KALMAN FILTERING APPROACH TO OPTIMIZE OFDM DATA RATE

Sashi Prabha Wunnava

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APPROVED:

Parthasarathy Guturu, Major Professor
Kamesh Namuduri, Committee Member
Bill P. Buckles, Committee Member
Shengli Fu, Director of Graduate Studies
Murali Varanasi, Chair of the Department of Electrical Engineering
Costas Tsatsoulis, Dean of the College of Engineering
James D. Meernik, Acting Dean of the Toulouse Graduate School

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This study is based on applying a non-linear mapping method, here the unscented Kalman filter; to estimate and optimize data rate resulting from the arrival rate having a Poisson distribution in an orthogonal frequency division multiplexing (OFDM) transmission system. OFDM is an emerging multi-carrier modulation scheme. With the growing need for quality of service in wireless communications, it is highly necessary to optimize resources in such a way that the overall performance of the system models should rise while keeping in mind the objective to achieve high data rate and efficient spectral methods in the near future. In this study, the results from the OFDM-TDMA transmission system have been used to apply cross-layer optimization between layers so as to treat different resources between layers simultaneously. The main controller manages the transmission of data between layers using the multicarrier modulation techniques. The unscented Kalman filter is used here to perform nonlinear mapping by estimating and optimizing the data rate, which result from the arrival rate having a Poisson distribution.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The growing demand for short distance wireless solutions which offer high data rate, low power consumption, low cost and reasonably good quality of service support are giving rise to the increase in more efficient digital devices and better techniques of using them. All these digital devices are exemplified by various resources such as power, spectrum, changing topology and other varying consumer needs. These are the characteristics which have become the reason for developing the upcoming digital devices being worked on currently while keeping in mind factors like efficiency and quality of service.

1.2 Problem Statement

The goal is to offer the required performance expected by the consumer by minimizing the total power consumption of the device. This in turn triggers the need to exploit the systems' run-time performance. This run time performance can then be compared to obtain an optimized performance trade-off by using a feedback filter system.

1.3 Thesis Overview and Contribution

With the goal in mind, such a type of optimization is effectual only if the system is cross-layer. By cross-layer, I mean across layers of the stack of protocol. Traditionally, the design of cross-layer enables exchange of enough data between different communication layers. Hence, this kind of information exchange between layers leads to optimization across the device components and also improves stability of the system. Therefore a Kalman filter is scheduled here in modeling the system that transfers information between layers. The filter enables the estimation of the data rate iteratively thereby leading to an optimization technique which is explained in detail in later chapters.

1.4 Organization of the Thesis

The thesis starts with an introduction to the Kalman filtering as discussed in Chapter 2. Chapter 3 describes the problem definition and solution approach adopted in the thesis. The simulation results are presented in Chapter 4. Chapter 5 discusses about the conclusions and future work.

CHAPTER 2

KALMAN FILTER

2.1. Basic Concept of Kalman filter

Kalman filter is an algorithm based on recursive estimation that was developed as early as 1960 by Rudolph E. Kalman. The Kalman filter (KF) is one of the most extensively used methods for tracking and estimation due to its simplicity, optimality, tractability and robustness [1], [7].

The basic idea of Kalman filter is to estimate a process by using a type of feedback control. This estimation tool consists of a set of measurement values and previous states of a system while keeping the estimation error minimal. The filter estimates the states at some point in time and then tries to obtain feedback in the form of noisy measurements. A recursive loop is used in the Kalman filter to obtain more accurate results.

If the noises are Gaussian even after some linear transformations, then the Kalman filter is applied to give us a solution to the conditional probability density functions (PDFs) and therefore this filter provides a minimum covariance estimate [4], [10]. In case the noise is non-Gaussian, then the Kalman filter would serve as the best linear estimator. In other words, it would have the smallest error covariance among all linear filters. The Kalman filter is noted for optimality because it is often derived for Gaussian noise where we find that the calculations are performed directly and thus optimality follows straight away.

The Kalman filter assumes that the posterior density at every time step is Gaussian and hence is parameterized by a mean and covariance [2]. The Kalman filter is implemented in a two-step procedure: prediction step and filtering or correction step. The dynamics of the system are handled in the prediction step while the measurements are integrated in the system in the filtering step. As mentioned earlier, the estimation of states at some point in time is done by the filter and then the feedback is obtained in the form of noise measurements. Thus the equations of Kalman filter also fall into the same two groups stated above: prediction step equations and filtering step equations. While prediction step equations are accountable for putting forward the present state and error covariance estimates in time, to attain the apriori estimates for the subsequent step, filtering step equations on the other hand are accountable for the feedback, i.e., for putting forward a novel measurement into the a-priori estimate in order to achieve a better a-posteriori estimate. While the former set of equations are considered as predictor equations, the latter are considered as corrector equations. Hence the final algorithm looks like that of a predictor-corrector algorithm for unraveling numerical problems.

2.2. Kalman Filter

In general, to define a filtering problem, the state-space model may be formulated along the subsequent lines. Let

$$x\,(k + 1) = f(x(k),\, u(k), w(k)) \qquad (2.2.1)$$

$$y(k) = h(x(k),\, v(k)) \qquad (2.2.2)$$

denote the state dynamics of a non-linear time-varying system, where

- $x(k) \in Q^n$ is the system state vector,
- $f(.,\, .,\, .)$ symbolizes the system's dynamics,
- $u(k) \in Q^m$ is the control vector,
- w is the vector that symbolizes the system error sources,
- $y(k) \in Q^r$ is the observation vector,
- $h(.,\, .,\, .)$ is the measurement function,

- v is the vector that symbolizes the measurement error sources.

The following set of variables are given

- *f, h*, the noise characterization, the initial conditions,
- set of controls, $u(0), u(1), \ldots, u(k-1)$,
- set of measurements, $y(1), y(1), y(2), \ldots, y(k)$,

To obtain: best estimate of $x(k)$.

To understand the state- space model, a state vector, $x(k)$ is considered which holds all information about the system, at time k, used to find out its future behavior when the input is given. The dynamics of the state determines how the system changes over time. If the inputs of the system are given by $u(k)$ and process noise $w(k)$, where $w(k)$ is an unpredictable input modeled as a stochastic process, then the state of the system at time $u(k+1)$ is related to the state at time $u(k)$, and the inputs of the system $u(k)$ and $w(k)$ by the above mentioned relation

In case of linear time-varying dynamic system, the above mentioned equations reduce to

$$x_{k+1} = A_k x_k + B_k u_k + C w_k \quad k \geq 0 \tag{2.2.3}$$

$$y_k = D_k x_k + v_k \tag{2.2.4}$$

where $x(k) \in Q^n$, $u(k) \in Q^m$, $w(k) \in Q^n$, $v(k) \in Q^r$, $y(k) \in Q^r$, $\{w_k\}$ and $\{v_k\}$ are sequences of white Gaussian noise whose mean is zero.

Steps in the Kalman filter algorithm

Initial conditions:

$$\begin{aligned} \tilde{x}(0\vdots-1) &= \bar{x}_0 \\ P(0\vdots-1) &= \bar{\xi}_0 \end{aligned} \tag{2.2.5}$$

Filtering step

$$
\begin{aligned}
\tilde{x}(k \vdots k) &= \tilde{x}(k \vdots k-1) + k(k)[y(k) - D_k \tilde{x}_{k:k-1} \\
k(k) &= P(k \vdots k-1) D_k^T [D_k P(k \vdots k-1) D_k^T + Q]^{-1} \\
P(k \vdots k) &= [I - k(k) D_k] P(k \vdots k-1)
\end{aligned} \tag{2.2.6}
$$

Prediction step

$$
\begin{aligned}
\tilde{x}(k+1 \vdots k) &= A_k \tilde{x}(k \vdots k) + B_k u_k \\
P(k+1 \vdots k) &= A_k P(k \vdots k) A_k^T + C_k M C_k^T
\end{aligned} \tag{2.2.7}
$$

Let k: =k+1 and repeat from step 2.

The aforesaid algorithm is a complete way of implementing the Kalman filter. However, if the estimation problem is not stated clearly, some numerical problems may arise. Firstly, covariance becomes indefinite. Second, the symmetry of the covariance matrices may be lost. While verifying the latter is easy, the former is more complex to verify. Therefore a solution to this problem is to use square- root implementation approach in which the square-root of the covariance matrix is sent. This assures symmetry and definiteness. One such method of implementation is the unscented Kalman filter which is discussed in Section 2.5.

The Kalman filter is limited in application to linear models alone. However, most real-time applications are non-linear models. The most common approach designed by Julier and Uhlman is to make use of the extended Kalman filter (EKF) which merely linearises every nonlinear model in such a way that the long-established linear Kalman filter can be applied [1].

2.3. Linearized Kalman Filter

The Kalman filter satisfies two criterions. It not only results in the average of the state estimate be equal to the average of the true estimate, but also results in the smallest possible variation in the state estimate. Technically, Kalman filter is an estimator that results in smallest possible error variance [5].

In order to implement the Kalman filter on nonlinear models, we assume that the nominal trajectory x_{nom} is known and therefore we linearize the system along this trajectory so as to apply the Kalman filter over the linearized model. This is usually known as the linearized Kalman filter.

2.4. Extended Kalman Filter

The extended Kalman filter is developed to overcome the problem of lack of nominal trajectory. This filter is used when dynamics of the system (state and observation) is non-linear. The extended Kalman filter (EKF) presents an approximation of the optimal estimate. The system's non-linearities such as the state and observation are approximated by a linearized edition of the non-linear system model about the final state estimate. In order to prove that this approximation is legitimate, this linearization ought to be a fine approximation of the non-linear model in the entire uncertainty domain associated with the state estimate.

The drawback associated with the usage of extended Kalman filter is that, this filter is harder to analyze than the traditional linearized Kalman Filter. However, it is proven to work well in many applications.

2.5. Unscented Kalman Filter

The unscented transform has been proposed by Julier and Uhlman [1] to overcome the poor performance of the Extended Kalman Filter in case of highly non-linear systems. The extended Kalman filter can handle only limited amount of nonlinearity and also the posterior mean and the covariance may be corrupted due to the analytical way of state distribution.

Therefore, a novel technique which does not use any gradients has been introduced. This is the unscented Kalman filter (UKF) which uses a deterministic sampling approach [6].

In deterministic sampling approach, a set of carefully chosen points called the sigma points are considered. These are the occurrences of the early stochastic process. The basic idea of the unscented Kalman filter is that the sigma points contribute in obtaining true mean and covariance when propagated through a typical non-linear system during filtration.

This section talks about the improvement to the unscented Kalman filter prepared by Eric A. Wan and Rudolph van der Menve over the UKF proposed by Julier and Uhlman [6].

Let us consider the propagation of random variable *x* (dimension M) through *y = f(x),* which is a nonlinear function [6]. Then *x* is assumed as the mean and Q_x as the covariance. To compute y, a matrix *X* of *2M + 1* sigma vectors ψ_t (with corresponding weights W_t) is framed and then the following equations are used:

$$
\begin{aligned}
X_0 &= \tilde{x} \\
X_t &= \tilde{x} + (\sqrt{(M+\phi)P_x})t \quad t = 1,\ldots\ldots,M \\
X_t &= \tilde{x} - (\sqrt{(M+\phi)P_x})_{t-M} \quad t = M+1,\ldots\ldots,M \\
\phi \; &= \sigma^2(M+i) - M \textit{ is a scaling parameter}
\end{aligned}
\qquad (2.5.1)
$$

where *i* is a scaling parameter usually set to 0, and

σ is the parameter that evaluates the stretch of the sigma points about $\tilde{x}$.

By transmitting the sigma points through the dynamic of the model, the time update equations are obtained.

The UKF is explained in detail in the following algorithm.

Steps in the unscented Kalman filter algorithm

Initial conditions:

$$\tilde{x}(0\vdots 0) = \bar{x}_0$$
$$P(0\vdots 0) = \bar{\xi}_0 \qquad (2.5.2)$$

Choosing S sigma points, while dividing X_i^t *as*

$$X_i^t = \begin{pmatrix} x_i \\ w_i \\ f_i \end{pmatrix} \textit{ and } w_i^t \qquad (2.5.3)$$

Filtering step

$$\begin{aligned}
&\tilde{x}_{i:i} = \tilde{x}_{i:i-1} + A_i\,(z_i - \tilde{z}_i) \\
&P_{i:i} = P_{i:i-1} - K_i\, B_i\, K_i^t \\
&\textit{where} \\
&z_i^t = k\,(x_i^t\,, f_i^t) \qquad (2.5.4) \\
&\tilde{z}_i = \sum_{t=-q_x}^{q_x} w_i^t\, z_i^t \\
&B_i = \sum_{t=-q_x}^{q_x} w_i^t\,(z_i^t - \tilde{z}_i)\,(z_i^t - \tilde{z}_i)^i \\
&A_i = \sum_{t=-q_x}^{q_x} w_i^t\,(x_{i:i-1}^t - \tilde{x}_{i:i-1})\,(z_i^t - \tilde{z}_i)^i
\end{aligned}$$

Prediction step

$$\begin{aligned}
&\tilde{x}_{i:i-1}^i = \sum_{t=-q_x}^{q_x} w_i^t\, x_{i:i-1}^t \\
&P_{i+1:i} = \sum_{t=-q_x}^{q_x} w_i^t\,(x_{i:i-1}^t - \tilde{x}_{i:i-1})\,(x_{i:i-1}^t - \tilde{x}_{i:i-1})^i \qquad (2.5.5) \\
&\textit{where } x_{i:i-1}^t = g(x_{i-1:i-1}^t, w_i^t)
\end{aligned}$$

Let i: = i+1 and repeat from step 2.

CHAPTER 3

PROBLEM DEFINITION AND SOLUTION APPROACH

3.1 Previous Approaches for Optimization and Problem Statement

Cross layer techniques are typically applied with intent of obtaining an optimized power and throughput for a time-varying channel while also achieving a fixed bit error rate (BER). Myriad adaptive techniques for orthogonal frequency division multiplexing channel (OFDM) have been presented in [3]. Cross-layer optimization of the different layers has demonstrated to be the for the most part beneficial than the typical ways of optimizing [3]. Hence the approach of link optimization has been agreed for wireless personal area network (WPAN) that denotes the standards of various layers for high rate WPANs that offer quality of service and also maintain ad-hoc network between systems.

Based on a typical channel state information (CSI), the transmitter can adopt three types of adaptation schemes, namely adaptive coding (AC): technique that acclimatizes code rate based on the state of the channel, adaptive bit loading (ABL): technique in which group size of modulation alphabet is tailored to the immediate channel features and adaptive power loading (APL): technique that adjusts the power of the transmitter with respect to attenuation of the channel [3].

To illustrate the approach of cross-layer optimization comprehensively [3]: A network with numerous nodes which are joined by T links has been taken into consideration. Of this, one node is chosen as the vital controller, which assembles information from one layer and uses it to aid in decision making in another layer. The vital controller node VC broadcasts information between links by means of the CSI for every link. Figure 3.1 from [3] depicts an arrangement of 3 nodes of which, one is named as a vital controller. This figure gives details on the flow of data from every layer to the vital controller. It is implicit that every link witnesses slow fading, so that the channel is not varying (approximately constant) for any of the coding symbols, however varies from one symbol to another. The average channel strength observed by the i^{th} link and s^{th} symbol is represented by $£_i(s)$.

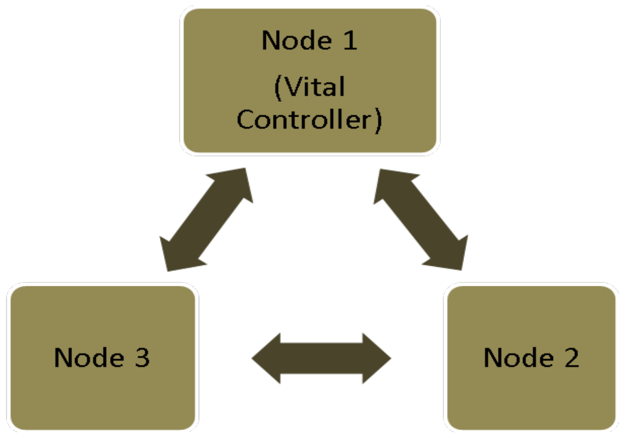

Fig. 3.1 Flow of information between nodes of the system.

The system considers the diverse power on every link to offer multiple-link diversity. One layer presents a set of data rates $D_i(s)$ to the vital controller where data may be sent out on the i^{th} link with symbol s. This gives rise to the function $P_R(d_i(s), B)$, which gives the power received, that is required to obtain a BER for a rate $d_i(s) \in D_i(s)$. This function is sent to the controller. The function can be obtained in any way, i.e., it may be logical, pre-simulated, or approximated from true data. The data transmission and receiving is done by the nodes directly from the vital controller through a separate channel. Every node specifies when the transmission has to be done while showing the receiver node, the average slow fading $£_i(s)$. This second layer sends the vital controller with the delay and queue size. The controller makes use of these parameters and average channel strength to decide which link is triggered, and this is represented by the following function.

$$M_i\ (s) = \begin{cases} 0 & i(s) \neq i \\ 1 & i(s) = 1 \end{cases} \tag{3.1.1}$$

The fullness of the buffer, indicated as $B_i(s)$ for the queue of the i^{th} link at the s^{th} symbol is denoted as

$$B_i(s+1) = B_{i(}(s) + R_i(s+1) - d_i(n)M_i(s) \tag{3.1.2}$$

Here $R_i(s)$ denotes the rate of arrival of data which is a Poisson distribution for i^{th} link.

Another optimizing parameter is the quantity of channel accesses from the time when a link was most recently activated for transmitting data, denoted as $Q_i(s)$. Therefore, the link that is activated at (s+*1) iteration* is denoted as

$$Q_i(s+1) = [1 + D_{i(}(s)][1 - M_i(s)] \tag{3.1.3}$$

Thus the controller is fed with the parameters $D_i(s)$, $R_i(s)$, $B_i(s)$ *and* Qi*(s)*.

Once the controller gets to know these parameters, an optimizing parameter for every link is formulated. The optimization is then divided into two phases. Firstly, an exchange amid delay and multiple-link is prepared; this gives the link the improved optimizing parameter. Next, the maximum data rate which can be sustained is utilized.

The underlying function that decides which link is to be examined is figured as:

$$Opt_i = \begin{cases} 0 & for\ \ d_i = \zeta \ \ or\ \ B_i = 0 \\ \mu D_i + \eta B_i + \omega Q_i & else \end{cases} \tag{3.1.4}$$

The parameters ω, μ and η for delay in time ever since the last activation of the link, data rate and fullness of the buffer respectively, have a huge impact on deciding the value of the Optimum function Opt_i. The Opt_i values will be compared in the process so as to obtain an optimized value of the data rate. The acquired output value will be sent to another function which generates bits in accordance with the overall quantity of links and among these, that link with the best optimized value shall be activated in order to be used while leaving aside the remaining links for the later sequence.

Thus the two factors of time varying nature of the channel and the arbitrarily distributed rate of arrival have consequently impacted in nonlinearity in these structures. This drawback calls for a modeling technique that provides an enhanced estimate of the optimization results.

Based on the aforesaid design objectives, the algorithm which involves a filtering method has been simulated. The unscented Kalman filter, originally proposed by Julier and Uhlmann [1] is used here as the filtering method. The filter iteratively estimates the data rate at a given time. Thus the filter is employed to give the best selection of the iteration that provides the maximum data rate.

3.2 Solution Approach and Optimization Model

In order to implement the design approach mentioned in the earlier section 3.1, the following model is adopted to obtain the best estimate for achieving maximum data rate. The rate of arrival which is a Poisson distribution is used as the input (noise) for the model. The data rate, fullness of buffer and delay are the parameters sent to the unscented Kalman filtering, which is a non linear mapping method, so as to tune the parameters for optimization ω, μ and η. The output obtained from this block of the model is compared to the earlier computed values and also the rate of arrival. The whole model algorithm is shown in the figure below.

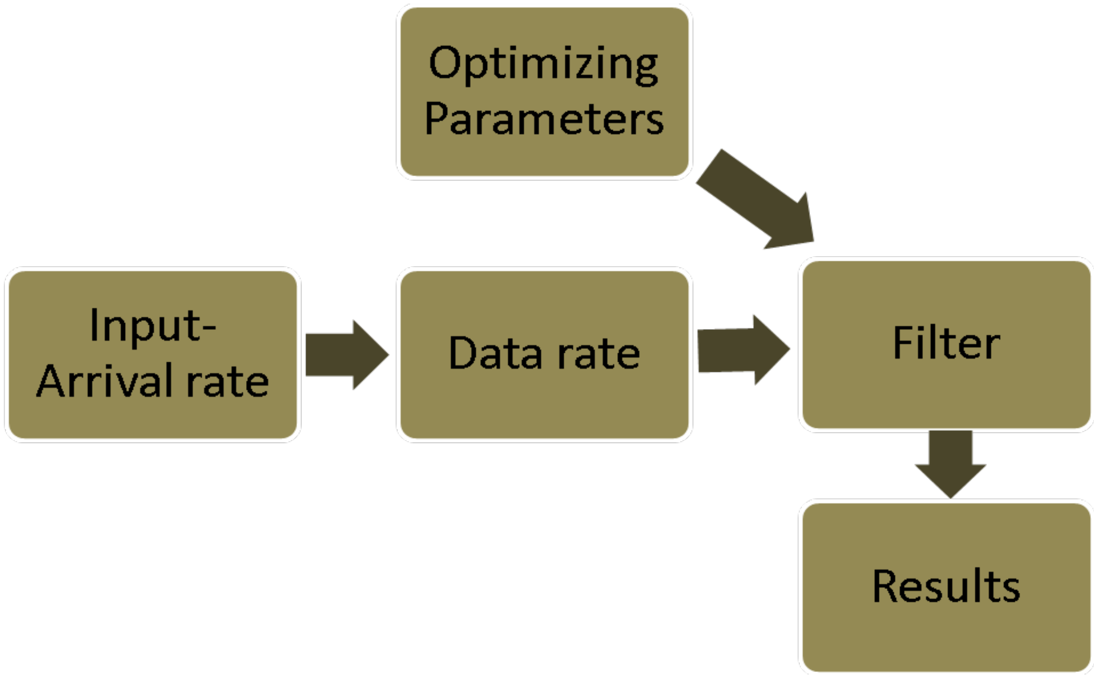

Fig. 3.2 Optimization model

3.3 Unscented Kalman Filtering Algorithm

To implement the algorithm, the model in [3] is applied. The simulation result obtained from the OFDM transmission system [3] is considered here. The OFDM system with $SC_N = 256$ subcarriers is employed, with a common rate-1/2 convolution code that has generators 133_{oct} and 171_{oct}. This extensively employed code actually has 64 states along with what is called the Hamming free distance $d_{free} = 10$. Also a channel that is time-varying, which has eight time-domain taps and a maximum Doppler of $f = 0.05$ is considered. Then for the variations in time, Jakes' model is considered. The BER = 10^{-4} and Number of links T = 20 links are employed for the computations. These first set of simulations have been obtained from the already conducted experiments in [3]. By making use of the existing parameters ω, μ and η, their respective values for delay Q_i, rate of data D_i, and fullness of buffer B_l versus the rate of arrival is computed. From these simulation results, the covariance and mean are computed so that they are sent to the model as calculated value. The whole process is thus repeated iteratively to make estimations of the rate of data.

The basic idea is that the filter will enhance the estimation of the states by the filter, or to calculate approximately the states of the links and additional measurements. This extra work of computing the states will further add to the enhancement of the filter. This implies that the kalman filter performs three tasks. First, it computes the states of the links in accordance with the model shown in the figure. Then, it calculates in accordance with the additionally computed measurements. Lastly, it makes a comparison between measurements of these states that were obtained as a function of rate of arrival of Poisson distribution, to that of the states of the links of the model and then formulates a weighted average of both of these results.

To stimulate this, the primary state and also its covariance have to be defined. Thus the rest of the unidentified states as well as the mean obtained from the measurements resulting out of the simulated model are all initialized to zero. Also, at the start, the covariance of both the model and the simulated model's measurements are all initialized.

The time phase or the prediction phase states are computed by the model. Thus the model computes the state x_k at a certain later time k, by making use of inputs u and previous state x_{k-1}. However, the value of the previous state is vague; since only the estimate and the covariance **P** is known. Therefore, the next state ought to be computed by making use of the possible states contained in that covariance. However, this step is not entirely mandatory to perform. Instead, the carefully chosen sigma points of the kalman filter are used here on the boundary of the covariance.

CHAPTER 4

SIMULATION RESULTS

The optimizing function is as follows:

$$
\begin{aligned}
OPT &= \omega Q + \mu D + \eta B \\
&= D \text{ with } \mu = 1, \omega, \eta = 0 \qquad (4.1)
\end{aligned}
$$

The algorithm uses the UKF equations (2.5.1) through (2.5.5) and mapping of the variables/parameters discussed in Section 3.3.

The algorithm for this case is given as:

Formulate measured mean queue
Formulate measured covariance
For i = no of iterations
Initialize value for state parameter: data rate
Initialize value for variance
Formulate sigma points
For l=no of links
Calculate sigma points
Predict sigma points for i+1
Obtain predicted measurement mean
Obtain priori covariance
Obtain covariance of predicted measurement
Formulate UKF estimate
Put OPT = UKF_est(l)
End
Assess the UKF for all l
Select link with higher UKF_est
End

The following simulation depicts the data rate vs. rate of arrival of data.

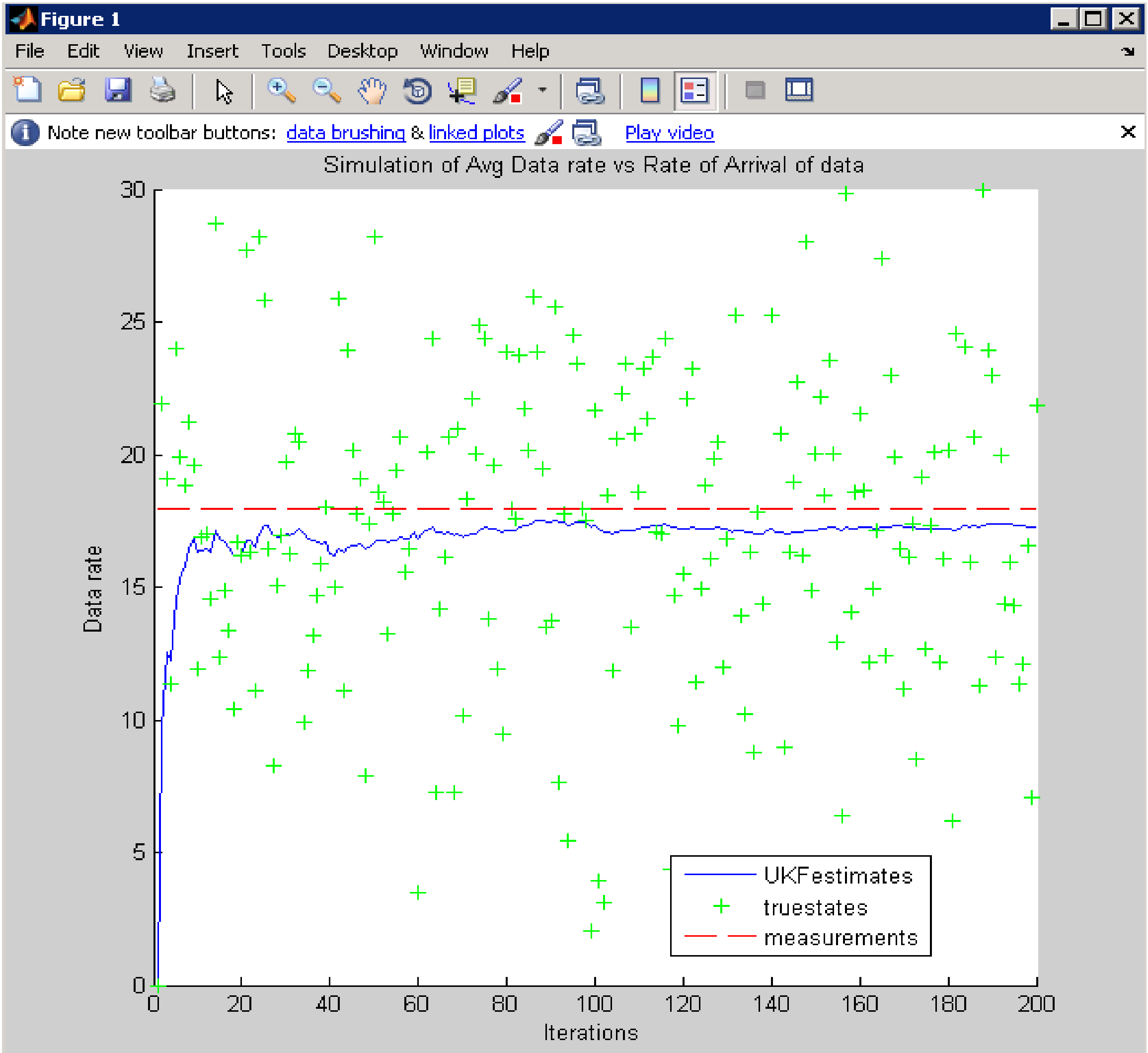

Fig. 4.1 Simulation depicting data rate vs. rate of arrival of data

The following simulation results from Fig. 4.2 - 4.5 were obtained for four different arrival sets.

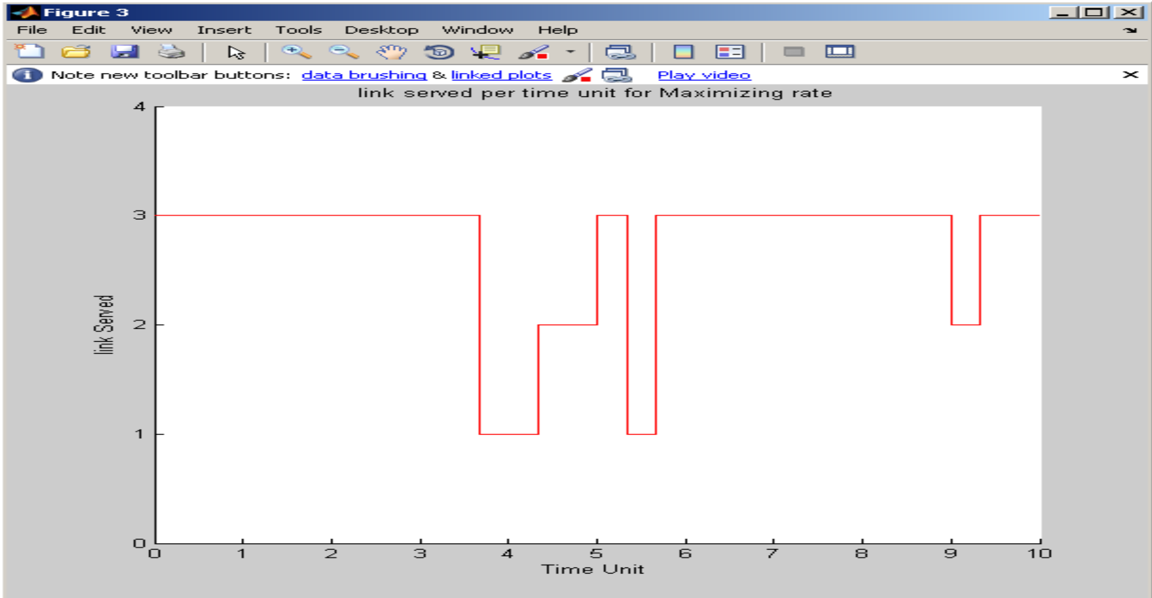

Fig. 4.2 Link served per time unit for maximizing rate

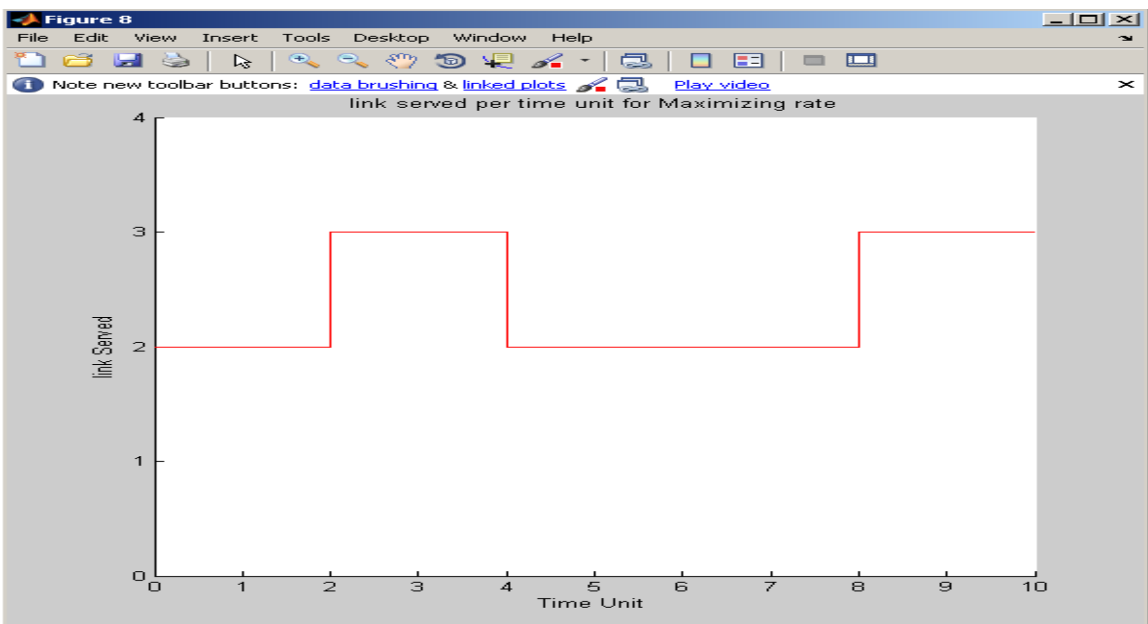

Fig. 4.3 Link served per time unit for maximizing rate

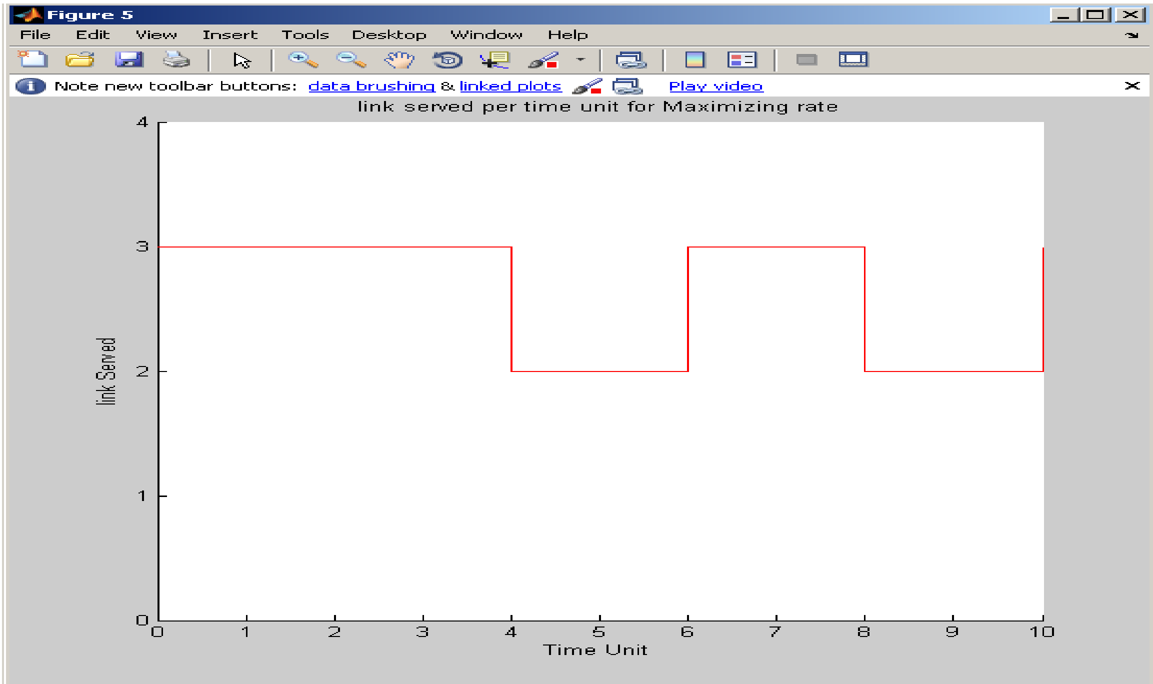

Fig. 4.4 Link served per time unit for maximizing rate

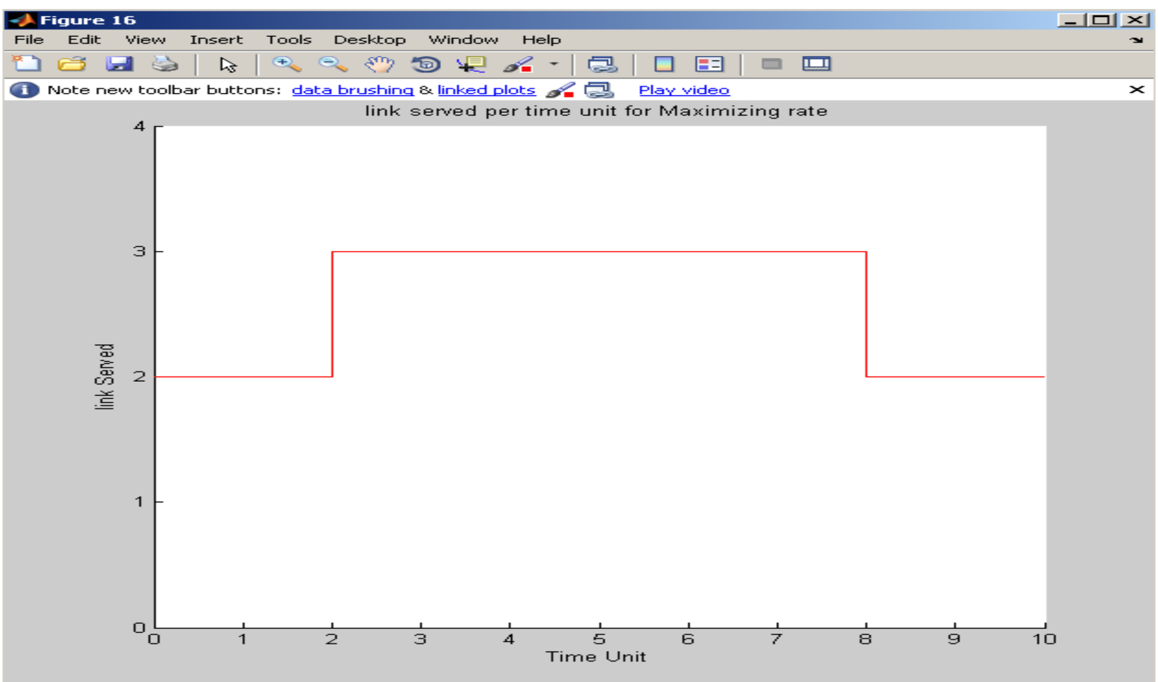

Fig. 4.5 Link served per time unit for maximizing rate

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

The algorithm is scheduled to obtain the link with better performance. After a few iterations, the filter converges to an average value based on the input. This means, as the number of iterations increases, the error covariance decreases. This is because the input covariance that is supplied to the filter is small. In summation, the link is so selected that the system is optimized based on arrival of data rate.

In the current thesis, an optimization method is implemented using the Kalman filter for OFDM transmission system. Thus the Kalman filter is used to optimize the data rate while transferring the data between different layers in a communication system. Simulations were performed to illustrate the link which results in the best estimate of maximizing data rate.

This work can be further enhanced by testing for efficiency using various other filters like particle filter. Moreover, other parameters like time delay can also be tested and optimized.

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