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Jinsuo Nie, Richard J. Morante, Manuel Miranda and Joseph Braverman

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Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

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ON THE CORRECT APPLICATION OF THE 100-40-40 RULE FOR COMBINING RESPONSES DUE TO THREE DIRECTIONS OF EARTHQUAKE LOADING

Jinsuo Nie, Richard J. Morante, Manuel J. Miranda, and Joseph I. Braverman
Energy Sciences & Technology Department
Brookhaven National Laboratory
Upton, NY 11973-5000, USA
Email: jnie@bnl.gov

ABSTRACT

The 100-40-40 rule is often used with the response spectrum analysis method to determine the maximum seismic responses from structural responses resulting from the three spatial earthquake components. This rule has been referenced in several recent Design Certification applications of nuclear power plants, and appears to be gaining in popularity. However, this rule is described differently in ASCE 4-98 and Regulatory Guide 1.92, consequently causing confusion on correct implementation of this rule in practice. The square root of the sum of the squares method is another acceptable spatial combination method and was used to justify the adequacy of the 100-40-40 rule during the development of the Regulatory Guide 1.92. The 100-40-40 rule, when applied correctly, is almost always conservative compared to the SRSS method, and is only slightly unconservative in rare cases. The purpose of this paper is to describe in detail the proper application of the 100-40-40 rule, as prescribed in ASCE 4-98 and in Regulatory Guide 1.92, and to clarify the confusion caused by the two different formats of this rule.

INTRODUCTION

The response spectrum analysis (RSA) method has been widely used in the seismic design/analysis of nuclear power plant structures, systems, and components in the United States. Historically, the most commonly applied method for combining structural responses resulting from the three spatial earthquake components, in order to obtain the maximum seismic responses for design, has been the square root of the sum of the squares (SRSS) method. Another method, commonly known as the 100-40-40 rule, has been referenced in several recent Design Certification applications, and appears to be gaining in popularity. Both methods are described in ASCE 4-98 “Seismic analysis of safety-related nuclear structures and commentary” [1], and are acceptable to the NRC if implemented in accordance with Revision 2 of Regulatory Guide (RG) 1.92 “Combining modal responses and spatial components in seismic response analysis” [2]. The application of the SRSS method is straightforward; however, these two documents prescribe the 100-40-40 rule in different formats.

Recent Design Certification applications that utilized the 100-40-40 rule have cited ASCE 4-98. However, in a number of cases, the technical review of these recent applications has raised the question whether the applicants have properly interpreted the 100-40-40 rule. Consequently, implementation of this rule has been a common subject of requests for additional information.

The 100-40-40 rule was originally recommended as an alternative to SRSS by Newmark [3] for spatial response combination. It was also observed in the same reference that the 100-40-40 rule is in general slightly conservative for most cases and its degree of conservatism is relatively small. There are discussions on whether the 100-40-40 rule is a reasonable method for spatial response combination compared to the SRSS method [4-7]. The conclusions of these discussions appeared to favor a differentiated treatment of collinear responses, and non-collinear and multiple responses. It should be pointed out that, although not prescribed in ASCE 4-98 and RG 1.92, there have been other methods developed for spatial response combination, some of which are more recent and potentially more advanced and accurate. For example, the original complete quadratic combination (CQC) rule [8, 9] for modal response combination has been extended to a new method called CQC3 [6, 10-15], to combine modal responses due to the three orthogonal earthquake components. These
more advanced combination methods are not evaluated in this paper.

The purpose of this paper is to describe in detail the proper application of the 100-40-40 rule, as prescribed in ASCE 4-98 and in RG 1.92, and to clarify the confusion caused by the two different formats of this rule.

TWO FORMATS OF THE 100-40-40 RULE

The ASCE 4-98 format considers all permutations of the responses resulting from the three directional seismic inputs. Assuming $R_x$, $R_y$, and $R_z$ are the maxima of one response (e.g. the axial force of a column) determined separately from the two horizontal and one vertical seismic input motions, the maximum response $R$ due to three seismic input motions can be found by all possible permutations (i.e. combinations as in ASCE4-98) of $R_x$, $R_y$, and $R_z$:

$$R = \pm [R_x \pm 0.4 R_y \pm 0.4 R_z], \text{ or}$$

$$\pm [R_y \pm 0.4 R_x \pm 0.4 R_z], \text{ or}$$

$$\pm [R_z \pm 0.4 R_x \pm 0.4 R_y].$$  

The underlying assumption is that when the maximum response from one earthquake component occurs, the responses from the other components are 40% of their corresponding maximum. It should be emphasized that Eq. 1 should be applied for one and only one response at a time. For multiple responses, such as the axial force and the moments of a column, Eq. 1 must be applied to each individual response separately to determine its maximum combined response.

ASCE 4-98 is specific on how to utilize the maximum responses in design if there are multiple response parameters. It requires the consideration of all possible combinations of the maximum responses, each of which is determined separately using Eq. 1. For example, if there are $M$ response parameters, all $2^M$ sets of response combinations shall be considered (i.e., combinations of all possible sign variations). An alternative method is also described in ASCE 4-98, which is less conservative than using all $2^M$ combinations in design calculation but is much more complicated. This step of combining different response parameters in design is the major step that is often missed in implementing the 100-40-40 rule.

In RG 1.92, the 100-40-40 rule is specified in a very different format compared to the ASCE 4-98 format. For the response of interest, the maximum responses $R_x$, $R_y$, and $R_z$ resulting from three earthquake components are sorted first, resulting in $|R_1| \geq |R_2| \geq |R_3|$. The maximum response $R$ due to three earthquake components can then be specified as:

$$R = |R_1| + 0.4|R_2| + 0.4|R_3|$$  

The responses in the RG 1.92 format are treated in the absolute sense, avoiding the multiple permutations as in Eq. 1. This format requires sorting the absolute values of the maximum responses; while the ASCE 4-98 format requires finding the maximum from the 24 permutations as shown in Eq. 1.

For design situations where multiple responses exist, the RG 1.92 does not specify how the individual maximum response $R$ is used. However, it is reasonable and practical to assume that the maximum responses should be used in similar ways as described in ASCE 4-98.

ASCE 4-98 also allows the 100-40-40 rule to be used with the time history method. RG 1.92 does not endorse the use of the 100-40-40 rule together with the time history analysis. It accepts the use of the SRSS method when each direction is analyzed separately. If statistical independence of the 3 input motions can be demonstrated, RG 1.92 also accepts the use of algebraic summation at each time step for determination of the maximum response. The algebraic summation is automatically achieved when three input motions are specified simultaneously in one dynamic analysis.

In several Design Certification applications, the applicant has indicated that the ASCE 4-98 format is used, and that it is equivalent to the RG 1.92 format. Although the equivalence of the two formats is mathematically obvious for cases of a single response, it is not so straightforward in achieving their equivalence for cases of multiple responses. The equivalence of the two formats in general requires some additional assumptions and often is implementation-dependent. Some clarifications are presented in the following paragraphs to establish the equivalence of these two formats.

For cases of a single response, the equivalence of the ASCE 4-98 format and the RG 1.92 format (i.e. ASCE 4-98 ⇔ RG 1.92) can be established using two implications as described in the following:

(1) ASCE 4-98 ⇒ RG 1.92

Since the 24 permutations in ASCE 4-98 are a complete enumeration, one of them must be the case of RG 1.92. The case corresponding to RG 1.92, should be the permutation that has all three response quantities as positive (with the help of sign flip) and the maximum of $|R_1|$, $|R_2|$, and $|R_3|$ is multiplied by 1.0. In implementing the RG 1.92 format, the ordering of $R_2$ and $R_3$ is not significant because both response quantities are multiplied by 0.4.

(2) RG 1.92 ⇒ ASCE 4-98

This implication is not automatically achievable without stating the underlying assumptions on how the combined response is used in design. These assumptions are not explicitly described in the two documents, but are easy to understand in a practical sense.
Assumption one: the positive $R$ in Eq. 2 is used as $\pm R$ in the appropriate design load combinations.

Assumption two: the extreme values of the seismic responses govern the design (with sign flips $\pm$). This assumption is implied in ASCE 4-98 by the use of $2^M$ sets of response combinations.

Assumption one indicates that the case of RG 1.92 covers the two extremes of the 24 possible combined responses generated by ASCE 4-98, because the former yields the positive maximum seismic response $R$. Assumption two states that any response between the two extremes ($\pm R$) does not govern the design. This assumption is understandable in most design situations. Under these two assumptions, the case of RG 1.92 implies a full cover of all possible 24 permutations in ASCE 4-98.

Implications (1) and (2) complete the proof of the equivalence of the two formats, for a single response.

For the case of multiple responses, the equivalence of the two methods requires that the maximum of each of the responses must be obtained separately from either Eq. 1 or Eq. 2. Since RG 1.92 does not specifically prescribe how the individually determined maximum responses shall be used, the authors conclude that the complete set of $2^M$ response combinations or the alternative method as described in ASCE 4-98 can be naturally assumed for RG 1.92. The mandatory requirement of response ordering in RG 1.92 automatically ensures that response parameters are treated separately. However, because the ASCE 4-98 format appears to resemble design load combinations, it can often lead to misapplications in which all responses in a structure are simultaneously treated using 24 design load combinations. This point will be demonstrated later in the discussion of a multiple response example.

Regarding the SRSS method, there is essentially no difference between ASCE 4-98 and RG 1.92. This method can be applied to each response separately or all responses simultaneously; both formats lead to the same maximum response parameters.

**COMPARISON TO SRSS**

The 100-40-40 rule is well recognized to be mostly conservative compared to the SRSS method. These two methods are herein compared in more detail. The RG 1.92 format is used here as the two formats of the 100-40-40 rule are equivalent. Without considering sign variations, the SRSS method can be written as:

$$R = \sqrt{R_1^2 + R_2^2 + R_3^2}$$  \hspace{1cm} (3)

Eqs. 2 and 3 can be further simplified without loss of generality to the following equations, because: (a) only the maximum response $|R_1|$ needs to be determined while $|R_3|$ and $|R_3|$ do not have to be ordered for Eq. 2 to be correct, (b) all responses can be assumed to be positive for ease of discussion, and (c) the direction indices in Eq. 3 are not critical. Dividing Eqs 2 and 3 by the maximum response leads to the following simplified formulas:

$$r = 1.0 + 0.4r_1 + 0.4r_2$$ \hspace{1cm} (4)

$$r = \sqrt{1.0 + r_1^2 + r_2^2}$$ \hspace{1cm} (5)

where $r = R/R_1, r_1 = R_2/R_1$, and $r_2 = R_3/R_1$ are normalized responses. A comparison can be made by the following equation:

$$\delta = (1.0 + 0.4r_1 + 0.4r_2)/\sqrt{1.0 + r_1^2 + r_2^2} - 1$$ \hspace{1cm} (6)

In Eq. 6, a $\delta$ greater than zero indicates the 100-40-40 rule is conservative compared to the SRSS method, otherwise it is unconservative. Figure 1 shows a contour plot of Eq. 6, with the two small shaded regions (in red) at the corners of (1, 0) and (0, 1) showing the unconservative areas for the 100-40-40 rule. It is easy to show that the maximum level of conservatism of the 100-40-40 rule compared to the SRSS method is 14.9% at $r_1 = r_2 = 0.4$, while the maximum level of unconservatism is slightly larger than 1.0% which occurs at the two corners in the shaded regions. In addition, assuming a uniform distribution for $(r_1, r_2)$, the probability that the 100-40-40 rule is unconservative is slightly less than 0.18%, which equals the area of the two shaded regions. In summary, compared to the SRSS method, the 100-40-40 rule is almost always conservative and is only slightly unconservative in rare
cases. This conclusion is unconditional because the comparison is purely analytical and numerical.

A similar comparison was performed in a numerical study to support the acceptance of the 100-40-40 rule in RG 1.92. It is the demonstrated conservatism of this rule that led to its adoption in RG 1.92, as an alternative to the SRSS method. Therefore, any implementation of the 100-40-40 rule that does not exhibit this conservatism should be carefully scrutinized. Such demonstrated conservatism can be utilized as a check for any claim that the two formats of the 100-40-40 rule in a particular implementation are equivalent.

**SINGLE RESPONSE EXAMPLE**

The equivalence of the ASCE4-98 format and the RG 1.92 format is obvious for the case of a single response of interest. An example of a truss member is utilized in this example to demonstrate this equivalence.

Let the axial forces calculated using RSA for the truss member be \( N_x = 100 \), \( N_y = 200 \), and \( N_z = 500 \), respectively for the orthogonal earthquake components \( X \), \( Y \), and \( Z \). These maximum values are assumed to be obtained using an appropriate modal combination method. Units are assumed to be consistent and are omitted without affecting the purpose of demonstration.

Using the RG 1.92 format, the maximum axial force due to three earthquake components is \( N = 1.0 \times 500 + 0.4 \times 200 + 0.4 \times 100 = 620 \).

Using the ASCE 4-98 format, the results of the 24 permutations are the same as shown in column “N” of Table 2. The extreme values of \( N \) are \( \pm 620 \), consistent with the result from RG 1.92. The equivalence of the two methods is automatically achieved.

**MULTIPLE RESPONSE EXAMPLE**

For cases where more than one response parameter is utilized to check a design criterion, such as in interaction equations, the equivalence of the two methods may not be automatically achievable. An example of a reinforced concrete column in a plane frame is presented below, to show the correct application of the 100-40-40 rule for the case of multiple response parameters.

Only the axial force and one moment are considered at a section in this example for simplicity. Units are assumed to be consistent as in the previous example. Table 1 shows the maximum axial force \( N \) and maximum moment \( M_y \), determined by RSA individually for each earthquake component \( X \), \( Y \), and \( Z \). These maximum values are assumed to be obtained using an appropriate modal combination method.

<table>
<thead>
<tr>
<th>Earthquake Directions</th>
<th>N</th>
<th>My</th>
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<tbody>
<tr>
<td>X</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Y</td>
<td>200</td>
<td>700</td>
</tr>
<tr>
<td>Z</td>
<td>500</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2 shows the results of the spatial response permutations using the ASCE 4-98 format, together with the results using the RG 1.92 format and the SRSS method. For the ASCE 4-98 format, the extremes obtained from the 24 permutations for the axial force and the moment are \( \pm 620 \) and \( \pm 748 \), respectively. These extremes should be used for the
design check of the column section, resulting in \( 2^2 = 4 \) load combinations.

For the RG 1.92 format, the positive maximum seismic responses attributable to the three orthogonal spatial earthquake components are 620 and 748, respectively for the axial force and the moment. These maximum responses agree with the extremes of the ASCE 4-98 format, showing the equivalence of the two formats. As shown by the shaded entries in Table 2, the spatial permutations for these maximum responses correspond to the ASCE 4-98 permutation numbers 17 and 9, respectively. These maximum responses should be used with the ± sign variations and result in the same number of load combinations as the ASCE 4-98.

For the SRSS method, the maximum seismic responses combining the effect of the three spatial earthquake components are 547.7 and 707.4, respectively for the axial force and the moment. Compared to the results from the SRSS method, the level of conservatism of the 100-40-40 rule can be calculated to be 13.2% and 5.7%, respectively for the axial force and the moment, both of which are below the theoretical maximum of 14.9%.

Figure 2 shows an illustration of the axial force-moment interaction diagram, which includes the 24 ASCE 4-98 permutations as dots (red), 4 design load combinations from the 100-40-40 rule as diamonds (blue), and 4 design load combinations from the SRSS method as squares (black). In this figure, the 24 ASCE 4-98 permutations are also annotated with their permutation number as shown in Table 2. A constant compression of 400 is assumed to account for dead and live loads. The curve in Figure 2 shows a postulated N-My interaction capacity of this column, which in its current position encloses the 24 ASCE 4-98 permutations but not the 4 design load combinations from either the 100-40-40 rule or the SRSS method. In other words, the design of this reinforced concrete column should not be based on the 24 ASCE 4-98 permutations.

Therefore, a correct implementation of the 100-40-40 rule using the ASCE 4-98 format should first determine the extremes of the individual response parameters and then use the combinations of the extremes for design. A common mistake is to use the 24 permutations to obtain 24 designs, which are used to determine the governing design.

MULTIPLE NON-INTERACTING RESPONSES

For multiple non-interacting responses at a location, each of them is normally treated as a single response. This effectively leads to the use of extremes of the response parameters, which can be obtained in a way similar to the case of multiple interacting responses. Therefore, regardless of whether a single response, multiple non-interacting responses, or multiple interacting responses are considered, the appropriate way to combine the spatial responses is to: (1) obtain extremes of each individual response, and (2) use the \( 2^M \) sets of load combination (or the more complicated but less conservative alternative method described in ASCE 4-98) for design. This process can be characterized as a column-first approach (as shown by blue arrows in Table 2). The inappropriate process that first performs 24 designs and then determines the final design can be characterized as a row-first approach (as shown by the red arrow in Table 2).

SUMMARY

A detailed examination of the 100-40-40 rule was performed in this paper, regarding the different formats described in ASCE 4-98 and RG 1.92. The 100-40-40 rule was also compared to the SRSS method in detail. The appropriate application of the 100-40-40 rule is illustrated through examples.

It is concluded that the two formats of the 100-40-40 rule are equivalent, provided that this rule is applied to a single response parameter at a time (i.e., following the column-first approach). In a design case where \( M \) response parameters exist, the \( 2^M \) sets of load combinations (or the less conservative but more complicated alternative) can be used according to ASCE 4-98. There is no specific guidance in RG 1.92 when multiple responses need to be considered in an interaction equation. However, since the RG 1.92 equation calculates the estimated maximum value of a single response quantity due to three directions of seismic input motion, the authors have concluded that either of the two methods in ASCE 4-98 is implied when multiple responses are considered.

The 100-40-40 rule, when applied correctly, is almost always conservative compared to the SRSS method, and is only slightly unconservative in rare cases. This conclusion was drawn from pure analytical and numerical comparisons and therefore is unconditional. Any implementation of the 100-
RG 1.92 endorses the 100-40-40 rule only for use with RSA but not with time history analysis. ASCE 4-98 allows the 100-40-40 rule to be used with both types of analysis.

REFERENCES

1. ASCE 4-98 (1999). Seismic Analysis of Safety-Related Nuclear Structures and Commentary, ASCE, Reston, VA.