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SHEAR STRENGTH OF SIMPLY SUPPORTED PRESTRESSED CONCRETE BEAMS HAVING WEB REINFORCEMENT

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SHEAR STRENGTH OF SIMPLY SUPPORTED PRESTRESSED CONCRETE BEAMS HAVING WEB REINFORCEMENT

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ABSTRACT

Tests were made on 24 simply supported, post-tensioned, end-anchored, prestressed concrete beams with and without web reinforcement. All of the beams were rectangular in cross section with over-all dimensions 6 inches by 12 inches by 10 feet-0 inch and were reinforced with straight wires bonded throughout their entire length. The beams were loaded at the third points on a 9-foot span. The variables included concrete strength, percentage of longitudinal steel, prestress in the steel, and percent of web reinforcement.

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The general direction of the investigation and supervision of the program were provided by Dr. Zwoyer.

The prestressing equipment and wire were furnished by Albuquerque Gravel Products Co., a former licensee of Prestressing Incorporated, San Antonio, Texas. Most of the testing equipment was developed by Dr. Zwoyer and myself.

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This report was written as a thesis in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering under the direction of Dr. E. M. Zwoyer. His assistance and helpful comments are gratefully acknowledged.

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CHAPTER I -- INTRODUCTION

1. Introduction

What is "Prestressed Concrete"? This question can best be answered by quoting the statement made by Dr. F. G. Thomas[1] in which he said, "Prestressing is the imposition of preliminary internal stresses to a concrete structure, before the working loads are applied, in such a way as to lead to a more favorable state of stress when these loads come into action."

During the last few years the use of prestressed concrete construction has become evident in this country, but the basic idea of prestressing concrete is not new. Although the development of prestressed concrete is credited to European engineers, American engineers conceived the principles of prestressed concrete in the nineteenth century, as disclosed in the patent applications of P. H. Jackson (1886),[2] but successful methods of prestressing concrete were not found until the 1920's.

The art of prestressed concrete has received its principal development, however, in recent years in Europe, where it has been made practically and economically usable in long-span structures through the pioneering work of Dischinger, Finsterwalder, and others, and most notably by Magnel and Freyssinet.[2]

European leadership in prestressed concrete construction seems to be due mainly to the relatively high cost and scarcity of materials, especially steel, and the low wage rates for and abundance of labor. In the United States, our mass-production methods and vast steel production have provided steel and many other materials in large volume at low cost, whereas labor has been both scarce and expensive. However, with recent developments of efficient prestressing methods, the use of prestressed concrete has become economical in the United States, and numerous structures have been built in this country as well as in Europe.

Although prestressed concrete is rapidly coming into its own in this country, the American engineer is still reluctant to use it in lieu of reinforced concrete because of a lack of knowledge of its behavior under load. What we know about the behavior and load-carrying capacity of a concrete structure as a whole has, in the large part, been derived from tests on individual members. It is generally felt that a thorough understanding of the basic units that make up a structure is a prerequisite to any further knowledge we may gain of the structure as a whole.

In January 1958, "Tentative Recommendations for Prestressed Concrete"[3] was reported by ACI-ASCE Joint Committee 323, covering recommended practices in design and construction of linear prestressing, flexural members, and buildings and bridges. This report constitutes a recommended practice, not a building code or a specification. Before these tentative recommendations or others like them are adopted as a code or specifications, a greater knowledge must be gained regarding mode of failure, ultimate strength, and general behavior under load of structural members.

In order to increase our knowledge of prestressed concrete, many research projects have been initiated both in the United States and in Europe. Much of this research is concerned with the load-carrying capacity of beams, since beams are more frequently prestressed than any other type of member.

Beams are generally considered to be flexural members; however, they sustain shear stresses as well as flexural stresses, and they may fail either in a region of pure flexure or in a region of combined shear and flexure. As a matter of fact, many beams are so loaded that there is no region of pure flexure and thus must fail in a region of combined shear and flexure. The proportions of these beams may be such that the shearing stresses do not affect the magnitude of the ultimate load; on the other hand, the presence of shear can lower the ultimate load and can even change the mode of failure if sufficient web reinforcement is not provided. This is true for both ordinary reinforced concrete and prestressed concrete.

In the case of ordinary reinforced concrete, the ultimate strength theories for beams failing in pure flexure are well established and have been verified by numerous comparisons with the results of tests. These theories can be extended to include prestressed concrete by incorporating the effects of the prestress force and the different stress-strain relationships of the steel. One such theory was developed by D. F. Billet and J. H. Appleton.[4] Theories so established must be verified by experiment.

The problem of shear strength in prestressed concrete beams is analogous to that in ordinary reinforced concrete, since there is very little difference between reinforced concrete and prestressed concrete beams in the cracked condition. Until very recently, the knowledge of the shear strength of reinforced concrete members and prestressed concrete members was wholly statistical. Developments in the last few years have advanced it to the semiempirical level,[5, 6, 7] but our knowledge of the effect that shear has on the ultimate strength of reinforced and prestressed concrete members is still far from complete.

2. Object and Scope of Investigation

Contemporary design philosophy has reverted to the earliest concept: design based on ultimate strength. In these terms, universality and efficiency in prestressed or reinforced-concrete design become synonymous with a thorough and rational understanding of the effect of shearing forces in such members. At present, the total elimination of all empirically determined factors seems far in the future and dependent on the solution of several attendant problems in connection with the state and distribution of stress in a cracked beam. On the other hand, it appears immediately possible to obtain a well-defined picture of the problem and anticipate a majority of the pitfalls of design with semiempirical theories covering the most common ranges of the variables.

A survey of the literature revealed only limited information on the ultimate shear strength of prestressed concrete beams.[6,7] This information pertained only to the ultimate shear strength of prestressed concrete beams without web reinforcement.

From our knowledge of ordinary reinforced concrete beams, it is known that the ultimate strength of beams failing in shear varies with the amount of longitudinal reinforcement, the quality of the concrete, the ratio of shear span to depth of beam, the geometric properties of the cross section of the beam, and the amount of web reinforcement, if any. Prestressed concrete beams have an additional variable because of the presence of initial stress in the steel. Another important difference between ordinary and prestressed concrete is the type and quality of the reinforcing steel. Steel bars ranging in diameter from 3/8 inch to nearly 1-1/2 inches with a working stress varying from 16,000 to 70,000 psi[8] are used in ordinary reinforced concrete, whereas flexible wires of small diameters, wire cables, and bars of diameters up to 1-1/4 inches, with working stresses as high as 140,000 psi, are used in prestressed concrete. High working stresses cannot be utilized in ordinary reinforced concrete because deflection and width of cracks would be excessive at working loads. Since the absence of cracks is one of the features of prestressed concrete, high-strength steel is acceptable and economical. Furthermore, high-strength steel is necessary since a certain amount of the initial prestress is always lost due to plastic flow, creep, shrinkage, and slip of reinforcement in the end anchorage at time of transfer. The small amount of prestress possible in low-working-stress steels would almost entirely be lost after a few months' time.

The tests carried out in conjunction with this investigation were planned with the following objectives:

1. To determine the mode and characteristics of shear failures in simply supported prestressed concrete beams with web reinforcement.

2. To determine the effect of the following variables on the ultimate shear strength:

a. Concrete strength

b. Amount of longitudinal steel

c. Prestress in the steel

d. Percent of web reinforcement

3. To determine the difference, if any, between the modes of failure and ultimate strength of rectangular beams with web reinforcement and rectangular beams without web reinforcement.

4. To substantiate "Tentative Recommendations for Prestressed Concrete."[3]

5. To establish a basis for future tests involving additional variables.

A total of 24 beams was tested. All of the beams were rectangular in cross section with over-all dimensions 6 inches by 12 inches by 10 feet-0 inch. Of the 24 beams tested, 16 beams had web reinforcement varying from 4.16 to 8.15 percent. The beams were loaded at the third points on a 9-foot span. Drawings and photographs of these beams appear in Figures 1 and 2.

3. Outline of Tests

The rectangular beams tested were reinforced in tension only; they had no compression steel, and two thirds of the beams had web reinforcement. The beams without web reinforcement were designed to fail in shear, and the beams with web reinforcement were designed to fail in flexure and to yield information on the effects of certain variables on this mode of failure. The properties of the specimens are given in Table I.

The concrete strengths varied from 2435-5045 psi with the majority falling between 2600 and 3500 psi. It was the original intent of this investigation to eliminate concrete strength as a variable and use a concrete with a design strength of 3000 psi in all beams tested. Since transit-mixed concrete had to be used in this investigation, and only 1/2-cubic-yard batches were required for each pour, consistent concrete strength was difficult to maintain.

The quantity of longitudinal reinforcement used varied from 0.289 to 0.870 percent. This range includes both underreinforced and overreinforced beams. The lower percentage approaches a limiting value that would result in shear failures for the beams with the type of reinforcement used without using a low-strength concrete and a low prestress. The upper value represents the maximum amount of steel that the beams and the accessory equipment could accommodate.

The initial prestress in the steel was varied from 0-127,000 psi with two thirds of the beams having the steel prestressed to either 60,000 or 120,000 psi. The eight beams considered as having zero prestress actually had steel stressed from 2000-3000 psi. This was considered to be the minimum value necessary to hold the reinforcement in a straightened position, yet low enough not to have an effect on the mode of failure. It was introduced in order to compare the behavior under load of beams reinforced with high-strength steel and insignificant prestress to the behavior of ordinary reinforced concrete beams, and to study the effect of gradually increasing the prestress.

The ratio of shear span to depth (a/d) was held nearly constant, varying only from 4.18 to 4.80. The consistency of the ratio of shear span to depth was due to the fact that all beams were rectangular, of the same cross section, all tested with third-point loading, and all beams had the same length of shear span.

One type of reinforcement was used in the tests; its characteristics are discussed in Section 6(e). Since the ultimate steel stress in beams failing in shear is generally below some limiting proof stress, the type of steel has little effect on the ultimate load for a given amount of steel, except that a steel with a large elastic range will permit a larger range of shear failures than a steel with a small elastic range. Several of the beams tested which failed in shear had steel stresses in the elastic range or slightly into the inelastic range. More tests would be required to verify experimentally the dividing line between shear and tension failures for the type of steel used.

The grout core in the beams shown in Figure 4 was considered exceptionally large for the size of the beam in which it was used, but it was considered necessary in order to provide adequate space around the wires for complete bonding of the grout.

4. Definitions*

Prestressed Concrete -- Concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the working loads are counteracted to a desired degree. In reinforced concrete the prestress is commonly introduced by tensioning the reinforcement.

Post-Tensioning -- A method of prestressing reinforced concrete in which the reinforcement is tensioned after the concrete has hardened.

Bonded Reinforcement -- Reinforcement bonded throughout its length to the surrounding concrete.

End-Anchored Reinforcement -- Reinforcement provided at its ends with the end anchorages capable of transmitting the tensioning forces to the concrete.

Cracking Load -- The external load which, according to design calculations, would cause cracking of the concrete.

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*The definitions and notations used in this report are, as far as possible, consistent with those recommended by the Joint ACI-ASCE Committee 323.[3]

Creep -- Inelastic deformation of concrete or steel, dependent on time and resulting solely from the presence of stress and a function thereof.

Shrinkage of Concrete -- Contraction of concrete due to drying and chemical changes, dependent on time but not on stresses induced by external loading.

Shear Span -- The portion of a beam between the reaction and the load over which shear is a maximum.

5. Notations

Cross-sectional constants

A_c = area of entire concrete section (steel area not deducted)

A_s = total steel area, steel area in simply reinforced section

c.g.c = center of gravity of entire concrete section

c.g.s = center of gravity of steel area

h = total depth of section

d = effective depth of section

k_{ud} = depth of neutral axis at ultimate load

b = width of rectangular section

y_b, y_t = distance of bottom (top) fiber to c.g.c

e = eccentricity of c.g.c

I_c = moment of inertia of entire concrete section about c.g.c

Z_b, Z_t = section modulus of bottom (top) fiber, referred to c.g.c

n.a. = neutral axis of cracked section

Loads

w_D = total dead load per unit length

P_{cr} = total concentrated load at first cracking

P_{ult} = total concentrated ultimate load

M_{cr} = bending moment due to cracking load

M_D = bending moment due to w_D

M_{ult} = bending moment due to ultimate load

V = shear

V_{ult} = shear due to ultimate load

Notations relating to prestressing only

F_i = initial prestress force

F_o = prestress force after release

F = effective prestress force after deduction of all losses

Stresses

Concrete

f'_c = cylinder strength at age of test

f'_{ci} = cylinder strength at the age of prestressing

f_c = compressive stress generally

f^b_{Fi}, f^t_{Fi} = stress at bottom (top) fiber due to initial prestressing only

f^b_F, f^t_F = stress at bottom (top) fiber due to effective prestressing only

f^b_D, f^t_D = stress at bottom (top) fiber due to total load w_D only

f^b_{FD}, f^t_{FD} = stress at bottom (top) fiber due to effective prestressing, F, and total dead load

f_{cu} = compressive stresses at ultimate load

f_t = tensile stress generally

f_{tp} = permissible tensile stress

f_{tc} = tensile stress at cracking load

f_r = modulus of rupture

v = shearing stress

E_c = modulus of elasticity of concrete

Steel

f'_s = ultimate strength of steel

f_{sp} = permissible tensile stress

f_s = steel stress generally

f_{si} = steel stress due to initial prestressing

f_{so} = steel stress due to prestressing after release

f_{se} = steel stress due to effective prestress force after deduction of all losses

f_{su} = steel stress at failure

f_y = yield stress of mild steel

E_s = modulus of elasticity of steel

Strains

Concrete

ϵ_c = compressive strain generally

ϵ_{ce} = compressive strain in concrete at the level of steel due to effective prestress

ϵ_u = ultimate compressive strain of concrete

ϵ_{cu} = strain in concrete adjacent to the steel at point of maximum moment

$K_1\epsilon_u$ = average compressive strain on top surface of beam over length of the shear span for ultimate load

$K_2\epsilon_{cu}$ = average strain in the concrete at the level of the steel over the length of the shear span for ultimate load

Steel

ϵ_{se} = steel strain at effective prestress

ϵ_{su} = ultimate steel strain at point of maximum moment

ϵ_y = steel strain of mild steel at first yielding

Miscellaneous

k_1 = ratio of area of the true stress block for ultimate load to the area of a rectangle of base, $k_3f'_c$, and altitude k_ud

k_2 = ratio of the depth of the centroid of the stress block to the depth to the neutral axis

k_3 = ratio of strength of concrete in the beam to the strength obtained from tests of standard 6- by 12-inch cylinders

$K = K_1/K_2$

$Q^1 = E_sp/k_1k_3f'_c$

$k_u = f_{su}p/k_1k_3f'_c$

CHAPTER II -- MATERIALS, FABRICATION, AND TEST METHODS

6. Materials

(a) Cements

Ideal Type I portland cement, manufactured by the Ideal Cement Co., El Paso, Texas, was used in all 24 of the post-tensioned beams. Ideal Type III portland cement was used in the grout mixtures for the beams.

(b) Aggregates

The coarse aggregates used were Rio Grande River gravels. The fine aggregate used in casting the beams was a Rio Grande River sand. The maximum size gravel used in the beams was 1 inch. The fine aggregate used in the grout mixtures of beams S-1 to S-13 was a fine plaster sand obtained from Albuquerque Gravel Products. The fine aggregate used in the grout mixtures of beams S-14 to S-24 was Ottawa Standard silica sand. The major constituents of the coarse aggregate were quartzite and limestone. The sand consisted largely of quartz. The absorption of both fine and coarse aggregate was about 2 percent by weight of surface-dry aggregate. Sieve analyses of the aggregates used are given in Table II.

(c) Concrete Mixtures

Concrete mixtures were designed by the Albuquerque Gravel Products Co. The average cylinder strengths obtained ranged from 2600-5045 psi. The beams were cast with 1-inch maximum size gravel, sand, and Type I portland cement; these mixtures were designed to have a slump of 1-1/2 to 3 inches. The concrete strengths are based on standard 6- by 12-inch control cylinders. Table III contains the proportions of the mixes, slumps, and compressive strengths for the concrete used in each beam.

(d) Grout Mixtures

The grout used in the beams to provide bond between the wire and the surrounding concrete was a mixture of equal proportions by weight of fine plaster sand or Ottawa Standard and Type III portland cement, with water-cement ratios as given in Table II.

Previous tests of beams reported in the First Progress Report of the Investigation of Prestressed Concrete for Highway Bridges[9] indicated that the shrinkage of the grout mixtures not containing aluminum powder was excessive and resulted in bond failures. No aluminum powder was added to the grout used on beams S-1 to S-3, and bond failures occurred in these when tested. About 4 grams of aluminum powder per 100 pounds of cement was added to the plaster sand grout used in beams S-4 to S-13.

Since bond failures continued to occur, it was decided to use Ottawa Standard silica sand in the grout for the remaining beams. About 4 grams of aluminum powder per 100 pounds of cement was used to grout beams S-14, S-15, S-18, and S-19. About 15 pounds of a commercial nonshrink grout known by the trade name of "Embeco" was added to the grout used in beams S-16, S-17, and S-20 to S-24. The consistency of the grout when pumped into the beam was that of a thick fluid. The problem of bond failures in the beams is explained further in Section 10.

Four 2- by 4-inch cylinders were cast from the grout mix used in each beam and tested at ages of 3-6 days. The average compressive strengths of these cylinders are given in Table IV.

(e) Reinforcing Wire

High-tensile-strength wire was used as tension reinforcement for the beams. The wire was of the high-carbon, cold-drawn, stress-relieved type and had a maximum ultimate strength of 225,500 psi.

The wire used was manufactured by Union Wire Rope, a subsidiary of Armco Steel Corporation. The wire was 0.250-inch diameter and is manufactured under the trade name of "Tufwire." The following steps were involved in its manufacture: (1) hot rolling, (2) heat treating, (3) cooling in air, lead, or other media, (4) cold drawing to 0.250-inch diameter, and (5) stress relieving. The heat treating was done to give the wire a distinctive and uniform microstructure prior to cold drawing. The wires were cold drawn or cold worked to a diameter tolerance of ±0.001 inch. It is the cold working of the metal that nearly doubles the tensile strength of the raw product to the finished product.

The wire was composed of the following chemical constituents:

Carbon, percent, .72 to .93
Manganese, percent, .40 to 1.10
Phosphorus, maximum percent, .040
Sulphur, maximum percent, .050
Silicon, percent, .10 to .35

To improve the bond characteristics, the surfaces of all wires were rusted slightly by placing the wires in a moist room for several days. This produced a slightly pitted surface which improved the bond characteristics. All wires were cleaned with a wire brush to remove loose rust.

Nine specimens of the wire were tested in a 60,000-pound-capacity Reihle hydraulic testing machine, and the strains were measured with a Berry-type strain gage having a gage length of 2 inches. The strain gage had a range of 4 microinches. All readings were recorded manually. The stress-strain characteristics of the various specimens were so similar to each other that an average stress-strain diagram was used in all computations. The stress-strain characteristics of the wire are shown in Figure 5.

7. Description of Specimens

The concrete beams tested were post-tensioned rectangular beams. Only longitudinal tension reinforcement, bonded to the concrete, was used; it extended in a straight line between the ends of the beams. The properties of the reinforcement are described in Section 6(e).

The rectangular beams were nominally 6 by 12 inches in cross section and 10 feet long as shown in Figure 1. Although the beams were cast in metal forms, the dimensions of the beams varied slightly. The measured dimensions of the beams are given in Table I.

The beams were cast with a hole, circular in cross section, in the lower part of the beam to provide a channel for the reinforcement. As explained in Section 11, this hole was later filled with grout to provide bond between the reinforcement and the surrounding concrete. The hole for beams S-1 to S-3 was approximately 1-1/2 inches in diameter with its center 8-1/2 inches below the top of the beam. The hole was formed using 1-1/2-inch-diameter B-X tubing. The hole for beams S-4 to S-8 was approximately 1-1/4 inches in diameter with its center 8-1/2 inches below the top of the beam. The hole was formed with the aid of a core consisting of a 1-1/4-inch-outside-diameter Neoprene tube pulled over a 7/8-inch rod. The hole for beams S-9 to S-24 was approximately 3-3/16 inches in diameter with its center 8 inches below the top of the beam. The hole was formed with the aid of a core consisting of eleven 5/8-inch rubber tubes placed longitudinally around the perimeter of a 1-1/4-inch pipe over which was pulled a 3-inch-diameter rubber tube which had a 3/16-inch wall thickness. The rod was held in position by a steel template at each end of the beam. The form was easily removed from the beam by first pulling out the rod or pipe and then removing the tubes. A 5/8-inch-diameter access hole from the top of the beam to the grout channel was located about 1 foot from one end of the beam. The access hole was formed with a 5/8-inch-diameter tube with a 1/4-inch rod down the center of the tube. After the reinforcement was placed in the channel and initially stressed, grout was pumped through a hole in the prestress hardware at one end of the beam until it came out the access hole at the other end of the beam. Details of the grout channel and access hole are shown in Figures 3 and 4.

The method used to stress and anchor the reinforcement in the beams is explained in Section 10.

8. Web Reinforcement

The web reinforcement used in the beams consisted of 1/4-inch-diameter smooth stirrups at 4 inches on center and 3/8-inch-diameter, deformed stirrups at 4-1/2 inches on center. The stirrups were 4- by 10-inch closed-type stirrups and were caged prior to being placed in the form by tack welding the top of each stirrup to two 1/4-inch round continuous rods. The stirrups were caged in this manner to insure that the stirrups remained vertical after vibration of the concrete. The placement of the stirrups is as shown in Figure 3.

9. Casting and Curing

Beams S-1 through S-8 were cast at the same time in groups of three at Albuquerque Gravel Products batch plant. These beams were cast from a 3/4-cubic-yard batch which was dry batched at the plant and then mixed in a rotary drum transit-mix truck. Beams S-9 through S-24 were cast one at a time in the University Strength of Materials Laboratory. These beams were cast from a 1/2-cubic-yard batch of transit-mix concrete.

Transit-mix concrete was used due to the fact that a concrete mixer of adequate size was not available at the University. The use of ready-mix concrete also guaranteed that the concrete in each beam would be uniform throughout.

The beams were cast in the following manner: The forms were cleaned and oiled. The grout channel form was then made up and inserted through the stirrup cage. The stirrup cage and the grout channel form were then placed in the form and held in place by circular holes in the end plates of the forms. The top of the stirrup cage was then tied with wire to cross bars on the top of the form to insure proper clearance of the stirrups in the beam. The concrete was placed in approximately three equal lifts, and each lift was vibrated with a mechanical vibrator, being careful not to disturb the alignment of the stirrups. The concrete on the top of the beam was screeded off flush with the top of the form and given a smooth trowel finish.

Five 6- by 12-inch cylinders were cast with each beam and were also vibrated with the mechanical vibrator. Three of the cylinders were tested 2 or 3 days before testing the beam in order to determine the strength of the concrete in the compression zone so that the ultimate load capacity of the beam could be estimated prior to testing. The remainder of the cylinders were tested the same day as the beam.

Beams S-1 to S-8 were cast at Albuquerque Gravel Products and were not moist-cured due to a mechanical failure of sprinkler and pumping equipment. Beams S-9 through S-24 were cast at the University. These beams were removed from the forms 2 or 3 days after being cast and were placed in a constant-temperature moist room. After remaining in the moist room for at least 21 days, they were removed and stored in the laboratory until they were tested.

Lifting hooks of 1/4-inch-diameter wire located 18 inches in from each end of the beam were cast into the beams. An A frame and a four-wheel dolly were used to transport the beams about the laboratory. A 3/4-inch rod threaded at both ends was placed through the grout channel in each beam and tightened up against bearing plates to provide reinforcement to the beams when lifting them out of the forms. It was necessary to handle the beams with extreme care since no reinforcement was provided for the dead load and impact stresses developed when the beams were being handled prior to prestressing. Two beams were broken and had to be recast.

10. Prestressing

(a) End Details of Wires

One of the major problems in the construction and testing of prestressed concrete beams is the development of suitable equipment for tensioning and properly anchoring the highly stressed reinforcement of the beams. In post-tensioned beams, three major classes of anchorages are used in practice: (1) wedge grips, (2) rivetlike heads formed on the ends of the wires, and (3) threaded connections of the ends of the wires.

Section B of First Progress Report of the Investigation of Prestressed Concrete for Highway Bridges[9] contains a discussion of the relative merits of the various methods used to anchor the ends of highly stressed reinforcement of the beams. The wires used throughout these tests were threaded at one end and had button heads at the other end. Since all of the reinforcement was bonded, it was necessary only to develop the prestress force in the end anchorage. Any increase in stress in the reinforcement was transferred to the surrounding concrete through bond and not through anchorage. The advantages of threaded connections not necessarily realized through the use of wedges or rivetlike ends are: (1) simplicity in anchoring the wires, (2) compact arrangement of the wires with a relatively small spacing between them, and (3) practically no loss of prestress when the stress in the wire is transferred from the jack to the bearing plate.

Specially heat-treated, 1/4-inch, 28-threads-to-the-inch chasers in an automatic threading machine were used to cut the threads on the end 2-1/2 inches of the wires. Even though the chasers were heat treated and had a hardness of Rockwell C-60 to C-65 as compared to the wire which was about Rockwell C-40, the chasers became dull after threading 50 to 60 wires and required resharpening. The threads on the wires were cut to provide a medium fit with the threads in the nuts. The nominal diameter of the wire was 0.250 inch.

The nuts were specially made in the University machine shop. The nuts were made from 1/2-inch hexagonal mild steel bar stock and were cut in 5/8-inch lengths. The nuts were subdrilled with a No. 3 tap drill and tapped with a standard 1/4-inch, 28-threads-to-the-inch tap. The nuts were hardened by the "Cyanide Process," using the following procedure: (1) The nuts were heated until cherry red with an acetylene torch. (2) The nuts were then rolled in a box containing powdered cyanide until thoroughly coated. (3) The nuts were then quenched in a bucket of cold water.

The hardening of the nuts by the above process proved quite satisfactory. Five specimens of nuts and wire were tested in a 60,000-pound-capacity Reihle testing machine. Failure of the specimens tested occurred by rupture of the wire and not stripping of the threads on all five specimens.

(b) <u>Tensioning Apparatus</u>

The tensioning equipment used was the type developed by Prestressing Incorporated of San Antonio, Texas, and furnished by Albuquerque Gravel Products, a licensee of Prestressing Incorporated.

The tensioning equipment consisted of: (1) a 60-ton Simplex center-hold hydraulic ram operated by a 10,000-psi-capacity Simplex pump to tension the reinforcement, (2) a jacking frame made of 5/8-inch steel plate, (3) a pulling rod and eight-wire stressing adapter, (4) an eight-wire stressing assembly, and (5) anchorage assembly, split holding rings, and bearing plates. Details of the complete assembly and the stressing assembly are shown in Figure 6.

The jacking frame rested against the bearing plate and was held in place by the pulling rod over which the jacking frame and the jack were placed, and secured by a nut on the end of the pulling rod. The jacking frame resting against the bearing plate provided a reaction for the jack;

the bearing plates reacted against the beam. To tension the wires, the ram reacted against the frame and the 1-3/4-inch pulling rod. The thrust was transferred from the ram to the rod through a nut, and from the rod to the wire through the stressing adapter and stressing assembly. When the wire was tensioned to the desired stress, shim plates of the correct thickness were inserted between the anchorage assembly and the split holding rings. The load on the jack was bled off, and the shims were held in place by the prestressing force on the anchorage assembly. The shims varied in thickness from 1/8 to 1/2 inch and were as shown in Figure 6. Figure 9 is a photograph of the apparatus in place during the prestressing of a beam.

The bearing plates used at the end of the beam where the dynamometer was placed are shown schematically in Figure 7, and in place in Figure 10. The 6- by 6- by 1-1/2-inch plates are heavy enough so that a fairly uniform bearing pressure is produced on the ends of the beam. The heavy bearing plates were used in order to eliminate the need for special reinforcement near the ends of the beam and proved to be satisfactory in this respect.

(c) Measurement of Tensioning Force

The tensioning force in the wires was determined simultaneously by measuring the compressive strain in 3-1/2-inch-diameter cylindrical aluminum dynamometers placed around the wires between the nuts, the 4-1/2- by 4-1/2-inch bearing plate, and the 6- by 6-inch bearing plate at the end of the beam opposite that at which the tension was applied. These dynamometers are shown in Figures 6 and 7. They consisted of a 3-1/2-inch-outside-diameter aluminum cylinder 2 inches in height and 1/4-inch wall thickness. Strains were measured by means of four Type A-7, SR-4 electric strain gages. These gages were mounted on the outside wall of the dynamometer, two gages horizontal and two gages vertical, and placed diametrically opposite each other. The gages were wired in series with the vertical gages giving a strain reading which was the average of the two gages while the horizontal gages compensated for the Poisson effect, provided temperature compensation, and increased the signal output by 33 percent. The dynamometer rested in a 1/8-inch recessed groove in each bearing plate, thereby eliminating the inducement of small eccentric loads when the wires were tensioned. The dynamometers were calibrated using the 60,000-pound range of a 60,000-pound-capacity Reihle hydraulic testing machine. The calibration of the dynamometers was nearly the same; the strain increment necessary to measure 120,000 psi in eight 0.250-inch wires was about 240 x 10^5 microinches. This large increment of strain allowed a fairly precise measurement of stress in the wires, since the strain indicator had a sensitivity of 3000 x 10^5 microinches.

At the dynamometer end of the wires, the prestress was transferred from the wire to the dynamometer through the nuts to the 4-1/2- by 4-1/2-inch bearing plate and from the dynamometer to the beam through the 6- by 6-inch bearing plate. Figure 7 shows a schematic drawing of this arrangement.

(d) Tensioning Procedure

Prior to inserting the wires into the grout channel, the 6- by 6- by 5/8-inch bearing plate was secured to one end of the beam with a thin layer of gypsum plaster. Before inserting the wires into the grout channel, the threaded end of each wire was threaded through the dynamometer bearing plate and secured with a nut. The anchorage housing was then placed behind the second button head and the insert slipped into place, being sure to keep the wires in their respective positions. The wires were then wiped down with acetone, and all of the wires, button head end first, were pulled through the beam. The dynamometer bearing plate was then secured to the beam with a thin layer of gypsum plaster. The nuts were then removed, the dynamometer was slipped over the wires, the end dynamometer bearing plate was placed over the wires, and the nuts were put back on each wire. At the button head end of the wires, the split holding ring was inserted between the anchorage assembly and the bearing plate, and the nuts at the opposite end were tightened with a wrench until the split holding ring was secured in place and the curvature in the wires had been removed. Before commencing to tighten the nuts, the dynamometer was wired to the Young SR-4 strain indicator in order to determine the amount of stress induced into the wires prior to tensioning with the jack. The stressing assembly was placed in position around the wires and pulled up snug against the first button head. The stressing adapter and pulling rod were threaded onto the stressing assembly after which the jacking frame and center-hole ram were slipped over the pulling rod and secured in place with the 1-3/4-inch nut threaded onto the pulling rod. The tensioning force was applied to the ram by the hydraulic pump; after the desired prestress had been applied, the split shims were inserted between the anchorage assembly and the split holding rings, after which the load on the ram was released. Since all of the wires were tensioned simultaneously, it was assumed that the prestress in each individual wire was approximately the same.

11. Grouting

Following the tensioning of the reinforcement, grout was pumped into the beam to provide bond between the wires and the surrounding concrete. The grout was pumped into the grout channel through the hole in the anchorage assembly located at the button head end of the beam as shown in Figure 11. Pumping was continued until grout was forced out the 5/8-inch hole in the top of the beam at the opposite end.

The grout pump is shown in Figure 11. It was constructed of a 6-inch-diameter cylinder about 36 inches long with a piston welded to a threaded shaft and driven by a handwheel. The piston apparatus was made by welding a 5-7/8-inch-diameter piston head onto an irrigation-gate handwheel assembly and was attached to the cylinder with six 3/8-inch cap screws which screwed into the top flange of the cylinder. A hole was cut in the bottom of the cylinder, and a 3/4-inch coupling was welded to the cylinder, into which was screwed a 3/4- by 6-inch pipe, which was reduced to 1/2 inch to fit a 6-foot piece of garden hose that had a 1/2-inch piece of pipe attached to the end. The grout was poured into the pump through a spout at the top of the cylinder and pumped out by forcing the piston downward by turning the handwheel in a clockwise direction. Before the pump could be refilled, the piston had to be returned to its uppermost position, which made the grouting operation rather lengthy.

It was found after grouting beams S-7 and S-8 that it was necessary to plug with gypsum plaster the holes in the dynamometer bearing plates not occupied by wires, as the dynamometer would become filled with grout. The grout expanded upon setting and greatly reduced the strain reading on the dynamometer.

The grout was mixed in a countercurrent, horizontal tub mixer of 1-cubic-foot capacity. The proportions of the grout mixture and the properties of the materials are shown in Table IV.

12. Electric Strain Gages

(a) Gages on the Reinforcing Wire

Due to the erratic strain readings obtained, SR-4 strain gages were attached only to the wires of beams S-1 through S-8. The strain gage used was the A-7 gage which had a nominal gage length of 1/4 inch and a minimum trim width of 3/16 inch; it was chosen for its narrow width and short length.

The surface of the wire was prepared for the gage by removing the rust with emery cloth and cleaning the surface thoroughly with acetone. Duco cellulose acetate cement was used as the bonding agent. The gage was attached to the wire by placing the gage on the cement and pressing the gage down with the forefinger and middle finger to fit the curvature of the wire. The gage was then held in this manner for about 10 minutes until the glue had set up. This method was quite simple and worked well since the pressure from the two fingers combined forced the excess glue out from under the gage. The gage was then allowed to air-dry for 24 hours, after which the resistance to ground of the gage was checked with a voltmeter to see if the gage was well bonded.

Three different types of materials for waterproofing the strain gages on the wires were tested: (1) Armstrong A-1 cement, (2) 3M adhesive EC-1294, and (3) Seymours Clear Plastic. Armstrong A-1 cement is produced by Armstrong Products Company, Adhesives Division, Warsaw, Indiana. It consists of an epoxy resin base and an activator which form an extremely strong and waterproof coating when cured. The adhesive is self-polymerizing at room temperature. It will develop half its strength in 16 hours at room temperature and full strength after about 1 week. By heating to 160°F, the curing time could be shortened to 1 hour, but since the gages were wrapped with rubber tape, no heat could be applied. The major objection with this cement was drying time.

EC-1294 is manufactured by Minnesota Mining and Manufacturing Co., Detroit 2, Michigan. It consists of an epoxy resin and an EC-1468 activator, which, when cured, has high shear strength, good acid and alkali resistance, and is water resistant. For the first 2 hours the material is quite viscous and tends to run off a curved surface such as the wire. Curing time at room temperature is 24 hours. After testing the wire in tension, the EC-1294 was found to be cracked due to the tension applied to the wire.

Seymours Clear Plastic Spray is produced by Seymour of Sycamore Inc., Sycamore, Illinois. It was composed of a clear plastic material and lacquer and came prepared in a spray can applicator. The clear plastic spray had to be applied in several applications as it was a thin liquid, but, since it dried in about 1 hour, this was no problem. After the wire had been tested in tension, no cracking was found, indicating good ductility. This material was used to waterproof the gages on the wires.

Before being sprayed with the waterproofing material, the gages were first padded with several layers of gauze and wrapped with three layers of Minnesota Mining and Manufacturing "3M" electrical tape.

The lead wires were attached to the strain-gage lead wires with solder and then taped to the prestress wire so that they could not move and break the waterproof seal.

The lead wires from the gages were carried down the grout channel to the end of the beam where they were brought out of one of the wire holes in the anchorage assembly. After the wires had been inserted in the beam and secured, one gage out of every three mounted on the wire was found to be inoperative. This was caused by the gages being scraped against the walls of the grout channel due to the initial curvature in the wires when they were inserted in the beam.

(b) Gages on the Concrete Surface

Shortly before the initial set of the concrete occurred, the top surface of the beam was struck smooth with a finishing trowel. When this surface was later ground smooth and polished with a portable grinder, it was suitable for mounting SR-4 gages. Only the small area to be occupied by the gage was ground.

From the results of previous tests conducted on ordinary reinforced and prestressed concrete beams, it was decided that the Type A-1, SR-4 electric strain gage was the best suited for measuring the strain in the concrete on the top surface of the beams. The A-1 gage has a nominal gage length of 13/16 inch and accurately described the distribution of strain on the top surface of the beam.

The ground surfaces on the beam where the gages were to be mounted were first sealed with two applications of Duco cement to prevent any moisture that might possibly be in the top of the beam from affecting the gages. The sealed surfaces were cleaned with acetone, and the gages were attached with Duco cement. Light weights were applied to the gages while the cement was drying. Heat was not used to hasten the drying period since it can be detrimental to the concrete.

13. Strain and Deflection Measurements

Strains were measured on the steel reinforcement of beams S-1 to S-8 only and on the top of concrete on all beams. Deflections were measured at midspan and under the load points in all beams.

The strains in the concrete on the top surface of the beams were measured with Type A-1, SR-4 electric strain gages as described in Section 12(b).

No attempt was made to measure the strains along the vertical sides of the beams.

Strains in the steel reinforcement were measured with Type A-7, SR-4 electric strain gages as described in Section 12(a).

Deflections were measured at midspan with a Reihle 0.0005-inch dial indicator and the load points with Ames 0.001-inch dial indicators. To be sure that the dial indicator push rods were constantly in contact with a smooth surface, 1/8- by 1- by 4-inch steel plates were glued to the bottom of the beams at load points and midspan. Before testing commenced, a gage line was marked on the beam at midspan. This gage line was used for measuring the deflections of the beam with a yardstick in advanced stages of loading after the dial indicators had been removed. Although the dial indicators could have been used up to ultimate load, it was decided to remove them in the advanced stages of loading to prevent them from being damaged when the beam ruptured.

Strains in the dynamometers were measured at various increments of load to determine if any increase in stress in the wires occurred at the end of the beam, indicating possible bond failure.

Cracks were marked at each load increment, and the development of each crack was recorded by marking the number of the increment at the top of the crack. Photographs were made of the beams, showing the crack patterns after failure had occurred.

14. Testing Machine and Loading Apparatus

All of the beams were tested in an 80,000-pound-capacity closed rectangular loading frame. The loading frame is shown in Figure 1. The load was applied with a 30-ton hydraulic jack and measured with a 60,000-pound-capacity load cell.

The load cell consisted of a 4-1/2-inch aluminum alloy cylinder which had a wall thickness of 3/8 inch and 5-1/2-inch diameter with 1-inch steel bearing plates, top and bottom. The load was measured by means of four A-1, SR-4 electric strain gages mounted in the same manner as used on the dynamometers, except that the strain gages were mounted on the inside surface of the cylinder. The lead wires were soldered to the gages and secured to the wall of the cylinder with Duco cement, after which the gages were padded with gauze, covered with "3M" electrical tape, and sprayed with Seymours Clear Plastic Spray to make them waterproof. The bearing plates were attached to the cylinder with eight 6-32 flathead machine screws which were tapped into the cylinder walls. The lead wires were lead out of the load cell through two 3/8-inch-diameter semicircles located at the bottom of the load cell. The load cell is shown in Figure 6.

The load cell was calibrated in a 60,000-pound-capacity Reihle testing machine at 500-pound increments. The equivalent strain reading on the load cell for a load of 500 pounds was 2.5 microinches x 10^5, and the change in strain for each load varied as a straight line.

The loading apparatus is shown in Figure 1. The load blocks were steel plates 6 inches by 2 inches by 6 inches. Two pieces of leather were inserted between the beam and each load block so as to leave a surface about 2 inches wide along the centerline free of contact with the steel. This measure was taken for the correct functioning of the electric strain gage to be applied at that location. The leather pieces were 4 inches long in the direction of the span and 1/4 inch thick. The ball and roller combination provided for the movements that occurred at the bearings during the latter stages of the tests when the deflections were large.

15. Test Procedure

The tests were made with about 10 increments of load, with deflection readings at the third points and at midspan between each increment. Strain and deflection readings were recorded, and cracking was observed and marked on the side of the beam. Usually there were about three equal increments of load up to the cracking load. Thereafter, increments were based on strain and deflection readings rather than load. A certain amount of dropoff in load and increase in deflection occurred while strain and deflection readings were being recorded. The maximum load, which occurred when the jacking was stopped, was recorded, and the deflection readings were taken immediately. The beams were loaded until they ruptured completely and would no longer carry any appreciable load. The length of time required to test the beams usually varied from 2 to 3 hours.

CHAPTER III -- ANALYSIS AND CALCULATION OF ULTIMATE LOADS

16. Discussion of Modes of Failure of Bonded Beams

A knowledge of the modes of failure is one of the most important results to be gained from ultimate load tests of concrete structural members. Some types of failures are ductile and progress gradually; others are brittle and progress rapidly. Codes usually provide a higher factor of safety against the latter type of failure since they are generally considered to be more dangerous and less desirable.

(a) Flexure Failures

Flexure failures may be grouped into three categories: (1) failure by fracture of the reinforcement, (2) failure by crushing of the concrete after the steel has entered the inelastic range, and (3) failure by crushing of the concrete while the steel strain is still in the elastic range. Beams in the first two groups are underreinforced, and beams in the third group are overreinforced. The range of the percentage of steel, for a given concrete strength, that may be used in a beam for each of the three types of failure depends on the stress-strain characteristics of the steel. That is, for steels with a large ultimate elongation, the possibility of fracturing the reinforcement is less than for steels with a small ultimate elongation.

In general, concrete beams are "elastic" prior to cracking regardless of whether they are overreinforced or underreinforced; that is, the load may be applied repeatedly without altering the behavior of the beam. The load-deflection curve is a straight line and has the same slope each time the load is applied. The presence of prestress in a beam increases the cracking load and thereby extends this range of elasticity.

If the beam is grossly underreinforced, the reinforcement may rupture as soon as the concrete cracks and transfers all of the tension to the steel. With a little more reinforcement, the steel will not rupture until after considerable deformation of the beam and crushing of the concrete.

After first cracking of the beam and before the steel becomes inelastic, the beam still behaves "elastically," but its stiffness has changed. As a load is applied to the beam, the cracks open, and, when the load is removed, the cracks close, or nearly close, depending on the amount of prestress in the beam. The presence of prestress causes the cracks to close before the load is completely removed. Upon reloading, the cracks will open at a load below the original cracking load because the concrete no longer carries any of the tension. After the steel becomes inelastic, the beam will continue to deform with little or no increase in load. When the compression zone is destroyed by crushing of the concrete, which occurs at some limiting value of strain, the beam loses its capacity to carry load. In ordinary reinforced concrete beams, the load may fall off slowly, especially if compression reinforcement is used in the beam. In prestressed concrete beams, the compression zone seems to explode when it has crushed to the extent that it can no longer carry the maximum load. This is caused by the release of the energy stored in the steel due to tensioning of the steel. If the beam were unloaded after the strain in the steel has become inelastic but before the concrete crushes, there would be some permanent deformation. However, if the prestress has not been completely relieved by the inelastic deformations, the cracks may still close entirely. This type of failure is designated as an initial tension failure with a final compression failure.

In overreinforced beams, the steel is not stressed into the inelastic range, as is the concrete. The load may be increased until the strain in the concrete reaches its limiting value and the concrete starts to crush. The load may still be increased as crushing progresses, but, when crushing has developed to the extent that the compression zone can no longer carry the load, the beam fails. The ultimate deformation of such overreinforced beams, other than the concrete strain in the region of crushing, is small. The steel strains are still in the elastic range. This type of failure is the least ductile of all flexure failures and can occur with little warning because the ultimate deflection is small, and the crack widths are not large if the bond is good. It is designated as an initial compression failure.

Typical flexure failures are shown in Figures 12, 13, 14, and 15.

(b) Shear Failures

A beam that fails in shear exhibits the same degrees of elasticity in the early stages of loading as a corresponding beam failing in flexure. Behavior during the latter stages of loading is influenced by the shape of the cross section, amount of prestress, concrete strength, percentage of steel, and web reinforcement, if any.

Deformation of beams failing in shear is accelerated by the formation of diagonal tension cracks. These cracks progress gradually in beams with rectangular cross sections and high prestress, but, in any event, they progress more rapidly than the pure flexure cracks. In beams with a rectangular cross section and low prestress, the diagonal tension cracks may occur very suddenly over a considerable depth of the beam, but not before the formation of cracks in the region of pure flexure.

The diagonal tension cracks are directed toward the point of application of the load causing a "concentration" of rotation near this point. The concrete strain increases rapidly at the top surface of the beam at a location over the diagonal tension cracks, and, when the limiting value is reached, the concrete begins to crush. The behavior of the beam during the final stages of loading is quite similar to that of a beam failing in compression. This type of failure has been defined as a shear failure, but it might be more accurately described as failure by compression as a result of diagonal tension cracking.

One interesting feature of the shear failures observed in the beams tested is that, in the beams in which diagonal tension cracks formed, the diagonal tension crack became horizontal at its upper end and invaded the region of pure flexure where final failure occurred by crushing of the concrete.

Typical shear failures are shown in Figures 12, 13, and 14.

(c) Bond Failures

In ordinary reinforced concrete beams with web reinforcement, bond failures are highly improbable because of recent improvements made in the bond characteristics of ordinary reinforcing steel. However, if diagonal tension cracking is extensive or if the shear span is short, cracking at the level of the steel may extend nearly to the end of the beam, leaving only a few inches in which to transfer the entire steel stress to the concrete through bond. A bond failure while both the concrete and the steel are in the elastic range can occur in these beams unless provisions are made for some type of special anchorage.

Bond failures in prestressed concrete beams can occur in the same manner as ordinary reinforced concrete beams as a result of diagonal tension cracking. Bond failures can occur in pretensioned prestressed concrete beams without the formation of diagonal tension cracking by loss of the bond between the steel and the concrete. In post-tensioned, bonded, prestressed concrete beams, bond failures can occur without the formation of diagonal tension cracking by loss of the bond between the steel and the grout or loss of bond between the grout core and the grout channel.

The bonding characteristics of most wire used in prestressed concrete beams is poor, due to the fact that the wire is not deformed and is stressed to much higher values than ordinary reinforcing steel. The wire reinforcement used in the beams tested was intentionally rusted to form small pits in the wires to improve its bonding ability. After the wires were rusted, the rust was wiped off before the wires were inserted into the beams. End anchorages were provided on the wires so that further slip of the wire would be prevented if the bond failed to the end of the beam. It was found that the bond failures which occurred in the beams tested were not a result of loss of bond between the grout and the wires, but between the smooth grout core and the walls of the grout channels.

Bond failures may occur at any stage of loading, depending on the bond characteristics of both the steel and the concrete. Once the bond has been destroyed to the end of the reinforcement, the beam becomes a plain concrete beam and fails immediately.

17. Definition of Shear Failure

In this report, a shear failure is defined as follows: A beam fails in shear as a result of diagonal tension cracking before having developed its full flexural capacity. It had not been intended to include shear failures accompanied by bond failures in this report, but, due to the frequent bond failures occurring in the beams tested, they will be discussed.

18. Hypotheses of Failure

One of the objectives of this investigation was to increase our knowledge of the effect that web reinforcement has on the ultimate strength of prestressed concrete beams. This can partially be accomplished by establishing a criterion for predicting whether a beam will fail in shear or in flexure and at what ultimate load it will fail. The beams tested were all simply supported, subject to third-point loading, and reinforced in tension only. Two thirds of the beams tested had web reinforcement; the remaining one third had none. The theories previously developed[4, 6] applied specifically to this type of beam.

The following hypotheses were partly based on the mode of failure observed in the first few beams tested and were helpful in planning the remainder of the test program.

First Hypothesis: Overreinforced beams will fail in shear if sufficient web reinforcement is not provided to restrain the shear deformation and allow the beam to develop its flexural capacity. It can be seen in Figure 16 that the maximum shear and maximum moment occur at the same section in the beams tested. If adequate web reinforcement is provided in the region of shear, diagonal tension cracks are somewhat retarded and are distributed better than in beams having no web reinforcement. It appears that if diagonal tension cracks are not restrained, they should be at least as well developed as pure flexure cracks at loads near the ultimate for beams that do not fail in tension. This is shown in Figure 12 for typical shear failures. If this is true, the state of stress over the diagonal tension crack near the section of maximum moment and maximum shear is more critical than the state of stress over the cracks in the region of pure flexure.

Second Hypothesis: Underreinforced beams will fail either in tension or in shear. Beams grossly underreinforced do not require web reinforcement for the beam to develop its flexural capacity, but as p/f'_c is increased, web reinforcement is required to restrain the shear deformation and allow the beam to develop its flexural capacity. This is only qualitative and is somewhat of an extension of the first hypothesis. The limit between initial shear failures and initial tension failures has been established and is set forth in Reference 3, Section 210.2. The expression $pf'_s/f'_c = 0.3(f_{se}/f'_s)(b'/b)$ is based on limited experimental data and indicates that diagonal tension cracks will not form and web reinforcement will not be required if these conditions are satisfied. The expression was developed for use in I-sections, where b' = thickness of the web and b = width of flange corresponding to that used in computing p.

Third Hypothesis: Shear failures are caused by the propagation of diagonal tension cracks, but final failure occurs when the compression zone above the diagonal tension crack is strained to its ultimate capacity. A secondary effect of the diagonal tension crack, apt to occur unrecognized, is a bond failure. Short, deep beams are more vulnerable to bond failures because the diagonal tension crack at the level of the steel may be close to the end of the beam. This provides a very short length in which to develop the full tensile stress by bond. Bond failures produce unexpected and unpredictable deformations of the beam which cause a premature destruction of the compression zone before it has developed its ultimate capacity.

Fourth Hypothesis: The vertical shearing stresses do not cause distress in the compression zone of the beam. Final failure occurs when the concrete above a diagonal tension crack crushes. This is somewhat substantiated by the fact that, in the test beams having no web reinforcement as well as those having web reinforcement, the diagonal tension crack became horizontal at its upper end and invaded the region of pure flexure where final failures occurred by crushing of the concrete. The stress at which concrete fails in true shear is not well established.

19. Restrictions and Assumption of Analysis

This analysis is restricted to rectangular beams with or without web reinforcement failing in shear and flexure. Bond failures were not to be considered in this analysis, but, due to their frequent occurrence during testing, they will be discussed. The equations used were developed for simply supported rectangular beams with concentrated loads, reinforced with longitudinal forces only.

The following assumptions on which this analysis is based are similar to those made for ultimate shear and ultimate flexural theories.

(a) Crushing of Concrete at a Limiting Strain

The analysis is based upon the supposition that the maximum capacity of the beam is reached when the concrete crushes. This is a usual assumption made in shear and flexural analyses. For a flexural failure, the steel stress may be in either the elastic or inelastic range, whereas for a shear failure the steel stress will usually be in the elastic range or not far into the inelastic range. In the case of shear or flexural failures accompanied by bond failures, the stress in the steel will be in the same range as stated above for flexural failures and shear failures. It is assumed in the analysis that the concrete crushes at some limiting strain, determined from experiments to be somewhere in the neighborhood of 0.004.

(b) No Tension Resisted by the Concrete

It is assumed that the concrete resists none of the tension stresses in the beam. Actually, the concrete possesses tensile strength and does resist some tension. However, the contribution of the tension stresses to the ultimate load-carrying capacity is so small that it can safely be neglected.

(c) Properties of the Stress Block of the Concrete at Ultimate Load

The distribution of stress at ultimate load in the compression zone of a beam has been assumed by various investigators. In this investigation no attempt was made to determine the configuration of the stress block other than to define the depth to the neutral axis. The compression stress block at ultimate load is assumed to be defined by three parameters, k_1, k_2, and k_3, as shown in Figure 17. The range over which these parameters vary can be calculated or estimated fairly accurately by equations developed by Zwoyer[6] or Billet,[4] but each will have to be evaluated from the test data in order to predict the ultimate capacity of a beam failing at a limiting concrete strain.

(d) Bond Between Steel and Concrete

The analysis presented applies only to beams with the reinforcement bonded to the concrete throughout the entire range of loading. However, if the dimensions of the beam and the manner of loading are such that the lower end of the diagonal tension cracks is close to the end of the beam, and if as a result of this cracking the bond is destroyed to the end of the beam, then the analysis is still valid for predicting the ultimate load the beam will carry, if further slip of the reinforcement is prevented by some manner of end anchorage.

The distribution of cracks is controlled by the amount of bond available. It is probable that beams with completely unbonded reinforcement will not develop diagonal tension cracks; consequently, shear failures would not be likely in unbonded beams.

20. Ultimate Flexure and Shear Strength of Prestressed Rectangular Beams Reinforced in Tension Only

An expression is developed for the ultimate load-carrying capacity of rectangular prestressed concrete beams reinforced in tension only. The expression can be applied to ordinary reinforced concrete by setting equal to zero the terms relating to the effect of prestress.

Conditions at ultimate load at the critical section are illustrated in Figures 17 and 19. In Figure 17 the total tension in the steel at ultimate load is equal to $T = f_{su}pbd$, and the total compression on the section is equal to $C = k_1k_3f'_cbk_ud$. The meaning of all symbols is explained in Section 5. The ultimate moment produced by the internal resisting moment acting on the couple is

$$M_{ult} = T(d - k_2k_ud) = f_{su}pbd^2(1 - k_2k_u) \tag{1}$$

where f_{su} is the steel stress when the beam reaches maximum load. Or, by dividing by $k_1k_3f'_cbd^2$ to obtain a dimensionless form,

$$\frac{M_{ult}}{k_1k_3f'_cbd^2} = \frac{f_{su}p}{k_1k_3f'_c}(1 - k_2k_u) \; . \tag{2}$$

The ratio k_u of depth to the neutral axis to effective depth of the beam may be evaluated by equating the tensile force in the reinforcement to the compressive force in the concrete and dividing both sides by $k_1k_3f'_cbd$ giving

$$k_u = \frac{f_{su} p}{k_1 k_3 f'_c} . \tag{3}$$

Substituting Equation 3 into Equation 2

$$\frac{M_{ult}}{k_1 k_3 f'_c bd^2} = k_u(1 - k_2 k_u) . \tag{4}$$

Equations 1 through 4 are valid for all concrete beams in which the conditions implied by the stress block in Figure 17 are met since they were derived using only equations of equilibrium.

In Equation 4, only k_u is needed to compute the ultimate moment. In this equation, the quantities b, d, k_1, k_2, k_3, and f'_c represent the physical properties of the beam and the concrete. It is assumed that they are known for any specific beam. The quantity k_u is a measure of the ratio of the depth to the neutral axis to the depth of the beam. It is a function of the known quantities p and $k_1 k_3 f'_c$, and the unknown ultimate steel stress, f_{su}. The difference between Equation 4 applied to a flexure failure and Equation 4 applied to a shear failure is only in the calculation of f_{su} and k_u.

(a) Stress Conditions for Flexure

An expression for k_u, and therefore f_{su}, can be derived from strain conditions assumed to exist for beams with bonded reinforcement. Strain distribution in the concrete at two stages is shown in Figure 19. In the first stage, the effect of prestress only is considered. Strain in the concrete at the level of the steel is a compressive strain ϵ_{ce} and the steel strain is ϵ_{se}, corresponding to the effective tensioning stress f_{se}. As load is applied to the beam, concrete strain at the level of the steel decreases until it becomes zero; at this load, strain in the steel is ϵ_{se} + ϵ_{ce}. The third stage refers to strain distribution at ultimate load. Concrete strain at the top of the beam is the compressive limiting strain

ϵ_u, and strain in the concrete adjacent to the steel is ϵ_{cu}. Thus, total strain in the steel at ultimate load ϵ_{su} may be defined by the sum of three strains: steel strain due to prestress, changes between prestressing, and ultimate load.

$$\epsilon_{su} = \epsilon_{se} + \epsilon_{ce} + \epsilon_{cu} \text{ .} \tag{5}$$

Since it is assumed that there is a linear distribution of strain over the depth of the beam, ϵ_{cu} may be related to ϵ_u by

$$\epsilon_{cu} = \frac{\epsilon_u(1 - k_u)}{k_u} \text{ .} \tag{6}$$

Substituting Equation 6 into Equation 5,

$$\epsilon_{su} = \epsilon_{se} + \epsilon_{ce} + \frac{\epsilon_u}{k_u} - \epsilon_u \text{ .} \tag{7}$$

Solving Equation 7 for k_u and equating to Equation 3,

$$k_u = \frac{f_{su}p}{k_1k_3f'_c} = \frac{\epsilon_u}{\epsilon_{su} - \epsilon_{se} - \epsilon_{ce} + \epsilon_u} \text{ .} \tag{8}$$

(b) Stress Conditions for Shear

As stated in the preceding paragraph, it is commonly assumed that the average strain in the concrete in the region of pure flexure varies linearly from the top surface of the beam to the level of the steel for all stages of loading. Measurements have shown this to be a valid assumption. Thus, the quantity k_u or depth to the neutral axis can be found from the conditions of compatibility of strains.

Figure 18c illustrates the cracked state of the beam at the instant of failure in shear. The diagonal tension cracks rise to a greater height then the flexural cracks and are all directed toward the load. The fact that shear stresses are present in quantities large enough to produce diagonal tension eliminates the assumption that there is a linear distribution of strain in the region of combined shear and moment. This leaves no rational method of determining the depth to the neutral axis. In finding an empirical method of determining this quantity, use is made of the crack pattern shown in Figure 18c. Since the diagonal tension cracks are all directed toward one point on the compression side of the beam, most of the rotation and compressive strain in the concrete in this region of combined shear and moment will be concentrated at this location, which thus acts as a hinge. In Figure 18c, if the portion of the beam to the left of crack a-b is considered as a free body diagram, it is seen from the equation of equilibrium involving horizontal forces that the steel stress at b is controlled by the moment at a. These facts suggest that an average value of the concrete strain on the top surface and an average value of the concrete strain at the level of the steel can both be determined empirically and used to define the depth to the neutral axis.

The conditions of distribution of strain at the various stages of loading as set forth in paragraph (a) above are applicable for shear failures. Thus the total steel strain at ultimate will be

$$\epsilon_{su} = \epsilon_{se} + \epsilon_{ce} + \epsilon_{cu} \, . \tag{5}$$

By applying the empirical modifications to the conditions of compatibility of strain, an expression for the strain in the concrete at the level of the steel at ultimate load is:

$$\frac{K_1 \epsilon_u}{k_u d} = \frac{K_2 \epsilon_{cu}}{d - k_u d} \tag{9}$$

where

$K_1\epsilon_u$ = average concrete strain on the top surface of the beam over the length of the shear span for ultimate load,

$K_2\epsilon_{cu}$ = average concrete strain at the level of the steel over the length of the shear span for ultimate load, and

$K_1/K_2 = K.$

Thus

$$\epsilon_{cu} = K\frac{\epsilon_u(1 - k_u)}{k_u} . \qquad (10)$$

Substituting for k_u from Equation 3,

$$\epsilon_{cu} = K\epsilon_u \frac{k_1 k_3 f'_c}{f_{su} p} - 1 . \qquad (11)$$

The value of K is an empirical term which modifies the ultimate concrete strain in the equations expressing depth to the neutral axis in the shear region of the beam. This empirical modification is necessary since linear strain distribution does not exist on a section in a region of shear after the formation of diagonal tension cracks.

Substituting Equation 11 into Equation 5,

$$f_{su} = \frac{K\epsilon_u k_1 k_3 f'_c/p}{\epsilon_{su} - \epsilon_{se} - \epsilon_{ce} + K\epsilon_u} . \qquad (12)$$

Equations 8 and 12 represent one relationship between the steel stress and the steel strain at ultimate load for any given value of f'_c and p; the stress-strain diagram is another relationship. A detailed explanation of this relationship and the derivations of the ultimate load equations may be found in the thesis by Zwoyer[6] and the thesis by Billet.[4]

CHAPTER IV -- PRESENTATION AND ANALYSIS OF RESULTS

21. Comparison of Measured and Computed Cracking Load

One of the features of prestressed concrete is that cracks can be eliminated at working loads.

For beams with a low percentage of steel and a low prestress, it was a simple matter to observe the cracking load of the beam since the crack usually opened to a height of 3 to 6 inches above the bottom of the beam and caused a sudden deformation of the beam which could be observed by a jump in the deflectometer reading. As the percentage of steel and prestress was increased, it became more difficult to determin the cracking load visually during the test because the cracks opened slowly and progressed slowly with further increase in load. These beam were therefore closely scrutinized during the loading procedure for signs of first cracking. Deflection readings were taken at small increments of load, and, if the cracking load was not observed during the test, it could be found rather reliably from the load-deflection curve.

(a) Flexural Cracking Load

As was expected, the first cracking appeared in the region of pure flexure. Comparison of observed and computed flexural cracking loads is given in Table V. The observed flexural cracking load is, then, the load somewhere in between the load at first deviation from linearity of the load-deflection curve and the load at which the first crack that is visible by the naked eye opens up. This is a comparatively small range, not large enough to affect the usefulness of comparison between observed and computed flexural cracking loads, the latter being the load which causes tensile stress equal to the modulus of rupture at the bottom fiber.

The flexural cracking load can be computed if one knows the effective prestress in the beam, the tensile stress at which the concrete cracks, and the properties of the cross section.

In the beams tested, the effective prestress at the end of the beams was measured with the dynamometers. It is possible that the effective prestress in the middle of the beam was different from that at the end of the beam due to a nonuniform loss in prestress.

Perhaps the greatest source of error in calculating the cracking load occurs in evaluating the stress at which the concrete cracks. No flexural specimens were cast to determine the modulus of rupture of the beams tested. It was found from previous tests conducted on prestressed concrete beams[4,6,7] that the moduli of rupture obtained from the tensile specimens gave very different values, none of which was necessarily as low as the weakest section in the tension zone of the beam tested. For this investigation, the stress at which the concrete cracks was calculated from $f_r = 0.08f'_c$.

The properties of the cross section of the beams were known rather accurately. However, the calculations were complicated somewhat by the presence of the grout channel. At the time the prestress was applied, the grout channel was empty, giving the beam a slightly reduced area and section modulus; however, when the beam was tested, the grout channel had been filled, and the section modulus was based on the gross cross-sectional dimensions. It was assumed that the grout core acted integrally with the rest of the beam during the entire range of loading. It is believed this would have been true if it had not been for the occurrence of bond failure between the grout core and the concrete.

The cracking load was computed as follows: It was assumed that cracking occurred when the stress at the bottom of the beam became equal to the modulus of rupture, f_r. This can be expressed as

$$\frac{M_{cr} y_b}{I_c} + f^b_{FD} = f_r \qquad (13)$$

where the first term is the stress caused by the applied moment at first cracking, M_{cr}, and the second term is stress caused by the prestress force and the dead load. The moment at cracking is determined by solving Equation 13, and cracking load, P_{cr}, is then obtained from the moment.

The average value of the ratio of the observed to computed cracking load for the 24 beams tested was 1.115, and the standard deviation was 0.116. Except in a few instances, the observed cracking load was greater than computed cracking load. This could have been due to the fact that the nuts on the wires at the dynamometer end of the beam were tightened to their limit by hand to remove the initial curvature in the wire and no strain reading was recorded prior to prestressing. This caused an additional stress in the steel of from 3000-4000 psi more than the actual recorded prestress. Also, two of the beams were cracked slightly from top to bottom during handling at the time of prestressing.

(b) Diagonal Tension Cracking Load

The diagonal tension cracking load has assumed importance in the analysis of reinforced or prestressed concrete beams especially in connection with the effect on mode of failure of the ratio of shear span to effective depth of a beam. Whatever the ratio of steel percentage to concrete strength may be, if the flexural ultimate resisting moment is reached before enough shear is developed to cause diagonal tension cracking, the beam will not have a shear-initiated failure. In structures loaded at fixed points, a reliable knowledge of the diagonal tension cracking load may eliminate the necessity of shear reinforcement entirely. The same information may make the design of web reinforcement in beams with moving loads more rational and economical. This has been partially substantiated in Section 210 of Tentative Recommendations for Prestressed Concrete.[3]

As it stands now, there seem to be several barriers to a rational approach to the diagonal tension cracking load. In a beam loaded at fixed points, the cracks may start at almost any point along the shear span and at any depth, depending on the properties of the cross section, the ratio of shear span to effective depth, anomalies in the pour, and presence of flexure cracks in the shear span. Another complication arises from the fact that the diagonal tension cracks observed in the tests of beams with or without web reinforcement had angles of inclination with the horizontal varying from 30 to 70 degrees.

As a prestressed beam is tested to failure, the diagonal tension cracks have been observed to form in a variety of ways and shapes. They may be initiated by a flexural crack which slowly develops into a diagonal tension crack as shown in Figures 12 and 13. In other cases they may start as a horizontal crack at the level of the grout core in the region of shear and develop into a single crack or form a radiating pattern as shown in Figures 13 and 15. In both cases the principal diagonal tension crack, i.e., the one that causes failure, may be preceded by a steeper crack close to the load block. However, it appears that, in the beams tested, the position of the main diagonal tension crack is independent of the prestress or percent of web reinforcement, and this distance is virtually constant for the same type of loading.

The shears corresponding to the first detection of diagonal tension cracking were compared to the concrete cylinder strength and observed flexural cracking loads. The results were found to be quite erratic and of no value. Since no precise definition of the diagonal tension crack has been postulated, it is very difficult to pinpoint this shear. Since sometimes the crack forms very gently, there is no way of detecting it other than by inspection; the observed value may be off by a significant amount.

In general terms, it is concluded from the tests that the diagonal tension cracking shear increases with the prestressing force and that the addition of web reinforcement prevented the diagonal tension cracks from opening as wide as the flexural cracks.

22. Distribution of Concrete Strain on Top Surface of Beam

The deformations of a beam failing in shear are considerably different from those of a beam failing in flexure. If the beam possesses a region of pure flexure, the cracks in this region are nearly vertical, and, if the bond is good, they are spaced closely together as noted in Figure 15. The strains will be rather uniform in this region of constant moment. In the region of combined shear and flexure, the cracks are inclined and converge to some point near the application of load. The section of concrete above the ends of these cracks acts like a hinge; most of the rotation from the region of combined shear and flexure is "concentrated" in this section.

The strain in the concrete on the top surface of the beam at any section is proportional to the amount of rotation that occurs at the section. Strains were measured at several points along the top of beams S-4 through S-9 and S-15 through S-24. The strain distribution at ultimate load for typical beams is shown in Figures 23, 24, and 25. It can be seen that the strain is not proportional to the moment as would be expected in a homogeneous, elastic material. For beams failing in shear (Figure 23), the highest strains were localized over the tops of the diagonal tension cracks, and the strains in the region of pure flexure were rather uniformly distributed and were usually not as large as those over the diagonal tension cracks. For the beams failing in flexure (Figure 24), the highest strains occurred in the region of crushing over the pure flexure cracks and were not quite as localized as for the shear failures.

23. Average Strain at First Crushing

The theory of failure presented in Chapter III is one of limiting strain. It is based on the assumption that the capacity of the beam is reached when the concrete crushes at some limiting strain.

As explained previously, strain measurements were made along the top surface of the beam with 1-inch gage length SR-4 gages. In the region of the loads, where the concrete crushed, the gages were spaced closely together so that the crushing strain could be measured. It was not always possible to read the strain when the concrete first crushed. In these instances, the strain was determined by extrapolation to maximum load from load-strain curves or from deflection-strain curves.

The crushing strains are plotted against the compressive strength of the test cylinders in Figure 26. There is a rather large scatter, and there seems to be no consistent variation with the strength of the concrete. The average value of the strain at which the concrete crushed is 0.00355. This is slightly lower than the results obtained by other investigations which found the limiting strain of the concrete at crushing to range from 0.0038 to 0.0040.

24. Evaluation of k_1, k_2, and k_3

In Figure 17, the distribution of stress in the concrete at ultimate load is defined by the parameters k_1, k_2, and k_3. The quantity k_3 is the ratio of the strength of the concrete in the beam to the strength obtained from tests of the standard 6- by 12-inch cylinders. The quantity k_1 is the ratio of the area of the stress block to the area of the rectangle of altitude k_u and base $k_3f'_c$, and k_2d is the depth to the centroid of the stress block.

To determine k_2 exactly, the shape of the stress block must be known. Since there is no practical method of finding the relationship between stress and strain in the concrete at ultimate load, it is impossible to determine the shape of the stress block exactly. Fortunately, k_2 has the narrow range of from 0.333 for a triangular stress block to 0.500 for a rectangular stress block, and the effect of varying k_2 over this range is small. For convenience, an average value of 0.42 was chosen for all conditions.

The terms k_1 and k_3 always appear as a product and cannot be determined separately. For the purpose of this investigation, they were determined from the equations of equilibrium in the following way:

$$M_{ult} = C(1 - k_2 k_u)d$$

$$C = k_1 k_3 f'_c b k_u d \ ,$$

then

$$k_1 k_3 = \frac{M_{ult}}{(1 - k_2 k_u) f'_c b k_u d^2} \qquad (14)$$

where k_2 was assumed to be 0.42, and all other quantities were either known or were measured in the tests.

The values obtained from Equation 14 are recorded in Table VI and are shown graphically in Figure 27 as a function of the cylinder strength of the concrete. Figure 27 compares the $k_1 k_3$ versus f'_c relationships determined by Billet[4] and Zwoyer[6] for rectangular beams without web reinforcement failing in flexure and shear, respectively, with values measured from the beams with and without web reinforcement failing in shear, flexure, and combined shear and flexure. A combined shear and flexure failure is referred to in the plotted data as a balanced failure.

Figure 28 compares the $k_1 k_3 f'_c$ versus f'_c relationships determined by Billet[4] and Zwoyer[6] as mentioned in the preceding paragraph, with the values measured from the beams tested.

The measured values of $k_1 k_3$ were scattered, but, for the most part, plotted between the curves of Billet[4] and Zwoyer.[6] As a result of this, a dashed straight line shown in Figure 27 was drawn through the measured data. Due to the fact that bond failures occurred in the majority of the beams tested, doubt exists as to the validity of the results. Therefore, no rigorous analysis of the results was made.

Since the majority of the beams tested had a concrete strength ranging from 3000-3500 psi, and since, at present, the only factor that k_1k_3 has been found to depend on is the concrete strength, there seems to be no rational reason why k_1k_3 should vary for different types of failure. It is quite possible that k_1k_3 is not a function of concrete strength at all but is dependent upon k_u or Q'.

This reasoning is somewhat substantiated by the fact that in "Tentative Recommendations for Prestressed Concrete"[3] the factor k_2/k_1k_3 may be taken equal to 0.6 for rectangular sections and flanged sections in which failure by flexure would occur. This factor is based on the results of numerous tests.

25. Evaluation of $K\epsilon_u$

In the ultimate load theory developed in Chapter III, the depth to the neutral axis is based on an empirical modification of the distribution of strains in the shear region of the beam. Reference to Equations 9, 10, 11, and 12 reveals that the empirical quantity K appears only as a coefficient of the ultimate compressive strain in the concrete ϵ_u; the basis for this has been discussed previously.

The following equation developed by Zwoyer[4] was used to compute values of $K\epsilon_u$ for rectangular beams without web reinforcement failing in shear and combined shear and flexure.

$$K\epsilon_u = \frac{\epsilon_{su} - \epsilon_{se} - \epsilon_{ce}}{\frac{1}{k_u} - 1} . \tag{15}$$

The terms ϵ_{su}, ϵ_{se}, ϵ_{ce}, and k_u were either measured or computed from measured quantities; thus $K\epsilon_u$ could be computed from measured test data. The computed values of $K\epsilon_u$ are recorded in Table VI for beams which failed in shear and combined shear and flexure. The values computed were quite erratic and did not vary consistently with the effective prestress as had been determined by Zwoyer[4] in his tests from which he developed the following empirical equation:

$$K\epsilon_u = 0.00040 + 5 \times 10^{-9} f_{se} \tag{16}$$

for computing $K\epsilon_u$.

It has been stated by E. Zwoyer[4] in his thesis on the shear strength of prestressed concrete beams that shear failures involve a drastic disturbance in the linearity of strain distribution over the depth of the section. Figure 23 agrees with the phenomenon for rectangular beam tested which failed in shear. It appears from Figure 25 that a certain degree of nonlinear strain distribution exists in rectangular beams having web reinforcement which failed in combined shear and flexure. It is conceivable that the presence of web reinforcement could affect the linearity of the strain distribution and that a new empirical equation modifying ϵ_u would be plausible for beams of this type.

26. Comparison of Measured and Computed Depth to the Neutral Axis

The ultimate moment expressions developed in Chapter III make extensive use of the depth to the neutral axis since it is the concrete above the neutral axis which must sustain the entire compression of shear on flexural stresses.

The depth to the neutral axis was computed from the following equations:

$$k_u d = \text{depth of the neutral axis,}$$

where

$$k_u = \frac{f_{su}p}{k_1k_3f'_c} \tag{3}$$

and

$$f_{su} = \frac{K\epsilon_u k_1k_3f'_c/p}{\epsilon_{su} - \epsilon_{se} - \epsilon_{ce} + K\epsilon_u} \text{ (shear)} \tag{12}$$

or

$$k_u = \frac{f_{su}p}{k_1k_3f'_c} = \frac{\epsilon_u}{\epsilon_{su} - \epsilon_{se} - \epsilon_{ce} + \epsilon_u} \text{ (flexure) .} \tag{8}$$

The measured depth to the neutral axis was found by measuring with a scale the distance from the top of the beam to the top of the crack over which failure occurred. This assumes that the tension zone above the crack is negligibly small, which is quite valid at ultimate load. As crude as this method may seem, it probably yields better results than could be obtained from strain measurements made in a region of combined shear and flexure and seriously distorted by diagonal tension cracks and flexure cracks.

Both the measured and computed values of the coefficient k_u are given in Table VI together with the ratio of the measured to computed depth to the neutral axis. The ratio for the beams failing in shear, flexure, and combined shear and flexure is 0.925 with a standard deviation of 0.122. For the most part the measured values were less than the computed values, which could have been due to the constant occurrence of bond failures.

27. Comparison of Measured and Computed Ultimate Load

The ultimate moment was computed from the following equation, which was developed in Chapter III:

$$\frac{M_{ult}}{k_1 k_3 f'_c bd^2} = k_u(1 - k_2 k_u) . \tag{4}$$

The terms in this equation have been previously discussed.

The measured ultimate moment was, of course, calculated from the measured ultimate load.

Both the measured and computed values of $M_{ult}/k_1 k_3 f'_c bd^2$ are given in Table VII, together with the ratio of the measured to computed moment. The average ratio of measured to computed moment is 0.964, and the standard deviation from the average is 0.122.

The majority of the beams tested failed at an ultimate load less than the computed ultimate load. This is believed to have been caused by the bond failure, because, just prior to failure or just after failure, an increase in strain was noticed in the dynamometer at the end beam.

28. Nominal Shear Stresses

Present design practice has endorsed the use of principal tension stresses as a measure of the susceptibility to shear failure of prestressed concrete more so than the nominal shear stress.

The shearing stresses for the beams tested were computed from the celebrated equation

$$v = \frac{V}{bjd} \tag{17}$$

where j is based on a transformed section and calculated by the well-known straight-line theory.

The shearing stresses computed from Equation 17 and the ratios of the nominal ultimate shear stress to the cylinder strengths of the concrete and the allowable shearing stress v_c from the present ACI Building Code (ACI 318-56) are shown in Table VIII. No use is made of these data in the interpretations of the test results other than to note that in some of the beams failing in shear the nominal ultimate shearing stress was in excess of that allowed by the code.

29. Bond Failures

As discussed in Chapter III, the occurrence of a bond failure in a beam will produce unexpected and unpredictable deformations and cause a premature destruction of the compression zone before the beam has developed its ultimate capacity. This was noted in the beams tested, as a number of the beams failed at loads lower than the computed ultimate loads.

It was noted from the beams tested in this investigation that the occurrence of bond failures did affect the behavior of k_1k_3, k_u, and ϵ_u. It was found that the values of k_1k_3 and k_u were less than the computed values and that ϵ_u was much lower than the values established from previous tests.

The type of bond failure which occurred in this investigation was caused by loss of bond between the grout channel wall and the smooth grout core.

It is believed that this type of bond failure could be prevented if a 1/8- by 3-inch rubber strip is wrapped around the outside of the grout channel form before the beam is cast. The rubber strip would be wrapped spirally around the form and lapped a short distance back over each previous turn. When the form is removed, a spiraled groove will be left in the wall of the grout channel due to the lapping of the rubber strip. This would provide somewhat of a keyed joint between the grout and the beam.

30. Effect of f_{se}, r, and p/f'_c on Mode of Failure

Since shear failure in a beam can occur quite suddenly due to the formation of diagonal tension cracks, it is essential that the development of diagonal tension cracks be reduced in order that the beam can develop its full flexural capacity. The development of diagonal tension cracks can be retarded by the use of web reinforcement.

The amount of web reinforcement necessary to develop the required ultimate flexural capacity is a function of the difference between the diagonal tension cracking load and the ultimate load in flexure. This difference varies rather widely as a function of the prestress force, beam cross section, percent of tensile reinforcement, concrete strength, and the shear/moment ratio, but is usually smaller for prestressed concrete than for conventional reinforced concrete.

Since the main object of this series of tests was to determine the effect that web reinforcement had on the ultimate shear strength of rectangular prestressed concrete beams, the percentage of web reinforcement, the effective prestress, and p/f'_c were varied systematically over a range which would produce shear and flexural failures. This was done to determine what effect these variables had on the mode of failure and to attempt to verify the following assumptions:

1. For given values of f_{se} and r and variable values of p/f'_c, beams having low values of p/f'_c will tend to fail in flexure, and beams having high values of p/f'_c will tend to fail in shear.

2. For given values of r and p/f'_c and variable values of f_{se}, beams having low values of f_{se} will tend to fail in shear, and beams having high values of f_{se} will tend to fail in flexure.

3. For given values of f_{se} and p/f'_c and variable values of r, beams having low values of r will tend to fail in shear, and beams having high values of r will tend to fail in flexure.

From the results of the beams tested in which f_{se} and r were constant and p/f'_c was varied, it was found that beams having low values of p/f'_c did fail in flexure. For beams with high values of p/f'_c, only those beams with no web reinforcement failed in shear, while those beams with increasing percentages of web reinforcement failed either in combined shear and flexure or in flexure.

Of the beams tested in which r and p/f'_c were constant and f_{se} was varied, only those beams having no web reinforcement and low prestress failed in shear. For the beams with high prestress, shear failures also occurred in those beams having no web reinforcement.

For the beams tested in which p and f_{se} were constant and r was varied, shear failures occurred for beams having low values of r. The beams having high values of r failed in flexure.

For the most part, the beams tested agreed with the previous assumptions. It is apparent from these results that some web reinforcement is necessary in beams having high values of p/f'_c and relatively high prestress if shear failures are to be prevented. The percentage of web reinforcement required for each particular case would, of course, be dependent on the effective prestress and the ratio of the percentage of tensile reinforcement p to concrete strength f'_c.

It is conceivable that a dividing line might exist between high and low values of p/f'_c and f_{se}, below which flexural failures will occur and above which shear failure might occur and web reinforcement would be required to prevent these shear failures.

CHAPTER V -- SUMMARY, FUTURE TESTS, AND CONCLUSIONS

31. Summary

The objects of this investigation were stated to be the following:

1. To determine the mode and characteristics of shear failures in simply supported prestressed concrete beams with web reinforcement.

2. To determine the effect of the following variables on the ultimate shear strength:

   a. Concrete strength

   b. Amount of longitudinal steel

   c. Prestress in the steel

   d. Percent of web reinforcement

3. To determine the difference, if any, between the modes of failure and ultimate strength of rectangular beams with web reinforcement and rectangular beams without web reinforcement.

4. To substantiate Tentative Recommendations for Prestressed Concrete.[3]

5. To establish a basis for future tests involving additional variables.

A total of 24 beams were tested. All of the beams were rectangular in cross section with over-all dimensions 6 inches by 12 inches by 10 feet-0 inch. Of the 24 beams tested, 16 beams had web reinforcement varying from 4.16 to 8.15 percent. All of the beams tested were post-tensioned and grouted. The beams were loaded at the third points on a 9-foot span. The main variables in these tests were the concrete strength, the percentage of longitudinal tension reinforcement, the prestress in the steel, and the percentage of web reinforcement.

The tests have been described, and the results have been presented in both tabular and graphical form.

32. Future Tests

Since our present knowledge of shear failures in prestressed concrete is somewhat limited and continues to remain in the theoretical stage, the following tests are recommended for future study:

1. Continued testing of rectangular beams with and without web reinforcement over a wide range of prestresses to determine, if possible, a way of predicting the dividing line between shear and flexure failures for this type of beam.

2. A series of tests of rectangular beams with varying percentage of web reinforcement to further substantiate the minimum requirements for web reinforcement as set forth in Reference 3.

3. A series of tests on I beams with web reinforcement to determine the effect on ultimate load and mode of failure caused by the addition of web reinforcement.

4. A series of tests on rectangular beams with depths greater than 12 inches, with and without web reinforcement, to determine the effect on the ultimate load and mode of failure caused by the addition of web reinforcement.

5. A series of tests on simple beams having web reinforcement with uniformly distributed loads to determine the effect on ultimate load and mode of failure of a uniform load in contrast to concentrated loads.

33. Conclusions

The following conclusions are based on the test results for the 24 beams and the studies which are presented in the body of this report:

1. The presence of shear in a simple beam of prestressed concrete has the effect of causing diagonal tension cracks. The following modes of failure are possible in this type of beam:

a. For beams with low values of p/f'_c, regardless of the percentage of web reinforcement, the pure flexure cracks will develop faster than the diagonal tension cracks, the steel will yield in the region of pure flexure, and final failure will occur by crushing of the concrete above the flexure crack. This mode of failure is called an initial tension failure with a secondary compression failure.

b. For the beams with higher values of p/f'_c and no web reinforcement, the diagonal tension cracks, once they start, develop faster than the pure flexure cracks, and final failure will occur by crushing of the concrete above the diagonal tension cracks before the steel yields. This mode of failure is called a shear failure; it occurs at a load lower than the flexural capacity of the beam.

c. For the beams with higher values of p/f'_c with web reinforcement, the flexure cracks develop approximately as fast as the diagonal tension cracks which are retarded by the presence of the web reinforcement, and final failure occurs by crushing of the concrete simultaneously over the flexure and diagonal tension cracks. This mode of failure is called a balanced failure.

d. At any stage during the loading of the beam, the bond between the concrete and the steel can fail. If the steel has an end anchorage, further slip of the reinforcement will be prevented when the bond has finally failed to the end of the beam; if there is no end anchorage, the beam will collapse when the bond fails since there is no way of transferring the tension from the concrete to the steel. The formation of diagonal tension cracks increases the bond stresses which must be developed because these cracks shorten the distance to the end of the beam over which all the tension must be developed. From an inspection of the beam, it is sometimes difficult to determine between a shear failure and a bond failure unless some method has been devised to measure the slip in the reinforcement.

2. The ultimate shear strength and the ultimate flexural strength were found to be lower than the computed values. This was due to the frequent occurrence of bond failures in the beams.

3. The depth to the neutral axis at ultimate load, and thus the ultimate load, was found to increase with the effective prestress in the steel, p, f'_c, and percent of web reinforcement. It was found that the measured depth to the neutral axis was generally less than the value computed from the empirical equations developed by Zwoyer[6] and Billet.[4]

4. It was found that for the beams tested, pf'_s/f'_c was greater than $0.3\,(f_{se}b'/f'_s b)$. According to Section 210.2.1,[3] diagonal tension cracks will form, and web reinforcement will be required under these conditions. This was found to be true. The presence of web reinforcement in rectangular prestressed concrete beams was found to be effective in retarding diagonal tension cracking and preventing shear failures. The amount of web reinforcement used in the beams tested was found to be in excess of the minimum amount required by the equation $Av = 0.0025bs$ as set forth in Section 210.2.3 of Reference 3.

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TABLE I

Properties of Specimens

Mark	Concrete strength f'_c (psi)	Type of reinforcement	Width b (in.)	Effective depth d (in.)	Area of steel A_s (in.^{2})	Longitudinal reinforcement p (%)	Effective prestress f_{se} (psi)	Shear span a (in.)	Beam span L (ft)	Percent of web reinforcement r
S-1	4736	1/4" dia. "Tufwire"	6.00	8.50	0.147	0.289	0	36	9	0.0
S-2	4700	1/4" dia. "Tufwire"	6.00	8.25	0.147	0.298	0	36	9	4.16
S-3	5045	1/4" dia. "Tufwire"	6.00	8.50	0.147	0.289	0	36	9	8.15
S-4	2640	1/4" dia. "Tufwire"	6.00	8.62	0.147	0.285	0	36	9	0.0
S-5	2600	1/4" dia. "Tufwire"	6.00	8.32	0.147	0.296	0	36	9	4.16
S-6	3200	1/4" dia. "Tufwire"	6.00	8.25	0.147	0.298	0	36	9	8.15
S-7	3100	1/4" dia. "Tufwire"	6.00	8.25	0.147	0.298	58,000	36	9	0.0
S-8	2910	1/4" dia. "Tufwire"	6.00	8.50	0.147	0.289	58,000	36	9	4.16
S-9	3746	1/4" dia. "Tufwire"	6.00	7.75	0.196	0.423	116,400	36	9	8.15
S-10	3024	1/4" dia. "Tufwire"	6.00	7.75	0.147	0.317	116,400	36	9	0.0
S-11	4901	1/4" dia. "Tufwire"	6.00	7.62	0.196	0.428	116,400	36	9	0.0
S-12	4174	1/4" dia. "Tufwire"	6.00	7.84	0.343	0.730	116,000	36	9	0.0
S-13	2718	1/4" dia. "Tufwire"	6.00	7.92	0.245	0.516	118,000	36	9	4.16
S-14	2794	1/4" dia. "Tufwire"	6.00	8.00	0.294	0.613	117,000	36	9	8.15
S-15	3225	1/4" dia. "Tufwire"	6.00	7.80	0.294	0.629	60,500	36	9	8.15
S-16	3337	1/4" dia. "Tufwire"	6.00	7.85	0.294	0.625	64,500	36	9	8.15
S-17	4226	1/4" dia. "Tufwire"	6.00	7.50	0.392	0.870	117,200	36	9	8.15
S-18	3314	1/4" dia. "Tufwire"	6.00	7.75	0.294	0.633	58,200	36	9	0.0
S-19	3502	1/4" dia. "Tufwire"	6.00	7.68	0.294	0.637	61,200	36	9	4.16
S-20	3155	1/4" dia. "Tufwire"	6.00	7.50	0.294	0.654	59,600	36	9	4.16
S-21	3485	1/4" dia. "Tufwire"	6.00	8.00	0.294	0.613	68,000	36	9	0.0
S-22	3060	1/4" dia. "Tufwire"	6.00	7.62	0.294	0.644	119,000	36	9	4.16
S-23	3140	1/4" dia. "Tufwire"	6.00	7.50	0.294	0.654	114,000	36	9	8.15
S-24	4376	1/4" dia. "Tufwire"	6.00	7.87	0.392	0.830	127,000	36	9	4.16

TABLE II

Sieve Analysis of Aggregates

Sand

Lot	Percentage retained on sieve No. 4	8	16	30	50	100	Fineness modulus
9	4.6	16.1	29.8	54.8	84.6	94.9	2.48
12	4.7	17.3	32.7	57.5	87.1	95.6	2.94
15	4.6	18.1	32.7	54.5	83.3	94.4	2.87
16	3.1	9.5	18.8	42.9	81.5	94.4	2.50
18	4.7	18.8	23.5	48.3	86.1	96.1	2.71
19	4.7	13.4	23.7	46.8	81.9	94.8	2.65
20	4.3	18.8	24.1	44.2	83.0	95.1	2.63
21	2.9	9.1	17.9	37.4	83.4	95.9	2.46
22	2.9	9.5	19.3	40.8	77.0	93.1	2.42
23	3.9	11.0	22.0	44.2	83.4	95.7	2.60
24	4.5	11.8	22.9	46.0	82.3	94.7	9.62

Gravel

Lot	Percentage retained on sieve No. 3/4	1/2	3/8	1/4	4	8	Fineness modulus
1	16.1	69.2	92.6	----	99.2	99.5	----
9	7.3	41.1	65.6	91.9	95.2	97.1	----
15	14.7	51.2	82.4	98.8	99.2	99.5	----

TABLE III

Properties of Concrete Mixtures

Beam	Cem: sand: gravel by weight	Water / cement	Slump (in.)	Concrete strength f'_c (psi)	Age (days)
S-1	1:1.9:2.6	0.47	6	4734	90
S-2	1:1.9:2.6	0.47	6	4500	98
S-3	1:1.9:2.6	0.47	6	5045	101
S-4	1:3.8:4.2	0.78	4	2640	61
S-5	1:3.8:4.2	0.78	4	2600	56
S-6	1:3.8:4.2	0.78	4	3200	70
S-7	1:3.8:4.2	0.78	2	3100	180
S-8	1:3.8:4.2	0.78	1	2910	182
S-9	1:3.8:4.2	0.78	3	3746	57
S-10	1:3.9:4.3	0.85	4	3024	53
S-11	1:3.9:4.3	0.81	2	2091	51
S-12	1:3.9:4.3	0.81	1-1/2	4174	59
S-13	1:3.8:4.2	0.75	8	2718	56
S-14	1:3.9:4.2	0.79	3	2794	55
S-15	1:3.8:4.2	0.85	3	3225	83
S-16	1:4.0:4.3	0.91	3	3337	84
S-17	1:3.7:4.0	0.86	1	4226	90
S-18	1:3.8:4.1	0.76	1	3314	43
S-19	1:3.7:4.0	0.73	1	3502	42
S-20	1:3.8:4.1	0.81	2	3155	71
S-21	1:3.8:4.1	0.81	3	3485	65
S-22	1:3.8:4.1	0.81	2	3060	70
S-23	1:3.8:4.1	0.78	1-1/2	3140	64
S-24	1:3.9:4.3	0.78	0	4376	69

TABLE IV

Properties of Grout Mixtures

Mark	*Ratio sand/cement	*Ratio water/cement	*Ratio alum. powder/cement (gr/100 lb)	**Compressive strength f'_c (psi)	Age at test (days)
S-1	1.00	0.61	----	2860	4
S-2	1.00	0.61	----	2910	4
S-3	1.00	0.61	----	2570	4
S-4	0.89	0.51	4.0	1962	6
S-5	0.89	0.51	4.0	2120	6
S-6	0.89	0.51	4.0	3134	6
S-7	1.00	0.42	6.0	----	-
S-8	1.00	0.42	6.0	----	-
S-9	1.00	0.58	4.6	----	-
S-10	1.00	0.58	4.6	----	-
S-11	1.00	0.58	4.6	----	-
S-12	1.00	0.62	4.0	----	-
S-13	1.00	0.62	4.0	----	-
S-14	1.00	0.62	4.0	----	-
S-15	1.00	0.55	4.5	----	-
S-16	0.92	0.49	†16.0	3562	2
S-17	1.00	0.45	†12.5	----	-
S-18	1.00	0.55	4.5	----	-
S-19	1.00	0.55	4.5	----	-
S-20	0.92	0.49	†16.0	4112	2
S-21	0.92	0.49	†16.0	3275	2
S-22	1.00	0.45	†12.5	----	-
S-23	1.00	0.45	12.5	----	-
S-24	1.00	0.50	6.0	----	-

*Ratio by weight.
**Strength from tests on 2- by 4-inch cylinders.
†Pounds of Embeco/100-pound cement.

TABLE V

Computed and Observed Flexural Cracking Loads

Mark	Area of steel A_s (in.^{2})	Effective prestress force F (lb)	Stress due to D. L. + prestress f^b_{FD} (psi)	Cracking load P_{cr} Computed (lb)	Observed (lb)	$\frac{P_{cr} \text{ - obs.}}{P_{cr} \text{ - comp.}}$
S-1*	0.1475	------	-378	3,020	2,800	0.926
S-2*	0.1475	------	-360	2,880	2,800	0.972
S-3*	0.1475	------	-404	3,220	3,200	0.994
S-4*	0.1475	------	-211	1,690	2,400	1.420
S-5*	0.1475	------	-208	1,667	2,000	1.182
S-6*	0.1475	------	-256	2,040	3,400	1.670
S-7	0.1475	8,550	-423	3,390	4,000	1.180
S-8*	0.1475	8,550	-408	3,260	4,400	1.350
S-9	0.1965	22,400	-857	8,570	8,600	1.004
S-10	0.1475	17,200	-647	5,160	4,200	0.815
S-11	0.1965	22,900	-900	7,200	7,400	1.030
S-12	0.343	29,800	-1,375	10,900	8,800	0.806
S-13	0.245	8,900	-956	7,600	9,600	1.265
S-14	0.294	34,400	-1,114	8,900	9,000	1.010
S-15*	0.294	17,800	-669	5,350	8,400	1.570
S-16	0.294	18,950	-755	6,040	4,300	0.895
S-17	0.392	46,000	-1,555	12,420	12,000	0.965
S-18	0.294	17,100	-676	5,400	5,800	1.074
S-19	0.294	18,000	-719	5,600	7,000	1.250
S-20	0.294	17,500	-677	5,420	7,400	1.365
S-21	0.294	20,000	-772	6,160	6,450	1.045
S-22**	0.294	35,000	-1,154	8,750	9,200	1.048
S-23**	0.294	33,600	-1,120	8,950	9,000	1.007
S-24	0.392	50,000	-1,777	14,100	13,200	0.935
					Avg. =	1.115
					Std. dev. =	0.116

*Nuts on wires were tightened to limit prior to prestress to remove curvature in wire. Strain reading of dynamometer due to initial force of tightening was not recorded.
**Beams were cracked slightly from top to bottom before prestressing due to handling.

TABLE VI

Empirical Terms k_1k_3, $K\epsilon_u$, and k_u

Mark	Concrete strength f'_c (psi)	Longitudinal steel p	Percent of web reinforcement r	$\frac{E_s p}{k_1 k_3 f'_c}$ Q'	Effective prestress f_{se} (psi)	k_1k_3	$K\epsilon_u$ x 10^{-5}	Depth to neutral axis k_u Measured	Depth to neutral axis k_u Computed	$\frac{k_u \text{ measured}}{k_u \text{ computed}}$	Mode of* failure
S-1	4736	0.00289	0	26.3	0	0.695	78.5	0.132	0.147	0.896	F/B
S-2	4700	0.00298	4.16	19.0	0	0.975	87.0	0.128	0.158	0.811	F/B
S-3	5045	0.00289	8.15	44.8	0	-----	42.6	0.130	0.142	0.915	F/B
S-4	2640	0.00285	0	33.7	0	0.960	79.6	0.151	0.153	0.989	S
S-5	2600	0.00296	4.16	31.0	0	1.100	114.5	0.172	0.210	0.820	S-F-B
S-6	3200	0.00298	8.15	21.2	0	1.320	131.0	0.151	0.186	0.815	S-F/B
S-7	3100	0.00298	0	51.5	58,000	0.560	34.2	0.182	0.190	0.957	F/B
S-8	2910	0.00289	4.16	37.7	58,000	0.790	54.5	0.171	0.199	0.860	F/B
S-9	3746	0.00423	8.15	35.1	116,400	0.965	69.6	0.226	0.233	0.970	S-F-B
S-10	3024	0.00317	0	35.6	116,400	0.878	25.0	0.182	0.196	0.930	F
S-11	4091	0.00428	0	26.9	116,400	1.170	44.4	0.174	0.221	0.790	S/B
S-12	4174	0.00730	0	57.5	116,000	0.890	97.0	0.350	0.346	1.010	S
S-13	2718	0.00510	4.16	57.2	118,000	0.985	252.0	0.375	0.368	1.020	F
S-14	2794	0.00613	8.15	63.4	117,000	1.040	42.6	0.326	0.367	0.890	F
S-15	3225	0.00629	8.15	52.0	60,500	1.120	94.6	0.320	0.350	0.915	F-B
S-16	3337	0.00625	8.15	53.5	64,500	1.050	197.0	0.334	0.318	1.050	S-F/B
S-17	4226	0.00870	8.15	62.7	117,200	0.985	181.0	0.425	0.421	1.010	F
S-18	3314	0.00633	0	61.0	58,200	0.940	93.5	0.276	0.328	0.845	S-F/B
S-19	3502	0.00637	4.16	64.1	61,200	0.850	141.0	0.326	0.327	0.998	F
S-20	3155	0.00654	4.16	51.0	59,600	1.200	158.0	0.300	0.346	0.870	F-B
S-21	3485	0.00613	0	57.3	68,000	0.920	98.9	0.282	0.200	1.090	S-B
S-22	3060	0.00644	4.16	49.7	119,000	1.270	100.0	0.317	0.366	0.866	F/B
S-23	3140	0.00654	8.15	63.0	114,000	0.990	74.0	0.345	0.359	0.960	F/B
S-24	4376	0.00830	4.16	55.8	127,000	1.020	151.0	0.387	0.412	0.940	F/B
									Avg. =	0.925	
									Std. dev. =	0.122	

*S, F denote shear and flexure failures, respectively.
S/B, F/B denote shear and flexure failures preceded by bond failure.
S-B, F-B denote shear and flexure failures accompanied by bond failures.

TABLE VII

Measured and Computed Ultimate Capacity

Mark	Concrete strength f'_c (psi)	Longitudinal steel p	Percent web reinforcement r	$\frac{E_s p}{k_1 k_3 f'_c}$ Q'	Effective prestress f_{se} (psi)	Measured depth to neutral axis k_u	a/d	Ultimate load P_{ult} (lb)	$\frac{M_{ult}}{k_1 k_3 f'_c bd^2}$ Meas.	$\frac{M_{ult}}{k_1 k_3 f'_c bd^2}$ Comp.	$\frac{M_{ult} - \text{Meas.}}{M_{ult} - \text{Comp.}}$	Mode of failure*
S-1	4736	0.00289	0	26.3	0	0.132	4.24	9,900	0.125	0.137	0.912	F/B
S-2	4700	0.00298	4.16	19.0	0	0.128	4.42	12,900	0.124	0.148	0.837	F/B
S-3	5045	0.00289	8.15	44.8	0	0.130	4.24	5,800	0.124	0.134	0.925	F/B
S-4	2640	0.00285	0	33.7	0	0.151	4.18	8,600	0.141	0.155	0.910	S
S-5	2600	0.00296	4.16	31.0	0	0.172	4.34	10,600	0.160	0.192	0.834	S-F-B
S-6	3200	0.00298	8.15	21.2	0	0.151	4.36	13,800	0.124	0.153	0.815	S-F/B
S-7	3100	0.00298	0	51.5	58,000	0.182	4.42	6,600	0.168	0.175	0.960	F/B
S-8	2910	0.00289	4.16	37.7	58,000	0.171	4.24	8,800	0.159	0.183	0.869	F/B
S-9	3746	0.00423	8.15	35.1	116,400	0.226	4.65	15,000	0.208	0.210	0.990	S-F-B
S-10	3024	0.00317	0	35.6	116,400	0.182	4.65	10,700	0.202	0.180	1.122	F
S-11	4091	0.00428	0	26.9	116,400	0.174	4.73	15,000	0.162	0.150	1.080	S/B
S-12	4174	0.00730	0	57.5	116,000	0.350	4.60	22,600	0.296	0.250	1.170	S
S-13	2718	0.00510	4.16	57.2	118,000	0.375	4.50	21,000	0.368	0.362	1.020	F
S-14	2794	0.00613	8.15	63.4	117,000	0.326	4.50	17,500	0.293	0.317	0.925	F
S-15	3225	0.00629	8.15	52.0	60,500	0.320	4.62	20,600	0.280	0.292	0.960	F-B
S-16	3337	0.00625	8.15	53.5	64,500	0.334	4.56	20,600	0.287	0.276	1.040	S-F/B
S-17	4226	0.00870	8.15	62.7	117,200	0.425	4.80	27,100	0.347	0.346	1.002	F
S-18	3314	0.00633	0	61.0	58,200	0.276	4.65	15,200	0.244	0.282	0.865	S-F/B
S-19	3502	0.00637	4.16	64.1	61,200	0.326	4.69	16,400	0.282	0.282	1.000	F
S-20	3155	0.00654	4.16	51.0	59,600	0.300	4.30	19,000	0.269	0.296	0.910	F-B
S-21	3485	0.00613	0	57.3	68,000	0.282	4.50	17,000	0.248	0.200	1.240	S-B
S-22	3060	0.00644	4.16	49.7	119,000	0.317	4.72	21,000	0.280	0.310	0.904	F/B
S-23	3140	0.00654	8.15	63.0	114,000	0.345	4.80	17,200	0.294	0.305	0.964	F/B
S-24	4376	0.00830	4.16	55.8	127,000	0.387	4.57	29,000	0.324	0.362	0.895	F/B
										Avg. =	0.964	
										Std. dev. =	0.122	

*S, F denote shear and flexure failures, respectively.
S/B, F/B denote shear and flexure failures preceded by bond failures.
S-B, F-B denote shear flexure failures accompanied by bond failures.

TABLE VIII

Nominal Shear Stresses

Mark	Concrete strength f'_c (psi)	Longitudinal steel p	Percent of web reinforcement r	a/d	Ultimate shear V_{ult} (lb)	Nominal ultimate shear stresses $v = \frac{V_{ult}}{bjd}$ (psi)	$\frac{v}{f'_c}$	Allowable shearing* stress V_c (psi)	ACI-ASCE** $\frac{pf'_s}{f'_c}$ =	$0.3 \frac{f_{se}b'}{f'_s b}$
S-1	4736	0.00289	0	4.24	4,950	103	0.0218	132	------	
S-2	4700	0.00298	4.16	4.42	6,450	138	0.0284	564	------	
S-3	5045	0.00289	8.15	4.24	2,900	60	0.0119	604	------	
S-4	2640	0.00285	0	4.18	4,300	89	0.0337	76	------	
S-5	2600	0.00296	4.16	4.34	5,300	114	0.0438	312	------	
S-6	3200	0.00298	8.15	4.36	7,500	162	0.0505	384	------	
S-7	3100	0.00298	0	4.42	3,300	72	0.0232	93	0.218	0.0765
S-8	2910	0.00289	4.16	4.24	4,400	93	0.0320	348	0.226	0.0765
S-9	3746	0.00423	8.15	4.65	7,500	177	0.0472	449	0.256	0.154
S-10	3024	0.00317	0	4.65	5,350	125	0.0413	90	0.238	0.154
S-11	4091	0.00428	0	4.74	7,500	177	0.0433	123	0.238	0.154
S-12	4174	0.00730	0	4.60	11,300	286	0.0685	126	0.397	0.153
S-13	2718	0.00516	4.16	4.55	10,500	224	0.0825	325	0.427	0.156
S-14	2794	0.00613	8.15	4.50	8,750	211	0.0755	335	0.499	0.154
S-15	3225	0.00629	8.15	4.62	10,300	250	0.0770	390	0.443	0.0799
S-16	3337	0.00625	8.15	4.56	10,300	255	0.0765	399	0.426	0.085
S-17	4226	0.00870	8.15	4.80	13,550	366	0.0859	510	0.465	0.155
S-18	3314	0.00633	0	4.65	7,600	186	0.0562	99	0.462	0.076
S-19	3502	0.00637	4.16	4.69	8,200	206	0.0587	420	0.414	0.0805
S-20	3155	0.00654	4.16	4.30	9,500	242	0.0768	378	0.472	0.0786
S-21	3485	0.00613	0	4.50	8,500	201	0.0576	105	0.400	0.0896
S-22	3060	0.00644	4.16	4.72	10,500	265	0.0865	366	0.479	0.157
S-23	3140	0.00654	8.15	4.80	8,600	224	0.0714	376	0.474	0.150
S-24	4376	0.00830	4.16	4.57	14,500	358	0.0820	525	0.432	0.168

*According to ACI Building Code (ACI 315-5).

**"Tentative Recommendations for Prestressed Concrete," ACI-ASCE Joint Committee 323.

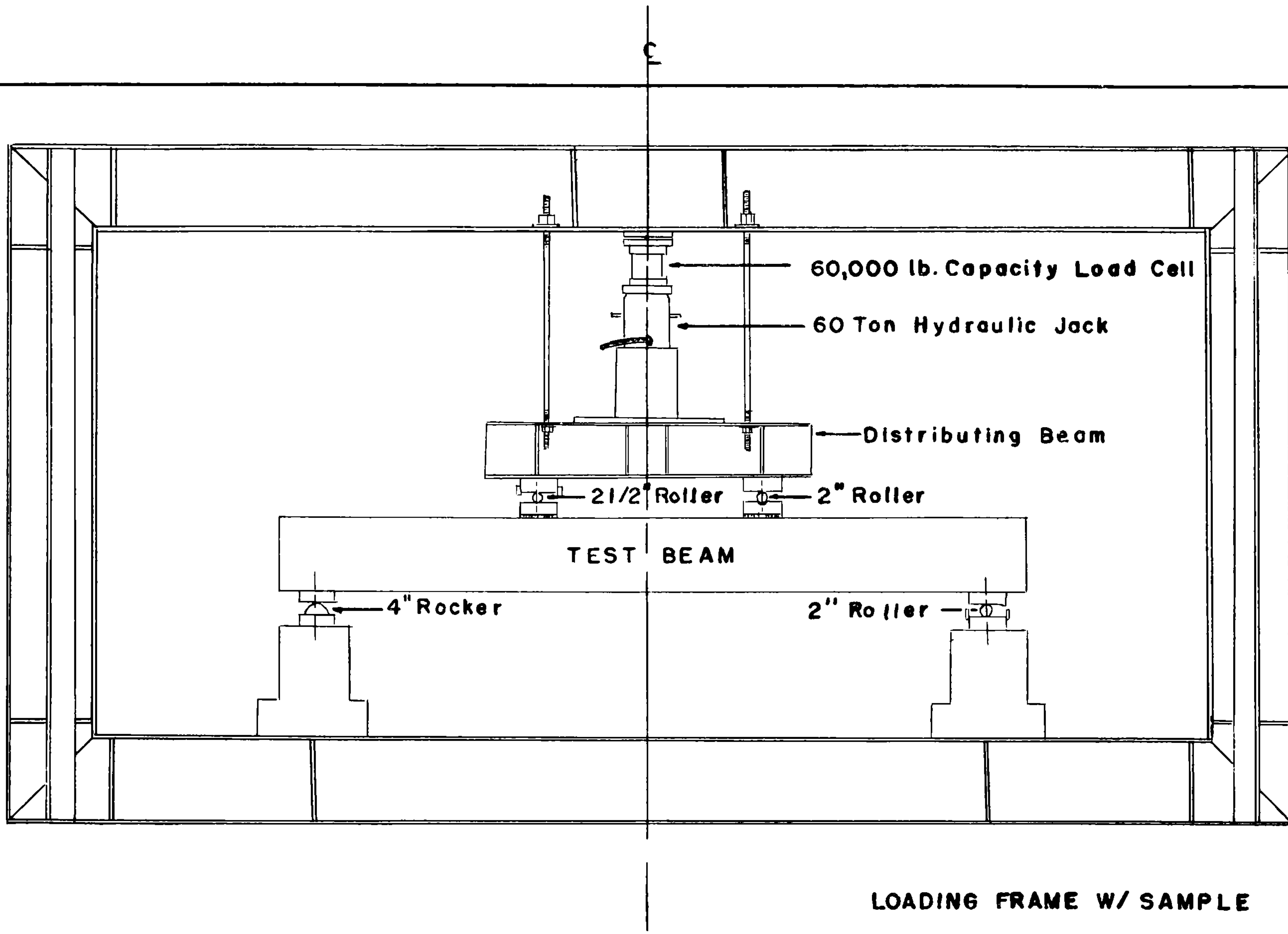


FIG. I TESTING APPARATUS

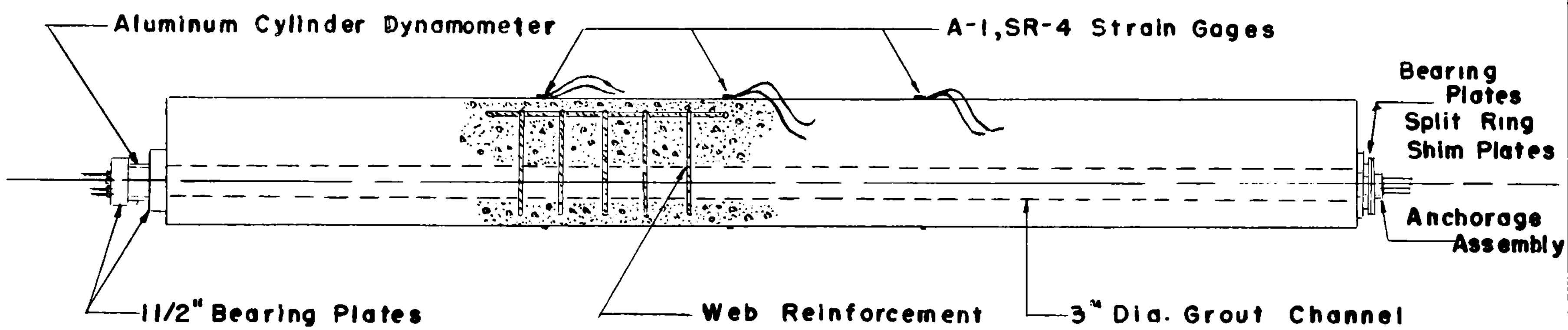


FIG.2 TYPICAL TEST BEAM

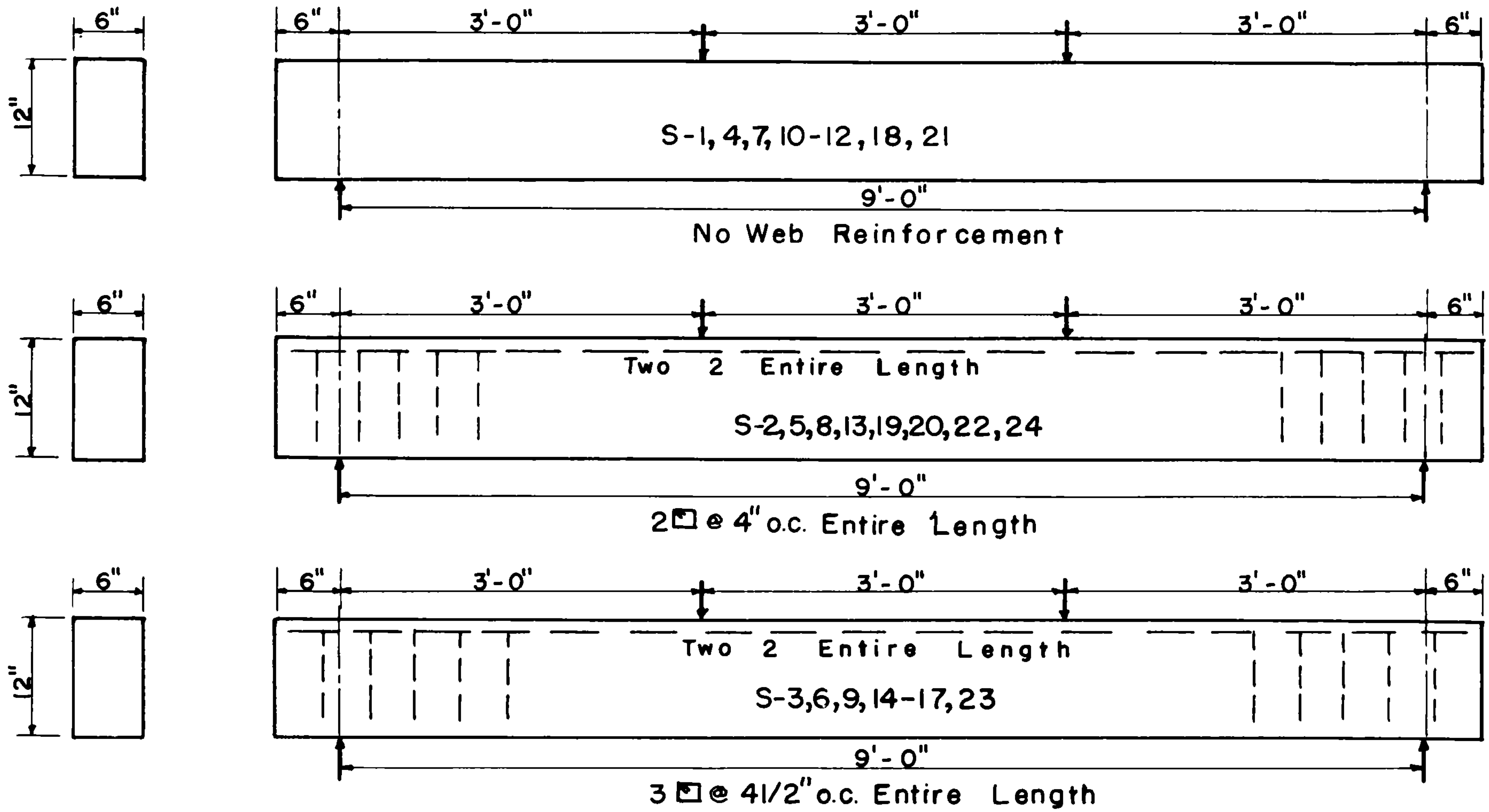


FIG. 3 DIMENSIONS, WEB REINFORCEMENT AND LOADING ARRANGEMENT FOR BEAMS

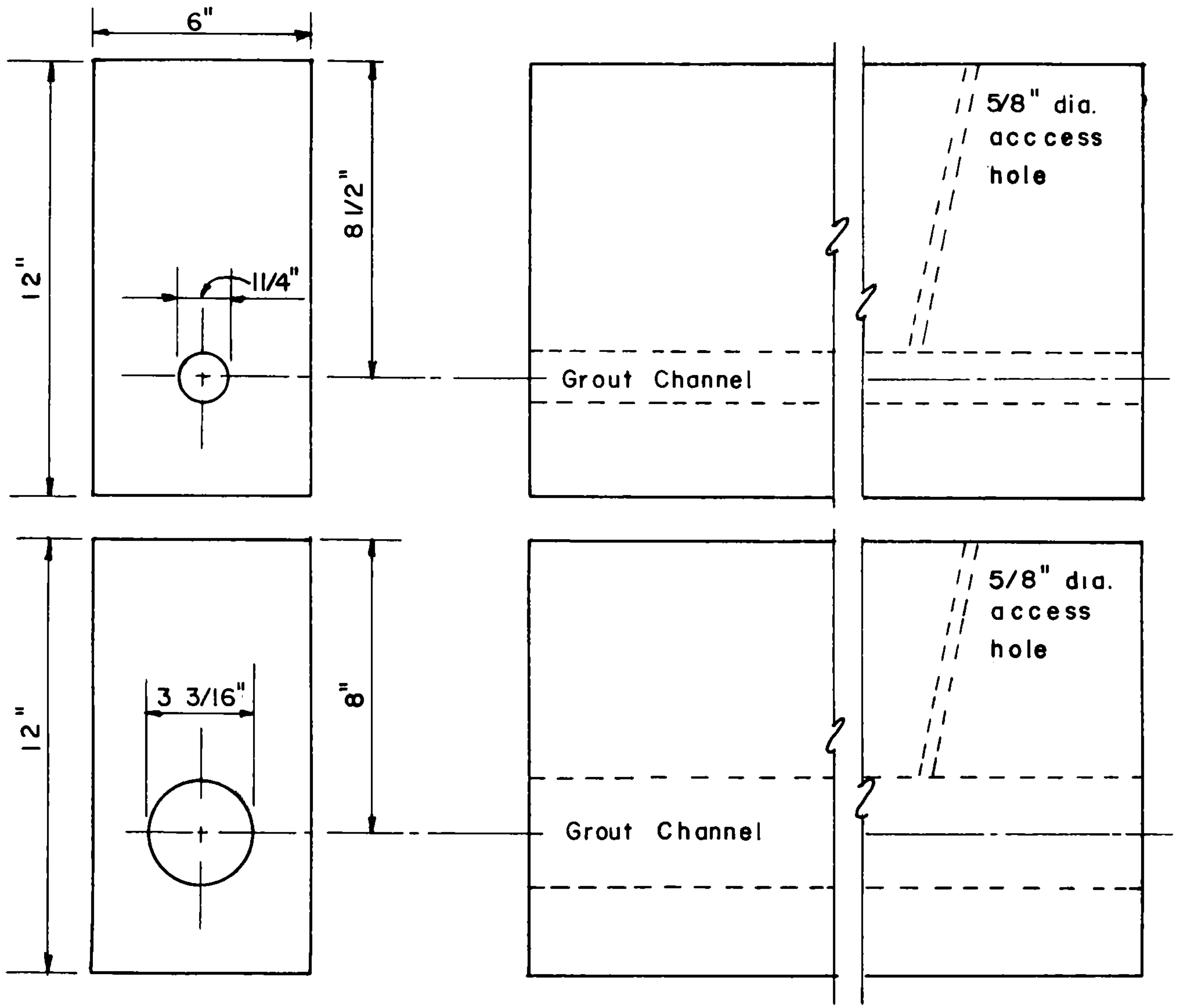


FIG. **4** DETAILS OF GROUT CHANNELS

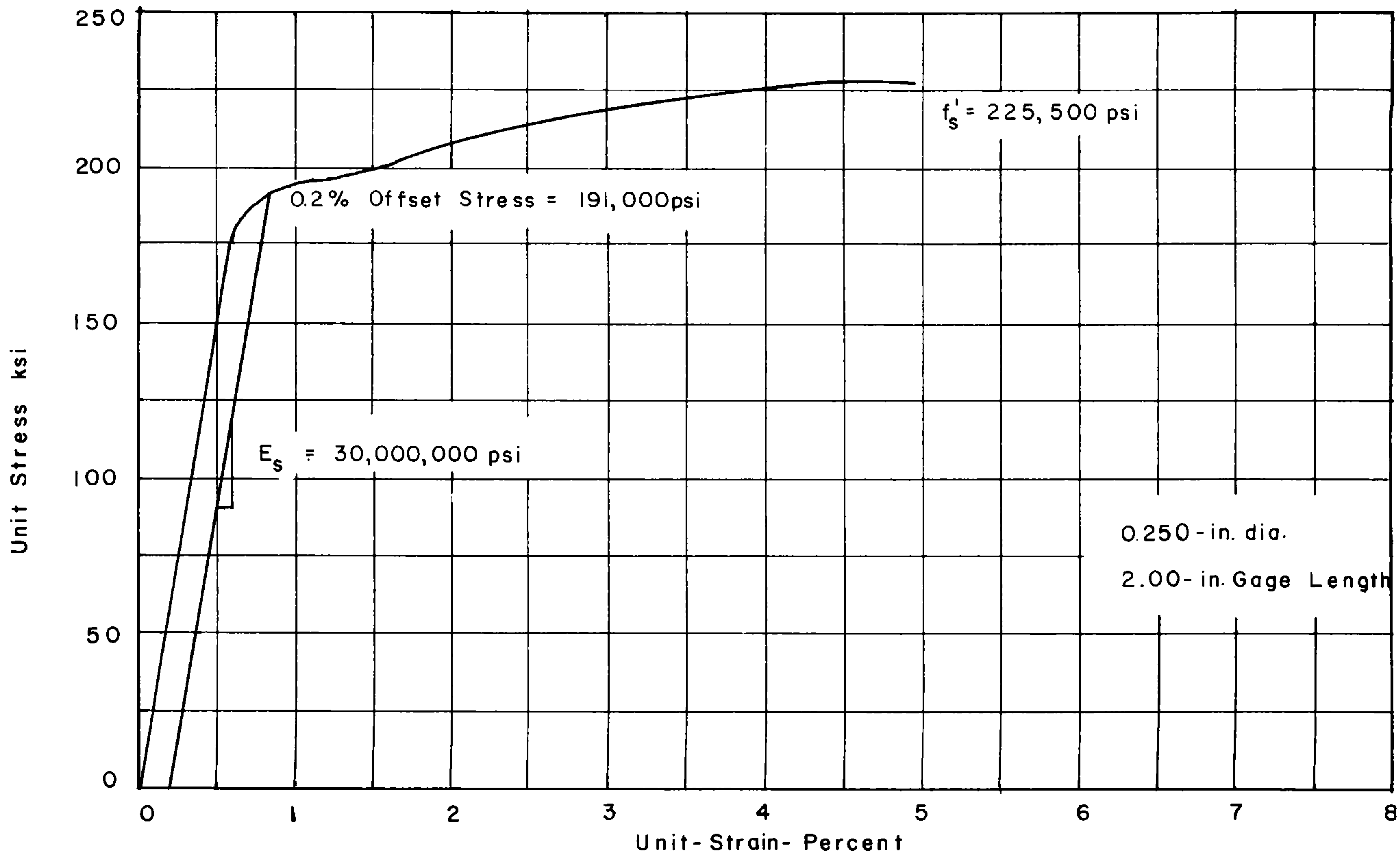
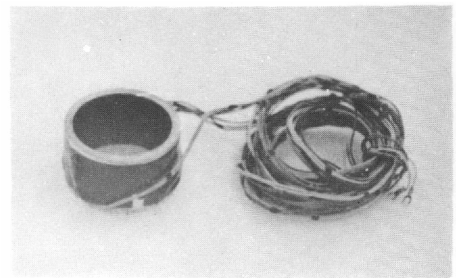
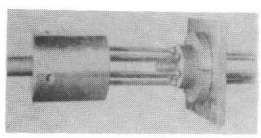


FIG. 5 STRESS STRAIN RELATIONSHIP FOR .250-in. Dia. "TUFWIRE"

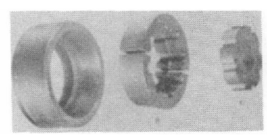
Load Cell



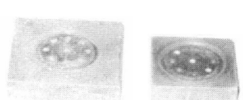
Dynamometer



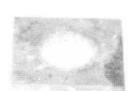
Complete P. I. Assembly



Stressing Assembly



Dynamometer Bearing Plates



P.I. Bearing Plate

Shim Plates

FIG. 6 VIEW OF LOAD CELL, DYNAMOMETER, PRESTRESSING ASSEMBLY, BEARING PLATES AND SHIM PLATES

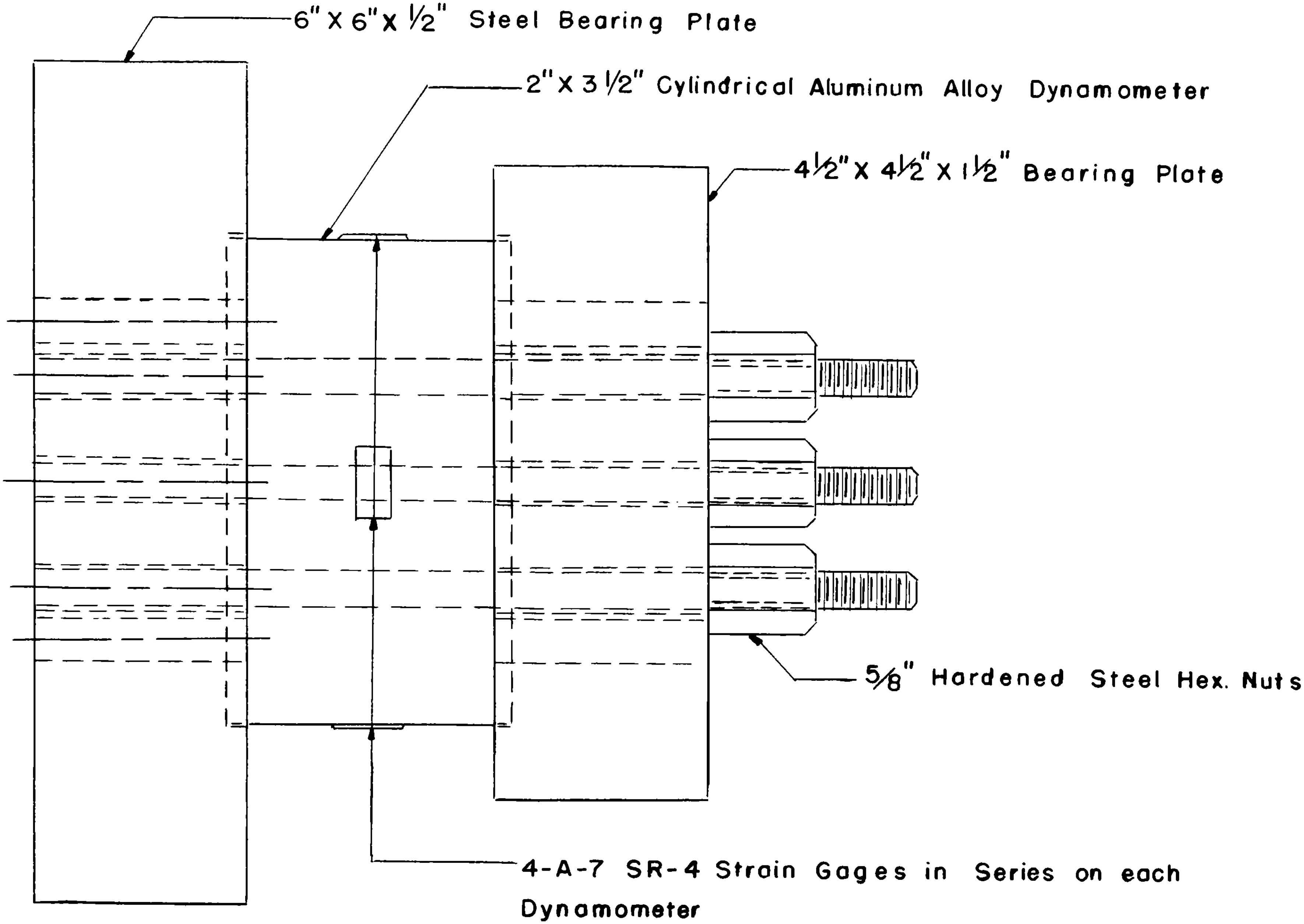


FIG. 7 DETAIL OF REINFORCEMENT ANCHORAGE AT DYNAMOMETER END OF BEAM

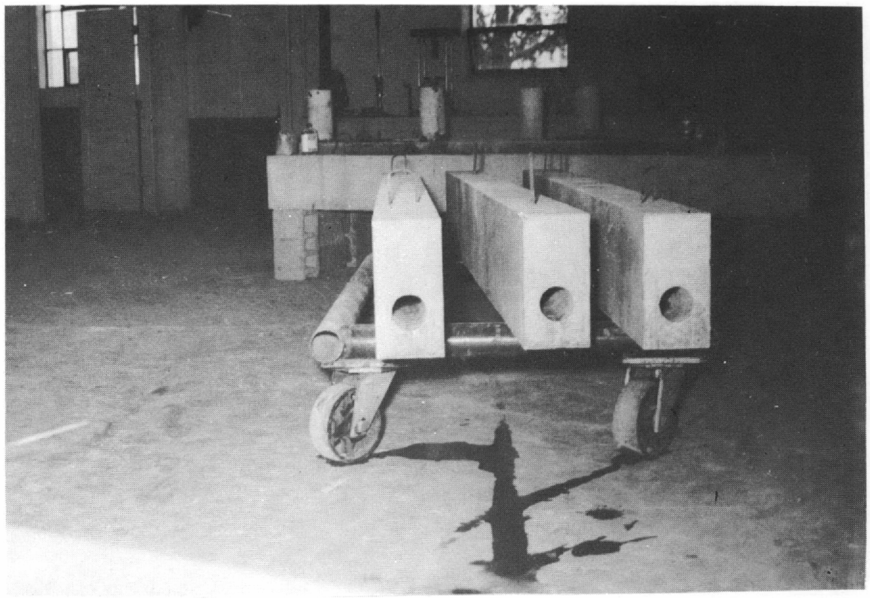


FIG. 8 VIEW OF GROUT CHANNEL

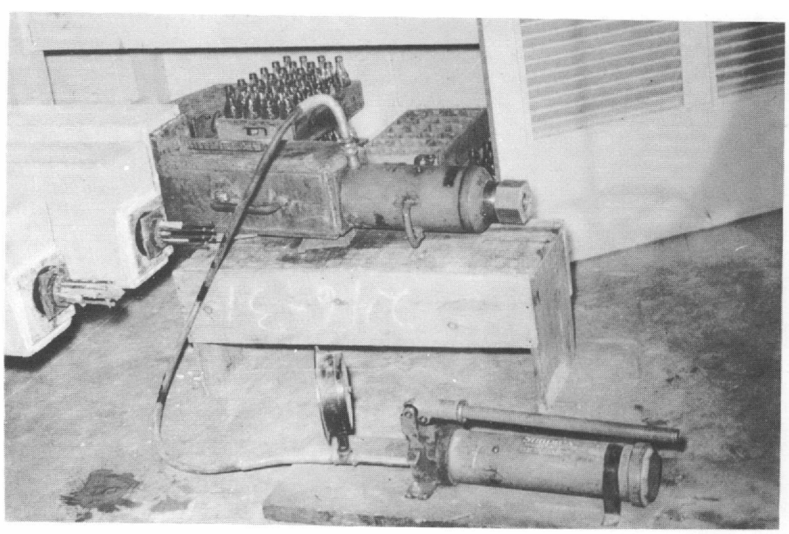


FIG. 9 VIEW OF PRESTRESSING APPARATUS

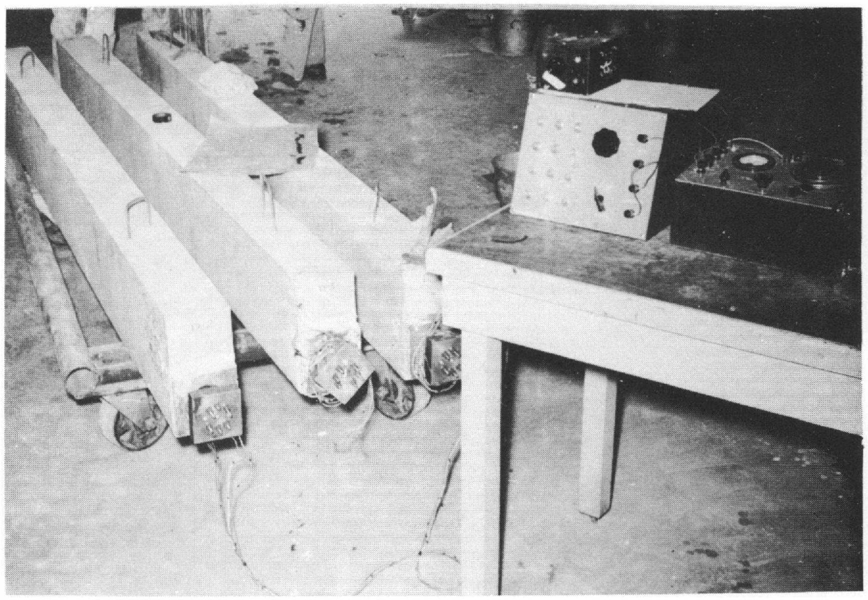


FIG. 10 VIEW OF DYNAMOMETER END OF BEAMS

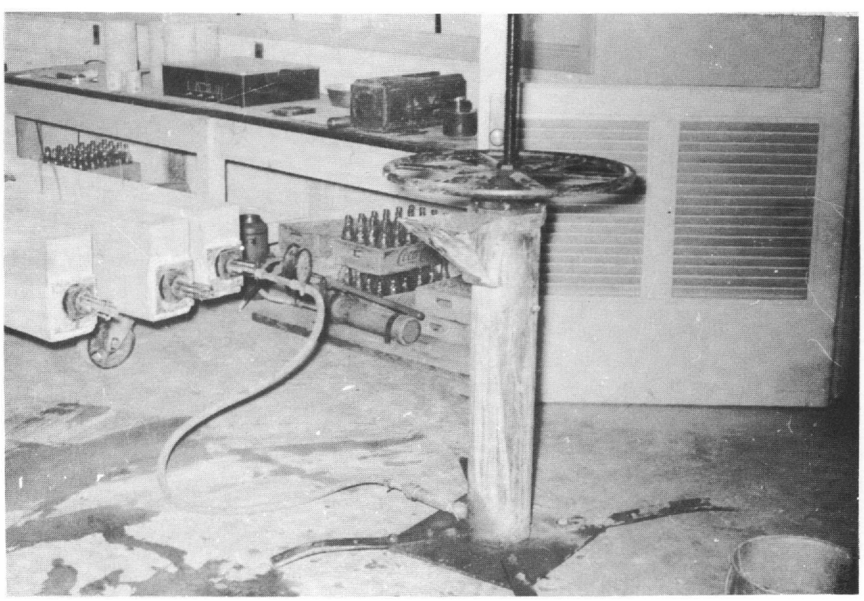
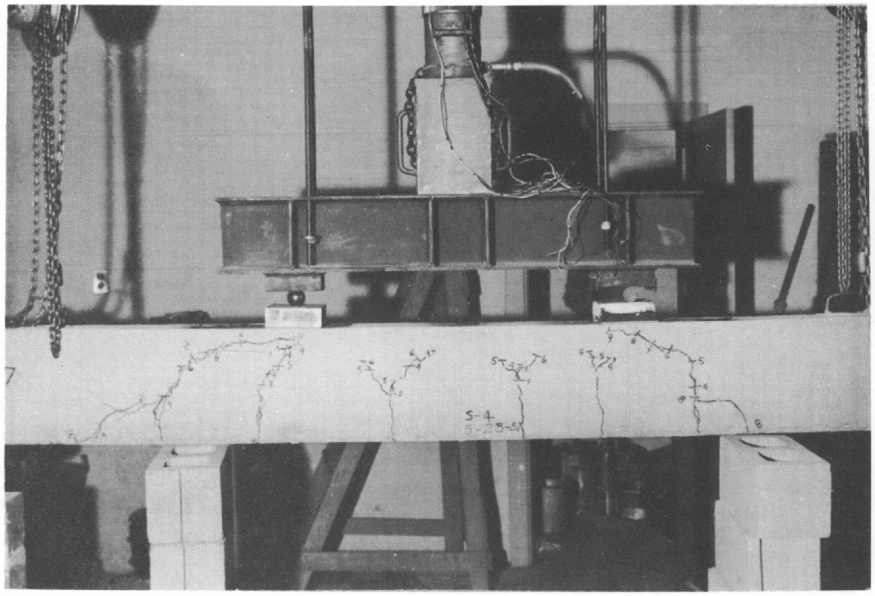


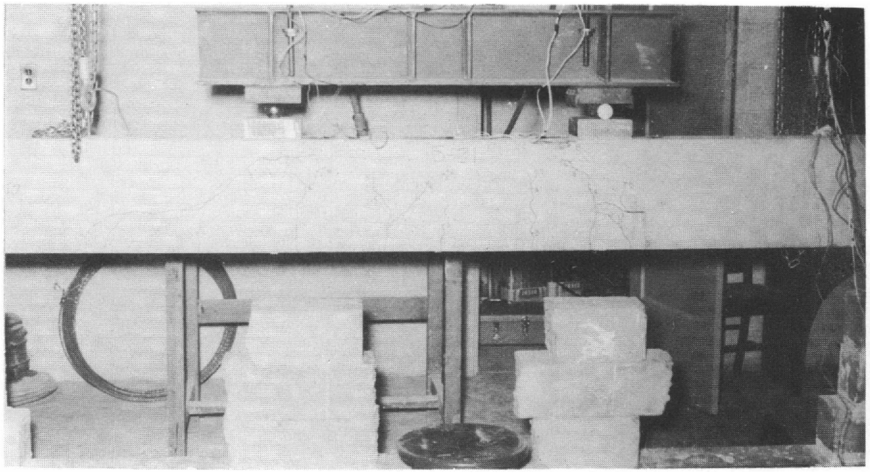
FIG. 11 VIEW OF GROUTING EQUIPMENT



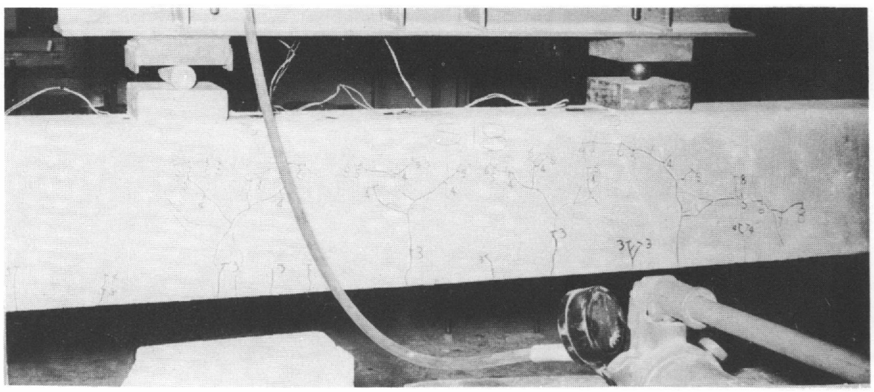
Typical Shear Failure — f_{se} = 0 psi, r = 0 %

Typical Balanced Failure — f_{se} = 0 psi, r = 4.16 %

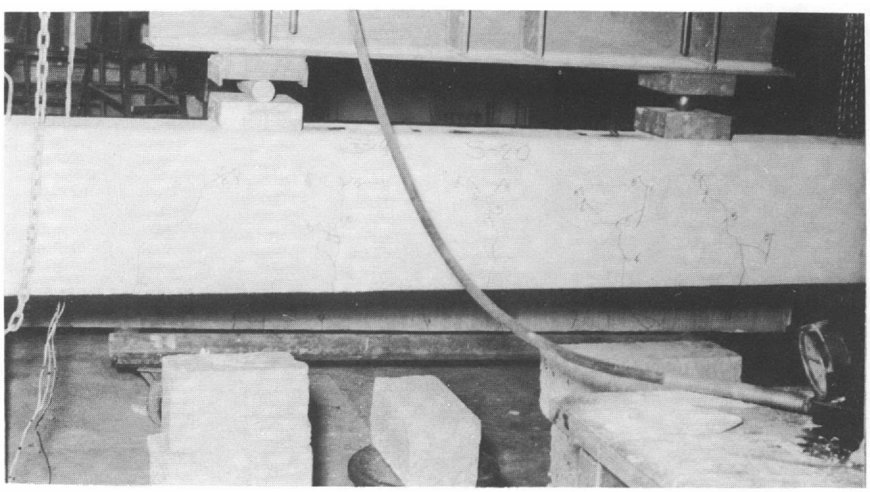
FIG. 12 VIEWS OF BEAMS AFTER TESTING



Typical Shear Failure — f_{se} = 60,000 psi, r = 0 %

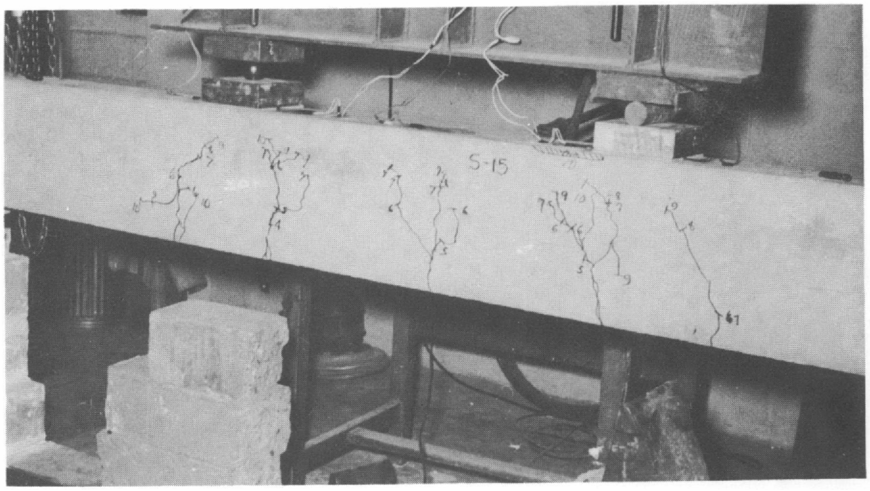


Typical Balanced Failure — f_{se} = 60,000 psi, r = 0 %

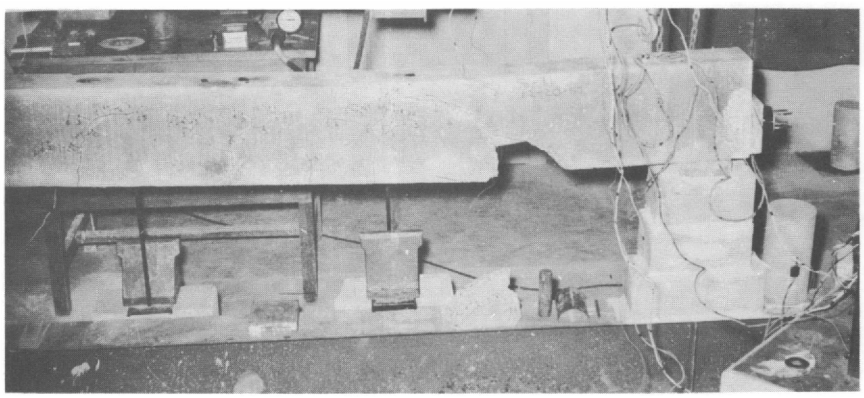


Typical Flexure Failure — f_{se} = 60,000 psi, r = 4.16 %

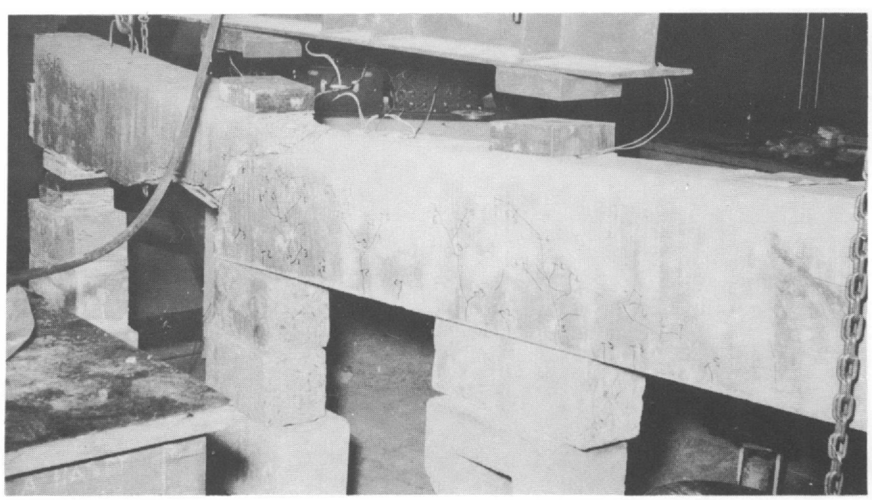
FIG. 13 VIEWS OF BEAMS AFTER TESTING



Typical Flexure Failure — f_{se} = 60,000 psi, r = 8.15 %

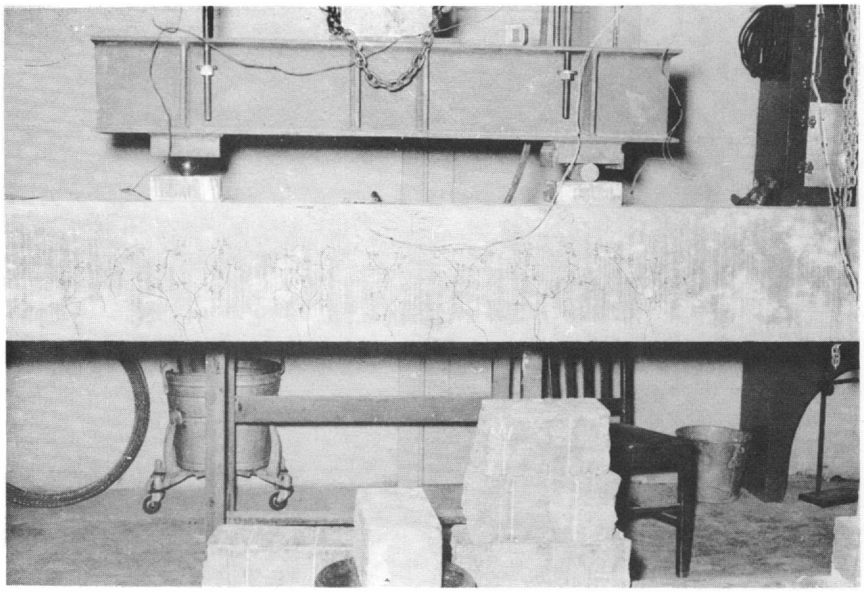


Typical Shear Failure — f_{se} = 120,000 psi, r = 0 %

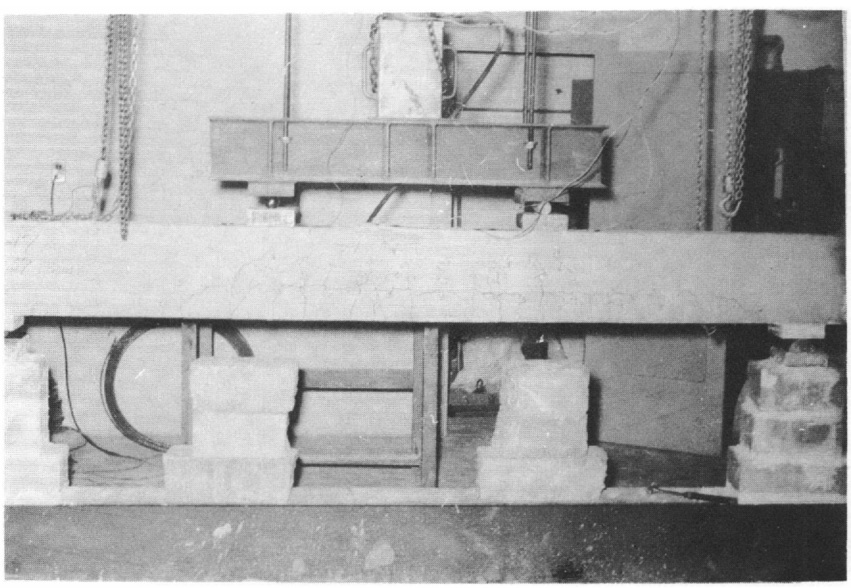


Typical Shear Failure — f_{se} = 120,000 psi, r = 0%

FIG. 14 VIEWS OF BEAMS AFTER TESTING



Typical Flexure Failure — f_{se} = 120,000 psi, r = 4.16 %



Typical Flexure Failure — f_{se} = 120,000 psi, r = 8.15 %

FIG. 15 VIEWS OF BEAMS AFTER TESTING

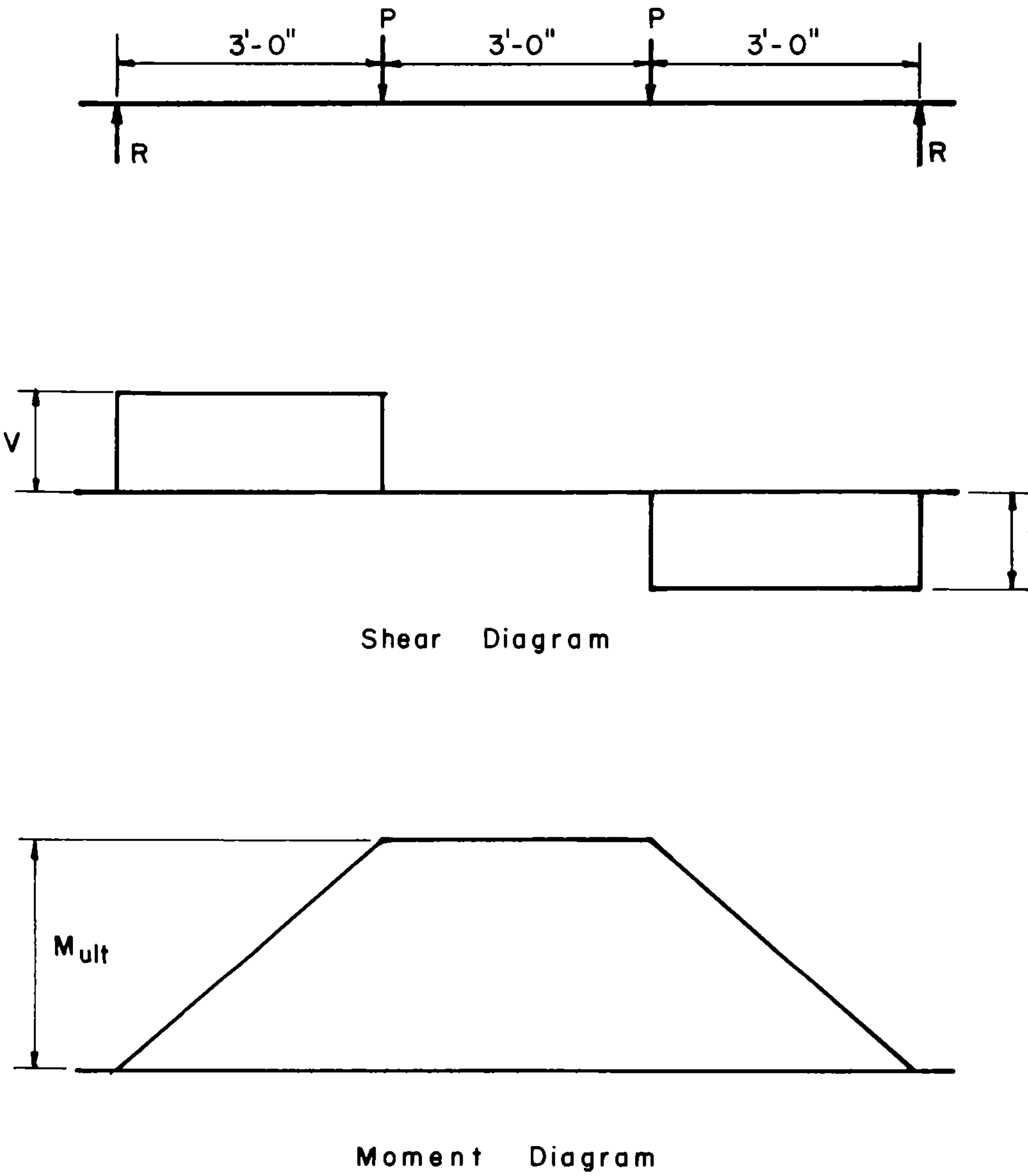


FIG. 16 SHEAR AND MOMENT DIAGRAMS

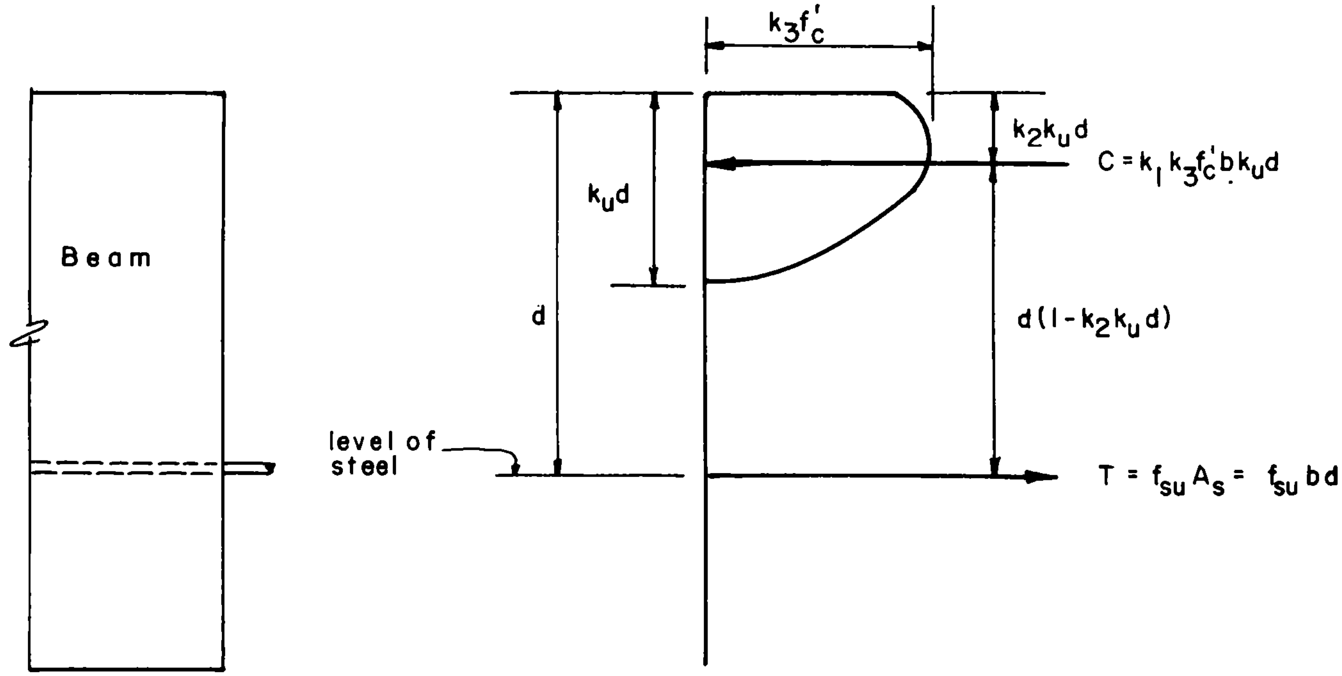


FIG. 17 PROPERTIES OF COMPRESSIVE STRESS BLOCK AT ULTIMATE LOAD

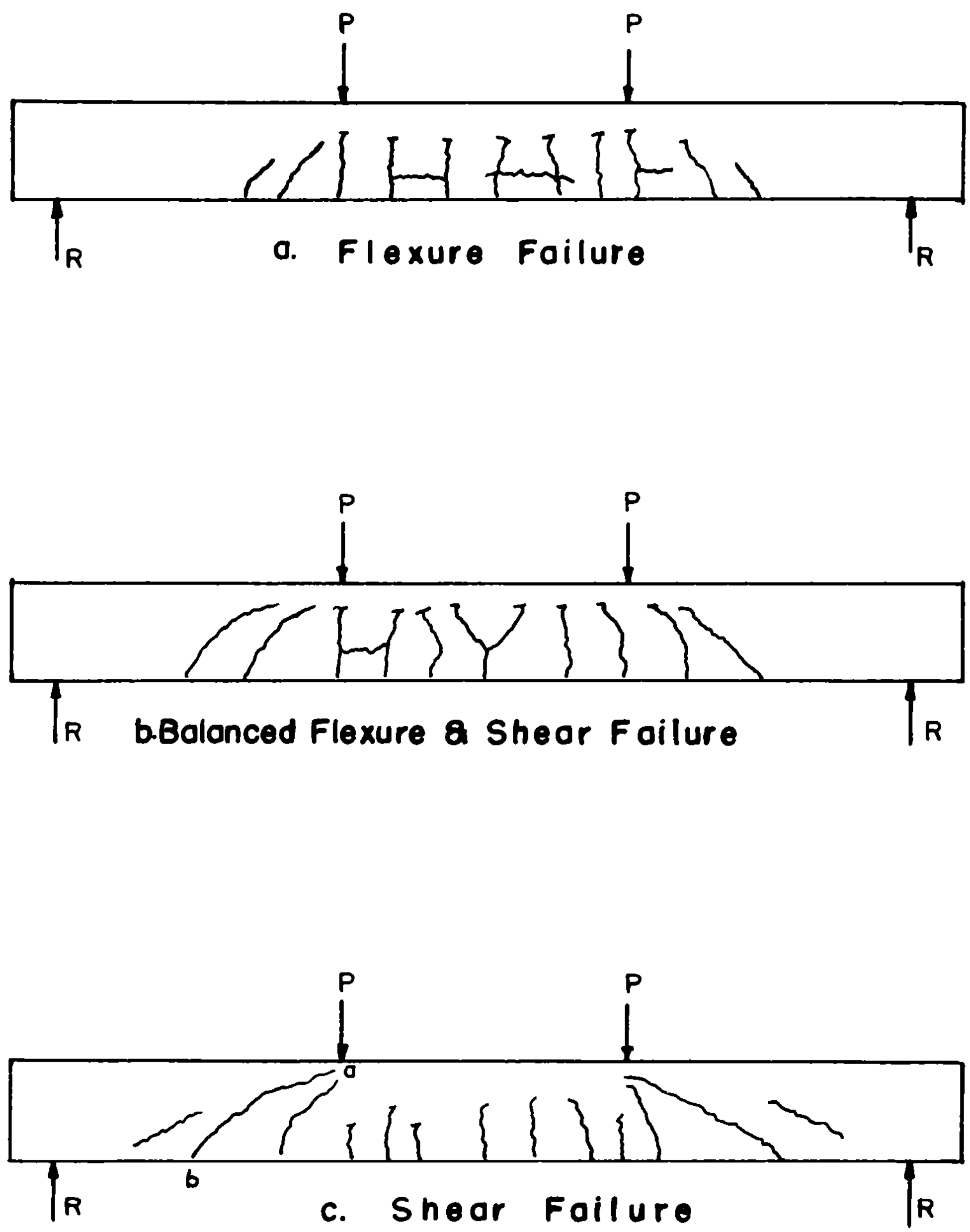


FIG 18 CRACKS IN BEAM AT ULTIMATE LOAD

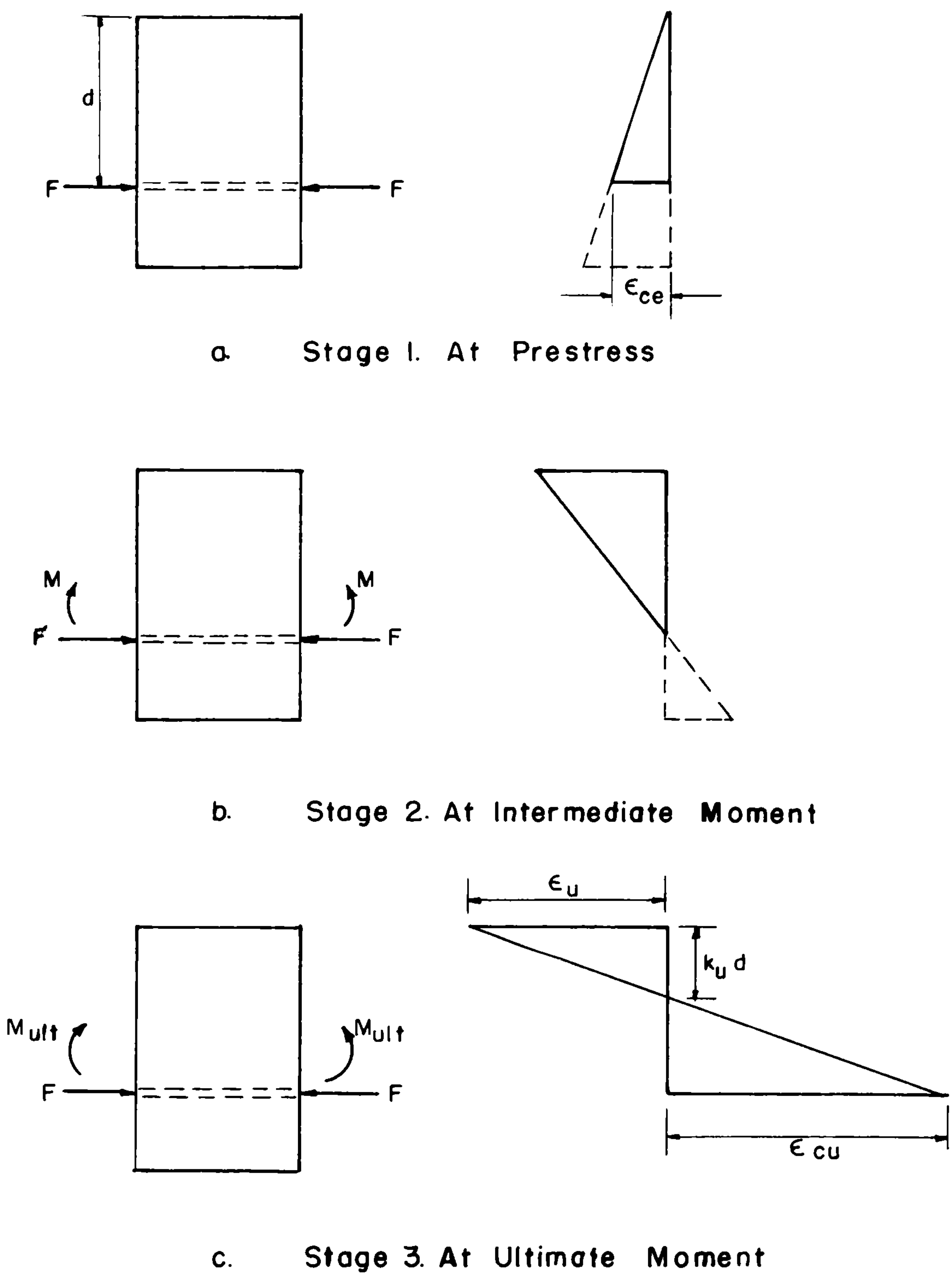


FIG. 19 STRAIN DISTRIBUTION IN THE BEAM AT VARIOUS STAGES OF LOADING

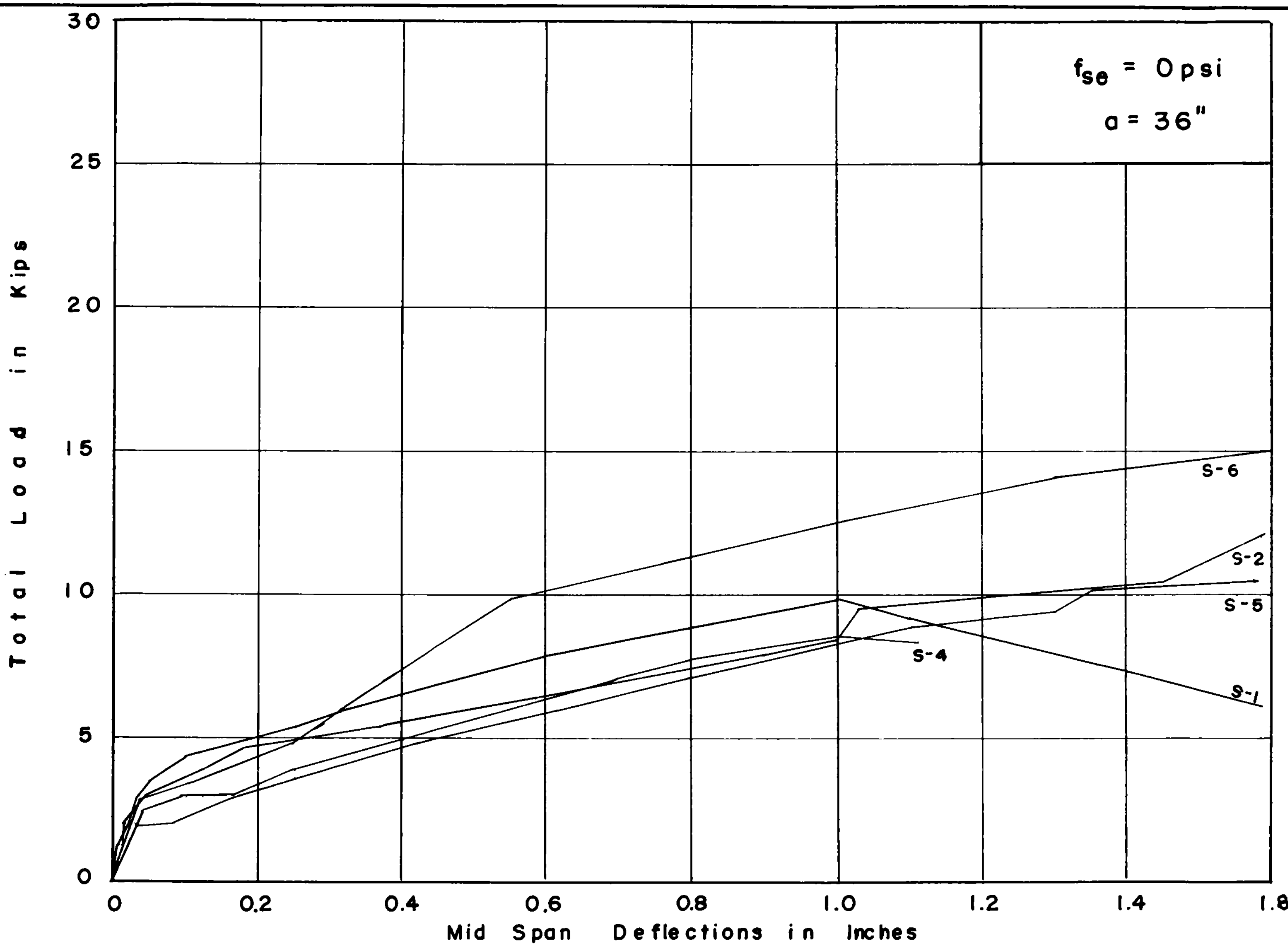


FIG. 20 LOAD DEFLECTION RELATIONSHIPS

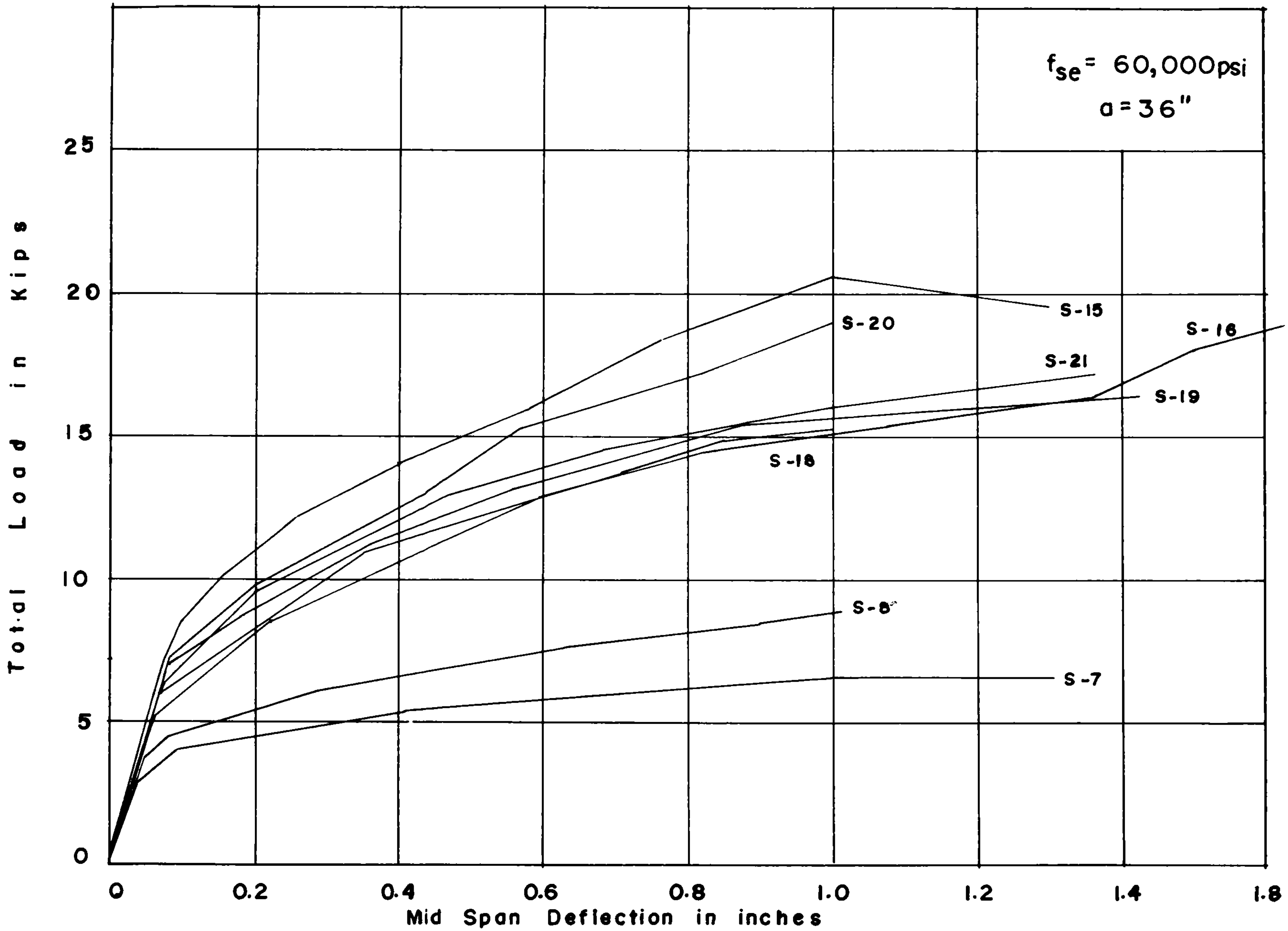


FIG. 21 LOAD DEFLECTION RELATIONSHIPS

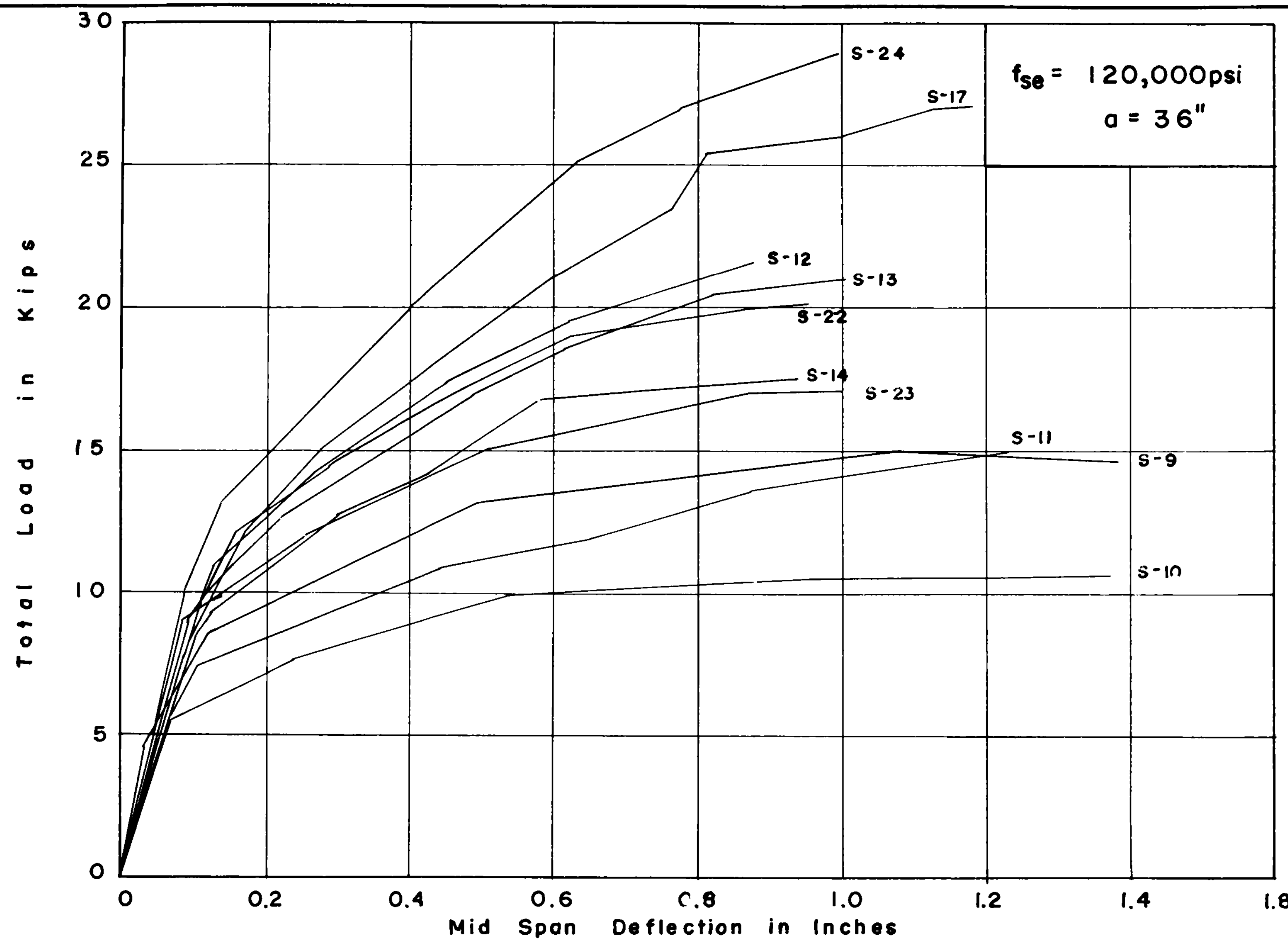


FIG. 22 LOAD DEFLECTION RELATIONSHIPS

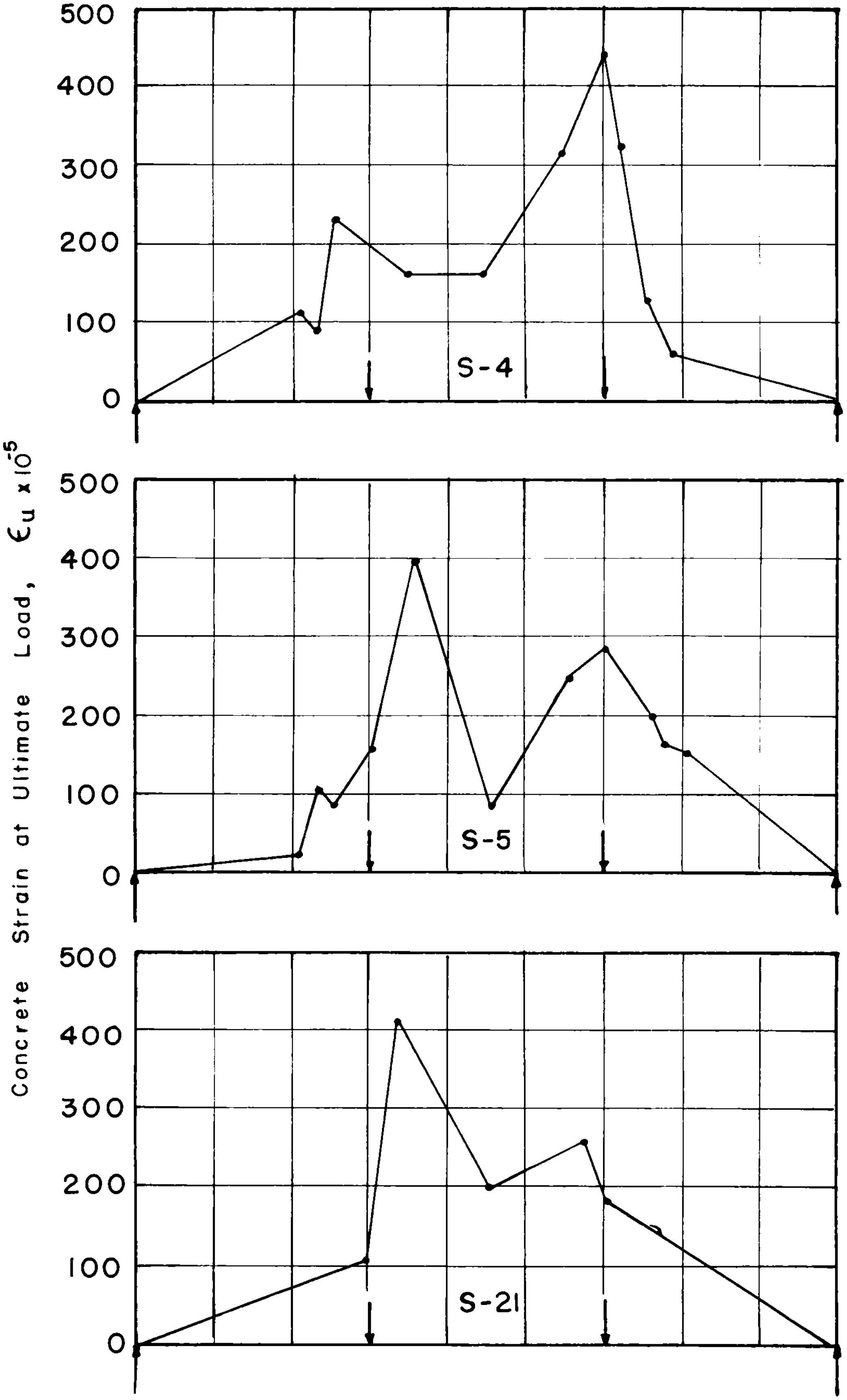


FIG. 23 DISTRIBUTION OF STRAIN AT ULTIMATE LOAD ALONG TOP SURFACE OF BEAMS FAILING IN SHEAR

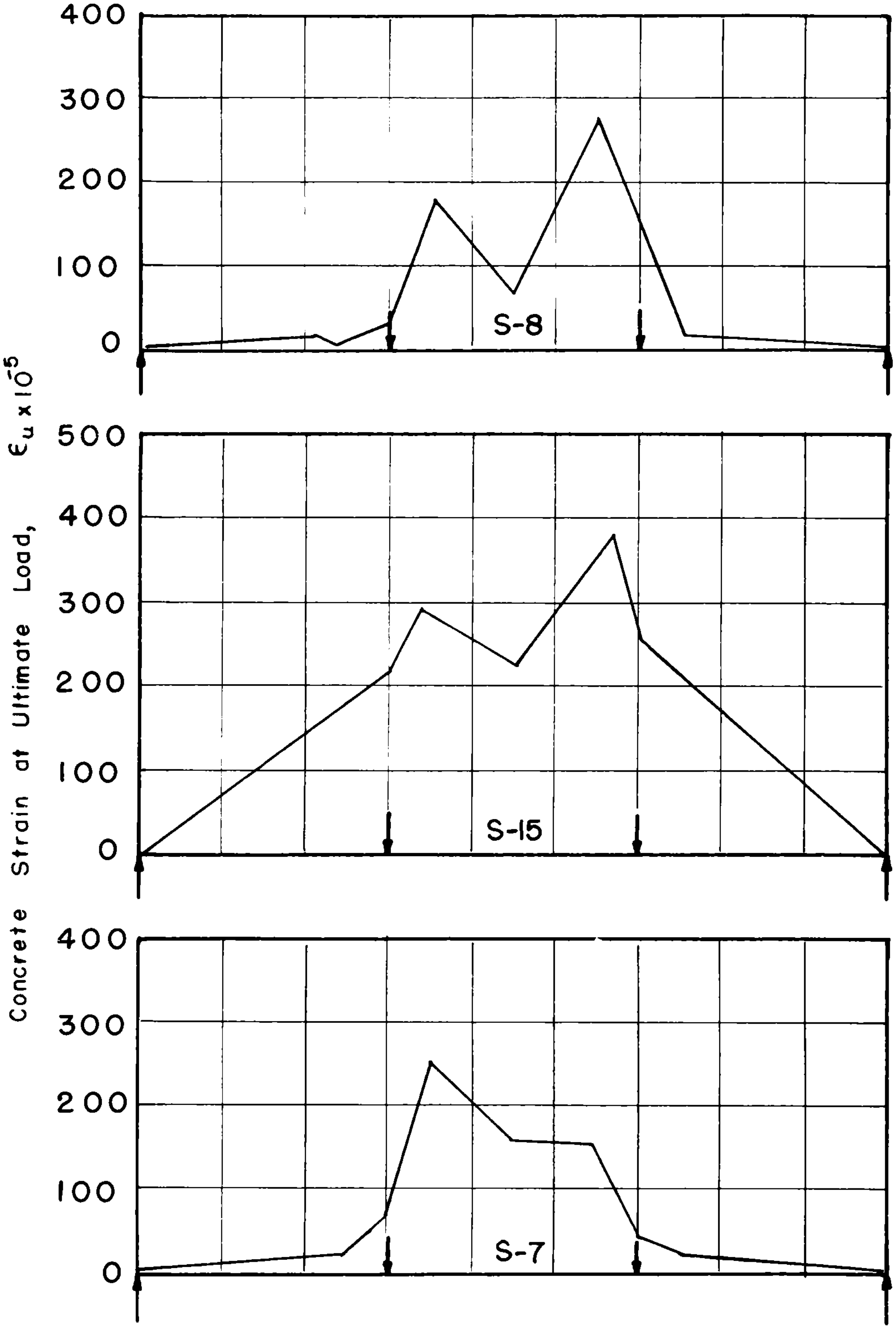


FIG.24 DISTRIBUTION OF STRAIN AT ULTIMATE LOAD ALONG TOP SURFACE OF BEAMS FAILING IN FLEXURE

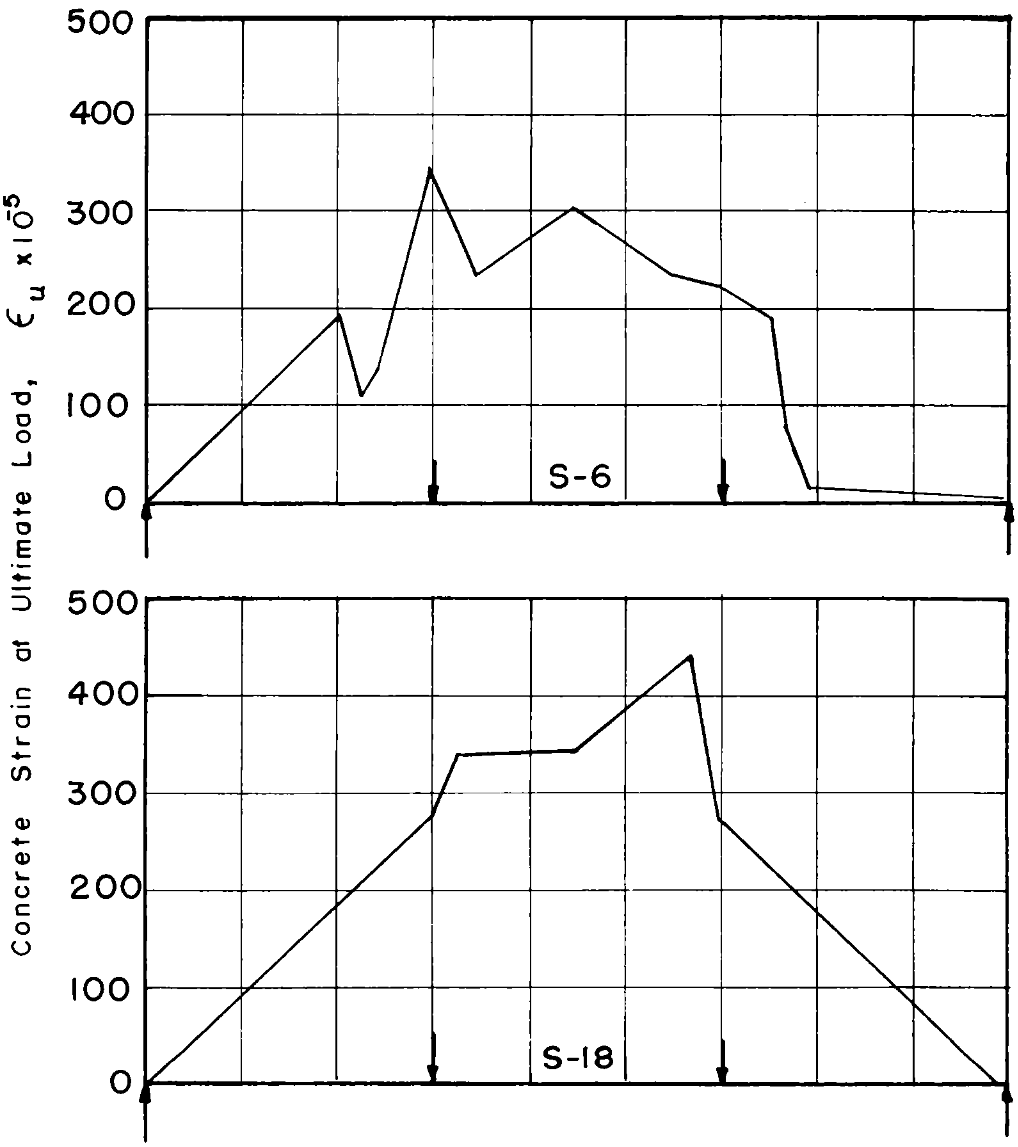


FIG. 25 DISTRIBUTION OF STRAIN AT ULTIMATE LOAD ALONG TOP SURFACE OF BEAMS FAILING IN SHEAR AND FLEXURE

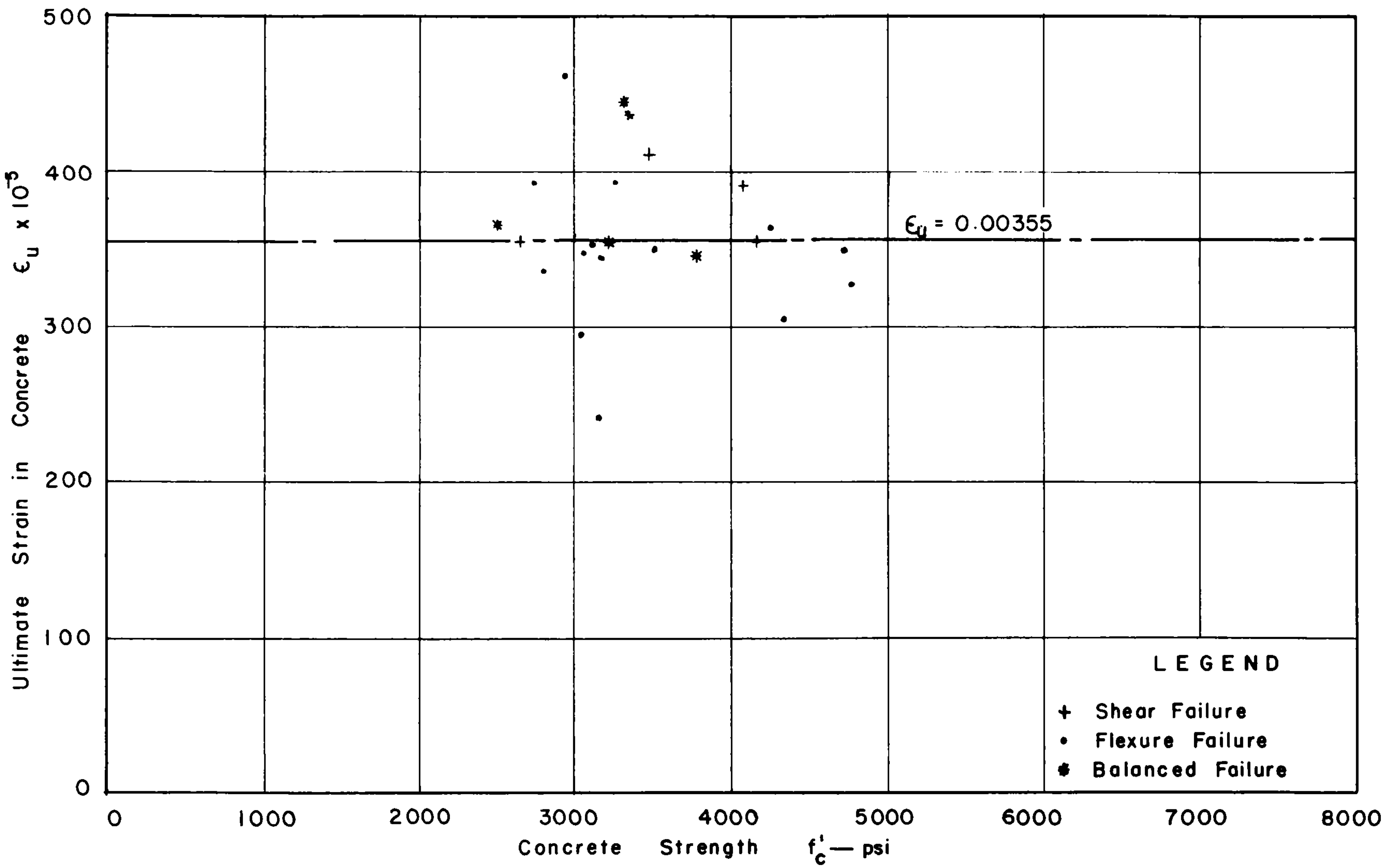


FIG. 26 COMPARISON OF ULTIMATE CONCRETE STRAIN WITH CONCRETE STRENGTH

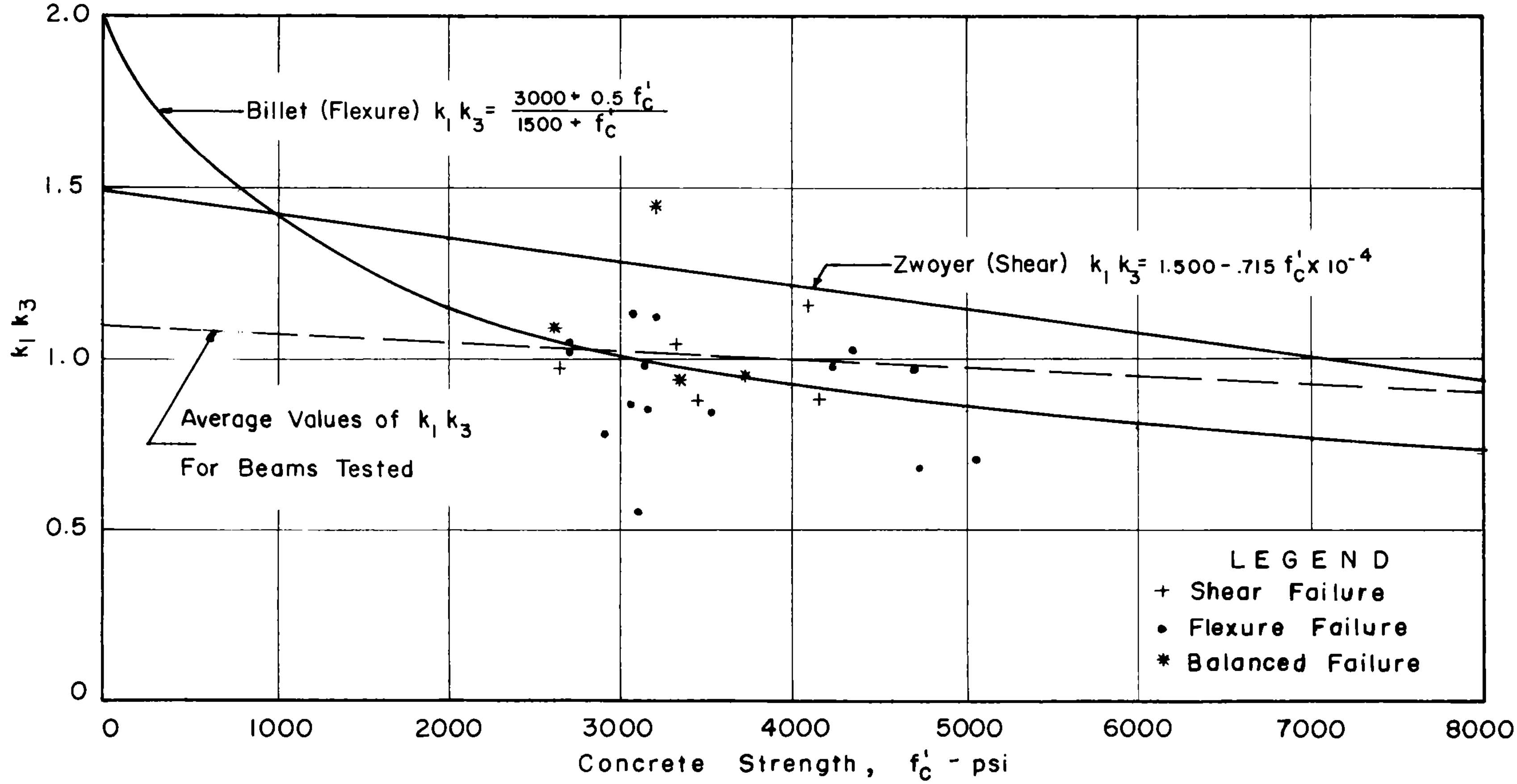


FIG. 27 COMPARISON OF $k_1 k_3$ WITH CONCRETE STRENGTH

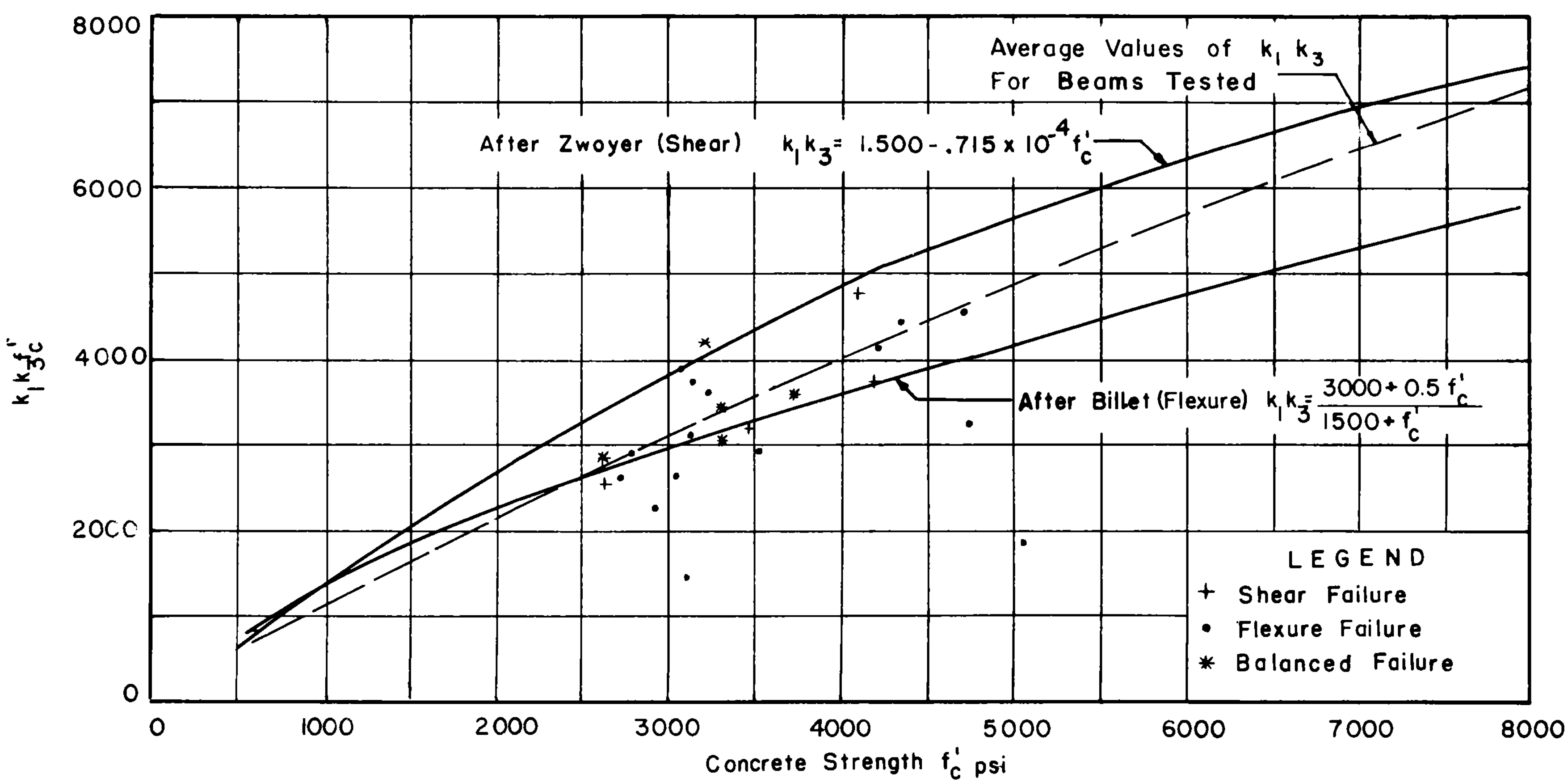


FIG. 28 COMPARISON OF VALUES OF $k_1 k_3 f'_c$ MEASURED FOR RECTANGULAR BEAMS WITH WEB REINFORCEMENT, WITH EXPRESSIONS OBTAINED FOR RECTANGULAR BEAMS WITHOUT WEB REINFORCEMENT, FAILING IN FLEXURE AND SHEAR

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