} J_{k+2}(ur)$$

Thus,

$$R_k = (-1)^{k+1} \left(\frac{2}{u} \right)^{k+1} \left\{ \frac{1}{u} \left[T^{(k+1)}(s) \cdot r^{k+2} J_{k+2}(ur) \right]_0^1 - \left(\frac{2}{u} \right) \int_0^1 T^{(k+2)}(s) \cdot r^{k+3} J_{k+2}(ur) \cdot dr \right\}$$

$$R_k = \frac{1}{u} \left\{ (-1)^{k+1} \left(\frac{2}{u} \right)^{k+1} \int_0^1 T^{(k+1)}(s) \cdot J_{k+2}(u) \right. \\ \left. + (-1)^{k+2} \left(\frac{2}{u} \right)^{k+2} \int_0^1 T^{(k+2)}(s) \cdot r^{k+3} J_{k+2}(ur) \cdot dr \right\} \quad (B-10)$$

Then (B-10) in (B-8) gives

$$h(u) = \frac{1}{u} \left\{ T(1) J_1(u) - \left(\frac{2}{u} \right) T'(1) J_2(u) + \left(\frac{2}{u} \right)^2 T''(1) J_3(u) - \dots \right. \\ \left. + (-1)^k \left(\frac{2}{u} \right)^k T^{(k)}(1) J_{k+1}(u) \right. \\ \left. + (-1)^{k+1} \left(\frac{2}{u} \right)^{k+1} T^{(k+1)}(1) J_{k+2}(u) \right\} + R_{k+1} \quad (B-11)$$

Thus, if (B-9) holds to k terms, it holds also to $(k+1)$ terms. But, since by (B-7), it holds for $k=1$, it therefore holds for $k=2$, etc.

To complete the discussion we should compute the error incurred in dropping the remainder term, R_k . This has not been done. If the asymptotic expansion is used in a particular case, the remainder should be checked for that case.

APPENDIX C

Evaluation of the Integral

$$h(u) = \int_0^1 r(1-r^2)^m J_0(ur) dr$$

We can start with Example 23, p. 382, of Reference 11. We have, for $n > m > -1$,

$$J_n(x) = \frac{x^{n-m}}{2^{n-m-1}(n-m-1)!} \int_0^{\frac{\pi}{2}} J_m(x \sin \theta) (\cos^{2n-2m-1} \theta) (\sin^{m+1} \theta) d\theta \quad (C-1)$$

Let $r = \sin \theta$; $\theta = \sin^{-1} r$

$$dr = \cos \theta d\theta$$

$$\cos^{2n-2m-1} \theta \cdot d\theta = \cos^{2n-2m-2} \theta \cdot \cos \theta \cdot d\theta$$

$$= (\cos^2 \theta)^{n-m-1} \cdot dr$$

$$= (1-r^2)^{n-m-1} dr$$

$$\sin^{m+1} \theta = r^{m+1}$$

Integration limits: $\theta = 0$ implies $r = 0$

$$\theta = \frac{\pi}{2} \text{ implies } r = 1$$

Thus, (C-1) becomes

$$J_n(x) = \frac{x^{n-m}}{2^{n-m-1}(n-m-1)!} \int_0^1 J_m(xr) \cdot (1-r^2)^{n-m-1} \cdot r^{m+1} \cdot dr \quad (C-2)$$

Let $m = 0, n-1 = p$. Then

$$J_{p+1}(x) = \frac{x^{p+1}}{2^p(p!)} \int_0^1 J_0(xr) \cdot (1-r^2)^p \cdot r dr$$

Finally, setting $x = u, p = m$, and re-arranging

$$h(u) = \int_0^1 J_0(ur) \cdot (1-r^2)^m r dr = \frac{2^m(m!) J_{m+1}(u)}{u^{m+1}} \quad (C-3)$$

which is Eq. (21)

It is interesting to obtain the asymptotic series for this function, using the results of Appendix B. We have

$$T(s) = (1-s)^m$$

$$T^{(k)}(s) = (-1)^k \frac{m!}{(m-k)!} (1-s)^{m-k}$$

Thus $T^{(k)}(1) = 0, m \neq k$

$$= (-1)^m (m)!, k = m$$

Substituting these results in (B-8) and (B-9) yields

$$\begin{aligned} h(u) &= \frac{1}{u} \left\{ (-1)^m \left(\frac{2}{u} \right)^m T^{(m)}(1) J_{m+1}(u) \right\} \\ &= \frac{1}{u} \left\{ (-1)^m \left(\frac{2}{u} \right)^m \left[(-1)^m (m)! \right] J_{m+1}(u) \right\} \\ &= \frac{2^m(m!) J_{m+1}(u)}{u^{m+1}} \end{aligned}$$

in agreement with (C-3). Note, in particular, that the remainder in (B-8) is zero.

APPENDIX D

Proof That The Stationary Points of $\Lambda_n(x)$ Coincide With The Zeros of $\frac{J'_{n+1}(x)}{J_n(x)}$.

We have

$$\Lambda_n(x) = 2^n(n!) \frac{J_n(x)}{x^n} \quad (D-1)$$

At a stationary (i.e., maximum or minimum) point, x_m ,

$$\Lambda'_n(x_m) = 0 \quad (D-2)$$

From (D-1),

$$\Lambda'_n(x_m) = 2^n(n!) \left[\frac{J'_n(x_m)}{x_m^n} - n \frac{J_n(x_m)}{x_m^{n+1}} \right] = 0 \quad (D-3)$$

For $x_m \neq 0$, (D-3) implies

$$x_m J'_n(x_m) - n J_n(x_m) = 0 \quad (D-4)$$

We have the standard relation [9]

$$J'_n(x) = \frac{n}{x} J_n(x) - J_{n+1}(x) \quad (D-5)$$

Using (D-5) in (D-4)

$$x_m \left[\frac{n}{x_m} J_n(x_m) - J_{n+1}(x_m) \right] - n J_n(x_m) = 0$$

which reduces to

$$x_m J_{n+1}(x_m) = 0$$

which, for $x_m \neq 0$, implies

$$J_{n+1}(x_m) = 0 \quad (D-6)$$

Furthermore, by repeating the preceding substitution it is easily shown that

$$\Lambda_n''(x) = 2^n(n!) \frac{J_{n+2}(x)}{x^n} \quad (D-7)$$

But if $J_{n+1}(x_m) = 0$, $J_{n+2}(x_m) \neq 0$, since zeros of successive orders of Bessel functions of the first kind alternate, - i.e., do not coincide [11] .

Therefore x_m is not a point of inflection of $\Lambda(x)$. It follows that it is a stationary point of $\Lambda_n(x)$ and, simultaneously, by (D-6), a zero of $J_{n+1}(x)$.

APPENDIX E

DERIVATION OF EQ. (33)

From Section 11.3.30, p. 484 of Reference 10 we have the indefinite integral

$$\int^z \frac{C_{\mu+1}(t) D_{\nu+1}(t) dt}{t^{\mu+\nu+1}} = - \frac{1}{2(\mu+\nu+1)z^{\mu+\nu}} \left\{ C_{\mu}(z) D_{\nu}(z) + C_{\mu+1}(z) D_{\nu+1}(z) \right\} \quad (E-1)$$

where C and D are any two cylinder functions. Let

$$\mu = \nu = m$$

$$C_{\mu} = C_m = D_{\nu} = D_m = J_m(z)$$

Substituting in (E-1),

$$\int^z \frac{J_{m+1}^2(t) dt}{t^{2m+1}} = - \frac{1}{2(2m+1)z^{2m}} \left\{ J_m^2(z) + J_{m+1}^2(z) \right\} \quad (E-2)$$

In Eq. (30) of the main text, let

$$\alpha\theta = t ; \theta = t/\alpha$$

Then (30) becomes

$$p_{av} = 2 \left[\left(\frac{2}{\alpha} \right)^{m+1} (m+1)! \right]^2 \frac{\int_{\alpha\theta_{min}}^{\alpha\theta} \frac{J_{m+1}^2(t) \cdot dt}{t^{2m+1}} (\alpha^{2m})}{(\theta^2 - \theta_{min}^2)}$$

$$= \frac{2^{2m+3} [(m+1)!]^2}{\alpha^2 (\theta^2 - \theta_{\min}^2)} \left\{ -\frac{1}{2(2m+1)} \right\} \left\{ \frac{J_m^2(z) + J_{m+1}^2(z)}{z^{2m}} \right\}^2 \theta$$

$$p_{av} = \frac{2^{2m+2} [(m+1)!]^2}{(2m+1) \alpha^2 (\theta^2 - \theta_{\min}^2)} \left\{ \frac{J_m^2(z) + J_{m+1}^2(z)}{z^{2m}} \right\}^{\alpha \theta_{\min}} \quad (E-3)$$

$$\left. \begin{aligned} \text{Let } u_{\min} &= \alpha \theta_{\min} \\ U &= \alpha \theta \end{aligned} \right\} \quad (E-4)$$

Then (E-3) becomes

$$p_{av} = \frac{2^{2m+2} [(m+1)!]^2}{(2m+1) (U^2 - u_{\min}^2)} \left\{ \frac{J_m^2(z) + J_{m+1}^2(z)}{z^{2m}} \right\}^{\frac{u_{\min}}{U}} \quad (E-5)$$

Write

$$R_m(u) = \frac{J_m^2(u) + J_{m+1}^2(u)}{u^{2m}} \quad (E-6)$$

Then (E-5) is

$$p_{av} = \frac{2^{2m+2} [(m+1)!]^2}{(2m+1) (U^2 - u_{\min}^2)} \left\{ R_m(u_{\min}) - R_m(U) \right\} \quad (E-7)$$

From (E-6) we get

$$R_m(u) = \left[\frac{J_m(u)}{u^m} \right]^2 + \left[\frac{J_{m+1}(u)}{u^m} \right]^2$$

which, by Eq. (22), main text, may be written

$$R_m(u) = \left[\frac{\Lambda_m(u)}{2^m(m)!} \right]^2 + \left[\frac{u \Lambda_{m+1}(u)}{2^{m+1}(m+1)!} \right]^2$$

$$= \left[\frac{1}{2^{m(m+1)!}} \right]^2 F_m(u) \quad (E-8)$$

where

$$F_m(u) = (m+1)^2 \Lambda_m^2(u) + \left(\frac{u}{2}\right)^2 \Lambda_{m+1}^2(u) \quad (E-9)$$

Then (E-8) in (E-7) yields

$$p_{av} = \frac{4}{2m+1} \frac{F_m(u_{min}) - F_m(U)}{U^2 - u_{min}^2} \quad (E-10)$$

where $F_m(u)$ is given by (E-9).

APPENDIX F

DETAILS LEADING TO Eq. (47)

Start with Eq. (37), main text:

$$G_m(u) = \left[P_m(u) \cos \phi_m - Q_m(u) \sin \phi_m \right]^2 + \left[P_{m+1}(u) \cos \phi_{m+1} - Q_{m+1}(u) \sin \phi_{m+1} \right]^2 \quad (F-1)$$

where

$$\phi_{m+1} = \phi_m + \frac{\pi}{2}$$

so that

$$\left. \begin{aligned} \cos \phi_{m+1} &= -\sin \phi_m \\ \sin \phi_{m+1} &= \cos \phi_m \end{aligned} \right\} \quad (F-2)$$

Using (F-2) in (F-1) and expanding,

$$\begin{aligned} G_m(u) &= P_m^2 \cos^2 \phi_m - 2 P_m Q_m \sin \phi_m \cos \phi_m + Q_m^2 \sin^2 \phi_m \\ &\quad + P_{m+1}^2 \sin^2 \phi_m + 2 P_{m+1} Q_{m+1} \sin \phi_m \cos \phi_m + Q_{m+1}^2 \cos^2 \phi_m \\ G_m &= (P_m^2 + Q_{m+1}^2) \cos^2 \phi_m + (P_{m+1}^2 + Q_m^2) \sin^2 \phi_m \\ &\quad + 2 (P_{m+1} Q_{m+1} - P_m Q_m) \sin \phi_m \cos \phi_m \\ &= \frac{1}{2} \left\{ (P_m^2 + Q_{m+1}^2) + (P_{m+1}^2 + Q_m^2) \right\} \\ &\quad + \frac{1}{2} \left\{ (P_m^2 + Q_{m+1}^2) - (P_{m+1}^2 + Q_m^2) \right\} \cos 2\phi_m \\ &\quad + (P_{m+1} Q_{m+1} - P_m Q_m) \sin 2\phi_m \\ &= \overline{G_m}(u) + \tilde{G}_m(u) \sin (2\phi_m + \sigma) \end{aligned} \quad (F-3)$$

where

$$\overline{G}_m(u) = \frac{1}{2} \left[(P_m^2 + Q_m^2) + (P_{m+1}^2 + Q_{m+1}^2) \right] \quad (F-4)$$

$$\left. \begin{aligned} \tilde{G}_m(u) \cos \sigma &= P_{m+1} Q_{m+1} - P_m Q_m \\ \tilde{G}_m(u) \sin \sigma &= \frac{1}{2} \left[(P_m^2 + Q_{m+1}^2) - (P_{m+1}^2 + Q_m^2) \right] \end{aligned} \right\} \quad (F-5)$$

We have the asymptotic expansion of the Hankel function [10, Section 9.2.7]

$$H_m^{(1)}(u) = \sqrt{\frac{2}{\pi u}} \left[P_m(u) + j Q_m(u) \right] e^{j\phi_m} \quad (F-6)$$

where $j = \sqrt{-1}$. The square of the magnitude is

$$\left| H_m^{(1)}(u) \right|^2 = \frac{2}{\pi u} \left[P_m^2 + Q_m^2 \right] = M_m^2 \quad (F-7)$$

where, according to Sections 9.2.16 and 9.2.28, loc. cit.,

$$\begin{aligned} M_m^2 \sim \frac{2}{\pi u} & \left\{ 1 + \frac{1}{2} \frac{q-1}{(2u)^2} + \frac{1.3}{2.4} \frac{(q-1)(q-9)}{(2u)^4} \right. \\ & \left. + \frac{1.3.5}{2.4.6} \frac{(q-1)(q-9)(q-25)}{(2u)^6} + \dots \right\} \end{aligned} \quad (F-8)$$

and

$$q = 4m^2$$

Thus, if we write

$$T_p^2 = P_p^2 + Q_p^2 = \frac{\pi u}{2} M_p^2 \quad \text{by (F-7),}$$

then (F-4) is written

$$\overline{G}_m(u) = \frac{1}{2} \left[T_m^2 + T_{m+1}^2 \right]$$

where T_m is just the bracketed expansion in the right number of (F-8)

The third term in this series is negligible if it is less than, say, 0.05. Thus, if m is not too large, we can approximate M_m by the first two terms provided

$$\left| \frac{3(4m^2-1)(4m^2-9)}{128 u^4} \right| \leq \frac{1}{20}$$

which leads to

$$u \geq 0.83 \left[|(4m^2-1)(4m^2-9)| \right]^{\frac{1}{4}} \quad (\text{F-9})$$

Table II of the main text results from substituting successive values of m in (F-9), and also from using Eq. (44).

REFERENCES

1. S. Frankel, Underwater Acoustic Imaging: Preliminary Feasibility Study, Sandia Laboratories Report, SC-CR-68-3545.
2. S. Frankel, Effect of Target Curvature on the Strength of Acoustic Image Signals, Report to Sandia Corporation, FA-138, July 11, 1968.
3. S. Silver, Microwave Antenna Theory and Design, Vol. 12 of MIT Rad. Lab. Series, McGraw-Hill, New York, 1949.
4. H. Cramér, Mathematical Methods of Statistics, Princeton University Press, Princeton, 1946.
5. P. Jacquinot and B. Roizen-Dossier, Apodisation, in "Progress in Optics", Vol. III, ed. E. Wolf, Interscience Publishers, New York.
6. T. T. Taylor, Design of Line-Source Antennas for Narrow Beamwidth and Low Side Lobes, I.R.E. AP-3, pp. (16-28), (January, 1955).
7. T. T. Taylor, Design of Circular Apertures for Narrow Beamwidth and Low Side Lobes, I.R.E., AP-8, pp. (17-22) (January, 1960).
8. R. C. Hansen, Tables of Taylor Distributions for Circular Aperture Antennas, I.R.E., AP-8, pp. (23-36), (January, 1960).
9. E. Jahnke and F. Emde, Tables of Functions, Dover, New York, 1945.
10. M. Abramowitz and I. A. Stegun, editors, Handbook of Mathematical Functions, U.S. Department of Commerce, NBS, Applied Mathematical Series No. 55, U.S. Gov't. Printing Office, Washington, D.C., June, 1964.
11. E. T. Whittaker and G. N. Watson, Modern Analysis, Cambridge University Press, Cambridge, 1940.

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Underwater Acoustic Imaging: Preliminary Feasibility
Study

Final Report to Sandia Laboratories

S. Frankel

May, 1969

SUMMARY

This report is, itself, a summary of the work performed on this project after preparation of the initial feasibility report (Reference 5). Details are given in Appendices A through H.

The main preoccupation of this effort has been with the problem of diffraction spill-over from target highlights into the image areas of diffuse-reflecting elements. It is concluded that, although an ideally-constructed lens/apodiser system could probably meet minimal imaging requirements, current practice suggests that, in fact, a satisfactory approximation to the ideal is not within the state-of-the-art.

It is suggested that, for the present at least, the most feasible approach probably would employ a planar phased array with beams formed by appropriate divider and phasing networks, and with each "beam channel" assigned a different frequency in a frequency division multiplex system. Alternatively, in place of the phased array, a Luneberg lens design might be considered.

1. Introduction

In the summer of 1967 Sandia Laboratories contracted with us to enter into a study to "...develop a comprehensive analytical design model for signature recognition systems using parallel input transducer arrays. The analysis should determine the ultimate detection sensitivities, spatial resolution, and maximum ranges feasible by utilizing this methodology..."*

The term "parallel transducer arrays" refers to the general class of arrangements by which two-dimensional acoustic pressure fields applied to an acoustic image plane are converted into electrical signals for further processing and ultimate presentation on a two-dimensional optical display. The "array" is conceived as consisting of an actual array of discrete transducer elements with faces embedded in the image plane [1]**, or of a single continuous transducer covering the entire image area, but capable in effect, of behaving like a (more-or-less well defined) number of separate transducers by virtue of decoupling as a function of lateral displacement in the image plane [2, 3] .

When, in 1967, we initiated contacts to acquaint ourselves with the state-of-the-art in the development of acoustic imaging systems for underwater use*** we found the emphasis to be in the area of component development, rather than in systems analysis, with particular emphasis on the development of image transducers [1, 2, 3] . If those components turn out to be useful for some

* Contract Number 72-6019, August 30, 1967.

** Numbers in [ ] correspond to those in Reference List

*** Our primary area of interest

Introduction (Concluded)

types of acoustic imaging systems, we doubt, on the basis of the study reported here and in earlier interim reports [4 - 13] *, that they can be used practically for simultaneous parallel imaging. This conclusion does not preclude the extension of conventional sonar techniques to the synthesis of high resolution images using sequential imaging techniques.**

The remainder of this report is divided into sections as follows: Section 2 discusses the basic nature of the system as an information gathering device; Section 3 describes the progress of the study and the results obtained; Section 4 deals with some recommendations for future investigation.

2. Basic Nature of Imaging System

For our purposes the image required is two-dimensional and of very low grade compared to typical television images or amateur photographs. We are concerned with the production of an image of a known object, recognizable in the presence of a variety of disturbances such as random acoustic noises, dispersive factors in the propagating medium, and other objects that, in the absence of adequate detail, could be mistaken for the desired object.

Shannon's 1948 paper [14] stimulated a point of view toward information gathering systems which we can employ gainfully, at least in its qualitative aspects, for the present study. Thus, the final output of an imaging system is to be considered as a two-dimensional array of independent information elements, the size of each element depending on the resolution of the system. In fact, the terms "independent" and "resolution" are vague, but complimentary terms; that

* References 6 - 13 appear among Appendices A - H to this report.

** Or of other methods capable of filtering target-highlight crosstalk from diffuse image regions [5, 9, 12, 13] . See also Section 3.3 below.

Basic Nature of Imaging System (Continued)

is, two adjacent information elements may be said to be independent when they are resolved by the system, and vice versa.

Each final image element may be considered to be the final output of a single information channel; the total number of information channels, in general, equals the number of image elements. However, in specific physical realizations, the elements may use a common carrier in part, or in the limit, in all of their respective transmission channels. Figure 1 shows some examples. In all cases there is a source, or insonifier, or transmitter; an object to be visualized; a receiving device; an electro-acoustic transducer; an amplifier system; and finally, an electro-optical transducer, indicated in all cases as a cathode-ray tube, although alternatives are possible. In systems using the Sokolov or Lockheed transducers, all channels are separate except for the common amplifier between transducers. In Litton tube systems all channels are separate except for the transfer from the separate amplifiers to the electron beam of the CRT. In this case, the display could consist of an array of miniature lamps, in which case every channel would be completely independent, with no common carriers. In the high-resolution sonar system, the target "elements" are insonified sequentially as in conventional sonar systems. In this case, every channel uses the same common carrier throughout.

Which type of system is preferable for a specific application depends on trade-offs among a large number of parameters, - required information rate, target range, available power and power-handling capability of the propagation medium, ambient and system noise, interchannel crosstalk, and so on. Relationships among these factors, and others, were investigated and are reported in the next section.

FIGURE 1. Examples Of Image Information Systems

3. Course of Study and Results Obtained

3.1 Single Channel Behavior. As a result of the point of view expressed in the preceding section, it was natural to concentrate initially on the physical behavior of a single channel with a view, ultimately, to combine the effects of all channels. This corresponds, crudely, to finding the response of the system to a point object and then integrating the effects of such points over the complete object.

As an initial task, with the help of a great many assumptions, we derived range equations for a beam reflected from a small region of an extended surface [4, or App. C of 5] . Equations were obtained for the ratio of power received to power transmitted for the cases of specular reflection and for diffuse reflection. The analysis was mainly adapted from the book by Beckmann and Spizzichino [15] .

This initial study disclosed, sharply, the limitations in trying to implement a practical system. To begin with, for minimum-viewing ranges of interest (order of 30 meters), optimum operating frequencies are in the neighborhood of one mHz, and transmission above ten mHz over any useful range is essentially impossible. But in this frequency range, typical machined (but not polished) objects have a surface roughness less than 1/100 of a wavelength. The corresponding roughness of a surface at optical frequencies is a few Angstrom units.

As a result, we estimated that, for reflections from a plane surface, diffuse reflection could be of the order of 100 db below the specular component.* Of course, the likelihood of dealing with a plane surface is not

* A private discussion with one of the Lockheed personnel confirmed this estimate.

Single Channel Behavior (Concluded)

very great, and the probability of experiencing specular reflection from such a surface is also small. The most likely surfaces are doubly-curved surfaces, for which the specular component can be reduced considerably, depending on the radii of curvature [9]. In the end, a figure of 75 db for the specular/diffuse ratio seemed more suitable, and we settled on this value as a design constraint.

3.2 Diffraction Effects

Any acoustic radiation field produced by a source of finite dimensions (e.g., a field emerging from a finite aperture) must exhibit the phenomenon of diffraction rings or side lobes in its radiation pattern. [16, 17; also footnote, page 66, App. C, Ref. 5] .

Reciprocally, an element in the receiver image plane can receive signals incident from directions of its side lobes, in addition to the desired signal on its central, or main, lobe. If the main lobe is receiving diffusely reflected energy from an object while a side lobe is receiving specular energy from another portion of the object, 75 db higher in level, then, unless the side lobe response is down from the main lobe response by at least 75 db, the channel will respond erroneously. The error is particularly grievous if the main lobe is looking in a direction off the object, so that the specular reflection is obscuring the outlines of the object, and information concerning the shape of that outline is consequently lost.

Now an aperture with uniform distribution of acoustic pressure has its first side lobes*about 17 db down from the main lobe, with the level varying roughly inversely with angular displacement from the main beam.**

* i.e., side lobe immediately adjacent to the main lobe.

** For narrow beams. See Refs. 16 and 17, and pp. 28 ff, Ref. 5.

Diffraction Effects (Continued)

Clearly this represents useless performance. Considerable improvement may be obtained by introducing non-uniformity to the aperture distribution, and a great deal of theoretical work has been done in this respect. [17 - 21] . However, in practice it turns out that for electromagnetic systems, radiators with first side lobes lower than about 30 db are unusual. When some of the reasons for this limitation are understood, there is no reason to conclude that an improvement of three or four orders of magnitude needed to realize first side lobes of -75 db can be attained for acoustic systems [22; 12, 13] .

However, it is not absolutely necessary to insist that the effects of a specular element be confined to its own central beam. Specular crosstalk may, under many conditions, spill over into several adjacent elements without seriously reducing the probability of correct object identification. Accordingly, we decided to modify the constraints by demanding that the fourth or fifth side lobes be down by 75 db. The following sub-sections discuss the consequences of this modified requirement.

3.2.1 Axial Element Pattern Study: Shaded Aperture Distributions

In Reference 12 we reported the results of a pattern study of a well-known class of aperture distributions, in which we extended previously published results to higher order side lobes and computed patterns for additional values of the parameter characterizing the class. The patterns are for a primary source at the focus of the imager, and it is assumed that the required aperture distribution is obtained by an appropriate combination of primary feed pattern and lens apodisation [18; 13] . The results are summarized in Table I and Figure 2 taken from Reference 12 (Table II and Figure 1 of that report, respectively). The aperture distribution is characterized by the function

Axial Element Pattern Study: Shaded Aperture Distributions (Cont'd)

$$f = (1 - r^2)^m \quad 0 \leq r \leq a$$

$$= 0 \quad \text{otherwise} \quad (1)$$

where r is the radial distance from the center of the (circular) aperture to a point of the aperture, a is the radius of the aperture, and m is a positive integer. Table I, for $1 \leq m \leq 7$, gives values of successive side lobe maxima, and their angular separation, Ψ , in half beamwidths, from the center of the main lobe. It can be seen that it takes a value of $m = 4$ to attain the desired side lobe level at the fourth or fifth side lobe. The corresponding Ψ is about 8 half beamwidths.

Table I. Side Lobe Maxima vs. m

m	Side Lobe Number									
	1		2		3		4		5	
	Ψ	db	Ψ	db	Ψ	db	Ψ	db	Ψ	db
1	3.4	24.6	5.2	33.6	7.0	39.8	8.7	44.4	10.4	48.3
2	3.5	30.6	5.1	42.7	6.7	49.6	8.2	55.6	9.7	60.6
3	3.6	36.0	5.1	48.9	6.5	58.0	7.9	65.4	9.2	71.5
4	3.8	40.9	5.1	55.4	6.4	65.8	7.7	74.3	8.9	81.6
5	3.9	47.7	5.2	61.1	6.4	72.9	7.6			
6	4.0	50.1	5.3	66.8	6.4	79.6				
7	4.1	54.3	5.3	74.0						

Side Lobe Level
(db)

FIG. 2: AVERAGE SIDE LOBE LEVEL IN
THE INTERVAL $\psi_{\min} \leq \psi \leq \psi$

Axial Element Pattern Study: Shaded Aperture Distributions (Cont'd)

On the other hand, if we impose the less restrictive requirement that the average energy at angles beyond a specified minimum value of Ψ be at least 75 db down, Figure 1 shows that $m = 3$ yields a $\Psi_{\min}$ between 8 and 9.

Thus, it would appear that an aperture distribution characterized by $m = 3$ or 4 might satisfy our requirements. However, at this point it is necessary to dig a little deeper before concluding that a feasible pattern can be realized. First, the patterns of sources in the image plane, but off the acoustic axis, need study; second, the effect of random errors due to construction limitations must be assessed [22]. These topics are discussed in the following sections.

3.2.2 Off-Axis Pattern Study: Shaded Aperture Distribution

The analytical procedure for this part of the study [13] was as follows: First, it was assumed that each source (or receiver) in the image plane had a radiation pattern resembling the Airy pattern for small, uniformly insonified, circular apertures [16]. Second, the lens apodisation function required to convert the primary pattern from the axial source to the distribution specified by Eq. (1) was introduced. Finally, the source was assumed to be translated a distance in the image plane, small compared to the focal length of the lens, but not necessarily small compared to a wavelength. An approximate expression for the radiation pattern for arbitrary values of azimuth and elevation (co-latitude) angle were obtained. This expression involved integrals which required machine computation for evaluation. It was decided to restrict this evaluation to pattern plots in the azimuth plane determined by the source and the acoustic axis. Patterns were computed for $m=4, 5$ [Eq. (1)], for a typical range of interest for relative source displacement from the system axis, for a range of lens f /numbers, and for a parameter, v , given by

$$v = \frac{ka\rho_1}{F} \quad [13]$$

$$= \pi \frac{n}{f} \left(\frac{a}{\lambda} \right) \quad (2)$$

where n is the ratio of source displacement to lens radius (a), $f/$ is the lens f /number, and $\frac{a}{\lambda}$ is an approximate inverse measure of the acoustic beam width. The data in the cited reference [13] is presented only for $f/$ equal to unity; for $m = 4, 5$; for $n = 0, 0.01, 0.03$, and 0.1 ; and for $v = 0, 1, 3, 10$, and 30 .

Generally, while this data shows some deterioration in the pattern of off-axis sources, the deterioration is not so severe (in this writer's opinion) as to suggest rejection of this design approach on the idealized theoretical grounds thus far presented.

3.2.3 Phase-Error Effects Due to Fabrication Limitations

It is not our intention to discuss the subject of phase-error effects, which has been well-covered in the literature [22, 23]. Practical results obtained with a wide variety of electromagnetic systems indicate a stone wall in side lobe performance in the range of -30 to -40 db. Although those figures pertain to all side lobes, it is difficult to imagine an actual capability of -75 db, even if one excludes the first 5 or 6 lobes adjacent to the main beam.

3.3 Discussion of Radiation Pattern Problems

The foregoing presentation concludes with a pessimistic prognosis regarding any attempt to produce a practical apodised-lens imaging system within the present state-of-the-art. Future developments in materials and fabrication procedures may alter this picture, since analysis indicates an idealized system may be capable of adequate performance.

Discussion of Radiation Pattern Problems (Cont'd)

In the meantime, however, are there other avenues of approach? Certainly, sequential methods of scanning, presently in use, can be extended and refined for better resolution and higher information rates. In this connection it should be noted that in sequential scanning with pencil beams, any side lobe interference suffers two-way attenuation, so that a requirement of 75 db reflection implies a one-way side lobe capability of only (!) 38 db, which is within striking distance of well-designed electromagnetic systems. (Looking at the problem in reverse, from the standpoint of existing operational sequential systems, a (simultaneous) lens system must have improved side lobes of the order of 35 db to compete).

But a major limitation of sequential systems is their low information rate. This limitation cannot be removed by the use of high prf modulation [10], because of the diffraction interference just discussed.

In Reference 5, Section 2.3.2.5, we listed several possible remedies for alleviating the specular crosstalk problem. Of those not discussed thus far in this report, the most attractive at the present time appears to be some sort of frequency division multiplex arrangement. If, say, we were contemplating a resolution of $10 \times 10 = 100$ elements, this would require 100 sources of power over a range of frequencies. These would be used to excite a planar phased array of piezoelectric elements through an appropriate matrix of delay networks to form a set of 100 low side lobe beams pointing in a cluster of pre-selected directions.* Having previously determined the range of the target by other means, the system would select an appropriate high pulse repetition frequency and duty cycle to accept target returns during transmitter quiescent periods. Receiving beams would be formed in the same way as transmitting beams. After possible preamplification, signals would be heterodyned down to lower frequencies for easier filtering. It would not be necessary to impose severe adjacent channel filtering requirements, since the high specular/diffuse contrast of concern should exist only for channels separated by at least a few beam widths.

* Alternatively, a Luneberg lens arrangement might be simpler to implement [16]

Discussion of Radiation Pattern Problems (Cont'd)

For a target at a range of 30 meters, typical parameters might be:

Number of Channels: 100
Pulse Rep. Freq.: 37.5/sec. [Reference 10]
RF Bandwidth/Channel
(including guard band): 250 Hz
Total RF Bandwidth: 25 kHz
Range of Carrier Freqs: 1,000 - 1,025 kHz

Following the examples of Fig. 1, a schematic arrangement of this type of system is shown in Fig. 3

Fig. 3. Frequency Division Multiplex Planar Array
Image Information System

3.4 Other Limitations

3.4.1 Other Interference Sources

In addition to the diffraction crosstalk discussed in the previous section, Reference 5 catalogs a considerable list of additional noise sources [Table I, p. 13, loc. cit.]. Except for thermal noise, which represents the lower limit of interference, the rest are dismissed as being of minor importance at the expected operating frequencies, or else are deferred for future consideration. The results of further study of some of these noise sources are reviewed in this section.

Physical limitations force compromises in system specifications which can lead to a decision to accept low-resolution images, in order to gain some other system advantage, such as visualization range, image frame rate (images per second), or reduced size and weight. The methods of producing these low-resolution images often result in undesirable distortions that go beyond the resolution limitations. One of the most disconcerting of these distortions results from displaying the quantized image without intermediate processing. Actually, because of the Fourier-transform relation between a source distribution and its Fraunhofer pattern, the quantization can be removed in a theoretically simple manner, as discussed in Reference 6. Certain "overshoot" effects, analagous to the response of sharp cut-off filters to suddenly-impressed signals, result from this type of smoothing, as discussed in Reference 8.

A second source of noise not discussed in the earlier report was that of reverberation. Very little information appears to exist on this subject for the megahertz region of interest. We made an elementary analysis of reverberation/signal ratio, which is different for our case than the usual, in that the element receiving beam is much smaller than the target, instead of much larger [11]. We had to conclude that more experimental data on the scattering strength of reverberating particles in the megahertz region was needed before any useful conclusions could be reached.

3.4.2 Platform Stability

Finally, one other limitation should be mentioned. One method of moderating the **severe** specular/diffuse ratio problem envisioned each image element channel operating as an independent sonar system complete with automatic power control, which would operate on the power level of an element transmitter whose received level greatly exceeded the average of the other elements in its vicinity [5]. Such a scheme is inoperative if relative motion between sonar and target results in the excessive signal failing to return to the appropriate receiver. This problem was considered briefly in reference 7, where it was shown that it could impose a severe restriction on the maximum relative velocity. (In a planar array using frequency division multiplex and beam-forming matrix, this limitation can be eased by forming wider receiving beams, at some sacrifice in gain and increase in filter requirements).

4. Recommendations for Continuing Effort

In order to establish a basis for continuing effort in this difficult field we need to answer two questions:

1. Which elements of basic information used in feasibility evaluation require further refinement because of wide variance in present estimates?
2. Which direction, or directions, show the greatest promise for successful equipment development at the present time?

In reviewing the studies performed on this project since its inception, it appears to us that large questions still remain in the realm of target scattering information. This applies not only to the desired targets, but to small particle reverberation scatterers as well.

Recommendations for Continuing Effort (Cont'd)

The theory of diffuse target return contains such a large number of simplifying assumptions that its acceptance without experimental verification is not justified [4, 5]. In particular, the assumption of perfect rigidity of the target probably exaggerates the ratio of specular-to-diffuse reflection. In addition, the assumption that correlation distances over random rough surfaces are large compared to a wavelength is not confirmed by measurement of typical machined surfaces. And so on.

On the subject of particle - or volume reverberation - we have a verbal expert opinion that at the higher frequencies of interest to us, the volume scattering strength should be taken as approximately -40 db. However, no experimental evidence appears to be available.

As for appropriate directions for ~~near~~-future development, our inclinations in this regard have already been discussed in Section 3.3. With our present information as a basis for judgment we favor the use of planar phased arrays* for beam forming, with each element beam operating in its own frequency channel, broad enough to permit high-repetition rate pulsing and filtering. The fact that such a scheme fails to yield unambiguous range information is not to be considered a serious limitation on its usefulness. High prf is desirable for realizing the largest possible signal/noise ratio within equipment and environmental limitations.

* Or a Luneberg Lens.

REFERENCES

Note: References identified with an asterisk (*) are included in the appendix to this report.

1. L. A. Noble, J. F. Hull, R. D. Culbertson; Hydrocon Acoustic Imaging System; Litton Industries (San Carlos); 27 April 1965.
2. P. S. Green, et. al.; Final Report on Underwater Acoustic Imaging; Lockheed (Palo Alto) Report 4-17-67-5; 29 June 1967.
3. R. W. G. Haslett, G. Pearce, A. W. Welch, and K. Hussey; The Underwater Acoustic Camera; *Acustica*, Vol. 17, No. 4, 187-203 (1966).
4. S. Frankel; Underwater Acoustic Back Scattering From Small Regions of Slightly Rough Surfaces; Report No. FA-132 to Sandia Laboratories; December 1967; later incorporated as Appendix C of Reference 5 (q.v.)
5. S. Frankel; Underwater Acoustic Imaging: Preliminary Feasibility Study; Sandia Laboratories Report No. SC-CR-68-3545; March 1968.
- *6. S. Frankel; Sampling Theorems and the Smoothing of Low-Resolution, Quantized, Acoustic Image Data; Sandia Laboratories Report No. SC-CR-68-3661; August, 1968. (See Appendix A of this report).
- *7. S. Frankel; Effect of Some Parameter Variations on Automatic Power-Level Control Signals; Report No. FA-136; July 9, 1968. (See Appendix C of this report).
- *8. S. Frankel; "Overshoot" Effects in the Smoothing of Two-Dimensional Quantized Signals; Report No. FA-137, June 11, 1968. (See Appendix B of this report).
- *9. S. Frankel; Effect of Target Curvature on the Strength of Acoustic Image Signals; Report No. FA-138; July 11, 1968. (See Appendix D of this report).

References (Continued)

- *10. S. Frankel; Some Thoughts Concerning Restrictions on Insonifier Modulation for Underwater Acoustic Imaging Systems; Report No. FA-139; July, 1968.
(See Appendix E of this report).
- *11. S. Frankel; Effects of Reverberation on Effectiveness of Acoustic Imaging at High Frequencies: Data Requirements; Report No. FA-145, October 23, 1968.
(See Appendix F of this report).
- *12. S. Frankel; An Assessment of Apodisation Requirements for Underwater Acoustic Imaging Systems; Report No. FA-147; December 1968. (See Appendix G of this report).
- *13. S. Frankel; Radiation Patterns of Apodised Acoustic Lenses With Off-Axis Image Plane Feeds; SC-CR-69-3248; May 1969. (See Appendix H of this report).
- 14. C. E. Shannon; Mathematical Theory of Communication; B.S.T.J.; July and October, 1948.
- 15. P. Beckmann and A. Spizzichino; The Scattering of Electromagnetic Waves From Rough Surfaces; Pergamon, London and New York; 1963.
- 16. M. Born and E. Wolf; Principles of Optics; Pergamon, London and New York; 1959.
- 17. S. Silver; Microwave Antenna Theory and Design; M.I.T. Rad. Lab. Series; McGraw-Hill, New York; 1949.
- 18. P. Jacquinot and B. Roizen-Dossier, Apodisation, in "Progress in Optics", Vol. III, edited by E. Wolf; North Holland Publishing Company (In U.S.A., Interscience Publishers, Division of Wiley, New York).

References (Concluded)

19. T. T. Taylor; Design of Line-source Antennas for Narrow Beamwidth and Low Side Lobes; I.R.E., AP-3; pp. (16-28); January 1955.
20. T. T. Taylor; Design of Circular Apertures for Narrow Beamwidth and Low Side Lobes; I.R.E., AP-8, pp. (17-22); January 1966.
21. R. C. Hansen; Tables of Taylor Distributions for Circular Aperture Antennas; I.R.E., AP-8, pp. (23-26); January 1960.
22. J. Ruze; Effect of Aperture Errors on the Antenna Radiation Pattern; Supplemento al Nuove Cimento, Vol. 9, No. 3, pp. 364-380 (1952) (In English).
23. L. L. Bailin and M. J. Ehrlich; Factors Affecting the Performance of Linear Arrays; Proc. I.R.E., Vol. 41, pp. 235-241; February 1953.

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