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A SPECIAL PURPOSE SLIDE RULE FOR COMPUTING SOLID ANGLES

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OAK RIDGE GASEOUS DIFFUSION PLANT

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DIVISION OF UNION CARBIDE CORPORATION

for the Atomic Energy Commission

Acting Under U. S. Government Contract W7405 eng 26

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A SPECIAL PURPOSE SLIDE RULE FOR COMPUTING SOLID ANGLES

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Oak Ridge Gaseous Diffusion Plant
Oak Ridge, Tennessee

Report Number: K-1345

Title: A SPECIAL PURPOSE SLIDE RULE
FOR COMPUTING SOLID ANGLES

Author: J. R. Knight

A B S T R A C T

The design and construction of a slide rule for calculating the solid angle subtended by a rectangle at a point located on a line perpendicular to the center of the rectangle are described. The application of the solid angle to the evaluation of safe spacing between individually subcritical containers of fissionable material is discussed, and a complete derivation of the solid-angle equation and a table of values used in marking the slide rule scales are given.

A SPECIAL PURPOSE SLIDE RULE FOR COMPUTING SOLID ANGLES

INTRODUCTION

The evaluation of the solid angle subtended by an object at a given distance from a point is often necessary in various types of problems. Of particular interest at the Oak Ridge Gaseous Diffusion Plant is the solid angle between separate units which contain fissionable material, since this is used as the basic criterion for determining the safe spacing of these units.¹ Although the calculation of solid angles is in general straightforward, it is frequently laborious, and it is thus desirable to find methods of reducing the time and labor involved in making a fairly large number of such determinations. In order to fulfill this need, a special purpose slide rule for calculating the solid angle subtended by a rectangular object at a point on a line perpendicular to its center has been developed. The slide rule described here yields values from a maximum of 2π steradians, which is the largest obtainable between a point and a rectangle, to a minimum of 0.005 steradian, which is the smallest solid angle that is considered to be significant in neutron interaction problems at the Oak Ridge Gaseous Diffusion Plant.

THEORY

A reasonably rigorous treatment of neutron interaction² shows that the critical spacing between identical and individually subcritical containers of fissionable material is a function of the average solid angle subtended by other containers at the most centrally located one, this average being taken over all points on the surface of the central container. Burton³ at the Oak Ridge Gaseous Diffusion Plant, and Pond⁴ of the Goodyear Atomic Corporation have developed values of the average solid angle between identical pairs of slabs and cylinders, respectively, and curves giving values of these average solid angles have also been included as an appendix to a more widely distributed report.²

Despite the fact that a more accurate evaluation of neutron interaction can probably be obtained from consideration of the average solid angle, the simple solid angle subtended by other containers at a central point of the most centrally located container is useful for nuclear safety evaluations. Not only is this value easily computed, but as will be discussed later in this report, the solid angle so determined is the largest that will be calculated for the given conditions. Thus, to meet a definite solid angle criterion, the spacing specified by this method will be greater, and therefore more conservative, than would be obtained by more rigorous calculations.

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- 1 Henry, H. F., Mallett, A. J., and Newlon, C. E., Basic Critical Mass Information and Its Application to Oak Ridge Gaseous Diffusion Plant Design and Operation, August 2, 1957, (K-1019, Fourth Revision), Oak Ridge Gaseous Diffusion Plant, Oak Ridge, Tennessee
 - 2 Henry, H. F., Knight, J. R., and Newlon, C. E., General Application of a Theory of Neutron Interaction, November 15, 1956, (K-1309) Oak Ridge Gaseous Diffusion Plant, Oak Ridge, Tennessee
 - 3 Burton, D. W., Average Solid Angles, Letter to H. F. Henry, April 30, 1956, Oak Ridge Gaseous Diffusion Plant, Oak Ridge, Tennessee
 - 4 Pond, J. A., Right Circular Cylinders, Solid Angle Calculations, June 5, 1956, (GAT-DM-455) Goodyear Atomic Corporation, Portsmouth, Ohio

Murray and Schmidt⁵ have given an expression for the solid angle subtended by a rectangle at a point on a line perpendicular to one corner. Obviously, this expression may readily be adapted to the problem of the solid angle subtended at a point on a line perpendicular to the center of the rectangle. A chart given in their report enables this type of calculation to be made rather rapidly, but does not yield as great an accuracy as was considered desirable for actual interaction calculations. In addition, it was found that the expression could be put in a somewhat more convenient form than that given by Murray and Schmidt. As a matter of completeness, the entire derivation of their solid angle formula is given in appendix 1, as is the expression used for developing the slide rule. This is as follows:

$$\Omega = 4 \sin^{-1} \frac{(a/2)(b/2)}{\sqrt{(a/2)^2 + h^2} \sqrt{(b/2)^2 + h^2}},$$

where a and b are the dimensions of the rectangle, and h is the distance of the point from its center. Rearranging the terms and expressing the product as the sum of the logarithms of the terms involved leads to:

$$\log \csc \Omega/4 = \log \sqrt{1 + \left(\frac{2h}{a}\right)^2} + \log \sqrt{1 + \left(\frac{2h}{b}\right)^2}$$

Using this formula, the logarithmic values may be added on the scales of a slide rule to obtain the desired answer. Three scales, one proportional to the $\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$, one proportional to the $\log \sqrt{1 + \left(\frac{2h}{b}\right)^2}$ (which is identical to the previous scale), and one proportional to the $\log \csc \Omega/4$ are required.

CONSTRUCTION OF THE SLIDE RULE

Tables of the logarithms of $\sqrt{1 + \left(\frac{2h}{a}\right)^2}$ and of $\csc \Omega/4$, listed in appendix 2, were prepared for appropriate values to be used in marking the scales. These values were plotted to a convenient scale and the total length of the scales was adjusted in the photographic step of the printing process to the slide rule size to be used. In practice, it was found desirable to use 2 ranges for the h/b and Ω scales to obtain the desired range of values. A diagram of a "homemade" slide rule, including regular C and D scales for obtaining the necessary h/a and h/b ratios, is shown in figure 1. This slide rule was constructed of transparent plastic to which the scales, printed on heavy paper, were glued. Other types of slide rule construction, such as the circular design, may also be adapted to this type of application.

5 Murray, R. L., and Schmidt, G. W., Methods of Determining Solid Angles, November 25, 1946, Declassified Jan. 18, 1956 (AECD-4056) Tennessee Eastman Corporation, Oak Ridge, Tennessee

APPLICATION TO NEUTRON INTERACTION PROBLEMS

It should be noted that the solid angle subtended by a rectangle at a point on a line perpendicular to its center is the largest solid angle that can be obtained for that rectangle at the given distance, both as compared to any point-to-plane determination, and as compared with any average plane-to-plane calculation. Accordingly, a calculation between planes using the point-to-plane technique described above will give conservative results. That is, the spacing will be greater than that obtained from what is probably the more accurate method involving the average solid angle. Such a method of calculation should also be adequate to take care of any possible nonconservatism that could result from the fact that the neutron flux density in a reactor is not uniform.

For cylinders, a similar comparison of the results obtained by using a simple point-to-rectangle relation with those obtained from the more detailed average solid angle calculations shows that the simple calculations will also yield a somewhat conservative value of the solid angle if the distance used is from the edge of one cylinder to the center of the other, and the dimensions of the equivalent rectangles are the length and diameter of the cylinders. Presumably, a similar conclusion applies to interaction between other geometrical forms, such as spheres, although this type of calculation has not been attempted. Thus, the slide rule discussed here has a wide general usefulness in safe spacing problems.

Figure 1
SOLID-ANGLE SLIDE RULE

FRONT

TO FIND THE SOLID ANGLE, Ω , (IN STERADIANS) SUBTENDED BY A RECTANGLE FROM A POINT P, AT A DISTANCE h FROM ITS CENTER:

1. Set arrow on slide at the value h/a .
2. Read the solid angle opposite the value of h/b , using $\Omega 1$ scale with $h/b1$ scale, or $\Omega 2$ scale with $h/b2$ scale.

BACK

NOTE: Scale markings as shown are only approximate.

APPENDIX 1

Consider a rectangle, dimensions a and b, with a point P at a distance h from one corner. The solid angle $d\Omega$ due to an element of area $dx dy$ at (x, y) is:

$$d\Omega = \frac{dx dy \cos \theta}{r^2} . \quad (1)$$

$$\cos \theta = \frac{h}{r} ; r = \sqrt{x^2 + y^2 + h^2} .$$

$$\therefore d\Omega = \frac{dx dy h}{\sqrt{(x^2 + y^2 + h^2)^3}} . \quad (2)$$

Thus, the total solid angle due to the rectangle is:

$$\Omega = \int_0^a dx \int_0^b dy \frac{h}{\sqrt{(x^2 + y^2 + h^2)^3}} \quad (3)$$

This gives, for the value of Ω^*

$$\begin{aligned} \Omega &= h \int_0^a dx \left[\frac{y}{(x^2 + h^2) \sqrt{x^2 + y^2 + h^2}} \right]_0^b \\ &= h b \int_0^a \frac{dx}{(x^2 + h^2) \sqrt{x^2 + h^2 + b^2}} . \end{aligned} \quad (4)$$

* Equation No. 122, Handbook of Chemistry and Physics, 32nd Edition, Chemical Rubber Publishing Co., Cleveland, Ohio

Equation (4) is simplified by the substitution:

$$u = x^2 + h^2,$$

which gives for Ω :

$$\begin{aligned}\Omega &= \frac{hb}{2} \int_{h^2}^{h^2 + a^2} \frac{du}{u \sqrt{u - h^2} \sqrt{u + b^2}} \\ &= \frac{hb}{2} \int_{h^2}^{h^2 + a^2} \frac{du}{u \sqrt{u^2 + (b^2 - h^2)u - h^2b^2}}.\end{aligned}\quad (5)$$

This integral can be expressed in the form:

$$\Omega = \frac{hb}{2} \int_{h^2}^{h^2 + a^2} \frac{du}{u \sqrt{A + Bu + Cu^2}}.\quad (6)$$

Integrating this equation yields:*

$$\Omega = \frac{hb}{2} \left[\frac{1}{\sqrt{h^2 b^2}} \sin^{-1} \left(\frac{u(b^2 - h^2) - 2 h^2 b^2}{u \sqrt{(b^2 - h^2)^2 + 4 h^2 b^2}} \right) \right]_{h^2}^{h^2 + a^2} \quad (7)$$

$$\begin{aligned}\therefore \Omega &= \frac{1}{2} \sin^{-1} \left[\frac{(h^2 + a^2)(b^2 - h^2) - 2 h^2 b^2}{(h^2 + a^2)(b^2 + h^2)} \right] \\ &\quad - \frac{1}{2} \sin^{-1} \left[\frac{h^2(b^2 - h^2) - 2 h^2 b^2}{h^2(b^2 + h^2)} \right].\end{aligned}\quad (8)$$

$$\Omega = \frac{1}{2} \sin^{-1} \left[\frac{a^2 b^2 - h^2 (a^2 + b^2) - h^4}{(h^2 + a^2)(h^2 + b^2)} \right] + \frac{1}{2} \sin^{-1} 1.$$

$$\Omega = \frac{1}{2} \sin^{-1} \left[\frac{a^2 b^2 - h^2 (a^2 + b^2) - h^4}{(h^2 + a^2)(h^2 + b^2)} \right] + \frac{\pi}{4}.\quad (9)$$

* Equation No. 167, Handbook of Chemistry and Physics, 32nd Edition
Chemical Rubber Publishing Co., Cleveland, Ohio

This expression is equivalent to one of the expressions given by Murray and Schmidt.⁵ However, by means of a few substitutions, equation (9) can be put in a more convenient form.

$$2 \Omega - \frac{\pi}{2} = \sin^{-1} \left[\frac{2 a^2 b^2 - \{h^4 + h^2 (a^2 + b^2) + a^2 b^2\}}{(h^2 + a^2)(h^2 + b^2)} \right]; \quad (10)$$

$$\begin{aligned} \sin \left\{ 2 \Omega - \frac{\pi}{2} \right\} &= \frac{2 a^2 b^2}{(h^2 + a^2)(h^2 + b^2)} - 1 \\ &= \sin 2 \Omega \cos \frac{\pi}{2} - \cos 2 \Omega \sin \frac{\pi}{2}. \end{aligned} \quad (11)$$

$$\therefore \cos 2 \Omega = 1 - \frac{2 a^2 b^2}{(h^2 + a^2)(h^2 + b^2)} = 1 - 2 \sin^2 \Omega. \quad (12)$$

$$\therefore \sin^2 \Omega = \frac{a^2 b^2}{(h^2 + a^2)(h^2 + b^2)}; \quad (13)$$

$$\Omega = \sin^{-1} \frac{ab}{\sqrt{(h^2 + a^2)(h^2 + b^2)}}. \quad (14)$$

Equation (14) has the advantages of compactness and simplicity of calculation. For a point P at a distance h from the center of a rectangle, we have:

$$\Omega = 4 \sin^{-1} \frac{(a/2) (b/2)}{\sqrt{(a/2)^2 + h^2} \sqrt{(b/2)^2 + h^2}} \quad (15)$$

$$\begin{aligned} \sin \Omega/4 &= \frac{(a/2) (b/2)}{\sqrt{(a/2)^2 + h^2} \sqrt{(b/2)^2 + h^2}}; \\ &= \frac{1}{\sqrt{1 + \left(\frac{2h}{a}\right)^2} \sqrt{1 + \left(\frac{2h}{b}\right)^2}} \\ \csc \Omega/4 &= \sqrt{1 + \left(\frac{2h}{a}\right)^2} \sqrt{1 + \left(\frac{2h}{b}\right)^2}. \end{aligned} \quad (16)$$

Equation (16) gives the cosecant of $\Omega/4$ in terms of the product of 2 terms whose values depend on the ratio of h/a and h/b ; thus the solid angle may

be computed on a slide rule by adding logarithmic scales, as follows:

$$\log \csc \Omega/4 = \log \sqrt{1 + \left(\frac{2h}{a}\right)^2} + \log \sqrt{1 + \left(\frac{2h}{b}\right)^2} . \quad (17)$$

APPENDIX 2

SCALE MARKINGS FOR THE SLIDE RULE

The following tables list the marks and scale position for the solid angle slide rule scales. The marks comprising the scales were of 3 lengths, the length of each mark being indicated in the tables by asterisks as follows:

1. Shortest marks - no symbol.
2. Medium marks*.
3. Longest marks**.
4. Longest marks, with numerals***.

SCALE MARKINGS FOR Ω SCALES

Ω	$\log \csc \Omega/4$	Ω	$\log \csc \Omega/4$	Ω	$\log \csc \Omega/4$
$2\pi^{***}$	0	.88	.6611	.3 ^{***}	1.1256
5 ^{***}	.0243	.86	.6709	.29	1.1400
4.5	.0447	.84	.6810	.28	1.1552
4 ^{***}	.0750	.82	.6913	.27	1.1709
3.9	.0821	.8 ^{***}	.7019	.26	1.1874
3.8	.0897			.25 ^{**}	1.2044
3.7	.0977	.78	.7127	.24	1.2221
3.6	.1060	.76	.7239	.23	1.2406
3.5 ^{**}	.1149	.74	.7353	.22	1.2581
3.4	.1242	.72	.7471	.21	1.2800
3.3	.1340	.7 ^{***}	.7592		
3.2	.1443			.2 ^{***}	1.3012
3.1	.1551	.68	.7716	.195	1.3122
		.66	.7845	.190 [*]	1.3235
3 ^{***}	.1665	.64	.7977	.185	1.3350
2.9	.1784	.62	.8114	.180 [*]	1.3469
2.8	.1910	.6 ^{***}	.8255	.175	1.3591
2.7	.2042			.170 [*]	1.3717
2.6	.2181	.59	.8328	.165	1.3847
2.5 ^{**}	.2328	.58	.8402	.160 [*]	1.3980
2.4	.2482	.57	.8477	.155	1.4118
2.3	.2645	.56	.8553		
2.2	.2817	.55 ^{**}	.8631	.15 ^{***}	1.4261
2.1	.3000	.54	.8710	.145	1.4408
2 ^{***}	.3193	.53	.8791	.140 [*]	1.4560
		.52	.8873	.135	1.4718
1.95	.3294	.51	.8957	.130 [*]	1.4882
1.9 [*]	.3396			.125	1.5052
1.85	.3505	.5 ^{***}	.9042	.120 [*]	1.5229
1.8 [*]	.3615	.49	.9129	.115	1.5414
1.75	.3730	.48	.9219	.110 [*]	1.5607
1.7 [*]	.3848	.47	.9310	.105	1.5809
1.65	.3969	.46	.9403	.10 ^{***}	1.6021
1.6 [*]	.4096	.45 ^{**}	.9498		
1.55	.4226	.44	.9595	.098	1.6109
		.43	.9694	.096	1.6198
1.5 ^{***}	.4362	.42	.9796	.094	1.6290
1.45	.4502	.41	.9900	.092	1.6383
1.4 [*]	.4648				
1.35	.4800	.4 ^{***}	1.0008	.09 ^{***}	1.6479
1.3 [*]	.4958	.39	1.0117	.088	1.6576
1.25	.5122	.38	1.0229	.086	1.6676
1.2 [*]	.5294	.37	1.0345	.084	1.6778
1.15	.5474	.36	1.0458	.082	1.6883
1.10 [*]	.5662	.35 ^{**}	1.0585		
1.05	.5859	.34	1.0711	.08 ^{***}	1.6990
		.33	1.0840	.078	1.7100
1.0 ^{***}	.6066	.32	1.0973	.076	1.7213
.98	.6152	.31	1.1110	.074	1.7329
.96	.6239			.072	1.7447
.94	.6329				
.92	.6421				
.9 ^{***}	.6515				

<u>Ω</u>	<u>$\log \csc \Omega/4$</u>	<u>Ω</u>	<u>$\log \csc \Omega/4$</u>	<u>Ω</u>	<u>$\log \csc \Omega/4$</u>
.07***	1.7570	.025**	2.2041	.007***	2.7570
.068	1.7696	.0245	2.2128	.0068	2.7695
.066	1.7825	.024*	2.2218	.0066	2.7825
.064	1.7959	.0235	2.2310	.0064	2.7959
.062	1.8097	.023*	2.2403	.0062	2.8097
		.0225	2.2498		
.06***	1.8239	.022*	2.2596	.006***	2.8239
.059	1.8312	.0215	2.2696	.0059	2.8312
.058	1.8386	.021*	2.2798	.0058	2.8386
.057	1.8462	.0205	2.2903	.0057	2.8462
.056	1.8539			.0056	2.8539
.055**	1.8617	.02***	2.3010	.0055**	2.8617
.054	1.8697	.0195	2.3120	.0054	2.8697
.053	1.8778	.019*	2.3233	.0053	2.8778
.052	1.8861	.0185	2.3349	.0052	2.8860
.051	1.8945	.018*	2.3468	.0051	2.8945
		.0175	2.3590		
.05***	1.9031	.017*	2.3716	.005***	2.9031
.049	1.9119	.0165	2.3846		
.048	1.9208	.016*	2.3979		
.047	1.9300	.0155	2.4117		
.046	1.9393				
.045**	1.9488	.015***	2.4259		
.044	1.9586	.0145	2.4407		
.043	1.9686	.014*	2.4559		
.042	1.9788	.0135	2.4717		
.041	1.9893	.013*	2.4881		
		.0125	2.5052		
.04***	2.0000	.012*	2.5229		
.039	2.0110	.0115	2.5414		
.038	2.0223	.011*	2.5606		
.037	2.0338	.0105	2.5809		
.036	2.0454				
.035**	2.0580	.010***	2.6021		
.034	2.0706	.0098	2.6108		
.033	2.0836	.0096	2.6198		
.032	2.0969	.0094	2.6289		
.031	2.1107	.0092	2.6383		
.03***	2.1249	.009***	2.6478		
.0295	2.1322	.0088	2.6576		
.029*	2.1397	.0086	2.6676		
.0285	2.1472	.0084	2.6778		
.028*	2.1549	.0082	2.6882		
.0275	2.1627				
.027*	2.1707	.008***	2.6990		
.0265	2.1788	.0078	2.7100		
.026*	2.1871	.0076	2.7212		
.0255	2.1955	.0074	2.7328		
		.0072	2.7447		

SCALE MARKINGS FOR h/a AND h/b SCALES

h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$	h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$	h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$
0***	.0000	2.5**	.7075	8***	1.2050
.05	.0022	2.55	.7158	8.2	1.2157
.1*	.0085	2.6*	.7239	8.4	1.2260
.15	.0187	2.65	.7319	8.6	1.2362
.2*	.0322	2.7*	.7397	8.8	1.2462
.25	.0484	2.75	.7474		
.3*	.0668	2.8*	.7550	9***	1.2559
.35	.0866	2.85	.7625	9.2	1.2654
.4*	.1074	2.9*	.7698	9.4	1.2748
.45	.1288	2.95	.7770	9.6	1.2839
				9.8	1.2928
.5***	.1505	3***	.7841		
.55	.1722	3.1	.7980	10***	1.3016
.6*	.1937	3.2	.8114	10.5	1.3227
.65	.2149	3.3	.8244	11*	1.3429
.7*	.2357	3.4	.8372	11.5	1.3621
.75	.2559	3.5**	.8495	12*	1.3806
.8*	.2757	3.6	.8615	12.5	1.3983
.85	.2950	3.7	.8732	13*	1.4153
.9*	.3137	3.8	.8845	13.5	1.4316
.95	.3318	3.9	.8956	14*	1.4474
				14.5	1.4626
1***	.3495	4***	.9065		
1.05	.3666	4.1	.9170	15***	1.4774
1.1*	.3832	4.2	.9272	15.5	1.4916
1.15	.3993	4.3	.9374	16*	1.5054
1.2*	.4150	4.4	.9473	16.5	1.5187
1.25	.4302	4.5**	.9569	17*	1.5317
1.3*	.4449	4.6	.9663	17.5	1.5442
1.35	.4593	4.7	.9756	18*	1.5565
1.4*	.4732	4.8	.9846	18.5	1.5683
1.45	.4868	4.9	.9935	19*	1.5799
				19.5	1.5912
1.5**	.5000	5***	1.0022		
1.55	.5127	5.1	1.0107	20***	1.6022
1.6*	.5254	5.2	1.0190	20.5	1.6129
1.65	.5376	5.3	1.0272	21*	1.6234
1.7*	.5495	5.4	1.0353	21.5	1.6336
1.75	.5611	5.5**	1.0432	22*	1.6436
1.8*	.5724	5.6	1.0509	22.5	1.6533
1.85	.5835	5.7	1.0586	23*	1.6629
1.9*	.5943	5.8	1.0661	23.5	1.6722
1.95	.6050	5.9	1.0734	24*	1.6813
				24.5	1.6903
2***	.6152	6***	1.0807		
2.05	.6253	6.2	1.0948	25**	1.6991
2.1*	.6352	6.4	1.1085	25.5	1.7077
2.15	.6449	6.6	1.1218	26*	1.7161
2.2*	.6543	6.8	1.1347	26.5	1.7243
2.25	.6637			27*	1.7324
2.3*	.6728	7***	1.1473	27.5	1.7404
2.35	.6817	7.2	1.1594	28*	1.7482
2.4*	.6905	7.4	1.1713	28.5	1.7559
2.45	.6990	7.6	1.1828	29*	1.7635
		7.8	1.1940	29.5	1.7709

h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$	h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$	h/a	$\log \sqrt{1 + \left(\frac{2h}{a}\right)^2}$
30***	1.7782	100***	2.3010	370	2.8692
31	1.7924	105	2.3222	380	2.8808
32	1.8062	110*	2.3424	390	2.8921
33	1.8196	115	2.3617		
34	1.8325	120*	2.3802	400***	2.9031
35**	1.8451	125	2.3979		
36	1.8574	130*	2.4150		
37	1.8693	135	2.4314		
38	1.8808	140*	2.4472		
39	1.8921	145	2.4624		
40***	1.9031	150***	2.4771		
41	1.9138	155	2.4914		
42	1.9243	160*	2.5052		
43	1.9345	165	2.5185		
44	1.9445	170*	2.5315		
45**	1.9543	175	2.5441		
46	1.9638	180*	2.5563		
47	1.9731	185	2.5682		
48	1.9823	190*	2.5798		
49	1.9912	195	2.5911		
50***	2.0000	200***	2.6021		
52	2.0170	205	2.6128		
54	2.0334	210*	2.6232		
56	2.0492	215	2.6335		
58	2.0645	220*	2.6435		
		225	2.6532		
60***	2.0792	230*	2.6628		
62	2.0934	235	2.6721		
64	2.1072	240*	2.6812		
66	2.1206	245	2.6902		
68	2.1335				
		250**	2.6990		
70***	2.1461	255	2.7076		
72	2.1584	260*	2.7160		
74	2.1703	265	2.7243		
76	2.1818	270*	2.7324		
78	2.1931	275	2.7404		
		280*	2.7482		
80***	2.2041	285	2.7559		
82	2.2148	290*	2.7634		
84	2.2253	295	2.7709		
86	2.2355				
88	2.2455	300***	2.7782		
		310	2.7924		
90***	2.2553	320	2.8062		
92	2.2648	330	2.8195		
94	2.2742	340	2.8325		
96	2.2833	350**	2.8451		
98	2.2923	360	2.8573

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