Electron Trappings in RHIC from a Debunched Proton Beam

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**Ion Trapping / Electron Trapping**

Proton Beam Completely Debunched

Gas Composition as derived in RHIC-PE-51

Densities:

<table>
<thead>
<tr>
<th>Molecules</th>
<th>Z</th>
<th>A</th>
<th>warm (25%)</th>
<th>cold (75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>2</td>
<td>2</td>
<td>2.1 x 10^7 cm⁻³</td>
<td>2.3 x 10^7 cm⁻³</td>
</tr>
<tr>
<td>He</td>
<td>2</td>
<td>4</td>
<td>-</td>
<td>2.3 x 10^7 cm⁻³</td>
</tr>
<tr>
<td>CO</td>
<td>14</td>
<td>28</td>
<td>2.1 x 10^7 cm⁻³</td>
<td>-</td>
</tr>
</tbody>
</table>

Ionization cross-section: \( \sigma_i \)

\[
\sigma_i = 1.8 \times 10^{-19} Z_i^2 \text{ cm}^2
\]

Ionization Rate for the ith species per proton:

\[
\frac{dn_i}{dt} = c \sigma_i n_i (\beta = 1)
\]
The ions (positively charged) are rejected by the potential barrier of the proton beam.

The electrons could be trapped.

Average energy of the electrons at production:

140 eV

which corresponds to a velocity:

\[ v_e \approx 10^7 \text{ cm/sec} \]
To "clear" the electrons one can create one or several gaps in the beam — these gaps though are to be no less than 300 μm in size if the electrons have to reach a wall 3 cm away with the velocity given above.

Assume the probe beam has a uniform density distribution with elliptical cross-section of semiaxis $a$, $b$—

$I$, beam current = \( N e \int_{0}^{a} \int_{0}^{b} N e dA \) = \( 57 \times 10^{12} \times 1.6 \times 10^{-19} \times 78.2 \times 10^{3} \) = 0.74 A

Electric Field produced by the beam:

\[
E_{x,2} = \frac{(120 \text{ Ohm}) I_{\text{amp}}}{\frac{(x,2)}{a,2}} \text{ Volt} \text{/m}
\]

The voltage distribution is:

\[
V = \frac{43 \text{ Volt}}{a + b} \left( \frac{x}{a} + \frac{y}{b} \right)
\]
The potential barrier depth is calculated by setting $x = 0_x$ and $e = 0_z$, and that is

$$\Delta V = 43 \text{ Volts}$$

Oscillation Frequency of the Trapped Electron

$$\omega^2_{x,z} = \frac{(120 \text{ ohm}) \cdot e \cdot c^2}{m_e c^2 \sigma_{x,z} (0_x + 0_z)}$$

where $m_e c^2 = 0.5 \text{ MeV}$ and $0_x, 0_z$ is in meter.

$$\sigma_{x,z} (0_x + 0_z) \approx 2 \sigma^2$$, $\sigma \approx 4 \text{ mm}$

As a result

$$\omega_{x,z} \approx 700 \text{ MHz}$$
Bestatron Tune Depression

$$\Delta \nu_{x,z} = \frac{e}{4 \pi m \beta^2 y c^2} \int ds \beta_{x,z}(s) \frac{\partial E_{x,z}(s)}{\partial x,z}$$

electron

$E_{x,z}$ is the electric field generated by the electrons in the background.

$$\frac{\partial E_{x,z}}{\partial x,z} = \frac{(120 \text{ eV}) T}{\sigma_{x,z} (\sigma_x + \sigma_z)} = \frac{\sigma \eta}{\sigma_{x,z} (\sigma_x + \sigma_z)}$$

electron

$\eta$, charge neutralization coefficient caused by the electron.

We have

$$\Delta \nu_{x,z} = \frac{120 \cdot eI}{4 \pi m \beta^2 y c^2} \int ds \frac{\eta(s) \beta_{x,z}(s)}{\sigma_{x,z} (\sigma_x + \sigma_z)}$$

$$= \frac{120 \cdot eI}{8 \pi m c^2 \beta^2 y} \frac{\eta}{\sigma^2(s)} \int \frac{\beta(s)}{\sigma^2(s)} ds.$$
\[ \Delta \nu = \frac{1.20 \, \text{TeV}}{4\pi \, mc^2 \gamma^2 \varepsilon \, R} \]

with
\[ \varepsilon = \frac{2 \sigma^2}{\beta} = 0.06 \, \text{mrad} \]

\[ R = 610.2 \, \text{m} \]
\[ \gamma = 100 \]
\[ mc^2 = 0.9383 \, \text{GeV} \]

We get
\[ \Delta \nu = 4.64 \, \eta \]

It is safe to keep \( \Delta \nu \leq 0.0025 \), therefore one requires
\[ \eta \leq 0.0005 \]
Removal of the Electrons

Electrons have a low radial drift. After a time $t$, they have travelled a distance

$$l = v_e t$$

where $v_e = 1 \times 10^7$ cm/sec.

We assume that after this distance they are removed. We take

$$t = \eta \tau_e$$

$\tau_e = \infty$, 2.21 sec. is the production time per proton

With $\eta = 0.0005$

$$l = 6,050 \text{ m}$$
Drift in a Dipole \((B_2)\)

This occurs at velocity \(v_D\)

\[
v_D = \frac{E_x}{B_2} \frac{\omega_L^2}{\omega_L^2 + \omega_X^2} = \times \frac{\omega_L \omega_X^2}{\omega_L^2 + \omega_X^2}
\]

\[
\omega_L = \frac{eB_2}{mc}, \quad 2\pi \times \text{Larmor frequency}
\]

\[
= 5.6 \times 10^{11} \text{ Hz}
\]

Therefore \(\omega_L > \omega_X\), and

\[
v_D = 0 \frac{\omega_X^2}{\omega_L^2} = 3.5 \times 10^5 \text{ cm/sec}
\]
Drift in a Quadrupole with Gradient $B^\prime$.

\[ V_Q = \frac{\omega_Q \omega_x^2}{\omega_Q^2 + \omega_x^2} \]

where

\[ \omega_Q^2 = \frac{1}{2} \left( \frac{eB^\prime}{mc} \right)^2 c^2 \]

with $B^\prime = 550 \text{ keV/m}$ and $\sigma = 4 \text{ mm}$.

$\omega_Q = 2.9 \times 10^{10} \text{ Hz}$

Therefore, $\omega_Q \gg \omega_x$ and

\[ V_Q = \sigma \frac{\omega_x^2}{\omega_Q} = 6.8 \times 10^6 \text{ cm/sec} \]
In conclusion the drift velocity is quite smaller in the dipoles. The calculations of page 7 to clear the electrons have to be repeated by taking

\[ V_e = V_0, \approx 3.5 \times 10^5 \text{ cm/sec} \]

We obtain then

\[ l = 2.12 \text{ m} \]

This corresponds to the length traversed in the dipoles; it is about 10 dipoles, that is 5 cells.

Solution: install pairs of clearing electrodes in the warm sections at the beginning and at the end of each long straight section. Install also pairs of clearing electrodes in the arcs equally spaced 4 cell cells apart. So there are in total 24 pairs, 12 in cold section and 12 in warm sections (see figure next page). Take a gap separation of 50 with the vacuum chamber and a length \( L = d \).
To sweep the electrons away the following voltage is required:

\[ V_c = mc^2 \left( \frac{dV_e}{cl} \right)^2 \]

which is quite negligible compared to \( \Delta V = 43 \text{ Volt} \).

It is quite safe to take \( V_c = 100 \text{ Volt} \).

- Cleaning Electrodes in cold section
- \( \times \) " " in warm section