

# Measuring Sparticles with the Matrix Element

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**Abstract.** We apply the Matrix Element Method (MEM) to mass determination of squark pair production with direct decay to quarks and LSP at the LHC, showing that simultaneous mass determination of squarks and LSP is possible. We furthermore propose methods for inclusion of QCD radiation effects in the MEM.

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The goal of the LHC at CERN, scheduled to start this year, is to discover new physics through deviations from the Standard Model (SM) predictions. After discovery of deviations from the SM, the next step will be classification of the new physics. An important first goal in this process will be establishing a mass spectrum of the new particles. One of the most challenging scenarios is pair-production of new particles which decay to invisible massive particles, giving missing energy signals. Many methods have been proposed for mass determination in such scenarios (for a recent list of references, see e.g. [1]).

In this proceeding, we report the first steps in applying the Matrix Element Method (MEM) in the context of supersymmetric scenarios giving missing energy signals. After a quick review of the MEM, we will focus on squark pair production, a process where other mass determination techniques have difficulties to simultaneously determine the LSP and squark masses. Finally, we will introduce methods to extend the range of validity of the MEM, by taking into account initial state radiation (ISR) in the method.

The Matrix Element Method is a procedure to measure a set of theoretical parameters from a sample of experimental events. It associates to each event  $x$  a weight  $P(x|\alpha)$ , which is a measure of the probability for an event  $x$  to be observed given a set of parameters  $\alpha$ . The computation of these weights is done by convoluting the production of a parton-level configuration  $y$ , given by the squared matrix element  $|M_\alpha|^2(y)$ , with the probability that the parton-level configuration  $y$  evolves into the reconstructed event  $x$ , as modeled by a transfer function  $W(x,y)$ . As a result, the weight of a specific event  $x$  can be written as

$$P(x|\alpha) = \frac{1}{\sigma_\alpha} \int d\phi(y) |M_\alpha|^2(y) dq_1 dq_2 f_1(q_1) f_2(q_2) W(x,y) \quad (1)$$

where  $f_1(q_1)$  and  $f_2(q_2)$  represent the parton distribution functions,  $d\phi(y)$  the phase space measure and  $\sigma_\alpha$  the total cross section. The normalisation by the cross-section ensures that  $P(x|\alpha)$  is a probability density:  $\int P(x|\alpha) dx = 1$ .

In principle, this definition provides the best possible discriminator on an event-by-event basis, and can be used to measure  $\alpha$ . With a sample of  $N$  events, we can construct a log-likelihood:

$$-\ln(L) = -\sum_{i=1}^N \ln(P(x_i|\alpha)) + N \int \text{Acc}(x)P(x|\alpha)dx \quad (2)$$

where the acceptance term  $\text{Acc}(x)$ , corrects for the bias introduced by detector acceptance and event selection. The set of parameters  $\alpha$  maximizing the likelihood correspond to the most probable value.

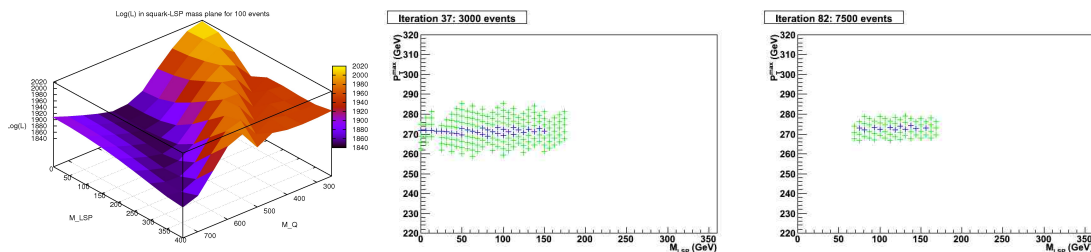
Although the MEM is conceptually very simple, the numerical evaluation of the weights is not straightforward due to the large variations of the integrand. Indeed, both the square matrix-element  $|M_\alpha|^2$  and the transfer function  $W(x,y)$  are highly non uniform on the phase-space. In consequence, a specific Monte Carlo phase-space integrator, designed to deal with the specific behavior of the integrand is required.

MadWeight [2] is a publicly available phase space generator dedicated to perform this type of integration. In order to find the best phase-space mapping, MadWeight applies a series of local changes of variables to promote, when possible, the invariant mass of resonant propagators as variables of integrations. In this way, MadWeight is able to find the best phase-space mapping for any decay chain and a large class of transfer functions.

## SQUARK MEASUREMENT

The MEM has been successfully applied for determination of the top quark mass at the Tevatron [3]. However, applying the MEM to supersymmetric scenarios brings new difficulties, including a large parameter space, a priori unknown order of particles in decay chains, and unknown masses of invisible particles. In the preliminary study presented here, we focus on the latter of these difficulties by studying squark pair production, where the squarks decay directly to quark and LSP, the lightest neutralino. The signature is two hard jets and large missing transverse energy. This scenario is interesting, because the  $m_{T2}$  “kink” method [4], as well as other methods for simultaneous reconstruction of the squark and neutralino masses often fails for two-body decays of the squarks.

As a first check that the Matrix Element method can indeed reconstruct masses in this scenario, we simulate squark pair production at the LHC, and study this process at parton level without cuts, with  $m_{\tilde{q}} = 561$  GeV and  $m_{\tilde{\chi}^0} = 97$  GeV. The resulting negative logarithmic likelihood function is shown for 100 events in Fig. 1a). The minimum of  $-\ln(L)$  forms a “valley” in the  $(m_{\tilde{q}}, m_{\tilde{\chi}^0})$  plane. This valley is closely aligned with the expression for the energy of the quark in the rest frame of the decaying squark, i.e., the maximum  $p_T$  of the quark produced by squark production at rest, given by  $p_T^{\max} = (m_{\tilde{q}}^2 - m_{\tilde{\chi}^0}^2)/2m_{\tilde{q}}$ . Fig. 1 b) and c) show the extent of the valley using 3000 and 7500 events, respectively (corresponding to an integrated luminosity of  $10 \text{ fb}^{-1}$  and  $25 \text{ fb}^{-1}$  respectively, assuming 100% branching ratio). With 3000 events, only an upper limit on  $m_{\tilde{\chi}^0}$  can be determined. For 7500 events, however, the mass range for the  $\tilde{\chi}^0$  is already quite restricted, and is determined to be between 65 GeV and 160 GeV. The  $p_T^{\max}$  value is very well determined, with an uncertainty of  $\pm 2$  GeV (less than 1%).



**FIGURE 1.** Matrix element  $-\log(L)$  for a) 100, b) 3000 and c) 7500 parton level  $q\bar{q} \rightarrow q\bar{q}\tilde{\chi}^0\tilde{\chi}^0$  events at the LHC. The true values are at  $(m_{\tilde{q}}, m_{\tilde{\chi}^0}) = (561, 97)$  GeV. b) and c) show the valley in the  $(m_{\tilde{\chi}^0}, p_T^{\max})$  plane. The blue crosses show parameter points with  $-\log(L)$  at most 4 above  $-\log(L)_{\min}$  (corresponding to approximately 90% CL).

Further studies indicate that these results are still valid when realistic cuts, and the effect of hadronization and detector simulation, are considered, although the efficiency is then reduced. See [5].

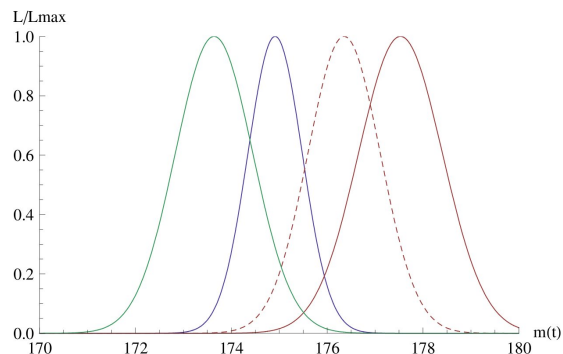
## INITIAL STATE RADIATION

One challenge associated with the Matrix Element Method is how to deal with initial state radiation (ISR). Energetic jets radiated from the initial state partons will be much more ubiquitous for high- $p_t$  physics at the LHC than at the Tevatron [6]. To simply ignore their effect leads to bad or impossible fits of the matrix element algorithm; on the other hand, a very strict veto on additional jets (besides jets expected from the primary hard process) would cause a significant reduction of signal statistics.

Conceptually, the most straightforward way of including events with sizeable ISR would be the use of matrix elements with additional partons in the final state. In practice, however, there is a limit to the multiplicity of the matrix elements due to the computing time for the matrix element generation and the phase-space integration for the weight computation. Therefore, in most cases, only one or two jets from radiation can be included.

This situation is illustrated in Fig. 2 for the example of 1000 simulated events of di-lepton top pair production at the LHC,  $pp \rightarrow t\bar{t} + nj \rightarrow b\bar{b}l^+l^- \nu_l \bar{\nu}_l + nj$ , where  $nj$  indicates  $n$  jets from ISR. The events have been generated with Pythia 6.4/PGS4 [7, 8]. Backgrounds have not been included. As is evident from the figure, the inclusion of matrix elements with one extra jet already significantly improves the agreement (peak of likelihood curve) with the true input value  $m_t = 175$  GeV and also the statistical precision (width of likelihood curve).

We also propose a different method to treat ISR. We observe that the most significant effect of ISR is on the kinematics of the events, since without proper inclusion of ISR the momentum balance would be violated. This effect can be taken into account by simply boosting the hard event by the momenta of the initial state jets. The longitudinal incoming momenta are integrated over in the computation of the likelihood (see (1)), so it is sufficient to perform the boost for the transverse coordinates only. Since the identification of the  $b$  jets from top decay is not unique, we sum over all permutations of



**FIGURE 2.** Reconstruction of the top quark mass from a matrix element likelihood fit to 1000 di-lepton top-antitop events at the LHC with  $\sqrt{s} = 14$  TeV. A top mass of  $m_t = 175$  GeV has been used for the event generation. *Blue*: “ideal” case without ISR in the event generation. *Red solid (rightmost curve)*: realistic event generation with ISR, but events with extra jets with  $p_T > 40$  GeV have been vetoed. *Red dashed*: same as above but also including  $t\bar{t}$  matrix elements with one additional jet. *Green*: boost method to deal with ISR in the events.

the four hardest jets in the event.

The result of a fit with this boost method is shown by the green curve in Fig. 2. It agrees with the true input value within statistical errors. The quality of the fit result is comparable to the computation with explicit  $t\bar{t} + j$  matrix element (red-dashed curve), but requires less computation time. Furthermore, the result of the boost method can be improved by including QCD splitting functions Sudakov form factors for the (cumulative) ISR on each leg [5].

In summary, we have here presented a first study in applying the Matrix Element Method to supersymmetric particle production with decay to massive invisible particles. In particular, we showed that it is possible to simultaneously determine the squark and LSP masses in squark pair production with exclusive decay to quark and LSP. We also proposed two methods to include the effects of initial state radiation in MEM calculations. For more details we refer to [5].

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