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OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

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"AN INVESTIGATION OF THE APPLICATION OF THE GAS GENERATOR-
FREE TURBINE CYCLE TO A NUCLEAR POWERED AIRCRAFT"

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August 1957

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PREFACE

In September, 1956, a group of men experienced in various scientific and engineering fields embarked on the twelve months of study which culminated in this report. For nine of those months, formal classroom and student laboratory work occupied their time. At the end of that period, these two students were presented with a problem in reactor design. They studied it for ten weeks, the final period of the school term.

This is a summary report of their effort. It must be realized that, in so short a time, a study of this scope can not be guaranteed complete or free of error. This "thesis" is not offered as a polished engineering report, but rather as a record of the work done by the group under the leadership of their advisor. It is issued for use by those persons competent to assess the uncertainties inherent in the results obtained in terms of the preciseness of the technical data and analytical methods employed in the study. In the opinion of the students and faculty of ORSORT, the problem has served the pedagogical purpose for which it was intended.

The faculty joins the authors in an expression of appreciation for the generous assistance which various members of the Oak Ridge National Laboratory gave. In particular, the guidance of the group consultants, W. T. Furgerson and A. P. Fraas, is gratefully acknowledged.

Lewis Nelson

for

The Faculty of ORSORT

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Lastly, to the ORSONT administration and faculty, our appreciation for the knowledge and education which enabled us to undertake the study.

SUMMARY

This study has investigated the feasibility of installing a gas generator-free turbine type power plant in the R3Y aircraft, using a circulating fuel reactor as a power source. Two variations of the cycle were considered. The split flow cycle bleeds high temperature, high pressure air from the gas generator directly to the free turbine in the wing. The through flow cycle partially expands the high temperature, high pressure air through the compressor turbine of the gas generator then directs the compressor-turbine exhaust air to the free turbine in the wing.

Design parameters of pressure ratio, radiator depth, radiation flow density, and hot gas duct size were optimized to give minimum weight per shaft horsepower of the complete power plant. The weight of a split flow power plant capable of supplying 22,000 shaft horsepower was found to be 116,600 pounds. The weight of a similar through flow power plant was found to be 119,900 pounds. The reactor power required in both cases was 70 megawatts. The nominal gross weight of the R3Y airplane is 175,000 pounds. With pay loads of approximately 20,000 pounds, either nuclear conversion will have a gross weight of 200,000 pounds.

It was found that either cycle could be installed in the R3Y aircraft; however, the installation of either would require major structural redesign. The split flow cycle with its smaller hot air ducts required the least amount of redesign.

A comparison of existing aircraft engines with a preliminary design of the split flow turbo-components indicated that the compressor and possibly the free turbine could be adapted from current engine components.

INTRODUCTION

Interest has been expressed in the rapid development of a nuclear powered medium performance aircraft through the utilization of the current state of reactor and aircraft technology. Such a project would have the advantage of developing personnel with design and operational experience in the nuclear propulsion field without the time lag inherent in the development of high performance nuclear powered aircraft.

A variety of thermodynamic cycles conceivably could be adapted to a medium performance nuclear powered aircraft. This study will investigate the feasibility of adapting the gas generator-free turbine cycle to a nuclear powered sea plane of the 200,000 gross weight class. It should not be concluded that this cycle is considered superior to other cycles. The results of this study should be compared with the results of other cycle studies before it is selected for a specific application. Two variations of the gas generator-free turbine cycle will be considered: split flow, in which a fraction of the air is bled from the gas generator to a free turbine; and through flow, in which all of the air from the gas generator is ducted to the free turbine. A reactor of the Aircraft Reactor Test Reactor (ART) type will be assumed for this study, and calculations will be based on current ART design values.

Feasibility requires the production of adequate power by a power plant whose total weight is sufficiently low to permit support in the airframe under consideration. Accordingly, a thermodynamic analysis will be made for each cycle, based on a number of design parameters. Component weight estimates based on these same parameters will be made, and an optimization study performed in order to determine the design configuration for minimum power plant weight per horsepower.

From the power plant design as fixed by the weight optimization, estimates will be made as to size and weight of components and methods of installation in the R3Y. Consideration will be given to possible adaptation of existing engine components to the cycle. A rough performance estimate will be made to aid in evaluation of the proposed system. Performance will be based on a cycle analysis with no consideration given to off design matching of turbo components.

Symbols

A	-	Area, ft^2 or activity, curies
C	-	Compressor or true velocity, ft/sec .
$\dot{C}$	-	Capacity rate, $\text{Btu}/\text{hr-deg. F}$.
c	-	specific heat, Btu/lb or chord, ft .
D	-	radiator depth, in
d	-	diameter, in .
$\frac{dH}{dt}$	-	rate of climb, ft/min
E	-	fin efficiency
eng	-	engine
F	-	fraction
f	-	friction factor
G	-	flow density
g	-	gravitational constant, ft/sec^2
H	-	total enthalpy, Btu/lb
h	-	stream enthalpy, Btu/lb , or local heat transfer coefficient, $\text{Btu}/\text{hr-ft}^2\text{-deg F}$.
HP	-	Horsepower
J	-	Joule's constant, 778 ft-lbs per Btu
k	-	ratio of specific heat at constant pressure to specific heat at constant volume or thermal conductivity, $\text{Btu}/\text{Hr-ft-deg F}$
KW	-	Kilowatts
L	-	Length, ft
L/d	-	ratio of length to diameter
M	-	Mach number
N	-	revolution per second
NaK	-	alloy of 56 percent sodium to 44 percent potassium
NTU	-	Number of heat transfer units
Nu	-	Nusselt number
P	-	total pressure, lbs/ft^2 or power, megawatts
p	-	stream pressure, lbs/ft^2
Q	-	volumetric flow rate, ft^3/sec
R	-	Radiator
Re	-	Reynolds number
s	-	stress, lbs/in^2

- SHP - shaft horsepower, HP
T - total temperature, deg. R
t - stream temperature, deg R or thickness, in
U - overall heat transfer coefficient, Btu/Hr-ft²-deg F or rotor tip speed, ft/sec
V - velocity, ft/sec
W - Weight, lbs or relative velocity, ft/sec
w - air flow rate, lbs/sec
Z - compressor pressure ratio
- η - efficiency or radiator effectiveness
 μ - viscosity, lb m/sec-ft
 ρ - density, lbs/ft³
 σ - density ratio
 ϕ - flow coefficient
 χ - degree of reaction
 ψ - head coefficient

Subscripts

A - air	G - gross weight
a - auxiliary or axial	H - hydraulic
Aux - auxiliary	h - hub
all - allowable	inc - inconel
acc - acceleration	m - mean
b - bleed	max - maximum
C - compressor	min - minimum
CT - compressor turbine	N - NaK inlet
cf - counterflow	P - power plant
D - duct	p - constant pressure
$\frac{dH}{dt}$ - rate of climb	R - radiator
E - engine	r - reactor or reduced
e - exit	s - stream
eq - equivalent	T - free turbine
ex - excess	t - total or tip
f - frontal	0, 1, 2, 3, etc - stations

Superscripts

* - sea level static
 ' - isentropic

PART I
WEIGHT OPTIMIZATION

General Considerations

Two separate thermodynamic cycles have been considered in this study. Fig. 1 displays schematic diagrams of the split flow and through flow cycles. In the split flow cycle, inlet air is compressed, heated in passing through the radiator, the necessary fraction is expanded to the atmosphere through the compressor turbine, and the remainder is bled off from the radiator outlet. Of this remaining fraction, a portion is utilized to drive aircraft and reactor auxiliaries, and the final portion is ducted through the aircraft and expanded through the free turbine in the wing.

In the through flow cycle, air is compressed and heated as before, a fraction is bled off from the radiator discharge for auxiliaries, and the remaining fraction expanded through the compressor turbine to a pressure such that compressor work just equals turbine work. The compressor turbine exhaust is then ducted through the aircraft and expanded through the free turbine in the wing.

The aircraft power plant was visualized as a single reactor supplying heat to four gas generator-free turbine engines, which would drive propellers through individual reduction gears. An artist's conception of the split flow cycle is presented in Fig. 2.

In order to fix the design parameters for radiators, compressors, turbines, and ducts; power plant weight per shaft horsepower was optimized. Operating parameters of sea level and a Mach number of 0.15 were chosen for this optimization, as take-off and wave-off conditions were assumed to be critical.

In all, five design parameters were considered:

1. Compressor pressure ratio, $\rho_c Z$
2. Radiator depth, D
3. Radiator flow density, G_R^*
4. Duct flow density, G_D^*
5. Effective length to diameter ratio of the duct, L/d .

Fig. 1
SCHEMATIC OF CYCLES

SPLIT FLOW

THROUGH FLOW

Fig. 2

NUCLEAR POWERED GAS GENERATOR-TURBOPROP SEAPLANE

Approximate equations for the relationships among the various design parameters were written and tabulated for solution on an IBM 650 or similar computer.

Power plant weight per horsepower was defined to include the weight of the installed reactor, shields, compressor, radiator with contained NaK, compressor turbine, free turbine and duct weight. Weights per horsepower of propellers and reduction gears were excluded as being essentially independent of the various parameters in the horsepower range desired.

Cycle Analysis (Split Flow)

Inlet Duct

Conditions at the entrance to the inlet duct are fixed by the operating parameters of altitude and Mach number. Total pressure and total temperature were related to the corresponding ambient values by the common relations:

$$T_0 = t_0 \left[1 + \frac{k-1}{2} M^2 \right]$$

$$P_0 = P_0 \left[1 + \frac{k-1}{2} M^2 \right]^{\frac{k}{k-1}}$$

Because of the importance of inlet duct efficiency on engine performance an inlet duct configuration was assumed and inlet pressure losses estimated in Appendix A. A three percent drop in total pressure was assumed for the adiabatic duct in these calculations.

$$P_1 = 0.97 P_0$$

$$T_1 = T_0$$

Compressor

Pressure ratio, Z , as an assumed parameter, determined the total pressure at compressor discharge:

$$P_2 = Z P_1$$

Total temperature, T_2 , was calculated from the common expression for the temperature rise of an isentropic compression, corrected by an assumed compressor efficiency:

$$T_2 = T_1 + \frac{T_1}{\eta_c} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

Radiator

Appendix B contains a detailed analysis of radiator performance. Because of the importance of radiator performance in this study, methods

of extrapolation and approximations made should be considered carefully.

Radiator performance was expressed in terms of heating effectiveness, η , defined as follows:

$$\eta = \frac{T_3 - T_2}{T_N - T_2}$$

From Appendix B, equations for the variation of heating effectiveness as a function of radiator depth and radiator flow density were written:

$$\eta = K_1 G^m + C_1$$

the constants K_1 , m and C_1 being functions of radiator depth. Assuming that stream temperatures and total temperatures through the radiator differ by negligible amounts, total temperature at the radiator outlet was then calculated:

$$T_3 = T_2 + \eta (T_N - T_2)$$

Drop in total pressure across the radiator was predicted by the relation determined in Appendix B. Total pressure at the radiator outlet was then calculated from the relation:

$$P_3 = P_2 - \frac{C_2 G_R^{1.8}}{\frac{P_2}{R(T_2 + T_3)} \left[1 + \sqrt{1 + \frac{R(T_2 + T_3) C_2 G_R^{1.8}}{P_2^2}} \right]}$$

where the value of C_2 was a function of radiator depth.

Since the design parameter G_R^* was based on operation at static, sea level conditions, a relation between G_R and G_R^* was required. Assuming fixed compressor and radiator design during variation in altitude and Mach number, continuity of mass flow requires:

$$G_C A_C = G_R A_f$$

$$G_C^* A_C = G_R^* A_f$$

Thus

$$\frac{G_C}{G_C^*} = \frac{G_R}{G_R^*} = \sigma_R$$

At the same compressor speed, volumetric flow rate, and hence axial air velocity must remain unchanged.

Hence,

$$V_C = V_C^*$$

$$\frac{G_C}{1} = \frac{G_C^*}{1^*}$$

$$\frac{G_C}{G_C^*} = \frac{1}{1^*} = \left(\frac{P_1}{P_1^*} \right) \left(\frac{T_1^*}{T_1} \right)$$

$$\frac{G_C}{G_C^*} = \frac{.97 P_o [1 + 0.2 M^2]^{\frac{k}{k-1}}}{.97 P_o^* [1 + 0]^{\frac{k}{k-1}}} \frac{t_o^* [1 + 0]}{t_o [1 + 0.2 M^2]}$$

$$\frac{G_R}{G_R^*} = \sigma_R = \sigma_o [1 + 0.2 M^2]^{\frac{1}{k-1}}$$

and

$$G_R = \sigma_R G_R^*$$

Compressor Turbine

The compressor turbine was assumed to exhaust to the atmosphere with an exhaust air velocity, C_e , of 300 feet per second. Enthalpy change per pound of turbine air was calculated as the enthalpy change resulting from expansion to a total pressure equal to ambient atmospheric, less the equivalent enthalpy of the leaving exhaust.

$$\Delta H_{CT} = \eta_{CT} \bar{c}_{pCT} T_3 \left[1 - \left(\frac{P_o}{P_3} \right)^{\frac{k}{k-1}} \right] - \frac{(C_e)^2}{2 g J}$$

By requiring that turbine work equal compressor work, the fraction of compressor air required by the compressor turbine in driving the

compressor turbine in driving the compressor was computed.

$$\Delta H_C w_C = \Delta H_{CT} w_{CT}$$

$$\frac{w_{CT}}{w_C} = \frac{\Delta H_C}{\Delta H_{CT}} = \frac{\bar{c}_{p_c} [T_2 - T_1]}{\Delta H_{CT}}$$

Auxiliaries

The remaining fraction of the compressor air was then available for ducting to the free turbine and auxiliary drives. The power required to drive pumps and auxiliaries in the stationary ART was estimated to be 1000 horsepower.¹ This auxiliary power required was arbitrarily tripled to account for aircraft auxiliaries and increased duct losses resulting from compression of the ART into an aircraft configuration. Preliminary calculations indicated that the 60 megawatt ART would produce approximately 20,000 horsepower with about one third of the compressor air being expanded through the free turbine. Accordingly, it was assumed that 15 percent of the bleed air or five percent of the compressor air would be ducted to the auxiliary turbines. The remaining fraction of compressor air was then bled through ducts to the free turbines. This fraction, F_b , was calculated as follows:

$$F_b = 1 - \frac{w_{CT}}{w_C} - \frac{w_a}{w_C}$$

$$F_b = 0.950 - \frac{w_{CT}}{w_C}$$

Bleed Duct

Drop in total pressure through the bleed ducts was computed from the incompressible pressure drop relation:

$$P_3 - P_5 = \Delta P = 4 f \frac{\rho_m}{2g} V^2 \frac{L}{d}$$

1. A. P. Fraas and A. W. Savolainen, Design Report on The Aircraft Reactor Test, ORNL-2095, May 1956 p. 12

where: f = friction factor

ρ_m = mean density in duct, lbs/ft³

V = air velocity in duct, ft/sec

L/d = equivalent length to diameter ratio of duct.

The friction factor for turbulent flow in the round ducts was assumed to follow the relation:²

$$f = \frac{0.054}{Re^{1/5}}$$

$$\text{where, } Re = \frac{\rho V d_H}{\mu} = \frac{G_D d_H}{\mu}$$

For round ducts:

$$d_H = d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 w_D}{\pi G_D}} = G_D^{-1/2} \sqrt{\frac{4 w_D}{\pi}}$$

Substituting:

$$Re = \frac{G_D^{1/2} \sqrt{\frac{4 w_D}{\pi}}}{\mu}$$

$$f = \frac{0.054 \mu^{0.20}}{G_D^{0.1} \left(\frac{4 w_D}{\pi} \right)^{0.1}}$$

$$\Delta P = \frac{0.108 \mu^{0.20} G_D^{1.9}}{\left(\frac{4 w_D}{\pi} \right)^{0.10} \rho_m g} \left(\frac{L}{d} \right)$$

From this relation it is seen that pressure drop through the ducts is an insensitive function of mass flow rate through the ducts. A value for w_D of 25 pounds per second was estimated from preliminary calculations.

Viscosity was determined from the Gas Tables³ at an estimated air temperature of 1950 degrees R. The mean density was assumed to be that at radiator outlet conditions. From these assumptions, an expression for duct discharge total pressure was written:

2 E. T. Vincent, The Theory and Design of Gas Turbines and Jet Engines, McGraw-Hill, New York, 1950, page 577.

3 J. H. Keenan and J. Kaye, Gas Tables, Wiley, New York.

$$P_5 = P_3 - \frac{0.0158 G_D^{1.9} T_3}{P_3} \quad (L/d)$$

It is desired to express this relation as a function of G_D^* , the duct flow density at sea level static conditions. For a fixed design configuration, continuity requires that:

$$G_R A_f F_b = G_D A_D$$

$$G_R^* A_f F_b^* = G_D^* A_D$$

$$\frac{G_D}{G_D^*} = \frac{F_b}{F_b^*} \quad \frac{G_R}{G_R^*} = \frac{F_b}{F_b^*} \quad \sigma_R$$

and,

$$G_D = \frac{F_b}{F_b^*} \quad \sigma_R \quad G_R^*$$

Assuming negligible heat loss from the duct, total temperature through the duct remained constant.

$$T_5 = T_3$$

Free Turbine

The shaft work of the free turbine in Btu per pound of turbine air was computed in the same manner as for the compressor turbine, again assuming an exhaust air velocity of 300 feet per second.

$$\Delta H_T = \eta_T \quad \frac{\bar{c}}{P_T} \quad T_5 \left[1 - \left(\frac{P_0}{P_5} \right)^{\frac{k}{k-1}} \right] - \frac{(C_e)^2}{2 g J}$$

Shaft horsepower per pound of compressor air was then calculated from the relation:

$$\frac{SHP}{w_c} = \Delta H_T \times F_b \times 1.415$$

where, $1.415 \text{ HP} = 1 \text{ Btu/sec}$

Cycle Efficiency

Cycle efficiency was defined as shaft horsepower per kilowatt of reactor power, and was calculated from the relation:

$$\frac{\text{SHP}}{\text{KW}} = \frac{\text{SHP}/\dot{W}_C}{\bar{c}_{P_R} [T_3 - T_2]} (1.054)$$

where, $1.054 \text{ KW} = 1 \text{ Btu/sec}$

Compressor Air Flow Rate

In order to determine a compressor air flow rate, a free turbine developing 5500 shaft horsepower was assumed. The compressor air flow rate was then given by:

$$\dot{V}_C = \frac{\text{SHP}}{(\text{SHP}/\dot{W}_C)} = \frac{5500}{(\text{SHP}/\dot{W}_C)}$$

Weight of Components per Shaft Horsepower (Split Flow)

Compressor and Compressor Turbine

The weight of the compressor plus the compressor turbine was assumed to approximate that of a conventional turbo-jet engine of equal air handling capacity. A weight analysis of current turbo-jet engines⁴ shows that the weight of engine per pound of air handled per second is essentially a constant of 24 pounds of air per second regardless of thrust rating or compressor ratio.

Thus:
$$\frac{\dot{W}_C + \dot{W}_{CT}}{\dot{W}_C} = 24$$

Free Turbine

The weight of the free turbine per pound of air per second was

4. K. Von Gersdorff, "Power Plant Review", The Aeroplane, March 15, 1957.

approximated by the empirical relation:⁵

$$\frac{W_T}{W_C} = 1313 F_b \sqrt{\frac{T_5}{P_5}} \log_e \left(\frac{P_5}{P_6} \right)$$

Radiator

The weight of a 12 inch deep radiator while fully loaded with NaK has been taken as 90 pounds per square foot of inlet frontal area in previous performance calculations.⁶ Radiator weight calculations based on a particular radiator design, Appendix C, indicate that the weight is more nearly 90 pounds per square foot of overall frontal area (including headers and manifolds). In this particular design the ratio of total frontal area to inlet frontal area is approximately 1.31. Accordingly the weight of the radiator per pound of air per second is calculated by:

$$\frac{W_R}{W_C} = \frac{(1.31)(90) \cdot \frac{D}{12} \cdot A_F}{v_C} = \frac{9.83 D}{G_R}$$

Bleed Duct

Preliminary calculations show that the maximum temperature of the air in the bleed ducts will be in the vicinity of 1500 degrees F. In this temperature range the material giving the best strength to weight ratio is Inconel X⁷. Inconel X also resists progressive oxidation in air at this temperature. The weight of the duct is given by:

$$\begin{aligned} W_D &= \left(\frac{P_3 d}{288 s_{all}} \right) (\pi) (d) (\rho_{inc}) (12)(L) \\ &= \frac{\pi d^2}{4} \frac{(48) P_3 \rho_{inc} L}{288 s_{all}} \\ &= \frac{A_D (24) P_3 \rho_{inc} L}{s_{all}} \\ &= 24 \frac{W_D}{G_D} \frac{P_3 \rho_{inc} L}{s_{all}} \end{aligned}$$

5. William W. Watchtl and Frank Rom, Analysis of Turbine Propeller Cycle for Propulsion of Low Speed Nuclear Powered Aircraft, NACA RM E52D02, May 1952 p. 5
6. W. S. Farmer, A. P. Fraas, H. J. Stumpf and G. D. Whitman, Preliminary Design and Performance Studies of Sodium-to-Air Radiators, ORNL-1509, Aug. 1953 p. 100

The weight of the duct per pound of compressor air is then:

$$\begin{aligned} \frac{W_D}{W_C} &= 24 \frac{W_D}{W_C} \frac{P_3 \rho_{inc} L}{G_D s_{all}} \\ &= \frac{24 F_b P_3 \rho_{inc} L}{G_D s_{all}} \end{aligned}$$

where,

$$G_D = \frac{F_b}{F_b^*} \sigma_R G_D^*$$

Fig. 3 shows the creep and stress to rupture characteristics of Inconel X as determined by 1000 hour tests. Between the temperatures of 1400 degrees F. and 1550 degrees F., which covers the range of interest, the stress to produce 0.2 percent total strain in 1000 hours is given by the linear equation:

$$s = 264,000 - 128.00 T_3$$

The stress in the wall of the duct is given by the hoop stress relation:

$$s = \frac{P_3 d}{2t}$$

Note that P_3 is the absolute pressure in the duct. The net radial pressure is $P_3 - P_0$. In the range of pressures encountered in this design, the use of P_3 instead of $P_3 - P_0$ introduces a factor of safety of approximately 10 percent, or 1.1. An additional factor of safety of two was added resulting in the following relation for the allowable stress:

$$s_{all} = 132,000 - 64.00 T_3$$

For an aircraft of the dimensions of the R3Y it is estimated that the ducts to the two outboard engines will be 67 feet in length and the ducts to the two inboard engines will be 53 feet in length. An average length of 60 feet will be taken to determine total duct weight per engine.

7. Metal Progress Data Sheet, June 1956, ASM, Cleveland, Ohio, p. 80-B.

Fig. 3

ELEVATED TEMPERATURE STRENGTH OF INCONEL X⁸
(Duration of Tests 1000 Hours)

Thus:
$$\frac{W_D}{W_C} = \frac{(24)(60) P_b P_3 \rho_{inc}}{G_D (132,000 - 64.00 T_3)}$$

Using $\rho_{inc} = 0.3$ pounds per cubic inch gives:

$$\frac{W_D}{W_C} = \frac{P_b P_3}{G_D (305.5 - 0.148 T_3)}$$

Weight of Engine Components per Horsepower

The weights of the various engine components per pound per second of compressor air have now been obtained. The weight of the engine per pound per second of compressor air is then:

$$\frac{W_E}{W_C} = \frac{W_C + W_{CT}}{W_C} + \frac{W_R}{W_C} + \frac{W_T}{W_C} + \frac{W_D}{W_C}$$

The weight of the engine per horsepower is then computed from the relation:

$$\frac{W_E}{SHP} = \frac{W_E}{W_C} \times \frac{1}{(SHP/W_C)}.$$

Weight of the Reactor and Shields Per Shaft Horsepower

Preliminary calculations indicated that sufficient shielding to reduce the crew compartment dose rate to 0.1 rem per hour would be feasible. For a 21 inch diameter core (diameter of the ART core) the weight of the reactor and shields in the range of power, P_r , of 50 to 100 megawatts may be expressed by⁹:

$$W_r = 69,000 + 209 P_r,$$

the reactor power in megawatts being computed from:

$$P_r = \frac{\text{Total SHP}}{(SHP/KW) (1000)}$$

9. E. P. Blizard and H. Goldstein (eds), Report of the 1953 Summer Shielding Session, ORNL-1575, June 1954, Table IV-I-2.

In this study a total shaft horsepower of 22,000 horsepower was assumed, giving:

$$P_r = \frac{22}{\text{SHP/KW}}$$

Weight of Power Plant Per Shaft Horsepower

The weight of the power plant per shaft horsepower, the quantity to be optimized, is:

$$\frac{W_P}{\text{SHP}} = \frac{W_r}{\text{Total SHP}} + \frac{W_E}{\text{SHP}}$$

The weight optimization equations as tabulated for solution by computer are shown in Table 1.

Selection of Optimum Values of Design Parameters

(Split Flow)

It was originally planned to use an IBM computer to solve the cycle and component weight equations for many combinations of design parameter values (see Table 1). A delay in getting computer time necessitated hand solution of the equations for the most promising values of the design parameters.

Table II shows the tabular calculations and the resulting weights of the power plant per shaft horsepower for the selected values of the design parameters. The results are shown plotted in Fig. 4. It can be seen that for the selected parameter values, a compressor pressure ratio of 8 to 10 with a flow density in the radiator of 15 pounds per square foot per second and a radiator depth of 24 inches, gives a minimum specific weight of the power plant of 4.49 pounds per horsepower. However a pressure ratio of eight and a flow density in the radiator of 10 pounds per square foot per second and a radiator depth of 24 inches gives only a slightly larger specific weight of 4.50 pounds per horsepower. It is felt that the lower flow density, even though it gives a slightly larger specific weight, is more desirable for two reasons. First, the radiator data was extra-

TABLE I

SUMMARY OF WEIGHT OPTIMIZATION EQUATIONS AND COMPUTER INSTRUCTIONS

Constants and Conversion Factors	Computer Instructions and Equations	Constants and Conversion Factors	Computer Instructions And Equations
	I. Assume Altitude: Alt. P_o t_o σ_o <u>S.L.</u> <u>2116</u> <u>519</u> <u>1.000</u> 10 1455.5 483.4 0.7384 18 1057 455 0.5697 36 474.6 392 0.2961		1. Same as split flow
	II. Assume Mach Number, M: @ Alt = <u>S.L.</u> ; <u>M</u> = 0, 0.15, 0.3, 0.6 @ Alt = 10 ; M = 0.15, 0.3, 0.6 @ Alt = 18 ; M = 0.15, 0.3, 0.6 @ Alt = 36 ; M = 0.15, 0.3, 0.6		2. Same as split flow
$k = 1.401$	III. Calculate T_1 : $T_1 = t_o [1 + 0.2 M^2]$		3. Same as split flow
$k = 1.401$ $\frac{P_1}{P_o} = 0.97$	IV. Calculate P_1 $P_1 = 0.97 P_o [1 + 0.2 M^2]^{3.51}$		4. Same as split flow
	V. Assume pressure ratio: $z = \underline{6}, \underline{8}, \underline{10}, \underline{12}$		5. Assume pressure ratio: $z = \underline{6}, \underline{8}, \underline{10}, \underline{12}$

Note: Underlined values were selected for hand calculation in this study

TABLE 1 (Con't)

<p>VI. Calculate P_2:</p> $P_2 = Z P_1$		6. Same as split flow
<p>$\eta_c = 0.86$ $k_c = 1.394$ $\bar{c}_{pc} = 0.242$</p> <p>VII. Calculate T_2:</p> $T_2 = \left[1.162 Z^{0.283} - 0.162 \right]$		7. Same as split flow
<p>VIII. Assume G_R^*:</p> $G_R^* = 5, \underline{10}, \underline{15}, 20$		8. Assume G_R^* :
<p>$k = 1.401$</p> <p>IX. Calculate σ_R:</p> $\sigma_R = \sigma_0 \left[1 + 0.2 M^2 \right]^{2.51}$		9. Same as split flow
<p>X. Calculate G_R:</p> $G_R = \sigma_R G_R^*$		10. Same as split flow
<p>XI. Assume D</p> $D = 18, \underline{24}, \underline{30}$		11. Same as split flow
<p>XII. Calculate heating effectiveness, η:</p> <p>1. @ $D = 18$</p> $= 1.025 - 0.0557 G_R^{0.484}$ <p>2. @ $D = 24$</p> $= 0.964 - 0.00644 G_R^{0.940}$ <p>3. @ $D = 30$</p> $= 0.933 - 0.0000745 G_R^{2.09}$		12. Same as split flow

TABLE 1 (Con't)

$T_N = 2000^\circ R$	XIII. Calculate T_3 : $T_3 = T_2 + \eta [2000 - T_2]$		13. Same as split flow
	XIV. Calculate P_3 : $P_3 = P_2 - \frac{C_2 G_R^{1.8}}{53.34 (T_2 + T_3) \left[1 + \sqrt{1 - \frac{53.34 (T_2 + T_3) C_2 G_R^{1.8}}{P_2^2}} \right]}$ <p> 1. @ $D=18$, $C_2 = 0.913$ @ $D=24$, $C_2 = 1.220$ @ $D=30$, $C_2 = 1.525$ </p>		14. Same as split flow
$F_a = \frac{w_a}{w_c} = 0.05$ $C_e = 300 \text{ ft/sec}$ $\bar{c}_{Pc} = 0.242$ $\eta_{CT} = 0.88$ $k_{CT} = 1.350$	XV. Calculate F_b : $F_b = 0.950 - 0.242(T_2 - T_1) \left[0.233 T_3 \left[1 - \left(\frac{P_0}{P_3} \right)^{0.259} \right]^{-1.80} \right]$	$k_{CT} = 1.340$ $\bar{c}_{Pc} = 0.242$ $F_D = 0.950$ $\bar{c}_{PcT} = 0.266$ $\eta_{CT} = 0.88$	15. Calculate P_4 : $P_4 = P_3 \left[1 - \frac{1.090(T_2 - T_1)}{T_3} \right]^{3.94}$
	XVI. Assume G_D^* : $G_D^* = 20, \underline{40}, 60$	$\bar{c}_{Pc} = 0.242$ $\bar{c}_{PcT} = 0.266$ $F_D = 0.950$	16. Calculate T_4 : $T_4 = T_3 - 0.958 (T_2 - T_1)$

TABLE I(Con't)

	XVII. Assume L/d : $L/d = 80, \underline{90}, 100$		17. Assume G_D^* : $G_D^* = 20, \underline{30}, 40$
	XVIII. Calculate G_D : $G_D = \left(\frac{F_b}{F_b^*} \right) \sigma_R G_D^*$ where $F_b^* = F_b$ at sea level and $M = 0$		18. Assume L/d : $L/d = \underline{50}, 60, 70$
$\mu = 2.97 \times 10^{-5} \frac{\text{lb}}{\text{ft-sec}}$ $v_d = 25 \frac{\text{lb}}{\text{sec}}$	XIX. Calculate P_5 : $P_5 = P_3 - \left(0.0158 G_D^{1.9} \right) \left(\frac{T_3}{P_3} \right) L/d$		19. Calculate G_D : $G_D = \sigma_R G_D^*$
$\eta_T = 0.88$ $\bar{c}_{PT} = 0.266$ $C_e = 300 \text{ ft/sec}$	XX. Calculate SHP/w_c : $\frac{\text{SHP}}{w_c} = F_b \left\{ 0.330 T_4 \left[1 - \left(\frac{P_0}{P_5} \right)^{0.259} \right] - 2.25 \right\}$	$\mu = 2.42 \times 10^{-5} \frac{\text{lb}}{\text{ft-sec}}$ $v_d = 80 \text{ lb/sec}$	20. Calculate P_5 $P_5 = P_4 - 0.01345 G_D^{1.9} \left(\frac{T_4}{P_4} \right) L/d$
$\bar{c}_{PR} = 0.2641$	XXI. Calculate SHP/KW $\frac{\text{SHP}}{\text{KW}} = \frac{\text{SHP}/w_c}{0.2784(T_3 - T_2)}$	$\eta_T = 0.88$ $\bar{c}_{PT} = 0.266$ $C_e = 300 \text{ ft/sec}$ $F_D = 0.95$	21. Calculate SHP/w_c : $\text{SHP}/w_c = 0.95 \left\{ 0.330 T_4 \left[1 - \left(\frac{P_0}{P_5} \right)^{0.259} \right] - 2.25 \right\}$

Table I (Con't)

	<p>XXII. Calculate $\frac{W_E}{w_c}$:</p> $\frac{W_E}{w_c} = 24 + \frac{9.83 D}{G_R}$ $+ \frac{1313 F_b \sqrt{\frac{T_3}{P_5}} \log_e \left(\frac{P_5}{P_0} \right)}{G_D (303.5 - 0.148 T_3)}$	$\bar{c}_{p_R} = 0.2641$	<p>22. Calculate $\frac{SHP}{KW}$:</p> $\frac{SHP}{KW} = \frac{SHP/w_c}{0.2784 (T_3 - T_2)}$
	<p>XXIII. Calculate $\frac{W_E}{SHP}$:</p> $\frac{W_E}{SHP} = \frac{W_E/w_c}{SHP/w_c}$		<p>23. Calculate $\frac{W_E}{w_c}$:</p> $\frac{W_E}{w_c} = 24 + \frac{9.83 D}{G_R}$ $+ 1247 \frac{\sqrt{\frac{T_4}{P_5}} \log_e \left(\frac{P_5}{P_0} \right)}{5}$ $+ \frac{50.6}{\sqrt{G_D}}$
SHP assumed to be 5500 HP for each free turbine	<p>XXIV. Calculate w_c:</p> $w_c = \frac{5500}{SHP/w_c}$		<p>24. Calculate $\frac{W_E}{SHP}$:</p> $\frac{W_E}{SHP} = \frac{W_E/w_c}{SHP/w_c}$

TABLE I (con't)

Total shaft horse power assumed to be 22000 HP	XXV. Calculate P_r $P_r = \frac{22}{\text{SHP/kw}}$	SHP assumed to be 5500 HP for each free turbine	25. Calculate w_c : $w_c = \frac{5500}{\text{SHP}/w_c}$
	XXVI. Calculate W_r : $W_r = 69000 + 209 P_r$	Total shaft horse power assumed to be 22000 HP	26. Calculate P_r : $P_r = \frac{22}{\text{SHP/KW}}$
	XXVII. Calculate $\frac{W_r}{\text{SHP}}$: $\frac{W_r}{\text{SHP}} = \frac{W_r}{22000}$		27. Calculate W_r : $W_r = 69000 + 209 P_r$
	XXVIII Calculate $\frac{W_P}{\text{SHP}}$: $\frac{W_P}{\text{SHP}} = \frac{W_E}{\text{SHP}} + \frac{W_r}{\text{SHP}}$		28. Calculate $\frac{W_r}{\text{SHP}}$: $\frac{W_r}{\text{SHP}} = \frac{W_r}{22000}$
			29. Calculate $\frac{W_P}{\text{SHP}}$: $\frac{W_P}{\text{SHP}} = \frac{W_E}{\text{SHP}} + \frac{W_r}{\text{SHP}}$

TABLE II

CALCULATION OF WEIGHT PER HORSEPOWER OF SPLIT FLOW POWER PLANT
(Sea Level, $M = 0.15$)

Line No.	Z	G^*_{R2} lbs/ft ² sec	D in.	G^*_{D2} lbs/ft ² sec	L/a	T_2 deg R	P_2 lbs/ft ²	T_3 deg R	P_3 lbs/ft ²
1	6	10	24	40	90	923	12,510	1899	12,034
2	6	10	30	40	90	923	12,510	1927	11,860
3	6	15	24	40	90	923	12,510	1867	11,520
4	6	15	30	40	90	923	12,510	1904	11,242
5	8	10	24	40	90	1006	16,680	1906	16,315
6	8	10	30	40	90	1006	16,680	1931	16,222
7	8	15	24	40	90	1006	16,680	1878	15,925
8	8	15	30	40	90	1006	16,680	1912	15,719
9	10	10	24	40	90	1080	20,850	1913	20,550
10	10	10	30	40	90	1080	20,850	1936	20,469
11	10	15	24	40	90	1080	20,850	1887	20,234
12	10	15	30	40	90	1080	20,850	1918	20,068
13	8	10	24	60	110	1006	16,680	1906	16,315

TABLE II (Con't)

Line No.	P_s	F_b	SHP/w_c	w_c	EFF HP/KW	w_e/HP	P_r MW	w_r	w_r/HP	w_p/HP
	lb/ft ²		HP/lb/sec	lb/sec		lb/HP		lbs	lb/HP	lb/HP
1	11,771	.342	76.0	72.3	.279	.713	78.6	85,500	3.89	4.60
2	11,587	.350	78.8	69.8	.283	.772	78.0	85,500	3.89	4.66
3	11,245	.320	68.8	79.9	.262	.658	84.0	86,000	3.91	4.56
4	10,954	.324	70.0	78.5	.246	.713	85.5	86,500	3.93	4.64
5	16,121	.306	78.5	70.1	.313	.698	70.2	83,700	3.81	4.50
6	16,027	.310	80.0	68.8	.311	.771	70.9	83,800	3.81	4.58
7	15,730	.287	71.0	77.0	.294	.642	74.7	84,600	3.85	4.49
8	15,517	.298	74.5	73.8	.295	.680	74.5	84,500	3.84	4.52
9	20,403	.265	73.6	74.8	.292	.748	69.5	83,650	3.80	4.55
10	20,320	.273	76.7	71.7	.322	.812	68.3	83,500	3.80	4.61
11	20,082	.253	69.0	79.8	.308	.667	71.1	84,000	3.82	4.49
12	19,913	.262	72.5	75.9	.311	.703	70.9	83,800	3.81	4.51
13	15,808	.306	77.5	71.0	.310	.683	71.1	83,900	3.82	4.50

TABLE II (Con't)

CALCULATION OF SPLIT FLOW BLEED FRACTION (SEA LEVEL, STATIC)

 $(G^*_D = 40 \quad , \quad L/d = 90)$

Line No.	Z	G^*_R lbs/ft ² sec	D in	T_2 deg R	P_2 lbs/ft ²	T_3 deg R	P_3 lbs/ft ²	F_b^*
1	6	10	24	919	12,300	1899	11,823	.337
2	6	10	30	919	12,300	1926	11,693	.343
3	6	15	24	919	12,300	1866	11,296	.312
4	6	15	30	919	12,300	1905	11,010	.319
5	8	10	24	1003	16,400	1906	16,023	.302
6	8	10	30	1003	16,400	1931	15,935	.307
7	8	15	24	1003	16,400	1876	15,635	.284
8	8	15	30	1003	16,400	1911	15,425	.294
9	10	10	24	1074	20,500	1914	20,199	.268
10	10	10	30	1074	20,500	1936	20,120	.275
11	10	15	24	1074	20,500	1885	19,876	.251
12	10	15	30	1074	20,500	1918	19,707	.261

Fig. 4
 VARIATION OF SPECIFIC WEIGHT OF SPLIT FLOW POWER PLANT WITH
 PRESSURE RATIO, RADIATOR FLOW DENSITY AND RADIATOR DEPTH

$$(G_D^* = 40, L/d = 90)$$

polated from a maximum test flow density of 2.5 pounds per square foot per second. The probable errors in the extrapolation make the data for the flow density of 10 more reliable than that of 15. Secondly, while no endurance tests have been made on full size radiators, the results of small scale tests¹⁰ indicate that radiator life time will be a critical factor in the design of NaK to air radiators. Further it would be expected that the radiator life time would be longer for lower flow densities.

The limited number of parameter values selected in the calculations do not prove conclusively that the design parameter values selected are the optimum. However, limitations on the size of radiators weighs heavily against flow densities lower than 10 and reliability of radiator data weighs against the use of higher flow densities. Fig. 4 does show that radiator depths greater than 24 inches are undesirable. It might be that a radiator depth of 18 inches would give better weight optimization; however, comparison with studies for turbo-jet cycles¹¹ indicates that for medium altitude, medium speed operation, performance begins to drop off for radiator depths less than 24 inches. Table II shows one calculation for a 50 percent increase in duct flow density to 60 pounds per square foot per second and the corresponding increase in duct length to diameter ratio to 110. This calculation shows no change in power plant specific weight at a pressure ratio of 8, a radiator flow density of 10 and a radiator depth of 24.

Weighing practical factors against theoretical calculations, as discussed above, the following design was selected to give the optimum power plant:

$$\begin{aligned}Z &= 8 \\G^*_R &= 10 \\D &= 24 \\G^*_D &= 40 \\L/d &= 90\end{aligned}$$

As more reliable radiator data becomes available, the assumptions made in the extrapolation of the radiator data used in this study should be reexamined. The optimum values of the design parameters may change if the radiator equations are shown to be appreciably in error.

10 W. S. Farmer, et al, op. cit., Appendix F.

11 William H. Sandeford, A Parameter Study of the Performance of Nuclear-Powered Turbojet Aircraft Using a Liquid-Metal Heat Transfer Medium, ORNL CF-55-8-198, August 1955, page 65.

Cycle Analysis(Through Flow)

The through flow cycle was identical to the split flow cycle up to and including station 3, the radiator discharge, as indicated in Fig. 1.

Compressor Turbine

The compressor turbine was assumed to expand all the compressor air, less the bleed to auxiliaries, to a total pressure such that turbine work was just sufficient to drive the compressor. Auxiliary bleed air was assumed to be five percent of the compressor air, as in the split flow cycle. Total pressure and temperature at station 4, the compressor turbine discharge were computed by equating compressor work to turbine work, as follows:

Compressor work = turbine work

$$w_C [H_2 - H_1] = w_{CT} [H_3 - H_4]$$

$$F_D = \frac{w_{CT}}{w_C} = 1 - \frac{w_a}{w_C} = 0.950$$

$$\text{Thus: } T_3 - T_4 = \frac{\dot{Q}_{PG} [T_2 - T_1]}{F_D \bar{c}_{pCT}}$$

$$\text{And: } T_4 = T_3 - \frac{\bar{c}_{pC}}{F_D \bar{c}_{pCT}} [T_2 - T_1]$$

$$\text{Also: } T_3 - T_4 = \eta_{CT} T_3 \left[1 - \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} \right]$$

$$\text{hence: } P_4 = P_3 \left[1 - \frac{T_3 - T_4}{\eta_{CT} T_3} \right]^{\frac{k}{k-1}}$$

Hot Gas Duct

Pressure drop through the ducts leading the energized air from the gas generator to the free turbine was computed as for the split flow case:

$$\Delta P = 0.108 \frac{\mu^{0.2} G_D^{1.9}}{\left(\frac{4 w_D}{\pi} \right)^{0.10} \rho_4^8} \left(\frac{L}{d} \right)$$

Preliminary calculations predicted an air temperature in the vicinity of 1400 degrees Rankine and an air mass flow rate of approximately 80 pounds per second.

Thus: $\mu = 2.42 \times 10^{-5} \frac{\text{Lb}}{\text{ft sec}}$

And: $\Delta P = 0.01345 G_D^{1.9} \left(\frac{T_4}{P_4} \right) L/d$

hence: $P_5 = P_4 - 0.01345 G_D^{1.9} \left(\frac{T_4}{P_4} \right) L/d$

Again it was desired to express the duct flow density in terms of flow density at sea level static conditions. From continuity:

$$0.95 G_R A_F = G_D A_D$$

$$0.95 G_R^* A_F = G_D^* A_D$$

$$G_D = \frac{G_R}{G_R^*} \quad G_D^* = \sigma_R G_D^*$$

Assuming negligible heat loss from the duct, total temperature through the duct remained constant.

$$T_5 = T_4$$

Free Turbine

The shaft work per pound of turbine air was computed as for the split flow cycle:

$$\Delta H_T = \eta_T \bar{c}_p T_5 \left[1 - \left(\frac{P_5}{P_4} \right)^{\frac{k-1}{k}} \right] - \frac{(C_5)^2}{2g}$$

Shaft horsepower per pound of compressor air was then calculated from the relation:

$$\frac{\text{SHP}}{W_C} = F_D \times \Delta H_T \times 1.415$$

Cycle Efficiency

Cycle efficiency was calculated as for the split flow cycle:

$$\frac{\text{SHP}}{\text{KW}} = \frac{\text{SHP}/w_C}{\bar{c}_{P_R} (T_3 - T_2)(1.054)}$$

Compressor Air Flow Rate

Compressor air flow rate was then calculated for a free turbine developing 5500 shaft horsepower:

$$w_C = \frac{5500}{(\text{SHP}/w_C)}$$

Weight of Components per Shaft Horsepower (Through Flow)

Compressor and Compressor Turbine

The weight of the compressor plus the compressor turbine was assumed to approximate that of a conventional turbo-jet engine of equal air handling capacity, as in the split flow analysis.

Thus:

$$\frac{w_C + w_{CT}}{w_C} = 24$$

Free Turbine

The weight of the free turbine per pound of air per second was approximated by the empirical relation used in the split flow analysis:

$$\frac{w_T}{w_C} = 1313 F_D \frac{\sqrt{T_5}}{P_5} \log_e \left(\frac{P_5}{P_6} \right)$$

Radiator

Radiator weight per pound of compressor air per second was computed from the same relation used in the split flow analysis:

$$\frac{w_R}{w_C} = \frac{9.83 D}{G_R}$$

Hot Gas Duct

Preliminary calculations showed that compressor turbine outlet temperatures would be in the vicinity of 1000 degrees Fahrenheit. Reducing

the extrapolated stress for 0.20 per cent total deformation (Fig. 3) by a safety factor of two, an allowable stress of 25,000 pounds per square inch was assumed. Using the duct weight equation as derived for the split flow cycle:

$$\frac{W_D}{W_C} = \frac{24 \frac{F_D P_4}{G_D} \rho_{inc} L}{s_{all}}$$

$$\frac{W_D}{W_C} = \frac{P_4}{60.8 G_D}$$

It was further assumed that 0.050 inches was a minimum practicable duct wall thickness, and an alternate duct weight equation was written.

Thus:

$$W_D = 144 \pi d t \rho_{inc} L \quad \text{lb}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \cdot W_D / G_D}{\pi}} \quad \text{ft}$$

$$\frac{W_D}{W_C} = 144 \pi t \rho_{inc} L \sqrt{\frac{4 F_D^2 W_C / G_D}{\pi}}$$

$$\frac{W_D}{W_C} = 7.75 L \sqrt{\frac{F_D}{W_C G_D}}$$

Substituting the assumed air fraction of 0.95, the assumed average duct length of 60 feet and the estimated compressor air flow rate of 80 pounds per second, the alternate duct weight equation yielded:

$$\frac{W_D}{W_C} = \frac{50.6}{\sqrt{G_D}}$$

For the range of parameters investigated, the alternate duct weight equation resulted in greater values and was therefore used in the weight analysis.

Weight of Engine Components Per Shaft Horsepower

Having obtained the weights of engine components per pound of compressor

air flow, the weight of the engine per pound of compressor air flow was computed:

$$\frac{W_E}{W_C} = \frac{W_T + W_{CT}}{W_C} + \frac{W_R}{W_C} + \frac{W_T}{W_C} + \frac{W_D}{W_C}$$

The weight of the engine per shaft horsepower was then computed:

$$\frac{W_E}{SHP} = \frac{W_E}{W_C} \left(\frac{1}{\frac{SHP}{W_C}} \right)$$

Weight of Reactor and Shielding per Shaft Horsepower

Weights of reactor and shields were calculated as for the split flow cycle, from the relations:

$$W_r = 69,000 - 209 P_r$$

$$P_r = \frac{22}{SHP/KW}$$

$$\frac{W_r}{\text{total SHP}} = \frac{W_r}{22000}$$

Weight of PowerPlant per Shaft Horsepower

The weight of the power plant per shaft horsepower, the quantity to be optimized, was then determined, as for the split flow cycle, from the relation:

$$\frac{W_P}{SHP} = \frac{W_r}{\text{total SHP}} + \frac{W_E}{SHP}$$

The weight optimization equations, and assumed values for the various constants were tabulated for solution by computer, as summarized in Table I.

Selection of Optimum Values of Design Parameters (Through Flow)

Hand calculations were performed for the through flow cycle optimization, because of the unavailability of computer time, as previously noted.

Table III presents tabular calculations and resulting power plant weights per shaft horsepower, for selected values of the design parameters. Results are displayed in Fig. 5. Based on consideration of theoretical and practical factors, as discussed in the split flow section, the following design parameters were selected:

$$Z = 9$$

$$G_R^* = 10$$

$$D = 24$$

$$G_D^* = 30$$

$$L/A = 50$$

It must be reemphasized that the above values may not be truly optimized. As more reliable radiator data becomes available, and the assumed radiator equations are verified or modified, a reexamination of the optimization calculations should be made, and a wider range of values of the design parameters should be investigated.

TABLE III

OPTIMIZATION OF COMPRESSOR PRESSURE RATIO (Through Flow)

Z	6	8	10	12
G^*_R	10	10	10	10
D	24	24	24	24
G^*_D	30	30	30	30
L/D	50	50	50	50
M	.15	.15	.15	.15
Alt	0	0	0	0
T_1	522	522	522	522
P_1	2085	2085	2085	2085
T_2	923	1006	1080	1140
P_2	12510	16680	20850	25050
T_3	1899	1906	1913	1920
P_3	12034	16315	20550	24794
T_4	1515	1442	1378	1327
P_4	4300	4510	4560	4520
P_5	4146	4370	4428	4392
SHF/ w_c	73.8	76.3	73.7	70.1
EW	63.3	63.7	63.3	63.1
w_e /HP	.858	.834	.859	.900
w_c	74.5	72.1	74.7	78.5
Eff	.271	.305	.318	.323
P_R	81.0	72.2	68.4	68.2
w_r	85,940	84,100	83,300	83,250
w_r /HP	3.91	3.825	3.790	3.790
w_p /HP	4.768	4.659	4.649	4.690

Fig. 5

VARIATION OF SPECIFIC WEIGHT OF THROUGH FLOW
POWER PLANT WITH COMPRESSOR PRESSURE RATIO

$$(G^*_R = 10, D = 24, G^*_D = 30, L/d = 50)$$

Weight of Reactor, Shields, Gas Generators,
Ducts and Free Turbine Per Shaft Horsepower
 $W_P/HP, \text{ lbs/HP}$

Compressor Pressure Ratio, Z

INSTALLATION OF OPTIMIZED SPLIT FLOW GAS GENERATOR IN R3Y AIRCRAFT

Selection of Airframe, Components, and Configuration

Selection of Airframe

The R3Y aircraft was selected for this installation study for the following reasons: 1) it is a medium speed aircraft with power requirements which can be met by current reactors, 2) it is a sea plane and can be operated entirely over water away from populated areas and 3) it is a suitable vehicle for installation of anti-submarine warfare equipment or aircraft early warning equipment or it can be used as a cargo carrier.

Selection of Reactor

In order to keep shield weights as low as possible and still keep radiation dose rates inside and outside the crew compartment down to realistic levels, a compact reactor must be used. The Aircraft Reactor Test reactor, a circulating fuel reactor, currently under construction at the Oak Ridge National Laboratory was selected as fulfilling the requirement for compactness. In addition, design and expected performance information was readily available. The design power of this reactor is 60 megawatts. If the ART is successful, it is predicted that the power density and fuel flow rates may be increased to give approximately 70 megawatts of power output without major design changes. This would be adequate power for a subsonic turbo-prop aircraft.

The ART and its aircraft type shield is approximately 11 feet in diameter and 14 feet high. This will just fit into the R3Y fuselage as shown in Fig. 2. Major structural changes will be necessary to support this concentrated load. In addition, the removal of the distributed fuel load from the wings of the aircraft and the substitution of free turbines for the heavier turbo-prop engines will increase the wing bending loads and probably necessitate strengthening of the wing structure.

Justification for Use of Gas Generators

In previous studies of nuclear powered turbo-prop aircraft it has been proposed that the intermediate coolant be piped out the wings to air

radiators installed in the engines. There are two problems in this system which can be simplified by using the gas generator concept. First, the intermediate coolant which has been tested sufficiently to insure reliable performance is 56 percent sodium and 44 percent potassium (NaK). The sodium in this alloy becomes activated in the primary heat exchanger and decays in the NaK lines and heat exchangers giving a strong source of gamma radiation. Coming from the wings, this radiation impinges on the sides of the crew compartment where the shielding is thinnest. By using gas generators the length of NaK lines, and thus the radiation, can be greatly reduced. In addition the direct radiation will strike the rear of the crew compartment where the shielding is thickest.

The second problem circumvented by using gas generators is the physical problem of piping NaK out the wings. The development of suitable expansion joints for such a line is a formidable problem which as yet has not been solved. In any event the piping of 2000 degree R. NaK out the vibrating wing of an aircraft is not a simple problem.

Placement of Gas Generators

From a radiation point of view it would be desirable to install the gas generators behind the reactor to use the reactor as a shadow shield between the radiators and the crew compartment. A scaled layout of the R3Y fuselage with the gas generators installed showed the exhaust of the two lower generators just above the waterline. This would cause serious operational problems. An examination of the structure of the R3Y fuselage showed that in order to bring the four inlet ducts through the top of the fuselage down to the compressor inlets, major structural redesign would be necessary. Also, the two lower generators would have to be set down into the cargo deck floor structure. Since the floor structure of the R3Y gives a large part of the fuselage strength this would involve major redesign.

It was decided that it would be better to place the gas generators at the top of the wing as shown in Fig.2. The extra radiation from the air radiators could be shielded against more easily than the fuselage could be redesigned. An estimation of the radiation dose rate at the crew compartment from the radiators was made in Appendix C and will be included in the shielding

calculations.

Design of Turbo Components

In Appendix D, preliminary designs of the compressor, compressor turbine and free turbine were made. As a result of these designs, overall dimensions of the turbo-components were estimated and utilized to assist in placement of system components in the cycle, as adapted to the R3Y. In addition the design information will be used to compare the turbo-component requirements with existing turbo components.

Compressor

The split flow cycle selected required a compressor having an air capacity of approximately 70 pounds per second and a pressure ratio close to eight to one. If a compressor of this air flow capacity and approximate pressure ratio exists, there should be no aerodynamic objections to use of the existing compressor component. Mechanical design details would have to be evaluated for each individual engine. It might be possible to adapt a compressor having other than the optimum pressure ratio, incurring some power plant weight penalty, as indicated in Fig. 4. Table IV, prepared from Wilkinson's compilation¹, lists existing engines within the range of air capacity and pressure ratio of possible interest.

The Pratt and Whitney T-34 turbo prop engine with an air capacity of 65 pounds per second and a pressure ratio of 6.7:1 appears to have a compressor with potentialities for adaptation to the split flow cycle. The thirteen stages result in an average pressure ratio per stage of 1.1 which is in the middle range of stage loading for modern aircraft engines. It might be feasible to increase the air capacity to the desired value by a slight increase in axial velocity with blade redesign. The non-optimized pressure ratio of 6.7:1 would result in a power plant weight penalty of 0.075 pounds per horsepower, as indicated in Fig. 4.

Two possible modifications to the cycle suggest themselves as a means of utilizing existing compressor components. The cycle might be modified to operate the power plant from two gas generators, each of which would supply bleed air to two of the free turbines. Such a modification would double the air requirement for each compressor. The J-71, J-73, or a scaled down version of the British Sapphire, (which has previously been scaled down to the J-65), seem to offer possibilities for adaptation to such a cycle. It is felt, however, that such a gas generator arrangement might be undesirable, since the most likely source of engine malfunction probably would be

¹ Paul H. Wilkinson, Aircraft Engines of the World, 1956, published by Author, 225 Varick Street, New York.

TABLE IV

SUMMARY OF AIRCRAFT ENGINES

MAKER	DESIGNATION	PRESSURE RATIO	AIR CAPACITY lb/sec	No. COMPRESSOR STAGES	RPM	TURBINE STAGES	THRUST OR HORSE POWER	TURBINE INLET TEMPERATURE
<u>TURBO - PROPS</u>								
P. & W	T-34	6.7:1	65	13	11000	3	5400	---
Allison	T-56	9.5:1	36	14	13820	4	3750	1780°F
Bristol	Proteus 700	7.2:1	42.5	12 axial 1 cent.	12000	4	3480	1535°F
Bristol	B.E. 25	10:1	86	10 spool ²	10500	4	4000	1394°F
Rolls Royce	Dart R.Da 6	5.5:1	20	2 cent.	14500	2	1990	1560°F
Rolls Royce	Dart R.Da 7	5.75:1	22	2 cent	15000	3	2105	---
<u>TURBO JETS</u>								
Allison	J-71	8.0:1	160	16	6100	3	10000	---
GE	J-47	5.5:1	102	12	7950	1	6000	1600°F
GE	J-73	7:1	150	12	8000	2	9200	---
Westinghouse	J-46	5.2:1	70	11	12500	2	6000	---
Bristol	Orpheus B.Or.3	4.8:1	80	7	9600	1	4850	---
Armstrong Siddley	Sapphire A.S. sa.7	9:1	180	13	8600	2	11000	---

radiator failure, resulting in loss of power to two of the four propellers.

Another possible modification would be to increase shaft horsepower requirements to a greater value, with corresponding increase in compressor air capacity requirements. Such a modification would result in the need for a larger air frame because of power plant weight and size increases, and was not investigated in this study.

Compressor Turbine

The preliminary design of the compressor turbine indicates that adaptation of any existing turbines to the compressor turbine probably would require major redesign. Matching the compressor turbine to compressor work and design rotational speed, at other than design turbine inlet conditions, and air capacities would certainly require blade redesign and probably annulus area modification. Additional stages most probably would be required for turbo jet turbines in order to match compressor work. It is felt that the compressor turbine should be designed specifically to match the compressor selected for the cycle.

Free Turbine

The free turbine could well be adapted from existing turbo prop turbines if such exist in the desired range of air capacity and turbine inlet conditions. The Rolls Royce Dart 7 turbo prop engine falls in this range and seems to show promise for use in the split flow cycle. Its air capacity and turbine work appear to be ideally suited for such an adaptation. Inlet conditions for the Dart turbine are at higher temperature and lower pressure than the cycle free turbine requirements. The adaptation of the Dart turbine would probably pose no major problems, assuming design Mach numbers are such that choking would not occur because of the lower inlet temperatures.

Placement of Bleed Ducts

Tab le II shows that for each engine to develop 5500 shaft horsepower, at sea level, $M = 0.15$, each compressor must handle 70.1 pounds of air per second. Of this total air flow, $(70.1)(0.306)$ or 21.5 pounds per second will be ducted to the free turbines. To give the selected design duct flow density of 40 pounds per square foot per second at sea level static a duct cross sectional area of 0.538 square feet is required. This is a circular duct 9.88 inches in diameter. Wrapping two inches of insulation around each

duct will give a duct outside diameter of approximately 14 inches. At the inboard engine station the leading edge of the R3Y wing forward of the front spar is a box approximately 36 inches deep at the spar and 26 inches wide at the mid depth. After redesign of the structure inside the wing leading edge, sufficient room should be available to allow the ducts to run out through the wing leading edge to the free turbines (See Fig. 2). With two inches of insulating material and at least one inch of air space between the duct and the wing skin the approximate skin temperature during sea level static operation was calculated to be 600 degrees R. This calculation assumed free air convection over the wing leading edge and forced air convection between the duct and front spar. The required forced convection would be small and could be supplied by a small auxiliary blower. This indicated that no special material would be required to replace the aluminum skin around the hot air duct.

Aircraft Weight Analysis

With the general configuration of the power plant determined, it is now possible to estimate weights of components, shields, and structure for installation of the optimized design as determined in Part I.

Weight of Filled NaK Lines

Assuming 4 inch NaK lines made of 316 stainless steel pipes and estimating that each hot NaK line is 20 feet long and each cold NaK line 6 feet long, the weight of the filled NaK lines was calculated to be 389 pounds per engine.²

Weight of NaK Pumps

The weight of a NaK pump plus drive was estimated as 500 pounds by comparison with the ART NaK pumps.

Weight of Propellers and Reduction Gear

The weight of the propellers plus reduction gear was taken as 0.633 pounds per horsepower³ or 3480 pounds per engine.

Weight of Gas Generators, Bleed Ducts and Free Turbines

The weight of the gas generators, bleed ducts and free turbines were calculated from the optimized weight per horsepower determined in Part I. For a 5500 shaft horsepower engine the weight is 3840 pounds.

Weight of Airframe

The weights of the airframe was estimated as 30 percent of a gross weight of 200,000 or 60,000 pounds.

2. W. S. Farmer, et al, op. cit., Appendix C

3. William W. Watchtl and Frank E. Rom, Op. Cit.

Weight of Reactor, Reactor Shield, and Crew Shield

The remaining weights to be determined are reactor and reactor shield, crew shield and useful loads. Fig. 6 shows the sum of reactor, reactor shield and crew shield weights as a function of dose rate 50 feet from the reactor for three dose rates in an eight foot diameter by 10 foot crew compartment.⁴ These curves were prepared for a 70.2 megawatt reactor with a 21 inch diameter core. A correction of 65 percent was added to the reactor weight given in the tables. The expected weight of the ART indicates that this correction is necessary.

Comparing reactor and shield weights from these curves with the total weight of all other components, a combined reactor, reactor shield and crew compartment shield weight of 83,760 pounds was selected giving a dose rate of 0.1 Rem per hour in the crew compartment and 10 Rem per hour at 50 feet from the reactor outside the crew compartment. Data used in preparing the curves of Fig. 6 considers only the radiation from the reactor proper. The radiation from the air radiators calculated in Appendix C gives a dose rate at the crew compartment of approximately 0.5 Rem per hour. Assuming this radiation will be attenuated by the crew compartment shield in the same manner as the reactor radiation, the dose rate in the crew compartment from the air radiators will be .005 Rem per hour. The total dose rate in the crew compartment will then be 0.105 Rem per hour.

Useful Load

The R3Y has been test flown at a gross weight of 200,000 pounds. This was selected as the design gross weight, giving a useful load of approximately 23,000 pounds.

The aircraft weight analysis is summarized in Table V.

Auxiliary Power

At sea level, five percent of the total air flow, assumed for aircraft and reactor auxiliaries, will supply the following power:

$$P_{Aux} = \frac{SHP/w_c}{F_b} \times w_c \times F_a$$

$$= \frac{78.5}{.306} \times 70.1 \times .05 = 900 \text{ HP/eng}$$

⁴ E. P. Blizard and H. Goldstein (eds), op. cit., Tables IV-I-1 to IV-I-4 and Table IV-H-1.

Fig. 6

SHIELD WEIGHTS FOR VARYING DOSE RATES
(70 Megawatt Reactor, 21 Inch Diameter Core)

TABLE V

SUMMARY OF AIRCRAFT WEIGHT ANALYSIS
(R3Y - SPLIT FLOW)

Props and Reduction Gear (3480 lbs/eng)	13,920
Gas Generators, Bleed Ducts and Free Turbine (3840 lbs/eng)	15,360
NaK Pumps and Drives (500 lb each)	2,000
Filled NaK Lines (389 lbs/eng)	1,560
Air Frame (0.30 x 200,000 lbs)	60,000
Reactor and Shields	83,760
Reactor	14,000
Reactor Shield	46,560
Crew Shield	23,200
Useful Load	23,400
GROSS WEIGHT	200,000

The total power available for auxiliaries will then be 3600 horsepower. As altitude is increased the power represented by five percent of the total air flow will decrease, but then the power required for reactor auxiliaries will decrease also.

Aircraft Performance

A very brief performance analysis of the nuclear R3Y conversion was made to give some idea of what could be expected from the aircraft at sea level and 36,000 feet. Water drag data was not available for take-off time and distance calculations.

Table VI lists thrust horsepower required, THP_R , at various speeds for a 175,000 pound tactical R3Y-1.⁵ From these data a drag polar was constructed as shown in Table VI and Fig. 7. By the use of Table VI and Fig. 7, the thrust horsepower required for a 200,000 pound R3Y at sea level and 36,000 feet was determined.

The shaft horsepower and the additional thrust horsepower from the eight turbine exhausts was calculated for three speeds and two altitudes as shown in Table VII. Assuming a gear box efficiency of 98 percent and a propeller efficiency of 86 percent at sea level and 82 percent at 36,000 feet, the shaft horsepower was modified to give the thrust horsepower available from the propellers. Adding the thrust horsepower from the turbine exhausts, the thrust horsepower available was obtained and plotted in Fig. 8.

Fig. 8 shows the horsepower characteristics of the nuclear R3Y conversion at a gross weight of 200,000 pounds. From Fig. 8 the minimum velocity, V_{min} , maximum velocity, V_{max} , maximum excess horsepower, $(HP_{ex})_{max}$ and velocity for maximum excess horsepower, $(V_{dH/dt})_{max}$, were obtained. The maximum rate of climb, $(dH/dt)_{max}$, was obtained from:

$$(dH/dt)_{max} = \frac{3300 (HP_{ex})_{max}}{W_G}$$

where the aircraft gross weight, W_G , was taken as 200,000 pounds. The aircraft performance is summarized in Table VIII.

5. Convair, San Diego, Nuclear Powered Sea Plane Study, Rpt. No. ZP-54-6001 April, 1954.

TABLE VI

THRUST HORSEPOWER REQUIRED FOR 200,000 GROSS WEIGHT R3 Y

V	175,000 - 2000 ft.			200,000 - Sea Level			200,000 - 36,000 ft		
	THP _R	C _D ^S	C _L ^S	C _L ^S	C _D ^S	THP _R	C _L ^S	C _D ^S	THP _R
knots	HP	ft ²	ft ²	ft ²	ft ²	HP	ft ²	ft ²	HP
80	---	---	---	9200	---	---	---	---	---
90	---	---	---	7320	370	2800	---	---	---
120	---	---	---	4090	233	4220	---	---	---
150	---	---	---	2630	168	5880	8810	550	5750
170	6400	132.4	1900	2050	137	7010	6870	335	5120
200	8300	105.6	1370	1480	110	9200	4950	268	6670
230	11000	92.4	1042	1120	92	11600	3760	218	8210
290	17700	73.6	650	702	75	19100	2350	154	11730
320	23050	71.8	534	576	72	24500	1930	133	13500
350	---	---	---	---	72	32200	---	---	---
395	---	---	---	---	---	---	---	---	---

Fig. 7

R3Y-1 DRAG POLAR

ORNL-LR-DWG.-23470
UNCLASSIFIED

TABLE VII

HORSEPOWER CHARACTERISTICS OF OPTIMIZED SPLIT FLOW POWER PLANT

LINE NO.	ALT FT	M -	T ₃ deg R	P ₃ lbs/ft ²	F _b -	P ₅ lbs/ft ²	SHP/w _c HP/lb/sec	w _c lbs/sec	SHP HP	THP HP	THP _A HP
1	SL	.15	1906	16,315	.306	16,121	78.5	70.1	5500	83	18,750
2		.3	1907	17,085	.312	16,882	81.5	72.5	5770	-46	19,300
3		.6	1904	20,549	.337	20,298	97.4	82.5	7050	-1099	20,100
4	36000	.15	1925	3,573	.462	3,557	118.5	20.7	2440	100	7,920
5		.3	1926	3,745	.467	3,729	123.0	21.5	2596	12	8,330
6		.6	1927	4,514	.484	4,495	137.0	24.5	3200	-860	9,930

Note: All calculations are per engine except THP_A which is for all four engines

ORNL-LR-Dwg.-23471

Fig. 8
HORSEPOWER CHARACTERISTICS
(Converted R3Y-1)

TABLE VIII
PERFORMANCE SUMMARY

Alt	V_{min}	V_{max}	$(dH/dt)_{max}$	$(V_{dH}^*/dt)_{max}$
FT	Kts.	Kts	ft/min	Kts
S.L	80	295	2540	100
36,000	150	244	627	160

PART III

INSTALLATION OF OPTIMIZED THROUGH FLOW GAS GENERATOR IN R3Y AIRCRAFT

Using the same reactor (ART) and general configuration described in Part II, the feasibility of installing a through flow gas generator in an R3Y airframe will now be considered. Calculations will be based on the optimum design as determined in Part I.

Air Capacity Requirements

From Fig. 5 it was determined that power plant weight per horsepower optimized at a pressure ratio of 9 for the through flow cycle. Compressor air requirements were determined by interpolation in Table III to be 73.4 pounds of air per second. Of this air flow, 95 per cent or 69.8 pounds per second was expanded through the compressor turbine and ducted to the free turbine.

Possible Use of Existing Turbo-Components

Fig. 5 indicates that power plant weight per horsepower is insensitive to compressor ratio within the range of 8 to 10. The Bristol B.E. 25 turbo-prop engine has a two spool, ten stage axial compressor producing a pressure ratio of ten to one and a design air capacity of 86 pounds per second, as indicated in Table IV. This compressor may offer possibilities for adaptation by scaling down to 73 pounds per second. By scaling up total horsepower requirements to 25,800 shaft horsepower, the required compressor air capacity would be increased to 86 pounds per second, and the Bristol compressor would seem to be ideally suited for adaptation to the cycle. Reactor power would then be scaled up to 80 megawatts, and power plant weight would be increased to 108,000 pounds. Such an adaptation might be desirable if the R3Y could be modified to fly with the increased power and a gross weight greater than the nominal value of 200,000 pounds.

Table IV indicates no existing turbo-components which appear to be readily adaptable for use as the compressor turbine in the cycle. In any event, major redesign of an existing component would be necessary to match the compressor turbine to the compressor.

The through flow cycle inherently requires a free turbine designed for considerably lower inlet pressures than existing aircraft engine designs. The required total pressure ratio of approximately 0.5 falls within the

range of current aircraft turbine design. However, extensive modification of annulus areas and blading would probably be required in order to adapt an existing turbine for use as the free turbine in the through flow cycle.

Placement of Hot Gas Ducts

The duct cross sectional area necessary to contain the selected design duct flow density of 30 pounds per second per square foot at sea level, static was calculated as follows:

$$A_D = \frac{W_D}{\sigma_R G_D^*} = \frac{69.8}{30.3} = 2.30 \text{ ft}^2$$

This is provided by a round duct, 20.5 inches in diameter. One of the ducts can be led out to its related free turbine through the leading edge of the wing. The other duct must go out either through the wing torsion box or aft of the rear spar. The wing structural drawings available do not show what is in the space just aft of the rear spar. It may be that there is sufficient space forward of the aileron control rods and flap actuating mechanisms to allow the duct to be carried out to at least the inboard engine. To get the duct from the rear of the wing to the turbine inlet without passing through the wing torsion box, a nacelle could be constructed under the wing to enclose the duct. It might even be possible to enlarge the free turbine nacelle atop the wing and carry the duct over the wing torsion box. The feasibility of these suggestions can be determined only with the aid of the airframe manufacturer's detailed drawings and structural calculations.

Weight Analysis

In a manner similar to that employed in Part II, the weights of the aircraft components for the optimum values of the design parameters were determined. Since weight calculations were not made for the optimum pressure ratio of nine, the component weights were determined by straight line interpolation between the results of the calculations for pressure ratios of eight and 10. The weight of the power plant, 102,420 pounds, calculated in this manner compared satisfactorily with the weight of the power plant, 102,300 pounds calculated from the power plant specific weight at a pressure ratio of nine from Fig. 5.

The aircraft weight analysis is summarized in Table IX.

Auxiliary Power

At sea level five percent of the total air flow, assumed for aircraft and reactor auxiliaries, will supply the following power:

$$\begin{aligned}
 P_{aux} &= \left[\eta_T \Delta H'_T - \frac{(300)^2}{2gJ} \right] v_C \times F_a (1.415 \frac{HP}{Btu/sec}) \\
 &= \left[(.88) (217.4) - 1.8 \right] 73.4 \times 0.05 \times 1.415 \\
 &= 983 HP/eng.
 \end{aligned}$$

The total power available for auxiliaries will then be 3932 horsepower.

Aircraft Performance

The aircraft performance at a gross weight of 200,000 pounds will be essentially the same as that shown in Part II for the installed split flow power plant.

TABLE IX

SUMMARY OF AIRCRAFT WEIGHT ANALYSIS
(R3v - Through Flow)

Props and Reduction Gears (3480 lbs/eng)	13,920
Gas Generators, Bleed Ducts and Free Turbine (4,680 lbs/eng)	18,720
NaK Pumps and Drives (500 lbs each)	2,000
Filled NaK Lines (389 lbs/ eng)	1,560
Air Frame (0.30 x 2000,000 lbs)	60,000
Reactor and Shields	83,700
Reactor	14,000
Reactor Shield	46,500
Crew Shield	23,200
Useful Load	20,100
	<hr/>
Gross Weight	200,000

PART IV

DISCUSSION OF RESULTS AND CONCLUSIONS

DISCUSSION OF RESULTS

Part I has shown that both the split flow and through flow cycles considered in this investigation are thermodynamically feasible for adaptation to a medium powered turbo-prop aircraft in the weight range of the R3Y. Because of the existing state of reactor technology, the reactor weights assumed are subject to errors which can only be evaluated by a specific shield design for a specific installation.

A potential source of error in the thermodynamic analysis was the assumed radiator performance, which has been previously discussed in detail. Existing technology is such that adequate testing and evaluation of radiator performance can be performed if desired.

The assumed allowance for auxiliary power is a quantity which should be evaluated in detail for the specific installation under consideration, as a change in auxiliary power requirements would result in appreciable changes in cycle efficiency as defined in this study. It is hoped that the auxiliary power of approximately 3,600 shaft horsepower at sea level, as obtained from the assumed five per cent bleed, was a conservative estimate.

A comparison of the split flow and through flow cycles may be made from the results as tabulated in Table X. Results indicate that the split flow cycle would be somewhat more desirable. The split flow cycle optimized with a reactor power of 70.2 megawatts and a total power plant weight of 116,600 pounds. The selected design parameters for the through flow cycle resulted in a reactor power of 70.3 megawatts and a total power plant weight of 119,900 pounds. As previously noted in the appropriate sections, slightly better optimization might be obtained by further investigation of the range of design parameters for either cycle. The larger diameters of the hot gas ducts in the through flow cycle impose major installation design problems.

It seems quite probable that existing compressors could be adapted to either cycle without imposing redesign problems of impracticable magnitude. The possibility of adapting existing turbines to either of the turbine components in either cycle seems more remote, with the possible exception

TABLE X

COMPARISON OF SPLIT FLOW AND THROUGH
FLOW CYCLES AS INSTALLED IN THE R3Y

	Split Flow	Through Flow
Aircraft Gross Weight, lbs	200,000	200,000
Total SHAFT Horsepower, HP	22,000	22,000
Number of Engines	4	4
Air Mass Flow Rate per Engine, lb/sec	70.1	73.4
Reactor Power, Megawatts	70.2	70.3
Auxiliary Power at Sea Level, M = 0.15, HP	3,600	3,932
Weight of Gas Generators, Bleed Ducts and Free Turbine, lbs	15,360	18,720
Weight of Reactor, Shields, Associated Pumps and NaK Lines, lbs.	87,320	87,260
Total Power Plant Weight, lbs.	116,576	119,900
Useful Load, lbs	23,400	20,100

TABLE X(Con't)

	Split Flow	Through Flow
Diameter of Hot Gas Ducts, Exclusive of Insulation, inches	9.9	20.5
Selected Design Parameters:		
Compressor Total Pressure ratio, Z	8	9
Radiator flow Density, G_R^* , lb/ft ² -sec	10	10
Radiator depth, D, inches	24	24
Duct Flow Density, G_D^* , lb/ft ² -sec	40	30
Duct Equivalent Length to Diameter ratio	90	50
Performance		
V_{min} , Sea level, knots	80	80
V_{max} , Sea level, knots	295	295
V_{min} , 36000 ft, knots	150	150
V_{max} , 36000 ft, knots	244	244

of the split flow free turbine. Detailed discussions of these observations have been made in the appropriate sections. The required range of pressure ratios, pressures, temperatures, and air flow rates is such that design of components requires no advance in the state of existing aircraft engine technology.

Adaptation of the R3Y air frame would require major structural redesign of the fuselage in order to accommodate concentrated loads imposed by installation of the reactor and shields. The absence of distributed loads in the wings (no fuel, lighter engines) would probably require structural redesign of the wing in order to compensate for increased bending stresses.

The placement of the engine components in the R3Y necessitated locating the gas generators above the wing. The concept of utilizing the reactor shield to shadow shield the activated NaK in the radiators had to be sacrificed due to space limitations.

As a result of the above considerations, the authors feel that the design of an integrated system, with each component designed for its specific application would be preferable to attempting to adapt various airframe and engine components for use in such an aircraft. It is emphasized, however, that the rapid development of a nuclear powered aircraft would be extremely desirable, if only to advance the state of the art and develop a nucleus of trained operational personnel.

This investigation has concerned itself essentially with full power operation, no consideration having been given to possible modes of power control. A detailed investigation of this aspect of the problem should be made for each specific cycle in order to further evaluate the feasibility of adaptation to a nuclear powered aircraft.

It is suggested that two modes of power control might be required; one mode for long term power changes, as for reduced power in the cruise condition, and one mode with the reactor operating at full power and the engines operating at reduced power, as necessary to satisfy requirements for transition from taxi to take-off, and landing to wave-off. The first mode could possibly be accomplished by manipulating control rods in conjunction with regulation of NaK by-pass rates. The second mode, with the reactor operating at or near full power, might be accomplished by throttling in the duct prior to

2.11.1

the free turbine entry, clam-shell control of the free turbine exhaust, by-passing the hot air around the free turbine to atmosphere, or any combination of the above. Both modes of control would require some method of speed control for the gas generators, probably in the form of a clam-shell control on the compressor turbine exhaust.

Conclusions

1. The adaptation of the split flow or through flow gas generator-free turbine cycle to a nuclear powered turbo-prop of 200,000 pounds gross weight is thermodynamically feasible.
2. The reactor power required to produce 22,000 shaft horsepower is 70 megawatts for each cycle.
3. The total power plant weight is estimated to be 116,600 pounds for the split flow cycle and 119,900 pounds for the through flow cycle.
4. The split flow cycle can be adapted to the R3Y. Major structural redesign of the fuselage and probable strengthening of the wing would be required.
5. Exclusive of the reactor and its associated shields, the production of the components for this cycle is a development problem, but required no advance in airframe or turbo machinery technology.
6. Existing compressors can be adapted for use as the compressor component of either cycle.
7. If either cycle should be utilized to power an aircraft, it would be preferable to design the airframe, reactor, shields, and engine components for their specific application, in order to form an integrated system, rather than attempting to modify existing components.

APPENDIX A

INLET DUCT PRESSURE LOSS

From shielding considerations it may be desirable to install the gas generators behind the reactor. In this event the feasibility of the design would depend to a large extent on the pressure loss sustained in the long, torturous inlet ducts required. In order to determine the approximate magnitude of the inlet duct efficiency, η_p , the pressure loss through one of the ducts was calculated as follows.

From an inlet above the wing leading edge to the compressor in the fuselage behind the reactor, the duct will be approximated by 30 feet of straight duct and three 90 degree bends, all of constant cross sectional area. Each duct will be required to handle approximately 100 pounds of air per second. A duct cross sectional area of four square feet will give a flow density of 25 pounds per square foot per second and a flow velocity of 337 feet per second corresponding to a flow Mach number of 0.297. These were considered reasonable values upon which to base a design.

At sea level for the critical design condition ($M = 0.15$) the following values are known or can be easily calculated:

$$\left. \begin{array}{ll} p_o = 2116 & t_o = 519 \\ p_o = 2150 & T_o = 521.3 \end{array} \right\} \begin{array}{l} \text{Just inside} \\ \text{inlet} \end{array}$$

$$\left. \begin{array}{ll} p_o = 2020 & t_o = 511 \\ p_o = 2150 & T_o = 521.3 \end{array} \right\} \begin{array}{l} \text{Just inside} \\ \text{inlet} \end{array}$$

$$R_e = \frac{\rho v d}{\mu} = \frac{(.0741)(337)(2)}{1.20 \times 10^{-5}} = 4,170,000$$

The total pressure loss in the straight portion of the duct is given by:¹

$$\Delta P = 2 f \rho \frac{v^2}{2g} \frac{L}{d}$$

1. John R. Henry, Design of Power-Plant Installations, Pressure - Loss Characteristics of Duct Components, NACA ARR No. L4F26, June 1944, p. 7.

$$= (2) (.0025) \left(\frac{.0741}{32.2} \right) (337)^2 \frac{30}{2}$$

$$= 19.6 \text{ lb/ft}^2$$

In order to reduce the pressure loss and to smooth out the flow in the 90 degree bends, vanes will be installed.

The following relation determines the optimum number of vanes required:²

$$n = \frac{2.5}{\bar{r}/W} - 1$$

where

$\bar{r}$ = mean radius of duct

W = width of duct

A square duct, two feet on a side, will be assumed. A mean radius of 2 feet will be chosen giving:

$$n = \frac{2.5}{2/2} - 1 = 1.5$$

Two vanes will be used in each 90 degree bend. With the vanes installed, the duct at the bends will be considered to be three individual ducts as shown in Fig. 1-A. MACA ARR No. L4P26 gives total-pressure-loss coefficient, $\frac{\Delta P_o}{q}$, for 90 degree bends, as a function of Re, radius ratio, $\frac{\bar{r}}{W}$, and aspect ratio, $\frac{h}{W}$. A cross plot of this data was made and is shown in Fig. 1-A. For the individual vane-ducts in the turns, $Re = 2,090,000$. The curves of Fig. 1-A are shown extrapolated from the highest experimental Re of 600,000 to the desired value. The pressure loss coefficient for the inside vane-duct will be taken as the over-all loss coefficient at $Re = 2,090,000$ giving $\frac{\Delta P_o}{q} = 0.086$. The pressure loss for three 90 degree bends is then:

$$P = \left(\frac{1}{2} \rho v^2 \right) (3) \left(\frac{\Delta P_o}{q} \right) =$$

$$P = \left(\frac{3}{2} \right) \left(\frac{.0741}{32.2} \right) (337)^2 (.086)$$

$$= 33.7 \text{ lb/ft}^2$$

2. Ibid. p. 12

Fig. 1-A

TOTAL PRESSURE LOSS FOR 90 DEGREE VANED BEND

The approximate entrance loss for a duct with a slightly rounded entrance edge is³:

$$\frac{\Delta P_o}{q} = 0.2$$

The resulting pressure drop is then:

$$P = (q)_{eq} \frac{\Delta P_o}{q}$$

$$P = \frac{.0741}{64.4} (337 - 167)^2 \quad (.2)$$

$$= 6.7 \text{ lb/ft}^2$$

where $(q)_{eq}$ = relative dynamic pressure at duct inlet, lbs/ft^2

The total pressure loss for the duct is then 60.0 pounds per square foot.

The duct efficiency is then given by:

$$\eta_D = \frac{P_1}{P_o} = \frac{2150 - 60.0}{2150} = .972$$

Making an allowance for the blockage of the duct by the vanes and allowing for a flow acceleration pressure drop entering the compressor, a conservative duct efficiency of 0.96 would be used for this duct configuration.

If the gas generators were placed atop the fuselage, the inlet duct would be shortened and the bends removed. A conservative estimate of the inlet duct efficiency for this configuration would be 0.98. A compromise value of $\eta_D = 0.97$ was used in the cycle calculations.

APPENDIX B

RADIATOR ANALYSIS

Heat Transfer

Experimental Data

The best experimental data available for liquid metal to air heat exchangers is that contained in ORNL CF-1509,¹ which presents test data for a series of sodium to air radiator test cells. The reliability of the data and results presented are somewhat questionable, and the range of test data taken required extrapolation through the transition range of Reynolds number for use in this study.

Tests are currently being made on a much larger NaK to air radiator of similar design, but data so far available are not in sufficient quantity to permit reliable conclusions to be drawn.

The analysis of the radiator in this study will be based on the data for radiator configuration Number 3, as presented in ORNL CF-1509, but the methods utilized should be equally applicable to the more recent test data as it becomes available.

A description of the radiator test cell is as follows:

- 1) The radiator is essentially a two-pass, cross-flow heat exchanger, with three banks of tubes per pass, so arranged as to most closely approach a counterflow configuration. A schematic diagram of the flow configuration is presented in Fig. 1-B.
- 2) A matrix of inconel tubes, containing the sodium, penetrates a parallel series of stainless steel fins.
- 3) Tubes are 3/16 inches outside diameter, on 2/3 inch square centers.
- 4) Fins are 0.01 inches thick with 15 fins per inch, interrupted every two inches.

Results from this and similar radiator configurations are reproduced² in Fig. 2-B, as a log-log plot of Nusselt number versus Reynolds number. Reynolds number in this figure was calculated from the relation:

$$R_e = \frac{G_R d_f}{\mu}$$

1. W. S. Farmer, et al., op. cit.

2. Ibid. p.61, Fig. 17

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FIG. 1-B
SCHEMATIC FLOW CONFIGURATION
FOR TEST RADIATOR

Aug. 1946

Fig. 2.B SODIUM TO AIR RADIATOR CORE ELEMENT
 HEAT TRANSFER PERFORMANCE, (Nu vs. Re)

where: G_R = flow density, lbs of air/ft²-sec of radiator frontal area
 d_f = twice the fin separation, ft
 μ = viscosity of the air at the mean of inlet and outlet air temperatures, lb/sec-ft

Nusselt number was computed from:

$$Nu = \frac{h d_f}{k_a}$$

where: h = local heat transfer coefficient, Btu/hr-ft²-deg F
 k_a = conductivity of air at mean of inlet and outlet air temperatures, Btu/hr-ft deg F

Extrapolation of Data

It should be noted that the NACA Lewis Laboratory is conducting tests on an ART radiator. Data have been received since the writing of this report which indicate the slope for extrapolation purposes to be approximately 0.78. These data were taken at mass velocities up to approximately 7.0 which greatly reduces the uncertainty of extrapolation. The difference between a slope of 0.78 and 0.80 as assumed would result in a turbine inlet temperature from 5 to 10 degrees lower than those given in this report.

Figure 2-B shows that the plot of Nusselt number versus Reynolds number correlates the test data quite well for the various configurations over a wide range of sodium inlet temperatures. In extrapolating this curve to the desired range of flow densities and liquid metal inlet temperatures, a straight-line extension of the Nusselt number versus Reynolds number curve was made at a slope determined by visually weighting the test data in the upper range of Reynolds number and the indicated data for an inconel tube definitely above the transition range of Reynolds number. A slope of 0.80 was assumed.

The potential error in this extrapolation is quite serious. Flow densities in the range of this study are such that Reynolds number are a factor of about ten greater than the highest test values. Should the slope of the curve beyond transition assume a lower value, radiator performance could be considerable reduced. Later data³ suggests that the slope may lie in the range from 0.5 to 0.8.

Values of k_a and μ were taken from the Gas Tables for the mean of the expected inlet and outlet air temperatures of 1100 and 1950 degrees R., respectively. Thus:

$$\mu = 2.56 \times 10^{-5} \text{ lb}_m/\text{sec-ft}$$

$$k_a = 0.037 \text{ Btu/hr-ft-deg F}$$

An equation was fitted to the extrapolated curve:

$$Nu = 0.0250 Re^{0.80}$$

$$\frac{h d_f}{k_a} = 0.250 \left(\frac{G_R d_f}{\mu} \right)$$

Thus:

$$h = 0.250 k_a (\mu)^{-0.80} (d_f)^{-0.20} G_R^{0.80}$$

$$h = 10.92 G_R^{0.80}$$

Fig. 3-B shows a plot of experimentally determined values of local heat transfer coefficient versus flow density and also shows the extrapolated curve as determined above. Note that basing the extrapolation on Nusselts number has the effect of raising the local heat transfer coefficient when the viscosity and conductivity of the air at the higher extrapolated temperatures are substituted.

Fin Efficiency

Fin efficiencies were then required to obtain overall heat transfer coefficient, U , from extrapolated values of local heat transfer coefficient, h , using the defining relation:

$$E = U/h$$

A theoretical solution discussed by Tate and Cartinhour⁴ was utilized to calculate fin efficiencies as a function of local heat transfer coefficient, h ; fin conductivity, k_f ; and fin geometry. Pertinent results of this report are reproduced in Fig. 4-B. The parameter M was calculated as follows:

$$M = 2 k_f t_f \sqrt{d_1} / \sqrt{d_2} (d_2 - d_1)^2$$

where: k_f = equivalent fin conductivity, 128 Btu/hr-ft-deg

t_f = mean fin thickness, 0.01 inches

d_1 = fin inside diameter, 0.1875 inches

d_2 = fin outside diameter, 0.667

These results were based on a fin of radial geometry. In the radiator under consideration, the fin geometry was essentially a series of parallel fins of uniform thickness, penetrated by a square matrix of tubes. The fin inside diameter, d_1 , was taken as the tube outside diameter, while the equivalent fin outside diameter, d_2 , was approximated by the center to center distance of the tube matrix.

4. G. E. Tate and J. Cartinhour, "Disk Extended Surfaces for High Heat Absorption Rates," Trans. ASME 1945

Fig. 3-B

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FIG. 4-B
THERMAL RESISTANCE OF DISK FINS

An equivalent fin conductivity of 128 Btu/hr-ft-deg F was calculated for a stainless steel clad copper fin of the same geometry.⁵ From these data, the parameter M was calculated:

$$M = 2 \times 128 \times \frac{0.01}{12} \sqrt{\frac{0.1875}{12}} \sqrt{\frac{0.667}{12}} \left(\frac{0.667}{12} - \frac{0.1875}{12} \right)^2$$

$$M = 70.2$$

Also: $d_2/d_1 = 0.667/0.1875 = 3.55$

Utilizing the parameters M and d_2/d_1 , values of fin efficiencies were determined for a range of values of local heat transfer coefficient, h, from Fig. 4-B. Fig. 5-B displays the variation of fin efficiency, E, with local heat transfer coefficient, h.

It was noted by Tate and Cartinhour that corrections to fin efficiencies for temperature gradients across fins of other than radial geometry were six tenths of one per cent for a value of M equal to 0.2967. The assumption has been made in this study, and apparently in ORNL-CF-1509, that variation in temperature gradient due to non-radial fin geometry would have negligible effect on fin efficiency. This assumption may be in error as the value of the parameter, M, was much larger than that discussed by Tate and Cartinhour, as noted above.

Overall Heat Transfer Coefficient

Values of overall heat transfer coefficient, U, were calculated for assumed values of G_R , using the extrapolated values of h, fin efficiency as determined from Fig. 4-B, and the defining equation:

$$U = E h$$

Results are presented in Fig. 6-B.

Number of Heat Exchanger Transfer Units

Heat exchanger heating effectiveness may be determined as a function of two dimensionless parameters which are sufficient to define the value of heating effectiveness. One of these common parameters, Number of Heat Exchanger Transfer Units, NTU, is defined as follows:⁶

$$NTU = \frac{A_t U}{w C_p}$$

Thus:

$$NTU = \frac{A_t U}{3600 A_r G_R C_p}$$

5. William H. Sandeford, op. cit., page 21.

6. W. M. Kays, A. L. London, and D. W. Johnson, Gas Turbine Power Plant Heat Exchangers, ASME, April 1951, page 17.

Fig. 5-B
FIN EFFICIENCY
($k = 128$)

Fig. 6-B

OVERALL HEAT TRANSFER COEFFICIENT

where: A_t = total heat transfer area on which calculations for U were based, ft^2

A_f = radiator frontal area

G_R = radiator flow density, lbs air/ft^2 of radiator frontal area

For radiator configuration No. 3 of ORNL CP-1509:

$$A_t = 14.24 \text{ D ft}^2$$

$$A_f = 0.0400 \text{ ft}^2$$

Selecting a value for c_p of 0.264, (averaged over 1100 to 1900 degrees R), a relation for NTU was written:

$$\text{NTU} = 0.0312 \frac{DU}{G_R}$$

Values of NTU over a range of radiator flow densities, G_R , for radiator depths of eighteen, twenty-four, and thirty inches were calculated as presented in Table I-B.

Heat Capacity Rate Ratio

The second heat exchanger parameter necessary to determine radiator effectiveness is Heat Capacity Rate Ratio, defined as follows:⁷

$$\frac{\dot{C}_A}{\dot{C}_N} = \frac{w_A c_{pA}}{w_N c_N}$$

Since it was assumed that all heat removed from the NaK was absorbed by the air,

$$w_N c_N \Delta T_N = w_A c_{pA} \Delta T_A$$

$$\text{Hence: } \frac{\dot{C}_A}{\dot{C}_N} = \frac{w_A c_{pA}}{w_N c_N} = \frac{\Delta T_N}{\Delta T_A} = \frac{\Delta T_N}{T_3 - T_2}$$

Radiator heating effectiveness is defined by the relation:

$$\eta = \frac{T_A}{T_N - T_2} = \frac{T_3 - T_2}{T_N - T_2}$$

Thus:

$$\frac{\dot{C}_A}{\dot{C}_N} = \frac{\Delta T_N}{\eta (T_N - T_2)}$$

7. Ibid., page 16.

TABLE I-B

CALCULATIONS OF COUNTERFLOW RADIATOR HEATING
EFFECTIVENESS

Radiator Depth D Inches	Radiator Flow Density G_R lb/ft ² -sec	overall heat transfer coef. BTU/hr-ft ² deg F	Number of Transfer Units NTU	Counterflow Heating Effectiveness η_{cf}
18	5	36	4.04	.930
	10	55	3.09	.877
	15	68	2.55	.836
	20	80	2.24	.803
	25	88	1.98	.772
	30	95	1.78	.742
24	5	39	5.39	.958
	10	55	4.12	.931
	15	68	3.40	.899
	20	80	2.99	.875
	25	88	2.64	.845
	30	95	2.38	.822
30	5	36	6.74	.959
	10	55	5.15	.957
	15	68	4.25	.935
	20	80	3.74	.916
	25	88	3.30	.892
	30	95	2.97	.872

The ART design report⁸ specifies design values for T_N of 1600 degrees F. and ΔT_N of 430 degrees F. Values of 2000 degrees R. for T_N and 430 degrees F for ΔT_N were assumed in this study.

The range of $T_N - T_2$ was calculated to vary from a low value of 870 degrees F. (at a pressure ratio of twelve) to a high value of 1080 degrees F. (at a pressure ratio of six). The range of radiator effectiveness was estimated to be from 0.80 to 0.95, with the expectation that the optimization process would discard the lower values of η . The corresponding range of heat capacity rate ratios was then calculated:

$$\frac{\dot{C}_A}{\dot{C}_N} = \frac{430}{0.80(870)} = 0.617, \text{ the highest value}$$

$$\frac{430}{0.95(1080)} = 0.420, \text{ the lowest value}$$

From these considerations, a value of 0.500 was selected for the capacity rate ratio parameter. An inspection of Table II-B shows that this value would result in errors in heating effectiveness of one and one half per cent or less for an expected range of NTU above two, except for the highest values of $\dot{C}_A/\dot{C}_N$, which would probably be discarded, as implied above.

Heating Effectiveness

Utilizing Table II-B⁹, values of heating effectiveness were determined by linear interpolation of values for NTU. Results are tabulated in Table I-B. Plots of counterflow heating effectiveness versus radiator flow density for different radiator depths were made.

Equations were fitted¹⁰ to these curves, resulting in the following relation:

$$\eta_{cf} = K_{cf} G_R^m + C_{cf}$$

The constants, K_{cf} , m , and C_{cf} had the following dependence on radiator depth:

D	K_{cf}	m	C_{cf}
18 inches	-0.0571	0.484	1.051
24 inches	-0.00660	0.940	0.989
30 inches	-0.0000764	2.09	0.957

8. A.P. Fraas and A.W. Savolainen, op. cit p.12

10. Dale S. Davis, Empirical Equations, and Nomenclature, McGraw-Hill, New York 1943

9. Reproduced from W. M. Kays, et al, op. cit. p.24

TABLE II-B

COUNTER FLOW HEAT EXCHANGER PERFORMANCE

Heat Exchanger Effectiveness (η_{CF}) as a Function of Capacity Rate Ratio $\frac{\dot{C}_A}{\dot{C}_N}$ and Number of Heat Transfer Units (NTU)

NTU	η_{CF} FOR INDICATED VALUES OF $\frac{\dot{C}_A}{\dot{C}_N}$				
	0	0.25	0.5	0.75	1.00
0	0	0	0	0	0
0.25	0.221	0.216	0.209	0.205	0.200
0.50	0.393	0.378	0.362	0.347	0.333
0.75	0.528	0.502	0.476	0.452	0.429
1.00	0.632	0.598	0.565	0.532	0.500
1.25	0.713	0.674	0.635	0.598	0.556
1.50	0.777	0.735	0.691	0.645	0.600
1.75	0.826	0.784	0.737	0.687	0.636
2.00	0.865	0.823	0.774	0.722	0.667
2.50	0.918	0.880	0.832	0.776	0.714
3.00	0.950	0.919	0.874	0.817	0.750
3.50	0.970	0.945	0.905	0.848	0.778
4.00	0.982	0.962	0.927	0.873	0.800
4.50	0.989	0.974	0.944	0.893	0.818
5.00	0.993	0.982	0.957	0.909	0.833
∞	1	1	1	1	1

The above expression thus expresses the heating effectiveness of a counterflow radiator as a function of radiator flow density and radiator depth. The radiator under consideration, however, was expected to have a lower performance than a true counterflow configuration. Results of a recent heat exchanger summary¹¹ are reproduced in Fig. 7-B, which shows the ratio of heating effectiveness of the indicated flow configuration to counterflow effectiveness. The equivalence of the flow configuration indicated in Fig. 7-B, and the flow configuration under consideration, as indicated in Fig. 1-B, has been noted in the above mentioned summary. Fig. 1-B represents the flow configuration of the radiator under investigation, assuming negligible mixing of the air during its passage through the radiator.

From Fig. 7-B, the correction factor was estimated to be very close to 0.975 for a capacity rate ratio of 0.500 in the range of NTU expected. Accordingly, heating effectiveness was reduced by this factor, resulting in the relation:

$$\eta = \left(\frac{\eta}{\eta_{cf}} \right) \eta_{cf} = K_1 G_R^m + C_1$$

where the modified constants are:

D	K ₁	m	C ₁
18 inches	-0.0557	0.484	1.025
24 inches	-0.00644	0.940	0.964
30 inches	-0.0000745	2.09	0.933

Fig. 8-B presents these relations graphically.

Pressure Drop Across the Radiator

Experimental Data

The experimental pressure drop data for the sodium to air test cell under consideration is presented in ORNL-CF-1509¹² as a plot of stream pressure drop times mean relative density ($\sigma_m \Delta p$) versus flow density

11. R. A. Stevens, J. Fernandez, and J. R. Woolf, "Mean Temperature Difference in One, Two, and Three-pass Cross Flow Heat Exchangers", Trans. ASME, Feb. 1957, p. 293, Fig. 9

12. W. S. Farmer, et al, op. cit. p. 59, Fig. 15-C

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FIG. 7-B

CORRECTION FACTOR TO COUNTERFLOW EFFECTIVENESS
FOR TWO PASS CROSSCOUNTERFLOW HEAT EXCHANGER,
BOTH FLUIDS UNMIXED

Fig. 8-B

RADIATOR EFFECTIVENESS

of the radiator, as reproduced in Fig. 9-B. Mean relative density, σ_m , was calculated as the ratio of mean density of the air in the radiator to standard atmospheric density. Mean density was determined by correcting the inlet air density with the ratio of inlet air temperature to the mean of inlet and outlet air temperatures.

Extrapolation

The air stream pressure drop through a heat exchanger may be expressed¹³ in the form:

$$\Delta P = \frac{G_R^2}{2g\rho_2} \left[\underbrace{(K_c + 1 - \sigma^2)}_{\text{entrance effect}} + 2 \underbrace{\left(\frac{\rho_2}{\rho_3} - 1\right)}_{\text{flow acceleration}} + r \underbrace{\left(\frac{A_t}{A_c} \frac{\rho_2}{\rho_m}\right)}_{\text{core friction}} - \underbrace{(1 - \sigma^2 - K_e) \frac{\rho_3}{\rho_2}}_{\text{exit effect}} \right]$$

where: A_c = free flow area

$$\sigma = A_c/A_f$$

The drop in total pressure from flow acceleration alone may be approximated for low inlet Mach numbers as follows:¹⁴

$$\Delta P_{acc} = \frac{k P_2 M_2^2}{2} \left(\frac{T_3}{T_2} - 1 \right)$$

or:

$$\Delta P_{acc} = \frac{k g P_2}{2} \frac{M_2^2 \rho_2}{2g} \left(\frac{\rho_2}{\rho_3} - 1 \right)$$

$$\Delta P_{acc} = \frac{\rho_2 v_2^2}{2g} \left(\frac{\rho_2}{\rho_3} - 1 \right)$$

$$\Delta P_{acc} = \frac{G_R^2}{2g} \left(\frac{\rho_2}{\rho_3} - 1 \right)$$

Also, the drop in stream pressure from flow acceleration alone is:

$$\Delta P_{acc} = \frac{\rho_3 v_3^2}{g} - \frac{\rho_2 v_2^2}{g}$$

or:

$$\Delta P_{acc} = \frac{G_R^2}{\rho_3 g} - \frac{G_R^2}{\rho_2 g} = 2 \left[\frac{G_R^2}{2g\rho_2} \left(\frac{\rho_2}{\rho_3} - 1 \right) \right]$$

13. W. M. Kays, et al, op cit., p. 20

14. F. W. Godsey, Jr. and Lloyd A. Young, Gas Turbines for Aircraft, McGraw-Hill, New York, 1949, p.56

$$\begin{aligned} \text{Thus: } \Delta P_{acc} &= \Delta p_{acc} - \frac{G_R^2}{2g\rho_2} \left(\frac{\rho_2}{\rho_3} - 1 \right) \\ \text{and: } \Delta P_{acc} &= \frac{G_R^2}{2g\rho_2} \left[(K_C + 1 - \sigma^2) + \left(\frac{\rho_2}{\rho_3} - 1 \right) + f \left(\frac{A_t}{A_c} \frac{\rho_2}{\rho_m} \right) - \right. \\ &\quad \left. (1 - \sigma^2 - K_e) \frac{\rho_3}{\rho_2} \right] \end{aligned}$$

Note that the flow acceleration term in the expression for the total pressure drop is one half that for the stream pressure drop. Kays suggests that the core friction term normally dominates the pressure drop in gas heat exchangers. Fig. 9-B has a linear character, tending to verify the assumption that core friction dominates the pressure drop. The slope of the curve has a value of 1.34, corresponding to a friction factor varying as $1/Re^{0.66}$.

Since a comparison of the relations for total pressure drop and stream pressure drop indicates that total pressure drop is even more dependent on core friction, it was assumed that total pressure drop could be extrapolated as though it were wholly due to core friction.

$$\text{Thus: } \Delta P = f' \frac{G_R^2}{2g\rho_m} \left(\frac{A_t}{A_c} \right)$$

where f' is a composite friction factor for the complex radiator air flow path. The experimental data may be fitted by the following equation:

$$\rho_m \Delta P \approx \rho_m \Delta p = 0.865 G_R^{1.34}$$

This corresponds to a pressure drop due to core friction alone, with the friction factor varying as $1/Re^{0.66}$, as previously noted. In order to extrapolate the experimental data to the higher flow densities of this study, it was assumed that the flow through the radiator was just becoming turbulent at the highest experimental flow densities ($G_R = 2.5$, $Re = 2000$). In the turbulent range of Reynolds numbers, it was assumed that f' would vary as $1/Re^{0.2}$.

$$\text{Thus: } \Delta P = \frac{C_2}{R_e^{0.20}} \frac{G_R^2}{2.5 \rho_m} \frac{A_t}{A_c}$$

$$\text{Or: } \rho_m \Delta P = C_2 \left(\frac{\mu_m}{\mu} \right)^{0.20} \left(\frac{A_t}{A_c} \right) G_R^{1.8}$$

To generalize to a radiator of any depth, substitute:

$$A_t = \frac{14.25 D}{12}$$

$$\text{Hence: } \rho_m \Delta P = C_3 \left(\frac{\mu_m}{\mu} \right)^{0.2} \left(\frac{D}{12} \right) G_R^{1.8}$$

where $\left(\frac{\mu_m}{\mu} \right)$ is the ratio of the mean viscosity of the air in the extrapolation to the mean viscosity of the test data. Beginning the extrapolation at a flow density of 2.5 and solving for C_3 gives:

$$\rho_m \Delta P = 0.567 \left(\frac{\mu_m}{\mu} \right)^{0.20} \left(\frac{D}{12} \right) G_R^{1.8}$$

Taking the mean temperature of the air in the test data as 900 degrees R., and the mean temperature of the air in the extrapolation as 1525 degrees R., the relation becomes:

$$\rho_m \Delta P = 0.608 \left(\frac{D}{12} \right) G_R^{1.8}$$

Fig. 10-B presents plot of $\rho_m \Delta P$ versus G_R for radiator depths of 18, 24, and 30 inches. These curves are fitted by the equation:

$$\rho_m \Delta P = C_2 G_R^{1.8}$$

$$\begin{aligned} \text{where } D = 18 \text{ inches, } C_2 &= 0.912 \\ D = 24 \text{ inches } C_2 &= 1.216 \\ D = 30 \text{ inches, } C_2 &= 1.520 \end{aligned}$$

Mean Density

Mean air density was then determined by the method used by Sandeford.¹⁶

$$\rho_m = \frac{P_m}{R T_m} = \frac{P_2 + P_3}{R(T_2 + T_3)}$$

$$\begin{aligned} \text{But: } P_3 &= P_2 - \Delta P \\ \rho_m &= \frac{2 P_2 - \Delta P}{R(T_2 + T_3)} \end{aligned}$$

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Fig. 10-B

DENSITY TIMES PRESSURE LOSS FOR NaK TO AIR RADIATOR

$$\text{or: } \Delta P = 2 P_2 - R (T_2 + T_3) \rho_m$$

$$\text{Since: } \Delta P = \frac{C_2 G_R^{1.8}}{\rho_m}$$

$$\frac{C_2 G_R^{1.8}}{\rho_m} = 2 P_2 - R (T_2 + T_3) \rho_m$$

$$\text{Thus: } \rho_m = \frac{P_2}{R(T_2 + T_3)} \left[1 \pm \sqrt{1 - \frac{R(T_2 + T_3) C_2 G_R^{1.8}}{(P_2)^2}} \right]$$

Since the mean density is closely approximated by the relation:

$$\rho_m = \frac{2 P_2}{R(T_2 + T_3)}$$

it is evident that the sign in the quadratic formula must be positive.

Pressure Drop

Using the relation for mean density, the drop in total pressure across the radiator was then written:

$$\Delta P = \frac{C_2 G_R^{1.8}}{\frac{P_2}{R(T_2 + T_3)} \left[1 + \sqrt{1 - \frac{R(T_2 + T_3) C_2 G_R^{1.8}}{(P_2)^2}} \right]}$$

It is reemphasized that this relation for pressure drop was determined from an extrapolation based essentially on the assumption that core friction caused all the pressure drop, and density variations were such that use of a mean density introduced negligible error.

Large scale radiator tests in the range of temperatures and flow velocities would be extremely desirable in order to verify or refute this assumption.

APPENDIX C

COCKPIT RADIATION DOSE RATE FROM ACTIVATED NaK

Shielding data used to determine the shield weights in this study takes into account only the radiation from the reactor.¹ In order to determine the total cockpit radiation dose rate it was necessary to estimate the radiation emanating from the NaK to air radiators.

It is estimated that the saturated activity of the NaK in the ART will be 1000 curies.² The reactor required for this design will be essentially an ART reactor with the fuel flow rate, and thus the reactor power, increased sufficiently to give 70.2 megawatts of power output instead of the 60 megawatts of the ART. Since the reactor core flux will be proportional to the reactor power, it is reasonable to assume that the leakage flux to which the fuel to NaK heat exchanger is subjected will also be proportional to the reactor power. The volume of NaK in the fuel to NaK heat exchanger will be unchanged, only the velocity of circulation will be increased. Since the circulation time will be very small compared to the half life of the NaK activity, it can be concluded that the saturated activity of the NaK in an ART type reactor of 70.2 megawatts will be:

$$A = 1000 \times \frac{70.2}{60} = 1170 \text{ curies}$$

In order to make an estimate of the effect of this activity on the cockpit radiation dose rate, it was necessary to lay out a reasonably detailed design of a NaK to air radiator. Such a design was made and is shown schematically in Fig. 1-C. From the geometry of the radiator the following were calculated:

Number of tubes, 3328
 Length of tubing, 6980 ft.₃
 Volume of tubing, 0.922 ft₃
 Length of headers, 48 ft.
 Length of manifolds, 20 ft.₃
 Volume of headers, 1.114 ft₃
 Volume of manifolds, 1.028 ft₃
 NaK volume in radiator 3.064 ft₃
 Volume of radiator (over-all), 23.0 ft₃
 Weight of radiator (at 90 lbs/ft₃), 1890 lbs.
 Weight of manifolds, 152.4 lbs.

1. E. P. Blizard and H. Goldstein (eds), op. cit.

2. A. M. Perry and H. W. Bertini, ANP Quar. Prog. Rep. March 10, 1956, ORNL-2061, Table 1.2.

Fig. 1-C

NAK TO AIR
RADIATOR

Fins : Stainless -
Clad Copper
0.010 Inches
Thick

Tubes : Inconel
 $\frac{3}{16}$ Inch OD
On $\frac{2}{3}$
Inch Square
Centers

Weight of headers, 176.6 lbs.
 Weight of NaK in radiator, 141 lbs.
 Weight of fins (considered as all steel) , 1100 lbs.
 Weight of inconel tubes, 320 lbs.

In order to determine how much of the NaK activity was actually in the radiators it was necessary to determine the fraction of the total NaK in the radiators. The NaK capacity of the system was estimated as follows:

Volume of NaK in radiators, 12.26 ft³
 Volume of NaK in lines leading from radiator to reactor, 9.20 ft³
 Volume of NaK in pumps, 3.00 ft³
 Volume of NaK in reactor, 2.97 ft³
 Total volume of NaK, 27.43 ft³

The fraction of NaK in the radiators is then 0.447. The activity in the radiator is then 0.447 x 1170 or 522 curies. The remainder of the calculations will be made for 130.5 curies of activity in each of the four radiators.

The induced activity will arise from the reactions shown in Fig. 2-C. The activation of K³⁹ was neglected because of the long half life of the decay. The activation of K⁴¹ was neglected because of its small abundance and branching ratio.

Due to the complex geometry of the radiator, it was decided to approximate the radiator by a homogeneous mass of the same composition and size as the radiator. The activity was then considered as being spread throughout the radiator. The equivalent isotropic source strength of the homogeneous mass was then calculated by³:

$$S_{eq} = p \lambda_c$$

where, S_{eq} = the equivalent surface strength of the radiator,
 photons/cm²-sec

p = source strength, photons/cm³-sec

λ_c = relaxation length in the homogenized radiator, cm.

The source strength was calculated as follows:

$$p = \frac{\text{Activity/Radiator}}{\text{Volume of Radiator}} = \frac{(130.5)(3.7 \times 10^{10})}{21 \times 2.832 \times 10^4} = 8.15 \times 10^6 \frac{\text{photons}}{\text{sec-cm}^3}$$

Table I-C shows the tabular calculation of the gamma attenuation coefficients for the two gamma rays considered. The relaxation lengths for the two gamma rays in the homogenized radiator are then:

3. Samuel Glasstone, Principles of Nuclear Reactor Engineering, D. Van Nostrand Co., New York, p. 603.

Fig. 2-C

Induced Activity Reactions of NaK

TABLE I-C

ATTENUATION COEFFICIENTS FOR ACTIVATED NaK GAMMA RAYS IN NaK-AIR RADIATORS

ELEMENT	WT IN RADIATOR	ρ_{APPARENT}	$(\mu/\rho)_{2.75}^4$	$\mu_{2.75}$	$(\mu/\rho)_{1.37}^4$	$\mu_{1.37}$
----	lbs	gm/cm ³	cm ² /gm	cm ⁻¹	----	---
Fe	1081	0.825	.0374	.0308	.0490	.0404
Ni	357	0.272	.0376	.0102	.0505	.0138
Cr	311	0.237	.0360	.0085	.0480	.0113
Na	79	0.060	.0364	.0022	.0498	.0030
K	62	0.047	.0376	.0018	.0510	.0024

4. Theodore Rockwell III, Reactor Shielding Design Manual, USAEC, Washington, p. 447 and p. 462

$$\lambda_c = \frac{1}{\mu_c} = \frac{1}{.0535} = 18.7 \text{ cm} \quad (2.76 \text{ Mev})$$

$$\lambda_c = \frac{1}{\mu_c} = \frac{1}{.0709} = 13.8 \text{ cm} \quad (1.38 \text{ Mev})$$

The equivalent surface source strength is approximated by:

$$S_{eq} = 8.15 \times 10^6 \times 18.7 = 1.525 \times 10^8 \frac{\text{photons}}{\text{sec-cm}^2} \quad (2.76 \text{ Mev})$$

$$S_{eq} = 8.15 \times 10^6 \times 13.8 = 1.125 \times 10^8 \frac{\text{photons}}{\text{sec-cm}^2} \quad (1.38 \text{ Mev})$$

The scattered radiation was neglected as being small compared to the direct dose rate in the cockpit. Due to the small frontal area of the radiator compared to the distance between the radiator and the cockpit, the radiation from the front of the radiator was approximated by a point isotropic source of radiation.

$$S = S_{eq} A_f$$

where, S = strength of equivalent isotropic point source, photons/sec

A_f = frontal area of radiator (headers and matrix), ft².

For the two gamma rays we get:

$$S = (1.525 \times 10^8)(11.1)(929) = 1.57 \times 10^{12} \frac{\text{photons}}{\text{sec}} \quad (2.76 \text{ Mev})$$

$$S = (1.125 \times 10^8)(11.1)(929) = 1.16 \times 10^{12} \frac{\text{photons}}{\text{sec}} \quad (1.38 \text{ Mev})$$

Before reaching the cockpit the radiation from the radiator will be attenuated by passing through the compressor section of the gas generator. The radiation from the center portion of the radiator will be strongly attenuated by the dense compressor rotor, but the radiation from the outer edges of the radiator and headers will be attenuated only by the compressor blades and casing. As an approximation it was assumed that all the radiation in the forward direction was attenuated by one inch of iron. The radiation dose rate at the cockpit, 50 feet in front of the radiator, was calculated by:⁵

$$D^0 = 5.765 \times 10^{-5} \frac{S}{4 \pi R^2} E_r (\mu_t / \rho) e^{-\mu t}$$

5. E. P. Blizard, Nuclear Radiation Shielding, ORNL, 1956 (to be published in McGraw-Hill Nuclear Reactor Handbook)

where, D = dose rate in tissue due to unscattered photons, rads/hr.

S = point isotropic source strength, photons/sec

R = distance from radiator to cockpit, cm.

E_γ = photon energy of source, Mev.

(μ_t/ρ) = energy-absorption mass attenuation coefficient for the source energy in tissue, cm^2/gm

μ = total mass attenuation coefficient for iron, cm^{-1}

t = thickness of iron through which radiation is attenuated, cm

For the two gamma rays we get:

$$D^0 = 5.767 \times 10^{-5} \times \frac{1.57 \times 10^{12}}{4 \pi (1525)^2} (2.76)(.0229) e^{-.0375 \times 7.85 \times 2.54}$$

$$D^0 = .0924 \text{ rad/hr} \quad (2.76 \text{ Mev})$$

$$D^0 = 5.765 \times 10^{-5} \times \frac{1.16 \times 10^{12}}{4 \pi (1525)^2} (1.37)(.0284) e^{-.0512 \times 7.85 \times 2.54}$$

$$D^0 = .0130 \text{ rad/hr} \quad (1.37 \text{ Mev})$$

The direct dose rate is then 0.1244 rad per hour for both gamma rays.

Considering the relative biological effectiveness, RBE, of gamma rays as one, dose rate, D , in rad equivalent man units, Rem, is:

$$D = D^0 (\text{RBE}) = .1244 \text{ Rem/hr}$$

From all four radiators the total direct dose rate is 0.4976 Rem per hour.

APPENDIX D

PRELIMINARY DESIGN OF TURBO MACHINERY

General

Preliminary designs of the compressor, compressor turbine, and free turbine were made in order to predict size of the components and permit possible comparison with existing turbo machinery components.

Detailed cycle calculations were performed, based on cycle design parameters as determined from the weight optimization studies. Station nomenclature is defined in Fig. 1-D.

Cycle Calculations

Free Turbine

From the weight optimization studies, duct discharge air (at the entrance to the free turbine scroll) was assumed to exist at the following condition:

$$P_1 = 15,837 \text{ lb. per sq. ft.}$$

$$T_1 = 1906 \text{ degrees R.}$$

A constant axial velocity of 600 feet per second was assumed. Turbine exhaust was assumed to have no tangential component, and was to be diffused through the exhaust nozzle to 300 feet per second. The design operating condition was selected to be sea level static. Turbine efficiency was assumed to be 0.88.

The turbine was designed to expand the air to a total pressure such that stream pressure in the exhaust discharge was just equal to sea level ambient pressure. Based on these assumptions, the total enthalpy drop through the turbine was calculated as described below.

1) A plot of total pressure versus total temperature at the exhaust discharge was made for the requirement that exhaust stream pressure, P_5 , was equal to ambient pressure, P_0 . Exhaust requirement calculations were made as follows:

FIG. 1-D

STATION NOMENCLATURE

- A) Assume values of T_5 .
- B) Determine corresponding total enthalpy, H_5 , and reduced pressure, P_{r5}] _t , from Gas Tables.
- C) Calculate corresponding stream enthalpy from:

$$h_5 = H_5 - C_e^2 / 2gJ$$
- D) Determine corresponding stream temperature, t_5 , and reduced pressure, P_{r5}] _s , from Gas Tables.
- E) Calculate total pressure at nozzle discharge:

$$P_5 = P_0 \frac{P_{r5}]_t}{P_{r5}]_s}$$
- F) Plot P_5 versus T_5 for the exhaust requirement, as displayed in Fig. 2-D.

2) A plot of total pressure, P_h , versus total temperature, T_h , was then made for expansion to assumed values of turbine discharge total pressure, P_h . Calculations were based on the assumed turbine efficiency of 0.88, and were made as follows:

- A) Assume values of P_h .
- B) Calculate the reduced pressure, P_{rh}] _t , from:

$$P_{rh}]_t = P_{r1}]_t \frac{P_h}{P_1}$$
- C) Determine isentropic enthalpy, H'_h , and the isentropic enthalpy change, $\Delta H'_T$, from the Gas Tables.
- D) Correct isentropic enthalpy change for turbine efficiency:

$$\Delta H_T = \eta_T \Delta H'_T$$
- E) Calculate total enthalpy at turbine discharge:

$$H_h = H_1 - \Delta H_T$$
- F) Determine the corresponding values of T_h from the Gas Tables. Plot P_h versus T_h for the free turbine, as shown in Fig. 2-D.

FIG. 2-D

DETERMINATION OF TURBINE DISCHARGE CONDITIONS

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Assuming negligible loss in the exhaust nozzle, the intersection of the exhaust and free turbine curves in Fig. 2-D determined the total head state of the air at turbine discharge.

$$\text{Thus: } T_h = 1231 \text{ degrees R.}$$

$$H_h = 299.22 \text{ Btu/lb}$$

$$\Delta H_T = 179.53 \text{ Btu/lb.}$$

Results of the free turbine cycle calculations are summarized in Table I-D

Compressor Turbine

Change in total enthalpy across the compressor turbine was calculated as for the free turbine, with the single exception that the compressor turbine inlet conditions were assumed, in accordance with the weight optimization studies to be:

$$P_1 = 16,023 \text{ lb. per sq. ft}$$

$$T_1 = 1906 \text{ degrees R}$$

The value of total temperature at the compressor turbine discharge was determined from the intersection of the compressor turbine and exhaust curves in Fig. 2-D, as before.

$$\text{Thus: } T_h = 1224 \text{ degrees R}$$

$$H_h = 297.43 \text{ Btu/lb}$$

$$\Delta H_{CT} = 181.32 \text{ Btu/lb}$$

Results are summarized in Table I-D.

Compressor

From the weight optimization studies, a total pressure ratio of eight to one was assumed. Inlet conditions, based on negligible inlet duct losses, were:

$$P_1 = 2116 \text{ lb. per sq. ft.}$$

$$T_1 = 519 \text{ degrees R}$$

TABLE I-D
SUMMARY OF CYCLE CALCULATIONS

QUANTITY	SYMBOL	FREE TURBINE STATION					COMPRESSOR TURBINE STATION					COMPRESSOR STATION			
		1	2	3	4	5	1	2	3	4	5	1	2	12	13
Total enthalpy, Btu/lb	H	478.75	418.91	359.07	299.22	299.22	478.75	418.31	357.87	297.43	297.43	124.03	133.80	231.55	241.32
Total pressure, lb/ft ²	P	15837	8950	4652	2155	2155	16023	9040	4665	2156	2156	2116	2762	14890	16928
Total temp., degrees R	T	1906.0	1686.4	1461.6	1231.0	1231.0	1906.0	1684.1	1457.1	1224.0	1224.0	519.0	559.8	962.0	1001.4
Change in total enthalpy across machine, Btu/lb	H	179.53					181.32					117.29			
Change in total enthalpy across single stage, Btu/lb	H _s	59.843					60.44					9.774			
Stage efficiency	s	0.861					0.862					0.880			
Isentropic change in total enthalpy across single stage, Btu/lb	H' _s	69.501					70.00					8.601			
Air mass flow rate, lbs/sec	w	21.62					46.14					71.33			
Air fraction	w/v _c	0.3031					.6469					1.000			

The reduced pressure at discharge was computed from the assumed pressure ratio, and total enthalpy at the compressor discharge was determined from the Gas Tables. Isentropic enthalpy rise across the compressor was then computed and corrected for an assumed compressor efficiency of 0.86 to obtain actual enthalpy rise across the compressor:

$$\Delta H_C = \frac{\Delta H'_C}{\eta_C}$$

$$\Delta H_C = 117.29 \text{ Btu/lb}$$

Results of the compressor calculations are summarized in Table I-D.

Air Mass Flow Rates

From values of enthalpy changes through the compressor and compressor turbine, the ratio of compressor turbine air to compressor air was calculated, based on the requirement that compressor work input must equal turbine work output:

$$\text{Compressor work} = \text{Turbine work}$$

$$w_C \Delta H_C = w_{CT} \Delta H_{CT}$$

$$\text{Thus: } \frac{w_{CT}}{w_C} = \frac{\Delta H_C}{\Delta H_{CT}} = \frac{117.29}{181.32} = 0.6469$$

From continuity requirements, the compressor mass flow rate must equal the sum of the free turbine, compressor turbine, and auxiliary bleed mass flow rates.

$$\text{Thus: } w_C = w_{CT} + w_T + w_a$$

$$\text{or: } \frac{w_T}{w_C} = 1 - \frac{w_{CT}}{w_C} - \frac{w_a}{w_C}$$

$$\text{and: } F_p = 1 - \frac{w_{CT}}{w_C} - F_a$$

Assuming the auxiliary bleed fraction, $\frac{w_a}{w_c}$, to be 0.05, as in the weight optimization studies, the free turbine bleed fraction, F_b , was calculated to be 0.3031.

The air mass flow rate through the free turbine necessary to produce 5500 shaft horsepower at sea level static conditions was then computed from the relation:

$$w_T = \frac{SHP}{1.415 \frac{HP}{Btu/sec} \times \Delta H_T (Btu/lb)}$$

$$\text{Thus: } w_T = \frac{5500}{1.415 \times 179.53} = 21.62 \frac{lb \text{ air}}{sec}$$

Corresponding mass flow rates through the compressor and compressor turbine could then be computed:

$$w_C = \frac{w_T}{(w_T/w_C)}$$

$$w_C = \frac{21.62}{0.3031} = 71.33 \frac{lb \text{ air}}{sec}$$

$$w_a = 0.05 w_C$$

$$w_a = 0.05 \times 71.33 = 3.57 \frac{lb. \text{ air}}{sec}$$

$$w_{CT} = w_C - w_a - w_T$$

$$w_{CT} = 71.33 - 3.57 - 21.62 = 46.14 \frac{lb \text{ air}}{sec}$$

Results are summarized in Table I-D.

Compressor Design

Having established the mass flow rate, the total temperature and pressure at the compressor inlet, and total temperature and pressure at discharge, a preliminary compressor design was made.

After preliminary investigation, a twelve stage axial compressor was

selected. An axial air velocity of 600 feet per second was selected for the first stage, diffusing to 400 feet per second at the compressor discharge, and 417 feet per second at entry to the twelfth stage. A constant tip speed of 1100 feet per second was selected, with a hub to tip ratio of 0.60 in the first stage. Free vortex flow was assumed.

From conservation of energy, mass continuity, and Newton's Law as applied to angular momentum,¹

$$\Delta H_S \times J = \frac{1}{g} \left[C_{U_2} U_2 - C_{U_1} U_1 \right]$$

Assuming a constant radius of any stream surface through each stage (two dimensional flow):

$$U_1 = U_2 = U$$

$$\text{Thus: } g J \Delta H_S = U \left[C_{U_2} - C_{U_1} \right]$$

Since the velocity relative to the rotor blade is the vector difference of the rotor vector velocity and the absolute air vector velocity:

$$\vec{W} = \vec{C} - \vec{U} \quad (\text{See Fig 3-D})$$

$$W_U = C_U - U$$

$$W_{U_2} - W_{U_1} = C_{U_2} - C_{U_1}$$

$$g J \Delta H_S = \frac{U}{2} \left[(C_{U_2} - C_{U_1}) (W_{U_2} - W_{U_1}) \right]$$

$$g J \Delta H_S = \frac{1}{2} \left[(C_{U_2} - W_{U_2})(C_{U_2} + W_{U_2}) - (C_{U_1} - W_{U_1})(C_{U_1} + W_{U_1}) \right]$$

$$\text{Thus: } g J \Delta H_S = \frac{1}{2} \left[C_{U_2}^2 - C_{U_1}^2 + W_{U_1}^2 - W_{U_2}^2 \right]$$

$$\text{Or: } g J \Delta H_S = \frac{1}{2} \left[\underbrace{C_2^2 - C_1^2}_{\Delta h \text{ across stator}} + \underbrace{W_1^2 - W_2^2}_{\Delta h \text{ across rotor}} \right]$$

Δh across
stator

Δh across
rotor

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FIG. 3-D
COMPRESSOR VECTOR DIAGRAM NOMENCLATURE

Degree of reaction is defined as the ratio of enthalpy gain across the rotor to enthalpy gain across the stage:

$$\gamma = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} = \frac{W_1^2 - W_2^2}{(W_1^2 - W_2^2) + (C_2^2 - C_1^2)}$$

or

$$\gamma = \frac{(W_{U1} - W_{U2})(W_{U1} + W_{U2})}{(C_{U1} - C_{U2}) [(W_{U1} + W_{U2}) - (C_{U1} - C_{U2})]}$$

and:

$$\gamma = \frac{W_{U1} + W_{U2}}{2U} = \frac{W_{U \text{ avg}}}{U}$$

The requirement of radial balance of acceleration and pressure forces is satisfied by free vortex pattern of rotational component with a constant axial velocity component along the blade span. This pattern was chosen for simplicity.

Pressure rise coefficient, $\Delta p/q$, was taken as a design criteria with the requirement that in no case should $\Delta p/q$ exceed³ 0.60. Pressure rise coefficient was defined as the rise in stream pressure across the rotor or stator, divided by the dynamic head based on entry relative velocity.

$$\begin{aligned} \text{Thus: } \left[\frac{\Delta p}{q} \right]_{\text{rotor}} &= \frac{\frac{\rho}{2} (W_1^2 - W_2^2)}{\frac{\rho}{2} (W_1^2)} = \frac{W_1^2 - W_2^2}{W_1^2} \\ \left[\frac{\Delta p}{q} \right]_{\text{stator}} &= \frac{\frac{\rho}{2} (C_2^2 - C_1^2)}{\frac{\rho}{2} (C_2^2)} = \frac{C_2^2 - C_1^2}{C_2^2} \end{aligned}$$

From the above relations, velocity diagrams, pressure rise coefficients and relative entry Mach numbers were calculated, using various assumed values for the degree of reaction at the arithmetic mean blade diameter.

2. IBID, p. 366

3. Seymour Leiblien, Francis C. Schwenk, and Robert L. Broderick, Diffusion Factor For Estimating Losses and Limiting Blade Loadings In Axial Flow Compressor Blade Elements, NACA RM E53D01, June 8, 1953, p. 34, Fig. 3

Calculations for the compressor first stage are summarized in Table II-D and results are presented in Fig. 4-D.

From Fig. 4-D, it was seen that the largest value of $\Delta P/q$, occurring at the stator hub, was nearly independent of degree of reaction. A 45 per cent degree of reaction at the mean diameter was selected to minimize entry Mach numbers, favoring the rotor slightly.

Having selected the first stage vector diagram, the absolute entry velocity to the first stage at the mean diameter was used to calculate the stream pressure at station (1), following the pre-whirl vanes.

$$h_1 = H_1 - \frac{C_1^2}{2gJ}$$

$$h_1 = 124.03 - \frac{48.0 \times 10^4}{2gJ} = 115.63 \quad \frac{\text{Btu}}{\text{lb}}$$

From the Gas Tables:

$$t_1 = 483.9 \text{ deg. R.}$$

$$p_{r1}]_s = 0.9446$$

$$p_1 = p_1 \frac{p_{r1}]_s}{p_{r1}]_t}$$

$$p_1 = 2116 \frac{0.9446}{1.2065} = 1658 \text{ lb/ft}^2$$

Thus:

$$\rho_1 = \frac{p_1}{R t_1}$$

$$\rho_1 = \frac{1658}{53.34 \times 483.9} = 0.0644 \text{ lb/ft}^3$$

The volumetric flow rate was then calculated for the first stage:

$$Q_1 = \frac{w_c}{\rho_1}$$

$$Q_1 = \frac{71.33}{0.0644} = 1109 \frac{\text{ft}^3}{\text{sec}}$$

TABLE II-D

INTERNAL FLOW CONDITIONS COMPRESSOR FIRST STAGE

QUANTITY/RADIAL STATION	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	HUB	TIP
U , ft/sec	880	660	1100	880	660	1100	880	660	1100	880	660	1100	880	660	1100
$C_{U_2} - C_{U_1}$ ft/sec	277	369	221	277	369	221	277	369	221	277	369	221	277	369	221
Degree of Reaction	0.40	--	--	0.45	--	--	0.50	--	--	0.60	--	--	0.70	--	--
$W_{U_{avg}}$ ft/sec	-352	--	--	-396	--	--	-440	--	--	-538	--	--	616	--	--
W_{U_1} ft/sec	-485	-133	-783	-534	-199	-823	-578	-257	-858	-666	-375	-928	-754	-492	-999
W_{U_2} ft/sec	-217	227	-560	-258	169	-603	-302	112	-638	-390	-5	-710	-478	-124	-779
W_1^2 (ft/sec) ² x 10 ⁻⁴	59.4	37.7	97.2	64.5	39.9	103.8	69.5	42.6	109.4	80.3	50.1	122.0	92.8	59.2	135.6
W_2^2 (ft/sec) ² x 10 ⁻⁴	40.7	41.1	67.4	42.7	38.9	72.4	45.2	36.3	76.6	51.2	36.0	86.1	58.9	37.5	96.9
$\Delta P/q \int_r$	0.315	-0.090	0.323	0.338	0.026	0.303	0.349	0.124	0.300	0.363	0.282	0.294	0.365	0.367	0.286
C_{U_1} ft/sec	395	527	317	346	461	277	302	403	242	214	285	172	126	168	101
C_{U_2} ft/sec	663	887	530	622	829	497	578	772	462	490	655	392	402	536	321
C_1^2 (ft/sec) ² x 10 ⁻⁴	51.6	63.8	46.0	48.0	57.2	43.7	45.1	52.2	41.8	40.6	44.1	39.0	37.6	37.0	38.8
C_2^2 (ft/sec) ² x 10 ⁻⁴	79.9	114.5	64.1	74.6	104.5	60.6	69.6	95.5	57.2	60.0	78.8	51.4	52.1	64.7	46.3
$\Delta P/q \int_s$	0.354	0.443	0.282	0.357	0.452	0.278	0.352	0.453	0.269	0.323	0.440	0.241	0.278	0.201	0.279
W_1 ft/sec	--	--	988	--	--	1018	--	--	1048	--	--	1108	--	--	1161
h_1 Btu/lb	--	--	114.63	--	--	115.29	--	--	115.67	--	--	116.23	--	--	116.47
t_1 deg R	--	--	481	--	--	483	--	--	484	--	--	486	--	--	488
C_{s_1} ft/sec	--	--	1061	--	--	1063	--	--	1064	--	--	1066	--	--	1068
M_1	--	--	0.930	--	--	0.957	--	--	0.975	--	--	1.04	--	--	1.09

TABLE II-D (Con't)

C_2	ft/sec	--	1071	--	--	1021	--	--	979	--	--	889	--	--	805	--
h_2	Btu/lb	--	99.13	--	--	103.13	--	--	--	--	--	--	--	--	--	--
t_2	deg R	--	423	--	--	432	--	--	439	--	--	453	--	--	465	--
C_{S_2}	ft/sec	--	1010	--	--	1018	--	--	1027	--	--	1044	--	--	1057	
M_2		--	1.061	--	--	1.005	--	--	0.952	--	--	0.869	--	--	0.761	--

VECTOR DIAGRAMS

PRESSURE RISE COEFFICIENT - $\Delta p/q$

Annulus area, A_1 , was then calculated from the volumetric flow rate, allowing for a five per cent increase in area to accommodate the boundary layer:

$$A_1 = 1.05 \frac{Q_1}{C_{a_1}}$$

$$A_1 = 1.05 \frac{1100}{600} = 1.940 \text{ ft}^2$$

Tip diameter was then computed for the first stage based on the annulus area and the assumed hub to tip ratio of 0.60:

$$D_{t_1} = \sqrt{\frac{4A_1}{\pi[1 - (h/t)^2]}}$$

$$D_{t_1} = \sqrt{\frac{4 \times 1.940}{\pi[1 - (0.6)^2]}}$$

$$D_{t_1} = 1.965 \text{ ft}$$

Also: $D_h = \left(\frac{h}{t}\right) D_t$

$$D_h = 0.6 \times 1.965 = 1.179 \text{ ft}$$

From the assumed tip speed of 1100 feet per second, the rotational speed was calculated:

$$N = \frac{U_t}{\pi D_t}$$

$$N = \frac{1100}{\pi \times 1.965} = 178.1 \text{ rev per sec}$$

Compressor annulus area calculations are summarized in Table III-D.

For purposes of this study, intermediate stages were not investigated. The twelfth stage was analyzed in a manner similar to the first stage. Total enthalpy at entry to the twelfth stage was calculated from:

$$H_{12} = H_{13} - 4 H_s$$

Isentropic enthalpy rise was computed from the relation:

$$\Delta H'_s = \eta_s \Delta H_s$$

Stage efficiency was not calculated, but experience has shown that a stage efficiency of approximately 0.88 will compensate for reheat effects and result in a compressor efficiency of 0.86.⁴

$$\text{Thus: } H'_{13} = H_{12} + \Delta H'_s$$

$$\text{and: } P_{12} = P_{13} \times \frac{P_{r12}}{P_{r13}}$$

The above calculations are summarized in Table I-D.

Assuming an absolute velocity at entry to the twelfth stage, stream enthalpy with corresponding stream pressure and temperature were computed as before, using the Gas Tables. From the corresponding density, volumetric flow rate and annulus area of the twelfth stage were computed, as were the hub and mean diameters. Vector diagrams and pressure rise coefficients were calculated, and a degree of reaction selected. From the vector diagram, the absolute entry velocity at the mean diameter was determined, and the above series of computations were iterated.

Results of twelfth stage calculations are summarized in Tables III-D and IV-D, and displayed in Fig. 5-D.

With the aid of Fig. 5-D, a degree of reaction of 0.40 at the mean diameter was selected for the twelfth stage. Relative Mach numbers were subsonic in this stage.

Compressor vector diagrams for the two stages are displayed in Fig. 6-D. With the aid of the vector diagrams, the annulus dimensions, and an assumed blade aspect ratio of 2.5 in the first stage, the compressor was laid out in Fig. 7-D. Blade chords were held constant in the radial direction, and were identical for rotors and stators of all stages. Blades were laid out for the first four stages on the basis of the first stage vector diagrams, while the remaining stages were laid out on the basis of the twelfth stage vector diagrams, decreasing axial velocity by equal increments per stage. The axial plane projections of the chords at the hub, mean, and tip diameters

4. W. T. Furgerson, Interview, July 18, 1957

TABLE III-D
ANNULUS AREA CALCULATIONS

		Total enthalpy BTU/lb	Total pressure lb/r ²	(Absolute velocity) ² x 10	Stream enthalpy Btu/lb	Stream Temp. Deg R	Stream Pressure lb/r ²	Density lb/r ²	Volumetric flow rate ft ³ /sec	Annulus Area ft ²	Tip Dia. Ft.	Hub Dia. Ft.	Hub to Tip Ratio
TURBO-COMPONENT	STATION	H	P	C ₁ ²	h	t	D	ρ	Q	A	D _t	D _h	h/g
Compressor 10700 RPM	1	124.03	2116	48.0	115.63	483.9	1658	0.0644	1109	1.940	1.965	1.179	0.60
	2	133.80	2762	48.0	124.60	521.4	2195	0.0790	903	1.629	1.965	1.330	---
	12	231.55	14890	42.5	223.05	927.6	13050	0.264	270	0.6799	1.965	1.731	0.88
	13	241.32	16928	42.5	232.82	967.1	14930	0.291	245	0.644	1.965	1.740	---
	14	241.32	16928	16	238.12	988.5	16150	0.307	232	0.609	1.965	1.755	---
Compressor Turbine 10700 RPM	1	478.75	16023	36.0	471.55	1879.8	15160	0.1510	305	0.534	1.781	1.580	0.886
	2	418.31	9040	36.0	411.11	1657.3	8480	0.0960	481	0.841	1.890	1.580	0.836
	3	357.87	4665	36.0	350.67	1429.7	4340	0.0569	811	1.420	2.015	1.500	0.745
	4	297.43	2156	36.0	290.23	1195.8	1972	0.0309	1493	2.615	2.107	1.054	0.500
	5	297.43	2156	9.0	295.63	1216.9	2116	0.0326	1418	4.960	2.518	0	--
Free Turbine 15000 RPM	1	478.75	15837	36.0	471.55	1879.8	15000	0.1498	144.5	0.253	1.209	1.067	0.882
	2	418.91	8950	36.0	411.71	1659.6	8400	0.0952	227.8	0.398	1.282	1.067	0.831
	3	359.07	4652	36.0	351.87	1434.3	4360	0.0570	380	0.665	1.342	0.978	0.728
	4	299.22	2155	36.0	292.02	1202.8	1979	0.0308	704	1.231	1.445	0.723	0.500
	5	299.22	2155	9.0	297.42	1224.0	2116	0.0324	669	2.340	1.708	0	--

TABLE IV-D

INTERNAL FLOW CONDITIONS CO PRESSOR TWELFTH STAGE

QUANTITY/STATION	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	HUB	TIP	MEAN	TIP	HUB
U ft/sec	1034	968	1100	1034	968	1100	1034	968	1100	1034	968	1100	1034	968	1100
$C_{U_2} - C_{U_1}$ ft/sec	237	253	222	237	253	222	237	253	222	237	253	222	237	253	222
Degree of Reaction,	0.35	--	--	0.40	--	--	0.45	--	--	0.50	--	--	0.55	--	--
$W_{U_{avg}}$ ft/sec	-361	--	--	-414	--	--	-465	--	--	-517	--	--	-569	--	--
W_{U_1} ft/sec	-480	-376	-579	-533	-433	-629	-584	-487	-677	-636	-543	-726	-688	-599	-775
W_{U_2} ft/sec	-242	-123	-356	-295	-179	-406	-346	-233	-454	-398	-288	-502	-450	-344	-552
W_1^2 (ft/sec) ² x 10 ⁻⁴	40.4	31.5	50.9	45.8	36.2	56.9	51.4	41.1	63.2	57.9	46.9	70.2	64.8	53.3	77.4
W_2^2 (ft/sec) ² x 10 ⁻⁴	23.3	18.9	30.1	26.1	20.6	33.9	29.3	22.8	38.0	33.2	25.7	42.6	37.7	29.2	47.9
$\Delta P/q \big]_r$	0.424	0.400	0.410	0.430	0.431	0.405	0.430	0.445	0.399	0.427	0.453	0.394	0.419	0.453	0.381
C_{U_1} ft/sec	554	592	521	501	535	471	450	481	423	398	425	374	346	369	325
C_{U_2} ft/sec	792	845	744	739	789	694	688	735	646	636	680	598	584	624	548
C_1^2 (ft/sec) ²	48.0	52.4	44.5	42.5	46.0	39.6	37.7	40.5	35.3	33.2	35.5	31.4	29.3	31.0	28.0
C_2^2 (ft/sec) ²	80.2	88.9	72.6	72.0	79.5	65.5	64.8	71.4	59.2	57.9	63.5	53.1	51.5	56.3	47.4
$\Delta P/q \big]_s$	0.402	0.411	0.387	0.410	0.422	0.396	0.418	0.433	0.404	0.428	0.442	0.408	0.431	0.449	0.410
W_1			711			754			796			839			880
C_2		946			891			851			796			765	

FIG. 5-D

VARIATION OF INTERNAL FLOW CONDITIONS WITH DEGREE OF REACTION
COMPRESSOR TWELFTH STAGE

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FIG. 6-D

COMPRESSOR VECTOR DIAGRAMS

FIRST STAGE

TWELFTH STAGE

TIP

MEAN

HUB

FIG. 7-D
COMPRESSOR LAYOUT

were computed from the relations:

$$\begin{aligned} \text{rotor: } c_{\text{proj}} &= c \left(\frac{c_a}{\sqrt{c_a^2 + W_{U_{\text{avg}}}^2}} \right) \\ \text{stator: } c_{\text{proj}} &= c \left(\frac{c_a}{\sqrt{c_a^2 + c_{u_{\text{avg}}}^2}} \right) \end{aligned}$$

Table V-D summarizes axial plane projections of the rotor and stator blade chords.

Compressor Turbine Design

Preliminary calculations indicated that excessive Mach numbers would occur in a two stage turbine design. Accordingly, a three stage design was selected, with each stage taking one third the change in total enthalpy across the turbine.

$$\text{Thus: } \Delta H_s = 59.843 \text{ Btu/lb}$$

The stage efficiency differed from turbine efficiency as a result of reheat effects. The stage efficiency corresponding to a turbine efficiency of 0.88 was calculated by iteration, using the Gas Tables:

$$\eta_s = 0.862$$

Utilizing these efficiencies, total enthalpies and total pressures were calculated for stations (1), (2), (3), (4), and (5), as summarized in Table I-D. Stream enthalpies were then calculated from the relation:

$$h = H - \frac{c_a^2}{2gJ}$$

Corresponding stream pressures and temperatures were then determined using the Gas Tables. These calculations ignored any effects on density resulting from tangential velocity components since these will be essentially constant from stage to stage. Densities were computed from stream pressures and temperatures, and used in the calculation of volumetric flow rates and annulus areas at the various stations, using the following relations:

TABLE V-D

COMPRESSOR

CHORD PROJECTIONS,

$s = 0.393 \text{ ft}$

$AR_1 = 2.5$

$c = 0.1572 \text{ ft}$

AXIAL STATION	ROTOR OR STATOR	MEAN	TIP	HUB
1	R	.1325	.1012	.1572
	S	.1222	.1320	.1078
2	R	.1304	.0998	.1572
	S	.1212	.1311	.1059
3	R	.1292	.0982	.1572
	S	.1197	.1300	.1039
4	R	.1277	.0964	.1572
	S	.1182	.1288	.1023
5	R	.1244	.1128	.1366
	S	.1026	.1078	.0986
6	R	.1227	.1112	.1353
	S	.1007	.1058	.0967
7	R	.1212	.1093	.1340
	S	.0988	.1040	.0950
8	R	.1192	.1072	.1329
	S	.0967	.1020	.0927
9	R	.1178	.1052	.1320
	S	.0945	.1002	.0909
10	R	.1160	.1031	.1301
	S	.0924	.0978	.0834
11	R	.1147	.1014	.1281
	S	.0903	.0957	.0862
12	R	.1119	.0987	.1259
	S	.0878	.0933	.0839

$$Q = \frac{w_t}{\rho}$$

$$A = 1.05 \frac{Q}{C_a}$$

The indicated increase of five per cent of the annulus area was included to accommodate the boundary layer. Results are tabulated in Table III-D.

Fig. 8-D⁵ is a plot of head coefficient, ψ , versus flow coefficient ϕ , with regions of good turbine design delineated as indicated. Coefficients were defined as follows:

$$\text{Flow coefficient: } \phi = \frac{C_a}{U}$$

$$\text{Head coefficient: } \psi = \frac{g J \Delta H}{U^2} = \frac{C_{U_1} - C_{U_2}}{U}$$

Variations from a constant radius stream surface were ignored in using Fig. 8-D as a design criterion.

From the design rotational speed of 10,700 revolutions per minute, as fixed by the compressor, the tip diameter at station (1) was calculated for an assumed value of tip speed. The hub diameter necessary to result in the required annulus area was then calculated. Head and flow coefficients were then computed for hub, tip and mean diameters, and plotted in Fig. 8-D. The assumed value of tip speed was then adjusted to permit the plot of head and flow coefficients to fall within the region of good design, at the same time maintaining the hub to tip ratio as small as possible in order to reduce the percentage tip leakage losses. Final selected values are tabulated in Tables III-D and VI-D.

By a similar procedure, hub to tip diameters were selected for the inlets to the second and third stages. Results are tabulated in Tables III-D and VI-D and plotted in Fig. 8-D. Hub and tip diameters were then selected for station (4) as necessary to result in the required annulus area. The hub to tip ratio at station (4) was maintained at no less than 0.500 to help prevent excessive blade stresses.

Fig. 8-D

REGIONS OF BEST DESIGN
FOR TURBINES

(DATA FROM BUSHIPS 338, VOL. 10, p. 45)

TABLE VI-D

TURBINE INTERNAL FLOW CONDITIONS

COMPRESSOR TURBINE

AXIAL STATION	RADIAL STATION	BLADE SPEED U ft/sec	FLOW COEFFICIENT ϕ	HEAD COEFFICIENT ψ	DIAMETER D ft	HUB TO TIP RATIO h/t
1	Tip	1000	0.600	1.511	1.781	0.886
1	Hub	886	0.677	1.928	1.580	
2	Tip	1060	0.566	1.348	1.890	0.836
2	Hub	886	0.677	1.928	1.580	
3	Tip	1131	0.530	1.169	2.015	0.745
3	Hub	826	0.721	2.211	1.500	
4	Tip	--	--	--	2.107	0.500
4	Hub	--	--	--	1.054	
5	Tip	--	--	--	2.518	0
5	Hub	--	--	--	0	

FREE TURBINE

1	Tip	950	0.631	1.610	1.209	0.882
1	Hub	836	0.717	2.070	1.067	
2	Tip	1007	0.601	1.430	1.282	0.831
2	Hub	836	0.717	2.070	1.067	
3	Tip	1055	0.569	1.297	1.342	0.728
3	Hub	768	0.782	2.450	0.978	
4	Tip	--	--	--	1.445	0.500
4	Hub	--	--	--	0.723	
5	Tip	--	--	--	0	
5	Hub	--	--	--		

From the selected diameters and rotational speed, vector diagrams for the hub, mean, and tip diameters were constructed for the four stations, as calculated from the following relations:

$$\begin{aligned} U &= \pi N D, \quad (N \text{ in rev. per sec}) \\ c_{u2} &= 0, \quad (\text{axial entry to stator}) \\ c_a &= 600, \quad (\text{Constant through the turbine}) \\ \left[c_{u1} - c_{u2} \right] &= \frac{g J \Delta H_s}{U} = \psi U \\ c_u &= w_u + U \end{aligned}$$

Fig 9-D shows a typical turbine vector diagram. The above relationships maintain radial balance of forces by means of free vortex flow, since the product of the radius times the tangential component of absolute velocity remains constant over the length of the blades. Results of the vector diagram calculations are summarized in Table VI-D and presented in Fig. 10-D. The maximum entry relative Mach number, occurring at the third stage stator hub, was calculated to be 0.705.

With the aid of the vector diagrams, axial plane projections of the blade chords were estimated from the relations:

$$\begin{aligned} \text{rotor:} \quad c_{\text{proj}} &= c \frac{c_a}{\sqrt{c_a^2 + \left(\frac{w_{u1} + w_{u2}}{2} \right)^2}} \\ \text{stator:} \quad c_{\text{proj}} &= c \frac{c_a}{\sqrt{c_a^2 + \left(\frac{c_{u1}}{2} \right)^2}} \end{aligned}$$

Assuming an aspect ratio of 1.0 at first stage entry, a blade chord of 1.26 in was selected. This chord was maintained constant radially and for rotor and stator blades of all stages. Axial plane projections were calculated as summarized in Table VII-D, and the compressor turbine layout made as presented in Fig. 11-D.

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FIG. 9-D
TURBINE VECTOR DIAGRAM NOMENCLATURE

FIG. 10-D

COMPRESSOR TURBINE VECTOR DIAGRAMS

TIP

FIRST STAGE

HUB

TIP

SECOND STAGE

HUB

TIP

THIRD STAGE

HUB

TABLE VII-D

TURBINE AXIAL PLANE CHORD PROJECTIONS

COMPRESSOR TURBINE

$$C = 0.1005 \text{ ft}$$

$$S = 0.1005 \text{ ft}$$

$$AR_1 = 1.0$$

AXIAL STATION	RADIAL STATION	TIP	HUB
1 - Stator		0.0641	0.0689
1 - Rotor		0.0925	0.0990
2 - Stator		0.0642	0.0530
2 - Rotor		0.0865	0.9980
3 - Stator		0.0670	0.0546
3 - Rotor		0.0775	0.0987

FREE TURBINE

$$C = 0.1000 \text{ ft}$$

$$S = 0.071 \text{ ft}$$

$$AR_1 = 0.71$$

1 - Stator		0.0619	0.0572
1 - Rotor		0.0952	0.0998
2 - Stator		0.0643	0.0572
2 - Rotor		0.0887	0.0998
3 - Stator		0.0661	0.0540
3 - Rotor		0.0841	0.0962

FIG. 11-D
COMPRESSOR TURBINE LAYOUT

SCALE: 1 inch = 0.3 feet

Free Turbine Design

Preliminary free turbine design calculations were made in a manner similar to those for the compressor turbine. A rotational speed of 15,000 revolutions per minute was selected as being great enough to maintain hub to tip ratios within practical limits, yet low enough that undue reduction gear design problems should not occur.

Annulus area calculations are summarized in Table III-D. Calculations of vector diagrams and selection of head and flow coefficients are tabulated in Table VI-D. The vector diagrams are displayed in Fig. 12-D. Axial plane projections of the chords were calculated for an assumed uniform chord of 0.100 feet, as summarized in Table VII-D. From these values, the turbine layout was made, as presented in Fig. 13-D.

FIG. 12-D

FREE TURBINE VECTOR DIAGRAMS

FIG. 13-D
FREE TURBINE LAYOUT

SCALE: 1 inch = 0.3 feet

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