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THE EFFECT OF HYDROSTATIC ROLLING
CONDITIONS ON THE MALLEABILITY
OF BRITTLE MATERIALS

Dennis R. Floyd

THE DOW CHEMICAL COMPANY
ROCKY FLATS DIVISION
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Dennis R. Floyd

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THE EFFECT OF HYDROSTATIC ROLLING CONDITIONS ON THE MALLEABILITY OF BRITTLE MATERIALS

Dennis R. Floyd

Abstract. This report develops the relation between stress state and malleability for sheet rolling brittle materials. Improvements in the malleability of beryllium metal is achieved by altering the rolling conditions, but the effect of these conditions on stress state is unknown. A detailed analysis of the rolling process results in the following relationship between stress state, R , and the key rolling variables of friction coefficient (μ), contact length (ℓ), and thickness (t):

$$R = 1 - \frac{z}{e^z - 1}, \text{ where } z = \frac{\mu\ell}{t}.$$

This theory, confirmed by tests with mild steel, shows that all three variables contribute to the same degree in changing stress state. Thus, doubling the friction coefficient or contact length has the same effect on stress state as halving the initial thickness. The curve resulting from the above relationship has a steep slope for z values under 2, where most rolling is done. Thus a large increase in stress state can be achieved by relatively minor changes in rolling conditions.

INTRODUCTION

This report characterizes the improvements in malleability of brittle materials which can be achieved by increased hydrostatic pressure. Although the analytical work could have been simplified greatly by varying hydrostatic pressure in a tensile or compression test, a manufacturing process was used to effect changes in both hydrostatic pressure and malleability because of its practical value. If improvements in malleability could be achieved, then they could be applied directly to the process. Also a better understanding of the process variables which influence malleability would evolve from this work, whereas tensile or compression tests would merely indicate a direction to go but not how to get there.

The manufacturing process chosen is rolling of flat sheet metal. The brittle material used is beryllium metal. Malleability is defined as the true strain at fracture, which is directly related to reduction in thickness in rolling. Hydrostatic pressure is described in terms of stress state, R , where $R = \frac{S_1}{S_3}$. In rolling, S_1 is the principal stress in

the rolling direction and S_3 is the principal stress in the thickness direction. These stresses are further defined in Figure 1.

This report is presented in two sections; Preliminary Investigation, and Stress State Investigation. The preliminary work shows why an improvement in malleability is expected with an increase in stress state and how these changes in stress state might be caused by adjusting rolling variables. It concludes with tests on beryllium which do show improved malleability but also show a need for a better understanding of stress state and rolling conditions.

The major emphasis of the work shifts to an analysis of the rolling process in the Stress State section. A schematic flow diagram of the process variables is developed and used to formulate a quantitative theory for a stress state - variable relationship. This theory is confirmed with tests on mild steel. Application of this theory to beryllium remains as an item for future work.

This report is similar to a thesis program still in progress.¹ This work is concerned more with the role of physical metallurgy in improving malleability. It is being done with an Al-10% Cu alloy, and it shows that grain size is an important variable. Grain size is not considered in this work because the fracture behavior of beryllium is not strongly dependent upon the number of grains through the thickness of the material. Once a fracture is initiated, it will propagate through the thickness.

PRELIMINARY INVESTIGATION

This section describes the theoretical background for the investigation, and the early experimental work. The purpose is to explain why stress state should affect malleability, and to demonstrate how rolling conditions can cause stress state to vary.

The Effect of Stress State on Malleability

EXPERIMENTAL EVIDENCE - Extensive data support the premise that stress state influences ductility. Tungsten

FIGURE 1. Schematic View Showing a Piece of Metal Being Reduced in Thickness by Rolling. Lower View Shows an Element of Volume Removed at Angle θ from the Sample to Define the Directions of Principal Stresses and Strains.

carbide, for example, exhibits 0.17% elongation to failure in tension, whereas the same material will exhibit 1.1%

strain in compression prior to failure.² Biaxial tension causes failure at lower strains than uniaxial tension.

Extrusion is known to be a favorable method for decreasing diameters of brittle materials.³ Fracture during sheet-drawing can be suppressed by imposing a hydrostatic pressure on the work piece during drawing.⁴

THEORETICAL APPROACH — These observations have prompted the relationship between stress state and malleability shown in Figure 2.⁵ The stress state of any metal working or metal testing conditions can be described on this diagram. The ratio of principal stresses has been plotted on the abscissa. The ratio changes at $S_1 = -S_3$ to avoid the ratio becoming infinite at $S_1 = 0$. The function, however, is continuous within the limits shown on the graph.

The ordinate quantity, μ , is the Von Mises factor.⁶ It is based upon one of the three laws of classical plasticity — that the intermediate principal stress (S_2) lies in the same relative position between S_1 and S_3 as the intermediate principal strain lies between ϵ_1 and ϵ_3 . Thus, if $\epsilon_2 = \epsilon_3$ as in a tensile test, then $S_2 = S_3$. In rolling, ϵ_2 is half-way between ϵ_1 and ϵ_3 and equal to zero. This does not mean S_2 is also zero. The Von Mises condition defines relative positions only. Absolute positions must be defined by other criteria.

Note that malleability increases for operations nearer the lower left (triaxial compression) and decreases for operations nearer the upper-right (triaxial tension). The basic problem with this diagram is that there are very little data to help quantify what is now a qualitative observation. The objective of this work is to define that quantitative relationship for the case of rolling.

The rolling operation is found in a band at the left-center of Figure 2. As the rolling conditions cause the stress state to increase (move further left) the ductility should improve.

The Effect of Rolling Conditions on Stress State

EXPERIMENTAL EVIDENCE — Rolling conditions have been studied extensively. These studies are usually directed at effects on power and strength requirements of the rolling mill and not on malleability. Some of the data suggest means by which stress state can be varied. For example, Underwood reports that Pomp, Lieg, and Siebel⁷

FIGURE 2. Representation of Various Metal Working and Metal Testing Conditions Expressed as a Function of Stress State. As Conditions Move to the Left, and Down, Materials Exhibit More Ductility.

have shown that both the mill separating force and the specific roll pressure increase as:

- roll diameter increases
- initial stock thickness decreases
- roll speed increases
- friction increases
- reduction per pass increases
- yield strength increases

Here, specific roll pressure is the total separating force divided by the theoretical area of contact. This is only an approximation of the true stress distribution. The actual roll pressure varies along the length of contact. Also, the length of contact varies as the result of elastic flattening of the rolls. Underwood reports that Siebel and Lueg⁷ have measured the true pressure distribution (see Figure 3). The stress peak occurs at the "no-slip" point, which will be discussed. The area under this pressure-distance curve corresponds to the mill separating force per unit width of sample.

For this program the initial thickness was varied to effect changes in stress state. The data presented in

Figure 4 show that changing t_0 will change stress state. The data results from a computation of the pressure distribution (based on the Orowan method) for mild-steel of varying initial thickness which has been rolled to the same percent reduction.⁸ It can be seen from those data that although the thicker material has twice the contact area it requires only 1.62 times as much load as the thinner material, for the same 30% reduction. Also note that both the peak stress and the average stress increase with decreasing stock thickness. Thus stress state, whether computed from mean stress or peak stress, increases as the stock thickness decreases.

THEORETICAL APPROACH — Because the peak stress cannot be measured during this program, the influence of average stress on malleability will be studied. This study thus assumes that the principal stress in the thickness direction (S_3) does not vary along the length of contact, across the thickness, or through the thickness. This is not true. Typical variations which might be expected are shown in Figure 5. The deviations from theoretical in the width

FIGURE 3. Typical Stress Distribution Along the Arc of Contact for Rolling Aluminum to 45% Reduction. The Increase in Yield Strength, as the Result of Work Hardening, is Shown in Dotted Lines.

FIGURE 4. A Comparison of the Pressure Distribution Curves for the Same Reduction but Different Initial Thickness. Roll Diameter = 24-In. Material: Mild Steel. The Average Stress, P_s , Which is Shown as Being Uniform Across the Contact Length, is Obtained by Dividing the Total Load by the Theoretical Contact Length.

FIGURE 5. Variation of Principal Stress S_3 Which Probably Occurs Along the Contact Length, Across the Sample Width, and Through the Sample Thickness During Rolling.

and thickness directions can be explained by St. Venants' principle, which states that an effect dies out at a distance 1-3 critical dimensions from the source.⁹ Thus, the stress along the width will build up very rapidly, probably over a distance corresponding to one thickness. Experimental data show that the average roll pressure ($\bar{S}_3$) is not a function of width, for width-to-thickness ratios exceeding 7.5.¹⁰

The stress through the thickness will be uniform, except when the thickness is very large or the contact length is very short. The critical dimension will be the contact length; and if the behavior is as shown in Figure 5, the assumption of uniform stress through the thickness will be valid for contact-length-to-thickness ratios greater than one.

The variation along the contact length provides the greatest source of error. The assumed value for S_3 is initially too high, it becomes too low around the pressure peak, and is again too high as the sheet exits from the rolls. However if the ratio S_1/S_3 is truly the one which controls malleability, then it must be mentioned that S_1/S_3 varies along the contact length. Thus, even if S_3 were measured directly, the value of S_1/S_3 varies from 0 to a maximum at the pressure peak and back to zero as the sheet exits from the rolls. Cracking could occur at some unfavorable stress state along the arc of contact, regardless of the initial thickness, strain, etc. Thus conditions could be such as to cancel the effect of improved stress state on malleability, regardless of whether an average or peak stress is used to analyze stress state. Conversely, if an improvement in malleability is seen by altering the rolling conditions so as to alter stress state, then both peak and average stress will have changed. The stress state-ductility relationship will not be the same for both cases, but the trend will be the same.

Also S_1 and S_2 will vary somewhat with contact length, width, and thickness. But these stresses are so completely tied up with S_3 by way of the laws of plasticity as to make the above argument analogous. These laws of plasticity influence rolling as follows:

Shear Stress Law: Plastic flow will commence when the difference between the algebraically largest and smallest principal stresses equals the yield strength of the material: $S_1 - S_3 = S_0$

Von Mises Condition: The intermediate principal stress (S_2) will lie in the same relative position between S_1 and S_3 as the intermediate principal plastic strain (ϵ_2) lies between ϵ_1 and ϵ_3 .

FIGURE 6. Free Body Diagram Showing the Relationship Between S_3 and P_s (Specific Roll Pressure) as a Function of θ . This Represents Any Element of Volume Between the Entrance and the No-Slip Point. The Sign of the Friction Component Would Be Opposite for Any Element Between the No-Slip Point and the Exit, but the Relationship Between S_3 and P_s Would Be the Same.

Constant Volume Condition: The plastic strains will be such as to keep the volume of material constant, if the material does not crack. Thus $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$.

The principal stress in the thickness direction (S_3) is related to the pressure distribution (P_s) by way of the free-body diagram shown in Figure 6. Thus $S_3 = P_s(1 + \mu \tan \theta)$ and for small angles and/or low friction conditions S_3 and P_s are equal.

The reason why decreasing stock thickness causes an increase in stress-state is shown in Figure 7. Assuming the same Δt for two different values of t_0 , the forces acting on the surface of each specimen will be the same as shown in Figure 7. However, the area over which this force is distributed is less for the thinner stock. Thus S_1 increases as t_0 decreases; Δt being constant. Since S_3 is related to S_1 by $S_1 - S_3 = S_0$, then S_3 will also change. For example, for unit width and assuming a horizontal force of 10,000 lb the stress on a 0.2-in.-thick specimen would be:

$S_1 = -F/A = \frac{10,000}{0.2} = -50,000$ psi. If $S_0 = 50,000$ psi (assume no work hardening) then $S_3 = S_1 - S_0 = -50,000 - 50,000 = -100,000$ psi. Now assuming the same conditions except that $t_0 = 0.1$ in.,

$$S_1 = -F/A = -\frac{10,000}{0.1} = -100,000$$

$$S_3 = S_1 - S_0 = -100,000 - 50,000 = -150,000$$

$$\text{Thus for } t_0 = 0.2, S_1/S_3 = \frac{-50,000}{-100,000} = 0.500$$

$$\text{and for } t_0 = 0.1, S_1/S_3 = \frac{-100,000}{-150,000} = 0.667$$

Stress state increases as thickness decreases; all other variables being constant. This effect shall be called the stress state effect throughout this report.

ARC OF CONTACT – A cross-section of a typical rolling operation is shown in Figure 8. The arc of contact can be approximated by the horizontal projection $x = \sqrt{R\Delta t}$, which is valid for small reductions (error for a 10% reduction of 0.125-in. stock on 5-in.-diameter rolls is 0.01%). In actual rolling practice the rolls flatten elastically. This causes an increase in the area of contact, so the two effects tend to compensate. However, when rolling pressures on the order of 100,000 psi are encountered, the change in area of contact becomes greater than 25%. This effect has been analyzed by Hitchcock¹¹ who offers the following formula to compensate for roll flattening:

$$L' = \left[\left[\frac{8R(1-\gamma^2)P_s'}{\pi E} \right]^2 + R\Delta t \right]^{1/2} + \frac{8R(1-\gamma^2)P_s'}{\pi E}$$

Where γ = Poisson's ratio for the rolls
 E = Young's Modulus for the rolls
 P_s' = Specific Roll Pressure

This formula is solved by an iterative process until the arc length, L , used to determine $\bar{P}_s'$ (recall $\bar{P}_s' = \frac{F}{wL}$) becomes the same as the L' determined by the formula. In this report, specific roll pressure ($\bar{P}_s$) will be used to describe the average roll separation force divided by the theoretical area of contact, and true specific roll pressure ($\bar{P}_s'$) will be the value resulting when the true arc of contact (ℓ') is used.

Beryllium Tests

In this section of the report, the equipment, test procedures, and test results will be described. Both

FIGURE 7. Cross-Sectional View of Samples of Different Thickness Being Given the Same Reduction in Thickness. Free-Body Diagrams are Shown Below, and the Computation Assumes that Rolling Conditions are Such that $F_A = F_B$.

FIGURE 8. Cross-Sectional View of a Typical Rolling Operation. The Calculations Assume That, for Small Angles, the True Arc of Contact (L) is Equal to the Horizontal Projection (X).

beryllium and mild steel were rolled, but the mild steel is discussed in a later section.

EQUIPMENT AND TEST PROCEDURES – The tests were conducted on the Stanat TA-315, powered, 2-high rolling mill, shown in Figure 9. The mill separating force was measured with a high speed recorder capable of ± 125 lb accuracy and having a full-scale response time of 0.25 second. This recorder received a signal from either of two BLH 50,000 lb load cells mounted beneath the bearing chocks of either side of the mill frame, as shown in Figure 10. These load cells are accurate to ± 125 pounds. Gape* was measured with a dial indicator connected to the bearing chocks. The gape was often calibrated by rolling lead and measuring the thickness after several passes at a given gape.

*Gape – opening between the rolls.

The beryllium coupons were 1-in.-long by 2-in.-wide and the initial thickness varied from 0.125-in. to 0.025-in. All specimens were machined from the same sheet to within 0.010-in. of the desired thickness and etched to final thickness.

Etching eliminates machining damage, but causes as much as 0.002-in. variation in thickness.

The coupons were all rolled in the 1-in. direction. A constant roll surface speed of 43 ft. per min. was used. Rolling was done at ambient temperature on polished (10 microinch AA surface finish) rolls which were 4.984-in.-diameter. The rolls were ground flat and parallel within ± 0.0005 -inch. No lubricant was used. Specimens and rolls were degreased by appropriate solvents prior to rolling.

FIGURE 9. Photograph of the Stanat Rolling Mill Used to Roll the Samples.

Thickness was measured with point micrometers accurate to 0.0001-inch. Measurements were taken at the center of each specimen for calculation of thickness strains. Variation in thickness within each piece was also determined — both before and after rolling.

RESULTS — To study the effect of stress state on malleability, both quantities must be measured. Stress state is measured only indirectly. The separating force is determined and the area of contact is calculated, based on the actual change in thickness, Δt . The true area of contact is somewhat larger because of the elastic flattening of the rolls, and the Hitchcock equation is used to compensate for this effect. The separating force is divided by the adjusted area of contact to obtain the true specific roll pressure, $\bar{P}_s$. S_3 is essentially equivalent to $\bar{P}_s$. S_1 is determined from $S_1 - S_3 = S_0$, where S_0 is the yield strength adjusted for strain hardening. An average value of stress state is obtained by taking the ratio S_1/S_3 .

Malleability is defined as the true plastic strain in the thickness direction at fracture. The thickness can be measured simply enough, but the condition of fracture is

not so exact. Since beryllium is a brittle material, curling during rolling can cause cracking at thickness strains much lower than anticipated. This type of failure can be isolated because the cracks occur on the tension side only.

It is not unusual for edge splitting to result if the interference* is high. An example is shown in Figure 11. This failure is caused by a mechanical constraint resulting from elastic spring in the rolls. Another common type of failure is edge cracking, as seen in Figure 12. These cracks usually appear in the sides, transverse to the rolling direction, but occasionally are found in the leading or trailing edge. They are not indicative of a complete absence of malleability, but rather serve as a sign that the malleability in the bulk is nearly gone. The reason for these cracks is, possibly, the unfavorable stress state near the edges, and it is because of this non-uniform stress state that the cracks must be ignored.

*Interference — the difference between the stock thickness and gape prior to a pass.

SIDE VIEW (no scale)

FRONT VIEW (no scale)

FIGURE 10. Two Schematic Views Showing the Position of Load Cells Used to Measure Separating Force. Each Cell will Measure 1/2 of the Total Force if a Sample is Passed Through the Center of the Rolls.

FIGURE 11. Edge-Splitting Caused by Mechanical Restraint During Rolling. The Arrow Points in the Direction of Rolling.

FIGURE 12. Edge-Cracking Caused by a Lack of Malleability and Poor Stress State Near the Edges. Rolling Direction was Toward the Top of the Photograph.

Malleability is defined as the thickness strain when the bulk fails. A typical failure of a sample with a 0.105-in. initial thickness is shown in Figure 13. These cracks occurred spontaneously on the seventh pass at -5.75% strain. The cumulative thickness strain was -4.55% after the sixth pass. The true malleability is between these values, but the value reported will always be the higher number.

The failure mode changes as initial thickness decreases. The cracks in thinner material are shorter and not connected. A typical failure is shown in Figure 14. The cracks in the bulk appeared at -24.7% strain. The edge cracks occurred as early as -8% strain but did not propagate. Figure 15 shows material of intermediate initial thickness (0.049-in.) from that shown in Figures 13

FIGURE 13. Typical Failure Caused by Lack of Malleability in 0.105-In. Stock. The Arrows Point in the Direction of Rolling.

FIGURE 14. Typical Failure Caused by Lack of Malleability in 0.025-In. Stock. The Arrows Point in the Direction of Rolling.

FIGURE 15. Typical Failure Caused by Lack of Malleability in 0.049-In. Stock. The Arrows Point in the Direction of Rolling.

and 14. Note that the failure mode is a compromise between the previous two. This specimen failed at -9.25% strain.

The malleability of beryllium is shown as a function of the ratio of roll diameter-to-specimen thickness (D/t) in Figure 16. The general trend which was anticipated – increasing malleability with increasing D/t_0 – is shown. These tests were conducted before the load-measuring

FIGURE 16. Plot of Malleability (Thickness Strain at Fracture) Versus Diameter-to-Thickness Ratio. The Curve is Probably Not Linear but is Shown as Such Because of Insufficient Data.

capability existed, so stress state could not be plotted as the independent variable.

When the load cells were installed and stress state measurements were made, it became obvious that stress state was dependent upon many variables in addition to the diameter-to-initial thickness ratio. Some of these variables were as follows:

- interference
- change in interference per pass
- strain per pass
- cumulative strain
- strain rate
- springback of the rolling mill

The danger in proceeding without proper understanding of these variables was obvious. It was possible to get higher stress states with thicker material than with thinner material. If variables were not controlled, they could have different effects upon each thickness. A better understanding of rolling parameters became imperative. This understanding is such a major task that the remainder of the report is devoted to obtaining it.

STRESS STATE INVESTIGATION

At this juncture, the scope of the investigation changed. The original plan was to develop a quantitative relationship between stress state and malleability, but the best that could be developed from existing data was a relationship between the ratio of diameter-to-thickness (D/t_0) and malleability (ϵ_{tf}). The relationship between D/t_0 and Stress State (S_1/S_3) is extremely complicated, and this relationship must be known before the $S_1/S_3 - \epsilon_{tf}$ relationship can be obtained.

Thus a need for defining the rolling parameters emerges. Perhaps the best way to define these parameters is to discuss what occurs when a piece of metal is given one pass through a set of rolls. This discussion follows.

Rolling Narrative

The variables which are operative during rolling are shown schematically in Figure 17 and defined in Table I. Those

FIGURE 17. Schematic View Showing the Interdependency of Variables Which Determine Roll Force and Strain During the Reduction of a Sample by Rolling.

within the box are fundamental variables, whereas those outside the box are testing variables. The distinction between these types is that the testing variables are set before the test commences, and the fundamental variables are adjusted during the rolling operation. The solid lines connecting the variables show the inter-dependency of variables within the box; the dotted lines show which fundamental variables are determined by the testing variables. The diagram is easier to interpret if the word "determines" is used wherever a line connects two variables. For example, in the upper left of the diagram Δt "determines" ϵ_t .

The diagram results from analyzing the rolling process as follows: The interference (consult Figure 17 – top) is set and it determines the change in thickness, Δt , which, along with the roll diameter, determines the arc length, ℓ , via the equation $\ell = \sqrt{\Delta t R}$. Arc length combines with friction forces and initial thickness (See Figure 7) to determine S_1 , the principal stress in the rolling direction. S_3 is determined from S_1 by the Shear Stress law: $S_1 - S_3 = S_0$. The yield stress, S_0 , increases as the result of strain, and strain rate, as shown to the left of S_3 . S_3 determines the mill separation force, F , by the relationship $S_3 \approx \bar{P}_s = \frac{F}{A} = \frac{F}{\ell w}$. The lines for ℓ and w enter the diagram between S_3 and F to show that their influence on F is indirect – resulting from a calculation.

Springback, $Sp.B$, which is determined by F and the stiffness of the mill, results in less Δt than predicted by the interference. Roll Flattening, $R. Ftg$, results from elastic compression of the roll surface in the arc of contact and is dependent upon S_3 . Roll Flattening in turn causes an increase in contact length. Of course Δt determines the strain in the thickness direction, ϵ_t . Since $-\epsilon_t = \epsilon_\phi = \epsilon_\theta$ and $S_0 = A \epsilon_\theta^m$, ϵ_t determines S_0 .

There are phenomena operative in this rolling process which suggest that the stress state could become infinite. For example, roll flattening causes an increase in contact length, which increases both separating force and S_1 . When S_1 increases, S_3 increases, as does roll flattening and the separating force. Increased separating force causes increased friction force and thus more S_1 , etc., etc., ad infinitum. There are, however, opposite effects involved simultaneously. For example, increased force means increased springback which decreases the contact length and thus decreases S_1 . Also, the factors tending to increase the stress state are not of sufficient magnitude to sustain the effect. There are also boundary conditions which limit stress state—such as Δt never exceeding the set interference. Rolling is truly a steady state or equilibrium phenomenon, except for the initial contact of the workpiece with the rolls.

TABLE I. Definition of Variables Affecting Rolling

Variable	Symbol	Description	Variable	Symbol	Description
Interference	Int.	The difference between the initial stock thickness and the opening between the rolls.	Thickness Strain	ϵ_t	Plastic strain from rolling defined as $\epsilon_t = \ln t/t_0$. Also $ \epsilon_t = \epsilon_0$.
Change in thickness	Δt	The difference between the initial and final thickness as the result of rolling.	Roll Flattening	R. Ftg.	Elastic increase in the roll diameter (hence contact length) caused by the pressure developed in rolling (S_3).
Roll Diameter	D	Average diameter of the rolls. This is not the flattened roll diameter present during rolling.	Initial thickness	t_0	Sample thickness before rolling.
Contact length	ℓ	The approximate arc of contact between the rolls and sheet resulting from the calculation: $\ell = \sqrt{R\Delta t}$.	Friction coefficient	μ	Surface friction between the sample and the rolls.
Principal Stress in the rolling direction	S_1	Stress in the sheet is compressive and less than S_3 by an amount equal to S_0 .	Mill Separating Force	F	Force resulting from S_3 which acts to separate the rolls. This force is resisted by the mill yoke and measured by load cells beneath each yoke.
Principal Stress in the thickness direction	S_3	Stress through the thickness. It is always compressive.	Mill Stiffness	K	The rate of change of gape with increased separating force.
Yield Stress	S_0	Stress corresponding to 0.002-in./in. elongation in a tensile test.	Springback	Sp.B.	The change in gape encountered in any given pass.
Plastic Properties of the Metal	m,A	Chiefly the strain hardening behavior: $S_0 = A\epsilon_0^m$. Also includes strain rate effects when present.	Gape	Gp.	The opening between the rolls prior to rolling.
			Sample Width	w	Width of the sample before rolling.

But the significance in figure 17, to this report, is not in showing that rolling is an equilibrium process. Rather, its importance is in defining the key rolling variables and providing a qualitative picture of how they affect stress state. To provide a better understanding of this important diagram, the effect of changing some of the key variables upon stress state is described.

Stress State is, of course, defined as the ratio of $\frac{\bar{S}_1}{\bar{S}_3}$. It is an average figure — the quotient of two averages. This quotient will increase if either the numerator ($\bar{S}_1$) increases, or the denominator ($\bar{S}_3$) decreases*. Many factors will promote an increase in $\bar{S}_1$, and some of these will be analyzed below. Note, that whereas a decrease in S_3 will increase stress state mathematically, it will not do so in fact. This is because S_3 and S_1 are so related by the shear stress law ($S_1 - S_3 = S_0$) that a decrease in S_3 means a corresponding decrease in S_1 . But since S_1 is always a lower absolute value, the quotient is lowered more as a result of a given decrease in S_1 than by the same decrease in S_3 . Thus stress state is actually lowered by a decrease in S_3 and, conversely, increased by an increase in S_3 .

Thus variables which increase S_1 or S_3 will also increase stress state. One such variable is initial thickness, t_0 . If t_0 is lowered while the change in thickness is held constant, both S_1 and S_3 will increase. The increase in S_3 results from strain hardening. Since strain, ϵ_t , is defined as $\epsilon_t = \ln t/t_0 = \ln\left(\frac{t_0 + \Delta t}{t_0}\right) = \ln\left(1 + \frac{\Delta t}{t_0}\right)$ the lower t_0 gets for a given Δt , the greater (more negative) ϵ_t gets. This increased strain means increased yield strength caused by strain hardening. Since $S_1 - S_3 = S_0$, S_3 will increase as S_0 increases, even if S_1 remains the same.

S_1 will increase with decreasing t_0 for a number of reasons. The principal reason is that for a given contact length (thus friction forces are constant) the cross sectional area (wt) over which the frictional forces are distributed decreases — increasing the stress, S_1 . S_1 also increases because the increased S_3 causes more roll flattening than would have occurred. This flattening increases the contact length which not only increases S_1 because there is greater length over which to build-up frictional forces, but also because the mill separation force, F , increases still more because of the increased area ($w\ell'$) upon which S_3 acts. This increased F increases the friction forces (μF) and thus S_1 .

*An increase in S_1 is assumed in this report to be an increase in the absolute value of S_1 . This is truly a decrease, since S_1 becomes more negative. The same applies to S_3 .

This tendency for a changing variable to have a compounding effect on stress state could be called a "vicious circle." There are many such "vicious circles" operative during even a single pass reduction. These can be seen easily in Figure 17. Any place on the diagram where a closed loop is formed will result in a "vicious circle". Perhaps the simplest to see is the one going from F to the friction (dashed) line to S_1 to S_3 and back to F . This means that as F increases, so does μF and hence S_1 . Increased S_1 means increased S_3 which means more F and the circle is complete. Another circle would be from Δt to ℓ to the $S_3 - F$ line to F to SpB to Δt . As Δt increases, ℓ increases by definition. A larger ℓ means a greater separating force F because $F = S_3 w\ell$. But more F means more springback which means Δt is less than it would have been if the mill were stiff. Note that this last "circle" has a decreasing effect, whereas the previous "circle" has an increasing effect on S_1 .

Obviously the analysis of rolling, although simplified greatly by Figure 17, remains quite involved. A more careful — and in particular a quantitative — look at the rolling variables is needed. This careful examination should result in a theory explaining the effects of key variables on stress state. Experimental work could then be performed to verify the theory. Such an analysis follows.

Rolling Variables Affecting Stress State

The stated objective of this work is to develop a relationship between stress state and malleability for brittle materials. Stress State is defined as $\bar{R} = \frac{\bar{S}_1}{\bar{S}_3}$ and malleability is defined as the true thickness strain at fracture, ϵ_{tf} . While the strain determination is straightforward, the stress state determination requires a careful analysis of the rolling variables. The remainder of the report deals with this stress state analysis.

A good qualitative picture of the rolling variables is presented in Figure 17 which was discussed. But a quantitative relationship between stress state and the key testing variables is needed. In other words, Figure 17 shows that stress state depends upon such variables as contact length, thickness, or friction coefficient. But it does not show *how* stress state varies. This section is concerned with that quantitative relationship.

Figure 18 provides the means for developing the desired relationships between Stress State and the key testing variables. It is a free-body diagram containing an element of volume along the contact length of a specimen being rolled. The element of volume is chosen to provide the relationships between S_1 , the stress in the rolling

FIGURE 18. Free-Body Diagram of an Element of Volume in a Piece of Metal Being Reduced by Rolling.

direction, and l contact length. This is because S_1 is the key variable affecting stress state, and l is the key variable affecting S_1 . When the equilibrium of force condition is applied to this element of volume, the following relationships result:

$$S_1 = S_0 \left(1 - e^{-\frac{2\mu x}{t}} \right) \quad 0 \leq x \leq l/2 \quad (1)$$

$$S_1 - S_0 \left(1 - e^{-\frac{2\mu(l-x)}{t}} \right) \quad l/2 \leq x \leq l \quad (2)$$

Examination of the boundary conditions shows that when $x = 0$ or $x = l$, $S_1 = 0$; and when $x = l/2$, $S_1 = S_1$ maximum $= -K S_0$, where K is a positive number whose value depends upon μ , l , and t . This behavior is plotted in Figure 19a. The curve is symmetrical about the value $x = l/2$, where $|S_1|$ reaches a maximum.

Assuming the metal flows plastically according to the Shear Stress Law, $S_1 - S_3 = S_0$, S_3 can be determined from S_1 . In fact, as Figure 17 shows, S_3 is physically determined by S_1 . Thus

$$|S_3| = S_0 + S_1 = S_0 + S_0 \left(1 - e^{-\frac{2\mu x}{t}} \right).$$

This reduces the form

$$S_3 = -S_0 e^{-\frac{2\mu x}{t}} \quad 0 \leq x \leq l/2 \quad (3)$$

and for x from $l/2$ to l

$$S_3 = -S_0 e^{-\frac{2\mu(l-x)}{t}} \quad l/2 \leq x \leq l \quad (4)$$

This behavior is plotted in Figure 19b. Thus both S_1 and S_3 can be determined for any value x along the contact length. But the assumption has been made early in this work that an average value for stress state would be used rather than an instantaneous value which results when taking the ratio S_1/S_3 for a given x . The average chosen was the value for S_3 resulting when the mill separating force, F , was divided by the contact area, Wl .

$$\text{Thus } \bar{S}_3 \approx P_s = \frac{F}{Wl}$$

$$\text{Now analytically } F = \int S_3 (w dx)$$

and when the relationship for S_3 in equation (3) is integrated from $x = 0$ to $x = l/2$ (doubling for the remaining, symmetrical portion of the contact length) the following expression results:

$$\bar{S}_3 = \frac{S_{0t}}{\mu l} \left(1 - e^{-\frac{\mu l}{t}} \right) \quad (5)$$

and since $\bar{S}_1 - \bar{S}_3 = S_0$ (assuming the Shear Stress Law

holds for averages) $\bar{S}_1 = \frac{S_{0t}}{\mu l} \left(1 - e^{-\frac{\mu l}{t}} \right) + S_0$ or

$$\bar{S}_1 = S_0 \left[\frac{t}{\mu l} \left(1 - e^{-\frac{\mu l}{t}} \right) + 1 \right]$$

$$\text{Now } \bar{R} = \frac{\bar{S}_1}{\bar{S}_3}$$

$$\text{Therefore } \bar{R} = \frac{S_0 \left[\frac{t}{\mu l} \left(1 - e^{-\frac{\mu l}{t}} \right) + 1 \right]}{\frac{S_{0t}}{\mu l} \left(1 - e^{-\frac{\mu l}{t}} \right)}$$

which reduces to the form:

$$\bar{R} = 1 - \frac{z}{(e^z - 1)}$$

$$\text{where } z = \frac{\mu l}{t}$$

FIGURE 19. Plots of Principal Stresses as a Function of Position Along the Contact Length.

Closer examination of this expression shows that as z approaches zero, e^z approaches $(1 + z)$, and $\bar{R}$ approaches zero. Also when z approaches infinity, $\bar{R}$ approaches unity. These boundary conditions agree well with reality. This is because as z gets small, contact length is small and/or thickness is large — both conditions causing poor (near zero) stress state. Conversely, a large z value means that the contact length is large and/or the thickness is small — both conditions known to produce a favorable (near $\bar{R}$) stress state.

This behavior of stress state has been plotted as a function of z in Figure 20. The curve has the exponential shape predicted by equation (6). When $z = 10$, $\bar{R}$ has achieved 99.95% of its maximum possible value of unity. This corresponds to a stress state of nearly pure hydrostatic compression.

The most important observation to make about equation (6) is found in the nature of z . This is that the three key variables have the same effect upon z — they are weighted the same. Thus doubling the contact length will have the same effect on z as doubling the friction coefficient or halving the thickness. It is also important to note that the effect of thickness is inverse — stress state decreasing as thickness increases — while both contact length and friction coefficient increase stress state as they increase. Finally, position along the curve has a strong effect upon changes in stress state which can be expected as a result of changes in z . When z is a low value, doubling its value can have a strong effect upon stress state. When z is a large value, doubling it will show little increase in stress state.

In summary, in this section and in the preceding section, the theoretical aspects of stress state during rolling are discussed. First a qualitative picture was established which shows that there are numerous variables involved during rolling which influence stress state. Some of these are called testing variables — established by selection before the test commences. Examples of this type are specimen thickness, roll speed, material properties, and roll geometry. Other variables are called fundamental in that their value is determined during the test and they have a direct affect upon stress state. Examples of this type are the principal stresses, change in thickness, contact length, etc. The interrelation of these variables is seen to be complicated — involving many “vicious circles” which intensify a given effect.

Secondly, a theory was developed which provides a prediction of just how the key variables affect stress state. These key variables are contact length, initial thickness, and friction coefficient. They were chosen for analysis

FIGURE 20. Plot of Stress State ($\bar{R}$) Versus Stress Intensification Exponent, Z , for Rolling.

because they have the most direct affect upon S_1 , the principal stress in the rolling direction. S_1 is the key variable controlling stress state because the only other variable involved with stress state S_3 , the principal stress in the thickness direction, is chiefly dependent upon S_1 . The theory that was developed predicts that stress state will vary exponentially from 0 to 1 as a function of z , where $z = \frac{\mu l}{t}$ which can vary from 0 to approximately 10. When $z = 0$ (corresponding to a small contact length or friction coefficient or a large initial thickness), stress state is zero. When $z = 10$ (corresponding to a large contact length or friction coefficient or a small thickness) stress state is nearly 1. Thus as z increases, stress state increases.

The remaining challenge is, then, to verify these predictions. Does stress state vary exponentially with z ? Does contact length, friction coefficient, and initial thickness have the same weight in affecting stress state? The next section contains experimental work aimed at answering these questions.

Mild Steel Tests

The foregoing analysis suggests that stress state is a function of three key variables: friction coefficient,

contact length, and initial thickness. Although the interest of this report has been toward brittle materials, nothing about the analysis requires that a brittle material be used to verify the predictions of the previous section. In fact, a ductile material would be preferable, since there would be no limitation on the amount of reduction in thickness (hence contact length) which could be achieved. A brittle material, on the other hand, could fail at such low strains as to limit the range of contact length available. This limitation arises because the laws of plasticity used in the foregoing analysis would not be valid beyond the fracture strain.

Consequently, mild steel was chosen for the remaining experimental work. Since the influence of pressure on friction coefficient is not well-understood, it was decided to keep this variable as constant as possible. No lubricant was used on either the rolls or the samples, and each sample was ultrasonically degreased prior to rolling.

Thus with friction coefficient held constant, only initial thickness and contact length remain as independent variables to be used to verify the predictions of the previous section. Initial thickness can be controlled easily by choosing the appropriate stock thickness. In this work four thicknesses were used: 0.029, 0.049, 0.064, and 0.110-inch. Each specimen size was 2-in.-wide by

6-in.-long prior to rolling. A specimen was given only one pass through the rolling mill, so that a "fresh" piece was used for each test. This was an attempt to keep the plastic properties of the metal constant. As it was, some variation of hardness for each thickness of the as-received stock existed. These initial-hardness values are given in Table II.

TABLE II. Hardness versus initial thickness for mild steel used in this study. The variation in initial hardness is responsible for some variation in results.

Initial Thickness (in.)	Hardness, R_B
0.0295	48
0.0480	58
0.0640	54
0.1100	61

Contact length cannot be varied in such a straightforward manner as initial thickness. This is because one cannot predetermine a value for contact length. Instead, a value for interference is set, resulting in a certain change in thickness, Δt . The Δt value has a strong influence on contact length, but roll flattening also exerts a strong influence on ℓ . In this program, five levels of interference were used for the 0.049, 0.064, and 0.110-in.-thick material. These values were 0.010, 0.020, 0.030, 0.040, and 0.050-inch. Values for interference of 0.010, 0.020, and 0.030-in. were used for the 0.029-in. stock. (When the interference was 0.001-in. greater than the stock thickness, a slight pre-load was placed on the closed rolls to effect a gape reading of minus 0.001-inch)

These experiments were conducted in the same manner as the beryllium tests discussed earlier. The separating force was recorded and the final thickness was measured. Calculations were performed from these data to obtain values for average stress state $\bar{R} = \frac{S_1}{S_3}$ and true contact length (ℓ' —based on the Hitchcock equation). The results are discussed below.

RESULTS—This section contains the results of the experiments with mild steel. It contains mostly observations, whereas the significance of the results are considered more carefully in the next section.

Figures 21-23 contain the data resulting from the "single-pass" tests on mild steel of varying thicknesses and different levels of interference. Figure 21 is a plot of change in thickness as a function of different

interferences. The nature of these curves prompts the following observations:

1. The actual reduction in thickness is much less than the maximum possible reduction in thickness (interference), for every thickness and every interference.
2. Change in thickness is not dependent upon initial thickness for the lower values of interference (0.010, 0.020, and 0.030-in.); but Δt decreases as t_0 decreases for the larger values of interference (0.040 and 0.050-in.)
3. Change in thickness is strongly dependent upon interference.

Figure 22 is a plot of true specific roll pressure versus initial thickness for different values of interference. A calculated stress rather than a measured load is plotted along the ordinate to show a better measure of the effect of t_0 and interference on stress state. The major observations are as follows:

1. True specific roll pressure is strongly dependent upon interference.
2. For interference values of 0.010 and 0.020-in., true specific roll pressure is not strongly dependent upon initial thickness. (Scatter in the data for the 0.010-in. interference is due to hardness variation in the starting material and measurement error caused by the low values of load and change in thickness).
3. True specific roll pressure increases with decreasing thickness for values of interference over 0.030-inch. The rate of increase becomes greater as the interference becomes greater.

Figure 23 offers a different analysis of the data found in Figures 21 and 22. True specific roll pressure is plotted against true strain [$\epsilon_t = \ln(t/t_0)$] for each of the different initial thicknesses used. Thus the interference data from the previous two figures are not contained directly in this plot. The different strain values do result from the different interference values used, however. The strain hardening behavior of the mild steel is included in this plot. It is the line marked S_0 , and the curve corresponds to the relationship $S_0 = 100,000 \epsilon_0^{0.2}$ which was determined from tensile tests of the material used in this work. The chief observations are as follows:

1. Specific roll pressure increases faster with increasing strain than would have been predicted based upon strain hardening alone.
2. Although the data seem closely grouped, a trend exists. For strains from 0 to 8%, specific roll pressure increases as thickness increases. For strains from 8 to

FIGURE 21. Dependence of Change in Thickness (Δt) on Initial Thickness (t_0) for Various Interferences.

FIGURE 22. Dependence of True Specific Roll Pressure (P_s) on Initial Thickness for Various Interferences. Each Datum Point is for a Separate Specimen.

FIGURE 23. Plot Showing the Dependence of True Specific Roll Pressure on True Strain for Mild Steel of Varied Initial Thicknesses. The Strain Hardening Behavior is Also Plotted in This Figure.

16%, specific roll pressure increases in the following order of initial stock thickness: 0.029, 0.064, 0.049, and 0.110-inch. This corresponds to the order of increasing hardness of the starting material. Finally, for strains above 16% the trends of the curve suggest that specific roll pressure will increase with decreasing stock thickness. This effect is masked by hardness differences.

The observations made about the data in Figures 21-23 obviously require further comment. The present form of these data does not provide answers to the predictions of the stress analysis sections above. In fact, the variable of interference is not one of the key variables contained in that analysis. The key variable of contact length must be extracted from all the variables influenced by interference in such a way as to isolate the effect of contact length upon stress state. This task is undertaken below.

SIGNIFICANCE OF RESULTS – The background of stress analysis developed in previous sections (with the aid of Figure 17) is helpful in analyzing the results presented in the preceding section. To begin with, the fact that the actual reduction in thickness is much less than the interference means that the rolling mill is not stiff. (Separate measurements have shown that the spring in the mill frame is a linear function of the separating force.) Related to this is the fact that the rolls themselves are not rigid. In some cases the elastic deformation of the rolls was enough to double the contact length.

If the rolling mill were completely stiff and the rolls perfectly rigid, the curves in Figure 21 would be horizontal with $\Delta t = \text{interference}$. But Δt is not equal to the interference because the mill is not stiff. There are three reasons why the curves are not horizontal with decreasing thickness.

1. Mild steel strain hardens. If the curves were horizontal, then strain would increase for decreasing thickness because Δt would be the same for each thickness. This would cause an increase in $|S_3|$ as the result of an increase in S_0 (from strain hardening) and, because of the Shear Stress Law ($S_3 = S_1 - S_0$), an increase in $|S_3|$ would increase the mill separating force (F) and thereby increase springback. Increased springback decreases Δt . Thus as t_0 decreases, Δt decreases for a given interference.
2. $|S_1|$ increases as t_0 decreases. This occurs because there is less area ($= wt$) over which to distribute the surface friction forces in any given deformation. Increasing $|S_1|$ increases $|S_3|$ via the shear stress law

($S_3 = S_1 - S_0$) and the argument becomes analogous to 1 above.

3. Since S_3 increases for reasons 1 and 2, roll flattening increases. This results in a longer contact length over which the friction forces build up and thus a greater $|S_1|$. Increased $|S_1|$ will have the same effect as mentioned in 2. Also, increased contact length means increased F , since $F = S_3 w\ell$. This means more springback and less Δt .

Arguments 1-3 above show why Δt will decrease as t_0 decreases for a given interference. There is an opposing effect operative also. This is, as Δt decreases, the contact length ($\ell = \sqrt{R\Delta t}$) also decreases; and as ℓ decreases, S_1 , S_3 , and Δt all decrease. This simply means that the trends predicted by 1-3 above will not be as potent as they would have been without this opposing effect. It should also be noted that the arguments broached in 1-3 above are more likely to have a greater effect for larger interferences since intensification of a large number makes for a larger result.

For a stiff mill with rigid rolls, the trend observed in Figure 22 is expected. True specific roll pressure (S_3) should increase with decreasing t_0 for a given interference. This is because such a mill produces the same Δt for each t_0 , thus increasing both strain and strain hardening as t_0 decreases. Since $S_3 = S_1 - S_0$, a larger S_0 means a larger (more negative) S_3 .

The stress state effect (increasing S_1 because of less area over which friction forces are distributed) results in an increasing S_3 as t_0 decreases also. This occurs regardless of whether the mill is stiff or the rolls rigid, for a given interference, as long as the thickness is decreased.

Since the mill used for these tests is not stiff and the rolls are not rigid, the question arises whether the behavior observed in Figure 22 is still expected. The answer is yes for the following two reasons:

1. The flexible rolls result in a larger contact length (and hence increased S_1 and S_3) than would occur with rigid rolls for the same t_0 and interference. Thus S_3 continues to increase with decreasing t_0 .
2. The nonstiff mill, although it results in a lower contact length as the result of increased springback, will result in some net increase in S_3 with decreasing t_0 . Otherwise springback would not have increased in the first place. Springback does increase, of course, due to the stress state effect.

The behavior noted earlier about Figure 23 – that stock thickness shows different effects upon S_3 for different ranges of strain – is the result of two opposing effects.

One of the effects is that S_3 decreases as t_0 decreases for a given strain. This can be seen from the definition of strain as $\epsilon_t = \ln t/t_0 = \ln \frac{t_0 + \Delta t}{t_0} = \ln(1 + \frac{\Delta t}{t_0})$. As t_0 decreases, Δt must also decrease to result in the same strain. This lower Δt lowers ℓ which lowers S_3 , thus resulting in lower S_3 with lower t_0 . This effect predominates in the strain range of 0-8%.

The other effect, which is opposing, is independent of strain. It is that decreasing stock thickness results in an increasing S_3 as the result of the stress state effect. This behavior would result in an inversion in the observed behavior from 0-8%. Such an inversion appears possible from the trend of the curves shown in dotted lines in Figure 22. This seems to occur at a strain of about 40%.

Between 8 and 16% the two opposing effects seem to cancel, and the trend of increasing S_3 is the same as would be predicted by the hardness data in Table II: 0.029, 0.064, 0.049, 0.110. Between 16-40% the observed effect is a mixture of the hardness and stress state effects.

Thus the behavior of mild steel in rolling agrees well with predictions resulting from Figure 17 and the preceding stress analysis sections. The trend of each curve supports the earlier theory of rolling. But the major challenge still remains. This is to determine whether the data will support the theory presented in Figure 20 — the R-z relationship. This is determined below.

One way to see if theory and practice agree is to calculate datum points for a dependent variable which can be later compared to measurements. A plot of S_3 versus ℓ' affords this possibility. S_3 is a derived quantity, shown earlier as:

$$S_3 = \frac{S_0 t}{\mu \ell'} \left(1 - e^{-\frac{\mu \ell'}{t}} \right)$$

S_3 can also be obtained from measurement of F as follows:

$$S_3 \approx \bar{P}'_s = \frac{F}{w \ell'}$$

Computation of S_3 proceeds as follows:

The contact length, ℓ' , used for the measured S_3 can also be used to obtain calculated values for S_3 .^{1,2} A value for μ of 0.1 is assumed to be independent of S_3 . An average thickness $t = t_0 + \Delta t/2$ is used for t . And S_0 is adjusted for strain hardening, using the equation $S_0 = 100,000 \epsilon_0^{0.2}$. The maximum value for S_0 is used in the calculation because S_0 is nearer this value over most of the contact length (See Figure 4).

The calculated values are compared to measured values in Figure 24. The measured data are obtained from Figures

21 and 22. Contact length is determined by the Hitchcock equation.

Rather good agreement between theory and experiment can be seen in Figure 24. The theoretical data points are shown as dotted lines. A line of best fit has been drawn between the theoretical and experimental data points.

Note that S_3 is dependent upon initial thickness for a given contact length. The greatest thickness has the lowest S_3 . This agrees with the nature of the expression

$$S_3 = \frac{S_0 t}{\mu \ell'} \left(1 - e^{-\frac{\mu \ell'}{t}} \right)$$

As t gets smaller, the

coefficients get smaller but the exponent gets larger. A mathematical limit analysis shows that the effect of the exponent predominates so that S_3 should increase.

Of course there is some disagreement between theory and practice, particularly for the larger S_3 values. This is not surprising because the development of the theory included many simplifying assumptions; such as, friction coefficient remaining constant, and both S_0 and t not varying with ℓ . Although these data were adjusted for S_0 and t , the integration would have yielded a different result had the adjustments been made prior to integration.

But the purpose of the curves in Figure 24 is not so much to show exact agreement between theory and practice as it is to show correct trends resulting from the theory. Figure 24 demonstrates that the important variables have been properly analyzed. Of course it would be nice to be able to predict stress state directly before a test is conducted based upon theoretical considerations alone. But it is more expedient to account for all variables by some scheme similar to that used in Figure 24 and then determine stress state and malleability values after a test has been conducted. Such an approach is necessary to complete the original goal of this thesis. But it is considered beyond the scope of this report to continue the investigation into this area.

SUMMARY

The original intent of this work was to develop a relationship between stress state (R) and malleability (ϵ_{tf}) for beryllium. The early work with beryllium proved to be too naive. It was hoped that changing t_0 would change R in a predictable manner. But because multiple passes were used and there was no control of contact length, strain, strain rate, or interference per pass, the desired R — t_0 relationship did not materialize. Instead a qualitative observation was made; that is, since decreasing

FIGURE 24, Plot Showing Experimental (Solid) and Theoretical (Dashed) Data Points for S_3 Versus l' . For a Given l' There are Two S_3 Values, One Experimental, One Theoretical.

thickness did increase malleability for beryllium at 25°C, it must have been due to an increase in stress state.

To develop the $R - t_0$ relationship, the rolling process was analyzed carefully, resulting in a flow diagram showing how the many variables affected one another. This diagram suggested that the three variables having the greatest influence on R were initial thickness (t_0), contact length (l), and friction coefficient (μ). But the degree of influence of these variables could not be obtained from the diagram. Thus a theory was developed from a stress analysis of a sheet being rolled, and this theory accounted for each of the key variables. This theory showed that each of the variables had the same degree of influence on R , since R was a function of z where $z = \frac{\mu l}{t_0}$; that is doubling μ had the same effect on R as doubling l or halving t_0 . Since the theory was developed without assuming any particular material was being rolled, it should be a general relationship — true for any metal which obeys the laws of plasticity.

Tests with mild steel were conducted to confirm the theory. Different values of z were obtained by changing both l and t_0 . There was good agreement between theory

and experiment regardless of whether z was changed by changing l or t .

Thus there are many ways to change the stress state during rolling. In addition to the key variables described (μ , l , and t), R will also change with the roll diameter, rigidity of the rolls, strain, and, for some metals, strain rate. It is important that each of these variables be taken into account before the stress state-malleability relationship can be determined.

CONCLUSIONS

1. The malleability of beryllium is improved by changing rolling conditions. When the initial thickness is decreased from 0.1 to 0.025-inch, such that the roll diameter-to-thickness ratio (D/t_0) increases from 50 to 200, malleability increases from -5 to -25%.
2. Diameter-to-thickness ratios are not direct measures of stress state, R , because when D/t_0 changes then numerous other variables, which affect R , also change.

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3. Stress State, R , varies with contact length (ℓ), thickness (t), and friction coefficient (μ) as follows:

$$R = 1 - \frac{z}{e^z - 1}, \text{ where } z = \frac{\mu \ell}{t}$$

4. Since most rolling conditions are such that $z = 1$, a relatively minor change in μ , ℓ , or t , can cause a substantial increase in R . For example, an increase in z from 1 to 2 results in an increase in R from 0.41 to 0.69
5. Each of the three key rolling variables: μ , ℓ , and t , exerts the same influence on stress state. Thus, doubling μ or ℓ results in the same change in R as halving t .

FUTURE WORK

This report shows that malleability can be improved by adjusting rolling conditions. It also shows that stress state

can be altered by changing rolling conditions. But the report does not show how much improvement in malleability can be expected for a given increase in stress state. This task remains as the major goal of any future work on this topic.

A probable starting place would consist of rolling samples of a given thickness of beryllium with gradually increasing values of interference until fracture occurs in one pass. Another approach would consist of using multiple passes on the same piece until failure occurs. (A preliminary look at multiple pass rolling is included in the Appendix.) Rolling parameters would be adjusted prior to each pass to maintain the same stress state during each pass until failure. This stress state and failure strain would constitute one datum point on the stress state malleability curve.

Once the curve is established, it can be repeated with another brittle material to see if the relationships are of a general nature. A likely material would be the Al-10% Cu alloy.

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APPENDIX

In anticipation of the need for multiple-pass rolling to get failure in brittle materials, mild steel was rolled using multiple passes. The results of this investigation are included in the appendix so as not to confuse the discussion surrounding the single-pass experiments contained in the main body of the thesis.

Four samples of 0.110-in. material were rolled, each with a different change in gape per pass. The gape was decreased by 0.001-in. after each pass for one sample, 0.002-in. for the second, 0.004-in. for the third, and 0.010-in. for the fourth. The change in thickness was recorded along with the mill separation force for each pass. The Hitchcock equation was used to compute the true contact length, ℓ' . True specific roll pressure was obtained by dividing the mill separation force by the

product of ℓ' and w , sample width. The resulting data are plotted in Figure A1.

The first general observation is that the specific roll pressure is greater for multiple passes than for single passes. Also the data suggest that the specific roll pressure is greater for lighter passes than for heavier, especially when the strain is less than 10%. But the fact that the SP-1 curve is so far displaced from the others suggests that the Hitchcock equation is not accurate for very light passes. Also the ratio of contact length to thickness is low for light passes. This means that the distribution of stress through the thickness may not be uniform, based on St. Venant's principle. The conclusion of this brief investigation is that a careful analysis of the effects of multiple passes will be required before such an approach is used to determine a stress-state-malleability relation.

FIGURE A1. Plot Showing How True Specific Roll Pressure Varies with True Thickness Strain, Various Conditions were Used as Shown in the Key.