AD/RHIC-PH-22

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April 17, 1987

Commissariat à l'Énergie Atomique

Centre d'Études Nucléaires
de SACLAY

INSTITUT DE RECHERCHE FONDAMENTALE
Département de Physique Générale

SERVICE DE PHYSIQUE THÉORIQUE

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ABSTRACT

We investigate the properties of the system of partons produced in the very beginning of ultra-relativistic heavy ion collisions. We propose simple criteria for characterizing the partons which get freed during the collision and which give the dominant contribution to the initial energy density. These partons are found to have an average transverse momentum which grows with the size of the colliding nuclei. Numerical estimates of their initial energy density are given.

* This work is supported in part by the US Department of Energy.

Submitted for publication to
"Nuclear Physics B"

Saclay PhT/87-022

1.INTRODUCTION.

In order to assess the possibility of producing quark-gluon plasmas in ultra-relativistic heavy ion collisions, it is important to understand what happens in the very beginning of such collisions. It is usually assumed that the quanta which are produced in the central rapidity region quickly reach a state of local thermodynamic equilibrium [1]. Such an assumption of thermalization is certainly a convenient one, as it leads for example to a simple dynamical model of the collisions based on hydrodynamics [2]. However, its validity remains to be checked. In particular, one would like to know how to characterize the quanta which have a chance to thermalize, on which time scale this thermalization takes place, what is the energy density of the system soon after thermalization, etc... To answer most of these questions would require solving a kinetic equation, given appropriate initial conditions and a detailed knowledge of the microscopic processes by which the quanta exchange energy, momentum, or possibly their number. Our goal in this paper will be more modest; we shall attempt to determine the properties of the quanta which turn into free particles during an ultra-relativistic heavy ion collision and which give the dominant contribution to the initial energy density.

Our starting point is a parton model. We assume that, in the center of mass frame, where they are fast moving objects, the colliding nuclei may be viewed as collections of quasi real particles, the partons, with lifetime much larger than the collision time. In the spirit of the "sudden approximation" of quantum mechanics, we assume that the nuclear wave functions are essentially unaltered by the collision; that is, just after the collision, the distribution of partons in phase space remains identical to what it was just before the collision. However, during the collision, some of the partons receive energy and momentum which, we assume, is just enough to put them on their mass shell. They then evolve as free particles, at least over a short period of time. Our purpose is to point out some properties of this system of free partons at the time it is formed. In particular, we shall find that the dominant contribution to the energy density comes from partons with transverse momenta growing like $p_T^2 \sim A^{1/3}$. This is in contrast for example with other parton models, such as the dual parton model [3], which use partons with limited transverse momenta. One may worry that the lack of partonic saturation in these models will lead to an overestimate of the soft parton contributions in collisions of large nuclei.

Recently, Hwa and Kajantie[4] presented an attempt to estimate the thermalization time in nuclear collisions. Eventhough we shall not adopt their view on the thermalization problem we would like to mention that our discussion in section 2 expands on their treatment of the kinematics of the parton distribution. We shall show in this section that the system of free streaming partons exhibits two different regimes as a function of time, the first regime being dominated by the longitudinal motion of the fast partons. Our arguments concerning the characterization of the partons which get freed during the collisions are developped in section 3. We shall present two fairly different scenarios which lead to the same A dependence of the partons transverse momenta. The last section of the paper contains numerical estimates of the formation time and the initial energy density.

2.THE FREE STREAMING REGIME.

We assume that, at least for a short period of time, the phase coherence between the partons can be ignored and we describe the system of partons by a classical distribution function $f(p,x)$, where x represents the space-time coordinates, and p the momentum. We shall assume that the distribution is uniform in the transverse direction, i.e. $f(p,x)=f(p_T,p_z,z,t)$. Longitudinal (p_z,z) and transverse (p_T) variables play different roles in the present discussion, and in order to express easily the consequences of longitudinal Lorentz boosts, it is convenient to transform the longitudinal variables. We define a "space-time rapidity" η and a proper time τ:

$$\eta=\frac{1}{2}\ln\frac{t+z}{t-z} \qquad \tau=\sqrt{t^2-z^2} \tag{2.1}$$

in terms of which one has $t=\tau\cosh\eta$, $z=\tau\sinh\eta$. Similarly, one introduces a "momentum rapidity" y:

$$y=\frac{1}{2}\ln\frac{p_0+p_z}{p_0-p_z}\qquad p_0=\sqrt{p_T^2+p_z^2}\tag{2.2}$$

such that $p_0=p_T\cosh y$, $p_z=p_T\sinh y$.

We shall ignore in this section the processes which may change locally the number of partons, and also the collisions which change the momenta. Then, the parton distribution obeys the following kinetic equation:

$$0=\frac{\partial f}{\partial t}+\frac{p_z}{p_0}\frac{\partial f}{\partial z}=\cosh(y-\eta)\frac{\partial f}{\partial\tau}+\sinh(y-\eta)\frac{\partial f}{\tau\partial\eta}\tag{2.3}$$

where we have assumed the partons to be massless. The general solution of this equation is easily seen to be of the form:

$$f(p_T,p_z,z,t)=f(p_T,w)\tag{2.4}$$

where the variables on the l.h.s. are defined in the center mass frame, while on the r.h.s. we have set [5]:

$$w=p_T\tau\sinh(y-\eta)=p_z t-p_0 z\tag{2.5}$$

The Lorentz invariant w may be given the following interpretation: w/τ is the longitudinal momentum measured in a frame moving with rapidity η with respect to the center of mass frame; on the other hand, $-w/p_T$ is the longitudinal coordinate z in a frame moving with rapidity y. Note that for a free parton emanating from the point $z=t=0$, $w=0$. The parton distribution function may then be calculated in terms of the momentum distribution $f_0(p_T,p_z)$ at $z=0$ and some initial time t_0:

$$f(p_T,w)=f_0\left(p_T,\frac{w}{t_0}\right)\qquad f_0(p_T,p_z)=f(p_T,p_z,z=0,t_0)\tag{2.6}$$

This way of defining the distribution function may look at first somewhat annoying since it introduces an arbitrary time scale t_0. However, we shall see soon that we can take the limit where t_0 goes to zero.

A particular choice for the initial distribution function, which is motivated by the y-independence of the momentum distribution,

see eq.(2.12) below, is the following:

$$f_0(p_T,p_z)=g(p_T)\,\theta(p_z^{max}-|p_z|)\tag{2.7}$$

where $g(p_T)$ is some transverse momentum distribution. The formula (2.7) assumes that for a given p_T, and in the plane $z=0$, the density of partons is independent of the longitudinal momentum p_z, up to a maximum value p_z^{max}. The equation (2.7), together with (2.6), implies:

$$f(p_T,p_z,z,t)=g(p_T)\,\theta(c-|w|)\tag{2.8}$$

where we have set $c=t_0p_z^{max}$. In fact, we shall argue below that c is a numerical constant of order 1.

Let us examine some of the implications of the ansatz (2.8). First, it is easy to show, using the formulae (2.5), that:

$$\theta(c-|w|)=\theta(\Delta-\eta+y)\,\theta(\Delta+\eta-y)\tag{2.9}$$

where we have set:

$$\Delta=\sinh^{-1}\left(\frac{c}{\tau p_T}\right)\tag{2.10}$$

In particular, the initial distribution at $t=0$ has the form:

$$f(p_T,p_z,z,t=0)=g(p_T)\,\theta\left(\frac{c}{p_T\cosh y}-|z|\right)\tag{2.11}$$

This initial distribution coincides with that advocated by Hwa and Kajantie. Its structure is easily understood. The partons with rapidity y are spread over a distance l/γ, where $l\approx 2c/p_T$ and $\gamma=\cosh y$ is the usual Lorentz contraction factor. The distance l may be taken to be of the order of the quantum spreading of the wave function of partons with longitudinal momentum $p_z\simeq 0$. Since p_T is the only energy scale in the problem, it is natural to take $l/2\sim\Delta z\sim 1/\Delta p_z\sim 1/p_T$, which implies $c\sim 1$. Now, the fastest partons occupy a longitudinal size $\sim 2c/p_z^{max}$; thus, by a time $\sim t_0=c/p_z^{max}$ no such partons remain in the plane $z=0$. This provides an interpretation of the time t_0 introduced in (2.6) and shows furthermore that, in the very high energy limit, t_0

is much smaller than any time in the problem.

Another important feature of the distribution (2.8) or (2.11) is illustrated by calculating the number of partons per unit rapidity at t=0:

$$\frac{dN}{d^2p_T\,dy} = \pi R^2 p_0 \int dz\; f(p_T, p_z, z, t=0) = 2\pi R^2 c\, g(p_T) \qquad (2.12)$$

where πR^2 is the transverse area of the colliding nuclei. This distribution is independent of y, as expected from the behaviour of the structure functions at small x. In fact, the relation between the phase-space distribution function and the structure function is given by:

$$xG_A(x, p_T^2) = \int_0^{p_T^2} \frac{dN}{dp_T'^2\,dy}\, dp_T'^2 = 2\pi R^2 c \int_0^{p_T^2} g(p_T')\, d^2p_T' \qquad (2.13)$$

where $G_A(x, p_T^2)$ is the parton (gluon) density of the nucleus, see section 3, and x is the fraction of the longitudinal momentum per nucleon carried by the parton.

Let us now evaluate the contributions to the energy density and the particle number density at $z=\eta=0$, of the partons with transverse momentum p_T. Using the fact that $dp_z = p_T \cosh y\; dy$, one easily finds:

$$\frac{dn(\tau)}{d^2p_T} = p_T g(p_T) \int_{-\Delta}^{\Delta} dy\, \cosh y = 2g(p_T) \sinh\Delta \qquad (2.14a)$$

$$\frac{d\epsilon(\tau)}{d^2p_T} = p_T^2\, g(p_T) \int_{-\Delta}^{\Delta} dy\, \cosh^2 y = \frac{1}{2} p_T^2 g(p_T)(2\Delta + \sinh 2\Delta) \qquad (2.14b)$$

These formulae lend themselves to a simple physical interpretation in the two limiting cases of short and long times.

At short time, the range of integration is large, $\Delta \sim \ln(l/\tau)$, and partons with all rapidities are found in the plane z=0. The formulae (2.14) give:

$$\frac{dn}{d^2p_T} \simeq p_T g(p_T) e^{\Delta} \qquad \frac{d\epsilon}{d^2p_T} \simeq \frac{1}{4} p_T^2 g(p_T) e^{2\Delta} \qquad (2.15)$$

Thus at very short time the dominant contribution to the energy density comes from the partons with a large rapidity. When $\tau \sim t_0$, the energy per particle is of the order of the maximum longitudinal momentum $p_z^{max} \sim p_T e^{\Delta}/2$: at this time, most of the energy is in the longitudinal motion.

For long time, Δ is small, $\Delta \sim l/2\tau$, and one gets:

$$\frac{dn}{d^2p_T} = 2p_T g(p_T)\Delta \qquad \frac{d\epsilon}{d^2p_T} = 2p_T^2 g(p_T)\Delta \qquad (2.16)$$

The quanta which remain at z=0 after a long time are those which carry little longitudinal momentum. In this regime, the density decreases as $1/\tau$, and the energy per particle is simply equal to the transverse momentum. Note that in both regimes, the particle density, eq.(2.14a), decreases as $1/\tau$ (see eq.(2.10)).

The crossing between the two regimes, i.e. the short and long time behaviours, takes place when the rapidity range of the partons which populate the region $z \sim 0$ is of order unity, i.e. $\Delta \sim 1$. In terms of time, this condition is equivalent to $\tau_0 p_T \sim 1$, that is $\tau_0 \sim \Delta z$. Thus the time τ_0 is, roughly speaking, the time it takes to the fast partons to leave the collision zone occupied by the slow ones. We shall show in the last section that τ_0 also turns out to be equal to the "formation time", that is to the time at which the partons get freed because of collisions.

3. APPROXIMATE CRITERIA FOR PARTONS SET FREE DURING THE COLLISION.

In this section, we are going to propose simple criteria for deciding which quanta are freed during a head-on relativistic heavy ion collision. These are the partons contained in the distribution (2.11) which suffer a hard enough interaction to allow them to convert their momentum into physical particles. We shall make estimates of the transverse energy released during the collision in the central unit of rapidity. We shall also estimate the energy density at the time the partons are freed from their initial wave function. As we shall see, the partons which dominate the energy density have p_T about 1 GeV so that the dynamics is marginally in the weak coupling regime, $\alpha \approx 1/3$. We might expect perturbation theory to serve as a reasonable guide although we would be hard pressed to certify our estimates reliable within a factor of 2

To begin, we imagine a head-on heavy ion collision, say in the center of mass system, as described above. We take our partons to be gluons since these are the quanta which dominate the semi-hard collisions with which we are concerned. Then the inclusive gluon "jet" cross section is:

$$x \frac{d\sigma}{dp_T^2 dx} = 2 \int_0^1 \frac{dx_1}{x_1} x_1 G_A(x_1, p_T^2)\; xG_A(x, p_T^2) \frac{d\hat{\sigma}}{dp_T^2} \Theta(x_1 xs - 4p_T^2) \quad (3.1)$$

where $G_A(x,p_T^2)$ is the usual gluon density of the nucleus. We shall ignore possible correlation effects in the nucleus which could make x greater than 1. Also, in our estimates we shall take $G_A(x,p_T^2) = A\, G(x,p_T^2)$ with A the number of nucleons and $G(x,p_T^2)$ the gluon number density of the nucleon, $\hat{\sigma}$ is the gluon-gluon cross section given by [6]:

$$\frac{d\hat{\sigma}}{d\hat{t}} = \left(\frac{\alpha C_A}{\pi}\right)^2 \frac{\pi^3}{2\hat{s}^2} \left(3 - \frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{\hat{u}\hat{s}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right) \quad (3.2)$$

where the "^" indicates variables directly related to gluon-gluon scattering ($\hat{s} = x_1 xs$, etc) , and $C_A = 3$. Because the $\frac{dx_1}{x_1}$ integration in (3.1) is logarithmic, the dominant contribution comes from small angle scattering, i.e. the $1/\hat{t}^2$ term in (3.2). Thus we approximate:

$$\frac{d\hat{\sigma}}{dp_T^2} \approx \left(\frac{\alpha C_A}{\pi}\right)^2 \frac{\pi^3}{2p_T^4} \quad (3.3)$$

Defining

$$\sigma(p_T^2, x) = p_T^2 \frac{d\sigma}{dp_T^2 dx} \quad (3.4)$$

one finds:

$$\sigma(p_T^2, x) = \left(\frac{\alpha C_A}{\pi}\right)^2 \frac{\pi^3}{p_T^2} xG_A(x, p_T^2) \int_{4p_T^2/xs}^1 \frac{dx_1}{x_1} x_1 G_A(x_1, p_T^2) \quad (3.5)$$

Now, at small x the Altarelli-Parisi equation is:

$$p_T^2 \frac{\partial}{\partial p_T^2} xG_A(x, p_T^2) = \frac{\alpha C_A}{\pi} \int_x^1 \frac{dx_1}{x_1} x_1 G_A(x_1, p_T^2) \quad (3.6)$$

so that

$$\sigma(p_T^2, x) = \left(\frac{\alpha C_A}{\pi}\right) \frac{\pi^3}{p_T^2} xG_A(x, p_T^2)\; p_T^2 \frac{\partial}{\partial p_T^2} x_0 G_A(x_0, p_T^2) \quad (3.7)$$

where $x_0 = 4p_T^2/xs$. In the central unit of rapidity, we suppose that xG is independent of x, see(2.12), so that:

$$\sigma(p_T^2, x) \approx \left(\frac{\alpha C_A}{\pi}\right) \frac{\pi^3}{2p_T^2} p_T^2 \frac{\partial}{\partial p_T^2} \left(xG_A(x, p_T^2)\right)^2 \quad (3.8)$$

Now what should we take for p_T^2 ? That is, which parton transverse momenta are going to dominate the cross section σ and hence the produced transverse energy distributions? If p_T^2 is taken very large, $\sigma(p_T^2,x) \sim 1/p_T^2$ and the resulting cross section is very small. Such high transverse momentum gluons are simply virtual fluctuations which are not freed during the collision. According to (3.8), we should choose p_T very small to increase σ. However, (3.8) ceases to be valid when p_T^2 is too small since gluon saturation effects become important. Perharps it is worth reminding the reader of the physical idea behind gluon saturation [7] and why such effects are especially important in large ions.

To that end, consider a large nucleus having longitudinal momentum p per nucleon. We suppose that p is much greater than the nucleon mass, m. Then, according to (2.11), the valence quarks in the nucleus, belonging to the individual nucleons, are within a longitudinal region of size proportional to 2Rm/p with R the radius of the nucleus. However, the gluons and sea quarks, having a particular value of x, cannot be confined to a longitudinal size smaller than $1/px$ so that for $x \lesssim 1/2Rm$ these quanta overlap in longitudinal coordinate space. The transverse size of the gluons is $|\Delta x_T| \sim 1/p_T$ so that if p_T is very large such small quanta will not overlap in the full three dimensional coordinate space. However, as one considers gluons with smaller p_T overlapping configurations become more common. When $xG_A(x,p_T^2) \gtrsim p_T^2 R^2$ different gluons must begin to occupy the same spatial region. Since $xG_A(x,p_T^2) \simeq AxG(x,p_T^2)$ one sees that this dense configuration is enhanced in large nuclei with strong interactions expected between the quanta when $p_T^2 \sim \alpha A/R^2$. (The factor of α, to be derived below, reflects the fact that the overlapping gluons interact (recombine) with strength α.) Thus, the actual transition from a low density to a high density gluonic system occurs at $xG_A(x,p_T^2) \gtrsim p_T^2 R^2/\alpha$

Now let's try to make these ideas a little more precise. The usual Altarelli-Parisi equation (3.6) is appropriate for a low density system and expresses the fact that as one looks to smaller transverse sizes, larger p_T's, the gluon density increases because a gluon may actually be composed of two gluons of smaller transverse size. This aspect of the A-P equation is traditionally called gluon splitting or gluon emmission. However, as the gluon number density becomes large, we may expect the opposite process, gluon recombination, to become important. Gluon recombination, where two gluons combine to form a single gluon lowers the number density. This is formally expressed as a higher twist modification of the usual A-P equations. At small values of x this modified equation takes the form:

$$p_T^2 \frac{\partial}{\partial p_T^2} xG_A(x,p_T^2) = \frac{\alpha C_A}{\pi} \int_x^1 \frac{dx_1}{x_1}\, x_1 G_A(x_1,p_T^2) - \left(\frac{\alpha C_A}{\pi}\right)^2 \frac{\pi^3}{2p_T^2} \int_x^1 \frac{dx_1}{x_1}\, x_1^2 G_A^{(2)}(x_1,p_T^2) \qquad (3.9)$$

where $G_A^{(2)}$ is the two gluon distribution of the nucleus. For a spherical nucleus of independent nucleons [8]

$$x^2 G_A^{(2)}(x,p_T^2) = \frac{\left(xG_A(x,p_T^2)\right)^2}{8/9\ \pi R^2} = \frac{A^2\left(xG(x,p_T^2)\right)^2}{8/9\ \pi R^2} \qquad (3.10)$$

with R the radius of the nucleus and G the gluon distribution of the nucleon.

Clearly, as p_T^2 becomes smaller, recombination becomes more important. We expect the gluon number density to stabilize when $xG_A(x,p_T^2) \sim p_T^2 R^2/\alpha$ which corresponds to

$$p_T^2 \frac{\partial}{\partial p_T^2} xG_A \simeq xG_A \qquad (3.11)$$

The largest p_T^2 at which (3.11) holds should determine the p_T^2 to be used in (3.8) to give the freed transverse energy in the collision. However, (3.11) is a little hard to use with (3.9) because the x-integration in that equation is not limited to very small x. It is easier instead to use the weaker equation

$$x\frac{\partial}{\partial x} p_T^2 \frac{\partial}{\partial p_T^2} xG_A(x,p_T^2) = 0 \qquad (3.12)$$

as a criterion for the saturation region. Eq.(3.12) follows from the expectation that $xG_A(x,p_T^2)$ become independent of x in the saturation region. Eqs.(3.12) and (3.9) give

$$\left(\frac{\alpha C_A}{\pi}\right) \frac{\pi^3}{2p_T^2}\, x^2 G_A^{(2)}(x,p_T^2) = xG_A(x,p_T^2) \qquad (3.13)$$

or

$$AxG(x,p_T^2) = \frac{16}{9\pi} \frac{p_T^2 R^2}{\alpha C_A} \qquad (3.14)$$

This gives p_T^2 as

$$p_T^2 = \frac{9\pi}{16}\,\alpha C_A\,\frac{AxG}{R^2} \tag{3.15}$$

Differentiation of (3.14) with respect to p_T^2 and using (3.8) yields:

$$\sigma(p_T^2,x) = \frac{16}{9}\,\pi R^2\,A\,xG \tag{3.16}$$

Eq. (3.16) gives the number of produced gluons per unit rapidity in a head-on collision of two spherical nuclei as:

$$\frac{dN}{dy} = \frac{\sigma}{\frac{8}{9}\pi R^2} = 2\,A\,xG \tag{3.17}$$

and the transverse energy as:

$$\frac{dE_T}{dy} = 2\,p_T\,AxG \tag{3.18}$$

The factor 8/9 in eq.(3.17) comes from an averaging over impact parameter. Eqs.(3.17) and (3.18) are rather remarkable in that our determination of p_T^2 by the equality of emission and recombination leads to a produced number of gluons per unit rapidity exactly twice the number in the wave function. This factor of 2 is the factor explicitely exhibited in (3.1). In a frame where the measured gluon has small rapidity we interpret this factor of two as corresponding to the gluon coming from either the two different colliding nuclei. In any case, it is remarkable that the recombination calculation carried out in Ref.8 and the calculation done here using (3.3) exactly compensate leaving the simple expressions (3.17) and (3.18).

In order to obtain from eqs.(3.17-18) number and energy densities, we assume that the newly freed partons obey the same free streaming kinematics as described in the previous section. Thus partons with transverse momentum p_t occupy a volume $V=2c\pi R^2/p_T$, see (2.11). We shall furthermore assume, and will justify in the next section, that all the partons within a rapidity range $\Delta\sim1$ contribute to the densities in the plane z=0. We obtain thus a number density:

$$n = \frac{2\Delta}{V}\frac{dN}{dy} = \frac{3}{2}\sqrt{\frac{\alpha C_A}{\pi}}\left(\frac{AxG}{R^2}\right)^{3/2} \tag{3.19}$$

and an energy density:

$$\epsilon \approx \frac{2\Delta}{V}\frac{dE_T}{dy} = \frac{9}{8}\,\alpha C_A\left(\frac{AxG}{R^2}\right)^2 \tag{3.20}$$

While the explicit numbers which may be extracted from (3.19-20) should only be taken as rough estimates (see section 4), we believe the α and A dependences exhibited in (3.15)-(3.20) are correct predictions of QCD, at least for large enough A.

Before we go and discuss the time at which this energy is freed, we would like to make a somewhat different estimate of which gluons are freed in a head-on ultra-relativistic heavy ion collision. The mechanism which we are about to discuss is subleading, by a power of α, in the amount of energy freed and we are not able to give a systematic account of this order α correction. Nevertheless, because of its intuitive appeal and because the resulting estimates are not too much smaller than those contained in (3.19-20), we should like to outline this simple mean free path argument. We emphasize that this is not an alternate version of our previous estimates but a discussion of a separate physical mechanism.

Consider a gluon, say in the right moving nucleus, just before the collision. We suppose that this gluon can have wide angle scatterings with those gluons left moving with respect to it and occupying the one unit of rapidity bordering it. (Beyond this one unit of rapidity,the scatterings are predominantly small angle and are exactly those scatterings covered by (3.8).) Let us focuss on the mean free path λ of our right moving gluon. We have:

$$\lambda = \frac{1}{n_g\hat{\sigma}} \approx \frac{1}{\frac{AxG}{\pi R^2\Delta z}\,\hat{\sigma}} \tag{3.21}$$

where $\hat{\sigma}$ is the cross section for wide angle gluon-gluon scattering and Δz is the longitudinal width occupied by the gluons through which our right-moving gluon passes. This gluon should be freed if it has one wide angle scattering as it passes through the distance Δz. Thus $\lambda \leq \Delta z$ is the criterion for converting the gluon from virtual to real. This requires

$$\frac{A x G}{\pi R^2}\,\hat{\sigma} \geqslant 1 \tag{3.22}$$

We estimate $\hat{\sigma}$ by taking

$$\hat{\sigma} = |\Delta\hat{t}|\,\frac{d\hat{\sigma}}{d\hat{t}}\bigg|_{\hat{s}=-2\hat{t}=-2\hat{u}} \tag{3.23}$$

with $|\Delta\hat{t}| \simeq \hat{s}/2$. Then

$$\hat{\sigma} = \frac{27\pi}{16\hat{s}}\,(\alpha C_A)^2 \tag{3.24}$$

Substituting (3.24) into (3.22) and taking $\hat{t} = -p_T^2$ one finds

$$p_T^2 \leqslant (\alpha C_A)^2\,\frac{27}{32}\,\frac{A x G}{R^2} \tag{3.25}$$

in contrast to our previous result (3.15).

The p_T^2 given by (3.25) is higher order in α compared to that given by (3.15), however in practice there is only a factor of two difference between the two estimates. Since both gluons involved in the scattering described by (3.24) remain in about the same unit of rapidity we arrive at a transverse energy per unit rapidity

$$\frac{dE_T}{dy} \simeq 2\,p_T\,A\,xG \tag{3.26}$$

and, using the same volume and rapidity range as before, an energy density

$$\epsilon \simeq \frac{2 p_T^2 A x G}{\pi R^2} = \frac{27}{16\pi}\,(\alpha C_A)^2 \left(\frac{A x G}{R^2}\right)^2 \tag{3.27}$$

about a factor two less than that obtained in (3.20).

4. DISCUSSION.

We turn now to the question of the formation time τ_0, i.e. the time at which the partons are freed. In fact, it is easy to see that this time is of order $1/p_T$ in either of our dynamical estimates given in the previous section. This follows from the fact that a gluon which gets freed in the central unit of rapidity must have undergone a scattering of momentum transfer p_T with partons in the neighbouring rapidity slices. Thus τ_0 is the time during which the partons in the central rapidity unit overlap with those partons with which they may interact. Referring to the discussion at the end of section 2 we see that τ_0 is also the time bordering between the long and short time behaviours in the free streaming regime. Viewing the collision in a slightly more general frame, suppose the unit of rapidity which we are considering is centered about $y = y_0$. Then using (2.5) and (2.8) we see that at time t the partons in this rapidity unit are located within

$$z_{\pm} = t\,\tanh y_0 \pm \frac{t}{\cosh^2 y_0} \pm \frac{1}{p_T \cosh y_0} \tag{4.1}$$

where the first $\pm$ in (4.1) comes from the unit rapidity spread and the second $\pm$ comes from the original spread, due to the uncertainty relation, at $t=0$. At $t \sim \cosh y_0 / p_T$ we see that

$$z_{\pm} = \frac{1}{p_T}\,\sinh y_0 \pm \frac{1}{p_T \cosh y_0} \tag{4.2}$$

with the spread in Δz due to the differences in rapidity being comparable to the original uncertainty spread. Partons with rapidity greater than $y_0 + 1/2$ or less than $y_0 - 1/2$ have separated from those centered about y_0 at the time $t \sim \cosh y_0 / p_T$. Thus our physical picture holds together. By the time the collisions necessary to free the partons occupying a unit of rapidity have occured, these partons have physically separated, in the longitudinal direction, from the partons *corrsponding to different rapidity intervals.*

In order to get numerical estimates, let's take $A^{1/3}=6$, $R=1.2A^{1/3}fm$, $xG=3$ and $\alpha=1/3$, i.e. $\alpha C_A=1$ (the value $xG=3$ is reasonable, even traditional, but at this time it is not a well determined quantity, experimentally). Then (3.15) gives $p_T\approx 0.94GeV$, i.e. $\tau_0\sim 0.2fm/c$, and one finds:

$$\frac{dN}{dy}\approx 1300 \tag{4.3a}$$

$$\frac{dE_T}{dy}\bigg|_T\approx 1.2TeV \tag{4.3b}$$

$$n\approx 37/fm^3 \tag{4.3c}$$

$$\epsilon\approx 35GeV/fm^3 \tag{4.3d}$$

Our second estimate gives $p_T\approx 0.65GeV$, eq.(3.25), and hence $\tau_0\sim 0.3fm$, $n\approx 25/fm^3$, and $\epsilon\approx 17GeV/fm^3$. These large numbers reflect the large value of the optimum p_T in large nuclei. They correspond for $A=1$, that is for proton-proton or proton-nucleus collisions, to $p_T\sim 380MeV$ and $\epsilon\sim 1GeV/fm^3$ which look like reasonable values. The A dependence that we have found for the energy density is quite similar to that obtained in other models [9], but for quite different physical reasons. In our approach a non trivial A dependence is contained in the parton transverse momenta.

Finally, we have said nothing about the thermalization of the partons set free during a heavy ion collision. In fact we have little to say on this subject. Nevertheless, there is one amusing calculation which can be done at this stage to perhaps get an indication of how far from equilibrium our initial distribution is. For a free boson gas in equilibrium one has

$$\frac{\epsilon^{3/4}}{n}=\frac{\pi^{7/2}}{2\,\zeta(3)\;30^{3/4}}\approx 1.7$$

while from (3.19) one has $\epsilon^{3/4}/n\approx 1.3$ and from (3.24), (3.25) and (3.26) one has $\epsilon^{3/4}/n\approx 1.1$. Hwa and Kajantie [4] use a refined version of this type of comparison to argue about the thermalization time.

However, eventhough the moments of the distribution may not be too far from those of an equilibrium distribution, as the numbers we just gave seem to indicate, it is clear that the thermalization time must depend on a collision rate and can't be determined from kinematical consideration alone. In fact, we would like to argue differently and assume that the distribution function of the newly formed partons is much like the free streaming distribution (2.8) in the long time regime where the space-time rapidity η and the momentum space rapidity y are strongly correlated, i.e. $\eta\simeq y$. The way such a distribution approaches equilibrium has been studied by Baym [9], using a simple collision time approximation. He finds that after a time $\tau\sim 2\theta$, with θ the collision time, the energy density is within 20% of its local equilibrium value. As an order of magnitude, we may take our mean free path estimate, eq.(3.21), which gives $\theta\sim\lambda\sim 1/p_T$, with p_T given by (3.25). (Note that θ becomes infinite in the limit of vanishing coupling strength, as it should.) With the number given above, one thus finds $\theta\sim.3fm$, and a thermalization time $\tau_{th}\gtrsim 2\theta\sim 0.6fm$. Let us emphasize again that this is meant to serve only as a rough estimate and not as a substitute to a decent treatment of the thermalization problem. In any case, energy densities such as those given above are sufficiently high that heavy ion collisions involve new and interesting aspects of QCD independently or not a true equilibrium is reached.

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