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ELECTRONIC TEMPERATURE-FLATTENING OF THERMIONIC REACTORS

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REVISED ABSTRACT

A method for electronically temperature-flattening a "flashlight-type" thermionic reactor with both axial and radial heat generation nonuniformities is described. The method, which makes use of compensating electron cooling variation while satisfying the voltage- and current-matching constraints on the in-core series-parallel diode network, results in substantially uniform emitter temperatures throughout the reactor. Novel analytical techniques for temperature-flattening a nonuniform reactor are described and used to generate specific illustrative designs. It is shown that, even with severe nonuniformities, the temperature-flattened reactor exhibits almost as high a power density and conversion efficiency as a reactor using fuel-zoning for full power flattening. By eliminating the need for fuel-zoning, the concept described here reduces the critical size and system weight, and permits the use of thicker emitter walls for enhanced reliability.

ELECTRONIC TEMPERATURE-FLATTENING OF THERMIONIC REACTORS*

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1. INTRODUCTION

Uniformly fueled nuclear reactors, particularly small ones, tend to have a highly nonuniform fission density distribution because of peripheral neutron leakage. This presents a serious problem in in-core thermionic reactors, in which the thermionic converters are distributed throughout the volume of the reactor core. Since the performance of these converters is a very sensitive function of their operating temperature, severe nonuniformities in fission density would lead to intolerable performance degradation of the in-core converter network.

To avoid this performance degradation, thermionic reactor designers usually resort to fuel zoning, in order to reduce the heat generation nonuniformity. Thus, power flattening can be achieved by using full fuel loading only in those converters near the reactor periphery; and by removing progressively more fuel from the converters nearer the center of the reactor.

The center of the reactor, however, is precisely the region where fuel has the highest neutronic worth, i. e., where it contributes most effectively to maintaining the chain reaction. Therefore, to maintain criticality, the use of fuel zoning for power flattening usually requires a substantial increase in the size of the reactor and of the neutron and gamma shields. The resultant weight increase is undesirable, particularly in the case of a space power plant, a prime application for thermionic reactors.

Fortunately, an alternative exists which permits the use of uniform (i. e., maximum) fuel loading with only a modest degradation in the power

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output and conversion efficiency of the thermionic reactor. This alternative makes use of electron cooling effects, to produce uniform converter temperatures in spite of highly nonuniform heat generation rates.

Electronic temperature flattening is made possible by the fact that in efficient thermionic converters electron cooling normally consumes the biggest share of the emitter heat flux. Thus, current density variation between diodes can compensate for quite substantial differences in their heat generation rates. The use of this principle to compensate for the axial variation of heat generation in a thermionic fuel element has been described earlier, both theoretically [1] and experimentally [2]. The present paper extends this subject in two ways.

First, it shows that the method can be applied to radial as well as axial nonuniformities in a reactor core. Thus, in a uniformly fueled thermionic reactor consisting of "flashlight-type" fuel elements, heat generation nonuniformity in the axial direction can be compensated by varying the diode lengths within each fuel element, using shorter diodes near the reactor mid-plane and longer diodes near the reactor end faces; while radial nonuniformity can be compensated by varying the number of diodes per fuel element, using more diodes in elements near the core axis and fewer diodes in elements near the periphery of the reactor. However, since the diodes in the reactor are arrayed in a series-parallel network, the diode lengths and numbers must be adjusted so as to produce a constant current in all diodes in series; and a constant voltage in all converters in parallel.

Second, the present paper derives special analytical techniques for applying the electronic temperature-flattening concept to reactor designs, and presents a specific illustrative design to demonstrate the concept's feasibility and effectiveness in raising the reactor's power density and conversion efficiency.

2. PERFORMANCE MODEL

2.1 Thermionic Data Base

To analyze the effect of heat generation nonuniformities and to evaluate the merit of the electronic temperature flattening concept, a simple but reasonably accurate thermionic performance model is required. For the work reported here, this will be based upon a set of tabulated performance data which had been generated by others, by means of the SIMCON analysis program [3]. These data had previously served as the basis of various AEC-sponsored thermionic reactor design studies. Although the present author considers this set of data to be rather optimistic, its use here appears justified since the present paper is primarily concerned with the relative effects of nonuniform heat generation and electronic temperature-flattening, rather than their absolute magnitude. In fact, Section 4.6 shows that the use of a much more pessimistic set of performance data leads to very similar conclusions about the effectiveness of the electronic temperature-flattening concept.

The SIMCON-generated data used were based on a tungsten emitter with a bare work function of 4.75 eV, and a 0.25-mm interelectrode gap. The tables list values of the interelectrode voltage V_e and emitter heat flux q_e , as a function of emitter temperature T_e , collector temperature T_c , reservoir temperature T_r , and current density J . In the present study, we shall arbitrarily confine our attention to a collector temperature of 1000°K and a reservoir temperature of 630°K. The reservoir temperature has purposely been set rather high, to ensure thermionic stability and avoid thermionic burnout [4].

2.2 Data Base Conversion

With the above assumptions, there are two independent variables, T_e and J , and two dependent variables, V_e and q_e . Before proceeding, we wish to modify these dependent variables by applying corrections for the electrical and thermal losses through optimized emitter leads. Although this requires a rather circuitous digression, the delay is justified because the results greatly simplify the subsequent analysis.

A typical emitter lead will have an irregular shape, with nonuniform cross-sectional area A , and temperature-dependent electrical conductivity σ and thermal conductivity κ . Let V , T , and Q denote respectively the local voltage, temperature, and heat flow rate at position x , and let I be the total diode current passing through the emitter lead. We can then write the electrical conduction equation (Ohm's law),

$$dV = - \frac{I}{A(x) \sigma(T)} dx, \quad (1)$$

the corresponding thermal conduction equation,

$$dT = - \frac{Q}{A(x) \kappa(T)} dx, \quad (2)$$

and the energy balance equation, accounting for ohmic heat generation in the emitter lead (but neglecting radiation),

$$dQ = -I dV. \quad (3)$$

Inserting Equation (1) into (3) gives

$$dQ = \frac{I^2}{A(x) \sigma(T)} dx, \quad (4)$$

and dividing this by Equation (2) we obtain

$$dQ = - \frac{I^2}{Q} \frac{\kappa(T)}{\sigma(T)} dT. \quad (5)$$

The thermal and electrical conductivities of a metallic conductor obeying the Wiedemann-Franz law are related by

$$\frac{\kappa(T)}{\sigma(T)} = \frac{\pi^2 k^2}{3e^2} T, \quad (6)$$

where k is Boltzmann's constant and e the electronic charge.

Inserting Equation (6) into (5) yields

$$Q dQ = - \frac{1}{3} (\pi k I / e)^2 T dT. \quad (7)$$

We can now integrate Equations (3) and (7) over the length of the lead, from its emitter end E to the collector end C where it joins the next diode in series, obtaining

$$Q_c - Q_e = I(V_e - V_c), \quad (8)$$

and

$$Q_c^* - Q_e^* = \frac{1}{3}(\pi k I / e)^2 (T_e^3 - T_c^3), \quad (9)$$

from which Q_c can be eliminated to give

$$Q_e = I \left[\frac{\pi^2 k^2 (T_e^3 - T_c^3)}{6 e^2 (V_e - V_c)} - \frac{1}{2} (V_e - V_c) \right]. \quad (10)$$

Dividing Equation (1) by the emitter surface area, we obtain a relation between q_e , the emitter-to-lead heat loss rate per unit emitter area, and V_e , the emitter lead voltage drop

$$q_e = J \left[\frac{\pi^2 k^2 (T_e^3 - T_c^3)}{6 e^2 V_e} - \frac{1}{2} V_e \right], \quad (11)$$

where J is the emitter current density. Note that this equation is independent of lead geometry and diode dimensions.

Next, we wish to find the value of V_e which maximizes the net conversion efficiency η of the diode. This efficiency, i. e., the power output at the end of the leads divided by the total heat generation in the fuel-emitter, is given by

$$\eta = J(V_e - V_c) / (q_e + q_c), \quad (12)$$

which is maximized when

$$V_e = V_c \left\{ 1 + \left[1 + \frac{(e V_e / \pi k)^2}{T_e^3 - T_c^3} \left(\frac{6 q_e}{J V_e} - 3 \right) \right]^2 \right\}^{-1/2}. \quad (13)$$

We can now solve for $V = V_e - V_c$, the net diode voltage at the end of the leads,

$$\frac{V}{V_e} = 1 - \left\{ 1 + \left[1 + \frac{(e V_e / \pi k)^2}{T_e^3 - T_c^3} \left(\frac{6 q_e}{J V_e} - 3 \right) \right]^2 \right\}^{-1/2}, \quad (14)$$

and for what may be termed the diode heat flux,

$$q = q_e + q_c, \quad (15)$$

i. e., the total heat generation rate per unit emitter area. Combining Equations (11), (13), and (15), we obtain:

$$\frac{q}{q_e} = 1 + \frac{1 + \left[1 + \frac{(e V_e / \pi k)^2 (6 q_e / J V_e - 3) / (T_e^3 - T_c^3)}{1 - J V_e / q_e} \right]^2 (1 - J V_e / q_e)}{(e V_e / \pi k)^2 (6 q_e / J V_e - 3) / (T_e^3 - T_c^3)}. \quad (16)$$

Thus, by means of Equations (14) and (16), the tables of electrode performance data $V_e q_e$ can be transformed into corresponding tables of diode performance $V q$, with the results presented graphically in Figures 1 and 2. In each figure the electrode performance is represented by the dashed curves, and the diode performance (after lead losses) by the solid curves. For each of three emitter temperatures, Figure 1 shows curves of current density versus voltage, and Figure 2 presents curves of heat flux versus current density. Figure 3, which displays trade-off curves between power density and efficiency, illustrates the severe performance degradation caused by the lead losses.

The great value of transforming the original $V_e q_e$ data set into corresponding $V q$ data is that it permits the subsequent analysis to ignore the question of lead losses and specific diode dimensions.

2.3 Diode Performance Model

We now wish to apply the diode performance data derived above to analyze a thermionic reactor in which the electronic temperature-flattening concept is employed to minimize the effect of nonuniform heat generation. In such a reactor, all diodes operate at the same emitter temperature, which for illustrative purposes will be set at 2000°K in the present study. We, therefore, require analytical expressions relating J , V , and q at the prescribed operating temperature. Specifically, V and q will be expressed as functions of J .

A good approximation for the diode voltage V is given by

$$V/V_m = 2 - (J/J_m) + \beta (J_m/J), \quad (17)$$

where least-square analysis of our 2000°K data set yields values of $J_m = 35.91$ amp/cm², $V_m = 0.4006$ volt, and $\beta = 0.03818$. Figure 4 presents a plot of Equation (17) and of the SIMCON-generated data points. As can be seen, the curve fit is excellent in the voltage range of practical interest.

The heat generation rate q can be related to the current density J by the simple linear expression

$$q = q_0 + q'J, \quad (18)$$

where regression analysis yields values of $q_0 = 12.36$ watt/cm and $q' = 3.304$ volt. As shown by Figure 5, Equation (18) provides an excellent fit to the 2000°K data points.

Physically, q_0 may be interpreted as the emitter heat flux due to radiation and cesium conduction, while q' represents the sum of the electron cooling and the emitter lead heat loss, expressed as electron-volts per electron.

Finally, for future convenience, let us rewrite Equation (18) as

$$q/q_m = q_0/q_m + J/J_m, \quad (19)$$

$$\text{where } q_m = q'J_m. \quad (20)$$

Equations (17) and (19) constitute the basic performance model for the analysis of the temperature-flattened reactor. From these equations, the diode power density P is given by

$$P = J_m V_m \left[2(J/J_m) - (J/J_m)^2 + \beta \right], \quad (21)$$

and the diode conversion efficiency η is

$$\eta = \left(\frac{J_m V_m}{q_m} \right) \left[\frac{2(J/J_m) - (J/J_m)^2 + \beta}{(q_0/q_m) + (J/J_m)} \right]. \quad (22)$$

Of particular interest is the efficiency-versus-power density trade-off equation, which is obtained by eliminating J from Equations (21) and (22):

$$\eta = (P/q_m) \left\{ 1 + q_0/q_m \pm [1 + \beta - P/J_m V_m]^{1/2} \right\}^{-1}. \quad (23)$$

The maximum power density P_m , which occurs when $J=J_m$, is given by

$$P_m = J_m V_m (1 + \beta), \quad (24)$$

and the maximum efficiency η_m , which occurs when

$$\frac{J}{J_m} = \left[\left(\frac{q_0}{q_m} + 1 \right)^2 - (1 + \beta) \right]^{1/2} - \frac{q_0}{q_m},$$

is given by

$$\eta_m = \frac{2J_m V_m}{q_m} \left\{ \left(1 + \frac{q_0}{q_m}\right) - \left[\left(1 + \frac{q_0}{q_m}\right)^2 - (1 + \beta)\right]^{1/2} \right\}. \quad (25)$$

3. REACTOR DESIGN

3.1 Electrical Arrangement

In a "flashlight-type" thermionic reactor, each TFE (Thermionic Fuel Element) contains a number of series-connected diodes within a common cesium enclosure. Moreover, the number of diodes per TFE must be rather small, to minimize the danger of cesium breakdown and shorts to ground.

One electrical design scheme that has been considered envisages independently connecting each TFE to its own power conditioning unit. The major disadvantage of this scheme is that the power is transmitted from the reactor to the conditioners in the form of extremely high currents at very low voltages. Transmission losses therefore assume great importance, which generally forces the use of massive, high-conductance transmission cables, and favors locating the power conditioners as close as possible to the reactor, to minimize transmission losses and cable weights. Bringing the power conditioners nearer to the reactor, however, increases the required shield weight.

For these reasons, the author prefers an alternative design, in which many TFE's are connected in series to build up a high reactor output voltage, thereby minimizing transmission losses and weights. How this can be done while avoiding the danger of high-voltage cesium breakdown is described in another paper of these proceedings [5].

When TFE's are connected in series to build up the reactor output voltage, they must be arranged in a series-parallel network, with cross-connections to each TFE terminal in the reactor. This is necessary so that open-circuit failure (loss of cesium) of a given TFE will not materially affect the operation of the other TFE's in series. The current which would normally have passed through the failed TFE is redistributed via the cross-connections to the intact parallel TFE's, which continue to operate at a reduced voltage.

3.2 Core Layout

Typically, TFE's are arranged in a hexagonal-cell lattice, to form a quasi-cylindrical reactor core. The quasi-cylindrical core may be considered to consist of several coaxial rings of TFE's, with all TFE's within the same ring having essentially the same heat generation rate, but with major variations between TFE's in different rings.

It can be shown that series-connected converters are far more sensitive to heat generation nonuniformities than parallel converters, since converters in series must necessarily pass the same current. It therefore appears desirable to make all series connections between TFE's in the same ring, and nothing but parallel cross-connections between TFE's in different rings.

A possible configuration illustrating this design principle is shown in Figure 6. This depicts an illustrative layout for a core consisting of either 54 full-core-length TFE's or 108 half-core-length TFE's [5]. All TFE's

with the same letter designation are connected in series. Parallel cross-connections between rings are made between the terminals of TFE's bearing the same number designation. Thus, each ring contains 18 TFE's with approximately the same heat generation rate, and each TFE has two parallel partners to pass the current in case of open-circuit failure.

Note that all series connections in this design are made between adjacent TFE's, to minimize ohmic losses and conductor weights. Parallel connections are made between nearby but not necessarily adjacent TFE's. These connections can be designed with much lower conductances, since normally no current passes through them.

The design depicted in Figure 6 is illustrative only. The specific numbers of TFE's in series and parallel are not important; only the basic core layout and electrical network arrangement of the reactor.

4. ANALYSIS

Consider a quasi-cylindrical reactor core consisting of R coaxial rings of TFE's. The relative heat generation rates in the reactor will depend upon the composition and dimensions of the reactor core and of its axial and radial reflectors. For a specific design, these rates can be computed by multi-group transport theory. For the present analysis, we shall merely assume that the axial heat generation profiles of all TFE's in the reactor can be approximated by a chopped cosine function with a maximum-to-minimum ratio μ . Thus, a TFE in the ring r has a heat generation profile given by

$$q = q_r \cos[(\arcsin \mu) z / Z], \quad (26)$$

where z is the distance from the reactor midplane, Z is half the core height, and q_r is the maximum value of q in ring r . For convenience, let us rewrite Equation (26) as

$$q = q_r \cos \alpha \xi, \quad (27)$$

where $\alpha = \arcsin \mu$ and $\xi = z/Z$.

4.1 Smearred Diode Approximation

To retain maximum generality, the subsequent analysis employs a technique we shall designate as the "smearred-diode" approximation. In this approximation, each TFE is analyzed as though it consisted of an infinite number of diodes of infinitesimal length. This is a very useful approach, which makes it possible to analyze the TFE without specifying the number of diodes it contains. The results are therefore quite general. The validity of this approximation is demonstrated in Section 4.4, which shows that it is in excellent agreement with an analysis based on a finite number of finite-length diodes. Of course, that validity is only possible because electrical and thermal lead losses have already been accounted for in Section 2.2.

With the above assumptions, we can solve for the average current density,

$$\bar{j}_r = \int_0^1 j d\xi, \quad (28)$$

and the average power density,

$$\bar{p}_r = \int_0^1 p d\xi, \quad (29)$$

of a TFE in ring r . Combining Equations (19), (21), and (27), with Equations (28) and (29), we obtain

$$\bar{J}_r/\bar{J}_m = (q_r/q_m) \sin \alpha/\alpha - (q_o/q_m), \quad (30)$$

$$\frac{\bar{P}_r}{\bar{P}_m} = \left[2 \frac{q_r}{q_m} \left(1 + \frac{q_o}{q_m} \right) \frac{\sin \alpha}{\alpha} - \frac{1}{2} \left(\frac{q_r}{q_m} \right)^2 \left(1 + \frac{\sin 2\alpha}{2\alpha} \right) - \frac{q_o}{q_m} \left(2 + \frac{q_o}{q_m} \right) + \beta \right] / (1 + \beta). \quad (31)$$

From Equations (23), (30), and (31), we obtain the average diode voltage $\bar{V}_r = \bar{P}_r/\bar{J}_r$ in ring r :

$$\frac{\bar{V}_r}{V_m} = 2 \left(1 + \frac{q_o}{q_m} \right) - \frac{\frac{1}{2} (q_r/q_m)^2 (1 + \sin 2\alpha/2\alpha) - (q_o/q_m)^2 - \beta}{(q_r/q_m) (\sin \alpha/\alpha) - (q_o/q_m)}. \quad (32)$$

It should be emphasized that $\bar{J}_r$ and $\bar{V}_r$ represent quite different kinds of averages, a point easily overlooked because of the smeared-diode approximation used here. In effect, $\bar{J}_r$ represents the current density of all diodes averaged by weighting the value for each diode by the length of that diode. By contrast, $\bar{V}_r$ represents the average diode voltage with all diodes weighted equally. If a TFE in ring r consists of N_r diodes, then its output voltage is $N_r \bar{V}_r$, and its output current per unit area of emitter surface in the TFE is $\bar{J}_r/N_r$.

By optimizing $\bar{q}_r$ in Equation (31), we obtain an expression for the largest possible value of the TFE's average power density $\bar{P}_r$:

$$\left(\frac{\bar{P}_r}{\bar{P}_m} \right)_{\max} = \left[2 \left(1 + \frac{q_o}{q_m} \right)^2 \left(\frac{\sin^2 \alpha/\alpha^2}{1 + \sin 2\alpha/2\alpha} \right) - \frac{q_o}{q_m} \left(2 + \frac{q_o}{q_m} \right) + \beta \right] / (1 + \beta). \quad (33)$$

Figure 7 presents a plot of $\bar{P}_r$ versus $\bar{V}_r$ for different values of the heat generation max/min ratio μ . It shows that the power density maximizes when $V_r/V_m \approx 1$, and that electronic temperature-flattening permits a high power output, even from a TFE with a highly nonuniform heat generation profile.

In a reactor with radially nonuniform heat generation, it is of course impossible for all TFE's to operate at their maximum power density. But this is not serious, since the reactor designer usually seeks to compromise between maximizing reactor power and efficiency, particularly in the case of a space power plant where radiator weight is also important.

4.2 Radial Temperature Flattening

Thus far, we have examined how all diodes in a TFE can be designed to operate at the same emitter temperature, by varying the diode length to compensate for heat generation variation in the axial core direction. Similarly, variation in the radial direction can be compensated by varying the number of diodes per TFE. With all TFE's forced to operate at the same voltage by their parallel cross-connections, placing more diodes into the TFE rings nearer the core axis decreases their voltage, which increases their current density and therefore their electron cooling rate. By proper choice of parameters, it is possible to achieve a flat temperature distribution throughout the reactor core.

All TFE's in the reactor operate at the same output voltage V_{TFE} . By specifying that voltage and N_r , the number of diodes in ring r , we can solve for the average diode voltage $V_r = V_{TFE}/N_r$ in that ring. Combining this with Equation (32), we obtain a quadratic equation for q_r :

$$\left(\frac{1}{2} + \frac{\sin 2\alpha}{4\alpha}\right)\left(\frac{q_r}{q_m}\right)^2 + \left[\frac{V_{TFE}}{V_m N_v} - 2\left(1 + \frac{q_o}{q_m}\right)\right]\left[\frac{\sin \alpha}{\alpha}\left(\frac{q_r}{q_m}\right) - \frac{q_o}{q_m}\right]\left(\frac{q_r}{q_m}\right) - \left(\frac{q_o}{q_m}\right)^2 - \beta = 0. \quad (34)$$

After solving for q_r , Equation (31) is solved for $\bar{P}_r$. Once this has been done for all R rings, it is easy to compute the reactor's average power density,

$$\bar{P} = \sum_{r=1}^R \bar{P}_r / R, \quad (35)$$

the average heat generation rate per unit emitter area,

$$\bar{q} = \frac{\sin \alpha}{\alpha R} \sum_{r=1}^R q_r, \quad (36)$$

and the reactor conversion efficiency,

$$\bar{\eta} = \bar{P} / \bar{q}. \quad (37)$$

The above process is repeated with different values of V_{TFE} and sets of N_r 's until a solution is found with q_r 's in the specified ratios and with a reasonable compromise between maximizing the power density $\bar{P}$ and the conversion efficiency $\bar{\eta}$. A computer program for carrying out this process has generated numerous reactor design solutions illustrating the electronic temperature flattening concept. Because of space limitations, only one of these will be presented here.

4.3 Illustrative Design

The illustrative solution described below is based on the TFE design concept presented in Reference [5], and on the specific core layout depicted earlier in Figure 6. Each core-half contains three rings of 18 TFE's. Rings 1, 2, 3, respectively contain 6, 5 and 4 diodes in each semi-core-length TFE. Thus, the reactor contains a total of 108 TFE's and 540 diodes. At its illustrative operating point, the reactor produces 97 volts at its terminals, corresponding to an output of 2.68 volts per TFE. This low TFE voltage is consistent with the concept for preventing cesium breakdown and internal shorting described in Reference [5].

For illustrative purposes, it is assumed that the heat generation rate has a max/min ratio μ of 2.00 in the axial core direction, and relative values of q_r of 1.00, 0.80, and 0.51 for the three TFE rings. The radial heat generation profile is depicted in Figure 8. This shows that the relative heat generation rates listed above really imply a max/min ratio of $1.05/0.35 = 3.0$ between core axis and periphery. Thus, the overall max/min ratio of the reactor is $2.0 \times 3.0 = 6.0$, i. e., the illustrative design is deliberately based upon a reactor with a highly nonuniform heat generation rate, to provide a severe test of the electronic temperature-flattening concept.

With the above assumptions, Equations (34) to (37) yield an average power density $\bar{P}$ of 12.4 watt/cm², an average heat generation rate $\bar{q}$ per unit emitter area of 90.7 watt/cm², and a reactor conversion efficiency $\bar{\eta}$ of 13.7%.

The solution obtained can also be used to extract and display much additional information about the illustrative reactor design. For each TFE ring, normalized axial profiles of heat generation, current density, voltage, power density, and efficiency are presented in Figures 9 to 13, respectively. Clearly, high heat generation rates are compensated by large current densities and therefore high electron cooling rates. Regions near the center of the reactor core operate at high power densities. Those near the outside exhibit high conversion efficiencies.

4.4 Finite-Diode Analysis

Although the preceding analysis has been based on the smeared-diode approximation, which treats each TFE as consisting of an infinite number of infinitesimal diodes, the solutions obtained can be used to compute the finite length of each diode in the reactor, expressed as a fraction of the core length. Since all N_r diodes in the TFE ring r must pass the same current I_r , it follows that

$$\int_0^{\xi_n} J d\xi = (n/N_r) \int_0^1 J d\xi, \quad (38)$$

where ξ_n denotes the end point of the n 'th diode in the TFE. Inserting Equations (24) and (27) for J and integrating, we obtain

$$(q_r \sin \alpha \xi_n / \alpha - q_o \xi_n) / n = (q_r \sin \alpha / \alpha - q_o) / N_r. \quad (39)$$

By inserting successive values of the diode number n , Equation (39) can be used to compute all values of ξ_n in the r 'th TFE. The length Δz_n of the n 'th diode is related to the core length $2Z$ by

$$\Delta z_n / 2Z = \frac{1}{2} (\xi_n - \xi_{n-1}). \quad (40)$$

For our illustrative design solution, the relative lengths of all diodes in the reactor are pictured in Figure 14. It should of course be noted that the above analysis neglects the unfueled axial gaps between diodes, which would have to be taken into account in a more refined design analysis.

Our final task is to determine the validity of the smeared-diode approximation in analyzing the performance of a reactor with a finite number of finite-length diodes. To permit comparison with previous results, the finite-diode reactor to be analyzed is based upon the same TFE arrangements as the previous illustrative design, using the absolute heat generation rates and relative diode lengths computed earlier by means of the smeared-diode approximation. Temperature and voltage gradients within each diode will be neglected here. From Equation (27), the heat generation rate per unit emitter area of diode n in TFE ring r is given by

$$q = (q_r / \alpha) (\sin \alpha \xi_n - \sin \alpha \xi_{n-1}). \quad (41)$$

Once q is known, we can compute the current density J from Equation (19) and the diode voltage V from Equation (17). From these in turn the diode's power density P and conversion efficiency η are computed. Equations (13), (14), and (16) are then combined to solve for the emitter lead voltage drop,

$$V_L = \frac{\pi k}{2} \left[\frac{T_e^2 - T_c^2}{6q/2V - 3} \right]^{1/2}, \quad (42)$$

which is inserted in Equation (11) to give the emitter lead heat loss rate q_L . These values determine the electrode voltage V_e and the emitter heat flux q_e .

4.5 Analytical Results

All the design parameters derived thus far are independent of the actual diode and core dimensions. To complete our illustrative picture, let us now postulate a core height of 40 cm, an emitter diameter of 2.25 cm, and a center-to-center spacing of 3.00 cm between TFE's, corresponding to an equivalent core diameter of 26.7 cm. Thus, the quasi-cylindrical core has a length-to-diameter ratio of 1.5.

For the illustrative design described above, detailed results for each type of diode are summarized in Table I. The table also presents averaged or summed values for each TFE and for the entire reactor array.

The first thing to note about Table I is its internal consistency: i. e., all TFE's have the same output voltage V , and all diodes within each TFE have the same current I . This consistency confirms the validity of the smeared-diode analysis, based upon an infinite number of infinitesimal diodes. In fact, the numerical results of that analysis were found to be in excellent agreement with those obtained from the finite diode analysis.

As shown in Table I, the diode lengths vary from 2.7 to 6.9 cm; the lead resistances vary from 0.08 to 0.16 milliohm; and the diode currents in the three TFE rings are 754, 699, and 511 amperes, respectively. Altogether the reactor has a thermal power of 1385 kw and a beginning-of-life electrical output of 190 kw (1964 amperes at 97 volts), with a corresponding conversion efficiency of 13.7%.

An illuminating way of displaying the effectiveness of the electronic temperature flattening concept is illustrated in Figure 15, which presents plots of the power density ratio $\bar{P}/P_m$ versus the efficiency ratio $\bar{\eta}/\eta_m$. The outer curve shows the locus of operating points for a 2000°K diode, and the arrowheads on that curve represent the operating points for the 15 types of diodes in our illustrative design. Each diode point is identified by the index letter of the corresponding TFE ring. As shown, diodes in ring A operate near the maximum power density; those in ring C are near the maximum efficiency.

The inner curve shows the locus of operating points for a TFE with a heat generation max/min ratio of 2.0. The three points on that curve represent the TFE operating points in the three rings of the reactor.

The average power density and conversion efficiency of the entire reactor is represented by the asterisked point in Figure 15. As shown, the reactor operating point lies on the 45° diagonal, where $\bar{P}/P_m = \bar{\eta}/\eta_m$. In other words, the reactor output voltage was chosen so that the average power density $\bar{P}$ and efficiency $\bar{\eta}$ are each the same percentage of their respective maximums, namely 83.2%. This compares with a corresponding percentage of 87.9% for an isothermal 2000°K diode. (The reason why that percentage is not 100% is, of course, that a diode cannot operate simultaneously at maximum power and maximum efficiency.) Thus, it is seen that with electronic temperature flattening even a highly nonuniform reactor, with respective heat generation max/min ratios of 2.0 and 3.0 in the axial and radial directions, has a power output and conversion efficiency only 5% lower than the corresponding values for a perfectly power flattened reactor.

4.6 Effect of Assumed Performance Model

One final question will be examined here. As shown by the illustrative design summary in Table I, some of the diodes in the reactor operate at very high heat fluxes and current densities; others have very high conversion efficiencies. These high values, of course, are a consequence of the thermionic data base employed, and not of the electronic temperature flattening method. Nevertheless, it may be questioned whether the effectiveness of

electronic temperature flattening does not perhaps depend upon operating at very high heat fluxes and current densities; and whether the method would still prove effective if a much less optimistic set of thermionic data were employed.

To resolve these questions, a new thermionic performance model was employed, in which we arbitrarily reduced the curve fit parameter J_m (Equation 17) from 35.9 to 20.0 amp/cm² and increased the radiative and conductive heat loss parameter q_c (Equation 18) from 12.36 to 20.0 watt/cm². The above assumptions reduce the maximum power density P_m from 14.9 to 8.3 watt/cm², and the maximum efficiency η_m from 16.5 to 11.9%, as shown in Figure 16.

Based on this more pessimistic model, a new illustrative design case was computed, with the principal results summarized in Figure 17 and Table II. As before, the axial max/min ratio is taken as 2.00. In the radial direction, the three TFE rings were assumed to have relative heat generation rates of 1.00, 0.85, and 0.64, for which a plot similar to Figure 9 yields a radial max/min ratio of 2.00. This is again quite severe, though somewhat less so than in the previous case.

At the selected operating point, and with the same core dimensions as before, the reactor's BOL power output is reduced from 190 to 111 ekw (1261 amperes at 88 volts), and its conversion efficiency is reduced from 13.7 to 10.4%. As shown in Table II and Figure 17, the design point again represents an even compromise between maximizing power and efficiency: $\bar{P}/P_m = \bar{\eta}/\eta_m = 0.874$. This corresponds to a reduction of only 7% from the maximum value of 0.942 attainable with a perfectly power flattened reactor. Thus, it appears that the effectiveness of the electronic temperature flattening concept does not depend on the use of a particularly optimistic performance model.

5. SUMMARY AND CONCLUSIONS

- In a "flashlight-type" thermionic reactor, with axially and radially nonuniform heat generation, the judicious use of compensating electron cooling can result in substantially uniform emitter temperatures throughout the core, while satisfying the required voltage- and current-matching constraints of the in-core series-parallel diode network.
- Analytical techniques for designing a reactor with electronic temperature flattening are greatly simplified by employing the "smeared diode approximation", in which the finite number of finite-length diodes in each fuel element is replaced by an infinite number of infinitesimal diodes.
- The use of the "smeared diode approximation" depends on another useful technique, in which the data base correlating current density, voltage, and heat flux is transformed into a new data base that accounts for electrical and thermal losses in optimized emitter leads.
- The effectiveness of the electronic temperature flattening concept does not depend on very optimistic assumptions about diode performance.

- A highly nonuniform thermionic reactor employing electronic temperature flattening will exhibit an average emitter power density and an overall conversion efficiency only a few percent lower than those of a perfectly power-flattened reactor operating at the same emitter temperature.
- Electronic temperature flattening eliminates the need for thermal power flattening by fuel zoning. The resultant increase in fuel volume fraction, particularly at the center of the reactor core where fuel has the highest neutronic worth, reduces the critical size and system weight, and permits the use of thicker emitter walls for enhanced reliability.

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TABLE I: ILLUSTRATIVE DESIGN BASED ON SIMCON-DERIVED PERFORMANCE MODEL

Diode	Length	Lead	q_e	q	J	I	V_e	V	P/P_m	η/η_m
-	cm	μohm	W/cm^2	W/cm^2	A/cm^2	Amp	Volt	Volt	-	-
A1	2.71	84	120	142	39.4	754	.440	.376	.991	.632
A2	2.77	85	119	140	38.5	754	.451	.386	.995	.648
A3	2.91	87	114	133	36.6	754	.473	.408	.999	.680
A4	3.17	91	107	124	33.7	754	.510	.442	.996	.731
A5	3.65	96	95	109	29.2	754	.566	.494	.967	.805
A6	4.78	102	76	86	22.3	754	.654	.577	.862	.909
A	20.00	91	105	118	32.0	754	.516	2.683	.958	.736
B1	3.24	102	98	113	30.5	699	.550	.479	.978	.785
B2	3.35	103	96	110	29.5	699	.563	.491	.969	.801
B3	3.62	105	90	103	27.3	699	.590	.516	.945	.835
B4	4.17	109	80	91	23.7	699	.636	.560	.889	.889
B5	5.63	114	63	70	17.6	699	.716	.636	.749	.965
B	20.00	107	86	94	24.7	699	.611	2.682	.888	.857
C1	3.99	155	64	72	18.1	511	.709	.630	.763	.960
C2	4.23	156	62	69	17.1	511	.722	.643	.736	.970
C3	4.86	158	55	61	14.9	511	.753	.672	.669	.980
C4	6.92	159	43	47	10.4	511	.816	.737	.515	.998
C	20.00	157	56	60	14.4	511	.751	2.682	.649	.979
ABC	20.00	--	82	91	23.7	1964	--	2.582	.832	.832

TABLE II: ILLUSTRATIVE DESIGN BASED ON "PESSIMISTIC" PERFORMANCE MODEL

Diode	Length	Lead	q_e	q	J	I	V_e	V	P/P_m	η/η_m
-	cm	μohm	W/cm^2	W/cm^2	A/cm^2	Amp	Volt	Volt	-	-
A1	2.63	119	85	101	24.6	457	.375	.320	.948	.654
A2	2.69	121	84	99	24.0	457	.388	.332	.961	.675
A3	2.84	125	81	95	22.8	457	.415	.358	.981	.719
A4	3.11	131	77	89	20.8	457	.459	.399	.998	.786
A5	3.64	139	69	79	17.7	457	.527	.463	.988	.877
A6	5.10	148	56	62	12.7	457	.639	.571	.871	.983
A	20.00	131	75	84	19.4	457	.467	2.444	.950	.789
B1	3.12	138	75	86	20.1	442	.475	.414	.999	.809
B2	3.23	140	73	84	19.4	442	.491	.429	.999	.881
B3	3.51	143	69	79	17.8	442	.525	.461	.989	.875
B4	4.12	149	62	70	15.2	442	.583	.517	.944	.940
B5	6.02	154	49	54	10.4	442	.690	.622	.778	.999
B	20.00	145	66	72	15.6	442	.553	2.444	.920	.896
C1	3.79	186	58	65	13.5	361	.621	.553	.898	.971
C2	4.03	187	56	62	12.7	361	.639	.571	.871	.983
C3	4.71	188	50	56	10.9	361	.680	.612	.799	.999
C4	7.48	184	39	43	6.6	361	.776	.709	.582	.956
C	20.00	186	51	54	10.2	361	.679	2.446	.751	.976
ABC	20.00	-	64	70	15.1	1261	-	2.445	.874	.874

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Fig. 2: EFFECT OF OPTIMIZED LEAD LOSS ON HEAT FLUX

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Fig. 16: COMPARISON OF SIMCON-BASED AND "PESSIMISTIC" PERFORMANCE MODELS

Fig.17: SUMMARY OF ILLUSTRATIVE DESIGN BASED ON " PESSIMISTIC" PERFORMANCE MODEL

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