STIRLING ENGINE NODAL ANALYSIS PROGRAM

A. Schock

Fairchild Space and Electronics Company
Germantown, Maryland

Summary

A general-purpose program for the nodal analysis of Stirling-cycle devices or other cyclic, regenerative machines is described. It employs finite-difference, explicit-forward solutions, and is readily adaptable to various machine topologies. The machine is divided into specifiable numbers of fluid and solid nodes. The program solves three coupled gas-dynamic differential equations (mass, momentum, and energy balances) for each fluid node, and an energy balance for each solid node. Interactions between these are represented by heat transfer and fluid friction terms. The kinetic energy of the fluid and the inertial resistance to flow acceleration are accounted for. Leakages through seals and ports are included in the analysis. Special schemes for enhanced mathematical stability and for accelerated convergence to a steady-state cycle are described. The program is applicable to both disciplined-piston and free-piston engines. It can automatically generate very detailed outputs, both tabular and graphic (Eulerian, LaGrangian, and 3-dimensional) to help provide insight and understanding. Extensive illustrations and discussions are presented.

CIA # 8330
CAC # 31-01

Introduction

SNAP (Stirling Nodal Analysis Program) is a digital computer program designed to provide detailed analyses of Stirling-cycle devices, such as Stirling engines or refrigerators. The program was developed by Fairchild Industries under a contract from the Department of Energy's Division of Advanced Systems and Materials Production, in conjunction with a parallel free-piston Stirling engine development contract to Mechanical Technology, Inc. [1].

Like earlier programs (e.g., [2]), the SNAP program employs finite-difference, explicit-forward solutions, and is written in Fortran, to permit easy modification and adaptation to various machine topologies. The machine's fluid passages are represented by a one-dimensional nodal string, consisting—in sequence—of a single expansion-space node, specifiable numbers of heater nodes, regenerator nodes, and cooler nodes, and a single compression-space node.

In addition to the above working fluid nodes, there are corresponding parallel strings of metal wall nodes and regenerator matrix nodes. Interactions between adjacent fluid nodes and metal nodes are represented by appropriate heat transfer and fluid friction terms.

A typical example of such an array, with a total of 25 working-space nodes, is illustrated schematically in Figure 1. Nodes 26 and 27 represent gas springs required for resonant piston and displacer motions in the free-piston engine to which the program was applied. For the time being, they will be ignored in the present discussion.

Fig. 1

For each gas node, the program solves the full set of fluid dynamic equations, consisting of mass, momentum, and energy balances. The energy balance includes a kinetic energy term, and the momentum balance includes a term accounting for inertial resistance to fluid acceleration. No other program incorporating those terms was available in the public literature at the inception of this study. This is no longer so, since the publication of an excellent PhD thesis by I. Urieli at the University of Witwatersrand, South Africa [3].

An alternative program in which inertial forces are ignored was also developed, and will be discussed later in this report. That program, in effect, assumes instantaneous quasi-equilibrium between the local pressure gradient and the friction drop due to fluid flow. It provides equally detailed though less accurate results than the full inertial program, but its operating speed is one to two orders of magnitude higher. The loss in accuracy depends on machine dimensions, operating frequency, and other design parameters, and its magnitude can only be assessed by ultimate comparison of results from both programs for the same design.

While the analysis discussed in this report deals with an engine with prescribed piston and displacer motions, the program has also been applied to a problem with free-piston dynamics, in which the displacer and piston motions are purely the result of forces exerted by the gas and the mechanical load.

Program Structure

Both programs consist of a pre-processor, a main program, and a post-processor. The pre-processor inputs the basic machine dimensions, fluid and metal properties, the desired numbers of nodes for the heater, regenerator, and cooler, and the operating parameters and initial conditions*. From these inputs it computes the length, cross-sectional area, wall and matrix surface areas, and wall conductance for each node; and also initializes the fluid parameters in each gas node.

*Note: For most of our computer runs, we used 6 heater nodes, 11 regenerator nodes, and 6 cooler nodes, for a total of 25 gas nodes, and started the computation with a uniform pressure, zero flow, and identical linear temperature profiles for the regenerator wall, matrix, and gas.

Starting from the above values, the main analysis program carries out the required integration of the simultaneous differential equations, stores the computed time- and space-dependent results for post-processing, and extracts the desired summary information, such as cyclic heat flows, power outputs, and efficiency. These operations are discussed in detail in the next section.

Finally, there is an extensive post-processor program which prepares output tables that summarize the problem parameters, the computational parameters used, and the computed results. Most important, it automatically prepares a very wide variety of graphic outputs, in Eulerian, LaGrangian, and 3-D formats. These graphic outputs, which are made possible by the detailed nodal analysis of this program, are designed to provide insight and facilitate understanding of the engine's operation.

Analytical Model

The analytical model used is Eulerian (node boundaries fixed in space) rather than LaGrangian (moving node boundaries enclosing fixed gas masses). However, once the Eulerian solution is obtained, the results can easily be displayed in LaGrangian form. Both types of output graphs will be presented: cyclic variations of fixed space points, and cyclic variations of specific gas particles.

The nodes in the finite-difference model are numbered in ascending order from the expansion space ($i=1$) to the compression space ($i=n$). Consider a typical gas node i of length L_i , cross-sectional area* A_i , wall surface area* S_{wi} , matrix surface* S_{mi} , and hydraulic diameter D_i , given by

$$D_i = 4A_i / [(S_{wi} + S_{mi}) / L_i] . \quad (1)$$

Node i has a wall temperature T_{wi} , matrix temperature T_{mi} , and contains a gas at mean pressure P_i , density ρ_i , and temperature T_i . For a perfect gas with gas constant R , these are related by

$$P_i = \rho_i R T_i . \quad (2)$$

*Where there are multiple ducts in parallel, their combined cross-sectional and surface areas must be used.

The gas in node i has a mean velocity u_i , with a corresponding volumetric flow rate

$$V_i = A_i u_i \quad (3)$$

and mass flow rate

$$F_i = V_i \rho_i \quad (4)$$

Note that u_i , V_i , and F_i are defined as positive when the local flow is in the direction from the expansion space toward the compression space.

In addition to the node-center variables defined above, let us define $\bar{A}_i$, $\bar{P}_i$, $\bar{\rho}_i$, $\bar{T}_i$, $\bar{u}_i$, $\bar{V}_i$ and $\bar{F}_i$ as the corresponding variables at the boundary between node $i-1$ and node i . How these are computed in the program will be discussed later.

The gas dynamic differential equations in the program solve for the time variation of the mass m_i , momentum M_i , and energy E_i of the gas in each node i . These three variables are defined by:

$$m_i = A_i L_i \rho_i \quad (5)$$

$$M_i = m_i u_i \quad (6)$$

$$E_i = m_i (C_v T_i + \frac{1}{2} u_i^2) \quad (7)$$

where C_v is the constant-volume specific heat of the gas at temperature T_i .

Finite-Difference Equations for Mass and Momentum

The rate of change of mass in node i is simply equal to the net mass flow rate into the node, so that

$$dm_i/dt = \bar{F}_i - \bar{F}_{i+1} \quad (8)$$

The rate of change of momentum in a typical node is equal to the net momentum flow rate across its two boundaries, plus the net pressure force on the node (in the axial direction), minus the retarding force due to shear at the wall and matrix surfaces. Thus,

$$\begin{aligned} dM_i/dt = & (\bar{F}_i \bar{u}_i - \bar{F}_{i+1} \bar{u}_{i+1}) \\ & + (\bar{A}_i \bar{P}_i - \bar{A}_{i+1} \bar{P}_{i+1}) - (\bar{A}_i - \bar{A}_{i+1}) P_i \\ & - A_i (f_i L_i / D_i) (\frac{1}{2} \rho_i |u_i| u_i) \end{aligned} \quad (9)$$

where f_i is the D'Arcy friction factor, which is four times the Fanning friction factor. Note that Eq. (9) uses the product $|u_i|u_i$ instead of u_i^2 , since the friction force direction is always opposite to the flow direction.

One of the shortcomings of the present program is that, even though the fluid flow in the Stirling machine is reciprocating, the computations of the friction factor f_i and the heat transfer coefficient h_i for each node and each time step are carried out by means of steady-state correlations. The program performs these computations in a separate subroutine, to facilitate substitution of cyclic fluid flow and heat transfer correlations when available.

It is not expected that errors introduced by use of steady-state correlations have very much effect in the overall results. Sensitivity studies carried out with the SNAP program indicate that even factor-of-two changes in the correlations for friction factor and heat transfer coefficient had only minor effect on the predicted power output and efficiency.

At present, friction factors and heat transfer coefficients are computed as functions of the instantaneous local Reynolds number,

$$Re_i = D_i u_i \rho_i / \mu_i \quad (10)$$

where the viscosity μ_i and gas conductivity k_i are functions of the local gas temperature T_i . Standard f_i -versus- Re_i correlations are used for the heater and cooler tubes, and experimental data in Kays and London [4] were used for the friction in the regenerator matrix. For an 83%-porous matrix consisting of stacked fine-wire screening, the Tong and London data can be satisfactorily represented by

$$f_i = 0.354 + 81.9 / Re_i \quad (11)$$

Twice per cycle the gas flow in the engine reverses, the velocity u_i and Reynolds number Re_i pass through zero, and the friction factor (in the heater, cooler, and regenerator) goes to infinity. However, this does not cause any computational problems, since the product $f_i u_i$ in Eq. (9) remains finite.

Finite-Difference Equation for Energy

In addition to the mass balance and momentum balance presented in Eqs. (8) and (9), we require an energy balance for each node. The rate of change of energy in a typical node i is equal to the net flow rate of (internal and kinetic) energy across its two boundaries, plus the net rate of PdV work done on the node by the incoming and outgoing gas streams, plus the heat transfer rates from the adjacent wall and matrix over the length of the gas node. Therefore,

$$\begin{aligned} dE_i/dt = & \left[\bar{F}_i (C_v \bar{T}_i + \frac{1}{2} \bar{u}_i^2) - \bar{F}_{i+1} (C_v \bar{T}_{i+1} + \frac{1}{2} \bar{u}_{i+1}^2) \right] \\ & + \left[\bar{P}_i \bar{V}_i - \bar{P}_{i+1} \bar{V}_{i+1} \right] \\ & + h_i \left[S_{wi} (T_{wi} - T_i) + S_{mi} (T_{mi} - T_i) \right]. \end{aligned} \quad (12)$$

The mass balance (Eq. 8), momentum balance (Eq. 9), and energy balance (Eq. 12) are the three basic gas dynamic equations for each gas node. It would, of course, be possible to combine these equations and manipulate them to obtain alternative formulations, e.g., equations for the time derivatives of density ρ_i , temperature T_i , and velocity u_i . But there seems to be no obvious advantage in doing so, since the system of equations will in any case have to be solved by numerical techniques. In fact, it is important that the program be able to accommodate abrupt changes in cross-sectional area at the interfaces between expansion space, heater, regenerator, cooler, and compression space. This is much more readily done by keeping the three balance equations in their present form.

Finite-Difference Equations for Metal Temperatures

In addition to the three gas dynamic equations, we also need heat balance equations for the wall nodes and matrix nodes. For the wall nodes, the variation of temperature T_{wi} is given by

$$dT_{wi}/dt = \left[h_i S_{wi} (T_i - T_{wi}) + K_{w,i-1} (T_{w,i-1} - T_{wi}) + K_{wi} (T_{w,i+1} - T_{wi}) \right] / C_{wi} \quad (13)$$

where K_{wi} is the wall's thermal conductance between the center of node i and the center of node $i+1$, and C_{wi} is the heat capacitance of wall node i .

For the corresponding matrix heat balance, the axial conduction term was omitted. For a typical fine-wire matrix (such as steel wool or stacked screens), with most of the wires oriented normal to the flow direction, it was

felt that the axial conduction is negligible compared to the wall conduction and to the convective heat transport. Therefore, the variation of matrix temperature is given by

$$dT_{mi}/dt = h_i S_{mi} (T_i - T_{mi}) / C_{mi} , \quad (14)$$

where C_{mi} is the heat capacitance of the regenerator matrix in node i .

Integration Technique

The five balance equations (8, 9, 12, 13, 14) discussed above, together with their ancillary algebraic equations (2 through 7), constitute the basic system of coupled differential equations to be integrated by the program. A number of different numerical integration techniques (e.g., Newton and fourth-order Runge-Kutta) were tried out, but the one which seemed to work most stably was a modified "leap-frog" technique, similar to one used by Physics International in their PISCES code [5]. In essence, it uses the values of the variables at time t to compute time derivatives. These derivatives are then used to compute the variables values at time $t+dt$ from their values at time $t-dt$.

The above leap-frog technique was used both with a fixed time step and with a variable time step. As in all explicit finite-difference computations, there is a maximum time step beyond which the program becomes mathematically unstable and the problem blows up (diverges). For our main program, which uses the full momentum balance (including inertial resistance to flow acceleration), this maximum time step is primarily dictated by the sonic transit time across the length of a node. Stable operation was obtained as long as the time step did not exceed $\sim 30\%$ of the smallest value of "node length divided by local sound velocity" ($L_i / \sqrt{\gamma_i R T_i}$).

Node-Boundary Values of Flow Rate and Pressure

Consider a time t for which we have previously computed the mass m_i , momentum M_i , and energy E_i for each node. From these, we can calculate the density

$$\rho_i = m_i / A_i L_i , \quad (15)$$

the velocity

$$u_i = M_i / m_i , \quad (16)$$

the temperature

$$T_i = (E_i / m_i - \frac{1}{2} u_i^2) / c_v , \quad (17)$$

the pressure

$$P_i = \rho_i R T_i , \quad (18)$$

the volumetric flow rate

$$V_i = A_i u_i , \quad (19)$$

and the mass flow rate

$$F_i = V_i \rho_i \quad (20)$$

at the center of each node.

In addition to these, the three gas-dynamic equations (8, 9, 12) also require the values of these variables at the interface boundaries between adjacent gas nodes. For the volumetric flow rate at the boundary between nodes $i-1$ and i , we employ a volume-weighted average of the adjacent node-center values:

$$\bar{V}_i = V_{i-1} + (V_i - V_{i-1}) \nu_i , \quad (21)$$

where ν_i is a volume fraction defined by

$$\nu_i = A_{i-1} L_{i-1} / (A_{i-1} L_{i-1} + A_i L_i) . \quad (22)$$

More refined interpolation schemes, using parabolic and cubic curve-fits of nearby node-center values, were tried initially. However, the results did not warrant the additional computing effort required.

Equation (21) for $\bar{V}_i$ is applied for all boundaries except for the expansion space/heater interface and the compression space/cooler interface. For these it cannot be used, since the flow rates and at the center of the end nodes are essentially undefined*. Instead, the boundary flow rates for those two nodes are computed by extrapolation:

$$\bar{V}_2 = 2V_2 - \bar{V}_3 , \quad (23)$$

$$\bar{V}_n = 2V_{n-1} - \bar{V}_{n-2} . \quad (24)$$

*Note: The momentum balance (Eq. 9) is not applied to the two end nodes.

The pressures at typical nodal boundaries are also computed as volume-weighted averages, analogous to Eq. (21):

$$\bar{P}_i = P_{i-1} + (P_i - P_{i-1}) v_i . \quad (25)$$

This applies to all internal boundaries within the heater, regenerator, and cooler, where no change of area occurs. However, at the entrance and exit to the heater and cooler an additional expansion or contraction term is included, to account for the pressure change due to an abrupt area change.

Stability Problem

So far, we have discussed how the program computes node-boundary values of the volumetric flow rate $\bar{V}_i$ and pressure $\bar{P}_i$ from the nearby node-center values of those variables. In addition, the gas-dynamic equations (8, 9, 12) require node-boundary values for gas temperature and density to compute the mass flow rates, momentum flow rates, and energy flow rates across the nodal boundaries.

The simplest and seemingly most logical way of computing the above node-boundary variables is by interpolation of the adjacent node-center values of those variables. For example, a volume-weighted averaging scheme similar to Eqs. (21) and (25) could be employed. However, when this was first tried, a condition of mathematical instability was encountered. This took the form of spatial oscillations of various variables. For example, the nodal temperature profile at a given time would exhibit alternating high and low values around a smooth mean curve. This pattern would persist over many time steps, grow in amplitude, and eventually blow up.

The above-described oscillations problem is not new, but has been observed before in other finite-difference programs (e.g., [4]). Something must be added to such programs to stabilize them and avoid blow-ups.

Stabilization Schemes

One such stabilization scheme, used in PISCES and in other programs, is the "donor-cell" concept. In the present case, this means that, at each time step, the values of $\bar{T}_i$ and $\bar{p}_i$ at each node boundary are set equal to the variables' node-center values at the adjacent upstream node. For each node boundary, the program tests for the sign of the previously computed volumetric flow rate to determine which is the upstream node. Physically, this scheme appears to be artificial and make less sense than the previously mentioned averaging schemes, since the actual node-boundary values must generally be intermediate between the adjacent node-center values. But mathematically, the donor-cell scheme worked (i.e., gave stable solutions); the simple averaging schemes did not.

Because of concern about the physics of the donor-cell concept, an alternative stabilization scheme was conceived and tried out in our program. Let us call this the "variables-smoothing" scheme. Suppose a given variable has developed an oscillatory spatial profile, with its nodal values alternating above and below some mean curve. Since the above oscillations are merely a numerical artifact of the finite-difference scheme and are not physically real, we wish to smooth out these oscillations, while preserving the shape of the mean curve. Our scheme for doing this is illustrated in Figure 2 for a number of nodal positions.

Fig. 2 Variables Smoothing Scheme

The open circles represent the computed node-center values, exhibiting typical oscillatory behavior. The open squares represent the node-boundary values, obtained by taking a volume-weighted average of the adjacent node-center values (as in Eq. 21). By then taking an arithmetic average of the two boundary values for each node, we obtain corrected node-center values, which are represented by the solid circles in Figure 2. In practice, when this stabilization scheme is used at every time step, the oscillations never have a chance to build up to any significant amplitude. As a result, the ratios of corrected-to-uncorrected center-node values are all very close to unity.

Both the "donor-cell" scheme and the "variables-smoothing" scheme were tried in our program, and each was found effective in preventing the build-up of noticeable oscillations or instabilities. The two stabilization schemes appeared to yield identical results. In addition to their individual use, the two schemes can be combined for added stability. The high degree of stability was evidenced when the program was applied to a free-piston problem in which piston collisions temporarily caused violent oscillations in the gas flow rates, but the solution did not diverge.

Program Output

The integration of the system of coupled differential equations described above is carried out on an IBM-370/165 computer. At desired time intervals, the program lists the results in tabular form, one page per time step, and one line per node, with each line containing the variables of principal interest. In addition, the program extracts and prints the summary information on heat flows and piston power needed to compute the cyclic energy balances, and to check the results for internal consistency.

The data needed for post-processing operations is then transmitted via telephone to a Tektronix-4051 unit, which carries out the subsequent computations and automatically prepares the various output figures presented in subsequent sections. It is of interest to note that the post-processing program is actually larger than the main analytical program.

Engine Geometry

Computer-generated output for a typical disciplined-piston problem is presented in Figures 3 through 23. Figure 3 presents summary tables which define the physical problem and its mathematical representation. It is a "fresh-start" problem; i.e., not a continuation of an earlier problem. (Such restart capability exists in the program). The problem deals with a two-piston single-cylinder machine, with the machine dimensions and prescribed piston motions defined below in Figure 3:

Fig. 3

SUMMARY

Problem No. 581, Page 2

Case No. 581 (Fresh Start)
Date of Run: 1/6/78
Data on Tape No. 3, File No. 1

Prescribed Piston Motion:

2.54 cm piston stroke
2.66 cm displacer stroke
1.775 cm between mid-stroke positions
3.19 cm between cylinder head and displacer at midstroke
21.9 cycles/sec frequency
78.4 degree phase angle

Machine Geometry

5.71 cm piston diameter
2.21 cm piston rod diameter

22 Parallel Heater Tubes:
8.26 cm inner diameter
28.4 cm length

Annular Regenerator:
6.1 cm inner diameter
7.37 cm outer diameter
6.49 cm length
0.085 cm matrix wire diameter
91 percent matrix porosity

135 Parallel Cooler Ducts:
0.85 cm width
0.38 cm thickness
8.45 cm length

Problem Parameters:

593 degree C heater-wall temperature
31 degree C cooler-wall temperature
72 degree initial cyclic angle
60 bar initial pressure
8 kg/sec initial mass flow rate

Computational Parameters:

6 heater nodes
11 regenerator nodes
6 cooler nodes
2 microsecond fixed time step

A schematic view of the machine topology and of the nodal arrangement was presented earlier in Figure 1. Although that figure shows only a single string of gas nodes, in the actual machine there are 22 parallel heater tubes and 135 parallel cooler ducts. These are connected to a single annular regenerator, filled with a wire matrix of 0.05 mm diameter and having a matrix porosity of 91%.

The power piston has a stroke of 2.54 cm, and the displacer stroke is 2.66 cm. The two pistons move with a frequency of 21.9 hertz, and the displacer leads the piston by a phase angle of 70.4 degrees. With both pistons at their midstroke position, there is a gap of 1.775 cm between the two pistons, and a gap of 3.19 cm between the displacer and the cylinder head. In addition to the volumes which can be swept by the pistons, there is a residual volume of 4.0 cc in the expansion space when the displacer meets the cylinder head; and a residual volume of 23.5 cc in the compression space when the two pistons meet.

The prescribed piston motions are illustrated in Figure 4. The lower curve represents the power piston, the middle curve the displacer piston, and the upper bold line represents the cylinder head. The expansion space lies between the displacer and the cylinder head, and the compression space between the power piston and the displacer. As can be seen, during one part of the cycle the power piston and displacer almost touch and the compression space almost vanishes (except for the residual volume).

Fig. 4

Problem Definition

As shown in Figure 3, the problem was started with a uniform pressure of 60 bars, with the fluid initially at rest in all nodes, and with an initial cyclic angle of 72 degrees. This was an arbitrary choice. The fluid-flow solutions rapidly home in on a steady-state cycle, regardless of starting angle and initial flow rate.

Figure 3 also shows a wall temperature of 593°C for the heater and expansion space; and a wall temperature of 31°C for the cooler and compression space. Initially, the regenerator wall and matrix were assumed to have a linear temperature gradient between their hot and cold ends.

In general, the program treats the temperatures of the hot walls and cold walls as fixed, and allows the temperatures of the regenerator's wall nodes and matrix nodes to float. Occasionally, the program was run with floating heater wall temperatures, with prescribed heat input rates.

Cyclic Variation

When the program is applied to the above problem with arbitrary starting conditions, the solution rapidly homes in on a repetitive cycle. This is illustrated in Figure 5, in which mass flow rates at various locations are plotted versus cyclic angle, for the first two cycles.

These plots will be discussed in more detail later. The thing to note for now is how rapidly the solution assumes a repetitive shape, after starting from the arbitrarily chosen initial angle of 72 degrees. Cycle 3 (not shown) is visually indistinguishable from Cycle 2; and even the latter half of Cycle 1 is almost the same as the corresponding portion of Cycle 2. This apparent rapidity with which the solution settles down was unexpected and came as quite a surprise.

Fig. 5

Another noteworthy item in Figure 5 are the rather violent startup oscillations, which die out within about 60 degrees. These have a period approximately equal to the round-trip transit time of a sound wave through the total gas passages. Sometimes these sonic oscillations, in much milder form, also make a brief appearance during the steady-state cycle, after reversals of the flow direction.

Inertia-Less Analysis Program

The rapidity with which the various cyclic plots of a given solution settle down to repetitive shapes, with no discernible difference between consecutive cycles, might lead one to conclude that these early plots represent the steady-state

cycle. But this is only partially true. It is true that the cyclic plots of flow rates, pressures, and temperatures vary very little from their ultimate, steady-state form. And, as will be shown, the mechanical power output per cycle assumes an almost constant value after only a very few cycles. But while the steady-state power output can be determined in only a few cycles, the heat input and the efficiency cannot. This is so because the massive regenerator wall nodes, and -- to a lesser extent -- the matrix nodes, have much higher heat capacitances than the corresponding gas nodes, and therefore take a long time to equilibrate from the initial linear temperature profiles we had arbitrarily assumed.

To find the complete steady-state solution, with stabilized metal temperature profiles, a much more efficient -- even if less accurate -- method of solution is required, to avoid excessive computation times. One way of achieving this is to use an inertia-less formulation of the basic equations for rapidly homing in on the steady-state cycle. Once the metal temperatures have stabilized, the more exact inertial equations can then be used for obtaining the final steady-state solution. The inertia-less equations can be solved much more rapidly than the inertial equations, because the maximum stable time step is no longer limited by the sonic transit time per node.

In the inertia-less formulation, the basic differential equations for conservation of mass (Eq. 8) and energy (Eq. 12) are the same as in the inertial case; but the momentum balance [Eq. (9)] is replaced by its steady-state equivalent. In this replacement, the nodal pressure drop at any instant is assumed to equal the nodal friction loss resulting from the instantaneous flow rate. Thus,

$$\bar{P}_i - \bar{P}_{i+1} = (f_i L_i / D_i) (\frac{1}{2} \rho_i |u_i| u_i), \quad (26)$$

with appropriate corrections for abrupt expansions or contractions, where necessary. Replacement of Eq. (9) by Eq. (26) is tantamount to assuming that the inertial forces are negligible compared to the friction forces. This assumption is likely to be more valid at low frequencies than at high.

With the above change, the previous method of solution can no longer be employed, since Eq. (9) for the rate of change of momentum (or flow rate) has been eliminated. The remaining equations could be solved in an implicit manner, but this can be quite time-consuming, particularly if matrix-inversion or successive-approximation solutions are employed.

Fortunately, by taking advantage of the fact that spatial pressure variations are small compared to the temporal variations, it was possible to devise an algorithm for solving the inertia-less set of differential equations, together with their ancillary algebraic relationships, in an expeditious manner. The algorithm uses an explicit, non-iterative method of solution for each time step. Since the time step size in this formulation is governed by the energy equation rather than the momentum equation, considerably larger time steps can be used.

For the problem presented earlier, the inertial solution was limited to a time step of 2 microseconds, while the inertia-less solution was stable with a 36-microsecond time step. On an IBM 370-165 computer, with a virtual memory, the comparative running times for our 27-node problem were 7 minutes per cycle and 0.35 minutes per cycle. It is expected that considerably shorter running times could be achieved on a scientific computer, such as a CDC-6600; not only because of its higher speed, but also because it would not require the use of double-precision computations. This time-consuming mode is necessary on the IBM computer, because of its lower inherent precision.

The 20-fold increase in computing speed resulting from the use of the inertia-less model is useful not only for finding the steady-state solution with stabilized metal temperatures, but also for preliminary design studies, where we wish to compare a large number of design alternatives and are willing to sacrifice absolute accuracy for speed. For the present problem, the solutions from the inertia-less program exhibit very good agreement with those of the full inertial program.

Even the higher speed of the inertia-less program, however, is not by itself adequate for equilibrating the wall and matrix temperatures in a reasonable number of cycles. While the engine's power output can be determined quite quickly, determination of its efficiency would require prohibitive computing time, unless an effective accelerating scheme is used.

Accelerated Convergence

The accelerating scheme evolved for the SNAP program is to artificially divide the heat capacity of the regenerator matrix and wall by acceleration factors which diminish with time. For example, we employed wall and matrix acceleration factors starting at values of 2500 and 250, respectively, at the beginning of the run; and diminishing parabolically to unity at the end of the 25th cycle.

The results of using this acceleration scheme, for the same problem as before, are shown in Figures 6, 7, and 8. These figures are for Cycle 27, i.e., two cycles after the acceleration process has ended and all heat capacities have returned to their normal values.

Figure 6 presents spatial wall temperature profiles, and Figure 7 displays spatial profiles of the regenerator matrix temperature. Each figure contains 36 curves, representing 10° intervals during the cycle. To reduce confusion, each curve is arbitrarily displaced by 15°C from its predecessor. Thus, the temperature scale shown is an absolute scale for only the first (topmost) curve, and is only a relative scale for all other curves.

Fig. 6

Fig. 7

As can be seen, the wall and matrix temperature profiles undergo relatively little change during the cycle. This is in contrast to the gas temperature profiles, which will be seen to exhibit considerable cyclic change.

It is of interest to note that the steady-state metal temperature profiles shown in Figure 6 and 7 exhibit only a slight curvature and tilt. Yet these small changes from the initially assumed linear temperature ramp are essential for achieving the essentially steady-state energy balance shown in Figure 8.

Fig. 8

ENERGY BALANCE FOR PROBLEM 502, CYCLE 27 (Average Flow Rates, in Watts)

$$\text{Efficiency} = 1238/4293 = 28.9\%$$

In the above computer-generated energy balance diagram, numbers attached to arrows denote energy flows during the cycle, and numbers within the boxes denote energy (or enthalpy) changes during the cycle. All quantities are expressed as rates of change (in watts) averaged over the cycle.

The significance of the energy flow rates is discussed in the next section. For the present, it can be seen that the heat flow rates to and from the regenerator matrix are essentially equal, and the rates of enthalpy change of the regenerator wall, matrix, and working fluid are all small, indicating good cyclic stability. Thus, the acceleration scheme described above results in quite rapid convergence to the steady-state cycle*.

*Even faster convergence could of course be achieved if the initial regenerator wall temperature profile, instead of being a linear ramp between heater and cooler, were based on previous experience. The program has the option of automatic restart from the end of a previous problem.

Energy Balance Summary

As shown above in Figure 8, during Cycle 27 the gas receives heat at an average rate of 3726 watts from the heater and expansion-space wall, and 172 watts from the regenerator wall. In the same period, the gas loses heat at an average rate of 2621 watts to the cooler and compression-space wall, and 12 watts to the regenerator matrix*. Thus, the working fluid receives a net heat input rate of 1265 watts, averaged over the cycle.

The gas does mechanical work at an average rate of 995 watts on the power piston and of 271 watts on the displacer, for a combined mechanical output rate of 1266 watts. Therefore, the energy flow rate from the gas during the cycle exceeds the energy flow rate to the gas by 1 watt. In the same interval, Figure 11 shows that the total (internal plus kinetic) energy of the gas decreased at an average rate of 3 watts. Thus, the overall energy balance for the gas closed within 2 watts, which is 0.05% of the 3898 watt heat input rate to the gas for Cycle 27.

In the same period, the regenerator wall - by axial conduction through the metal - received heat at an average rate of 557 watts from the heater wall, and lost heat at an average rate of 372 watts to the cooler wall. There is a net heat flow rate of 172 watts from the regenerator wall to the gas.

The four arrows denoting energy flows to or from the bounce space are all zero, as they should be for a problem in which all leak rates have been assumed to be zero. As will be shown later, in a problem with leakage through seals and ports it is possible to have zero mass flows (over a cycle) together with non-zero energy flows.

It should be noted that the energy flow rates of 995 watts and 271 watts shown in Figure 8 represent the rate at which the working fluid (Nodes 1 through 25) does mechanical work on the piston and displacer. The work done on the displacer

*The 12 watts represents the difference between the average heat flow rates of 14124 watts to the matrix and 14112 watts from the matrix during Cycle 27.

is surprisingly large, but this does not represent a loss to the system. Most of this work (243 watts) is transferred to the power piston. This power transfer takes place via the gas spring within the power piston (see Figure 1). Power transfer occurs because of the phase difference between the two piston motions.

Finally, the 29 watts of mechanical work seen leaving the displacer requires a brief explanation. In a free-piston engine, this quantity should be zero (except for friction). However, in the present problem we are using a disciplined-piston model, with prescribed piston motions, to analyze a free-piston engine. The fact that the displacer has a net power output of 29 watts indicates that the prescribed piston motions are not fully consistent with free-piston operation. But the fact that the anomalous output is no greater than 29 watts indicates that we are not too far off. Presumably, the 29 watts could be made to approach zero by appropriate adjustment of piston strokes and/or phase angle.

The best way to analyze a free-piston engine is by modeling the piston dynamics, instead of prescribing the piston motions. However, this would require modeling the load characteristics (e.g., electromagnetic response and non-linear damping), which was considered too design-specific to include in the present report.

Leakage Past Seals

As indicated earlier, in the problem discussed thus far the leakages between the expansion space, compression space, bounce space, and gas spring were assumed to be zero. Let us now examine a new problem, identical to the previous one except for more realistic assumptions about those leakages. The problem will consider both continuous-flow leaks, through piston and piston rod seals, and periodic leaks, through centering ports*.

*Such centering ports are required in free-piston engines, to prevent the displacer and piston motion from drifting off their desired mid-stroke positions.

The continuous leakage takes place through four annular seals, whose dimensions are tabulated below:

<u>Seal Between:</u>	<u>Diameter(cm)</u>	<u>Length(cm)</u>	<u>Gap Width(mm)</u>
Expansion and Compression Space	5.71	0.965	0.0244
Compression and Bounce Space	5.71	4.876	0.0231
Compression and Bounce Space*	2.21	3.49	0.0203
Gas Spring and Bounce Space*	2.21	2.04	0.0203

The periodic leakage takes place through two sets of centering ports. The ports connecting the compression space with the bounce space are aligned when the power piston is 0.343 ± 0.051 cm below its midstroke position; and the ports connecting the gas spring with the bounce space are aligned when the compression space length is 1.480 ± 0.132 cm. Each set of ports is aligned twice during each cycle. When the ports are aligned, they have a conductance equal to that of an orifice of 0.107 cm diameter.

The above problem, with leakage, was analyzed using the methodology described earlier. This consisted of an inertialess analysis (25 accelerated cycles plus 2 unaccelerated cycles) followed by an inertial analysis for an additional two cycles. The results for the last cycle of the latter analyses are presented in Figures 9 through 18.

Cyclic Pressure Profiles

The piston motions shown earlier in Figure 4 produce the cyclic pressure variation depicted in Figure 9. Actually, that figure presents seven different curves, representing pressures at different locations ranging from the expansion space to the compression space. However, these curves are so close together as to be

*At the center of the piston rod seal, there is a circumferential groove, which is connected to the bounce space (not shown in Figure 1).

indistinguishable on the scale of the figure. In other words, at the prescribed operating frequency of 22 Hz the spatial variation of pressure is very small compared to its temporal variation.

Fig. 9

The two vertical lines in Figure 9 (and also in Figures 4, 10, 11, 12, and 14) depict the points of maximum and minimum pressure. As shown in Figure 4, these are close to the points of minimum and maximum gas volume. However, they do not quite coincide, because of changes in the mean gas temperature. As can be seen, the pressure wave lags the power piston motion by 30 degrees.

Fig. 10

Figure 10, shows the time variation of the pressure difference between various gas nodes and the center of the regenerator. These pressure differences include the friction losses, the expansion and contraction effects at the ends of the heater and cooler, and the acceleration and deceleration forces required to overcome the fluid's inertia and cause the changes in fluid flow rates.

Cyclic Mass-Flow-Rate Profiles

The resultant cyclic variation in mass flow rates is shown in Figure 11 for nine different node boundaries: expansion space (1/2), heater (5/6), regenerator (9/10, 11/12, 13/14, 15/16, 17/18), cooler (21/22), and compression space (24/25).

Fig. 11

The plots show that, most of the time, the mass flow rate is far from uniform. In fact, there are times when positive and negative flows coexist. For example, at a cyclic angle of 155°, the gas volume is expanding and the pressures and densities are decreasing rapidly. As a result, gas is streaming out of both ends of the regenerator. The reverse situation exists at a cyclic angle of 350°. The figure shows that for all points in the engine, the period of positive flow (hot end to cold end) exceeds the period of negative flow.

As can be seen, flow reversal (in either direction) starts at the compression-space end and proceeds toward the expansion space. There are two points in the cycle when all mass flow rates are approximately equal. These occur near the points of maximum and minimum pressure, when the nodal gas densities are not changing.

Cyclic Temperature Profiles

Figure 12 shows the cyclic variation of gas temperatures in the expansion space, in two heater nodes (3, 6), five regenerator nodes (9, 11, 13, 15, 17), two cooler nodes (20, 23), and the compression space; as well as the variation of matrix temperatures at the same five regenerator locations. Heater and cooler wall temperatures are also shown in the figure.

Fig. 12

The expansion and compression-space temperature variations are primarily due to compression and expansion effects. Both temperatures reach a maximum near the time of maximum pressure, and a minimum near the time of minimum pressure. Since heat transfer rates to the expansion and compression space walls are relatively small, these spaces are close to adiabatic.

The variations of gas temperatures in the heater and cooler nodes displayed in Figure 12 show that during part of the cycle the gas in the heater is actually hotter than the heater wall; and during part of the cycle, the gas in the cooler is colder than the cooler wall. This is a source of inefficiency, which results from the fact that the thermal coupling between gas and wall in the expansion and compression spaces is relatively weak.

Finally, Figure 12 displays five solid curves, showing the variation of gas temperature in the regenerator (at nodes 9, 11, 13, 15, 17); accompanied by five short-dash curves, showing the variation of matrix temperature at the same five locations. The cyclic variation of the gas temperature and of the matrix temperature is essentially the same at all regenerator locations. During somewhat more than half of the cycle, the gas is hotter than the matrix, and heat is being stored in the matrix. During the remainder of the cycle, the warmer matrix returns the stored heat to the cooler gas. The relative temperature reversals do not occur simultaneously at all locations. In both directions, the temperature reversal starts at the cold end and propagates to the hot end.

The two slanting dashed lines in Figure 12 indicate the times of flow reversal at various points of the regenerator. As would be expected, the period of flow in the hot-to-cold direction tends to coincide with the period of matrix heating; and vice versa. But this coincidence is not perfect. In fact, at the time of flow reversals the gas temperatures are moving away from the matrix temperatures, and some time elapses before they reverse direction and cross over to the other side of the matrix temperature. This comes about because, at the time of flow reversal, the flow velocity and heat transfer are very low, and the gas is undergoing virtually adiabatic expansion or compression.

Inspection of Figure 12 reveals that, for the given regenerator design parameters, the matrix temperatures are far from constant. They undergo quite noticeable periodic swings, as the result of the cyclic heat addition and removal. The temperature fluctuation of the regenerator wall is much smaller, because of the latter's much lower surface-to-volume ratio. These temperatures have not been plotted, since their cyclic variation would not be discernible on the scale of the figure.

Spatial Temperature Profiles

In addition to the gas temperature's cyclic variation at various locations, it is also of interest to examine its spatial variation at various times in the cycle. Such spatial temperature profiles are displayed in Figure 13, as a function of nodal position. As before in Figures 6 and 7, thirty-six curves are shown, at intervals of 10 degrees of cyclic angle, with each curve offset from its neighbors by 15°C, for clarity.

Fig. 13

To help interpret the curves, flow from left to right is denoted by solid curves, and flow from right to left is depicted by dotted curves. Close-spaced curves indicate rising temperatures. Wide spacings denote diminishing temperatures. As can be seen, the spatial gas temperature profiles vary considerably during the cycle. This is particularly true in the heater and cooler. After flow is well established, the gas temperature rises in the flow direction in the heater, and falls in the flow direction in the cooler. This is not true immediately after flow reversal, and some time elapses before the proper temperature profiles are established.

Cyclic Heat Flow Profiles

Figure 14 shows the cyclic variations of the heat flow rates from various components to the working fluid. The dot-dash curve represents heat flow from the heater and expansion-space wall to the gas; the dashed curve represents heat flow from the cooler and compression-space wall to the gas; and the solid curve represents heat flow from the regenerator (matrix plus wall) to the gas.

Fig. 14

As can be seen, the heat flow to and from the regenerator is much greater than the heat flows from the heater and to the cooler. This is true of any well-designed Stirling engine. Also, we note that all three curves exhibit irregularities at the points of flow reversal, where the local flow velocities (and therefore the heat transfer coefficients) pass through zero. Finally, we see that the matrix-to-gas heat flow is briefer and has a higher peak than the gas-to-matrix heat flow.

Mechanical Power Output

Figure 15 presents the pressure-volume diagrams for the expansion space and the compression space. As shown by the arrowheads, time variation is clockwise along the expansion-space curve and counter-clockwise along the compression-space curve. During a cycle, the working fluid does net work in the expansion space, and has work done on it in the compression space. Since the area enclosed by the expansion-space curve is larger than that enclosed by the compression-space curve, the heat engine has a positive power output.

Fig. 15

In Figure 16 the force exerted by the working fluid on each piston is plotted versus the piston's position. As shown in Figure 1, these forces are exerted by the gas in the expansion space, the compression space, the gas spring, and the bounce space. The area enclosed by each curve represents the work which the gas does on the piston during the cycle.

Fig. 16

The cyclic motion along the power-piston curve is clockwise, corresponding to a net work input from the gas. The area enclosed by the displacer curve is close to zero, as it should be. Virtually all of the mechanical power goes to the power piston, for transmission to the load. The fact that the net force on the displacer is always close to zero shows that the disciplined-piston motions analyzed in this problem are consistent with free-piston operation, since in a free-piston machine the net displacer work must be zero.

Energy Balance

The overall energy balance for the above problem is summarized in Figure 17. As before in Figure 6, all cyclic energy changes are expressed as rates of change (in watts) averaged over the cycle.

Fig. 17

ENERGY BALANCE FOR CASE 1029, CYCLE 27 (Average Flow Rates, in Watts)

Efficiency = $932/4264 = 21.8\%$

It should be noted that Figure 17 differs significantly from Figure 6 for the leakage-free case. Comparison of those figures shows that the addition of leakage terms has relatively little effect on the heat input rate, but has a profound effect on the predicted power output of the engine. As shown, gas leakage lowers the output power from 1238 watts to 930 watts, and drops the engine efficiency from 28.7% to 22.1%. This demonstrates the importance of reducing the seals' gap widths to the lowest possible value, since leakage is proportional to the cube of the gap width.

Another significant difference between Figures 6 and 17 are the non-zero energy flows to and from the bounce space in the problem with leakage; and the fact that the gas spring receives more work from the displacer than it delivers to the power piston, with the difference in energy going to the bounce space.

These effects arise because instantaneously each leaking gas stream carries along a flow of energy. The amount of this energy flow depends on the temperature of the upstream gas node. For a steady-state cycle, the pressures and temperatures of the gas spring and bounce space must equilibrate, and the positive and negative mass flows through each leakage path must balance. For the problem whose energy balance is presented in Figure 17, all pressures have equilibrated so that the net mass flows from the bounce space to the gas spring and to the compression space are essentially zero by Cycle 27. But the net energy flows are not zero, because there has not been sufficient time for these temperatures to find their equilibrium level.

The bounce space temperature varies extremely slowly, because its volume is very large compared to the volume of gas leaked per cycle. Thus, Figure 17 can be interpreted in two ways: (1) To maintain the bounce space constant at the assumed cold-wall temperature of 31°C, heat would have to be removed by a coolant at a rate of 273 watts, as indicated in Figure 17; or (2), if the bounce space were truly adiabatic, then its gas energy (and temperature) would rise, initially at a rate of 273 watts, and this would continue at a diminishing rate until the bounce space reaches its equilibrium temperature.

Equilibration of Leakage Solution

To find this steady-state condition more rapidly, without excessive computation time, the following additional acceleration scheme was employed. After each cycle, the net energy changes of the gas in the bounce space and in the gas spring were determined. From these, the program computes what temperature changes in those two nodes would have been required to null the observed energy changes during the previous cycle. These two temperature changes are then introduced as step changes, before starting the new cycle. Of course, this scheme does not work instantaneously, because -- as a result of leakage -- the imposed temperature changes in turn affect the compression space and the rest of the working fluid. But if the scheme is applied at the end of each cycle, the gas spring and bounce space temperatures equilibrate rather quickly. The two acceleration schemes, i.e., the earlier one for equilibrating the wall temperatures and the one above for equilibrating the gas springs, can be applied simultaneously.

This was done for our previous problem, and the resultant energy balance is depicted in Figure 18. As can be seen from the very small enthalpy changes for the regenerator wall (1w), the working fluid (4w), and the regenerator matrix (4w), the cycle is quite close to convergence. As seen from the small energy discrepancy for the displacer (6w), the solution is consistent with free-piston operation. And the heat flow from the bounce space to the cold wall is zero, consistent with its assumed adiabatic operation.

Fig. 18

ENERGY BALANCE FOR CASE 1834, CYCLE 50 (Average Flow Rates, in Watts)

What is most surprising about the steady-state solution is the remaining large energy loss in the gas spring. The gas spring receives mechanical work at the average rate of 273w from the displacer, but delivers only 190w to the power piston. The mechanical power loss of 83w shows up as a useless heat flow, via gas leakage to the bounce space, and similarly from there to the compression space. It was expected that the gas spring and bounce space would equilibrate at such temperatures as to make those heat losses zero, but this is not the case. Even though the positive and negative gas flows through each leakage path balance over the cycle, the resultant energy flows do not. This comes about because the mean upstream gas temperatures are different for the forward flows and the reverse flows.

Comparison of Figures 17 and 18 shows that equilibration of the gas spring and bounce space temperatures has increased the performance degradation due to leakage. The predicted power output dropped from 932w to 881w, and the efficiency from 21.9% to 20.8%.

LaGrangian Output Graphs

As mentioned earlier, once a given problem has been analyzed by the Eulerian SNAP program, the results obtained can readily be used to generate a family of LaGrangian plots. Such plots are shown in Figures 19 through 21, for the same problem as presented earlier.

In Figure 19 the nodal positions of specific gas particles are plotted as a function of the cyclic angle. The heater, regenerator, and cooler are shown as fixed nodes (1 through 24). The expansion and compression spaces are depicted as variable nodes (of arbitrary scale), to represent their volume variation as the result of piston and displacer motions during the cycle. These volume variations cause the gas particle motions represented by the curves.

Fig. 19

The figure presents the nodal paths of 19 gas particles, at intervals of 5% of the total gas mass in the working space. Thus, the band between any two neighboring curves represents 5% of the working fluid. As can be seen, about 20% of the working fluid never leaves the compression space; and approximately 10% never leaves the expansion space. No single gas particle moves all the way from the expansion space to the compression space. In fact, for the problem at hand, no single gas particle traverses the entire regenerator during the cycle. One of the gas particle paths shown never leaves the regenerator.

Figure 20 shows the cyclic temperature history of some of the gas particles whose motion was depicted earlier in Figure 19. To help interpret the curves, a solid curve is used when the particle is in the regenerator, a dashed curve when the particle is in the heater or cooler, and a dotted curve when the particle is in the expansion or compression space. (Curves for the three coldest and two hottest gas particles have been omitted for clarity.)

The figure shows that gas particles traveling in the regenerator are alternately heated and cooled (depending on the direction of motion). Particles traveling through the heater are almost always heated, regardless of direction. The only exceptions are at a cyclic angle around 90 degrees, where the gas is hotter than the heater wall; and around 170 degrees, where the cooling effect due to expansion is greater than the heating effect of heat transfer from the wall. The dotted curves show the strong expansion and compression effects while the gas particles are in the almost adiabatic end nodes.

Figure 21 presents pressure-volume diagrams for eight selected gas particles. The volume plotted is the specific volume of each gas particle. Again, the particles are represented by solid curves while they are in the regenerator, by dashed curves in the heater and cooler, and by dotted curves in the expansion and compression space. In addition, there is a set of (dotted) hyperbolas, representing lines of constant temperature.

Fig. 21

The cyclic motion of each particle around its curve is clockwise, and the area within each loop may be interpreted as the specific work output of each of the particles during the cycle. The figure indicates that most of the power output is produced by a small fraction of the working fluid.

The program also generates LaGrangian plots of entropy versus cyclic angle as well as temperature-entropy diagrams for the specific gas particles. These plots have been omitted here.

Three-Dimensional Plots

Previously, we examined the variation of gas temperature with cyclic angle, in Figure 12; and the variation of gas temperature with nodal position, in Figure 13. For added clarity, these two types of plots can be combined in a single, three-dimensional (perspective) display, as illustrated by the computer-generated Figures 22 and 23.

Figure 22 displays the cyclic variation of gas temperature as a function of nodal position and of cyclic angle for a typical problem. In addition to the gas temperatures, the temperatures of the heater wall (T_H) and cooler wall (T_C) are also shown, for reference. The dotted curves represent isotherms corresponding to those two temperatures, and the areas enclosed by those isotherms shows the previously mentioned regions where the gas is cooled by the heater wall or heated by the cooler wall.

Fig. 22 3D Plot of Gas Temperature versus Position and Time

Finally, the three-dimensional plot of Figure 23 presents an even fuller picture, since it also shows the variation of gas volume with time. Here gas temperatures are plotted versus cyclic angle and nodal volumes. All node volumes are constant, except for the expansion space and compression space. These vary sinusoidally as the result of piston motions. The dashed lines indicate the time of local flow reversal, and the two arrows indicate the flow direction between flow reversals.

Fig. 23 3D Plot Showing Cyclic Variation of Node Volumes, Flow Direction, and Gas Temperatures

Thus, Figure 23 provides a combined illustration of piston motions and resultant volume variations; their effect on gas flow direction; the effect of expansion and compression on gas temperatures, particularly in the almost adiabatic expansion and compression spaces; and the effect of flow reversals on the instantaneous gas temperature profiles, particularly in the heater and cooler.

Conclusion

The program described and illustrated in this report provides a versatile and, it is hoped, useful tool for the analysis of cyclic thermodynamic machines. It should be readily adaptable to other machine topologies, and to future refinements of the analytical model.

Acknowledgment

The author takes pleasure in expressing his appreciation for the assistance of M.B. Eck, M. Mukunda, H. Sookiazian, and E. Vogel, all of Fairchild Industries, in carrying out the above-described work.

References

- [1] B. Goldwater and R.B. Morrow, "Demonstration of a Free-Piston Stirling Linear Alternator Power Conversion System," Proc. 12th IECEC, vol. 2, 1977.
- [2] T. Finkelstein, "Thermophysics of Regenerative Energy Conversion", Paper 67-216, AIAA 5th Aerospace Science Meeting, 1967.
- [3] I. Urieli, "A Computer Simulation of Stirling Cycle Machines," Ph.D. Thesis, University of the Witwatersrand, 1977.
- [4] L.S. Tong and A.L. London, "Heat-Transfer and Flow-Friction Characteristics of Woven-Screen and Crossed-Rod Matrixes," Trans. ASME, vol. 79, no. 10, 1957.
- [5] S.L. Hancock, "Finite Difference Equations for PISCES 2DELK, A Coupled Euler Lagrange Continuum Mechanics Computer Program," Physics International Company Technical Memo TCAM 76-2, 1976.