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BOUNDS FOR INTEGRAL RESPONSES OBTAINED FROM NEUTRON ACTIVATION MEASUREMENTS

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Theory

In the activation method for the determination of neutron flux spectra, certain integral responses which are obtained from a set of detectors are measured. If $\sigma_i(E)$ is the cross section of the i-th detector at energy E and $\phi(E)$ is the neutron flux, then the activities α_i are given by the integrals

$$\alpha_i = \int_0^\infty \sigma_i(E)\phi(E)dE \quad , \qquad i = 1, 2, \dots n. \tag{1}$$

The equations (1) represent a system of linear transforms which may be used to obtain information about the unknown spectrum $\phi(E)$. Considered as a system of linear equations (1) is highly underdetermined since there are

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only n equations for infinitely many values of $\phi(E)$, and thus there are infinitely many solutions of (1). It is even impossible to obtain valid bounds for the differential flux values $\phi(E)$.

Instead, we may consider the information given by a linear combination I_c of the α_i's; i.e.,

$$I_c = \sum_{i=1}^{n} c_i \alpha_i \tag{2}$$

with suitable coefficients c_i. The combinations (2) also represent integral responses

$$I_c = \int_0^\infty W_c(E)\phi(E)dE \tag{3}$$

with

$$W_c(E) = \sum_{i=1}^{n} c_i \sigma_i(E) \quad . \tag{4}$$

By choosing the coefficients c_i carefully, we may be able to obtain good approximations to those integral responses which reveal significant features of the neutron spectrum; for instance, the total flux energy in a certain range

$$I(E_1,E_2) = \int_{E_1}^{E_2} \phi(E)dE \tag{5}$$

or the total damage computed according to a known damage response function $D(E)$

$$I_D = \int_0^\infty D(E)\phi(E)dE \quad . \tag{6}$$

In general, let $T(E)$ be the desired test response and the corresponding integral I_T

$$I_T = \int_0^\infty T(E)\phi(E)dE \tag{7}$$

may be called a test integral. We are able to obtain bounds for these test integrals from activity measurements if we choose a set of coefficients c_i in (4) so that $W_c(E)$ is either consistently larger or consistently smaller that the test function $T(E)$. This leads to a linear programming problem which has already been described in 1965 by S. K. Mehta [1]. A more extensive mathematical discussion can be found in the book by B. W. Rust and W. R. Burrus [2].

This problem is theoretically infinitely dimensional, but since the cross section data are given only for a finite number of energy points, a finite dimensional version is sufficient. Furthermore, since the cross sections and activity measurements are of limited accuracy, upper and lower bounds $\overline{\sigma_i}(E)$, $\underline{\sigma_i}(E)$ and $\overline{\alpha_i}$, $\underline{\alpha_i}$, respectively, are used for the computations. Specifically, let T_ν be the value of the test response T(E) in the interval $(E_{\nu-1},E_\nu)$, c_i^+ the values of those coefficients which are positive and c_i^- the ones which are negative; otherwise, these values are set to zero. Using these definitions, the conditions for the coefficient set c_i of the upper bound $W_c(E) \geq T(E)$ is formalized in the set of inequalities

$$\int_{E_{\nu-1}}^{E_\nu} \sum_{i=1}^{n} [c_i^+\underline{\sigma_i}(E) + c_i^-\overline{\sigma_i}(E)]\hat{\phi}(E)dE \Big/ \int_{E_{\nu-1}}^{E_\nu} \hat{\phi}(E)dE \geq T_\nu, \; \nu = 1, \ldots k. \quad (8)$$

The values $\hat{\phi}(E)$ are a priori guesses of the true spectrum $\phi(E)$ and are used here to counteract effects of the finite subdivision of the energy range. Any set of coefficients c_i which satisfies (8) leads to an upper bound $\overline{I_T}$ obtained from the coefficients according to the formula (2). With only upper and lower bounds for the activities α_i, we have

$$\overline{I_T} = \sum_{i=1}^{n} (c_i^+\overline{\alpha_i} + c_i^-\underline{\alpha_i}) \quad . \quad (9)$$

The linear programming procedure determines the set of coefficients c_i^+, c_i^- which minimizes $\overline{I_T}$ in (9) giving the smallest upper bound for the test integral. An analogous procedure leads to the largest lower bound for the test integral.

Results

Meaningful results could be obtained so far only in the high energy range (above 1 MeV). The main reason for this is that all detectors have sizable integral responses $\sigma_i(E)dE$ in the high energy range masking contributions from low energy neutrons. Detectors which have responses in the low energy range (primarily fission type activities) provide very little resolution in this part of the spectrum, which, in addition, makes it impossible to decide what particular part of the spectrum generated the observed response. Fortunately, the high energy part of the spectrum is also the most interesting one since it accounts for most of the structural damage. We have investigated in detail the following test integrals:

1. The total number of displacements [3]

$$I = \int D(E)\phi(E)dE$$

where the test response is the damage cross sections given by

$$D(E) = \int \sigma_s(E)K(E,T)\nu(T)dT \quad .$$

2. The integral spectrum

$$I^{\infty}(E) = \int_{E}^{\infty} \phi(E)dE \quad , \qquad 0.1 \leq E \leq 10 \text{ MeV}.$$

3. Ideal detectors

$$I(E_1,E_2) = \int_{E_1}^{E_2} \phi(E)dE \quad , \qquad \text{for } E_1 = 10,\ E_2 = 20 \text{ MeV and } E_1 = 3,\ E_2 = 10 \text{ MeV}.$$

The approximations are based on a set of 13 reactions by R. Dierckx [4] and 14 reactions by W. N. McElroy [5]. No significant differences in the goodness of approximation between these two sets were found. In each case not more than 10 and frequently fewer cross sections were actually used to obtain the bounds. This point will be investigated further in order to make recommendations for optimal detector sets.

In all cases the investigated spectrum must have at least 90% of its total energy in the 1-20 MeV range. If this condition was satisfied, the upper bound was computed and came within 10-30% of the true value of the test integral. If 30% seems to be too large a deviation, then it should be remembered that this is an absolutely safe bound taking into account a ±5% error in the cross section measurements and also possible 3-10% errors in the activities. Furthermore, no a priori assumptions need to be made about the flux spectrum.*

Lower bounds were also computed and gave approximations of similar accuracy. These bounds may be used as an indication of how close the upper bound is likely to be to the real value of the test integral. If upper and lower bounds differ by more than a factor of 2, then these bounds are probably useless. Of course, the upper bound could also be compared with an estimation of the test integral obtained by one of the conventional unfolding methods.

One may safely assume that damage cross sections other than Linhard's model would lead to essentially the same results (see, for instance, Reference [6]). As for the ideal detectors, the energy ranges given above were the only ones for which meaningful approximations could be obtained. This seems to indicate that the foil detector method has practically no resolution below 3 MeV. A look at the graph of the window functions $W_c(E)$ is instructive (see Figures 1 and 2). $W_c(E)$ gives a fairly decent approximation of the test response down to about 0.1 MeV where it reaches a minimum. This point of the spectrum has the poorest coverage by any set of detectors. Below this point, $W_c(E)$ assumes a typical fission response with possibly a few resonance peaks. The low energy portion of the window function may be quite large in value but has little influence on the bound unless the integral flux $\phi(E)dE$ is very large at low energies. If this is the case, the integral bounds become useless.

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*This accuracy is also consistent with the uncertainties of neutron induced material changes (see W. N. McElroy and R. E. Dahl [5]).

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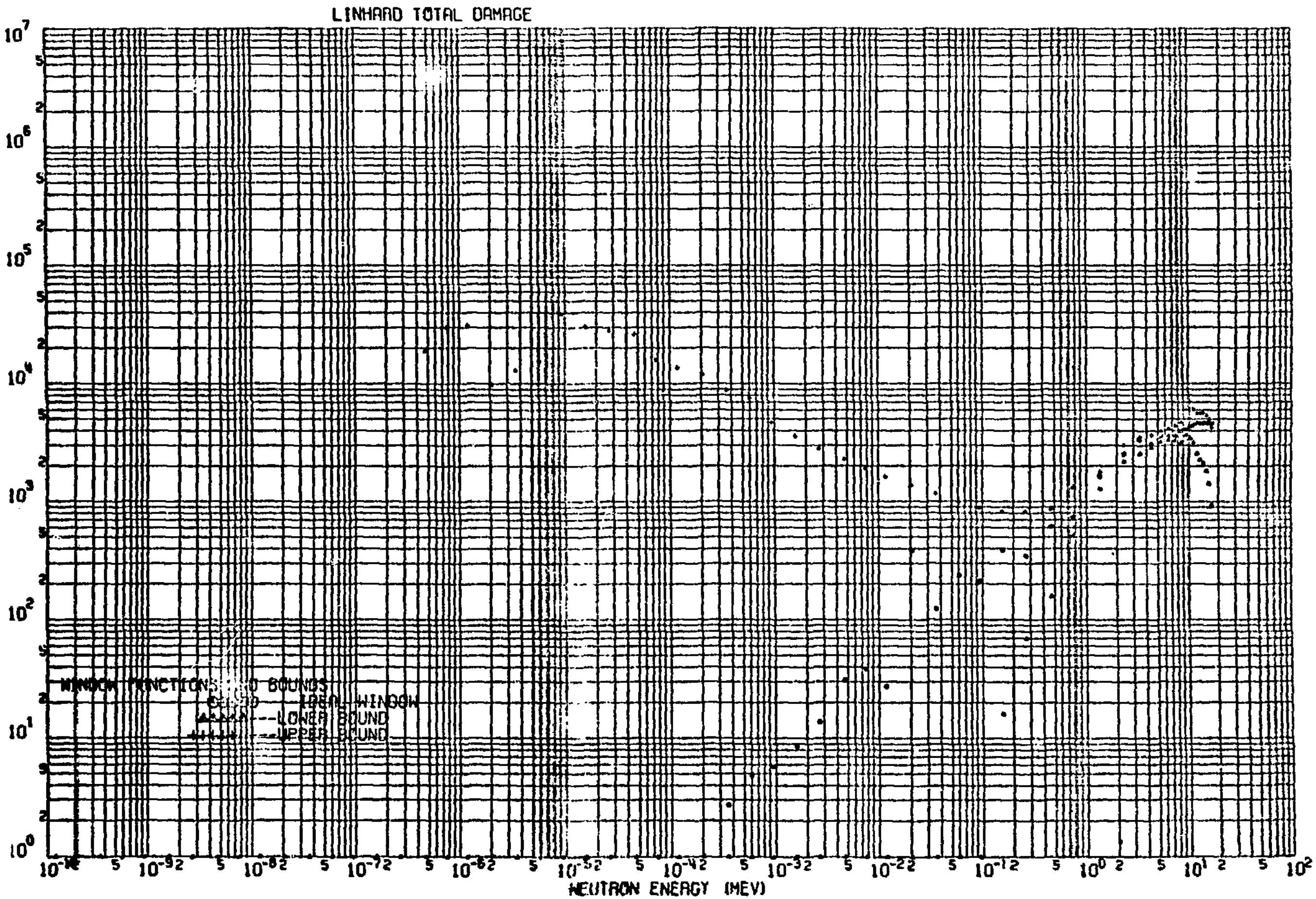

Fig. 1. Upper and lower approximations to a damage response function computed according to Linhard's model.

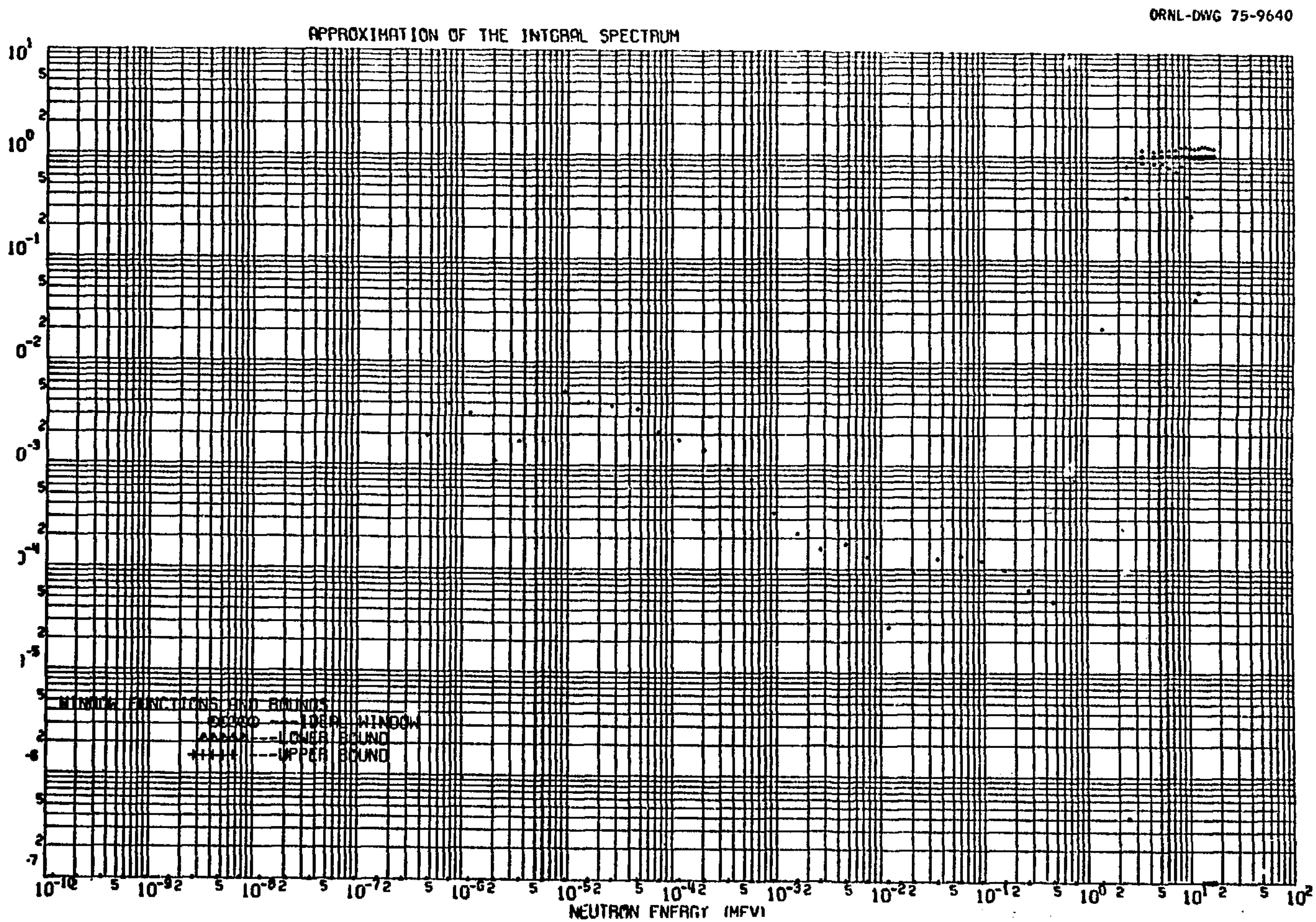

Fig. 2. Upper and lower approximations to the integral spectrum from 3 - 2o MEV.

References

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