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# **Bispectral Speckle Imaging Algorithm Performance on Specific Simulated Scenarios**

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# Bispectral Speckle Imaging Algorithm Performance on Specific Simulated Scenarios

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## Abstract

The purpose of this report is to describe the anticipated performance of LLNL’s bispectral speckle imaging algorithm on specific imaging scenarios and further evaluate the regime over which bispectral speckle imaging can be used to compensate for atmospheric turbulence. This includes investigating a number of relevant  $C_n^2$  cases and multiple wavelengths. The anticipated performance described here will be evaluated based upon simulated imagery. As with any simulation, it must be realized that the full truth of the matter will be determined when real data is analyzed. This report assumes some basic familiarity with bispectral speckle imaging<sup>1,2,3,4</sup>.

## 1.0 Introduction: Description of Scenarios

Certain imaging systems may be required to provide imaging capabilities at ranges up to 10 km over a horizontal ground path in the presence of strong, distributed atmospheric turbulence ( $C_n^2 = 2.0 - 5.0E-13 \text{ m}^{-2/3}$ ). Such operating scenarios are characterized by extended (non-point source) scenes and targets (tanks or trucks), and severe anisoplanatism and scintillation resulting from turbulence. The aperture for the system under consideration will be a maximum of 6” in diameter. Imaging resolution is specified for the mid-wave infrared band (MWIR, 3 – 5  $\mu\text{m}$ ), but this report will consider other spectral bands as well (visible/near infrared (VNIR, 0.4 – 1.0  $\mu\text{m}$ ), short-wave infrared (SWIR, 1.0 – 3.0  $\mu\text{m}$ ), and long-wave infrared (LWIR, 8 – 12  $\mu\text{m}$ )).

Three scenarios of interest are defined in Table 1. The top three rows in the table provide atmospheric and geometric parameter guidance for the simulations to be performed in this report.

	Scenario 1	Scenario 2	Scenario 3
Ground-level $C_n^2 \text{ (m}^{-2/3}\text{)}$	<2.0E-13	<2.0E-13	<2.0E-13
Range to target (km)	2	10	10
Slant path	At least 80% of path <10 m AGL	At least 50% of path <10 m AGL	Horizontal path
Desired resolution (MWIR)	<140 $\mu\text{rad}$	<140 $\mu\text{rad}$	<100 $\mu\text{rad}$

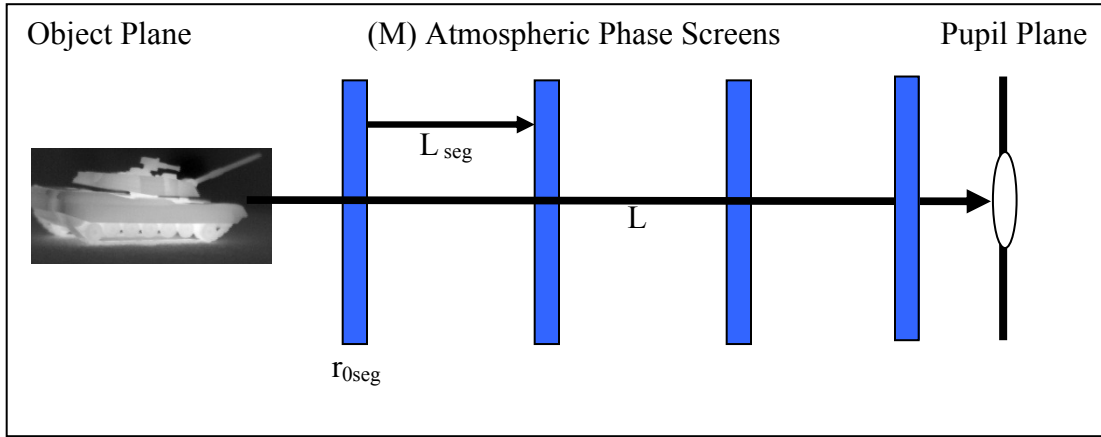
**Table 1: Typical Scenario Requirements**

## 2.0 Description of Anisoplanatic Incoherent Imaging Simulation method

### 2.1 The atmosphere

We model the distributed turbulence by splitting the imaging path into atmospheric layers of a certain length. At the center of each layer we insert a properly sampled Kolmogorov phase screen with a given  $r_0$  value. Figure 1 shows the geometry. Because  $r_0$  scales with wavelength to

the 6/5 power, the phase screen generator always generates the phase screen at a reference wavelength of 0.5 microns; which is then scaled to the imaging wavelength (e.g. 4 microns.)



**Figure 1: Illustration of the placement of atmospheric phase screens in the simulations**

For a horizontal path case, the atmosphere is assumed constant for the entire path, meaning that the value of  $C_n^2$  directly determines the value of the Fried parameter, or atmospheric coherence length,  $r_0$ .  $r_0$  can then be used to calculate the maximum length of the propagation until ray crossings occur. Once this maximum length is known, we then know how many atmospheric screens are needed and their corresponding  $r_0$ 's to simulate the distributed atmosphere.

For example, the spherical wave formulation for  $r_0$  is given by (1):

$$r_0 = 3.01 \left[ (2\pi / \lambda)^2 C_n^2 L \right]^{-3/5}, \quad (1)$$

where  $L$  is the range to target and  $\lambda$  is the wavelength. If a path with coherence length  $r_0$  is split into  $M$  segments, then the coherence length of each segment,  $r_{0seg}$ , is:

$$r_{0seg} = M^{3/5} r_0, \quad (2)$$

Now in order to avoid ray crossings, we need:

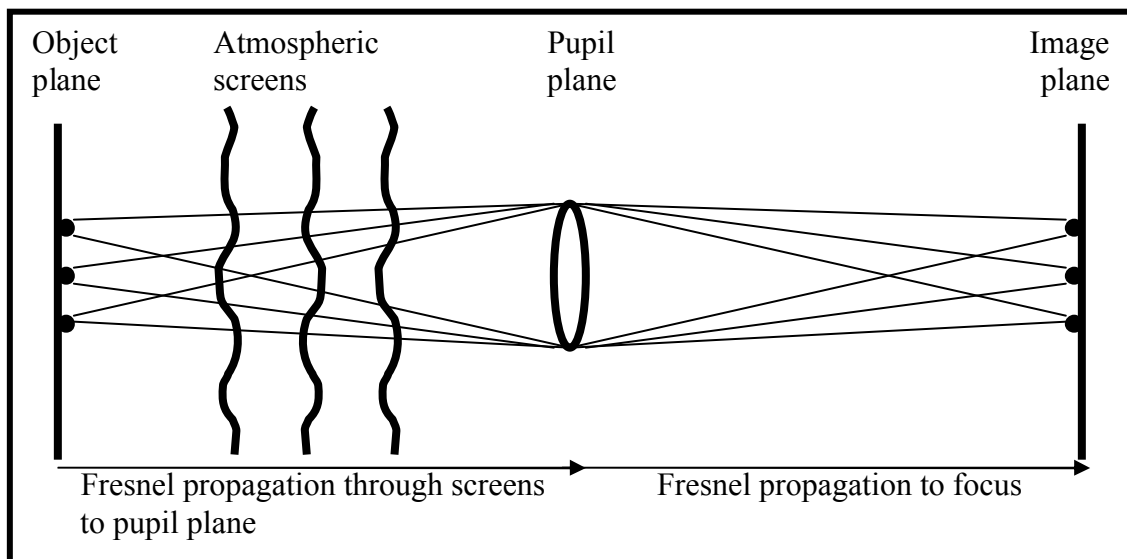
$$\frac{r_{0seg}}{L_{seg}} \geq \frac{\lambda}{r_{0seg}}, \quad (3)$$

where  $L_{seg}$  is the path length of the segment. The minimum number of phase screens is then given by  $M = L/L_{seg}$ , or,

$$M \geq (L\lambda / r_0^2)^{5/11}. \quad (4)$$

## 2.2 Propagation and image formation

The diagram in Figure 2 illustrates the full path of a light ray from the object to the image plane.



**Figure 2: Flow of light rays from object plane to image plane.**

### 2.2.1 Point source case

For the simple case where the object is a point source or a few point sources, using Fresnel propagation, we can propagate each point source from its origin through each phase screen to the pupil plane. Once the complex field reaches the pupil plane, we apply the aperture function, and Fresnel propagate through free space to the image plane position. The Fresnel propagation is realized in frequency space through the following relationship between the Fourier transform of the complex wavefront at a distance  $Z$ ,  $W_Z$ , and the Fourier transform of the starting complex wavefront,  $W_0$ :

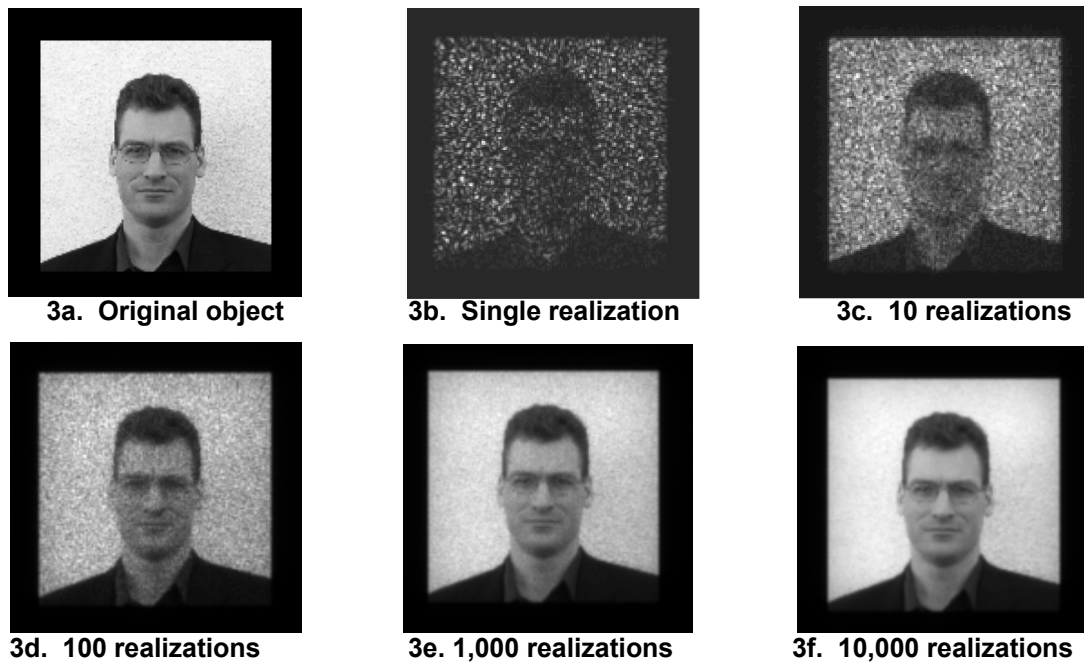
$$W_Z(f_x, f_y) = W_0(f_x, f_y) \exp[-j\pi\lambda Z(f_x^2 + f_y^2)]. \quad (5)$$

The intensity images of each point source (or point spread functions) at the camera are summed to create the multi-point source result. To generate the multiple frames needed for speckle imagery, we repeat the process many times but using newly generated atmospheric phase screens at each iteration.

### 2.2.2 Extended object simulation with method of random phases

In order to simulate an incoherent extended target of substantial size when the atmosphere is not fully isoplanatic, a method similar to that described in Section 2.2.1 would work, but for a 100x100 image, at least 10,000 propagations are required per timestep. Such simulation software is currently being developed and tested by us to determine if the point by point propagation method is feasible and can be made computationally more efficient for strong atmospheric cases. We have previously developed a random phase method that requires less than a few thousand propagations per timestep, which is described here. First, a complex field at the object plane is constructed which consists of the magnitude of the object and a uniformly distributed random phase over 0 to  $2\pi$ . Then the field is propagated in its entirety through the

atmospheric screens and an image is formed. This is repeated with new random object phase screens to form more realizations of the image. The resulting intensity images from these randomly phased object propagations are then added together. To create a single speckle image frame, it takes several thousand of these realizations to reduce the noise sufficiently introduced by the random phase. To generate multiple speckle images, a new set of atmospheric screens is created and the sequence is repeated. To better illustrate the random phase simulation method, the following set of images in Figure 3 shows the no atmosphere case with increasing numbers of realizations. As we see in the images, the noise is significantly reduced after 1000 realizations.



**Figure 3: Illustration of the dependence of the resulting image quality versus the number of random realizations for an extended object with no atmosphere.**

When doing Fresnel propagations through strong turbulence over long distances for substantial fields of view, there are certain sampling considerations to contend with. Ideally, we would like to be able to image any field of view at any range, but this can lead to intractable simulations due to the large numbers of pixels needed to represent the complex fields and not to mention the large number of phase screens.

Considering only the phase screen sampling requirements, in order not to alias the phase screens, the sampling interval will need to be less than  $r_0/2$ . With  $r_0$ 's in the range of a few mm to a few cm, we already can see that the problem size gets big in a hurry. For example, in order to get a 2 meter FOV with a 1 cm  $r_0$  requires 400 pixels square. Realizing that it is wise to have a substantial guard band around the objects, we are easily up to 800 pixels square.

Based on the Fresnel propagation, we can calculate the sampling interval that allows a point source the size of a single pixel ( $dx$ ) to fill one half the screen when propagated a distance  $L$  at wavelength  $\lambda$  from the relation:

$$Ndx = 2\left(\frac{\lambda L}{dx}\right) \quad (5)$$

Meaning that

$$dx = \sqrt{\frac{2\lambda L}{N}}, \quad (6)$$

where N is the desired number of pixels in x and y. The reason we want the diffraction pattern to fill up a good portion of the screen is because we want pixels off center to be “seen” by the aperture. If a point diffracts only a few pixels, then points too far off center will not get captured by the aperture which may only be 10’s of pixels across, thereby limiting the FOV. Let’s look at some sample calculations:

For N = 256,  $\lambda = 4\mu\text{m}$ , L = 10 km then dx = 1.77 cm/pixel.

For N = 512,  $\lambda = 4\mu\text{m}$ , L = 10 km then dx = 1.25 cm/pixel.

For N=1024,  $\lambda = 4\mu\text{m}$ , L = 10 km then dx = 0.884 cm/pixel.

For N=2048,  $\lambda = 4\mu\text{m}$ , L = 10 km then dx = 0.625 cm/pixel.

For N=4096,  $\lambda = 4\mu\text{m}$ , L = 10 km then dx = 0.442 cm/pixel.

These dx values can be fiddled with slightly, meaning if we use dx = 0.7 cm/pixel in the N=1024 case then a point will diffract slightly more than half the screen.

What we find for the Scenario 1 cases is that we need to have N=512 and sometimes N=1024. For the Scenario 2 and 3 cases we need N= 2048 to get part of a tank in the FOV or N=4096 to get a whole tank in our field of view. With such small sample intervals, we are quite oversampled for the diffraction limit of the aperture itself, which would only require  $\lambda L/2D$  meters per pixel (e.g.  $4\mu\text{m} * 10\text{km} / 2 * .15 \text{ m} = 13.3 \text{ cm/pixel}$  compared to 4 or 6 mm/pixel.)

Timing wise, with N=1024, 2000 random realizations, and 9 phase screens, it takes 50 minutes to create a single speckle frame running on our 16 processor machine. Therefore to generate 30 frames of speckle imagery takes 25 hours in that case. For the N=2048 and 4096 cases, it is necessary to run the simulations on LLNL’s institutional massively parallel machines. Using 256 CPUs on the N=4096 case, 5 to 8 speckle frames can be generated in 12 hours, depending on the number of atmospheric screens. Most of the job time (from days to weeks) is spent waiting in the queue to get available processors, though.

### 3.0 Scenario 1

This scenario considers imaging over a 2 km horizontal path with a maximum aperture diameter of 15 cm. Our study of this scenario considers mid-wave IR as well as other common spectral bands. The maximum aperture diameter is to be 15 cm, but we will take a look at a smaller diameter case for MWIR as well. Expected imaging performance will be ascertained both by looking at tables of pertinent calculations and results of simulations.

To avoid confusion, formulas used in the tables are listed below:

- Isoplanatic Angle:  $\theta_0 = \left[ 1.092 \left( \frac{2\pi}{\lambda} \right)^2 C_n^2 L^{8/3} \right]^{-3/5}$  [7]

- Total r0 (spherical):  $r_0 = 0.33 \left[ \frac{C_n^2 L}{\lambda^2} \right]^{-3/5}$  [8]

- Total r0 (planewave):  $r_0 = 0.18 \left[ \frac{C_n^2 L}{\lambda^2} \right]^{-3/5}$  [9]

- Diffraction limited resolution:  $\lambda / D$  [10]

- Uncorrected Resolution:  $\lambda / r_0$

[11]

For each simulation, at least 30 frames of image data are generated. The exposure times can be considered instantaneous and the spectral bandwidth equal to zero. In the following sets of simulations, a high-resolution image of a soldier aiming a gun (from the website <http://www4.army.mil/armyimages/>) acquired at a visible wavelength was used as the input object for all wavebands. Although it does not represent what one would expect to see in the IR from a radiometry standpoint, for the purposes of understanding the atmospheric effects and the speckle processing performance, this should be fine.

### 3.1 MWIR

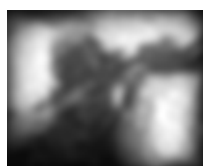
In Table 2 we show the MWIR input parameters (in BOLD), results of calculations needed for the simulation, instructive parameters, as well as a summary of the speckle processed result (if a simulation was performed.) Figures 4 and 5 show two sample frames from the simulation, the shift and add result from the 30 frames, a diffraction limited image, and speckle processed results with two different tile sizes. For the  $Cn2=2e-13$  case, the raw imagery is not too bad to start with, but the speckle processed result looks practically the same as the diffraction limited image. For the  $Cn2=5e-13$  case, the raw imagery is quite distorted and blurred, but the speckle processing is able to reconstruct a fairly decent image of the soldier.

<b>Scenario 1 MWIR</b>			
<b>parameters</b>	<b>Case 1a</b>	<b>Case 1b</b>	<b>Case 1c</b>
<b>Cn2</b>	<b>2.0e-13</b>	<b>5.0e-13</b>	<b>2.0e-13</b>
<b>Wavelength</b>	<b>4e-6</b>	<b>4e-6</b>	<b>4e-6</b>
<b>Range</b>	<b>2 km</b>	<b>2 km</b>	<b>2 km</b>
Isoplanatic angle	7.57 urad	4.37 urad	7.57 urad
Total r0 (spherical)	4.8 cm	2.8 cm	4.8 cm
Total r0 (planewave)	2.6 cm	1.5 cm	2.6 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>5 cm</b>
# screens used in sim.	4	5	-
R0 per screen (at $\lambda$ )	11 cm	7.3 cm	-
Diffraction-limited resolution	26.7 urad or 5.33 cm on target	26.7 urad or 5.33 cm on target	80.0 urad or 16.0 cm on target
Uncorrected Resolution	83.2 urad or 17 cm on target	144 urad or 29 cm on target	83.2 urad or 17 cm on target
D/r0	3.12	5.41	1.04
Did we simulate this?	Yes.	Yes	No, already nearly
If not, why not?	Good. Diffraction-	Okay. Some	at diffraction limit
If so, how did the speckle processing do?	limited performance.	improvement.	in raw images.

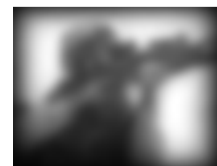
**Table 2: MWIR Scenario 1 cases**



**Sample frame**



**Sample frame**



**Shift and add 30 frames**





Figure 4: Simulated imagery and results for MWIR,  $Cn2 = 2e-13$ , range = 2 km, horizontal path.

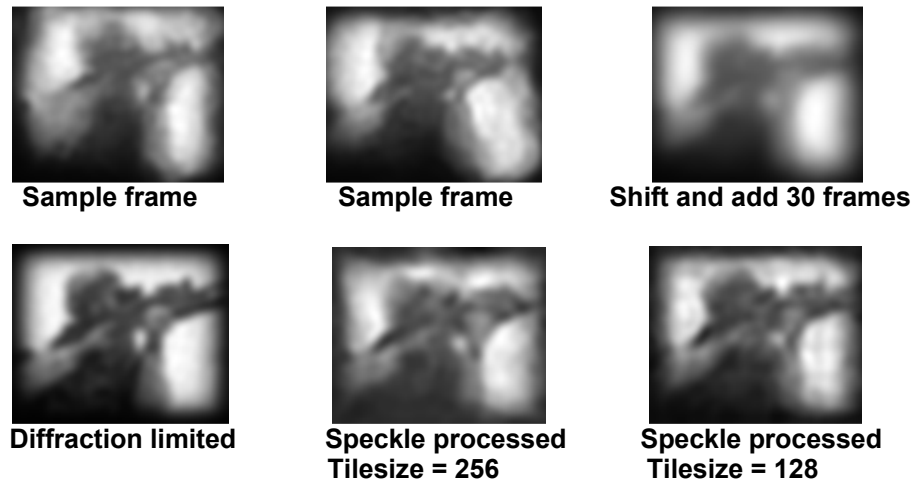


Figure 5: Simulated imagery and results for MWIR,  $Cn2 = 5e-13$ , range = 2 km, horizontal path.

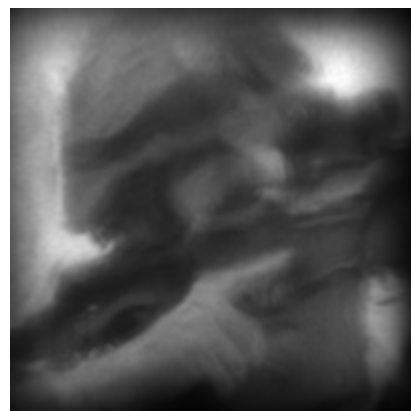
### 3.2 Visible

The cases listed in Table 3 were considered for visible imaging, and the results displayed in Figures 6 and 7.

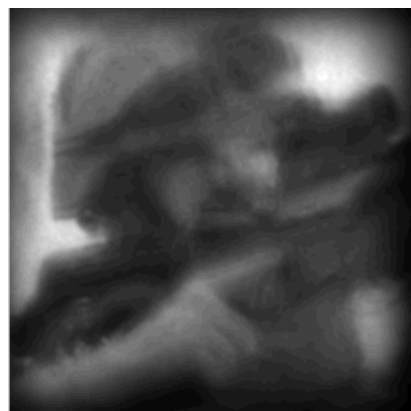
<b>Scenario 1 Visible parameters</b>			
	<b>Case 1j</b>	<b>Case 1k</b>	<b>Case 1d</b>
<b>Cn2</b>	<b>2.0e-14</b>	<b>6.0e-14</b>	<b>2.0e-13</b>
<b>Wavelength</b>	<b>0.5 um</b>	<b>0.5 um</b>	<b>0.5 um</b>
<b>Range</b>	<b>2 km</b>	<b>2 km</b>	<b>2 km</b>
Isoplanatic angle	2.49 urad	1.28 urad	0.625 urad
Total r0 (spherical)	1.6 cm	0.82 cm	0.40 cm
Total r0 (planewave)	0.86 cm	0.44 cm	0.22 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used	2	4	7
R0 per screen (at $\lambda$ )	2.4 cm	1.9 cm	1.3 cm
Diffraction-limited resolution	3.33 urad or 0.67 cm on target	3.33 urad or 0.67 cm on target	3.33 urad or 0.67 cm on target
Uncorrected Resolution	31.7 urad or 6 cm on target	61.2 urad or 12 cm on target	126 urad or 17 cm on target

D/r0	9.5	18.4	37.8
Did we simulate this?	Yes.	Yes	Tried. Had
If so, how did the speckle processing do?	Very Well. Diffraction-limited performance.	Okay. Some improvement.	sampling issues.

**Table 3: Visible Scenario 1 cases**



**Sample frame**



**Sample frame**



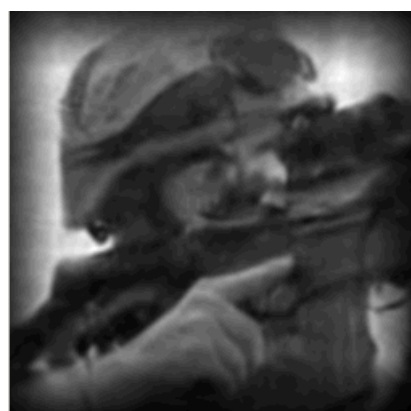
**Shift and add 30 frames**



**Diffraction limited image**

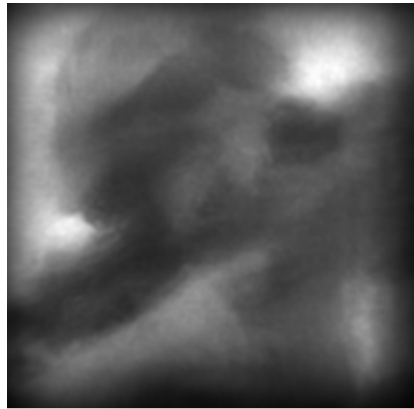


**Speckle processed (tilesize = 256)**

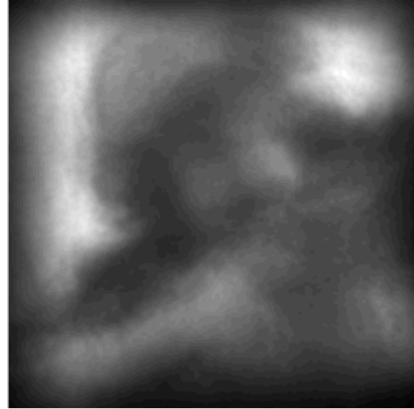


**Speckle processed (tilesize = 128)**

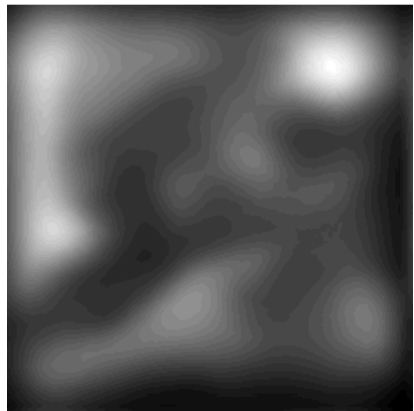
**Figure 6: Simulated Imagery and results for Visible, Cn2 = 2e-14, range = 2 km, horizontal path.**



Sample frame



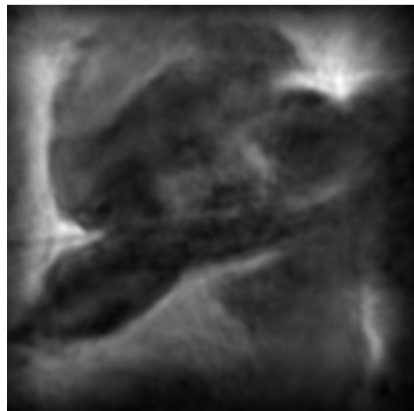
Sample frame



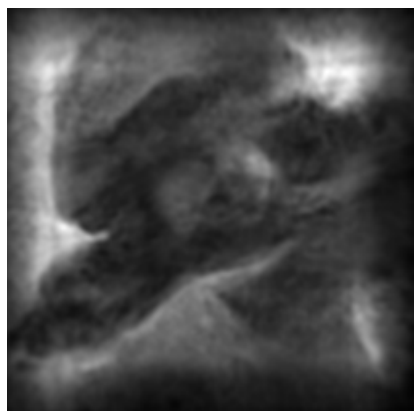
Shift and add 30 frames



Diffraction limited image



Speckle processed (tile size = 256)



Speckle processed (tile size = 128)

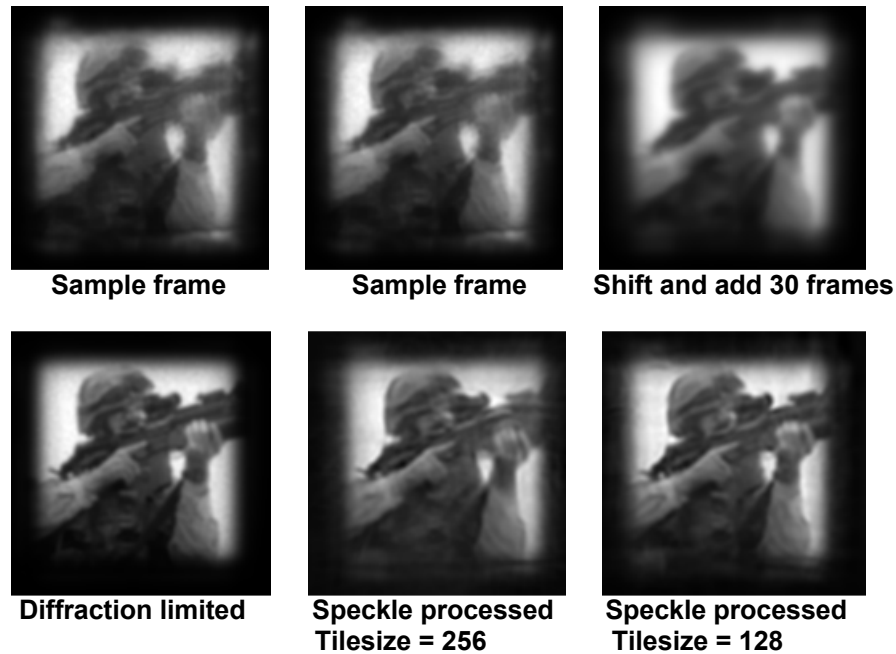
Figure 7: Simulated Imagery and results for Visible,  $C_n2 = 6e-14$ , range = 2 km, horizontal path.

### 3.3 SWIR

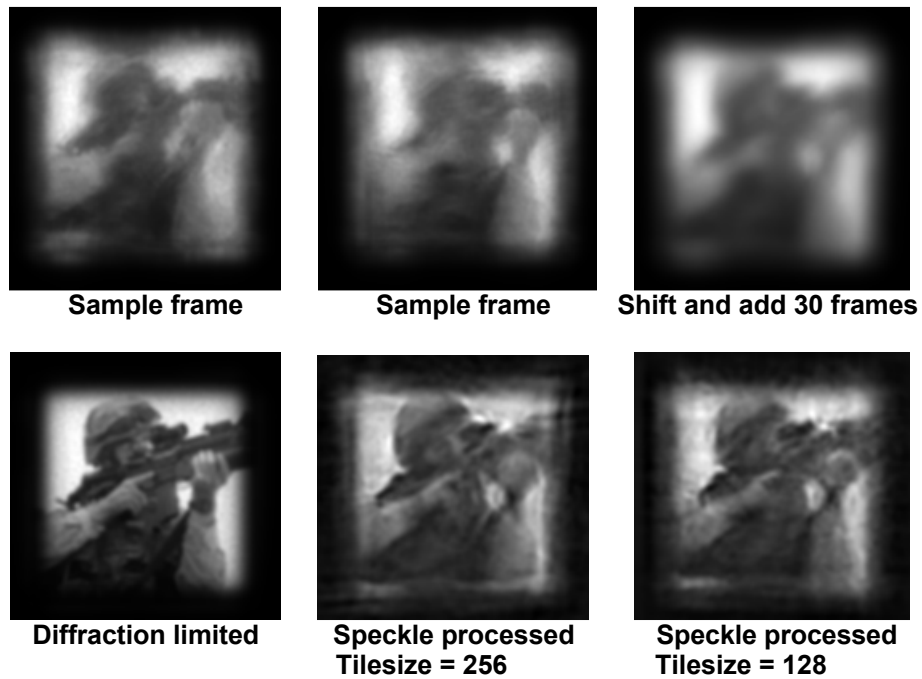
The cases listed in Table 4 were considered for SWIR imaging, and the results displayed in Figures 8, 9, and 10.

Scenario 1 SWIR parameters			
	Case 1l	Case 1f	Case 1g
<b>Cn2</b>	<b>6.0e-14</b>	<b>2.0e-13</b>	<b>5.0e-13</b>
<b>Wavelength</b>	<b>1.5 um</b>	<b>1.5 um</b>	<b>1.5 um</b>
<b>Range</b>	<b>2 km</b>	<b>2 km</b>	<b>2 km</b>
Isoplanatic angle	4.81 urad	2.34 urad	1.35 urad
Total r0 (spherical)	3.0 cm	1.5 cm	0.86 cm
Total r0 (planewave)	1.6 cm	0.80 cm	0.46 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used in sim.	2	4	6
R0 per screen (at $\lambda$ )	4.6 cm	3.4 cm	2.5 cm
Diffraction-limited resolution	10 urad or 2.0 cm on target	10 urad or 2.0 cm on target	10 urad or 2.0 cm on target
Uncorrected Resolution	49.1 urad or 10 cm on target	101 urad or 20 cm on target	175 urad or 35 cm on target
D/r0	5	10	17.5
Did we simulate this?	Yes.	Yes	.Yes
If so, how did the speckle processing do?	Very Well. Diffraction-limited performance.	Well. Good improvement.	Marginal. Slight improvement

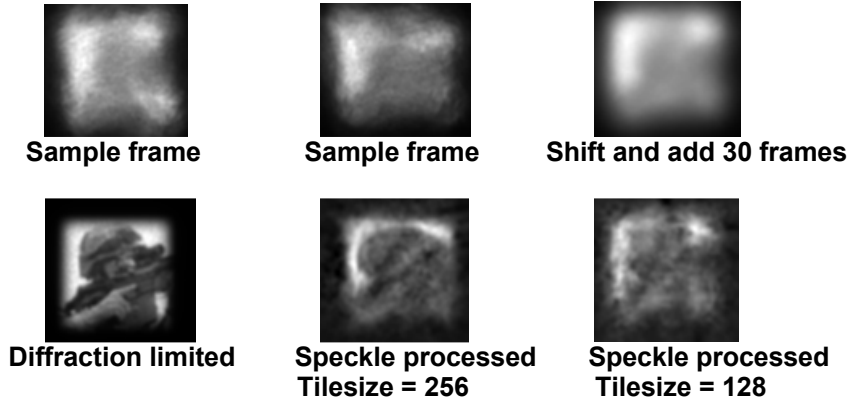
**Table 4: SWIR Scenario 1 cases**



**Figure 8: Simulated Imagery and results for SWIR, Cn2 = 6e-14, range = 2 km, horizontal path.**



**Figure 9: Simulated Imagery and results for SWIR,  $Cn2 = 2e-13$ , range = 2 km, horizontal path.**



**Figure 10: Simulated Imagery and results for SWIR,  $Cn2 = 5e-13$ , range = 2 km, horizontal path. This case looks different from the previous two cases because we had to change the sampling interval and further restrict the FOV in order to avoid aliasing/wrap around effects due to stronger turbulence.**

### 3.4 LWIR

In Table 5 we show the calculated parameters for the 2 km range cases of  $Cn2 = 2e-13$  and  $Cn2 = 5e-13$  at a 10 micron wavelength.

<b>Scenario 1 LWIR</b>		
<b>parameters</b>	<b>Case 1h</b>	<b>Case 1i</b>
<b>Cn2</b>	<b>2.0e-13</b>	<b>5.0e-13</b>
<b>Wavelength (<math>\lambda</math>)</b>	<b>10.0 <math>\mu</math>m</b>	<b>10.0 <math>\mu</math>m</b>
<b>Range</b>	<b>2 km</b>	<b>2 km</b>
Isoplanatic angle	22.7 urad	13.1 urad
Total r0 (spherical)	14.4 cm	8.3 cm
Total r0 (planewave)	7.8 cm	4.5 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used	1	2
R0 per screen (at $\lambda$ )	14.4 cm	12.6 cm
Diffraction-limited resolution	66.7 urad or 13.3 cm on target	66.7 urad or 13.3 cm on target
Uncorrected Resolution	69.2 urad or 14 cm on target	120 urad or 24 cm on target
D/r0	1.04	1.80
Did we simulate this?	No. No blurring	No. Very little
If so, how did the speckle processing do?	would occur.	blurring would occur.

**Table 5: LWIR Scenario 1 cases**

### 3.5 Analysis

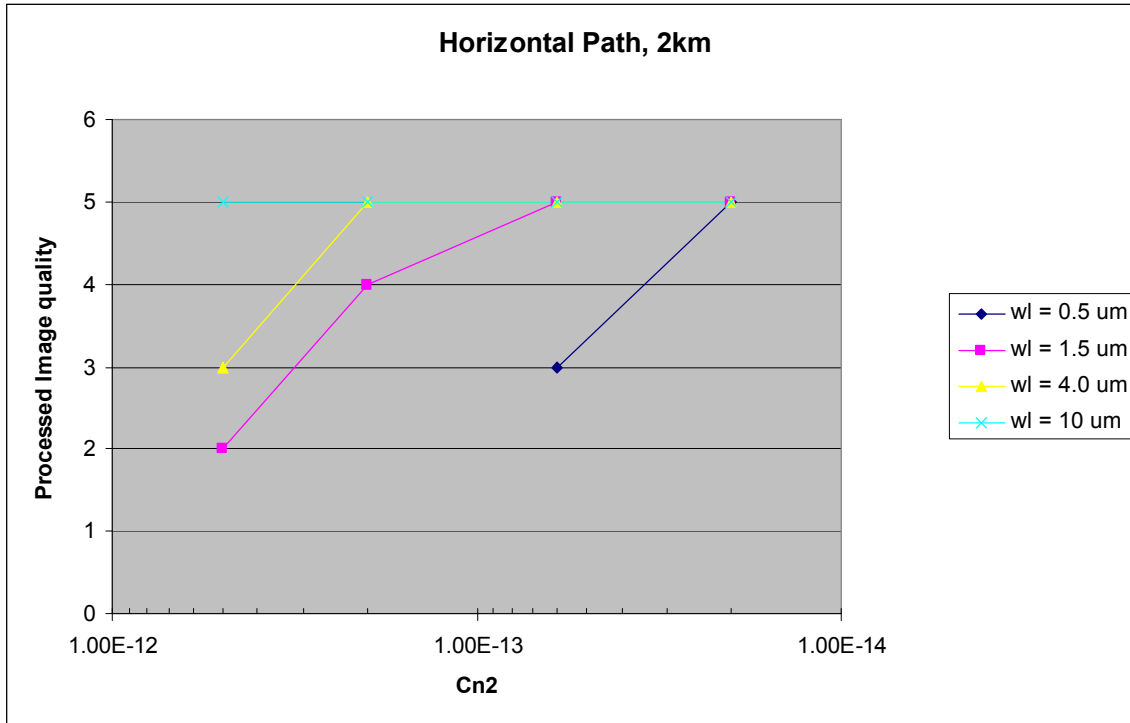
The effective resolution requirement in Table 1 is 140 urad in the MWIR. For the 2 km path in the MWIR, this requirement should be met with the raw data alone nearly into turbulence strength regimes of  $5e-13$ , assuming proper stabilization. The speckle processing will give roughly a factor of 3 improvement in the  $Cn2 = 2e-13$  case to get to the diffraction limited resolution for a 15 cm aperture of 26.7 urad. For a 5 cm aperture in the same conditions, there is very little possible resolution gain to be found with speckle processing because  $D/r_0$  is already at or near one, though it should be useful for eliminating the anisoplanatic warping effects.

For visible wavelength imaging, the resolution requirement is also met with the raw data up to  $Cn2$ 's near  $2.3e-13$ , but order of magnitude resolution improvements are possible to at or near diffraction limited with a 15 cm aperture when speckle processing is employed for  $Cn2$ 's less than  $5e-14$ .

For SWIR wavelength imaging, the resolution requirement is again met with the raw data up to  $Cn2$ 's near  $3.5e-13$ . Factors of several to ten improvements in resolution are obtainable with a 15 cm aperture by using speckle processing for  $Cn2$ 's below  $3e-13$ .

To visualize the subjective speckle imaging performance, Plot 1, displays a rating for the speckle processed image for each simulated case that was evaluated. The rating system is as follows:

- 5 = very good (diffraction limited performance)
- 4 = good (near diffraction limited performance)
- 3 = okay (decent and noticeable improvement over raw imagery)
- 2 = marginal (slight improvement)
- 1 = poor



**Plot 1: Summary of speckle imaging performance versus Cn2 for 2 km horizontal imaging at four different wavelengths for simulated imagery.**

#### 4.0 Scenario 2/3

Both Scenario 2 and 3 indicate a 10 km path, with the primary difference being that Scenario 2 allows for a slight slant path while Scenario 3 is purely horizontal and with a stricter resolution requirement. As it is simpler to understand and model the horizontal path case, it will be considered first and then the MWIR slant path will be considered next as a special case.

#### 4.1 MWIR

Table 6 lists the simulated cases for Scenario 3 in the MWIR. The object (see left side of Figure 1) used for this case was a tank image acquired with a thermal camera (courtesy of sierra pacific [www.imaging1.com](http://www.imaging1.com)). The sample interval was estimated and the image rescaled appropriately for the simulations. A diffraction limited image of the tank simulated is shown in Figure 11. For these simulations,  $N = 4096$ , sampled at 4 mm/pixel was used, which means they are 66.6x oversampled. Such a large  $N$  and small sample interval we needed to capture a large enough field of view and also be able to adequately sample the atmosphere. Images shown in Figures 11-15 are at 3.125% scale to show what Nyquist sampling of the diffraction limit would look like. Figures 12-15 show sample frames, shift-and-add, and the speckle processed results for four different Cn2 values.

<b>Scenario 3 MWIR</b>				
<b>parameters</b>	<b>Case 3m</b>	<b>Case 3n</b>	<b>Case 3o</b>	<b>Case 3a</b>
<b>Cn2</b>	<b>3.0e-14</b>	<b>7.0e-14</b>	<b>1.0e-13</b>	<b>2.0e-13</b>
<b>Wavelength</b>	<b>4e-6</b>	<b>4e-6</b>	<b>4e-6</b>	<b>4e-6</b>
<b>Range</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>
Isoplanatic angle	1.8 urad	1.08 urad	0.87 urad	0.58 urad
Total r0 (spherical)	5.7 cm	3.4 cm	2.8 cm	1.8 cm
Total r0 (planewave)	3.1 cm	1.9 cm	1.5 cm	1.0 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used in sim.	4	5	7	9
R0 per screen (at $\lambda$ )	13 cm	9.0 cm	8.9 cm	6.8 cm
Diffraction-limited resolution (FWHM)	26.7 urad or 27 cm @10km	26.7 urad or 27 cm @10km	26.7 urad or 27 cm @10km	26.7 urad or 27 cm @10km
Uncorrected Resolution	70.0 urad or 0.7 m on target	116 urad or 1.2 m at 10km	144 urad or 1.4 m at 10km	218 urad or 2.2 m at 10km
D/r0	2.6	4.4	5.4	8.2
Did we simulate this?	Yes.	Yes	Yes	Yes
If not, why not?	Very good	Good	Okay	Marginal
If so, how did the speckle processing do?				

**Table 6: MWIR Scenario 3 cases**



**Figure 11: Diffraction limited images for a telescope diameter of 15 cm. Image is roughly 8.5 meters on a side.**



**Figure 12: Simulated imagery and results for MWIR, Cn2 = 3e-14, Range = 10 km, horizontal path.**



**Figure 13: Simulated imagery and results for MWIR, Cn2 = 7e-14, Range = 10 km, horizontal path.**



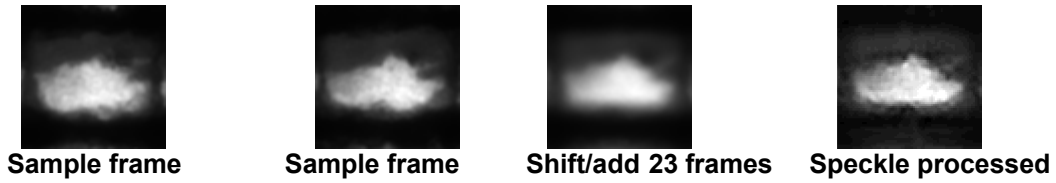


Figure 14: Simulated imagery and results for MWIR,  $Cn2 = 1e-13$ , Range = 10 km, horizontal path.



Figure 15: Simulated imagery and results for MWIR,  $Cn2 = 2e-13$ , Range = 10 km, horizontal path.

#### 4.2 Visible

Table 7 lists the sample cases in the visible considered. The object used for these simulations was again the “soldier aiming” from the 2 km simulations, but sampled appropriately for the longer range. Weaker turbulence cases were looked at in order to even be able to perform the distributed turbulence simulation, but the parameters for the  $Cn2 = 2.0e-13$  case are listed in the rightmost column of Table 7 for reference. A diffraction-limited image of the target we used for this simulation as viewed through a 15 cm diameter telescope is shown in Figure 16. Sample frames, shift and add image, and speckle processed results for the two  $Cn2$  cases of  $1e-15$  and  $5e-15$  are shown in Figures 17 and 18. We can see that a  $Cn2$  of  $5e-15$  atmosphere does not result in a diffraction limited reconstruction, but some improvement is still observed.

Scenario 3 Visible parameters	Case 3j	Case 3k	Case 3d
<b>Cn2</b>	<b>5.0e-15</b>	<b>1.0e-15</b>	<b>2.0e-13</b>
<b>Wavelength</b>	<b>0.5 um</b>	<b>0.5 um</b>	<b>0.5 um</b>
<b>Range</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>
Isoplanatic angle	0.435 urad	1.14 urad	0.047 urad
Total r0 (spherical)	1.4 cm	3.6 cm	1.5 mm
Total r0 (planewave)	0.75 cm	2.0 cm	0.8 mm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used	5	2	34 needed
R0 per screen (at $\lambda$ )	3.6 cm	5.5 cm	1.25 cm needed
Diffraction-limited resolution	3.33 urad or 3.33 cm @ 10km	3.33 urad or 3.33 cm @ 10km	3.33 urad or 3.33 cm @ 10km
Uncorrected Resolution	36.2 urad or 36 cm on target	13.8 urad or 14 cm on target	331 urad or 3.3 meters @10km
D/r0	9.5	18.4	37.8
Did we simulate this?	Yes.	Yes	No.
If so, how did the speckle processing do?	Very good. Diffraction limited.	Okay. Some improvement.	Computationally intractable.

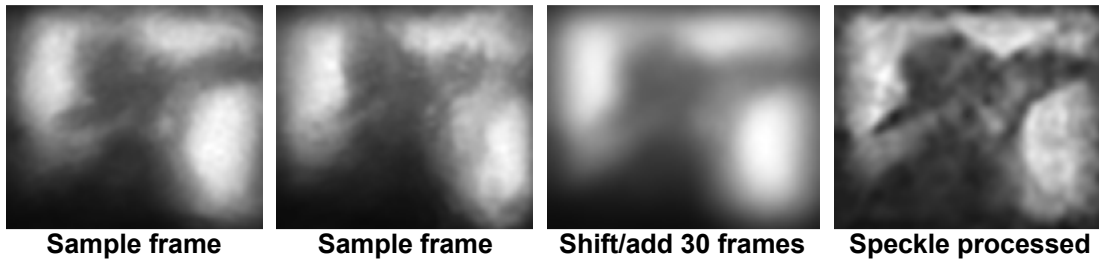
Table 7: Scenario 3 Visible wavelength cases



**Figure 16: Diffraction limited image of the target at 10 km through a 15 cm aperture. (10x oversampling needed for simulation; displayed here at 25% scale.)**



**Figure 17: Simulated imagery and results for visible wavelength,  $C_n2 = 1e-15$ , Range = 10 km, horizontal path.**



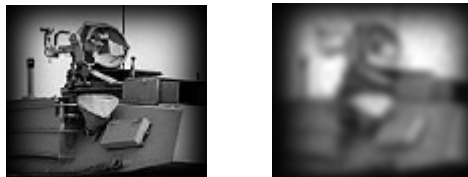
**Figure 18: Simulated imagery and results for visible wavelength,  $C_n2 = 5e-15$ , Range = 10 km, horizontal path.**

### 4.3 SWIR

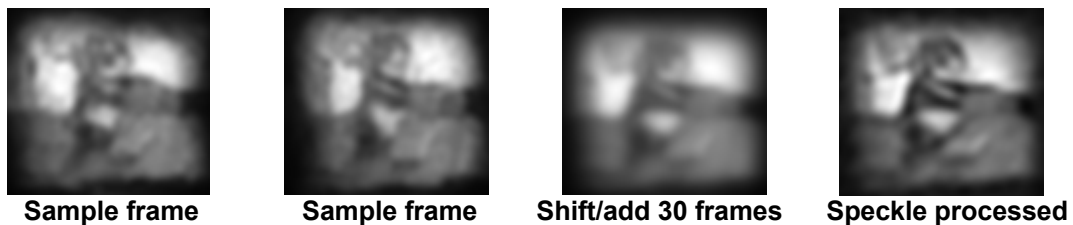
Table 8 lists the some sample cases in the SWIR. Weaker turbulence cases were looked at in order to be able to perform the distributed turbulence simulation, but the parameters for the  $C_n2$  of  $2.0e-13$  case are listed in the rightmost column of Table 8 for reference. A picture of the object and a diffraction limited image of the object used for this simulation as viewed through a 15 cm diameter telescope is shown in Figure 19. Sample frames, shift and add image, and speckle processed results for the three  $C_n2$  cases of  $5e-15$ ,  $1e-14$ , and  $3e-14$  are shown in Figures 20, 21, and 22. We see that the  $C_n2$  of  $5e-15$  case gives a nearly diffraction limited result while the  $C_n2$  of  $1e-14$  still gives substantial improvement over the raw frames or the shift and add image. The  $C_n2 = 3e-14$  is not particularly recognizable, but it is considerably sharper than the shift and add version.

<b>Scenario 3</b>				
<b>SWIR parameters</b>	<b>Case 3p</b>	<b>Case 3l</b>	<b>Case 3q</b>	<b>Case 3f</b>
<b>Cn2</b>	<b>5.0e-15</b>	<b>1.0e-14</b>	<b>3.0e-14</b>	<b>2.0e-13</b>
<b>Wavelength</b>	<b>1.5 um</b>	<b>1.5 um</b>	<b>1.5 um</b>	<b>1.5 um</b>
<b>Range</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>
Isoplanatic angle	1.6 urad	1.07 urad	0.55 urad	0.18 urad
Total r0 (spherical)	1.4 cm	3.4 cm	1.8 cm	5.6 mm
Total r0 (planewave)	0.75 cm	1.85 cm	9.6 mm	3.1 mm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used	4	4	6	17 needed
R0 per screen (at $\lambda$ )	11.8 cm	7.8 cm	5.2 cm	3.1 cm needed
Diffraction-limited resolution	10 urad or 10 cm @ 10km	10 urad or 10 cm @ 10km	10 urad or 10 cm @ 10km	10 urad or 10 cm @ 10km
Uncorrected Resolution	29 urad or 29 cm on target	44 urad or 44 cm on target	85 urad or 85 cm on target	331 urad or 3.3 meters @10km
D/r0	2.9	4.4	8.5	27
Did we simulate this?	Yes.	Yes	Yes	No.
If so, how did the speckle processing do?	Good. Nearly limited.	Okay. Some improvement.	Marginal	Computationally intractable.

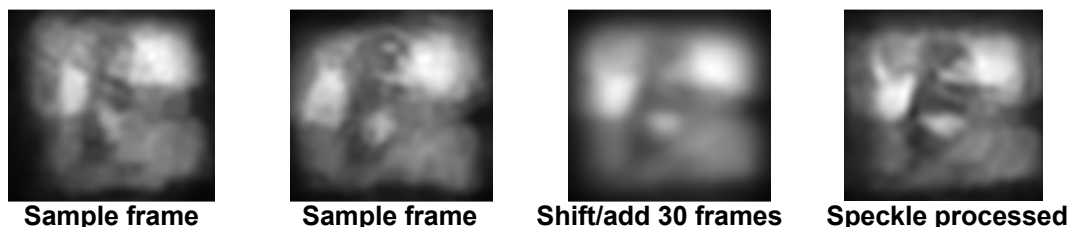
**Table 8: Scenario 3 SWIR wavelength cases**



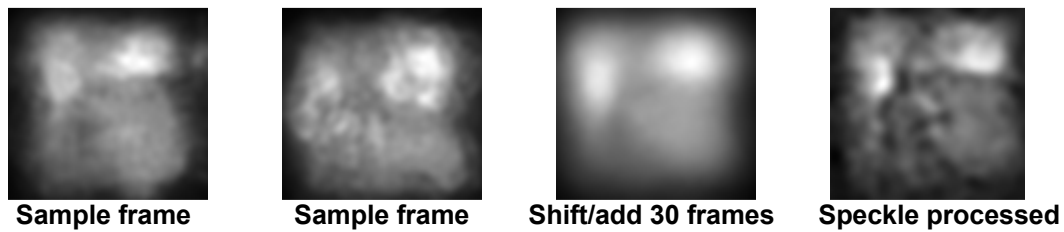
**Figure 19: High-resolution picture of object (at left) and diffraction limited image (at right) of the target at 10 km through a 15 cm aperture. (Note: 20x oversampling needed for simulation; displayed here at 25% scale.)**



**Figure 20: Simulated imagery and results for SWIR wavelength, Cn2 = 5e-15, Range = 10 km, horizontal path.**



**Figure 21: Simulated imagery and results for SWIR wavelength, Cn2 = 1e-14, Range = 10 km, horizontal path.**



**Figure 22: Simulated imagery and results for SWIR wavelength, Cn2 = 3e-14, Range = 10 km, horizontal path.**

#### 4.4 LWIR

Below in Table 9 is listed the LWIR cases considered for this study. With a 15 cm aperture, it is possible to meet the Scenario 3 resolution requirement of 100 urad, but if the aperture needs to be reduced below ~12 cm, it will no longer be possible to reach the Scenario 3 requirement due to the diffraction limit being too large at the 12 um end of the 8-12 um wavelength range. Given that a large enough aperture can be employed, and super-fine resolution much beyond the requirements are not needed, the situation is much better at these longer wavelengths. From the simulations, we see that minimal processing is required (e.g. shift-and-add may be sufficient) when the Cn2's are a few times  $10^{-14}$  or less to have nice quality imagery. But note that even with  $D/r_0 < 1$ , the isoplanatic angles are still small, meaning that the short exposure frames will exhibit some warping as we see in Figure 24. From the simulation of the tank with Cn2 equal to  $2e-13$ , Figure 25 shows that the LWIR speckle processed image allows near diffraction-limited imaging with the 15 cm optic. As the Cn2 approaches  $5e-13$ , the image quality begins deteriorating as demonstrated in Figure 26.

<b>Scenario 3 LWIR</b>			
<b>parameters</b>	<b>Case 3h</b>	<b>Case 3i</b>	<b>Case 3r</b>
<b>Cn2</b>	<b>2.0e-13</b>	<b>5.0e-13</b>	<b>3.0e-14</b>
<b>Wavelength (<math>\lambda</math>)</b>	<b>10.0 um</b>	<b>10.0 um</b>	<b>10.0 um</b>
<b>Range</b>	<b>10 km</b>	<b>10 km</b>	<b>10 km</b>
Isoplanatic angle	1.73 urad	1.0 urad	5.4 urad
Total r0 (spherical)	5.5 cm	3.2 cm	17.2 cm
Total r0 (planewave)	3.0 cm	1.7 cm	9.3 cm
<b>Aperture Diameter</b>	<b>15 cm</b>	<b>15 cm</b>	<b>15 cm</b>
# screens used	5	9	2
R0 per screen (at $\lambda$ )	14.4 cm	11.9 cm	26 cm
Diffraction-limited resolution	66.7 urad or 66.7 cm on target	66.7 urad or 66.7 cm on target	66.7 urad or 66.7 cm on target
Uncorrected Resolution	182 urad or 1.82 m on target	315 urad or 3.15 m on target	66.7 urad or 66.7 cm on target
D/r0	2.7	4.7	0.87
Did we simulate this?	Yes.	Yes.	Yes.
If so, how did the speckle processing do?	Good - near DL	Marginal.	Irrelevant. DL.

**Table 9: Scenario 3 LWIR wavelength cases**



Figure 23: High-resolution picture of object (at left) and magnified LWIR diffraction limited image (at right) of the target at 10 km through a 15 cm aperture. (Note: 70x over-sampling needed for simulation; displayed here at 10% scale.)

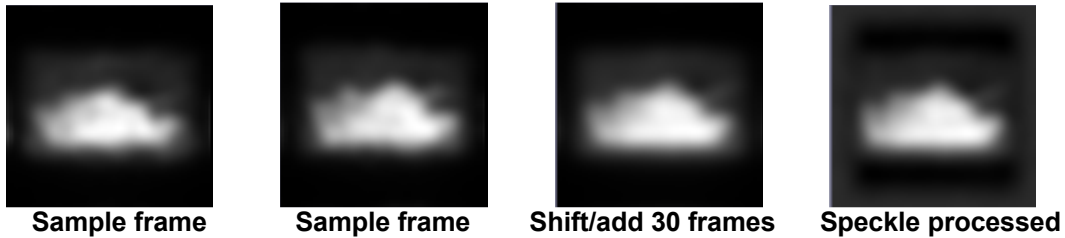


Figure 24: Simulated imagery and results for LWIR wavelength,  $C_n2 = 3e-14$ , Range = 10 km, horizontal path. Since  $D/r0 < 1$ , the shift and add and the speckle processed result look nearly identical.

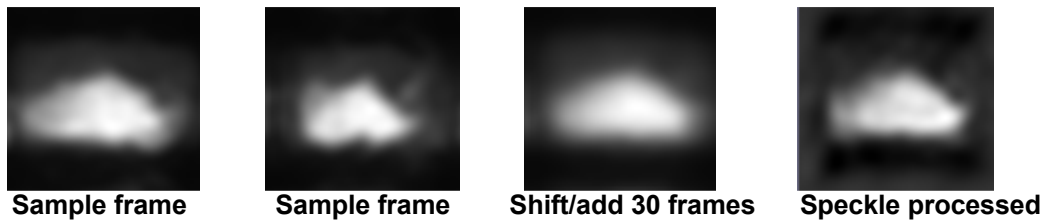


Figure 25: Simulated imagery and results for LWIR wavelength,  $C_n2 = 2e-13$ , Range = 10 km, horizontal path.

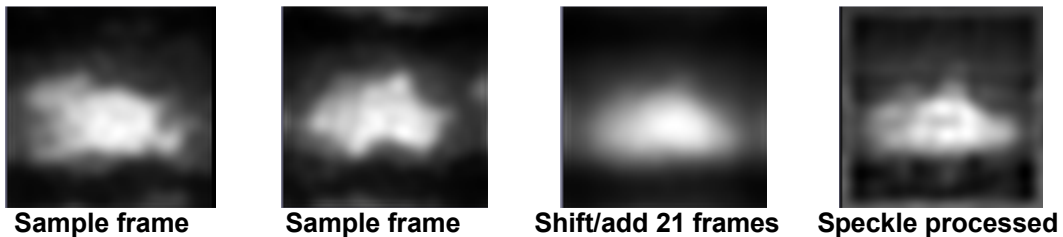
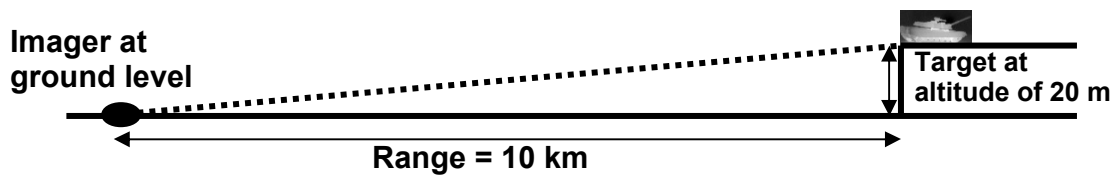


Figure 26: Simulated imagery and results for LWIR wavelength,  $C_n2 = 5e-13$ , Range = 10 km, horizontal path. Perhaps more frames would yield a better processed result.

#### 4.5 Slight slant-path with MWIR

The imaging geometry for which the slant-path calculations are performed is shown in Figure 27. The camera is at ground level with the target at an altitude of 20 m. This geometry meets the Scenario 2 requirements that 50% of the path must be less than 10 m above the ground.



**Figure 27: Slant-path imaging geometry used for simulations**

The simulation for this geometry will be broken into ten equally spaced atmospheric phase screens. The  $C_n^2$  at ground level ( $h=0$ ), or  $C_0$ , in a simple  $h^{-4/3}$  model for determining  $C_n^2$  as a function of altitude, or:

$$C_n^2(h) = C_0 h^{-4/3}, \quad (12)$$

where  $h$  is altitude above the ground.

We will consider two cases, with  $C_0 = 2e-13$  and  $C_0 = 5e-13$ . We can make a table of  $C_n^2$  versus range (translated to altitude) from which the  $r_0$  for each segment can be calculated. Solving the integrals in piecewise fashion and calculating intermediate quantities, it is possible to calculate the effective  $\theta_0$  and  $r_0$  for the full slant path. Table 10a and Table 10b list the results of the intermediate calculations for each case. Notice that even a slight slant path such as these reduces the burden on the imager, as the  $r_0$  value and the isoplanatic angles are significantly increased from the comparable horizontal cases.

The diffraction limited image of the part of the tank used in the simulation is shown in Figure 28 and results for the two  $C_n^2$  profiles shown in Figures 29 and 30. Notice that it is possible to obtain at or near diffraction limited imaging in both cases.

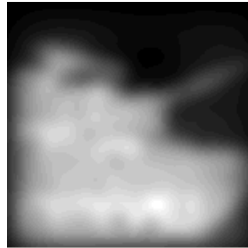
Range Segment #	Range position	Altitude at Range	$C_n^2$ at Alt	$r_0$ per screen
1	500	1	2E-13	0.0419
2	1500	3	4.62241E-14	0.1080
3	2500	5	2.33921E-14	0.1752
4	3500	7	1.49359E-14	0.2498
5	4500	9	1.06833E-14	0.3377
6	5500	11	8.17535E-15	0.4472
7	6500	13	6.54293E-15	0.5944
8	7500	15	5.4064E-15	0.8156
9	8500	17	4.57542E-15	1.2248
10	9500	19	3.9448E-15	2.5880

**Table 10a: 10 km slant path from 0 to 20 m altitude for  $C_0 = 2e-13$ . The effective isoplanatic angle is calculated to be 4.15 urad with an overall effective  $r_0$  of 3.56 cm.**

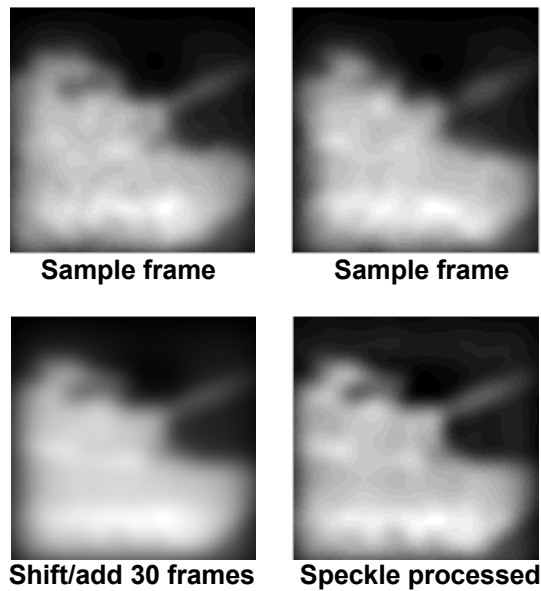
Range Segment #	Range position	Altitude at Range	$C_n^2$ at Alt	$r_0$ per screen
1	500	1	5E-13	0.0242
2	1500	3	1.1556E-13	0.0623
3	2500	5	5.84804E-14	0.1011
4	3500	7	3.73399E-14	0.1442
5	4500	9	2.67083E-14	0.1949
6	5500	11	2.04384E-14	0.2581

7	6500	13	1.63573E-14	0.3430
8	7500	15	1.3516E-14	0.4706
9	8500	17	1.14386E-14	0.7068
10	9500	19	9.862E-15	1.4935

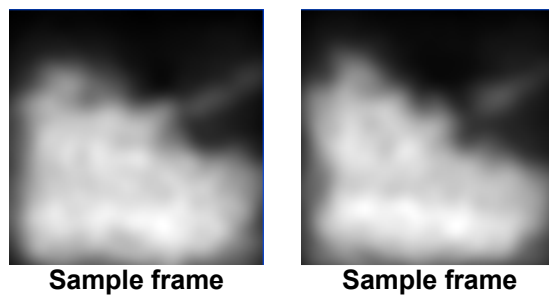
**Table 10b: 10 km slant path from 0 to 20 m altitude for  $C_0 = 5e-13$ . The effective isoplanatic angle is calculated to be 2.4 urad with an overall effective  $r_0$  of 2.0 cm.**

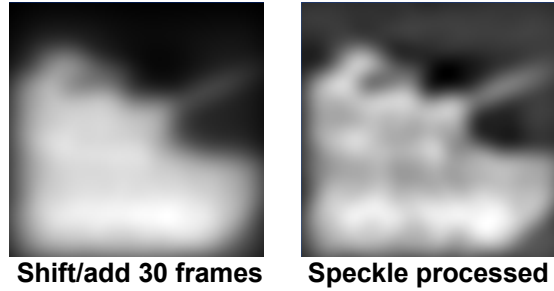


**Figure 28: Diffraction limited image of the target. This image is oversampled by 38x to meet sampling requirements, meaning there are only ~13 resolution cells across the image.**



**Figure 29: Simulated imagery and results for 0 to 20 meter high slant path with  $C_0 = 2e-13$ .**





**Figure 30: Simulated imagery and results for 0 to 20 meter high slant path with  $C_0 = 5e-13$ .**

#### 4.6 Analysis

To visualize the subjective performance, Plot 2 displays a rating for the speckle imaging performance for each simulated case that was evaluated. The rating system is as follows:

- 5 = very good (diffraction limited performance)
- 4 = good (near diffraction limited performance)
- 3 = okay (decent and noticeable improvement over raw imagery)
- 2 = marginal (slight improvement)
- 1 = poor

Another useful way to evaluate the speckle imaging performance is to compute the log amplitude variance versus  $C_n^2$  for each wavelength and note where on the plot the image degradation really begins compared to where scintillation is said to be dominant. We have defined log amplitude variance as <sup>5</sup>:

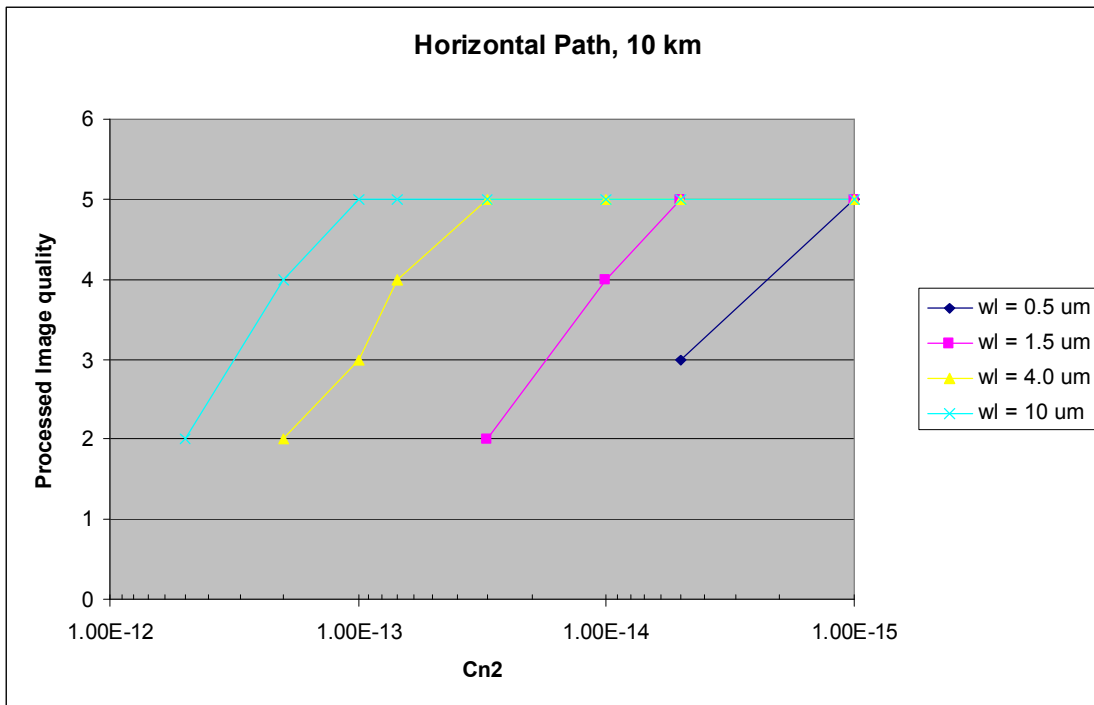
$$\sigma_\phi^2(L) = 0.124k^{7/6}L^{11/6}C_n^2 \quad (13)$$

Plot 3 displays this information. The black box drawn on the plot indicates the transition region from diffraction limited imaging to okay or marginal performance. The red line at  $\sigma_\chi^2 = .05$  marks the crossover from turbulence effects being mostly phase dominated to being amplitude dominated. What this plot indicates is that speckle imaging performs well more than an order magnitude into the scintillated regime, with a limited degree of performance available even two orders of magnitude into the scintillated regime as quantified by log amplitude variance.

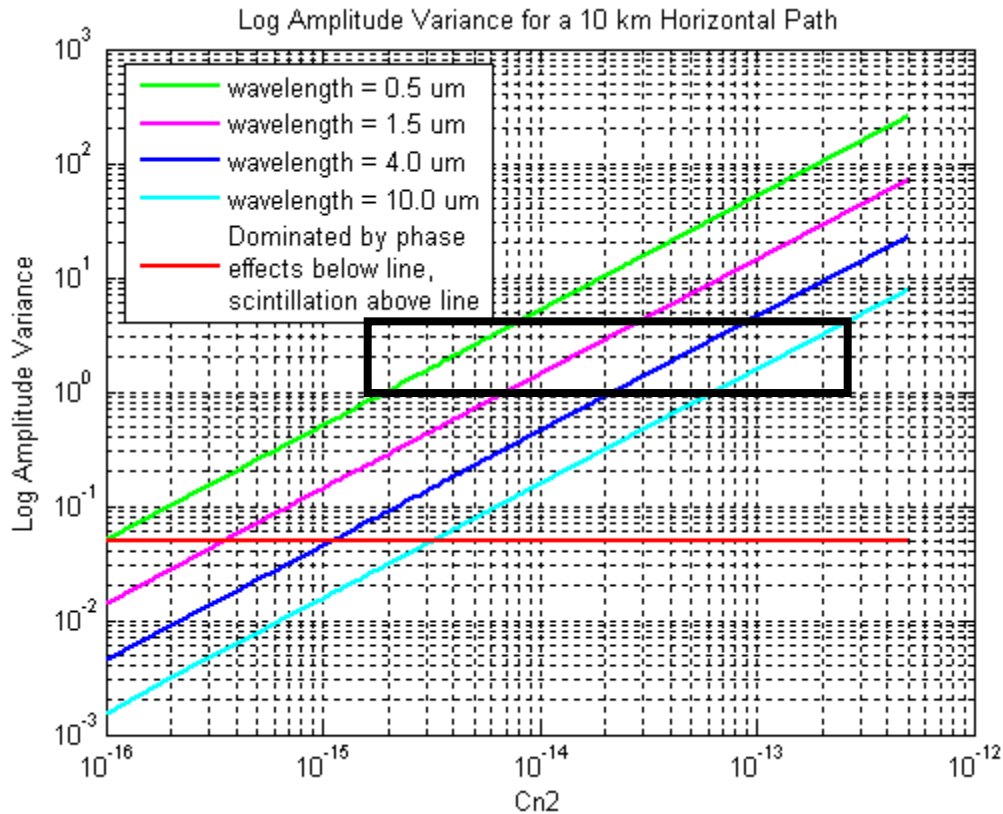
#### 5.0 Conclusions

Remembering that imaging simulations can really only be verified with real data, speckle imaging performance has been examined on simulated imagery for a wide range of horizontal path  $C_n^2$ 's as well as wavelengths. Bispectral speckle imaging can give at or near diffraction limited imaging over a large parameter space while giving some amount image resolution and accuracy improvement over an even larger parameter space. As for the MWIR scenarios considered, the Scenario 1 resolution requirements can be met or exceeded even in the strong turbulence  $C_n^2$  regime. For Scenario 2, simulations also indicate that for properly setup slant paths, the resolution requirements can also be met or exceeded through the strong turbulence regimes. For Scenario 3, simulations show that for  $C_n^2$ 's up to a factor of 2.5 below the target  $C_n^2$  value ( $2e-13$ ), at or near diffraction limited imaging is possible. Once the  $C_n^2$ 's approach  $2e-13$ , the image quality both before and after processing is substantially reduced, but still the speckle processing shows some image resolution enhancement.





**Plot 2: Summary of speckle imaging performance versus Cn2 for 10 km horizontal imaging at four different wavelengths for simulated imagery.**



**Plot 3: Log amplitude variance vs.  $C_n^2$  for 4 wavelengths for a 10 km horizontal path. Black box show transition region from where speckle imaging gives diffraction limited results (below the box) to where image quality begins to suffer (inside and above the box).**

## References

1. C. J. Carrano, "Speckle Imaging over Horizontal Paths", Proceedings of the SPIE -High Resolution Wavefront Control: Methods, Devices, and Applications IV, 4825, 109-120, (2002)
2. C. J. Carrano, J. M. Brase, "Horizontal and Slant Path Surveillance with Speckle Imaging", AMOS Technical Conference Proceedings, (2002)
3. C. J. Carrano, "Progress in horizontal and slant-path imaging using speckle imaging", Proceedings of the SPIE-LASE2003 - Optical Engineering at the Lawrence Livermore National Laboratory, **5001** (2003) pp. 56-64
4. T. W. Lawrence, J.P. Fitch, D. M. Goodman, N. A. Massie, R. J. Sherwood, and E. M. Johansson, "Extended-image reconstruction through horizontal path turbulence using bispectral speckle interferometry," Opt. Eng. **32**, 627-636 (1992)
5. R. R. Beland, "Propagation through atmospheric optical turbulence," in IR/EO Handbook (F. G. Smith, ed.), vol. 2, pp. 157-232, Bellingham, CA: SPIE Press, 1993