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PERMANENT MAGNET AND PITOT-STATIC PROBE
FLOWMETERS FOR LIQUID SODIUM

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ABSTRACT

A permanent magnet probe flowmeter and a Pitot-static bubbler flowmeter are considered for coolant flow measurement in the Fast Test Reactor. The permanent magnet probe offers a sensitivity on the order of tenths of a millivolt per foot/second flow velocity, approximate linearity of response, simplicity of operation, and no contact between liquid metal and electrical insulation. Its applicability is contingent on magnetic material having sufficient magnetization retention in high temperature and high neutron flux environments. The Pitot-static bubbler offers low sensitivity to nuclear radiation, use of off-the-shelf materials, and high though non-linear sensitivity. Positive displacement flow in the bubbler may eliminate plugging of the probe tubing by sodium oxide.

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PERMANENT MAGNET AND PITOT-STATIC PROBE
FLOWMETERS FOR LIQUID SODIUM

I. Introduction

Flow measurements in liquid metal systems are made difficult by high operating temperatures and chemical reactivity. Reactor liquid metal coolant flow measurement offers the additional complication of gamma and neutron fluxes at the flowmeter locations. The current concept for the Fast Test Reactor (FTR) requires locating the flow sampling device within the liquid sodium coolant. This report discusses two possible flowmeter concepts and their compatibility with FTR requirements: a permanent magnet probe flowmeter and a Pitot-static device.

II. Permanent Magnet Probe Flowmeter

A. Design and Operation

Permanent magnet flowmeters measure velocity by measuring electrical potential difference developed between two points in a conductor moving in a magnetic field. The design visualized here for a probe has a long cylinder permanently magnetized perpendicular to its axis and covered with a nonconducting surface layer. See Fig. 1. The magnet is enclosed in a tightly fitting cylindrical case or can impervious to liquid metal and oriented in the process tube with the cylinder axis parallel to the liquid flow. Two electrical leads contact the inner can walls at the ends of a diameter perpendicular to the direction of magnetization. The electrical leads exit from the end of the can through stainless steel tubing to a high resistance voltmeter calibrated to read velocity or volumetric flow rate. Both flow velocity and volumetric flow rate readings require temperature corrections for temperatures deviating significantly from calibration temperature.

Fig. 1. Permanent Magnet Probe Flowmeter.

Feasibility of this arrangement depends on sensitivity and reliability with available materials. We shall here examine the aspects of sensitivity which can be readily treated mathematically, thereby obtaining a sharper picture of materials requirements.

B. Mathematical Model

We shall take as a mathematical model an infinitely long cylindrical magnet magnetized uniformly in a direction perpendicular to its axis. The cylinder axis is oriented parallel to the flow of an infinite sea of liquid metal. The magnet of radius r_1 has a nonconducting layer of negligible thickness between it and a closely fitting clad of outer radius r_2 . See Fig. 2. The cylindrical cladding of conductivity σ_c is assumed to make a good electrical contact at radius r_2 with flowing liquid metal of conductivity σ . The cladding and liquid are assumed non-ferromagnetic.

For definiteness we will take the direction of magnetization as the x axis, the direction of liquid flow (and of the cylinder axis) as the z axis, and the angle ϕ positive for counterclockwise rotations about the z axis measured from the x axis. We then have three regions:

$r < r_1$	Region I, magnet
$r_1 < r < r_2$	Region II, cladding
$r > r_2$	Region III, liquid metal.

Fig. 2. Mathematical Model and Notation for Permanent Magnet Probe Flowmeter.

C. Magnetic Fields from Transversely Magnetized Cylinders

Maxwell's equations for steady state of non-polarizable materials are

$$\epsilon_0 \nabla \cdot \underline{E} = \rho \quad (1)$$

$$\nabla \cdot \underline{B} = 0, \quad (2)$$

$$\nabla \times \underline{E} = 0, \quad (3)$$

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \nabla \times \underline{M} \quad (4)$$

Here $\underline{E}$ is the electric force field, $\underline{B}$ is the magnetic force field, $\underline{M}$ is the magnetization intensity, $\underline{J}$ is the current density, ρ is the charge density, and ϵ_0 and μ_0 are the permittivity and permeability of free space.

For our cylinder uniformly magnetized parallel to the X axis we can take $\underline{B}$ inside to be

$$\underline{B} = B(\underline{u}_r \cos \phi - \underline{u}_\phi \sin \phi) \quad (5)$$

where $\underline{u}_r$ and $\underline{u}_\phi$ are the unit vectors in the directions of increasing r and ϕ . Outside the magnet, neglecting effects of local currents on the magnetic field and assuming no magnetic materials present, we have from (4)

$$\nabla \times \underline{B} = 0, \quad (6)$$

and hence

$$\underline{B} = -\nabla \phi, \quad (7)$$

where from (2), the magnetic potential ϕ satisfies Laplace's equation

$$\nabla^2 \phi = 0. \quad (8)$$

The solution to Laplace's equation having z-translational invariance is

$$\phi = \sum_{j=0}^{\infty} (A_j r^j + C_j r^{-j}) (D_j \cos j\phi + E_j \sin j\phi), \quad (9)$$

At the interface at r_1 , Eq. (2) implies that the radial component of $\underline{E}$ is continuous, while Eq. (4) and the absence of a surface current imply that the tangential component of $\underline{B} - \mu_0 \underline{M}$ is continuous. Hence at r_1 we have

$$B_r = B \cos \phi = -\frac{\partial \phi}{\partial r} \Big|_{r=r_1} = -\sum_{j=0}^{\infty} (jA_j r_1^{j-1} - jC_j r_1^{-j-1})(D_j \cos j\phi) + E_j \sin j\phi \quad (10)$$

and

$$(\underline{B} - \mu_0 \underline{M})_\phi = -(B - \mu_0 M) \sin \phi = -\frac{1}{r} \frac{\partial \phi}{\partial \phi} \Big|_{r=r_1} = -\sum_{j=0}^{\infty} (jA_j r_1^{j-1} + jC_j r_1^{-j-1})(-D_j \sin j\phi + E_j \cos j\phi). \quad (11)$$

Equations (10) and (11) must hold for arbitrary ϕ . Therefore

$$-A_1 D_1 + C_1 D_1 r_1^{-2} = E \quad (12)$$

$$+A_1 D_1 + C_1 D_1 r_1^{-2} = -(B - \mu_0 M) \quad (13)$$

and the other subscripted coefficients can be set to zero. We thus obtain for the $\underline{B}$ field in the region outside the cylinder

$$B_r = [(B - \frac{1}{2} \mu_0 M) + \frac{1}{2} \mu_0 M r_1^2 r^{-2}] \cos \phi \quad (14)$$

$$B_\phi = [-(B - \frac{1}{2} \mu_0 M) + \frac{1}{2} \mu_0 M r_1^2 r^{-2}] \sin \phi \quad (15).$$

Note that the B and M on the right hand side of (14) and (15) are the fields inside the cylinder. The portion of the B field in (14) and (15) which does not vanish as r becomes arbitrarily large must be interpreted as the externally applied field. If no externally applied field is present, the magnetic flux density and magnetic induction inside the magnet must according to (14) and (15) be related by

$$B = \frac{1}{2} \mu_0 M \quad (16)$$

or

$$\mu_0 H \equiv B - \mu_0 M = -B \quad (17)$$

We have thus determined the proportionality between B , H , and M inside the uniformly magnetized cylinder. The magnitude will depend on the properties and history of the material. The field outside the uniformly transverse magnetized cylinder with no externally applied field is according to (14) - (17)

$$\underline{B} = B r_1^2 r^{-2} (\underline{u}_1 \cos \phi + \underline{u}_2 \sin \phi) \quad (18)$$

D. Currents and Fields in the Presence of Flow

The current density $\underline{J}$ in a material obeying Ohm's law is

$$\underline{J} = \sigma (\underline{E} + \underline{v} \times \underline{B}) \quad (19)$$

where $\underline{v}$ is the velocity of the material relative to the frame of reference in which the electric and magnetic fields are $\underline{E}$ and $\underline{B}$. Equation (19) is applicable in region III, $r > r_1$. In region II we have

$$\underline{J} = \sigma_c \underline{E} \quad (20)$$

where σ_c is conductivity of the cladding. We assume that the thin insulating layer does not allow charge flow into the magnet from its clad. In regions II and III, (4) reduces to

$$\mu_0 \underline{J} = \nabla \times \underline{B} \quad (21)$$

and hence

$$\nabla \cdot \underline{J} = 0. \quad (22)$$

In region III, (22) and (19) say that

$$\nabla \cdot \underline{E} = -\nabla \cdot (\underline{v} \times \underline{B}) \quad (23)$$

$$= -(\nabla \times \underline{v}) \cdot \underline{B} + \underline{v} \cdot (\nabla \times \underline{B}) \quad (24)$$

$$= -(\nabla \times \underline{v}) \cdot \underline{B} + \mu_0 \underline{v} \cdot \underline{J} \quad (25)$$

For uniform flow velocity, the curl of $\underline{v}$ vanishes. From the symmetry of our system, we expect current density $\underline{J}$ to be in the xy plane, while $\underline{v}$ is in the z direction. Hence $\underline{v} \cdot \underline{J}$ will vanish also. Equation (25) becomes

$$\nabla \cdot \underline{E} = 0, \quad (26)$$

which is consistent with (1) and our intuitive idea that the highly conducting liquid metal should not support a free charge density in steady state. Note that while (22) holds in regions II and III and at boundary points, (26) holds in regions II and III but not necessarily at boundary points. Equation (3) implies that in steady state

$$\underline{E} = -\nabla V \quad (27)$$

where from (20) and (22) in region II and (26) in region III,

$$\nabla^2 V = 0. \quad (28)$$

We are thus seeking the solution of Laplace's equation (28) which is consistent with:

$$a. \quad \underline{E} \rightarrow 0 \text{ as } r \rightarrow \infty \quad (29)$$

$$b. \quad J_r(r_2 - , \phi) = J_r(r_2 + , \phi) \quad (30)$$

$$c. \quad E_\phi(r_2 - , \phi) = E_\phi(r_2 + , \phi) \quad (31)$$

$$d. \quad \int_{r_1}^{\infty} J_\phi dr = 0 \quad (32)$$

Relation (32) says that the current does not circle the entire cylinder, so that the net flow through a plane extending radially outward from the inner surface of the cladding is zero.

The solutions V to Laplace's equation having invariance with respect to z -translation for regions II and III we will write as

$$V_{II} = \sum_{j=0}^{\infty} (F_j r^j + G_j r^{-j}) (I_j \cos j\phi + K_j \sin j\phi) \quad (33)$$

$$V_{III} = \sum_{j=0}^{\infty} L_j r^{-j} (N_j \cos j\phi + P_j \sin j\phi) \quad (34)$$

where we have already imposed (29), but not yet (30) - (32). The field components are, from (27), (33), and (34)

$$E_r^{II} = \sum_{j=1}^{\infty} j (-F_j r^{j-1} + G_j r^{-j-1}) (I_j \cos j\phi + K_j \sin j\phi) \quad (35)$$

$$E_r^{III} = \sum_{j=1}^{\infty} j L_j r^{-j-1} (N_j \cos j\phi + P_j \sin j\phi) \quad (36)$$

$$E_\phi^{II} = \sum_{j=1}^{\infty} j (F_j r^{j-1} + G_j r^{-j-1}) (I_j \sin j\phi - K_j \cos j\phi) \quad (37)$$

$$E_\phi^{III} = \sum_{j=1}^{\infty} j L_j r^{-j-1} (N_j \sin j\phi - P_j \cos j\phi) \quad (38)$$

and the currents are from (19) and (20)

$$J_r^{II} = \sigma_c \sum_{j=1}^{\infty} j(-F_j r^{j-1} + G_j r^{-j-1})(I_j \cos j\phi + K_j \sin j\phi) \quad (39)$$

$$J_r^{III} = \sigma \sum_{j=1}^{\infty} j L_j r^{-j-1} (N_j \cos j\phi + P_j \sin j\phi) - \sigma v B r_1^2 r^{-2} \sin \phi \quad (40)$$

$$J_\phi^{II} = \sigma_c \sum_{j=1}^{\infty} j(F_j r^{j-1} + G_j r^{-j-1})(I_j \sin j\phi - K_j \cos j\phi) \quad (41)$$

$$J_\phi^{III} = \sigma \sum_{j=1}^{\infty} j L_j r^{-j-1} (N_j \sin j\phi - P_j \cos j\phi) + \sigma v B r_1^2 r^{-2} \cos \phi \quad (42)$$

We have assumed the magnetic field from the cylinder is unaltered by the cladding and flowing metal. Imposing (30) for arbitrary ϕ requires

$$-\sigma_c F_j I_j r_2^{j-1} + \sigma_c G_j I_j r_2^{-j-1} - \sigma L_j N_j r_2^{-j-1} = 0 \quad j = 1, 2, 3, \dots \quad (43)$$

$$-\sigma_c F_j K_j r_2^{j-1} + \sigma_c G_j K_j r_2^{-j-1} - \sigma L_j P_j r_2^{-j-1} = 0 \quad j = 2, 3, 4 \dots \quad (44)$$

$$-\sigma_c F_1 K_1 + \sigma_c G_1 K_1 r_2^{-2} - \sigma L_1 P_1 r_2^{-2} = \sigma v B r_1^2 r_2^{-2}. \quad (45)$$

Relation (31) for arbitrary ϕ requires

$$F_j I_j r_2^{j-1} + G_j I_j r_2^{-j-1} - L_j N_j r_2^{-j-1} = 0 \quad j=1, 2, 3, \dots \quad (46)$$

$$F_j K_j r_2^{j-1} + G_j K_j r_2^{-j-1} - L_j P_j r_2^{-j-1} = 0 \quad j=1, 2, 3, \dots \quad (47)$$

Relation (32) is the same as

$$\int_{r_1}^{r_2} J_\phi^{II} dr + \int_{r_2}^{\infty} J_\phi^{III} dr = 0 \quad (48)$$

or

$$\begin{aligned} & \sigma_c \sum_{j=1}^{\infty} (F_j (r_2^j - r_1^j) - G_j (r_2^{-j} - r_1^{-j}))(I_j \sin j\phi - K_j \cos j\phi) \\ & + \sigma \sum_{j=1}^{\infty} L_j r_2^{-j-1} (N_j \sin j\phi - P_j \cos j\phi) \\ & + \sigma v B r_1^2 r_2^{-1} \cos \phi = 0. \end{aligned} \quad (49)$$

For this to be true for arbitrary ϕ , we have

$$\sigma_c F_j I_j (r_2^j - r_1^j) - \sigma_c G_j I_j (r_2^{-j} - r_1^{-j}) + \sigma L_j N_j r_2^{-j} = 0. \quad j=1,2,3,\dots \quad (50)$$

$$-\sigma_c F_j K_j (r_2^j - r_1^j) + \sigma_c G_j K_j (r_2^{-j} - r_1^{-j}) - \sigma L_j P_j r_2^{-j} = 0 \quad j=2,3,\dots \quad (51)$$

$$-\sigma_c F_1 K_1 (r_2 - r_1) + \sigma_c G_1 K_1 (r_2^{-1} - r_1^{-1}) - \sigma L_1 P_1 r_2^{-1} = -\sigma v B r_1^2 r_2^{-1} \quad (52)$$

We now solve the set (43) - (47) and (50) - (52) for the coefficients. Equations (43), (46), and (50) give

$$\begin{aligned} F_j I_j &= 0 \\ G_j I_j &= 0 \quad j=1,2,\dots \\ L_j N_j &= 0. \end{aligned} \quad (53)$$

Equations (44), (47), and (51) give

$$\begin{aligned} F_j K_j &= 0 \\ G_j K_j &= 0 \quad j=2,3,\dots \\ L_j P_j &= 0. \end{aligned} \quad (54)$$

Equations (45), (52), and (47) for $j = 1$ give the nonzero coefficients of interest:

$$\begin{aligned} F_1 K_1 &= v B r_1^2 [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \\ G_1 K_1 &= v B r_1^4 [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \\ L_1 P_1 &= v B r_1^4 (1 + \frac{r_1^2}{r_1^2}) [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1}. \end{aligned} \quad (55)$$

The electric fields (35) - (38) are now specifically:

$$\begin{aligned} \underline{E}^{II} = & -vBr_1^2 \left(1 - \frac{r_1^2}{r^2}\right) [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \sin \phi \underline{u}_r \\ & -vBr_1^2 \left(1 + \frac{r_1^2}{r^2}\right) [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \cos \phi \underline{u}_\phi \end{aligned} \quad (56)$$

$$\begin{aligned} \underline{E}^{III} = & vBr_1^4 r^{-2} \left(1 + \frac{r_2^2}{r_1^2}\right) [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \sin \phi \underline{u}_r \\ & -vBr_1^4 r^{-2} \left(1 + \frac{r_2^2}{r_1^2}\right) [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1} \cos \phi \underline{u}_\phi \end{aligned} \quad (57)$$

We can now evaluate the difference in electrical potential between any pair of points in regions II and III. In particular, for the pair of points on the inner clad surface at which we hope to tap off the velocity-measuring potential we have from (33):

$$V(r_1, \frac{\pi}{2}) - V(r_1, -\frac{\pi}{2}) = 4vBr_1^3 [r_1^2 + r_2^2 + \frac{\sigma_c}{\sigma} (r_2^2 - r_1^2)]^{-1}. \quad (58)$$

The potential difference (58) which we will henceforth call E can be factored into a major contribution and a geometry and conductivity dependent correction:

$$E = 2vBr_1 K_1 \quad (59)$$

where

$$K_1 = 2 \left[1 + \left(\frac{r_2}{r_1}\right)^2 + \frac{\sigma_c}{\sigma} \left(\left(\frac{r_2}{r_1}\right)^2 - 1\right) \right]^{-1} \quad (60)$$

The potential difference given by (59) and (60) is the potential difference across the terminals of a high resistance voltmeter having low resistance leads making good contact with the inner can walls.

To evaluate the effects of leakage between clad and magnet, a different idealization is useful. For good electrical contact between the magnetized cylinder of conductivity σ_m and its clad, the potential difference across the inner clad surfaces is found to be

$$E = 2vBr_1K_2 \quad (61)$$

where

$$K_2 = 2[(1 + \frac{\sigma_m}{\sigma})(1 + (\frac{r_2}{r_1})^2) + (\frac{\sigma_c}{\sigma} + \frac{\sigma_m}{\sigma_c})(\frac{r_2}{r_1})^2 - 1]^{-1}. \quad (62)$$

This latter correction (62) reduces to the one for a magnet insulated from its clad in the limit $\sigma_m \rightarrow 0$.

E. Obtainable Sensitivity

The voltage expected across the voltmeter terminals is

$$E = 2vBr_1K$$

where K is a product of correction factors, including K_1 or K_2 as previously discussed. The saturation magnetization of Alnico VIII in an external magnetic field and subsequent removal of the applied field results in a flux density inside the transversely magnetized cylinder of (1)

$$B = .13 \text{ Weber/m}^2.$$

Preliminary results⁽²⁾ are consistent with a Curie temperature of 1585°F for Alnico VIII. Theory predicts that approximately⁽³⁾ 70% of saturation flux density would be retained at 1200°F.

The irreversible losses from temperature cycling and mechan-

we might hope to retain 50% of the low temperature saturation value, or approximately

$$B = .065 \text{ weber/m}^2.$$

For a 1 inch diameter magnetic cylinder, neglecting corrections, we obtain a sensitivity

$$\begin{aligned} \frac{E}{v} &= 2(.065 \text{ weber/m}^2)(.5 \text{ inch} \times 2.54 \times 10^{-2} \text{ m/inch}) \\ &= 1.65 \text{ millivolt/(meter/sec)} \\ &= .503 \text{ millivolt/(ft/sec)} \end{aligned}$$

or, in terms of volumetric flow rate $\dot{v}$,

$$\frac{E}{\dot{v}} = .015 \text{ millivolt/(gallon/minute)}$$

in the current FTR process tube design.

The corrections (60) and (62) due to finite current densities are tabulated in Table 1. For no current transfer between magnet and clad only the electrical conductivities of the sodium and clad are needed. We use for the resistivities at 1200°K for 304 stainless steel⁽⁴⁾

$$\rho_c = \frac{1}{\sigma_c} = 113. \text{ micro-ohm cm.}$$

and for sodium⁽⁵⁾

$$\rho = \frac{1}{\sigma} = 35.6895 \text{ micro-ohm cm.}$$

The latter was calculated from Epstein's equation.

For good electrical contact between magnet and can, the resistivity or conductivity of the magnetic material is needed as well.

Good values of resistivity of Alnico VIII were not readily available, so K_2 was tabulated for two values of magnet resistivity:

$$\rho_m = \frac{1}{\sigma_m} = 50.0 \text{ micro-ohm cm}$$

$$\rho_m = \frac{1}{\sigma_m} = 314.34 \text{ micro-ohm cm.}$$

The first of these magnet resistivities⁽¹⁾ is that of Alnico VIII at 25°C; the second is obtained by assuming a linear temperature coefficient of resistivity that is approximately that obtained by averaging those of the components with mass fraction weighting.

From the table we conclude that a thin clad is preferable and that electrical insulation between magnet and clad is helpful but not essential. The difference between the correction factors K_1 and K_2 does show the necessity of having the type of electrical contact between flowmeter and clad remain unchanged during operation.

TABLE 1.

Finite Current Density Correction Factors

(r_2/r_1)	K_1	$K_2(\rho_m=50 \text{ micro-ohm cm})$	$K_2(\rho_m=314.34 \text{ micro-ohm cm})$
1.00	1.00	.584	.898
1.06	.925	.505	.817
1.08	.901	.483	.792
1.10	.879	.462	.768
1.20	.776	.376	.664
1.30	.688	.313	.578

The development presented here has assumed a uniform sodium velocity outside the cylindrical magnet cladding. There will actually be a velocity profile with low velocity boundary layers adjacent to surfaces. The order of the effects of non-uniform velocity can be estimated roughly by the following idealization.

The boundary layer of sodium outside the clad will be considered as having a zero velocity. Its effects are then mathematically similar to those of the cladding can. We can replace the can thickness $r_2 - r_1$ by a value $r_2' - r_1$ where $r_2' - r_2$ is the boundary layer thickness, and we simultaneously replace the can conductivity σ_c by some effective value σ_c' between can and sodium conductivity. For the extreme case of a clad of negligible thickness, we take $\sigma_c' = \sigma$ (sodium conductivity) to obtain from (60)

$$K_1' = \left(\frac{r_1}{r_2'}\right)^2.$$

By examining information presented by Knudsen and Katz⁽⁶⁾ for lower Reynolds number flows, T. J. Bennett⁽⁷⁾ estimates that liquid sodium achieves 90% of a mean 17.2 ft/sec flow velocity in an annular region with 1." ID and 3.75" OD outside an inner boundary layer of less than .065" thickness. The corresponding correction is

$$K_1' = .783.$$

The choice of the 90% of mean flow rate as determining the boundary was arbitrary and conservative. It seems reasonable to expect to multiply the correction (60) or (62) for can conductivity by a boundary correction factor of order $(r_2/r_2')^2$, the latter factor on the order of 80% but somewhat velocity dependent, since boundary layer thickness depends on velocity.

F. Neglected Effects

Among the effects neglected in this treatment are

1. Finite length effects on the magnetic field.
2. Finite length current shunting effects.
3. Finite sodium flow-area effects.
4. Effects of external ferromagnetic materials.
5. Poor electrical contact effects between sodium and probe clad.
6. Effect of eddy currents on the magnetic field.

The finite length effects on the magnetic field and the current shunting effects around the end of the finite length cylindrical probe are difficult to assess. It seems plausible, however, that these effects are of the same order as in the flow-through permanent magnet flowmeter and hence that they become small for length to diameter ratios in excess of 4. It seems essential to determine these effects experimentally.

An estimate of the effect of the finite area of the sodium flow rather than an infinite sea seems mathematically tractable and will be attempted in further studies of the probe flowmeter. We anticipate that the relative correction will be on the order of the square of the ratio of magnet diameter to pipe inside diameter, since the B field outside the cylinder has an r^{-2} dependence.

The effects of external ferromagnetic materials will be difficult to include, but if the effect of the finite area of sodium flow proves to be small, the effects of external B fields in the radially outer flow areas will probably be small also. An attempt

will be made to estimate the effect on calibration of removal or reorientation of the magnet from adjacent instrument probe tubes.

The effects of poor electrical contact between can and sodium due to incomplete surface wetting or formation of oxide or bubble layers should not differ significantly in nature or size from corresponding effects in flow-through flowmeters.

The effects of eddy currents on the magnetic field are described qualitatively as sweeping of the B field downstream. This effect becomes significant in flow-through flowmeters only for large diameters, 5 inches or more.

G. Conclusions

The probe type permanent magnet flowmeter appears capable of yielding the desired sensitivity if the magnetic materials retain sufficient magnetization in neutron fluxes and temperatures characteristic of the reactor environment. Response is nearly linear with flow velocity, nonlinearities appearing because of velocity dependence of boundary layer and eddy current. Temperature correction, primarily because of temperature dependence of the magnetization, will be necessary. A major advantage of the permanent magnet probe envisioned here is that electrical leads never contact molten sodium, and hence insulation need be capable of withstanding only high temperature and not corrosive or wetting action.

III. Pitot-Static Bubbler Flowmeter

Bubbling positive displacement flows of argon cover gas inward through conventional Pitot-static probes may eliminate plugging of the probe tubing by sodium oxide precipitant. Annular Pitot and static ports milled into the upstream and downstream faces of a disc probe transverse to the flow (as in Fig. 3) could provide rugged simplicity, circumferential averaging of flow irregularities, and response symmetry under flow reversal conditions.

Key advantages of the Pitot-static bubbler flowmeter include low sensitivity to nuclear radiation exposure, use of off-the-shelf materials and auxiliaries, and high response sensitivity. For Fast Test Reactor design conditions (550 gpm liquid sodium, specific gravity 0.793 at 1200°F, flowing through 1" ID by 3.75" OD annulus pierced by 0.5" OD gas-sample tube), the Pitot-static pressure differential, given by Bernoulli's integral as half the density times the square of the velocity, is 1.64 lb/in², large enough to be easily measurable with good precision.

Twin positive displacement rotary vane pumps on one driveshaft should suffice to ensure matched flow rates of the argon cover gas through both the Pitot and static tubes, so that the dynamic pressure drop through each tube does not disturb the pressure differential. But the strong variation of sodium density with temperature (specific gravity 0.928 at 212°F) does imply the need for temperature-dependent calibration of the flowmeter over the operating range.

Figure 3. PITOT-STATIC MACHMETER FLOWMETER

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