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AGS-Booster Orbit and Resonance Correction

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Abstract

A large tune-spread due to space charge and strong random eddy current sextupoles exists at injection in the AGS-Booster. As a result, particles in the beam may cross several imperfection resonances that can be caused by random magnet errors. The correction scheme proposed deals with four half-integer stopband resonances, four third-order sextupole induced resonances and both the sum and difference linear coupling resonances. All of these resonances can have a chance to be swept through by the beam during injection, bunching and the early stage of acceleration. A system of correctors involving skew quadrupoles, trim quadrupoles and sextupoles is described with the capability of correcting all the major resonances involved, simultaneously. Also a closed orbit correction scheme is described which requires, to be effective, a cascade of local 3-magnet bumps. Several magnet imperfections and survey misalignment errors are investigated.

Orbit Correction

An accelerator lattice cannot be expected to be perfect and as an immediate consequence the same will be true for the closed orbit. Since more or less reliable assumptions can be made about realistic lattice errors, it is important to see how they translate into expected closed orbit distortions, and if the latter exceed acceptable levels to see how to correct them.

Among many possible sources of closed orbit distortions, we have selected four major types of lattice errors. They are the error in the integrated dipole field strength $\Delta(B)/B$, the axial tilt of the dipole $\Delta\theta$, and the lateral displacements of the quadrupole along the two transverse directions.

The RMS values of the lattice errors we have used are the following ones:

$$\Delta(B)/B = 0.3 \times 10^{-3}, \quad \Delta\theta = 0.3 \times 10^{-3} \text{ radians,}$$

$$\text{Lateral quad displacements } \Delta_Q X = \Delta_Q Y = 0.3 \times 10^{-3} \text{ m.}$$

A 2.5σ cut was imposed on all distributions of random errors. Sextupoles were modeled as thin lenses, but in all other aspects they were assumed perfect. Higher order multipole errors have not been included yet. Orbit correctors were assumed to be thin lenses. Both beam position monitors and correctors were assumed ideal, i.e. perfectly aligned with the axis going through an ideally placed quadrupole and monitors were assumed to have a perfect sensitivity.

The tracking/analysis code PATRIS was used to handle the simulation and analysis of closed orbit distortions and furthermore to correct them.

For a realistic closed orbit modeling, it is desirable to have a better scheme than that of a simple representation of lattice error effects by kicks. Moreover, one would like to see what happens with closed orbit distortions once a certain well-defined sort of correction is implemented. Both goals have been attained in PATRIS, which on the one hand has the capabilities of simulating the lattice errors by incorporating them realistically into its 7×7 transfer matrix, and which on the other hand can correct the orbit by engaging the Fermilab correcting scheme, based on the so-called three bump method. The details of this scheme can be found elsewhere^{1,2,3} and will not be repeated here. One thing we will emphasize here are placements of correctors. They are assumed to be BPM's at the same time and they have been placed beside

focusing quadrupoles where the relevant beta function is large. Another thing we would like to mention is that the scheme evaluates the strengths of the corrective kicks on the basis of observed (or simulated) on-momentum closed orbit displacements at three successive monitors (with nonlinearities included) and the transfer matrix in the absence of nonlinearities. As a result of the correction evaluated in this manner the scheme supplies kick strengths which (in the absence of nonlinearities) reduce orbit distortions to zero at all monitors, a fact fully confirmed by PATRIS, and which in the presence of nonlinearities reduce the orbit distortions by an order of magnitude or better. Finally, we would like to emphasize that the scheme avoids any need to invert matrices and is therefore fast and economical.

We performed calculations with 11 different sequences of random errors. Initially, we attempted to handle the problem at 10^{-3} level of RMS lattice error values, but it was too much and we reduced values to the presently accepted 0.3×10^{-3} . In all of these 11 cases, the corrected closed orbit, in the presence of chromaticity correcting sextupoles, fell within one millimeter. To examine the sensitivity of our result to a change of the input RMS lattice error values, we increased the latter for the case of the worst random error sequence that we had encountered. While this comparatively rather bad distribution yielded a corrected closed orbit that fell within one millimeter for the 0.3×10^{-3} errors, it reached 1.36 mm at one location for the 0.4×10^{-3} errors and it barely passed 2 millimeters at four (out of 48) monitors for the 0.5×10^{-3} errors.

These facts have given us the necessary confidence to claim that if a 0.3×10^{-3} level of RMS values is attained in a real machine, the Fermilab correcting scheme alone will be able to successfully handle the closed orbit distortions arising from the four types of errors we have discussed. The results of the run over the worst encountered distribution of random errors are shown in Figures #1 and #2. These figures represent the uncorrected and corrected closed orbit for the two planes. The reader will undoubtedly notice that the uncorrected closed orbit behaves in the horizontal plane much worse than in the vertical one. This is obviously a consequence of the particular set of random errors. We have inspected the results of all of the eleven runs and have found the closed orbit distortions bigger in the horizontal plane in four cases. It was just the opposite in another group of four cases, whereas in the remaining three runs the distortions were about equal in the two planes.

As far as the four types of magnet errors that we have discussed are concerned, our conclusion is clear: one should strive to achieve the 0.3×10^{-3} level of RMS values and this is very likely sufficient for the Fermilab correcting scheme to work well. Convinced by our analysis, we definitely support its implementation on the Booster. The implementation will require installing a beam position monitor, followed by a dipole corrector, beside each quadrupole. The maximum integrated kick strength for such a bump corrector, to be able to correct the orbit at the top magnetic rigidity of 18 T.m, is predicted to be about 55 Gauss. meters. This estimation may have to be somewhat changed in the future, once other relevant factors are brought in for analysis. But the essentials of the correcting scheme should remain unchanged.

Resonance Correction

To reduce beam loss, we will include a resonance correction system. The large space charge tune spread [4] at injection can cause particles in the beam to cross imperfection resonances. Furthermore, because the AGS-Booster is fast cycling, the dipoles will have large eddy current sextupoles⁵ giving rise to third integer resonances that must be corrected.

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The AGS-Booster consist of six superperiods and each superperiod contains eight correction trim coil assemblies of which four of these assemblies will contain the skew quadrupole correctors. Furthermore, each of the Boosters 48 quadrupoles contain trim coils for half integer stop-bandwidth correction and the 48 sextupoles contain trim coils for third integer resonance correction. Using these correctors we want to generate resonances that cancels those resonances excited by the errors.

The resonances that are important can be deduced from the tune diagram for the Booster shown in Fig. 3. The presence of the large tune spread is due to space charge effects.⁴ We propose a scheme for correcting imperfection resonances up to third order (excluding skew sextupole resonances). The sextupoles are expected to be important due to the eddy current effects in the dipole magnets [5]. This leads to three classes of resonances:

- (1) Quadrupole $2\nu_x = 9$, $2\nu_y = 9$
- (2) Skew Quadrupole $\nu_x + \nu_y = 9$, $\nu_x - \nu_y = 0$
- (3) Sextupole $3\nu_x = 14$, $\nu_x + 2\nu_y = 14$, $3\nu_x = 13$, $\nu_x + 2\nu_y = 13$.

We need to relate the corrector strength with the resonance strength [1,6,7]. To do this, we define the following functions related to the phase advance:

$$\mu_x(s) = \int_0^s \frac{dt}{\beta_x(t)} - \frac{2\pi}{C} \nu_x s, \quad \mu_y(s) = \int_0^s \frac{dt}{\beta_y(t)} - \frac{2\pi}{C} \nu_y s$$

where $\beta_x(s)$ and $\beta_y(s)$ are the betatron functions, ν_x and ν_y are the tunes and C is the circumference.

Due to the periodicity of the β and μ functions, the sums to find the resonance strengths can be simplified. First, we define

$$t_q = t_{(p-1)q}$$

then

$$t_{pq} = t_q + (p-1)C/6$$

which represents the position of the q 'th corrector in the p 'th superperiod.

We introduce the coefficients $f_p^{(n)}$ in which

$$K_{pq} = f_p^{(9)} K_q^{(9)}, \quad M_{pq} = f_p^{(0)} M_q^{(0)} + f_p^{(9)} M_q^{(9)}, \quad S_{pq} = f_p^{(14)} S_q^{(14)} + f_p^{(13)} S_q^{(13)}$$

are the strengths of the correctors (i.e. quadrupoles, skew quadrupoles and sextupoles respectively) so that the sum over the superperiods, p , can be separated. Additionally, by choosing $f_p^{(n)} = \cos[n\pi(p-1)/3]$, the different harmonic numbers, n , in each of the classes are orthogonal.

We can now write the resonance strengths in terms of "normalized" corrector strengths K_q , M_q and S_q as

- (i) $2\nu_x = 9$

$$A_9 = \frac{i}{4\pi} \eta(f_p^{(9)}, 9) \sum_{q=1}^8 K_q^{(9)} \beta_x(t_q) e^{i[2\mu_x(t_q) + 18\pi q/C]},$$

- (ii) $2\nu_y = 9$

$$B_9 = \frac{i}{4\pi} \eta(f_p^{(9)}, 9) \sum_{q=1}^8 K_q^{(9)} \beta_y(t_q) e^{i[2\mu_y(t_q) + 18\pi q/C]},$$

- (iii) $\nu_x - \nu_y = 0$

$$C_0 = \frac{i}{4\pi} \eta(f_p^{(0)}, 0) \sum_{q=1}^4 M_q^{(0)} \beta_x^{1/2}(t_q) \beta_y^{1/2}(t_q) e^{i[\mu_x(t_q) - \mu_y(t_q)]},$$

- (iv) $\nu_x + \nu_y = 9$

$$D_9 = \frac{i}{4\pi} \eta(f_p^{(9)}, 9) \sum_{q=1}^4 M_q^{(9)} \beta_x^{1/2}(t_q) \beta_y^{1/2}(t_q) e^{i[\mu_x(t_q) + \mu_y(t_q) + 18\pi q/C]},$$

- (v) $3\nu_x = m$

$$E_m = \frac{i}{48\pi} \eta(f_p^{(m)}, m) \sum_{q=1}^8 S_q^{(m)} \beta_x^{3/2}(t_q) e^{i[3\mu_x(t_q) + 2\pi m t_q/C]},$$

- (vi) $\nu_x + 2\nu_y = m$

$$F_m = \frac{i}{16\pi} \eta(f_p^{(m)}, m) \sum_{q=1}^8 S_q^{(m)} \beta_x^{1/2}(t_q) \beta_y(t_q) e^{i[\mu_x(t_q) + 2\mu_y(t_q) + 2\pi m t_q/C]},$$

where m is either 13 or 14 and

$$\eta(f_p, n) = \sum_{p=1}^6 f_p e^{i\pi n(p-1)/3}.$$

Since the resonance strengths A , B , C , D , E and F are complex quantities, then we have 4 conditions for the quadrupole and skew quadrupole resonances and 8 conditions for the sextupole resonances. The unknown variables are $K_q^{(9)}$ for $q = 1, 2, \dots, 8$, $M_q^{(0)}$ and $M_q^{(9)}$ for $q = 1, 2, 3, 4$ and $S_q^{(13)}$ and $S_q^{(14)}$ for $q = 1, 2, \dots, 8$. This is twice the number of unknowns for the available conditions. Thus, we impose the additional constraints:

$$K_1^{(n)} = K_4^{(n)}, \quad M_1^{(n)} = M_2^{(n)} \quad \text{and} \quad S_1^{(n)} = S_4^{(n)}.$$

We can now set up systems of equations to solve for the corrector strengths given the resonance strengths.

Preliminary simulations show that the corrector strengths required for the quadrupole and skew quadrupole correctors are less than 1% of the main quadrupole strengths and the strength of the sextupole correctors are less than 2% of the main sextupole strengths. Additionally, this system of correctors are orthogonal which means the tunes and chromaticities are not affected. Furthermore, simplified connections with a reduced number of power supplies is feasible.

References

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Fig. 1. Horizontal incorrected (+) and corrected (Δ) closed orbis for realistic error distribution.

Fig. 3. The tune diagram with the expected tune spread due to space charge.

Fig. 2. Vertical incorrected (+) and corrected (Δ) closed orbits for realistic error distribution.